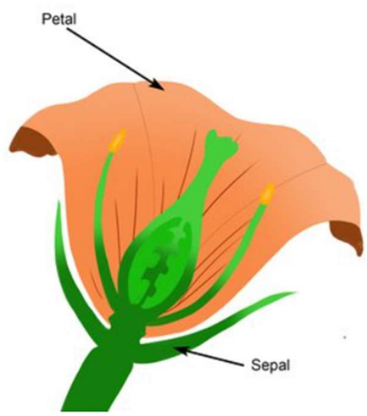
IC 272 - Data Science III

Assignment 2: Principal Component Analysis (PCA) and K-Nearest Neighbour (KNN)

Deadline: September 15, 2024: 23.59 Hr.

Dataset Description:

You are given a CSV file "**iris.csv**" containing the measurements of three types of Iris flowers:



Independent variables/ Attributes/ Features:

i. "`SepalLengthCm`": Sepal length in cm.

ii. " `SepalWidthCm`": Sepal width in cm.

iii. "`PetalLengthCm`": Petal length in cm.

iv. "`PetalWidthCm`": Petal width in cm.

Dependent variable/ Target Attribute/ Class:

i. "`species`": Type of the flower corresponding to a set of measurements.

Problem Statements:

I. Read the file using Pandas and do the following:

a. Extract the attributes as one matrix and the target attribute as an array, called **"true class labels"** or **y**.

b. Replace the outliers (if at all any) in the attributes with the median of the respective attributes. Let's call this outlier corrected data as **X**.

c. Reduce the dimension of X through Principal Component Analysis (PCA) by implementing Algorithm 1 in Python:

Algorithm 1 Data Dimension Reduction using Principal Component Analysis (PCA)

Require: Data matrix $\mathbf{X}$ having $N = 150$ number of samples. Each sample is of dimension $d = 4$.

Ensure: The reduced-dimensional data - dimension is reduced from $d = 4$ to $l = 2$.

1: Subtract the respective mean from each attribute (dimension) in data samples (tuples). Lets call the mean subtracted data as $\tilde{\mathbf{X}}$.

2: Compute the correlation matrix $\mathbf{C} = \tilde{\mathbf{X}}^T\tilde{\mathbf{X}}$.

3: Perform eigen analysis of $\mathbf{C}$ using `numpy.linalg.eig`.

4: Order the eigenvalues (λ's) of $\mathbf{C}$ such that $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_4$.

5: Consider $l = 2$ eigenvectors ($\mathbf{q}$'s) corresponding to the l significant eigenvalues and create a matrix $\mathbf{Q}$ of size 2×4.

6: Take the original data $\mathbf{X}$. Project each original sample $\mathbf{x}_i,\ i = 1, 2, ..., 150$ onto each of the l directions performing the following operation: $\hat{\mathbf{X}} = \mathbf{Q}\mathbf{X}$.

7: Return the dimension-reduced version $\hat{\mathbf{X}} \in \mathbb{R}^{150\times 2}$ of the original data $\mathbf{X}$.

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d. Draw a scatter plot of the dimension-reduced data generated in QI.(c). Superimpose the eigen directions with proper scaling (use in-built plotting functions).

e. Reconstruct the original data from dimension-reduced data generated in QI.(c) by implementing Algorithm 2 in Python:

Algorithm 2 Reconstruction of Data from Dimension-reduced Data

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Require: 1. Dimension-reduced data matrix $\hat{\mathbf{X}}$ having $N = 150$ number of samples, each of dimension $l = 2$.

2. Eigenvector matrix $\mathbf{Q}$.

Ensure: The reconstructed original data $\dot{\mathbf{X}} \in \mathbb{R}^{150\times 4}$.

1: Approximate each original sample $\mathbf{x}_i,\ i = 1, 2, ...150$ by performing the following operation: $\dot{\mathbf{X}} = \hat{\mathbf{X}}\mathbf{Q}$.

2: Return the reconstructed original data $\dot{\mathbf{X}}$ of size 150×4.

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f. Compute the RMSE between the original data (**X**) and its reconstruction (**Ẋ**) using your implementation in Assignment 1.

II. Consider the dimension-reduced data generated in QI.(c) and do the following:

a. Build a K-Nearest Neighbour (KNN) classifier by implementing Algorithm 3 in Python:

Algorithm 3 K-Nearest Neighbour (KNN)

Require: 1. Dimension-reduced data matrix $\hat{\mathbf{X}}$ having $N = 150$ number of samples, each of dimension $l = 2$.

2. True class label vector $\mathbf{y}$.

Ensure: A matrix containing the true class label (actual) and the predicted class label (predicted) of each test sample $\mathbf{x}_i^{test}$.

Split data $\hat{\mathbf{X}}$ into training set and test set using `sklearn.model_selection.train_test_split(`$\hat{\mathbf{X}}$`,y ,random_state=104, test_size=0.20, shuffle=True)`.

for each test sample $\mathbf{X}_i^{test}$ in the test set **do**

  for each training sample $\mathbf{X}_j^{train}$ in the training set **do**

    Compute the Euclidean distance between $\mathbf{X}_i^{test}$ and $\mathbf{X}_j^{train}$ as

$$dist = \sqrt{\sum_{l=1}^{2} (\mathbf{X}_{il}^{test} - \mathbf{X}_{jl}^{train})^2}$$

  end for

  Sort the training examples in ascending order of the distance to $\mathbf{X}_i^{test}$ (use in-built sorting function).

  Choose the first $K = 5$ examples in the sorted list.

  Assign $\mathbf{X}_i^{test}$ to the most common class among its 5 neighbours.

  Save the actual and predicted class labels of $\mathbf{X}_i^{test}$.

end for

Return the matrix of actual and predicted class labels of 150×2 samples.

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b. Compute the confusion matrix using the function from sklearn.metrics. Create an interpretable visual display of the confusion matrix using the function from sklearn.metrics and plot it using matplotlib.pyplot.