Introduction to electricity markets

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Agenda

1. Electricity features
   - Non-storability
   - Transport
2. Electricity markets microstructure
   - Intraday market
   - Day-ahead market
   - Forward market
3. Derivatives & Risk management
   - Power plants & tolling contracts
   - Energy storage & swing contracts
   - Other derivatives
4. Conclusion
5. References

Electricity features

Main electricity features

A local commodity

- Electricity is non-storable.
- Electricity transport satisfies specific laws.

Storability

Comments

- Generally, subscribed capacity consumption exceeds installed capacity.
- Example in France: $\approx$ 250 GW of subscribed consumption capacity vs 128 GW installed capacity.
- Present best way to store large volume of power:

Storability

Comments

- Generally, subscribed capacity consumption exceeds installed capacity.
- Example in France: $\approx$ 250 GW of subscribed consumption capacity vs 128 GW installed capacity.
- Present best way to store large volume of power: hydro-reservoir.
- Limited by pumping rate ≈ 0.74.

Consequences

On short-term (next hours)

- A too long excess of demand compared to production may first resolve in a decrease of frequency,
- ... and if not properly corrected, may lead to dramatic blackouts.
  - July 30th, 2012: India, 670 millions people.
  - August 13th, 2003: Ontario and North America, 50 millions people.
  - November 4th, 2006: UCTE, 15 millions people.
- $\Rightarrow$ Minute by minute real-time assessment of the equilibrium between consumption and production.
- The Transmission System Operator (TSO) is responsible for the electric system security and reliability.
- He manages the uncertainties on demand and production by a series of operating reserves.

Actors in the electricity field

There are four categories of actors in the field :

- producers,
- suppliers,
- transmission system operators,
- distributors.

Electricity producers

They produce electricity and inject it onto the network using their power plants (nuclear, hydroelectric, wind turbines, photovoltaic, coal, ...).

They can be great firms with many production units (e.g. EDF) up to private individuals equipped with a wind turbine.

What can be used to produce electricity?

- https://www.rte-france.com/eco2mix
- https://www.electricitymap.org/map

Electricity suppliers

They act as intermediaries between producers and final consumers: they buy electricity to producers on wholesale markets and deliver it to final consumers through the network run by transmission system operators and distributors.

Some examples in France: EDF (“historical” supplier), Engie, Enercoop, TotalÉnergies, E.ON Énergie, EGL, Enel, Leclerc, Iberdrola, Vattenfall...

Transmission system operator

It manages the high voltage network. It acts as a natural monopoly in its geographical zone (usually its country).

Electricity distributors

They manage the medium to low voltage networks, connected to final consumers.

Enedis (ex-ERDF), a subsidiary from EDF, manages 95% of French medium and low voltage networks in France. 160 other firms manage the 5% remaining ones.

Reserves

Operating reserves

- An operating reserve is a generation that can be mobilized with a short-term notification.
- Operating reserves vary by response time. Three kinds of reserve:
  - primary reserve: response time $\leq$ 20s. Automatic devices. $\approx$ 500 MW in France.
  - secondary reserve: response time $\leq$ 3mn. Automatic. $\approx$ 600 MW in France.
  - tertiary reserve: response time $\leq$ 15mn. Manuel. $\approx$ 1,500 MW in France.
- The volume of each reserve may vary depending on the nature of the uncertainties on a particular electric system.
- They tend to grow in $\sqrt{T}$, where T is the time to mobilization.

Consequences

On mid-term basis

- Reliability assessment analysis: is there enough capacity to fullfill demand in the next months within a certain default probability?
- Means: changing planned outage schedule, buy on the market, demand-side management policy
- Extreme way: load shedding.

On long-term basis

- Build new capacity to allow enough excess capacity.
- Demand-side management policy.
- Sound tarification.

Transport

Comments

- The transport of electricity satisfies Kirchhoff's laws.
- The intensity at each node should be zero and the tension in each loop should be also zero.

Consequences

- In a meshed electricity network, power will go from one point to another using all available paths.
- $\Rightarrow$ Electricity flow interference.

Interference between commercial flows and physical flows

Situation

A power producer G1 has client in node C whose consumption is 180 MW, while a power producer G2 has also a client in node C whose consumption is 90 MW. Each producer holds enough generation capacity and no production cost advantage.

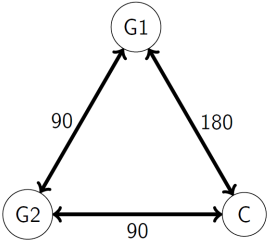
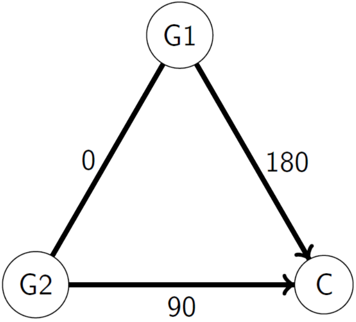
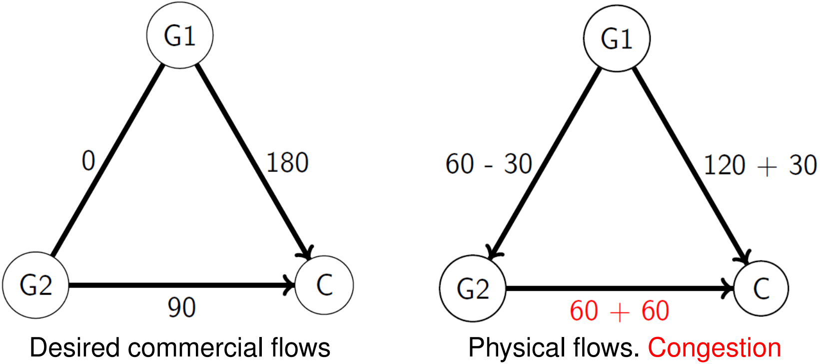


Figure: Network physical capacity limits.

Interference between commercial flows and physical flows



Interference between commercial flows and physical flows



Transport consequences on trading

Consequences

- $\Rightarrow$ Cross-border trading opportunities.
- Transfer capacities available for trading between countries need some generation hypothesis.
- In Europe, the available net transfer capacity (NTC) are managed and published by the ENTSO (European Network System Operator)
- Publicly available in her transparency platform (www.entso.net).

Electricity markets microstructure

Common market structure for a local commodity

Comments

- Electricity is a local commodity.
- As many electricity market as they are states:
  - USA: California (CAISO), MISO, New England (ISO-NE), New York (NYISO), PJM, Southeast, SPP, Texas (ERCOT)
  - Europe: EPEX SPOT, EEX, Nord Pool...
  - South America: Brazil energy market, Chili (1981)
  - Asia
  - Pacific
- Market microstructure highly depends on national regulation.
- Nevertheless, common structure emerges driven by the necessary equilibrium between consumption and production.

Common market microstructure

A sequence of markets ordered by time horizon

- The intraday market and/or balancing mechanism
- The day-ahead market
- The forward market

Day-ahead market mechanism

Example of Epex spot

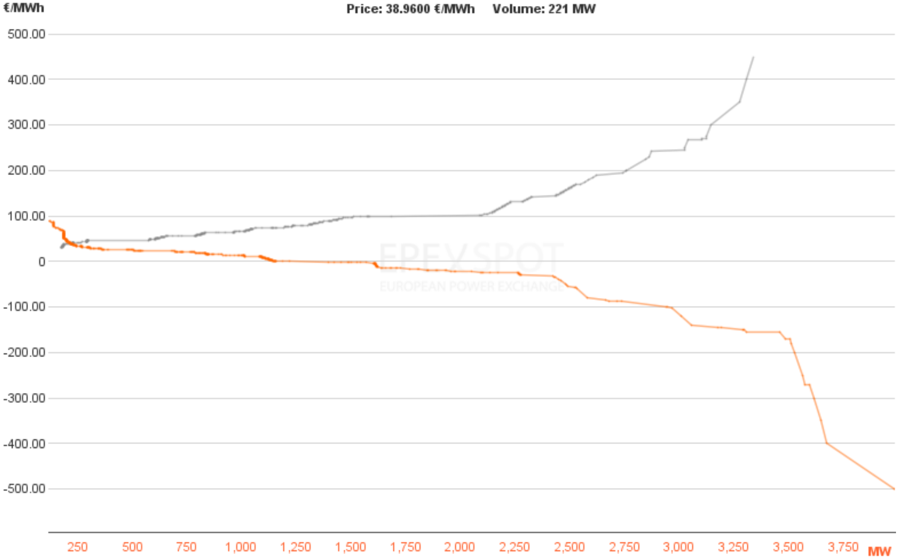
- Based on a fixed trading auction.
- Participants submit bids before a certain time (around 10:00).
- Bids can concern a particular hour of the next day or a set of hours (order block).
- Bids of market participants for a particular hour form a bid curve because she can submit a list of prices and quantities.
- Market organizer clears the market: she fixes a price for each hour of delivery and determines the seller and the buyers.
- Market players have then enough time to send production orders to their power plants and send their schedule to the TSO.
- *Note: the clearing process results in a non-convex optimization problem (block orders), for which defining a market price requires caution.*

Day-ahead market mechanism

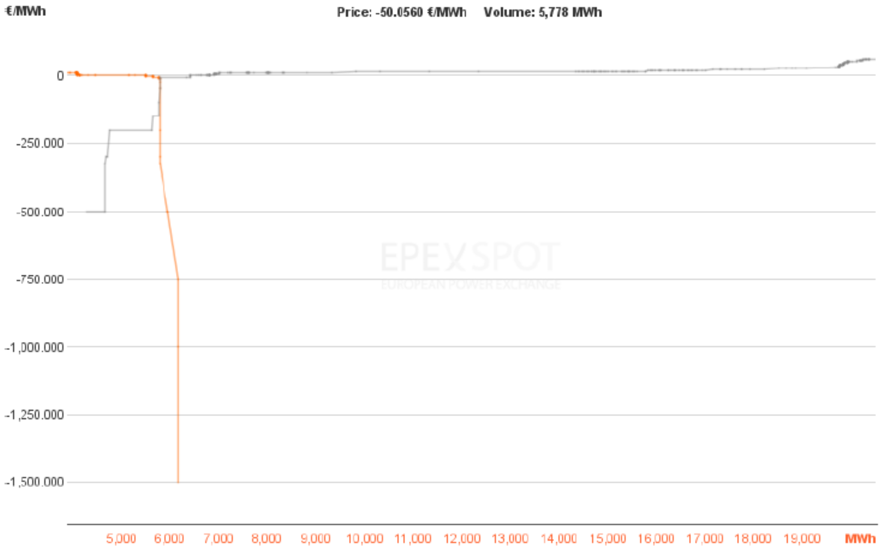
Trans-countries trading

- In continental Europe, each country has its own electricity day-ahead market cleared by her own firm.
- Without coordination, resulting quoted prices may provide the wrong signal when compared to transit flow between countries.
- Example given between France and Germany: flows would not follow spot prices difference between countries.
- Since quoted day-ahead prices by market organizer have a transparency function, mechanisms have been developed to ensure a consistent relation between cross-border transactions and local day-ahead prices.
- Market coupling: performing implicit auction mechanism.

Day-ahead market mechanism



Day-ahead market mechanism



Market Coupling

Implicit auction mechanism

- In each country, market participant do not have to care about finding a counterpart in neighboring countries.
- She has just to submit her bid in her country (sell or buy).
- Market organizers perform a clearing process with transport constraints implied by the available transfer capacity (ATC).
- If there is no binding transit capacity constraints, then there will be a single price for the clearing area.
- If there is at least one binding transit capacity constraint, two prices will emerge.

Day-ahead market prices

- Day-ahead markets prices exhibit seasonalities

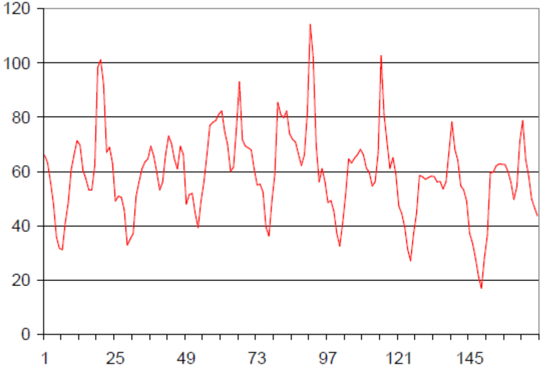
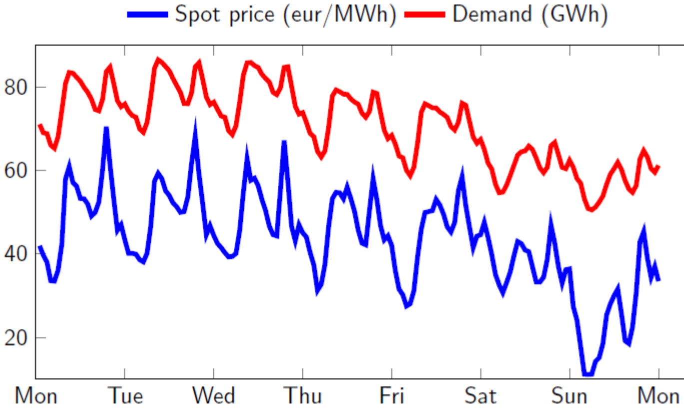


Figure: Daily and weekly seasonality. Epex hourly spot price, January, 4th to 10th 2010.

- Exhibit also annual seasonality

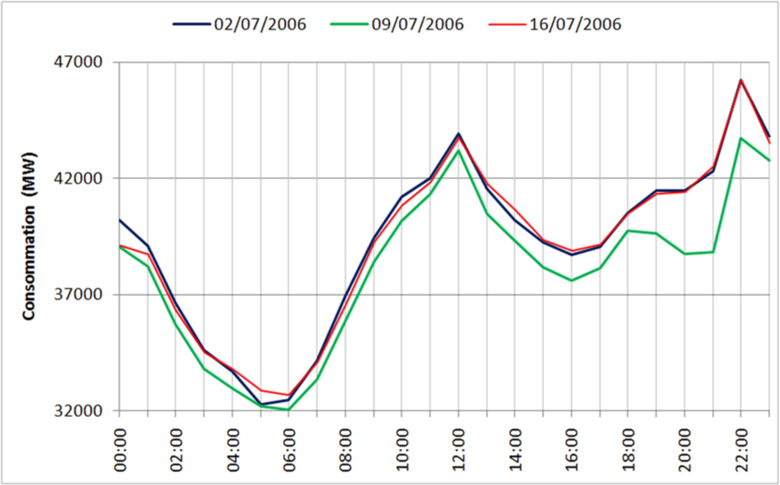
Day-ahead market prices

- Strong dependence with consumption



Day-ahead market prices

- Some specific event may considerably change usual consumption



Day-ahead market prices

- Day-ahead market price exhibits spikes and negative prices

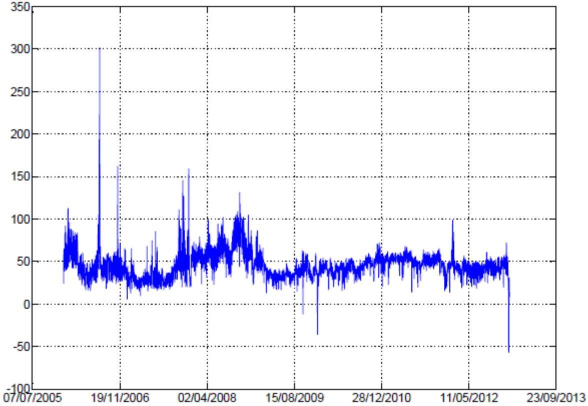


Figure: EEX day-ahead prices

Intraday

Commons

- Ensure the security of the system: Balancing Mechanism.
- Transparent market price for the cost of imbalance: Imbalance Settlement Price.

Remark

- May coexist at the same time a market for next hours where firms exchange power.

Balancing mechanism

Example French TSO adjustment market mechanism as of April, 2022.

- Balance Responsible Entities (BR) submit bids and offers to increase or decreases their production (or consumption).
- TSO selects offers based on economic precedence.
- BR are paid as bid.
- Every power available plant should be declared on the adjustment market
- Producers declare their price to increase their production
- System operator uses all these offers to insure real-time production consumption equilibrium
- But, some time later, each balance responsible entity receives the bill of her imbalances...

Balancing mechanism

Example French TSO adjustment market mechanism as of April, 2022.

- S represents the day-ahead price settled the day before for the hour of interest.
- P^d is the average price of the offers used by the TSO on the balancing mechanism to decrease the production (or increase the consumption).
- P^u is the average price of the offers used by the TSO on the balancing mechanism to increase the production (or decrease the consumption).

Imbalance mechanism

	Network Adjustment Trend Positive	Network Adjustment Trend Negative
Actor Imbalance Positive **Actor is paid**	$P^u(1-k)$	$P^d(1-k)$
Actor Imbalance Negative **Actor pays**	$P^u(1+k)$	$P^d(1+k)$

Reading key

- Network needs upward adjustment & Actor is producing too much $\Rightarrow$ Actor is paid $P^d(1-k)$.
- Network needs downward adjustment & Actor is not producing enough $\Rightarrow$ Actor pays $P^u(1+k)$.

Balancing mechanism

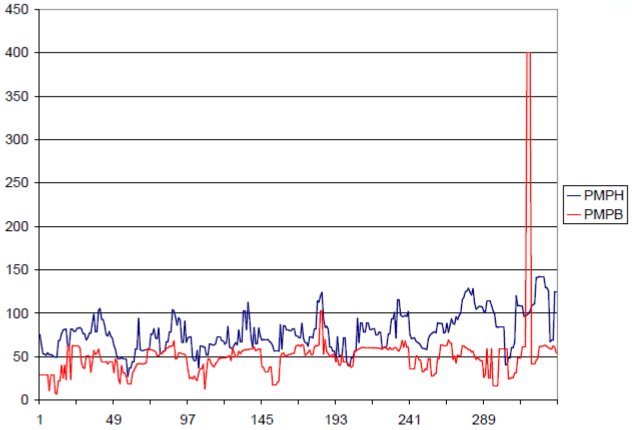


Figure: Weigthed Average Upward and Downward adjustment price in French power market from January 4th, to 10th 2010.

Imbalance prices

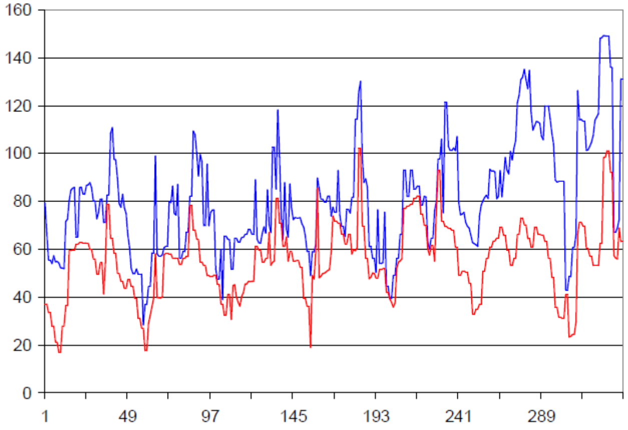
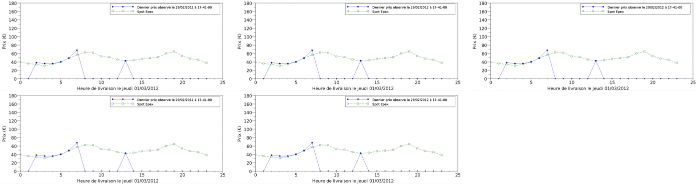


Figure: Imbalanced Settlement Prices (Upward and Downward) in French power market from January 4th, to 10th 2010.

Note

- Beside this balancing mechanism, an intraday market for energy delivery for the hours of the day or of the next day exists.



Epex intraday market prices

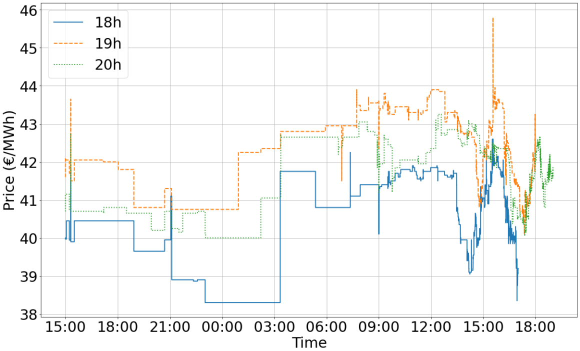


Figure: Some intraday transaction prices on July 11th, 2017

Source : Deschatre and G., 2023

Comment

Trading actions are mostly done the closest hours of the delivery

Epex intraday market prices

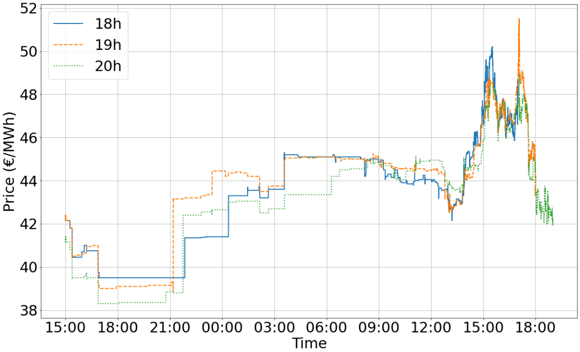


Figure: Some intraday transaction prices on August 30th, 2017

Source : Deschatre and G., 2023

Comment

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Epex intraday market prices

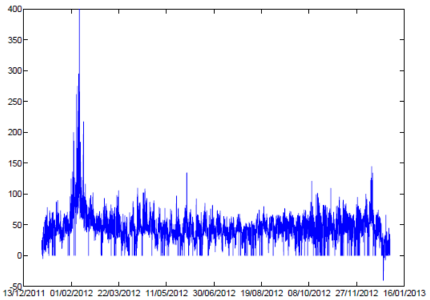

Figure: Epex prix horaires intraday sur l'année 2012.

Comments (data 2017)

- Since March 2017 : half hourly intraday prices
- 150GWh traded over 1 day in Germany, 10GWh in France
- Transactions at millisecond
- 40 robots (30 to 40% of the volume) for 220 actors

- Relation between intraday and day-ahead prices

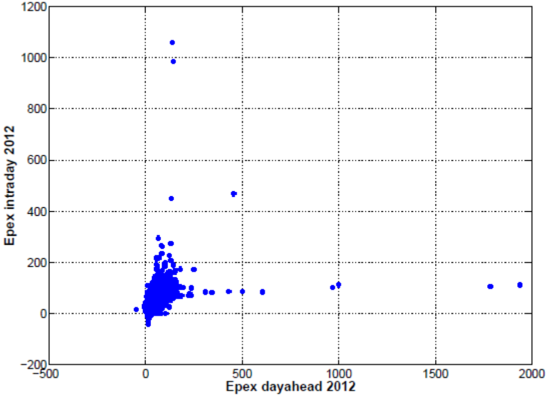

Figure: Epex intraday and dayahead hourly prices during 2012.

Day-ahead market prices

Comments

- High day-ahead price and low intraday: uncertainty resolved.
- Low day-ahead and high intraday: very short-term uncertainty realisation.

- Day-ahead market prices are referred to as the **spot prices**.

Future market

Forward market

- **Power specificity: delivery period**
- Different maturities and different delivery periods

	2011												2012												Time
2010	Q1			Q2			Q3			Q4			Q1			Q2			Q3			Q4			2013
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	

Forward market

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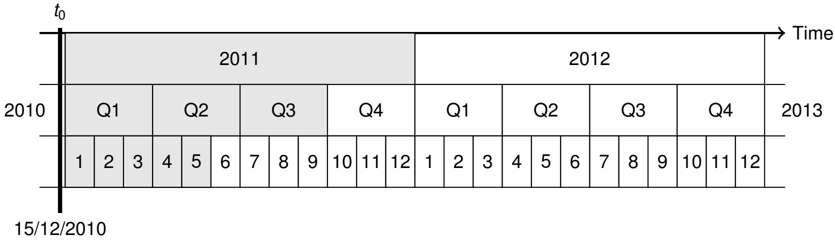

Future market

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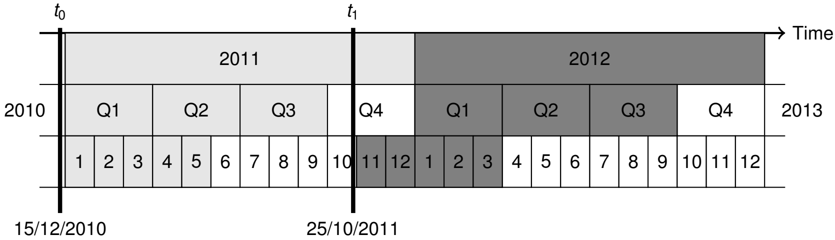

Future market

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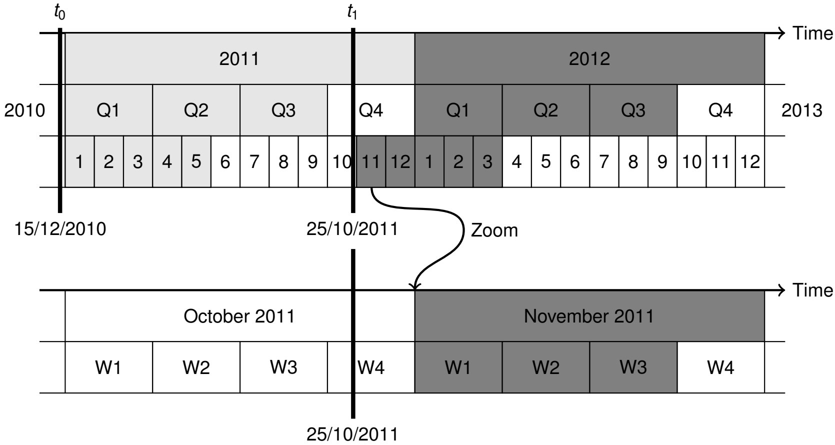


Available forward products

Example EEX (www.eex.com)

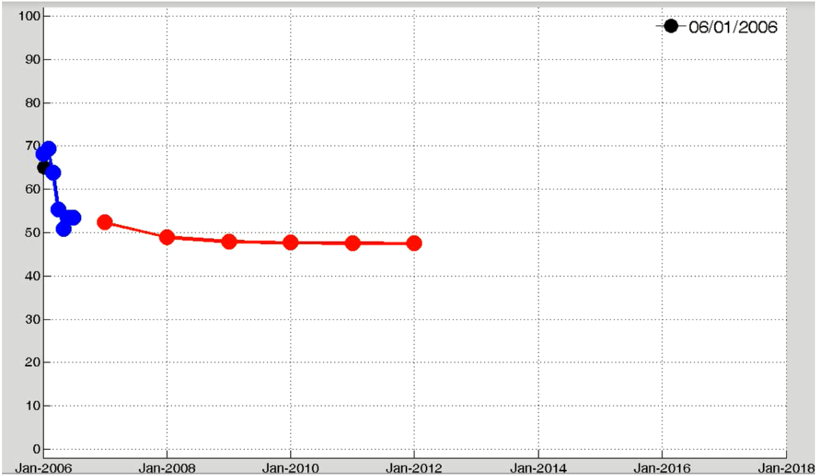
- Are available at the same time:
  - 6 calendars
  - 11 quarters
  - 10 months
  - 5 weeks
  - 2 weekends
  - 10 days
- In three flavours: baseload (each hour), peakload (07:00-20:00 Monday to Friday) and offpeak (complementary to peakload, not available for weeks, weekends and days).
- Thus, 132 contracts are available
- Compare with the 525,684 hours in the next six years.

Forward contract settlement

Settlement

- Electricity forward contracts implies a delivery during a period of time.
- Delivery of power every hour of the week, month, quarter or year (base load) or a set of hours frome Montay to Friday (peak load).
- Possible settlement at maturity or continuously during the delivery period.

German baseload forward curve dynamics

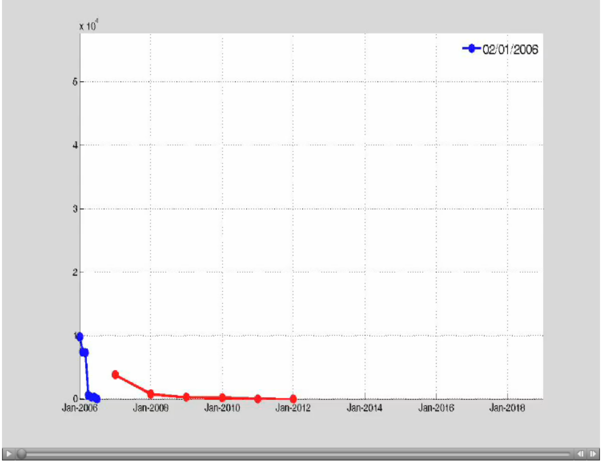


German baseload forward curve dynamics

Comments

- Very differentiated behaviour between spot, month and yearly contracts.
- Slow motion of yearly contracts. May exhibit report or deport configuration.
- Strong seasonal pattern of monthly contracts (blue dots).

German baseload open interest curve dynamics



German baseload open interest curve dynamics

Comments

- Close maturities catch all liquidity.
- Linear growth of closest maturity open interest.

French-German spread curve dynamic

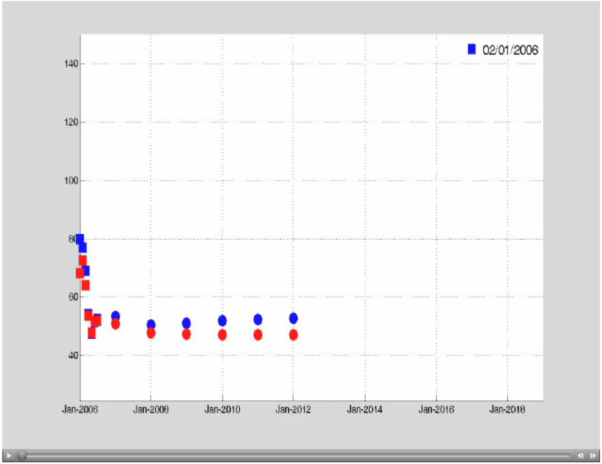


Figure: French is in blue, Germany is in red

French-German spread curve dynamic

Comments

- Strong dependence.
- Possible sign inversion with time and maturity.

Global vision of embedded Markets and physical mechanisms

Moyen Terme	Journalier	Infra journalier	Temps réel	Ex post

Les marchés électriques (entre acteurs du marché)

MT (1 semaine à 3ans)	J-1	Infra J	T	T+
• Marchés organisés: Contrats à terme • Options (peu utilisées) • Gré à gré (intermédié + direct)	• Epex spot (CWE, puis NWE et PCR • Gré à gré (intermédié + direct)	• Epex intraday (FRA, ALL/AUT, SUI) • Autres marchés organisés (ESP, UK) • Gré à gré		

Le physique : les mécanismes avec RTE

MT (1 semaine à 3ans)	J-1	Infra J	T	T+
• Enchères capacités d'intercos (N+1, M+1)	• Enchères capacités d'intercos journalieres • Intercos implicites pour CWE • Programme d'appel 16h30 (production, services système, offres ajustement)	• Capacités intercos IJ • Capacités intercos implicites pour Epex intraday • Guichets : mises à jour du programme d'appel chaque heure	• Mécanisme d'ajustement pour P=C ou congestion (utilisation des offres ajustement)	• Règlement des écarts : au préalable chaque point d'injection ou de soutirage (physique ou contrat) est rattaché à un responsable équilibre

New markets !

- **Capacity market (2016 in France)**
  - CRE gives a list of capacity certificates (in MW) for year n
  - RTE is estimating the need of capacity in a "worst case" for year n
  - Each actor must buy enough cerificates to ensure its electricity supply
  - 15 capacity auctions over 4 years ($n-4$ to $n-1$) before delivery year n...
  - ... and possibility to buy in peer-to-peer, but the price is constrained

- **Balancing market (2017 in France)**
  - To be continued...

Derivatives & Risk Management

The name of the game

Problems

- An electric utility is exposed to a whole set of risk factors:
  - Electricity prices
  - Fuel prices: coal, crude, gas,
  - Emission prices
  - Currencies
  - Consumption and market shares
  - Outtages and in flows
  - Climate
- Two kind of problems: local problems and global ones.

The name of the game

Local problems

- Simulating realistic spot prices.
- Pricing and hedging a particular structured product.
- Assessing the impact of a particular factor on the value of an asset.
- ...

Global problems

- Designing the models for a whole risk management system for a utility exposed to those risk factors.
- Need for one or several models to allow
  - realistic representation of prices and their dependencies,
  - fast and robust estimation procedures,
  - calibration to market prices,
  - fast computation of simulated forwards,
  - fast computation of derivatives,
  - fast computation of greeks.

General information about energy raw materials

- Nature of a commodity: a product from agriculture, forestry, fishing or minerals which has undergone the transformations required for international exchange.
- But requirement of minimum standardisation for the exchange. Example: copper: not the raw mineral extracted from the mine (between 0.5 and 6% of the mineral), but a concentrate containing 25 to 35% of copper. The transformation is carried out ex-mine.
- The ratio between the value per ton and the cost of transport determines the usage zone of a raw material. Let's take water and cement: these are not subject to international trading.
- A commodity is therefore a merchandise for which producers use a global market where competition bears solely on price. (P.-N. Giraud)
- Geographical characteristics are put aside, leaving only differences in quality, assessed in respect of a benchmark index.
- Comment: some prepared products become commodities (paper paste), as do services (freight, bandwidth). Some commodities may be surprising: cattle.

General information about energy raw materials

Difference with the stock market

- Consumer goods: applies to all human activities. The final purchaser wishes to include it in a process, or to consume it (energy needs to run machines), or to sell it in a transformed way (cocoa, metal, cereals...).
- Technological goods: result of an industrial production process, involving all the problems linked to industrial processes (breakdown, strike, delay, aging of materials and machines, evolving technologies...).
- Although upstream of other industrial processes, and producing waste in fine (recycled or otherwise, as with metals). Makes it possible to define commodity strands from a source, and for a specific purpose: for example for mining products: those for construction, those used in the chemical and metallurgical industry, or for energies, used in the energy conservation system.
- Asset used for a function, not for its value: if a producer finds a material meeting the same technical characteristics, but cheaper, it will switch to this new material: key concept of **substituability.**

General information about energy raw materials

Replaceability

- A commodity is consumed for its technical qualities. It may be exchanged by another (for example coal/ oil, gas lighting instead of electricity...)
- Example having far from negligible impacts on prices of iron minerals: the substitution of steel with carbonated compounds in automobile construction.
- Idem for changes in livestock feed.
- For all raw materials, there is at least one possible replacement product. Only the cost of replacement acts as a barrier to switching. Example: possible switch from the oil economy to the coal economy of oil prices sustainable reach a certain threshold (above $80)

General information about energy raw materials

Other key concepts

- **Exhaustion**: natural resources are finite (deposit or productivity): causes repeated controversy as to what should constitute "proper management" in the long term of these resources.
- Random geographical distribution of deposits of an extremely variable nature: causes two phenomena:
  - significant requirements for transport (deposits are not close to consumption centres). Central role of transport and networks (current example of the trans-Ukrainian gas pipeline).
  - the existence of a differential profit or mining profit (cost of extracting a barrel almost zero in SA, compared with $15 in the North Sea). Proportional growth curve growing with quantity. Creates geo-political tensions. Structures the strategy of industrial groups aware that their product is the same as that of their competitors (Total petrol runs as well in my car as BP petrol). The most significant margins lie here.

General information about energy raw materials

Other key concepts

- The fact that raw materials cannot be produced creates a phenomenon of dependency of states and the need for a policy of security of supply. Causes strategic behaviour by states and their systematic intervention, affecting price dynamics. Example of the oil shocks of 73 and 79.
- Broad price variations : In the short-term, the rigidity of production systems in the face of possible severe variations in demand causes broad price variations. Fundamental role of storage and information on the level of stocks and reserves.

Two different coupled markets

- **Physical market:** sell/purchase and physical delivery or the raw material
- **Paper market:** futures / forward market alowing the actors to hedge the prices' risks.

Derivatives

Problems

- Like any other commodity market, electricity markets have their options on quoted futures.
- But, the most challenging problems comes from the pricing, hedging and structuring of exotic tradable products linked to physical assets.
- Can be called Real Derivatives.

Real derivatives

- Embedded options in producers or retailers portfolio.
- Power plant $\rightarrow$ strip of call options on fuel spread.
- Tolling contracts $\Rightarrow$ structured contract counterpart.
- Water reservoir $\rightarrow$ strip of call options on calendar spread.
- Swings $\Rightarrow$ structured contract counterpart.
- Demand-side management ! strip of puts.

Power plants as derivatives

First approximation of power plants value

- Strip of calls on the spread between its fuel price and the electricity price.
- The value of a power plant on a period of time $[0; T]$ would then be given by :

$$\mathbb{E}\left[\int_0^T \left(S_t^e - hS_t^f\right)^+ dt\right] \quad (1)$$

  where S^e is the price of power, S^f the price of its fuel and h its heat-rate and $x^+ = max(0; x)$.
- Margrabe's 1978 closed-form formula for exchange options applies and gives value and Greeks.
- But, there are some problems with this approximation.

Difficulties with power plants as derivatives

First approximation of power plants value

- With emission price S^c, value reads

$$\mathbb{E}\left[\int_0^T \left(S_t^e - hS_t^f - gS_t^c\right)^+ dt\right] \tag{2}$$

- Margrabe's formula does not apply anymore (see later on...).
- But, it still neglects operational constraints: start-up cost act as non-zero strike price. Ramp-up time, minimum power...
- Taking constraints into account $\rightarrow$ stochastic control problem (optimal switching or singular control).

but another problem with electricity...

Difficulties with power plants as derivatives

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$$\mathbb{E}\left[\int_0^T \left(S_t^e - hS_t^f - gS_t^c\right)^+ dt\right] \tag{2}$$

- Margrabe's formula does not apply anymore (see later on...).
- But, it still neglects operational constraints: start-up cost act as non-zero strike price. Ramp-up time, minimum power...
- Taking constraints into account $\rightarrow$ stochastic control problem (optimal switching or singular control).

but another problem with electricity... **Electricity is not storable !!**

Difficulties with power plants as derivatives

Risk-neutral probability

- If the market was complete, the probability under which the former expectations are computed would not be ambiguous.
- At time 0, future prices would exist for each hour t of the year and the value of the power plant would be :

$$\mathbb{E}\left[\int_0^T \left(F^e(t,t) - hF^f(t,t) - gF^c(t,t)\right)^+ dt\right] \tag{3}$$

  where $F^e(0,t)$, $F^f(0,t)$, $F^c(0,t)$ are respectively the futures price for power quoted at time zero for delivery at hour t, the futures price for its fuel and the futures price for the emission permit.
- But, such information is not available.
- In realistic cases, the valuation of a power plant cannot avoid the problem of the incompleteness of the electricity market.

Valuation

- Theoretical difficulties of power plants valuation apply also for hydro power plants
- But, the limited ressource of fuel leads to a problem of storage management.
- Most simple hydrolic storage management problem deals with a single reservoir.
- The valuation problem writes

$$\begin{aligned} \sup_{q_t \in [0;\overline{q}]} \quad & \mathbb{E} \quad \left[\int_t^T q_u S_u^e du + g(S_T^e, X_T) \right], \\ dX_s^{t,x} \quad & = \quad \left(a_s^{t,a} - q_s \right) ds \end{aligned}$$

with S_t^e the electricity spot price, X_t current level of the water reservoir, as random in flows. X is subject to level constraints and should stay within $[\underline{x}; \overline{x}]$. The function g represents a final value for having a certain level of water at a final time.

Water reservoir and energy storage

Valuation

- Leads to stochastic control problems
- Intensive use of Dynamic Programming
- Main difficulties: dimension state.

Other derivatives

Retail contracts

- Tarification policy for different final consumers (industrial, small business and households)
- Financial risk embedded in a client load curve vs portfolio effect.
- Market share vs margin.

Weather derivatives

- Producers financial risk highly depends on weather.
- Weather derivatives on temperature, rain, wind procure insurance on bad outcomes.

Spread options

- **Commodity spread** (Power plant)

$$\left(S_t^{power} - hS_t^{fuel}(-h' S_t^{CO2}) - K\right)^+$$

  - (Clean) Spark spread: S_t^{fuel} is the gas spot price ("clean" if Carbon is taken into account)
  - (Clean) Dark spread: S_t^{fuel} is the coal spot price ("clean" if Carbon is taken into account)

- **Locational spread** (ex: 2 different gas hubs, National Balancing Point (NBP) and Zeebugge (ZEE))

$$\left(S_t^{NBP} - S_t^{ZEE} - K\right)^+$$

Other classical spread options in energy markets

- **Index spread** (typical for oil products: Crack spread)

$$\left(\frac{2}{3}S_t^{gasoiline} + \frac{1}{3}S_t^{heatingoil} - S_t^{crudeoil}\right)^+$$

- **Temporal spread**: used to approximate the value of a storage asset

$$\left(F_t^{NBP}(T_1) - F_t^{NBP}(T_2) - K\right)^+$$

Some analytical formulas

Margrabe formula

- An extension of the Black-Scholes formula for pricing “exchange” options ($K = 0$)
- Quite same assumptions as in the Black-Scholes model
- Objective: pricing an echange option of payoff

$$\left(S_T^1 - S_T^2\right)^+$$

at time T.

Assumptions

- The risk-free rate r is constant.
- The underlying assets follow a correlated ($dW_t^1 dW_t^2 = \rho$) Brownian motion under the risk-neutral measure

$$\frac{dS_t^i}{S_t^i} = r dt + \sigma_i dW_t^i, \quad i = 1, 2 \tag{4}$$

Margrabe's formula

Assuming two assets S_t^1 and S_t^2 following (4).
The price $V(t, T, S_t^1, S_t^2)$ of an exchange option of payoff $\left(S_T^1 - S_T^2\right)^+$ is given by the Margrabe's formula

$$V(t, T, S_t^1, S_t^2) = S_t^1 \Phi(d_1) - S_t^2 \Phi(d_2)$$

with

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T-t}} \ln\left(\frac{S_t^1}{S_t^2}\right) + \frac{\sigma}{2}\sqrt{T-t} \\ d_2 &= \frac{1}{\sigma\sqrt{T-t}} \ln\left(\frac{S_t^1}{S_t^2}\right) - \frac{\sigma}{2}\sqrt{T-t} \\ \sigma^2 &= \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 \end{aligned}$$

and Φ the distribution function of a $\mathcal{N}(0, 1)$.

Margrabe's formula

Remarks

- The Margrabe's formula is valid for pricing spread options with $K = 0$
- When $K \neq 0$, no closed-form solutions are known
- Analytical methods rely on approximation formulas

- **Recalling commodity spread** (Power plant)

$$\left(S_t^{power} - hS_t^{fuel} - h' S_t^{CO2} - K\right)^+$$

K is the fixed cost $\Rightarrow$ cannot be neglected.

Spread options valuation with analytical approximations

Review

- Kirk's formula
- Approximation of the spread distribution with a Gaussian distribution
- Case of mean-reversion in the risk-neutral probability
- (Alos, Eydelan & Lawrence (2011)) asymptotic expansion in the case of three assets

Kirk's formula (1995)

Kirk's formula

Consider r the constant risk-free rate and two assets following (4) in the risk-neutral probability.

Note $Y_t = \left(S_t^2 + Ke^{-r(T-t)}\right)$

The approximated Kirk's formula of a spread option with payoff $\left(S_T^1 - S_T^2 - K\right)^+$ is given by

$$\hat{V}(t, T, S_t^1, S_t^2) = S_t^1 \Phi(d_1) - Y_t \Phi(d_2)$$

with

$$\begin{aligned} d_1 &= \frac{1}{\sigma\sqrt{T-t}} \ln\left(\frac{S_t^1}{Y_t}\right) + \frac{\sigma}{2}(T-t) \\ d_2 &= \frac{1}{\sigma\sqrt{T-t}} \ln\left(\frac{S_t^1}{Y_t}\right) - \frac{\sigma}{2}(T-t) \\ \sigma^2 &= \sigma_1^2 + \sigma_2^2\left(\frac{S_t^2}{Y_t}\right)^2 - 2\rho\sigma_1\sigma_2\left(\frac{S_t^2}{Y_t}\right) \end{aligned}$$

and Φ the distribution function of a $\mathcal{N}(0, 1)$.

Conclusion

Complexity

- Power systems knowledge.
- Nested market microstructure.
- Complex products.
- Incompleteness & valuation.
- Numerical methods for stochastic control problems.

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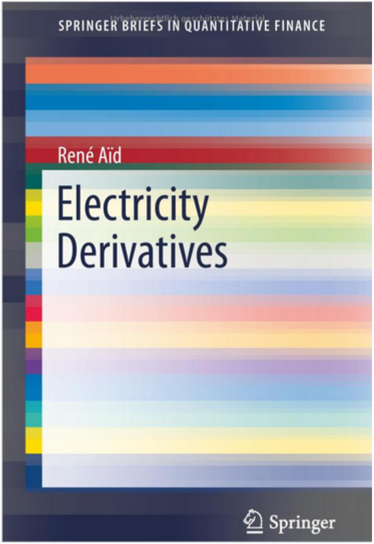
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