
Probleem 1: Wys dat $\mathcal{L}\{t^n\} = \frac{n}{s}\mathcal{L}\{t^{n-1}\}$ vir $n > 0$ en dus lei af deur induksie dat

Problem 1: Show that $\mathcal{L}\{t^n\} = \frac{n}{s}\mathcal{L}\{t^{n-1}\}$ for $n > 0$ and hence conclude by induction that

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}},\ n > 0.$$

Probleem 2: As $\mathcal{L}\{f(t)\} = F(s)$ en $a > 0$ konstant is, wys dat

Problem 2: If $\mathcal{L}\{f(t)\} = F(s)$ and $a > 0$ is constant, show that

$$\textbf{(a)}\quad \mathcal{L}\{e^{at}f(t)\} = F(s-a) \qquad \text{en/and} \qquad \textbf{(b)}\quad \mathcal{L}\{f(at)\} = \frac{1}{a}F\Big(\frac{s}{a}\Big).$$

Probleem 3: Gebruik bostaande resultate om te wys dat

Problem 3: Use the results above to show that

$$\textbf{(a)}\quad \mathcal{L}\{t^2e^{at}\} = \frac{2}{(s-a)^3}, \qquad \textbf{(b)}\quad \mathcal{L}\{\cos(kt)\} = \frac{s}{s^2+k^2}, \qquad \text{en/and} \qquad \textbf{(c)}\quad \mathcal{L}\{\sin(kt)\} = \frac{k}{s^2+k^2},.$$

Wenk: Onthou dat $\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}$ en $\mathcal{L}\{\sin(t)\} = \frac{1}{s^2+1}$.

Hint: Recall that $\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}$ and $\mathcal{L}\{\sin(t)\} = \frac{1}{s^2+1}$.

Probleem 4: Gebruik Laplace transforms om elk van die volgende aanvangswaardeprobleme op te los.

Problem 4: Use Laplace transforms to solve each of the following initial value problems.

(a) $2x' + x = 0, \quad x(0) = -3$

(b) $x'' - 4x' = -3e^{-t}, \quad x(0) = 1, \quad x'(0) = -1$

(c) $x'' + 9x = e^t, \quad x(0) = 0, \quad x'(0) = 0$

(d) $x' = x - 2y, \quad y' = 5x - y, \quad x(0) = -1, \quad y(0) = 2$

Probleem 5: Beskou die volgende aanvangswaardeprobleem wat 'n veer-massa stelsel beskryf:

Problem 5: Recall the following initial value problem from tutorial #5:

$$\frac{d^2x}{dt^2} + 2x = \sin(t), \qquad x(0) = 0, \qquad x'(0) = 0.$$

Los die aanvangswaardeprobleem met die Laplace transforms metode op.

Solve the initial value problem with the method of Laplace transforms.

Probleem 6: Los die volgende 2de-orde aanvangswaardeprobleem op met die metode van Laplace transforms:

Problem 6: Solve the following 2nd-order initial value problem with the method of Laplace transforms:

$$x'' + 8x - 3y = 0, \qquad x(0) = 1, \quad x'(0) = 0,$$
$$y'' - 4x + 4y = 0, \qquad y(0) = 0, \quad y'(0) = -1.$$

Probleem 7: Die **gamma funksie** is gedefinieer deur

Problem 7: The **gamma function** is defined by

$$\Gamma(\alpha) = \int_0^\infty u^{\alpha-1}e^{-u}\,du, \qquad \alpha > -1.$$

Gebruik die definisie en die verandering van die veranderlike $u = st$ om te wys dat

Use this definition and the change of variable $u = st$ to show that

$$\mathcal{L}\{t^\alpha\} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}}.$$
