Probleem 1: Vir elk van die vier gegewe lineêre outonome stelsels, klassifiseer die kritieke punt $(x, y) = (0, 0)$ as 'n stabiele nodus, onstabiele nodus, saalpunt, senter, stabiele spiraal of onstabiele spiraal. Moenie die stelsels oplos nie, gebruik eerder die figuur op bl. 398 in die handboek.

Problem 1: For each of the four given linear autonomous systems, classify the critical point $(x, y) = (0, 0)$ as a stable node, unstable node, saddle point, centre, stable spiral or unstable spiral. Do not solve the systems, rather use the figure on p. 398 of the textbook.

(a) $\dfrac{dx}{dt} = -2x - 2y, \quad \dfrac{dy}{dt} = -2x - 5y$

(b) $\dfrac{dx}{dt} = -x - 2y, \quad \dfrac{dy}{dt} = 3x + 4y$

(c) $\dfrac{dx}{dt} = -x + y, \quad \dfrac{dy}{dt} = 4x - y$

(d) $\dfrac{dx}{dt} = x - y, \quad \dfrac{dy}{dt} = x + y$

Probleem 2: Bepaal die eiewaardes en eievektore van die koëffisiëntmatriks in probleem 1(c) hierbo, en gebruik dit om tipiese oplossingskrommes in die fasevlak te skets.

Wenk: die eievektore van die koëffisiëntmatriks is $[1\ 2]^\top$ en $[-1\ 2]^\top$. Vind self die eiewaardes.

Problem 2: Determine the eigenvalues and eigenvectors of the coefficient matrix in problem 1(c) above, and use that to draw typical solution curves in the phase plane.

Hint: the eigenvectors of the coefficient matrix are $[1\ 2]^\top$ and $[-1\ 2]^\top$. Find the eigenvalues yourself.

Probleem 3: Los die stelsel in probleem 1(d) op, en skryf die oplossing in 'n vorm wat die klassifikasie van die kritieke punt $(0, 0)$ duidelik bevestig.

Wenk: die eievektore van die koëffisiëntmatriks in hierdie stelsel is $\mathbf{v}_1 = [1,\ -i]^\top$ en $\mathbf{v}_2 = [1,\ i]^\top$. Vind self die eiewaardes λ_1 en λ_2, skryf die oplossing as $\mathbf{x} = c_1 e^{\lambda_1 t}\mathbf{v}_1 + c_2 e^{\lambda_2 t}\mathbf{v}_2$, en vereenvoudig.

Problem 3: Solve the system in problem 1(d), and write the solution in a form that clearly confirms the classification of the critical point $(0, 0)$.

Hint: the eigenvectors of the coefficient matrix in this system are $\mathbf{v}_1 = [1,\ -i]^\top$ and $\mathbf{v}_2 = [1,\ i]^\top$. Find the eigenvalues λ_1 and λ_2 yourself, write the solution as $\mathbf{x} = c_1 e^{\lambda_1 t}\mathbf{v}_1 + c_2 e^{\lambda_2 t}\mathbf{v}_2$, and simplify.

Probleem 4: Beskou die Lotka-Volterra stelsel:

Problem 4: Consider the Lotka-Volterra system:

$$\frac{dx}{dt} = -3x + 8xy, \qquad \frac{dy}{dt} = 2y - 4xy.$$

Vind al die kritieke oplossings en probeer om elkeen van deur die stelsel te lineariseer en dan die figuur op bl.405 te gebruik. Pasop vir grensgevalle waar geen uitspraak oor die klassifikasie gemaak kan word nie.

Find all the critical solutions and attempt to classify them by linearizing the system and then using the figure on p.405. Be careful of borderline cases where nothing concrete can be said about the classification.

Probleem 5: Vind en klassifiseer die kritieke punte van die onderstaande DV stelsel. Gebruik hierdie informasie om die faseportret te skets.

Problem 5: Locate and classify the critical points of the DE system below. Use this information to sketch the phase portrait.

$$\frac{dx}{dt} = x^2 + y^2 - 1, \qquad \frac{dy}{dt} = x + y + 1$$

Wenk: Een manier om die rotasierigting van 'n stabiele of onstabiele spiraal by, sê nou maar, (x_1, y_1) te bepaal, is om die koordinate $(x_1 + \varepsilon, y_1)$ in die oorspronklike uitdrukking vir $\frac{dy}{dt}$ te vervang en na die resultaat se teken (m.a.w., $\pm$) te kyk.

Hint: One way to determine the direction of rotation of a stable or unstable spiral at, say, (x_1, y_1) is to substitute the coordinates $(x_1 + \varepsilon, y_1)$ to the original expression for $\frac{dy}{dt}$ and look at the sign (i.e., $\pm$) of the result.

Probleem 6: Los die volgende nie-lineêre DV:

Problem 6: Solve the following nonlinear DE:

$$\frac{dy}{dx} = \frac{x(2 + 3x^2)}{y}$$

Probleem 7: Gebruik die fasevlak metode om te wys dat $(0, 0)$ 'n senter van die nie-lineêre tweede-orde differensiaalvergelyking $x'' + x^3 = 0$ is.

Problem 7: Use the phase-plane method to show that $(0, 0)$ is a centre of the nonlinear second-order differential equation $x'' + x^3 = 0$.

Probleem 8: Die Schrödinger vergelyking vir die eendimensionele tyd-onafhanklike kwantum harmoniese ossilator is

Problem 8: The Schrödinger equation for the one-dimensional time independent quantum harmonic oscillator is

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega^2x^2\right)\psi(x)=E\psi(x).$$

Nie-dimensionaliseer deur 'n karakteristieke lengte $x = x_c\widehat{x}$ in te stel om aan te toon dat die DV geskryf kan word as

Nondimensionalise by introducing a characteristic length $x = x_c\widehat{x}$ to show that the DE may be written as

$$\left(-\frac{d^2}{d\tilde{x}^2}+\tilde{x}^2\right)\tilde{\psi}(\tilde{x})=\tilde{E}\tilde{\psi}(\tilde{x}),$$

waar $\tilde{\psi}(\tilde{x}) = \psi(x)$. Druk x_c en $\tilde{E}$ uit in terme van $\hbar, m, \omega,$ en E.

where $\tilde{\psi}(\tilde{x}) = \psi(x)$. Express x_c and $\tilde{E}$ in terms of $\hbar, m, \omega,$ and E.