

# Numerical modeling of the Radial Profiles of Axial Velocity and of the parietal Constraint in Transitory laminar flows in Conduits

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## Abstract :

This work deals with the modelling of the transient wall shear stress in transient laminar flow with the coupling the method for developing polynomial series of radial profiles of the instantaneous axial velocity and the method of characteristics to solve the water hammer problems in transient laminar flow. During the transport of fluids in pipe, the change of boundary conditions would result from deliberate or accidental shutoff valve and would lead to sudden jumps of pressure stress in the fluid. These sudden jumps in parameters of flow, such as pressure and average velocity, are called in hydraulic, commonly, water hammer phenomena. The results are in good agreement with those given by Hombloe and Rouleau [1], in laminar flow of Newtonian fluid.

**Mots clefs:** Velocity profiles, Newtonian laminar flow, Velocity profiles, Polynomial, radial profiles, expansion, Transient Shear stress, average velocity.

## 1. Introduction

The work presented concerns the theoretical and numerical modeling of radial profiles of axial velocity and shear wall stress in laminar and transient flows in pipes, to simulate water hammer phenomena. This numerical code proposed is based, more essentially, on a coupling technique of a method of development in polynomial series of the radial profiles of the instantaneous axial speed and the method of the characteristics in the space-time plane.

In transient pipe flow, the essential part of the energy dissipation comes from the pressure loss due to friction of the fluid on the wall of the pipe, Streeter and Wylie (2, 3).

The discrepancies are introduced by a difference in velocity profile, turbulence and transition from laminar to turbulent flow and vice versa. There are number of unsteady friction models which have been proposed in the literature. We can, principally, cite the works of Zielke (4), Trikha (5), Kagawa et al. (6), Brown (7), Suzuki et al. (8), Vardy et al. (9).

In this works, the friction terms dependent on instantaneous mean flow velocity  $V$  and weights for past velocity changes. In a different way, Brunone et al. (10), has expressed the friction term dependent function of instantaneous mean flow velocity  $V$ , instantaneous local

acceleration and instantaneous convective acceleration. Generally, difficulties arise in analysis of transient turbulent flow. Models from the literature are calibrated for certain flow conditions whereas the development of a general friction model in transient turbulent flow is a subject of intensive research worldwide.

Compared to these one-dimensional models Vardy et al. (11) developed a two-dimensional model. It has been shown that, contrary to the quasi-stationary regime; the velocity profiles are not parabolic.

## 2 . Initial and boundary conditions

The boundary conditions are in addition to the pressure imposed by the tank on the upstream end, the instantaneous closing of valve on the downstream.

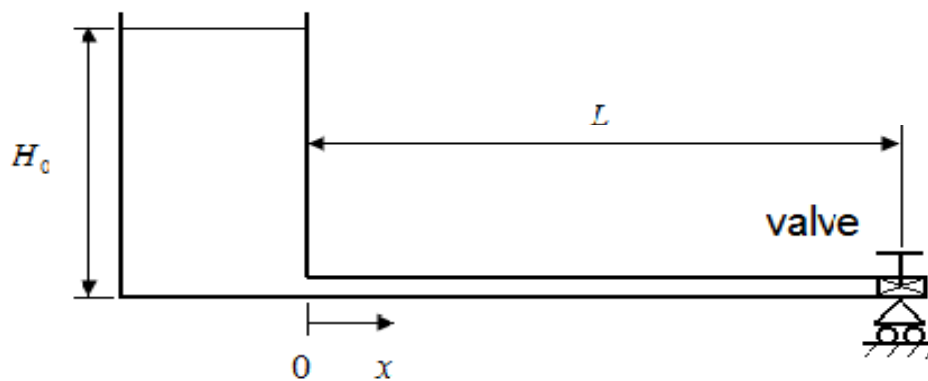


Figure 1. Diagram of the system studied

## 3 . ASSUMPTIONS AND BASIC EQUATION

This work is conducted under the assumption of ax symmetric unsteady flow of Newtonian, isentropic and compressible fluid. The deformation of the pipe wall is of low amplitude. The inertia terms are negligible and the pipe is modeled by a juxtaposition of independent rings without mass. The radius of the pipe is sufficiently negligible compared with its length enough that the current lines of fluid are straight. Assume, furthermore, that the longitudinal velocity gradients are very low compared with the transverse gradients.

We have to solve the following set of derivative partial equations:

$$\frac{\partial P}{\partial t} + \rho a^2 \frac{\partial V}{\partial x} = 0 \quad (1)$$

$$\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial V}{\partial t} = S_0 \quad (2)$$

$$\frac{\partial V_i}{\partial t} + \frac{1}{\rho} \frac{\partial P}{\partial x} = S_i \quad (3)$$

#### 4 . Application and results

The expressions for the shear stress  $\tau_p$  a the wall for the three cases are, respectively:

Option a ( $\dim(J_a) = 3$ )  $J_a = \{2, 8, 12\}$

Option b ( $\dim(J_b) = 4$ )  $J_b = \{2, 6, 10, 12\}$

Option c ( $\dim(J_c) = 5$ )  $J_c = \{2, 6, 8, 10, 12\}$

Option d ( $\dim(J_d) = 6$ )  $J_d = \{2, 4, 6, 8, 10, 12\}$

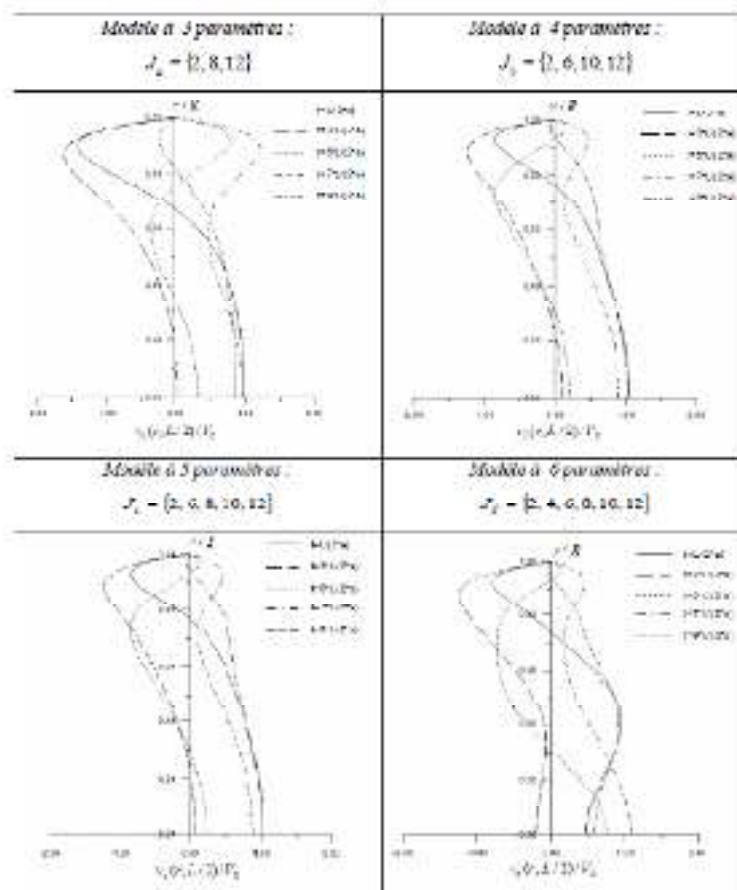


Figure 2 : Profils de la vitesse instantanée au milieu de la conduite pour les modèles :  $J_a$ ,  $J_b$ ,  $J_c$ ,  $J_d$

Figure presents the radial graphs of the profiles of the instantaneous axial speed in the middle of the pipes at times:  $t = \frac{L}{2a}$ ,  $t = \frac{L}{2a} + \frac{T}{4} = \frac{3L}{2a}$ ,  $t = \frac{L}{2a} + \frac{T}{2} = \frac{5L}{2a}$ ,  $t = \frac{L}{2a} + \frac{3T}{4} = \frac{7L}{2a}$ ,  $t = \frac{L}{2a} + T = \frac{9L}{2a}$ . Where  $\frac{L}{2a}$  is the time taken by the first pressure wave to get to the position  $x = \frac{L}{2a}$ , and  $T = \frac{4L}{2a}$  is the period of the wave.

## 5. CONCLUSION

This study shows that, compared with the quasi-stationary flow model, taking into account the unsteady nature of the velocity profiles has the advantage of providing a significant correction to the evaluation of the transient stress in laminar flows. and transients in pipes. This model has the advantage of being less prohibitive and requiring little calculation and therefore memory space to give a better representation of the parietal stress. From a practical point of view, this code can easily be used in existing codes to calculate transient laminar flows in pipes.

## 6. REFERENCES

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