Problem 1a

We are given the differential equation

$$\frac{\mathrm{dP}}{\mathrm{dt}} = -4P^4 + 4P^3 - 5P^2 + 2P,$$

which can be factorised as

$$\frac{\mathrm{dP}}{\mathrm{dt}} = P(-4P^3 + 4P^2 - 5P^1 + 2) = -P(P-2)(P-1)^2,$$

from the facrorised form we can tell our roots are

$$P = 0, P = 1, P = 2,$$

this is verified using the roots() function

```
%de = @(P) -P^4+4*P^3-5*P^2+2*P;
%de(3)
p = [-1; 4; -5; 2; 0];
p = roots(p)
```

```
p = 4×1 complex
   0.0000 + 0.0000i
   2.0000 + 0.0000i
   1.0000 + 0.0000i
   1.0000 - 0.0000i
```

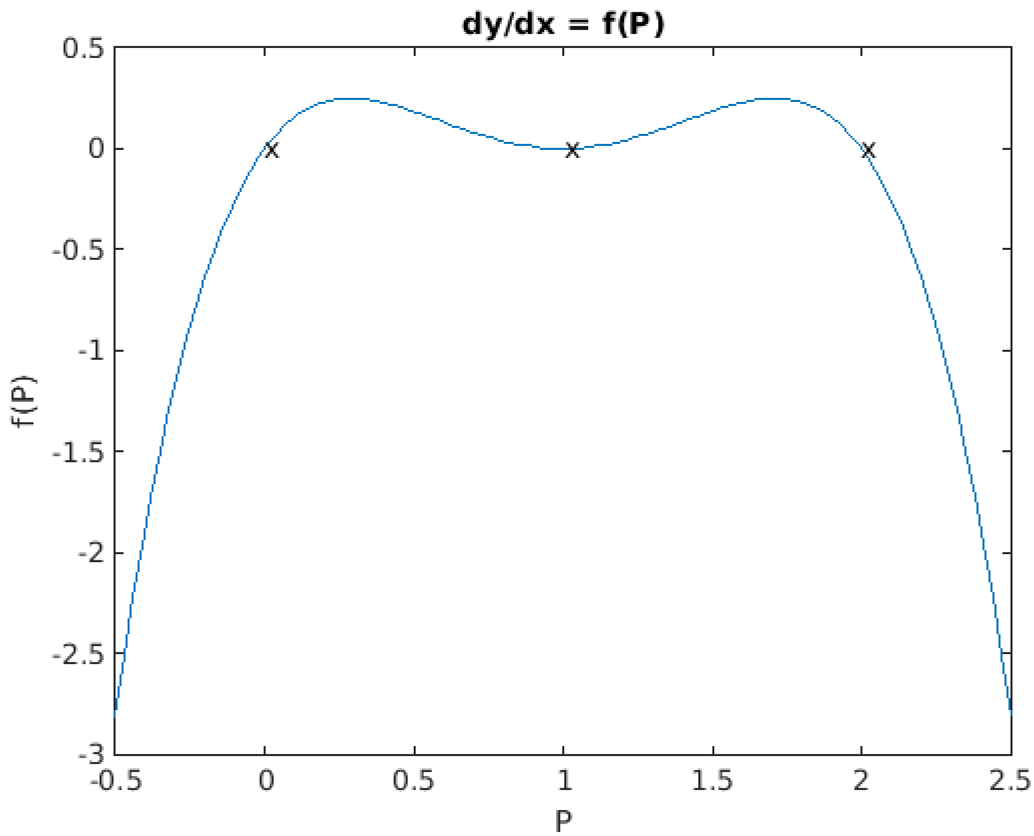
Problem 1b

Increasing or decreasing

Inorder to tell where the solution is inc/dec we need to tell were the DE is positive or negative. For this we will plot the DE and see which regions are (+) and which regions are (-)

```
de = @(P) -P^4 + 4*P^3 - 5*P^2 + 2*P; % original DE
x = linspace(-0.5, 2.5, 101);
y = [];
for i = 1:length(x)
    y(i) = de(x(i));
end

plot(x, y)
title("dy/dx = f(P)")
xlabel('P');
ylabel('f(P)');
text(0,0, "x")
text(2,0, "x")
text(1,0, "x")
```



From our plot we can see the follwing regions:

- P < 0: (-) => decreasing
- 0 < P < 1: (+) => increasing
- 1 < P < 2: (+) => increasing
- 2 < P   : (-) => decreasing

Concavity

Inorder to tell the concavity of our solution we first need to determine the 2nd derivative $\frac{d^2P}{\mathrm{dt}^2}$

$$\frac{d^2P}{\mathrm{dt}^2} = \frac{d}{\mathrm{dt}}\frac{\mathrm{dP}}{\mathrm{dt}} = \frac{d}{\mathrm{dt}}(-4P^4 + 4P^3 - 5P^2 + 2P)$$
$$= -16P^3\frac{\mathrm{dP}}{\mathrm{dt}} + 12P^2\frac{\mathrm{dP}}{\mathrm{dt}} - 10P\frac{\mathrm{dP}}{\mathrm{dt}} + 2\frac{\mathrm{dP}}{\mathrm{dt}}$$
$$= -16P^3 f(P) + 12P^2 f(P) - 10\mathrm{Pf}(P) + 2f(P)$$
$$= 64P^7 - 112P^6 + 168P^5 - 140P^4 + 82P^3 - 30P^2 + 4P$$

Now that we have $\frac{d^2P}{\mathrm{dt}^2}$ we need to calculate the roots

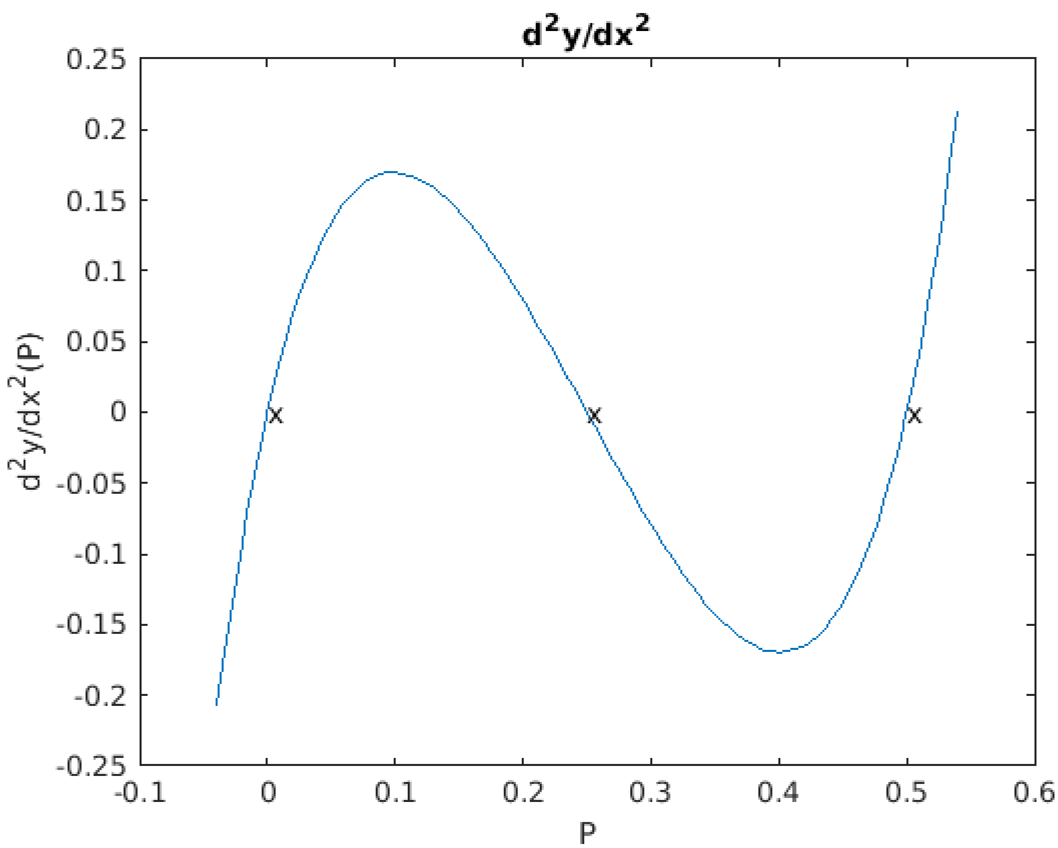
```
dde = @(P) 64*P^7 - 112*P^6 + 168*P^5 - 140*P^4 + 82*P^3 - 30*P^2 + 4*P; % original DE
p = [64; -112; 168; -140; 82; -30; 4; 0];
p = roots(p)
```

```
p = 7×1 complex
   0.0000 + 0.0000i
   0.2500 + 0.9682i
   0.2500 - 0.9682i
   0.2500 + 0.6614i
   0.2500 - 0.6614i
   0.5000 + 0.0000i
   0.2500 + 0.0000i
```

Now that we have the roots lets figure out our regions of concavity from a plot

```
x = linspace(-0.039, 0.54, 101);
y = [];
for i = 1:length(x)
    y(i) = dde(x(i));
end

plot(x, y)
title("d^2y/dx^2")
xlabel('P');
ylabel('d^2y/dx^2(P)');
text(0,0, "x")
text(0.25,0, "x")
text(0.5,0, "x")
```



From our plot we can see the follwing regions:

- $P < 0.00$: (-) => concave down ∩
- $0.00 < P < 0.25$: (+) => concave up ∪
- $0.25 < P < 0.50$: (-) => concave down ∩
- $0.50 < P$: (+) => concave up ∪

Problem 1c

Plot

Now that we have all of our data points lets attempt to plot the approximate solution family to our original DE

①

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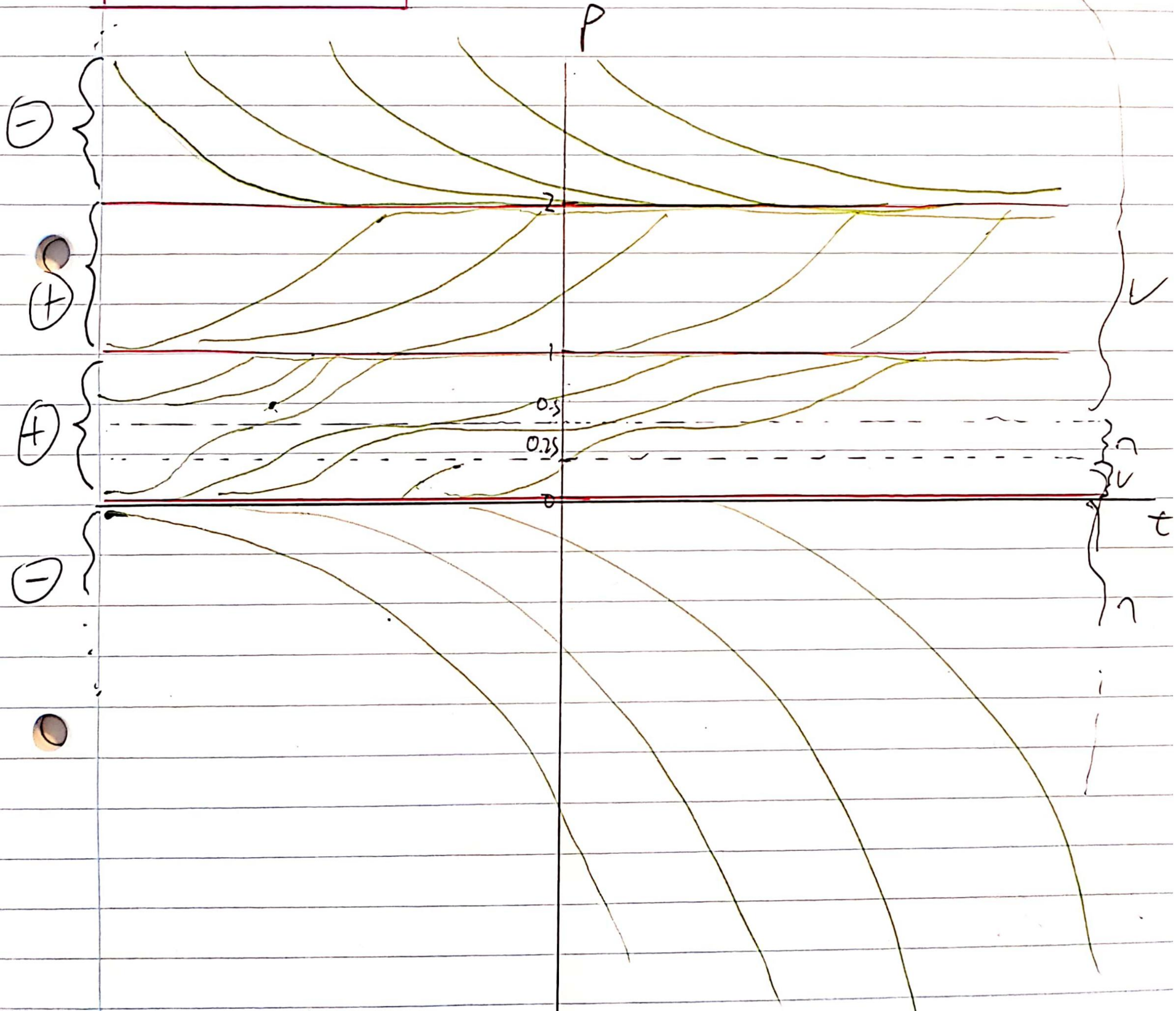
Problem 1.C



Problem 1d

Using the results of my scetch in the previous question I can now classify all of the critical points.

(fixme): If a point tends away from a critical point but still end up convering at another critical point are they still concidered unstable?

- P=0: This is an unstable point as both points above and bellow this point result in solutions being repelled away from P=0. The points bellow tend to $-\infty$ and the points above all tend to P=1
- P=1: This is a semi-stable point. The points bellow it tend to P=1 while the points above it tend to P=2
- P=2: This is a stable point. Both the points above and bellow tend to P=2

Problem 1e

FIXME:

- I think this question is wrong as the question is kind of posturing that the limits will be different for these 2 particular solutions
- I would like to use matlab's baked in limit calculation function

P(0) = 0.25

For this problem we are looking at a specific solution to

$$\frac{\mathrm{dP}}{\mathrm{dt}} = -4P^4 + 4P^3 - 5P^2 + 2P + c$$

Where

$$P(0) = -4(0)^4 + 4(0)^3 - 5(0)^2 + 2(0) + c = c = 0.25$$

Therefore the specific solution is

$$\frac{\mathrm{dP}}{\mathrm{dt}} = -4P^4 + 4P^3 - 5P^2 + 2P + 0.25$$

$$\lim_{t\to\infty}(P) = -\infty$$

P(0) = 1.4

For this problem we are looking at a specific solution to

$$\frac{\mathrm{dP}}{\mathrm{dt}} = -4P^4 + 4P^3 - 5P^2 + 2P + c$$

Where

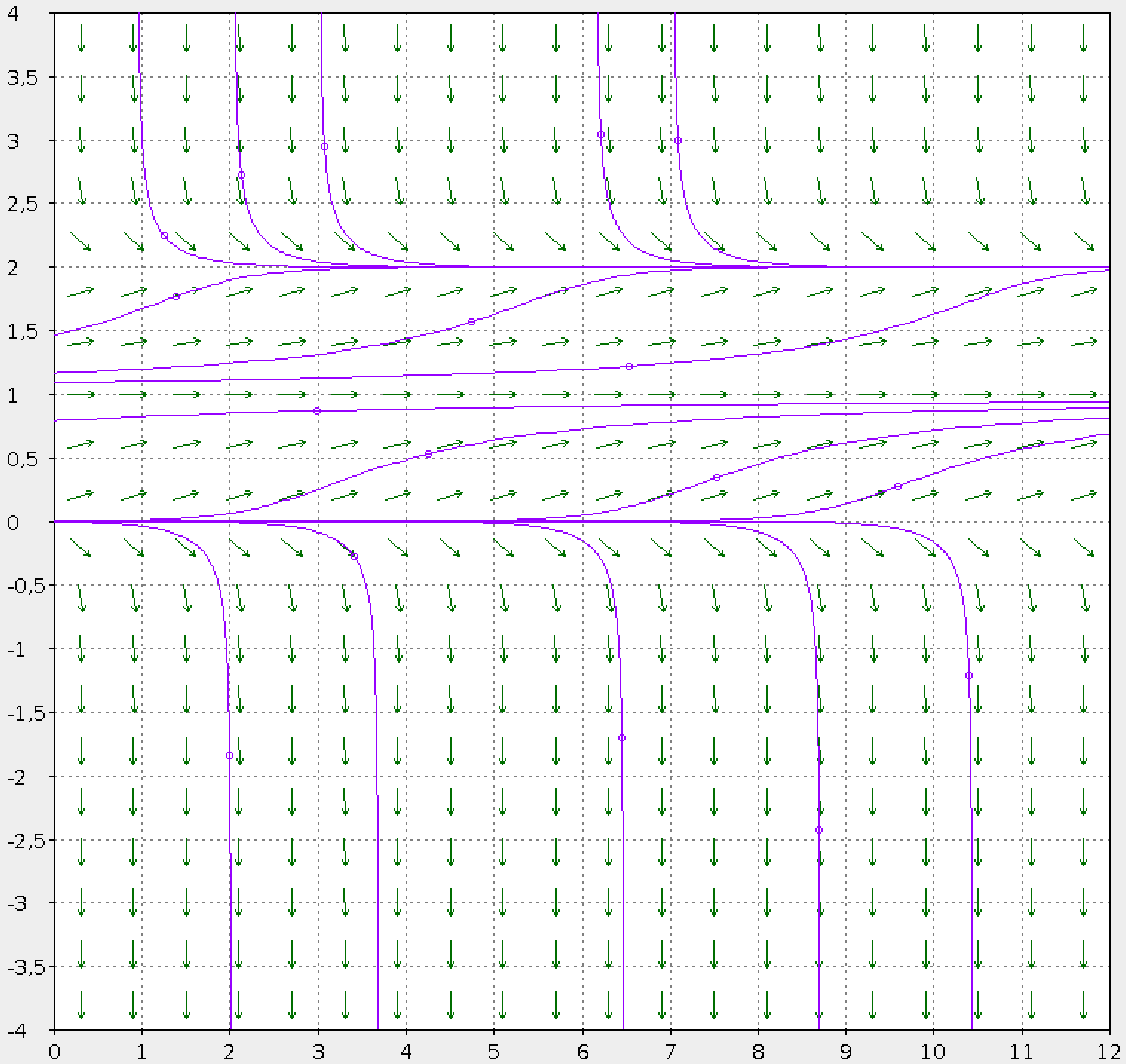
$$P(0) = -4(0)^4 + 4(0)^3 - 5(0)^2 + 2(0) + c = c = 1.4$$

Therefore the specific solution is

$$\frac{dP}{dt} = -4P^4 + 4P^3 - 5P^2 + 2P + 1.4$$

$$\lim_{t\to\infty}(P) = -\infty$$

Problem 1f



Above is the output `dfield.jar`. `Which very nicely coroberates my results.`