Dynamics analysis of a symmetrical on-board rotor the h-p version of the finite elements method

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Résumé :

La version h-p de la méthode des éléments finis est utilisée dans cet article pour l'analyse de la réponse dynamique non-linéaire des forces d'excitation extérieurs d'un rotor embarqué sur des supports indéformables mobiles et monté sur des paliers hydrodynamiques. Le disque et est supposé rigide et seul son énergie cinétique est considéré, l'arbre est déformable est traité avec la théorie des poutre d'Euler Bernoulli, le matériau utilisé est isotrope. L'élément poutre tridimensionnelle a deux nœuds est utilisé pour la discrétisation du rotor. Les fonctions de formes utilisées sont de type Hermite cubique qui respectent les conditions aux limites dans toutes les directions qui représentes la partie h des éléments finis et sont modifiées de façon qu'on peut faire la combinaison avec les fonctions de forme polynomiales K-orthogonale qui facilite la combinaison pour l'utilisation de la version h-p de la méthode des éléments finis. La méthode énergétique est utilisée pour la détermination des énergies cinétiques, et déformations pour l'ensemble du système rotor (arbre, disque, balourd).
Les étapes de calcul du comportement dynamique linéaire de l'analyse du système de rotor embarqué sont regroupés dans une application en MATLAB. Une comparaison des fréquences propres du système de rotor embarqué, obtenu par la version h-p de la méthode des éléments finis est faite avec les résultats obtenus en utilisant la version h.

Abstract:

The h-p finite element method is used in this paper for the dynamic analysis of a symmetrical on-board rotor on mobile dimensionally stable supports. The disk and the bearings are assumed to be rigid, with deformable shaft, the material is isotropic. A three dimensional beam element is used for the discretization of the rotor. In the standard h-version of the finite element, the used shape functions are cubic Hermit, which respect the boundary conditions in all directions, the shape functions-h are modified so they can make the combination between K-orthogonal polynomial shape functions facilitating combination to use the h-p version of the finite element method. Energy method is used for the determination of energy for the entire rotor system, The equations of motion of the rotor system are determined by the Lagrange method. The calculation steps for linear dynamic behaviour of on-board rotor system analysis are grouped in an application created using MATLAB programming language and validated with the work done previously by the classic version of the finite element method. In this paper we make a comparison of the natural frequencies of on-board rotor system, obtained by the h-p version of the finite element method with version h.

Keywords:
Symmetrical rotor, h-p version of FEM, on-board rotor, rotor dynamics, base excitation.

1- Introduction

The h-p version is a general version of the finite element method, a numerical method to solve partial differential equations based on approximations polynomial elements that use variable sized elements (h) and polynomial of degree (p). The origins date back to h-p-MEF innovative work of Ivo Babuška et al. [1] who discovered that the finite element method converges exponentially fast when the mesh is refined using an appropriate combination of refinements -h (dividing into smaller elements) and -p refinement (by increasing the polynomial degree). The exponential convergence makes the method a very attractive choice compared to most other methods of finite elements that converge at an algebraic rate. The exponential convergence of the h-p-MEF was not only theoretically predicted, but observed by many independent researchers.

The dynamic behaviour study of a rotor in the boarding effect is the study of the global behaviour of the rotor whose support is subject to any movements. This model is well suited to understand the movement of rotors phenomenon embedded in vehicles, aircraft etc. This phenomenon recently attracted the attention of researchers. M. Duchemin [3] made a detailed study based on the simple model of Rayleigh-Ritz, a rotor whose support is in motion. Various analytical studies were performed on simple movements, simple translation, sinusoidal translation, constant rotation, accelerated rotation. [M.Z. Dakel] [4] continued the work by adding the hydrodynamic bearings by the conventional finite element method. In the work of Boukhalfa [2] we find a study on behaviour of composite rotor materials by h p-version finite element. Based on the latter two works, in our case we will make a study of an on-board rotor, whose support is assumed to be rigid and mobile, with the application of the h p-version finite element.

2- Model of on-board rotor

In general, a rotor system is composed of a shaft supported by bearings, and having one or more disks. In this study, the solicitations considered are the imposed displacements of rigid base. The number of each component can be more than one, however, in this paper, the studied rotor has one shaft (single shaft), (see Figure 1)

The assumptions that we consider in this paper are: The shaft is deformable and modelled by homogeneous straight beams, isotropic, linear elastic with constants sections subjected to bending moment in two orthogonal directions (horizontal and vertical directions); The disks are assumed to be rigid; The bearings are assumed to be rigid; The rotor support is infinitely rigid; but mobile,

The consideration of the motion of the support can significantly affect the form of the equations of motion of a rotor in flexion with respect to those obtained in the case of a fixed support. To obtain the simplest possible model, the approach presented by Duchemin [3] is used. It offers the modelling of a rotor with mobile support by considering the motion of the rotor relative to the support and that of the support relative to the ground. This is to study the transverse deflections of the mid-line of the rotor shaft relative to a reference linked to the rigid support.

The steps for obtaining the equations of motion are inspired by the approach used by Lalanne and Ferraris [5], so it needs the description of the movement of the on-board rotor and vectors expressing rotations between them.

The differential equations of the bending motion of an on-board steady rotor are deducted from the equations LAGRANGE applied with respect to the generalized coordinates as follows:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) - \frac{\partial E}{\partial q_i} + \frac{\partial U}{\partial q_i} = \delta F_i \tag{2.1}$$

2.1- Equations of motion

The formulation of the equation of motion is determined by the computation of the energies of the components of the rotor, this formulation is detailed in the paper of SAIMI.A [6].

The modelling method used is that of the h-p version of the finite element method. The steps of calculating mass, gyroscopic, rigidity, and vector of external forces, which form the differential equation of motion, are well treated in the paper of SAIMI.A [6]

2.2- Elementary movement equation

Applying Lagrange equations to the discretised system by h p-version of the finite element method we obtain the following differential equations:

$$[M_{er}(t)]\{\ddot{q}_r\} + [C_{er}(t)]\{\dot{q}_r\} + [K_{er}(t)]\{q_r\} = \{F_{er}(t)\} \tag{3.12}$$

where:

- $[M_{er}(t)], [C_{er}(t)], \ and \ [K_{er}(t)]$ are respectively the elementary matrices, mass, damping and stiffness, with periodical parametric terms, and varying in time due to the geometric asymmetry of the rotating rotor and rotations of the rigid support.
- $\{\ddot{q}_r\}, \{\dot{q}_r\}, and \ \{q_r\}$ are respectively the acceleration vector, velocity and global displacement adapted to the connectivity of finite elements.
- $\{F_{er}(t)\}$ is the global vector of linear external forces containing excitations due to the rotational and linear movements of the support,

3- Simulation and results

A computer program for determining the natural modes of on-board rotor has been developed and for validate this program, we have the rotor shown in Figure 1.

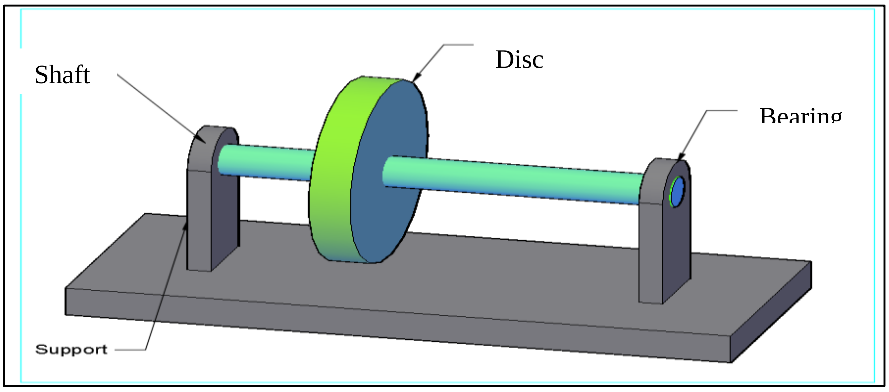


Figure 1 : On-board rotor

Table (Table 1) shows the characteristics of symmetric rotor used in the literary [22], [19], [20].

Tab 1 : Characteristics of symmetric rotor

Density of the disk material	$\rho_d = 7800 \ kg/m^3$
Outer radius of the disk	$r_d = 0.15 \ m$
Disk Thickness	$e_d = 0.03 \ m$
Position of the disk	$y_d = 0.4/3m$
Density of the shaft material	$\rho_a = 7800 \ kg/m^3$
Radius of the shaft	$r_a = 0.01 \ m$
Length of the shaft	$l_a = 0.4 \ m$
YOUNG Module of the shaft	$E_a = 2 * 10^{11} \ N/m^2$
POISSON coefficient of the shaft	$\upsilon_a = 0.3$

To demonstrate the basic phenomena occurring in rotor dynamics with the movements of the support and compare them to those related to the case of a fixed support, the analysis of the dynamic behaviour in continuous operation is performed through Campbell diagram.

3.1- Campbell diagram

- Convergences of results:

several examples were processed to determine the influence of angular perturbations of the support relative to the axes Ox and Oy and the locations of the disks on the rotor.
The advantage of using the h-p version of the finite element method in this paper is the ability to control both parameters of the previous versions (h-FEM and p-FEM). Knowing that if we fix the degree of polynomial in p=4, and we make vary the other one parametrizes (mesh degree h), the h-p version is converted into h version, and the found results are identical to those of the h version.
The same thing applies if we fix the mesh degree (h), and we make vary the polynomial degree (p), the found results are similar to those of the p-version of the FEM.
In Figures 2, 3, 4 we proceeded as follows: In Figure 2; we fix the number of elements (h-refinement) and we make vary the polynomial degree-p. In Figure 3; we fix the polynomial degree-p and we make vary the number of elements (h-refinement). In the case of the figure 4; we make vary simultaneously both parameters (h-refinement and polynomial degree-p) by using the following method:
$(p_i = p_{i-1} + 1 \; and \; h_i = h_{i-1} * 2)$

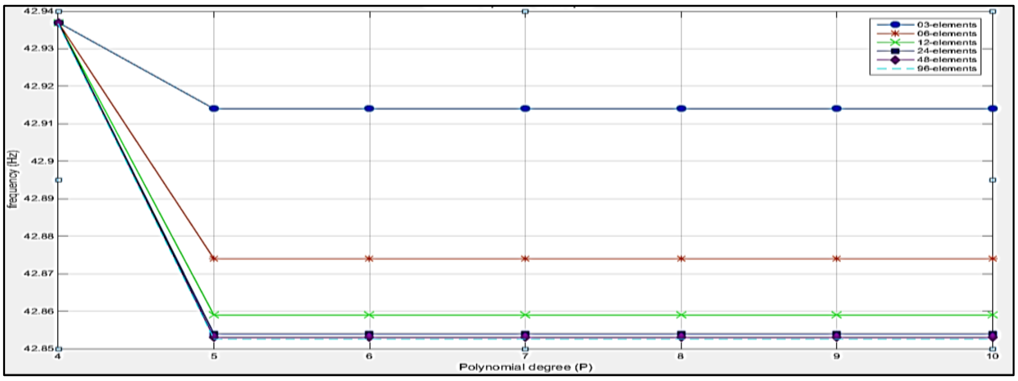


Figure 2: Convergence of the first frequency according to the number of p hierarchical shape functions with a fixed number of elements, at a speed of 1500 rpm

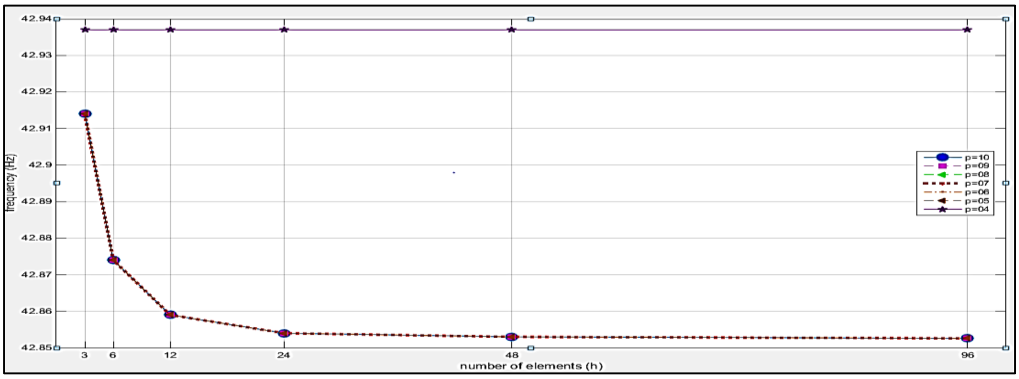


Figure 3: Convergence of the first frequency according to the number of elements (h), with a fixed polynomial degree (p), and a speed of 1500 rpm

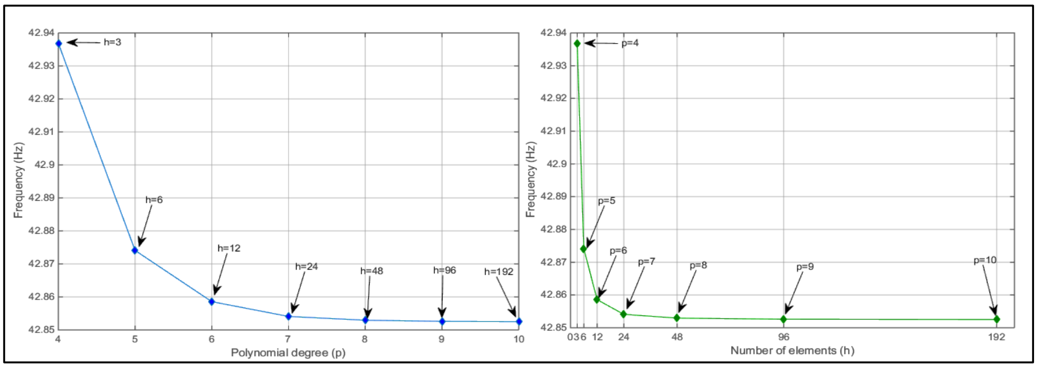


Figure 4 : Convergence of the first frequency with a combination of the mesh degree-h and the polynomial degree-p with ($p_i = p_{i-1} + 1$ and $h_i = h_{i-1} * 2$) in a 1500 rpm rotational speed

We notice that the convergence is very rapidly obtained for low frequencies (figures 2 and 3) by increasing both parameters (h and p) of the version h-p. The gap between the results is very small and can be neglected; so to have good results for low frequencies, increasing the elements number (h-refinement) and the polynomial degree (p-refinement) are not necessary; we need to fix a parameter (either the number of elements -h or the polynomial degree -p) and varying the other one until the convergence is obtained. This phenomenon is generally valid for the low frequencies, between the first and the fifth one. In the figure 8, the curves of convergence are traced by the fixation of the mesh degree h and the variation of the polynomial degree p, we notice that more we increase the number of elements (h-refinement) more the gap between the curves of convergence decreases. In case we fix the degree of polynomial p and we vary the mesh degree h (figure 3), we notice that the convergence is very low(weak) in the case or p=4 (which represents the h version of FEM), and in the case or p≥5 we have a fast convergence. In Figure 4 we see that convergence is exponentially fast, However, we cannot provide a typical combination.

- Interpretations of Campbell diagrams

Figure 5 shows the variation of the frequencies of the bending with versus the rotational speed (Campbell diagram) of the first two modes. The influence of the angular speed of the support taken in this example ($\omega^x = 0\ Hz, 5Hz, et\ 10Hz$) is small, so better visualized the results on the graph we used zooms in on two areas (see Figure 6):

We notice from these results that the influence of rotation of the support with respect to the O_x axis is very weak from the small angular rotations (5 Hz, 10 Hz), it is therefore concluded to have significant vibrations due to the rotation of the support along the O_x axis these rotations must be as important.

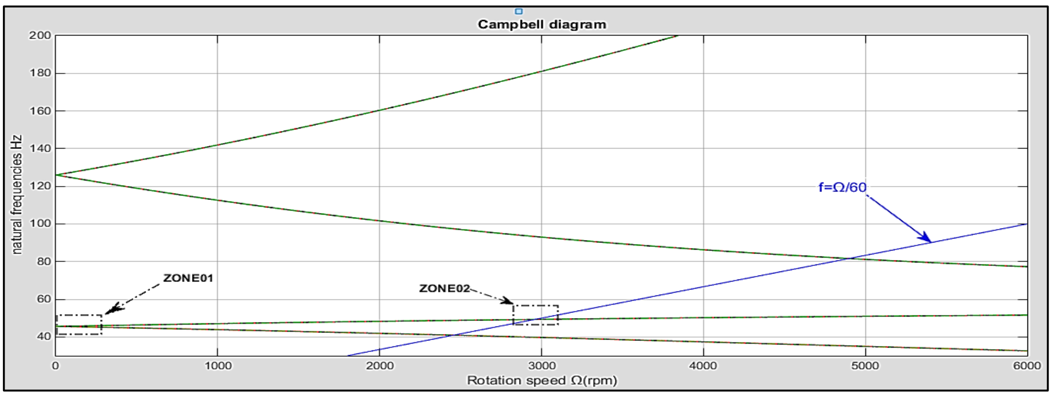


Figure 5 : Campbell diagram for the first four frequencies of the rotor under the effect of the angular velocity of the support ($\omega^x = 0\ Hz, 5Hz, et\ 10Hz$)

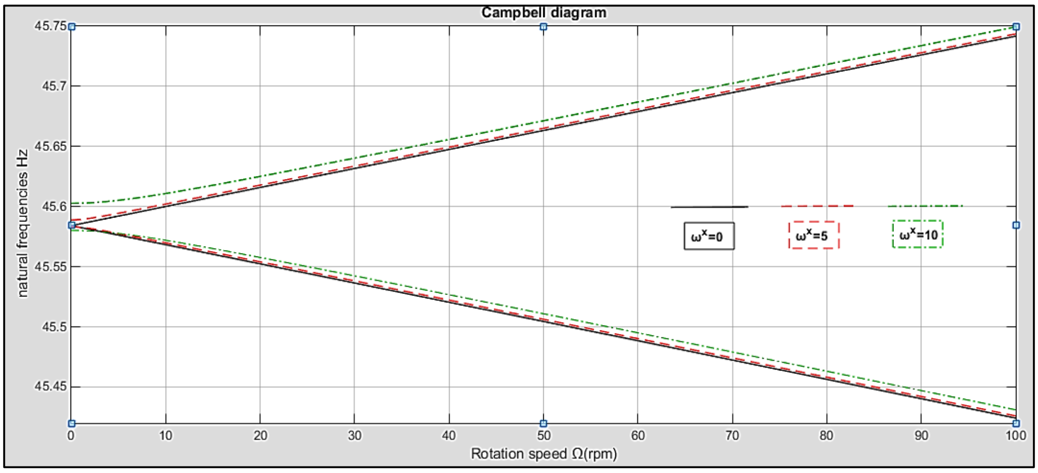
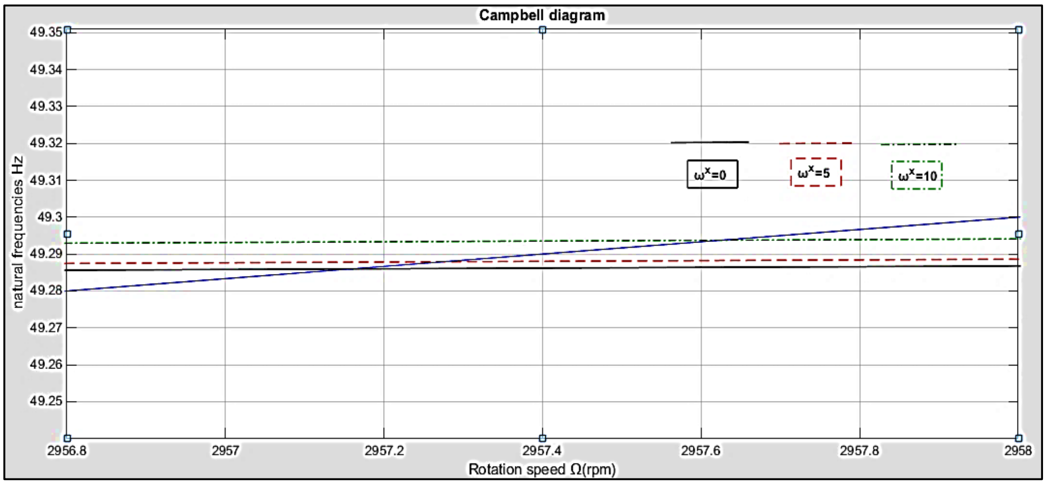


Figure 6 : Expansion in zone 01



Expansion in zone 02 in Figure 12

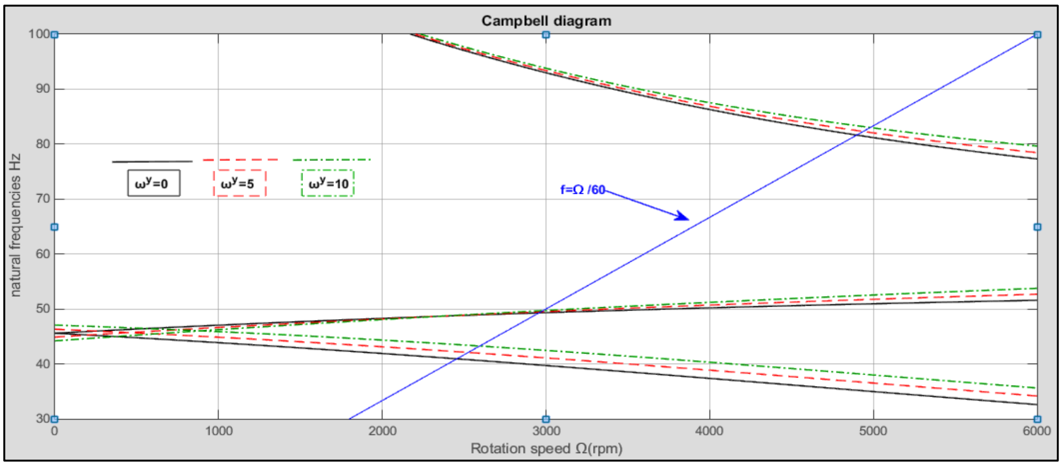


Figure 7 : Campbell diagram for the first four frequencies of the rotor under the effect of the angular velocity of the support ($\omega^y = 0\ Hz, 5Hz, et\ 10Hz$)

4- Conclusion

The vibrational behaviour of on-board rotors analysis using the method of the hierarchical finite elements (p version of the MEF) with functions with Legendre polynomials form, combined with the standard method of finite elements (h version of the FEM) is treated in this paper.
The calculation of energy different components of the on-board rotor, the application of the method of Lagrange, and introducing features of the h-p version of the finite elements, has given us the equations of motion of the system. The calculation is done on an element, then after assembly will have the overall equation of the system.
Several examples are treated and this has allowed us to determine the influence of different geometric parameters of on-board rotor and also the influence of the movement of the support on the behaviour of the rotor.
This work has allowed us to reach the following conclusions: The convergence of results can be controlled by increasing the polynomial degree (number of shape functions) and also the number of elements. The difference in results between the two methods (h-p version and the classic version of h) is very small, but the benefit of h-p version is that you can control both parameters of the element (the polynomial degree and number of elements). We conclude that the convergences can be classified in a following way: h-version: convergence in an algebraic way. h-p version: Convergence in an exponential way, or it can be improved by a suitable combination of the parameters (mesh degree h and polynomial degree p). The movement of the on-board rotor support amplifies the gyroscopic effect caused by the coupling of the displacements perpendicular to the axis of rotation, and creates an asymmetry in the movement of the rotor. In the case of disturbances in the neighbouring of the first critical speed, the influence of rotation of the support around the axis O_x (ω^x) is very low compared with the rotation of the support around the axis O_y (ω^y) From the results obtained in this work we conclude that the rotation of the support around to the axis parallel to the axis of rotation of the rotor has a great influence on the rigidity of the system and especially on the gyroscopic effect. The geometric parameters also have an influence on the vibration behaviour of on-board rotor. It is noted that it causes frequency variations and therefore variations of the critical speeds depending on the disk position.

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