FIN 460 – Dynamic Asset Pricing Theory Project

Aziz Chaouachi

Khawla Guesmi

e. 2 slides to present final model, key results and conclusions

Oussema Belkadhi

Amal Ben Hariz

Instructor: Dr Eymen Errais / Dr. Bechir Bouzid

Literature Review

The study of foreign exchange (FX) rates is crucial due to their impact on global financial markets, trade, and economic policy. With the advent of advanced statistical and machine learning models, the ability to predict and understand FX rate dynamics has significantly improved. Two such models are the Autoregressive Integrated Moving Average with Exogenous Inputs (ARIMAX) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. This literature review explores recent advancements in the application of these models for FX rate modeling.

ARIMAX

The ARIMAX model extends the traditional ARIMA framework by including exogenous variables, which are external factors influencing the time series. This model is particularly useful in FX rate modeling where economic indicators such as interest rates, inflation, and GDP can be significant predictors.

Effectiveness in FX Rate Prediction: Tay and Linn (2001) demonstrated that ARIMAX models, incorporating macroeconomic variables, outperform traditional ARIMA models in FX rate prediction. This improvement is attributed to the model's ability to account for external economic conditions.

GARCH

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model has been extensively employed to analyze volatility in various financial markets, including FX rates. For instance, researchers like Gronwald (2014) have empirically analyzed FX rates using an autoregressive jump-intensity GARCH model, which has been widely accepted in the empirical finance community. This model effectively captures the volatility dynamics inherent in FX rates, shedding light on the market's behavior and assisting in risk management strategies.

Volatility Clustering: One of the key strengths of the GARCH model is its ability to capture volatility clustering, where periods of high volatility tend to be followed by more high volatility, and low volatility follows low volatility. Engle and Rangel (2008) demonstrated that GARCH models are adept at modeling this phenomenon in FX rates, making them invaluable for understanding market behavior and for applications in risk management and derivative pricing.

Data description

Mean EURTND=2.9305
Mean USDTND=2.5976
The average EURTND over the period 29/05/2014 to 29/05/2024 is 2.9305 while The average USDTND over the period 29/05/2014 to 29/05/2024 is 2.5976 This gives insights about the central tendency of the database but issensitive to outliers.

Median EURTND=3.162
Median USDTND=2.747

Over the period 29/05/2014 to 29/05/2024, in 50% of the days,EURTND is higher than 3.162 and vise versa ,USDTND is higher than 2.747 and vise versa

Skewness EURTND=-0.7
Skewness USDTND=-0.49

The distribution is slightly skewed to the right, but as the skewness is close to 0, the distribution is relatively symmetrical but with a slightly longer tail on the right side.

Kurtosis EURTND=-1.17
Kurtosis USDTND=-0.97

The kurtosis is <3 which indicates a distribution more peaked and has more values clustered around the mean than a normal distribution.

STDEV EURTND=0.4403
STDEV USDTND=0.4349

The average deviation from the mean is equal to 0.4403 for EURTND and 0.434 for USDTND. which is relatively high considering an average rate of 2.9302 and 2.5976 respectively. This indicates a high volatility in USDTND and EURTND which can make it challengin to forecast future prices.

. Model Presentation

Arimax

To model the returns we used ARIMAX, The ARIMAX (AutoRegressive Integrated Moving Average with eXogenous variables) model is an extension of the ARIMA model that incorporates exogenous variables (predictors) into the time series analysis. This model is particularly useful for forecasting when external factors, such as economic indicators, weather conditions, or other relevant variables, are believed to impact the variable of interest. The ARIMAX model combines the autoregressive (AR) component, which uses the dependency between an observation and several lagged observations, the integrated (I) component, which makes the time series stationary through differencing, and the moving average (MA) component, which models the relationship between an observation and a residual error from a moving average model applied to lagged observations. By including exogenous variables, the ARIMAX model can account for additional information outside the primary time series, potentially improving the accuracy and robustness of forecasts.

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} + \beta_1 X_{1t} + \beta_2 X_{2t} + \cdots + \beta_k X_{kt} + \epsilon_t$$

- Y_t is the dependent variable at time t.
- ϕ are the coefficients for the autoregressive terms.
- θ are the coefficients for the moving average terms.
- ϵ_t is the error term at time t.
- X_{kt} are the exogenous variables at time t.
- β are the coefficients for the exogenous variables.

GARCH

To model the volatility, we used the GARCH model The GARCH model assumes that the variance of a financial time series is a function of its past variance and the past squared error terms. It can be represented as GARCH(p,q), where p and q are the order of the autoregressive and moving average components, respectively.

The GARCH model estimates the conditional variance of the time series by regressing the squared residuals of the mean equation on the past variances and squared residuals. The GARCH model assumes that the past volatility affects the current volatility, and the past squared error terms are used to capture any remaining volatility that is not explained by the past variances.

The GARCH model is useful for modeling the volatility of financial time series, as it can capture the persistence and clustering of volatility shocks that areoften observed in financial markets. It is widely used in finance for risk management and portfolio optimization, as it can provide accurate estimates of future volatility that can be used to construct optimal portfolios.

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

- σ_t^2 is the conditional variance at time t.
- ω is a constant term.
- α_i are the coefficients for the lagged squared residuals (ARCH terms)
- ϵ_t are the residuals (errors) at time t.

Data Transformation

1) Loading libraries, importing and declaring data as time series

```
library(quantmod)
library(forecast)
library(tseries)
library(dynlm)
library(readxl)
library (ggplot2)
library(TSA)
library(fGarch)
```

```
# Install and load required libraries
install.packages("quantmod")
install.packages("forecast")
install.packages("tseries")
install.packages("dynlm")
install.packages("TSA")
install.packages("fGarch")
```

2)Stationarity testing using KPSS:

```
Warning message:
In kpss.test(data) : p-value smaller than printed p-value
>
```

---> The data is not stationary

3) Differencing and confirming stationarity:

```
> #Differentiation
> ndiffs(data)
[1] 1
>
```

--->Only 1 differencing is needed

```
> #Seasonality
> nsdiffs(data)
[1] 0
```

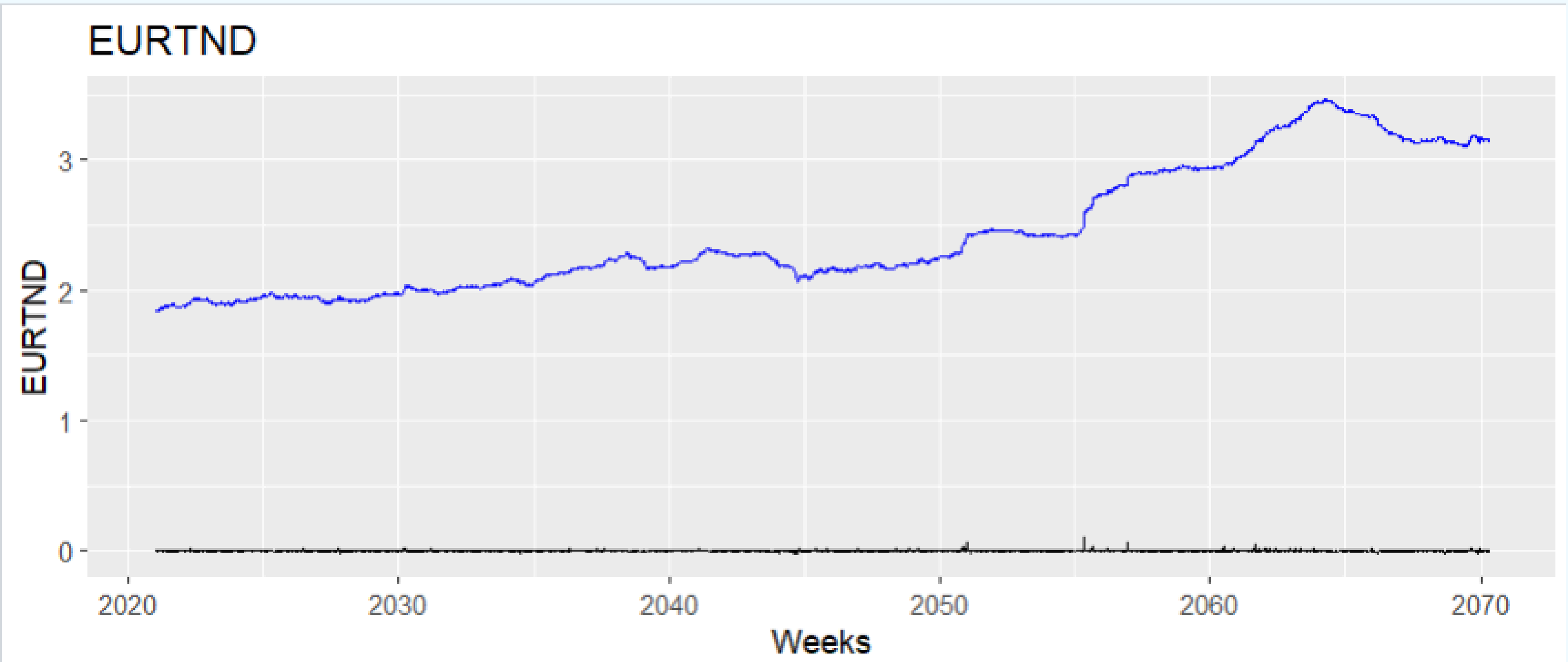
--->no seasonality in the data

```
Warning message:
In kpss.test(data1) : p-value greater than printed p-value
> adf.test (data1)

        Augmented Dickey-Fuller Test

data:  data1
Dickey-Fuller = -10.857, Lag order = 13, p-value = 0.01
alternative hypothesis: stationary
```

--->The data is now stationary

4) Plotting the original and differenced data

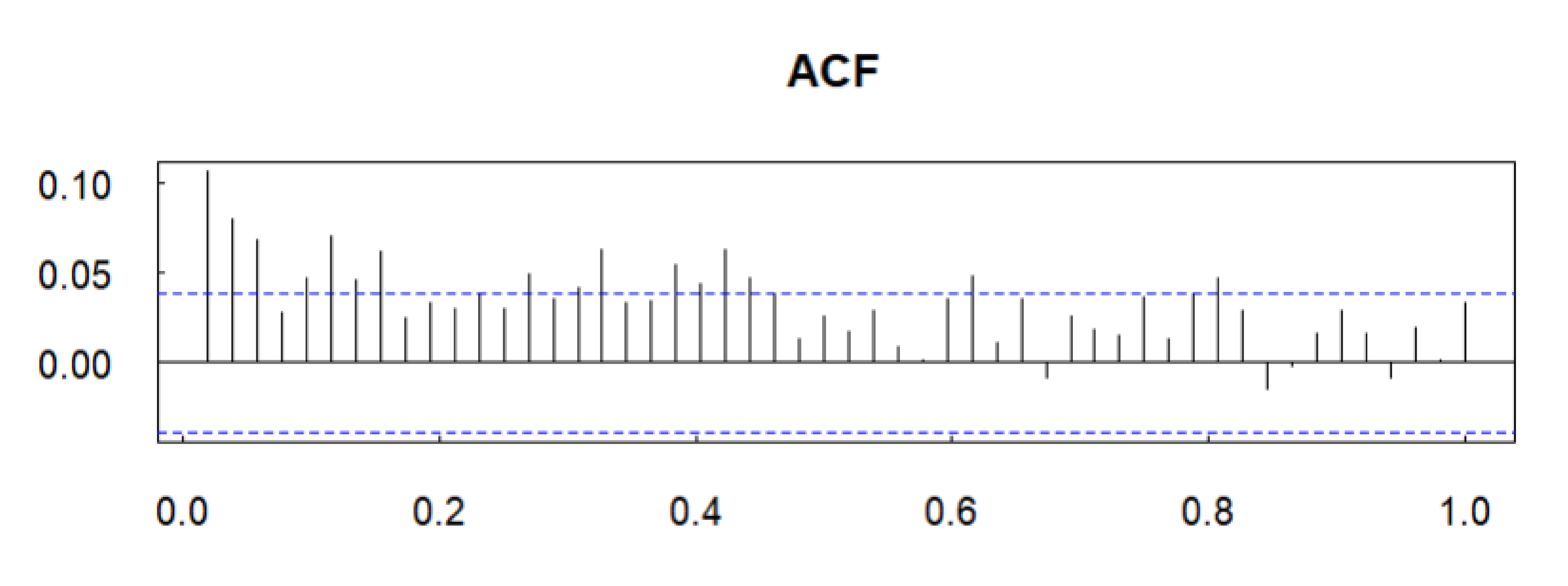
```
#Plot
autoplot(data) +
  ggtitle("EURTND") +
  ylab("EURTND") +
  xlab("Date")
```

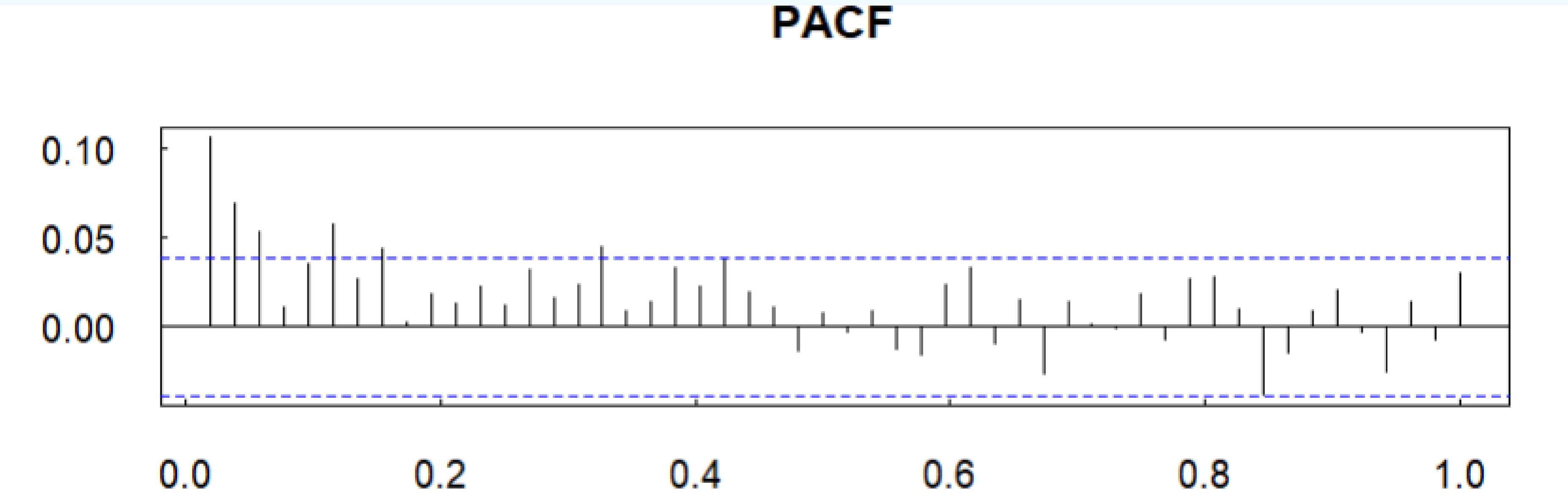


ARIMAX Models

5)ACF and PACF

```
#ACF
acf(data1,lag=52,tck=0.02,xlab="",ylab="",main="ACF",las=1)
#PACF
pacf(data1,lag=52,tck=.02,xlab="",ylab="",main="PACF",las=1)
```





6) Short-process candidate models

```
fit <- arimax(data, order = c(0, 0, 1), seasonal = list(order = c(1, 0, 0)),
           xreg = exogenous_data, fixed=c(0.8501, 0.9545, 1.1392987, -3.9146052, 0.0118908, 4.9917415))
summary(fit)
fit2 <- arimax(data, order = c(0, 1, 1), seasonal = list(order = c(1, 0, 0)),fixed=c(0.8501, 0.9545, -3.9146052, 0.0118908, 4.99174
           xreg = exogenous_data)
summary(fit2)
fit3 <- arimax(data, order = c(0, 1, 0), seasonal = list(order = c(1, 0, 0)),fixed=c(0.9545,-3.9146052, 0.0118908, 4.9917415),
           xreg = exogenous_data)
summary(fit3)
```

7) Long-process candidate models

```
fit4 <- arimax(data, order = c(12, 1, 0),seasonal = list(order = c(1, 0, 0)),fixed=c(0.0388, 0.0216, 0.0276, -0.0067, -0.0265 ,0.0425 ,0.0200 ,0.0131
           ,0.0026 ,0.0131 ,0.0177, 0.0359, 0.0468,-3.9146052, 0.0118908, 4.9917415),xreg = exogenous_data)
summary(fit4)
fit5 <- arimax(data, order = c(0, 1, 13), seasonal = list(order = c(1, 0, 0)),fixed=c(0.0398,0.0236 ,0.0307 ,-0.0053,-0.0300,0.0403 ,0.0203 ,0.0129
           ,0.0075 ,0.0091 ,0.0126 ,0.0401 ,-0.0131 ,0.0471 , -3.9146052, 0.0118908, 4.9917415),
           xreg = exogenous_data)
summary(fit5)
fit6 <- arimax(data, order = c(0, 1, 12), seasonal = list(order = c(1, 0, 0)),fixed=c(0.0388, 0.0216, 0.0276, -0.0067, -0.0265 ,0.0425 ,0.0200 ,0.0131
           ,0.0026 ,0.0131 ,0.0177, 0.0359, 0.0468,-3.9146052, 0.0118908, 4.9917415),
           xreg = exogenous_data)
summary(fit6)
```

Arimax(0,1,0):Best short-process model because lowest AIC

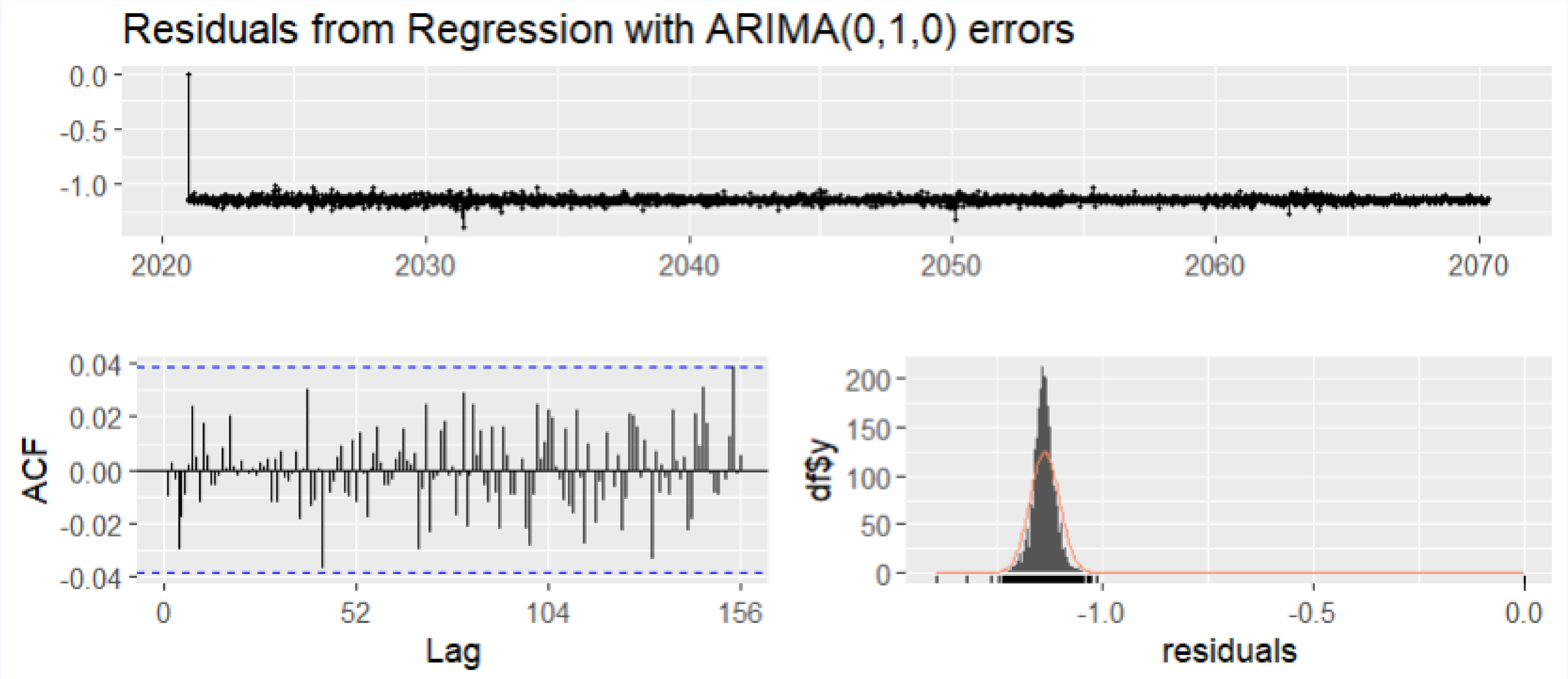
Model	Akaike Informtion Criterion
ARIMAX(0,0,1)	-4758.49
ARIMAX(0,1,1)	-6120.06
ARIMAX(0,1,0)	-9541.4
ARIMAX(12,1,0)	-11080.34
ARIMAX(0,1,13)	-11081.81
ARIMAX(0,1,12)	-11081.36

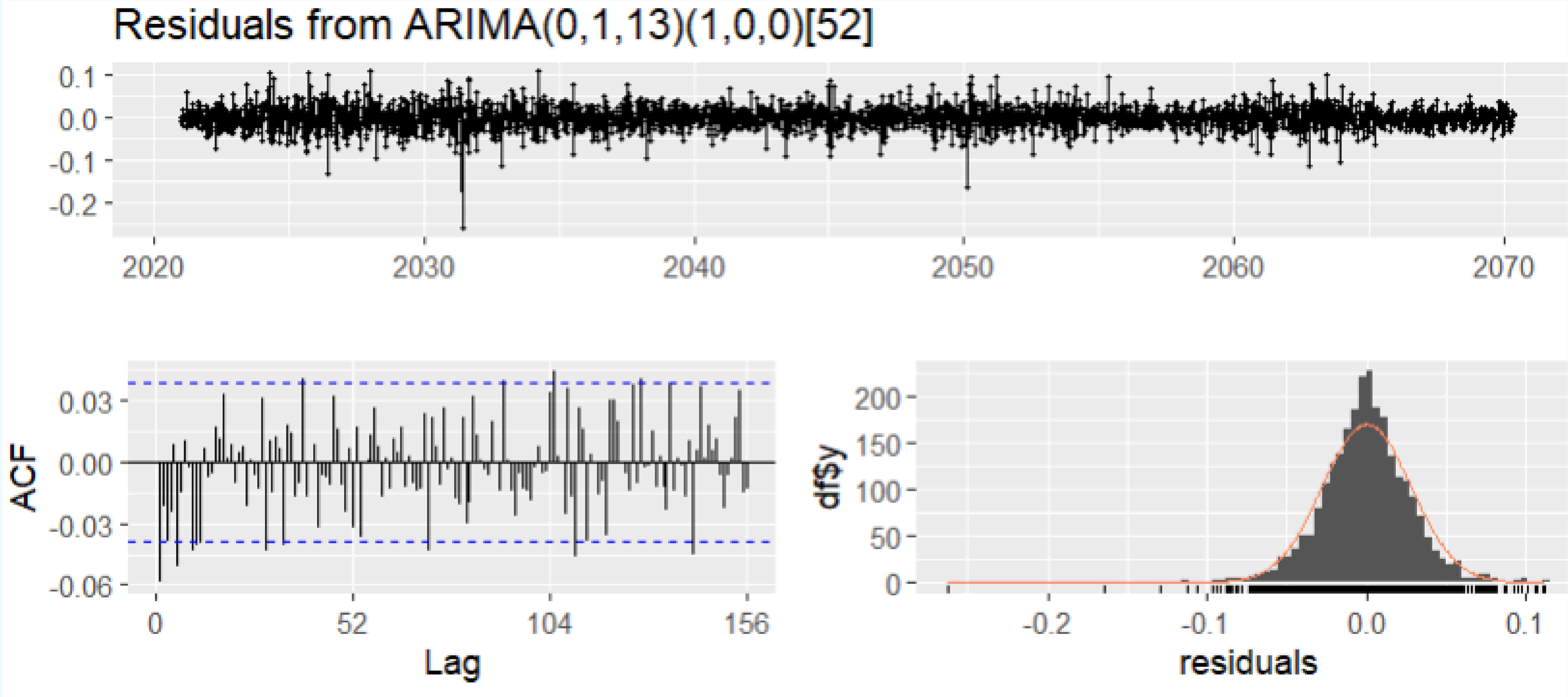
Arimax(0,1,13):Best long-process model because lowest AIC

8)Auto.arima

```
> arimax_model <- auto.arima(data, xreg = as.matrix(exogenous_data),fixed=c(1.1392987, -3.9146052, 0.0118908, 4.9917415))
> summary(arimax_model)
Series: data
Regression with ARIMA(0,1,0) errors

Coefficients:
   drift    GBPUSD    JPYUSD    EURUSD
  1.1393   -3.9146    0.0119    4.9917
```

8) Checking residuals:



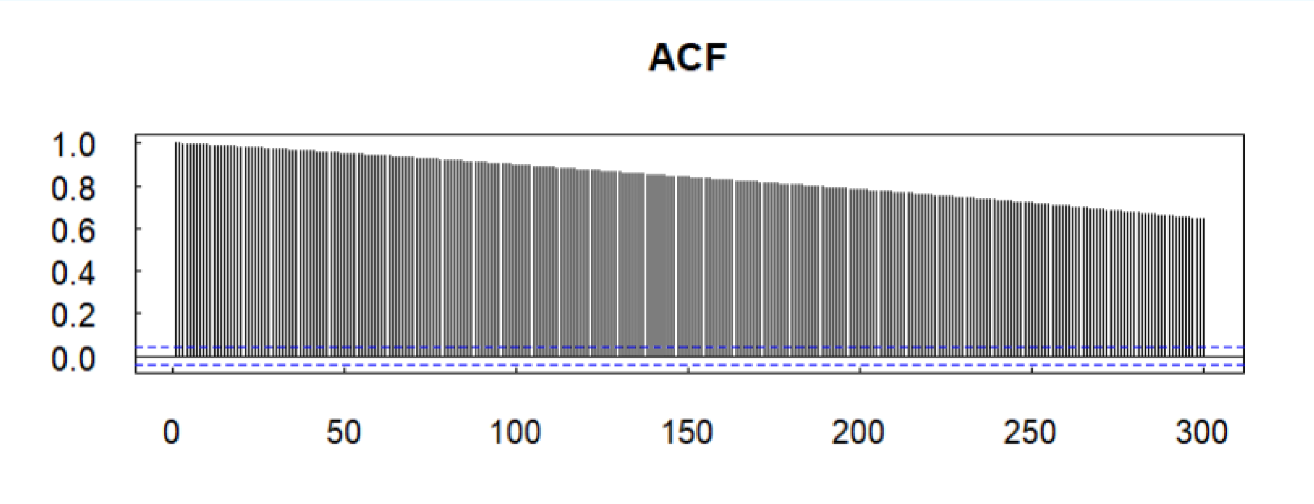


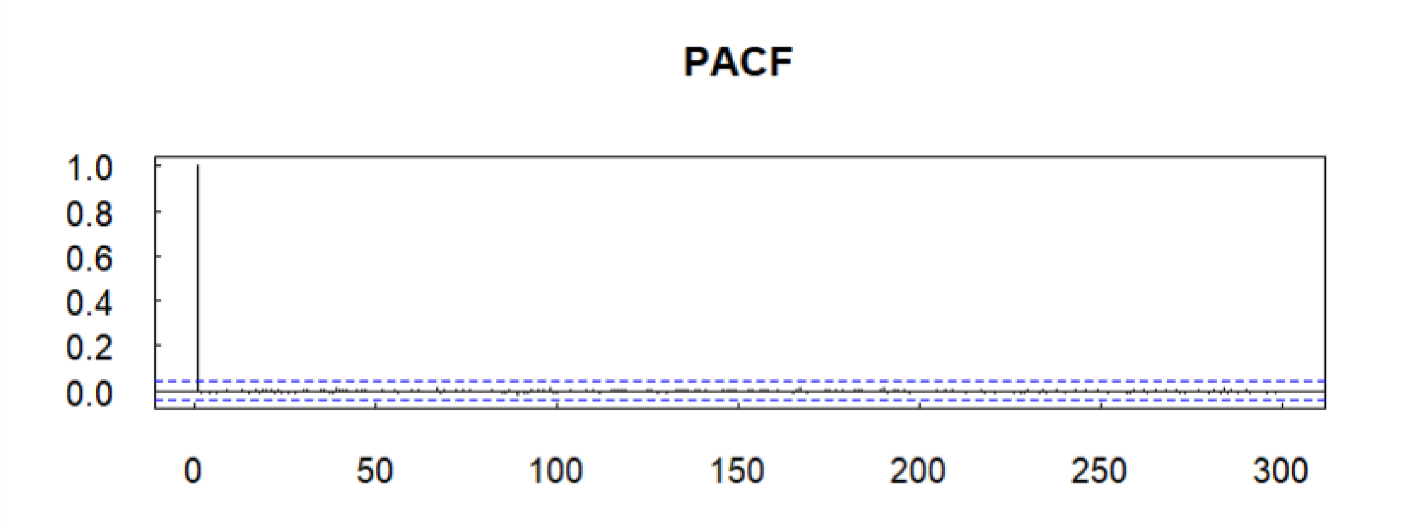
Both models have non-correlated residuals so we can move on to checking if there is an ARCH effect

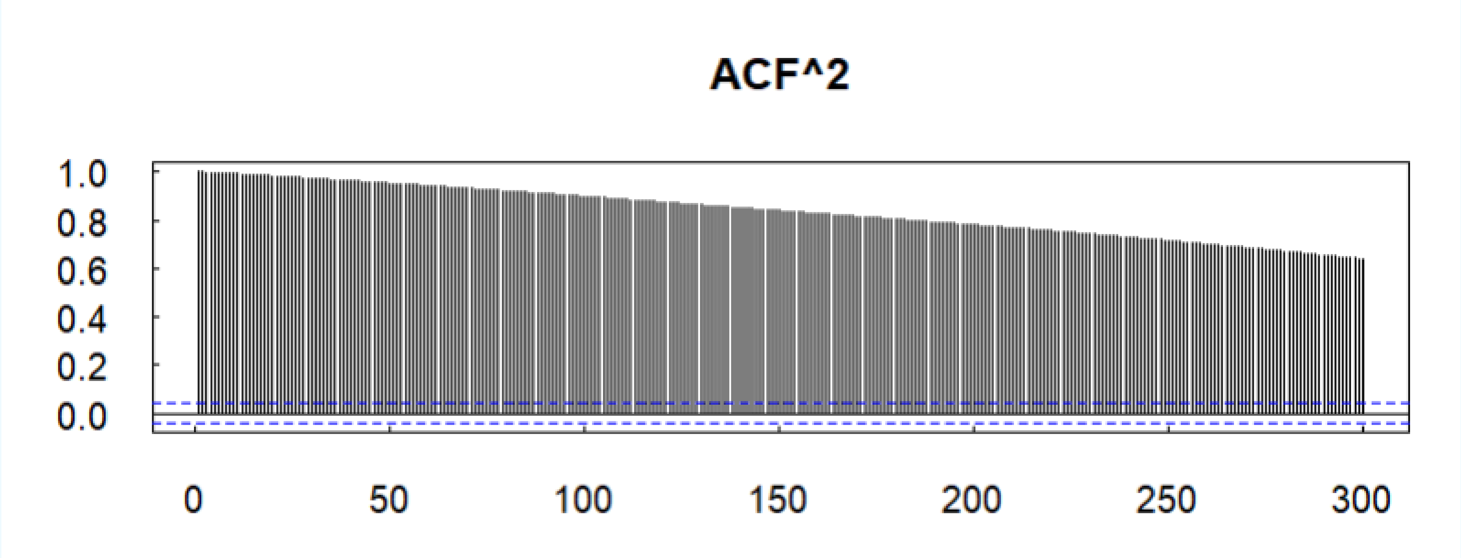
9)Checking ARCH Effects:

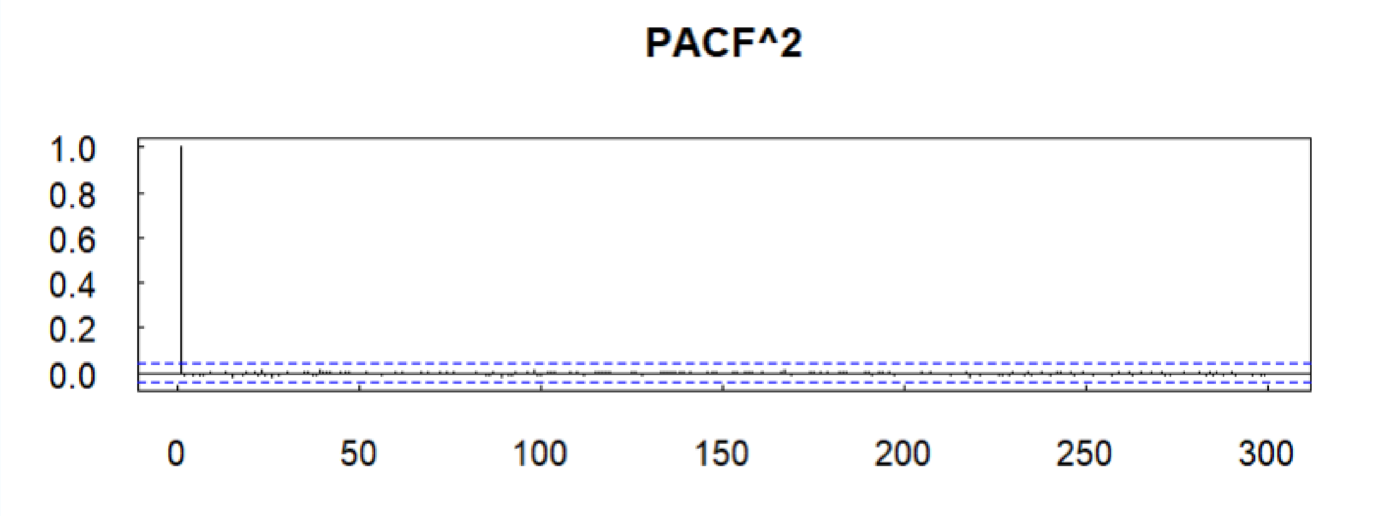
```
EURTND <- log(1+dataset$EURTND)
#ACF
acf(EURTND,lag=300,tck=.02,xlab="",ylab="",main="ACF",las=1)
#PACF
pacf(EURTND,lag=300,tck=.02,xlab="",ylab="",main="PACF",las=1)

#ACF^2
acf(EURTND^2,lag=300,tck=.02,xlab="",ylab="",main="ACF^2",las=1)
#PACF^2
pacf(EURTND^2,lag=300,tck=.02,xlab="",ylab="",main="PACF^2",las=1)

#Testing correlation of residuals
Box.test(EURTND,lag=300,type="Ljung-Box")
Box.test(EURTND^2,lag=300,type="Ljung-Box")
```

According to ACF, PACF and Ljung-Box test, there is significant dependence between residuals so we will use a GARCH model to predict the volatility of returns









10) Fitting short-process GARCH

```
m1=garchFit(~arma(0,0)+garch(1,2),data=data,trace=F)
summary(m1)
m2=garchFit(~arma(0,0)+garch(1,1),data=data,trace=F)
summary(m2)
m3=garchFit(~arma(0,0)+garch(2,1),data=data,trace=F)
summary(m3)
```

11) Fitting long-process GARCH

```
m4=garchFit(~arma(0,13)+garch(10,6),data=data,trace=F)
summary(m4)
m5= garchFit(~arma(0,13)+garch(26,1),data=data,trace=F)
summary(m5)
m6=garchFit(~arma(0,13)+garch(6,20),data=data,trace=F)
summary(m6)
```

Model	Akaike Informtion Criterion
GARCH(1,2)	-0.5437020
GARCH(1,1)	-0.07292372
GARCH(2,1)	-0.4978642
GARCH(10,6)	-4.21509
GARCH(26,1)	-4.398042
GARCH(6,20)	-4.396131

In conclusion, the best short-process model is GARCH(1,2) and the best long-process model is GARCH(26,1)

Forecasting

12) Forecasting currency pairs for the next 4 weeks:

```
#Forecast future values
future_exogenous1 <- forecast(auto.arima(exogenous_data[1]), h = 30)
future_exogenous2 <- forecast(auto.arima(exogenous_data[2]), h = 30)
future_exogenous3 <- forecast(auto.arima(exogenous_data[3]), h = 30)
future_exogenous_data<- cbind(future_exogenous1,future_exogenous2,future_exogenous3)
forecasts <- forecast(arimax_model, xreg = as.numeric(future_exogenous_data), h = 30)# Adjust 'h' for the desired forecast horizon
plot(forecasts, main = "EURTND Forecast", ylab = "EURTND", xlab = "Date")
print(forecasts)
accuracy(forecasts)
```

Key Results & Conclusions