Quantum Phase Estimator

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1 Introduction

Quantum Phase Estimation (QPE) is designed to estimate the phase (or eigenvalue) of an eigenvector of a unitary operation with remarkable precision. This process is not only fundamental to quantum algorithms like Shor's Algorithm, which has the potential to revolutionize cryptography by efficiently factoring large numbers, but also extends its applications to quantum chemistry, where it can simulate molecular structures and predict chemical reactions with unprecedented accuracy.

2 Quantum Phase Estimation (QPE) Circuit Structure

The Quantum Phase Estimation (QPE) circuit is structured into multiple layers of quantum gates, each playing a crucial role in extracting the phase information from a unitary matrix. The process begins by initializing the circuit, where the target qubits are set in the $|1\rangle$ state. Hadamard gates are then applied to all control qubits, creating a superposition that enables phase accumulation.

Next, a unitary matrix U is applied to the target qubits. The power of this matrix increases exponentially with each subsequent control qubit, meaning that the first control qubit applies U, the second applies U^2, the third applies U^4, and so on. This controlled evolution encodes the eigenphase of U into the quantum state.

In this case, the unitary matrix has a phase of $\theta = \frac{3}{5}$, and its controlled operations influence the qubits in a decreasing order. Finally, to extract the phase information, an **Inverse Quantum Fourier Transform (IQFT)** is applied to the control qubits, converting the encoded phase information into a measurable binary representation. The result is then measured, revealing an approximation of the phase associated with the eigenvalue of U.

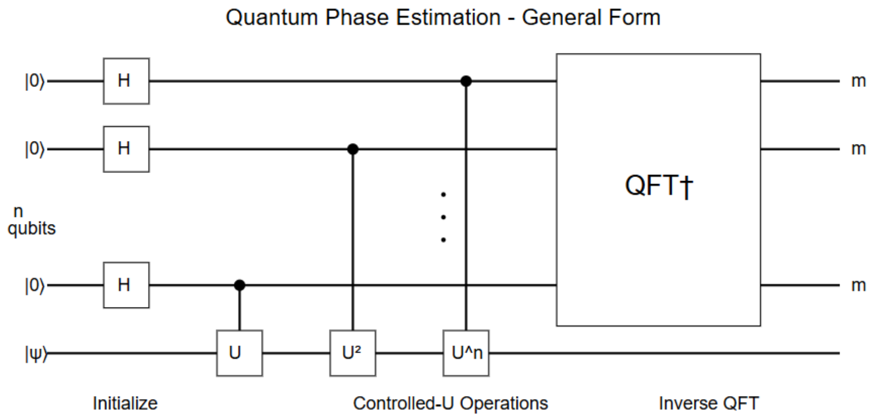

Figure 1: Quantum Phase Estimation (QPE) Circuit

3 Results

In this experiment, the objective was to estimate $\theta = 0.6$ with a precision of 10^{-5} using the **Quantum Phase Estimation (QPE)** algorithm. **Figure 2** represents the measured values as the number of qubits increases. Additionally, for each circuit, the deviation between the true value of θ and the estimated value is analyzed.

The first set of circuits used **between 3 and 6 qubits**. In this range, the circuit depth varied from **10 to 66**. For these circuits, the estimated value had a deviation between **0.015112304687500022** and **0.005395507812499978** from the true value. Given this circuit depth, the QPE algorithm required additional operations to further reduce the deviation.

For the circuit with **7 qubits**, we observed an improved performance with a circuit depth of **130**. The measured θ value was **0.6013717651367188**, with a precision of **0.0013717651367187722**. As the number of qubits increased from **8 to 10**, the estimated θ value continued to converge towards the true value. However, at **10 qubits**, the precision slightly deteriorated compared to the **9-qubit circuit**:

- **9-Qubit Circuit:**
  - **Measured** θ: 0.5998897552490234
  - **Precision**: 0.0001102447509765403
- **10-Qubit Circuit:**
  - **Measured** θ: 0.6001157760620117
  - **Precision**: 0.00011577606201174095

From this, we observe that the **9-qubit and 11-qubit circuits** demonstrated better performance compared to the **10-qubit circuit**.

Beyond **11 qubits**, the precision showed greater stability, with the following results:

- **12-Qubit Circuit Precision**: $6.809234619142845 \times 10^{-5}$
- **13-Qubit Circuit Precision**: $2.3698806762673108 \times 10^{-5}$
- **14-Qubit Circuit Precision**: $3.969669342018811 \times 10^{-6}$
- **Measured θ for 14 Qubits**: **0.599996030330658**

4 Conclution

The reason for obtaining better results as the number of qubits increases is that **the circuit achieves higher precision due to improved phase resolution**. Additionally, with a larger number of qubits, the probability of measuring the correct eigenphase increases, leading to more accurate estimates of θ.

$$\tilde{\theta} = \frac{m}{2^n} \tag{1}$$

where n is the number of control qubits and m is the measured integer. As n increases, the denominator 2^n gets larger, allowing for finer precision in estimating θ. This leads to a reduced deviation from the true value.

Furthermore, increasing the number of qubits enhances the probability of measuring the correct eigenphase, as the probability distribution becomes sharper. However, at specific qubit counts (e.g., 10 qubits), the precision might temporarily deteriorate due to numerical artifacts, algorithmic errors, or minor simulation effects.

Thus, from **11 qubits onward**, the results show greater stability, demonstrating that increasing the number of qubits significantly enhances the accuracy of phase estimation.

Figure 2: Results of (QPE) Circuit for theta = 3/5.