A) $y'' - 2y' + y = 0$

i) $y = e^{mt} \Rightarrow y' = me^{mt},\ y'' = m^2 e^{mt}$

$\Rightarrow e^{mt}(m^2 - 2m + 1) = 0$

$\Rightarrow m^2 - 2m + 1 = (m-1)^2 = 0 \Rightarrow m = 1$

$\Rightarrow \boxed{y_1 = e^t,\ y_2 = te^t}$

ii) $W(t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

$= e^t(te^t + e^t) - te^t(e^t) = e^{2t} > 0\ \forall t.$

$\boxed{W(t) \neq 0 \Rightarrow \text{fundamental solution.}}$

B) $y'' - 2y' + y = 0,\ y(0) = 4,\ y'(0) = 2.$

$y_c = c_1 e^t + c_2 e^t t$

$y(0) = c_1 + 0.c_2 = 4 \Rightarrow c_1 = 4$

$y'(0) = c_1 e^t + c_2(te^t + e^t)\big|_0 = c_1 + c_2 = 2$

$\Rightarrow c_2 = 2 - c_1 = -2.$

$\Rightarrow \boxed{y(t) = 4e^t - 2te^t = e^t(4 - 2t)}$

C) $y'' - 2y' + y = 4te^{-t},\ y(0) = 4,\ y'(0) = 2$

Try $y_p = (A + Bt)e^{-t} \Rightarrow y_p' = -(A + Bt)e^{-t} + Be^{-t}$

$= (B - A - Bt)e^{-t}$

$y_p'' = -(B - A - Bt)e^{-t} - Be^{-t}$

$= (A - 2B + Bt)e^{-t}$

$\Rightarrow y_p'' - 2y_p' + y_p = e^{-t}((A - 2B + Bt) - 2(B - A - Bt) + (A + Bt))$

$= e^{-t}([4A - 4B] + 4Bt) = 4te^{-t}$

$\Rightarrow \begin{cases} 4A - 4B = 0 \\ 4B = 4 \end{cases} \Rightarrow A = B = 1$

$\Rightarrow \boxed{y_p = (1 + t)e^{-t}}$

Not the same as in B!

$$y(x) = y_c(x) + y_p(x)$$
$$= d_1 e^t + t d_2 e^t + (t+1)e^{-t}$$
$$\to y'(x) = d_1 e^t + d_2(1+t)e^t + (\cancel{1} - t - \cancel{1})e^{-t}$$

$$\to y(0) = 4 = d_1 + 0.d_2 + 1. \quad \Rightarrow d_1 = 3$$
$$y'(0) = 2 = d_1 + d_2 \quad \Rightarrow d_2 = -1.$$

$$\Rightarrow \boxed{y(t) = 3e^t - te^t + (1+t)e^{-t}}$$

D) $$G(x,t) = \frac{y_1(t)y_2(x) - y_1(x)y_2(t)}{W(t)}$$
$$= \frac{e^t x e^x - t e^t e^x}{e^{2t}} = e^{x-t}(x-t).$$

E) $$y'' - 2y' + y = \frac{e^t}{t}$$

$$y_p(x) = \int^x G(x,t)f(t)\,dt = \int e^{(x-t)}(x-t)\frac{e^t}{t}\,dt$$
$$= e^x \int^x \frac{x-t}{t}\,dt = e^x\left[\int^x \frac{x}{t}\,dt - \int^x 1\,dt\right]$$

$$y_p(x) = e^x x \log x - e^x x$$

$$\boxed{\begin{aligned} \to y(t) &= c_1 e^t + t c_2 e^t + t e^t \log(t) - t e^t \\ &= d_1 e^t + d_2 t e^t + t e^t \log(t) \end{aligned}}$$

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TW244 2021 TT06

Toegepaste wiskunde - Applied mathematics - 244

You can preview this quiz, but if this were a real attempt, you would be blocked because:

This quiz is not currently available

Information

- A reminder that numerical answers should be entered to four significant figures.
  (Intermediate rounding may cause larger errors in the final answer!
  Try not to round/convert to decimal until the very end.)
  Onthou dat numeriese antwoorde vier beduidende syfers moet bevat.
  (Tussentydse afronding kan lei tot groter foute in die finale antwoord!
  Probeer om eers aan die einde af te rond/na desimale om te skakel.)

- Correct answers without working (when asked for) will not receive full marks.
  As jy 'n korrekte antwoord gee sonder enige bewerkings (wanneer daarvoor gevra word), sal jy nie volpunte vir daardie antwoord kry nie.

- Partial credit may be given for correct working but incorrect answers.
  Gedeeltelike krediet kan gegee word vir korrekte bewerkings maar verkeerde antwoorde.

TW244 2021

Question **1**

Not yet answered

Marked out of 3.00

For each question, give your working in Q2. | *Gee jou bewerkings vir elke vraag in V2.*

A) [2 marks | *punte*] Consider the homogeneous DE $y'' - 2y' + y = 0$.
Beskou die homogene DV ... .

Find two solutions of the DE and show that they are *fundamental* solutions.
Vind twee oplossings van die DV en wys dat hulle fundamentele oplossings is.

(Hints: Step 1: Find y_1 and y_2 using the auxiliary equation. Step 2: Compute the Wronskian.)
(Wenke: Stap 1: Vind ... en ... met behulp van die hulpvergelyking. Stap 2: Bereken die Wronskiaan.)

B) [1 mark | *punt*] Consider the initial value problem $y'' - 2y' + y = 0, y(0) = 4, y'(0) = 2$.
Beskou die aanvangswaardeprobleem ... .

The solution is given by $y(t) =$ 4 $\times e^t +$ 2 $\times te^t$.

Die oplossing word gegee deur ...

C) [2 marks | *punte*] Consider the initial value problem $y'' - 2y' + y = 4te^{-t}, y(0) = 4, y'(0) = 2$.
Beskou die aanvangswaardeprobleem ... .

The solution is given by $y(t) =$ 3 $\times e^t +$ -1 $\times te^t + y_p(t)$,

Die oplossing word gegee deur ...

where | waar $y_p(t) =$

○ $2t\exp(-t)$ ✓ $(t+1)\exp(-t)$ ○ $2\exp(-t)$ ○ $t^2\exp(-2)$ ○ other|ander ○ $(t+2)\exp(-t$

D) [1 mark | *punt*] Determine the Green's function for the DE $y'' - 2y' + y = 0$.
Bepaal Green se funksie vir die DV ...

E) [1 mark | *punt*] Find the general solution to the inhomogeneous DE $y'' - 2y' + y = \frac{e^t}{t}$.
Vind die algemene oplossing vir die nie-homogene DV ...

A) $y'' - 2y' + y = 0$

Try $y = e^{mt} \Rightarrow y' = me^{mt},\ y'' = m^2 e^{mt}$

$\Rightarrow m^2 e^{mt} - 2me^{mt} + e^{mt} = 0 = e^{mt}(m^2 - 2m + 1) = e^{mt}(m-1)(m-1)$

i) $\Rightarrow m = 1, 1 \Rightarrow \boxed{y_1 = e^t,\ y_2 = te^t}$

ii) To show fundamental, consider the Wronskian:

$W(t) = \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix} = y_1(t) y_2'(t) - y_2(t) y_1'(t)$

$= e^t(e^t + \cancel{te^t}) - (\cancel{te^t}) e^t = e^{2t} \neq 0$

$\therefore$ $\boxed{y_1 \text{ and } y_2 \text{ are fundamental solns}}$

B) $y'' - 2y' + y = 0,\ y(0) = 4,\ y'(0) = 2$

$\rightarrow y(t) = c_1 e^t + c_2 te^t \quad , \quad y'(t) = c_1 e^t + c_2(e^t + te^t)$

$y(0) = 4 = c_1 + 0.c_2 \Rightarrow c_1 = 4$

$y'(0) = 2 = c_1 + c_2 \Rightarrow c_2 = 2 - c_1 \Rightarrow c_2 = -2$

$\Rightarrow \boxed{y(x) = 4e^t - 2te^t} = 2e^t(2 - t).$

C) $y'' - 2y' + y = 4te^{-t},\ y(0) = 4,\ y'(0) = 2$

$y(t) = y_c(t) + y_p(t) = d_1 e^t + d_2 te^t + y_p(t)$

Guess: $y_p(t) = Ae^{-t} + Bte^{-t}$

$\Rightarrow y_p'(t) = -Ae^{-t} + B(e^{-t} - te^{-t}) = e^{-t}(B - A) - Bte^{-t}$

$y_p''(t) = +(A - B)e^{-t} - B(e^{-t} - te^{-t}) = (A - 2B)e^{-t} + tBe^{-t}$

$y_p'' - 2y_p' + y_p = 4te^{-t} = [(A - 2B)e^{-t} + tBe^{-t}]$
$\quad - 2[(B - A)e^{-t} - Bte^{-t}]$
$\quad + [Ae^{-t} + Bte^{-t}]$

$= e^{-t}[A - 2B - 2B + 2A + A] + te^{-t}[B + 2B + B]$

$= e^{-t}\underbrace{[4A - 4B]} + \underbrace{4B}te^{-t} = \underbrace{0} \times e^{-t} + \underbrace{4}te^{-t}$

$\Rightarrow A = B,\ B = 1$

$\Rightarrow \boxed{y_p(t) = (1 + t)e^{-t}}$

$y(0) = 4,\ y'(0) = 2$

$y(t) = d_1 e^t + d_2 te^t + (1 + t)e^{-t}$

$y'(t) = d_1 e^t + d_2(e^t + te^t) + e^{-t} + -(1 + t)e^{-t}$

$y(0) = 4 = d_1 + 1 \Rightarrow d_1 = 3$

$y'(0) = 2 = d_1 + d_2 + \cancel{1 - 1} \Rightarrow d_2 = 2 - d_1 = -1$

$\boxed{y(t) = 3e^t - te^t + (1 + t)e^{-t}}$

D) $G(x, t) = \dfrac{y_1(t) y_2(x) - y_1(x) y_2(t)}{W(t)}$ $\quad W(t) = e^{2t}$

$= (e^t x e^x - e^x te^t) e^{-2t}$

$= \boxed{e^{x-t}(x - t)}$

E) $y'' - 2y' + y = e^t / t$

$y_p(x) = \int^x G(x,t) f(t)\, dt = \int^x e^{x-t}(x - t) \frac{e^t}{t}\, dt$

$= \int^x e^x \frac{(x - t)}{t}\, dt$

$= e^x \left[x \int^x \frac{dt}{t} - \int^x 1\, dt \right]$

$= e^x [x \log(x) - x] = xe^x \log(x) - xe^x$

$\Rightarrow y(t) = c_1 e^t + c_2 te^t + te^t \log(t) - te^t$
$= c_1 e^t + d_2 te^t + te^t \log(t)$

Question **2**

Not yet answered

Marked out of 4.00

Give your working here | *Gee u werk hier.*

Paragraph

Path: p

Maximum size for new files: 2GB

Files

You can drag and drop files here to add them.

◄ TT05 video

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Tut test 07 - 13 Oct @ 4PM ►