Probleem 1: Gestel die aantal visse in 'n vissery kan met die volgende aangepaste logistiese model beskryf word:

Problem 1: Suppose the number of fish in a fishery can be described by the following modified logistic model:

$$\frac{dP}{dt} = 5P - P^2 - 4, \quad P(0) = P_0.$$

Hier word $P(t)$, die aantal visse op tyd t (in jare), in duisende gemeet. P_0 is die aanvanklike aantal visse (ook in duisende). Die (-4)-term dui daarop dat die visse teen 'n konstante tempo van 4 duisend per jaar ge-oes word.

(a) Gestel die vispopulasie word ook aangevul teen 'n konstante tempo van 10 duisend per jaar. Pas die DV dienooreenkomstig aan.

(b) Met die aanpassing van deel (a), bepaal of die populasie in 'n eindige tydperk kan uitsterf, <u>sonder</u> om enige DV op te los.

Wenk: analiseer die DV kwalitatief.

(c) Los die DV van deel (a) op, en gebruik die oplossing om aan te toon dat, indien $P_0 > 0$, die visse nooit sal uitsterf nie.

(d) Wat is die dravermoë van die meer?

Here $P(t)$, the number of fish at time t (in years), is measured in thousands. P_0 is the initial number of fish (also in thousands). The (-4) term indicates that the fish are harvested at a constant rate of 4 thousand per year.

(a) Suppose the fish population is also replenished at a constant rate of 10 thousand per year. Modify the DE accordingly.

(b) With the modification of part (a), determine whether the population can go extinct in a finite period of time, <u>without</u> solving any DE.

Hint: analyze the DE qualitatively.

(c) Solve the DE from part (a), and use the solution to show that, if $P_0 > 0$, the fish will never go extinct.

(d) What is the carrying capacity of the lake?

Probleem 2: Zill & Wright: bl. 101, nr. 9:

Problem 2: Zill & Wright: p. 101, nr. 9:

9. Two chemicals A and B are combined to form a chemical C. The rate, or velocity, of the reaction is proportional to the product of the instantaneous amounts of A and B not converted to chemical C. Initially, there are 40 grams of A and 50 grams of B, and for each gram of B, 2 grams of A is used. It is observed that 10 grams of C is formed in 5 minutes. How much is formed in 20 minutes? What is the limiting amount of C after a long time? How much of chemicals A and B remain after a long time? At what time is chemical C half-formed?

Probleem 3: Gestel 'n klip word vertikaal opwaarts gegooi met aanvanklike snelheid $v_0 > 0$. Aanvaar geen lugweerstand (sodat swaartekrag die enigste krag is wat op die klip inwerk).

(a) Skryf 'n aanvangswaardeprobleem neer en los dit op, om 'n uitdrukking vir die snelheid $v(t)$ van die klip te bepaal.

(b) Vind 'n uitdrukking (in terme van v_0 en swaartekragversnelling g) vir die tyd wanneer die klip maksimum hoogte bereik.

Problem 3: Suppose a rock is thrown vertically upwards with initial velocity $v_0 > 0$. Assume no air resistance (so that gravitational pull is the only force acting on the rock).

(a) Write down an initial value problem and solve it, to determine an expression for the velocity $v(t)$ of the rock.

(b) Find an expression (in terms of v_0 and gravitational acceleration g) for the time when the rock reaches maximum height.

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Probleem 4: (a) 'n Stroom vul 'n 100 liter watertenk teen 'n konstante tempo van 2 liter per minuut. As die tenk aanvanklik vol is en water word teen 'n konstante tempo van 3 liter per minuut uitgepomp, skryf 'n AWP wat die volume water $v(t)$ (liter) in die tenk op enige gegewe tyd t (minute) beskryf, en los die AWP.

(b) Neem nou aan dat die water in die tenk vuil is, met 'n aanvanklike konsentrasie van $0.2kg$ vuilgoed per liter. Die water wat vanaf die stroom inkom is skoner, en bevat slegs $0.1kg$ vuilgoed per liter. Deur aan te neem dat die water in die tenk goed gemeng is, skryf die AWP neer wat die massa vuilgoed in die tenk op enige gegewe tyd t beskryf, en los dit op.

(c) Gestel die water word drinkbaar wanneer die konsentrasie vuilgoed in die water onder $0.15kg/\ell$ is. Hoe lank sal dit neem tot die water drinkbaar is?

Probleem 4: (a) A 100 litre water tank is filled by a stream at a constant rate of 2 litres per minute. If the tank starts off full and water is pumped out at a constant rate of 3 litres per minute, write down an IVP describing the volume of water $v(t)$ (litres) in the tank at any given time t (mins), and solve the IVP.

(b) Assume now that the water in the tank is dirty, with an initial concentration of $0.2kg$ of dirt per litre. The water coming in from the stream is cleaner, and only contains $0.1kg$ of dirt per litre. Assuming that the water in the tank is well-mixed, write down and solve an IVP describing the mass of the dirt contained in the tank at any given time t.

(c) Suppose the water becomes drinkable when the concentration of the dirt in the water drops below $0.15kg/\ell$. How long before the water is drinkable?

Probleem 5: Beskou die DV: $y'' - y' - 12y = 0$.

- **(a)** Bevestig dat $y_1 = e^{4x}$ en $y_2 = e^{-3x}$ oplossings is.
- **(b)** Wys dat die oplossing in (a) fundamentele oplossings is, deur die gepaste Wronskiaan te bereken.
- **(c)** Vorm nou die algemene oplossing, en bevestig dat dit die DV bevredig.
- **(d)** Los op die aanvangswaardeprobleem

Problem 5: Consider the DE: $y'' - y' - 12y = 0$.

- **(a)** Verify that $y_1 = e^{4x}$ and $y_2 = e^{-3x}$ are solutions.
- **(b)** Show that the solutions in (a) are fundamental solutions, by calculating the appropriate Wronskian.
- **(c)** Now form the general solution, and verify that it does satisfy the DE.
- **(d)** Solve the initial value problem

$$y'' - y' - 12y = 0, \qquad y(0) = 1,\ y'(0) = 0.$$

Probleem 6: Beskou die DV: $y'' - y' - 12y = -10e^{2x}$.

- **(a)** Wys dat $y = e^{2x}$ 'n oplossing is.
- **(b)** Met behulp van Probleem 5c, los op die aanvangswaardeprobleem

Problem 6: Consider the DE: $y'' - y' - 12y = -10e^{2x}$.

- **(a)** Show that $y = e^{2x}$ is a solution.
- **(b)** With the aid of Problem 5c, solve the initial value problem

$$y'' - y' - 12y = -10e^{2x}, \qquad y(0) = 3,\ y'(0) = 2.$$

Probleem 7: Beskou die DV: $y'' - 4y' + 4y = e^x$.

- **(a)** Wys dat $y = e^{2x}$ en $y = xe^{2x}$ fundamentele oplossings vir $y'' - 4y' + 4y = 0$ is.
- **(b)** Bevestig dat $y = e^x$ 'n oplossing vir $y'' - 4y' + 4y = e^x$ is.
- **(c)** Los op die aanvangswaardeprobleem

Problem 7: Consider the DE: $y'' - 4y' + 4y = e^x$.

- **(a)** Show that $y = e^{2x}$ and $y = xe^{2x}$ are fundamentals solution to $y'' - 4y' + 4y = 0$.
- **(b)** Verify that $y = e^x$ is a solution to $y'' - 4y' + 4y = e^x$.
- **(c)** Solve the initial value problem

$$y'' - 4y' + 4y = e^x, \qquad y(0) = 0,\ y'(0) = 1.$$