

Simulation and calculation of the initial crack propagation in HDPE under biaxial loading in mixed mode

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Résumé :

Dans cet article, une analyse numérique est effectuée sur des plaques 2-D en polyéthylène haute densité (PEHD) contenant une fissure centrale initiale inclinée (CCT) avec un angle afin de déterminer l'angle initial de propagation sous chargement biaxial. L'angle initial de propagation est évalué numériquement en fonction du facteur de la densité d'énergie de déformation (SED) autour de pointe de fissure, en utilisant le code de calcul par éléments finis Ansys APDL (Ansys Parametric Design Language). Selon l'analyse numérique, la fissure a une tendance à se propager perpendiculairement à la direction du chargement le plus dominant. Les valeurs minimales S_{min} du facteur S sont atteintes aux points correspondant à la direction de propagation de la fissure. Ces résultats suggèrent que le concept du facteur de la densité d'énergie de déformation peut être utilisé comme un indicateur de la direction de propagation de la fissure.

MOTS CLÉS :

Densité d'énergie de déformation, mode mixte, chargement biaxial, HDPE, amorçage de fissure.

1. Introduction

As known, the major source of failure of structural components is crack growth for such reason, defects and precracks may be introduced in HDPE pipelines during installation, repair or by accident. Crack propagation was not considered to predict its trajectory. Nevertheless, at present, studying crack propagations' direction has been made possible by Finite Element Method (FEM), and the prediction of crack initiation angle in components is accessible by the fracture mechanics approaches, according to these approaches, the fracture process is assumed to be initiated from the preexisting defects cited previously [1]. Therefore, it is necessary to develop a procedure that allows the prediction of crack initiation angle and the propagation direction in HDPE. In linear elastic fracture mechanics, the various fracture criteria for cracks subjected to mixed mode loading have been introduced for the determination of the propagation direction and the critical stress, such as maximum tangential stress criterion [2,3-5], maximum principal tangential stress criterion [3], maximum strain criterion [6-7], and strain energy density criterion [3,8]. All these criteria are almost postulated that crack initiation will occur at the crack tip and propagate towards the radial direction.

2. Strain energy density theory

SED theory is based on the idea that a continuum can be seen as an assembly of small building blocks, each of which contains a unit volume of material and can store a finite amount of energy at a given moment [9].

The SED fracture criterion locally focuses on the continuum element ahead of the crack and is based on the notion of weakness or severity experienced by the local material. Failure occurs when a critical amount of strain energy dW is accumulated within the element volume dV and the crack is then advanced incrementally in the corresponding direction [3,10]. The strain energy density function (dW/dV) is assumed to have the following form

$$\frac{dW}{dV} = \frac{S}{r} \quad (1)$$

Where S is the strain energy density factor and r is the distance from the crack tip. The minimum of the strain energy density factor S_{min} around the crack tip determines the likely direction of crack propagation.

The strain energy density can be determined directly from the relationship.

$$\frac{dW}{dV} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \quad (2)$$

Where σ_{ij} and ε_{ij} are the stress and strain components respectively [10]. It states that the direction of crack initiation coincides with the direction of minimum strain energy density values.

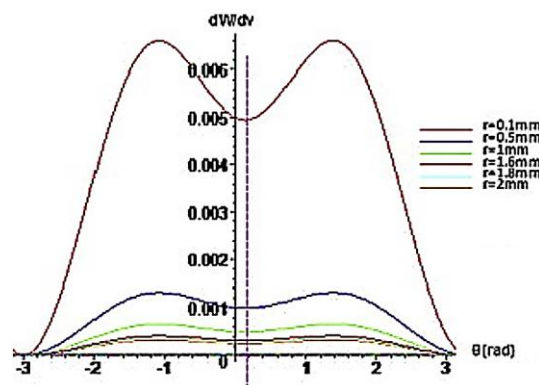


Figure 1. Strain energy density density as a function of the angle θ

3. Numeric analysis

In this part of study, we examined the failure of HDPE plates under biaxial loadings under plane stress assumptions. In this model, we consider a thin square plate of dimensions 100x100 mm². The plate is subjected under a biaxial loading of traction-traction type along the axes (Ox) and (Oy), perpendicular to each other. Figure 2.a shows the geometric model of the precracked plate subjected to biaxial tension. The square-shaped plate is meshed by quadratic elements of 8 nodes, particularly a mesh with special elements with nodes at quarter of the element sides have been used to characterize the singularity at vicinity of the crack (Figure 2.b). The mesh of the central zone containing the two extremities of the crack is preserved to manipulate different angles α , using a program in APDL language.

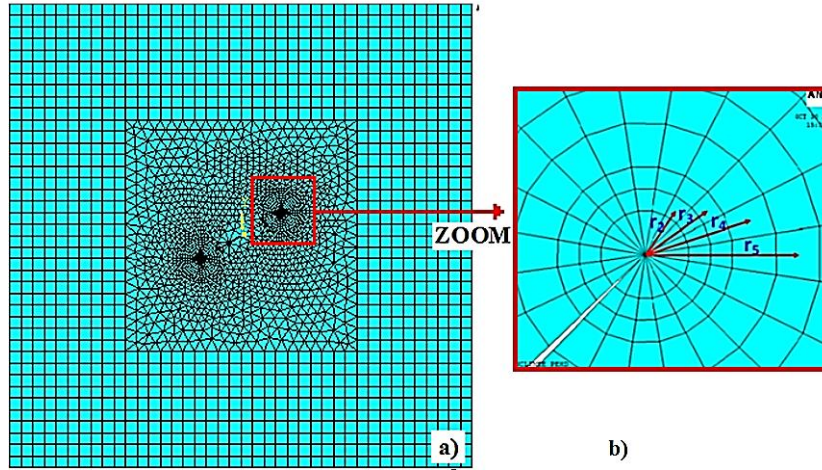


Figure 2. Example on the mesh of the pre-cracked square plate ($\alpha = 45^\circ$)

a. Full mesh of the plate. b. Elements surrounding the crack tip

In this analysis, the biaxial loading is characterized by the biaxility ratio $\frac{d_x}{d_y}$ (with $d_x = 1\text{mm}$. $d_x > d_y$ and d_y represent displacements along the axes (Ox) and (Oy), respectively. The variation of the ratio is monitored by the variation of the displacement d_x . We note that a true stress-strain curve obtained experimentally by [11] considered as elastic-plastic behavior has been used in this study chosen to perform the analysis in HDPE plates subjected on biaxial tension.

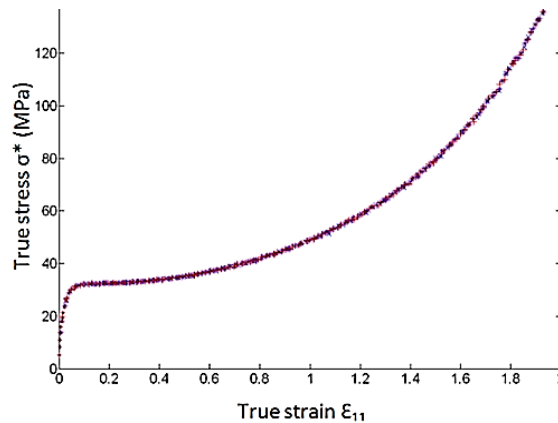


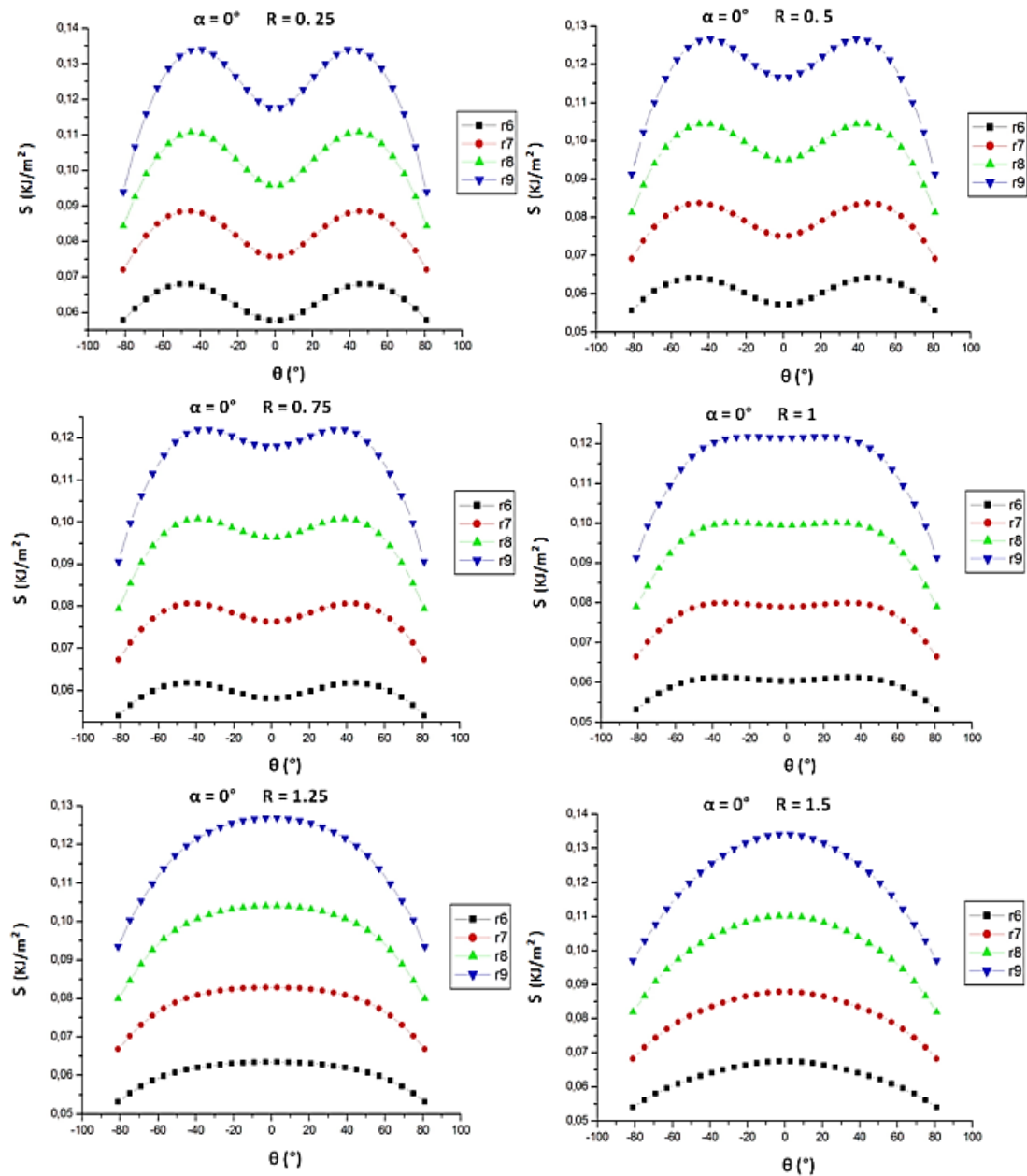
Figure 3. Stress-strain curve obtained from a tensile test [11]

Due to the high concentration of stress in vicinity of the crack tip resulting from the loading applied to the square plate and the non-linear behavior of HDPE, the estimation of the factor S was avoided at the level of the first rows (from r_1 to r_5). In this part of the study, we calculated the variation of the factor S for different loadings levels characterized by the ratio R and for different distances r measured from the crack tip. The calculation is carried out for four cases of crack inclination angles ($\alpha = 0^\circ, 30^\circ, 45^\circ$ and 60°). Examples of calculation are shown in Figures (4, 5) illustrating the evolution of the factor S as a function of the radius r and the polar

angle θ . The results obtained are plotted for an inclined central crack with angles $\alpha = 0^\circ$ and 45° , respectively.

4. Results and discussion

The obtained results in the figure 4. represent the variation of the strain energy density factor S as a function of the angle θ and the radius r (r varies from r_6 to r_9). The curves are plotted for different ratios R (with $R = 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75$ and 2) for the angle $\alpha = 0^\circ$



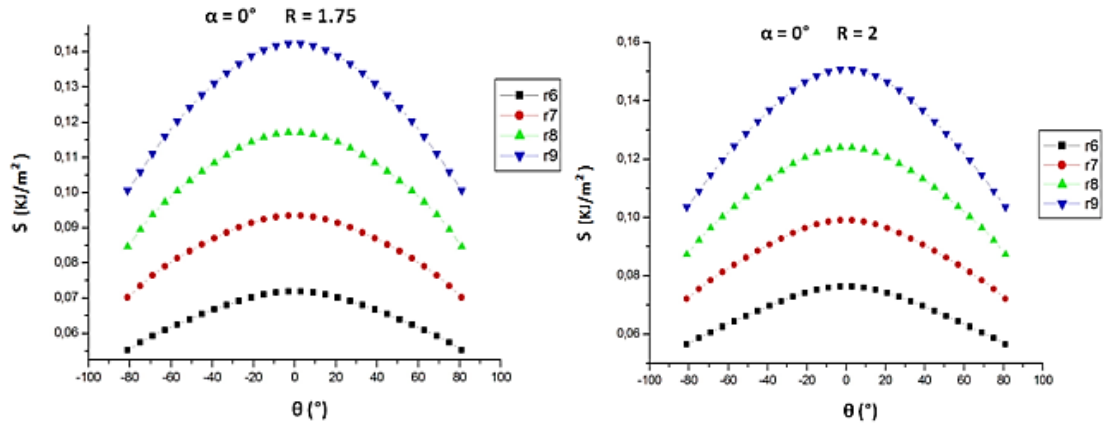
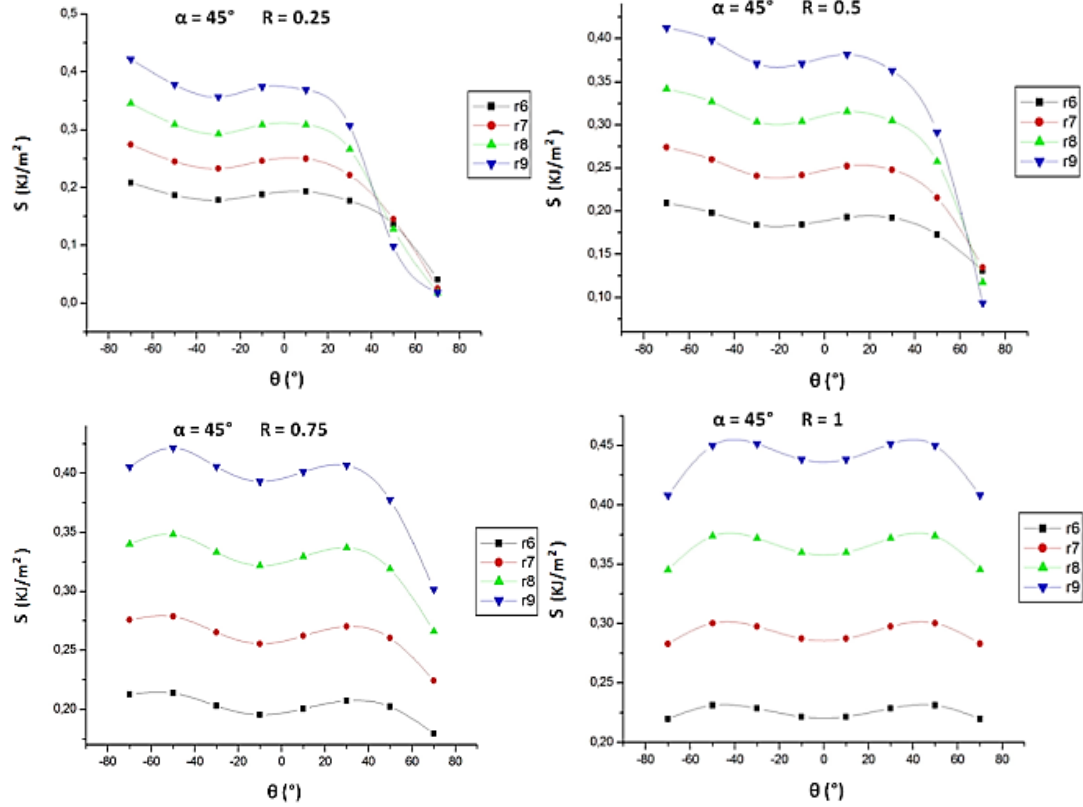


Figure 3. Influence of meshing on the SED criterion for $\alpha = 0^\circ$

The results represented in the Figure 4. allow identifying the following points:

For a biaxiality ratio $R < 1$ (with $\alpha = 0^\circ$), the angle θ_0 takes null values independent of the ratio R . This can be explained by the fact that the crack propagates horizontally according to the mode I (mode opening of the crack). The direction of propagation is perpendicular to the direction of the largest load. For $R = 1$ ($d_x = d_y$), the minimum of the factor S is reached a value of $\theta_0 = 0^\circ$, i.e. The probable direction of crack propagation would always be horizontal. For $R > 1$ ($d_x > d_y$), the plotted curves show the total absence of the local minimum of the factor S . In this case, no crack propagation can be considered.



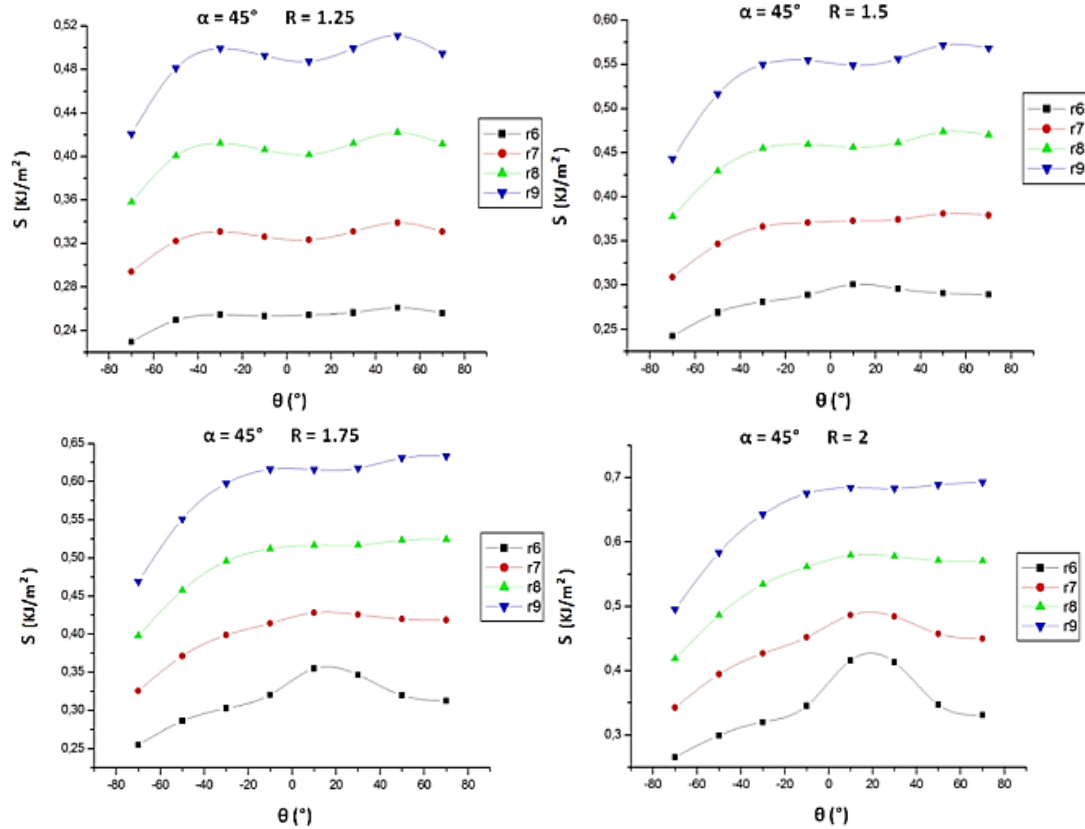


Figure 5. Variation of S as a function of θ for different values of R (with $\alpha=45^\circ$)

The numerical calculations carried out by the FEM lead to summarizing the following conclusions:

Outside of a certain area around the crack tip, the minimum of S is reached a value (S_{\min}) for a well-defined value of R , i.e. each inclined crack has a value of θ_0 depends on the biaxiality ratio R .

Each inclined crack having a critical ratio R_{crit} in which the crack changes its direction of propagation.

In the case of $\alpha = 45^\circ$ (with $R = 1$), the plate verifies a symmetry conditions for the geometry and the loading about the plane of the pre-crack. In this case, the minimum of the factor S is reached a value of $\theta_0 = 0^\circ$, i.e. the crack propagates in the direction parallel to the plane of symmetry (plane of the pre-crack).

For better showing the effect of the biaxiality ration R on the determination of the propagation direction, we have illustrated on the figure(III.20), the variation of the kinking angle as a function of the ratio R . the obtained results are given for 4 angles of inclination α (with $\alpha = 0^\circ, 30^\circ, 45^\circ$ and 60°).

The figure below shows that:

For a crack supposed to initiate horizontally ($\alpha = 0^\circ$) and if $R \leq 1$, the crack will propagate according to the opening mode (mode I). In the opposite case ($R > 1$), no propagation can be considered.

For an inclined crack ($\alpha \neq 0^\circ$), the estimate of the initial angle of propagation θ_0 is strongly related by the ratio R . Each pre-crack has a value of θ_0 that depends on R . In figure (III.20), the plane corresponding to the angle $\theta_0 = 0^\circ$ represents the critical plane in which the crack changes its direction of propagation.

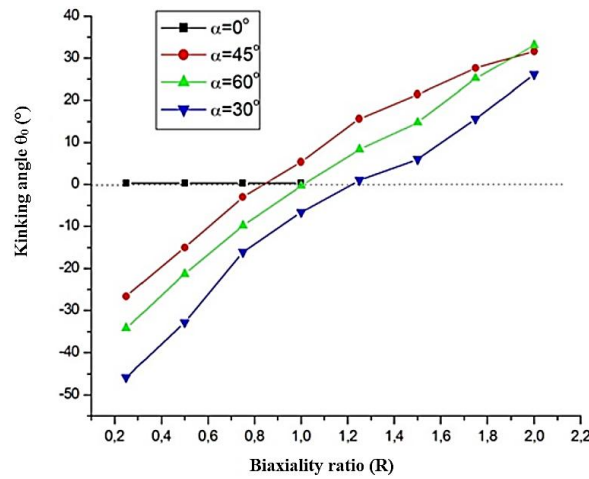


Figure 6. Effect of the ratio R on the initial angle of propagation θ_0

4. Conclusion

This study conduct to summarize the following equation

- 1- The crack initiation angle in mixed mode depends strongly on the angle the initial crack inclination and the biaxiality ratio.
- 2- In the case of $\alpha = 0^\circ$ no crack bifurcation is noticed when the biaxiality ration less than 1, because of the opening mode condition, but no crack initiation angle can be predicted for the raison that the domination of the loading component which lead to the closing of the crack.
- 3- The crack has a tendency to propagate perpendicular to the dominate load's axis. the increase in loading influence on the prediction of the angle of initiation, i.e. a high loading causes a great singularity in the area of the crack tip which will give less accuracy of results at this zone

5. Bibliography

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