
Problem 1:

(a) 4th-order, linear **(b)** 3rd-order, nonlinear **(c)** 2nd-order, nonlinear

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Problem 2:

The DE is linear in the dependent variable y, and nonlinear if the dependent variable is x.

Problem 3:

$y = x + 4\sqrt{x+2} \implies y' = 1 + 2(x+2)^{-1/2}$.

Now, $(y-x)y' = [4\sqrt{x+2}][1+2(x+2)^{-1/2}] = 4\sqrt{x+2} + 8 = y - x + 8$, so the DE is satisfied.

The domain of $y = x + 4\sqrt{x+2}$ is $[-2, \infty)$ but, for y to be a solution of the DE, dy/dx must exist and be continuous which implies that $x \neq -2$. The largest interval I of definition is therefore $(-2, \infty)$.

Problem 4:

$$\frac{d}{dx}(-2x^2y + y^2) = \frac{d}{dx}(1) \implies -2[x^2\frac{dy}{dx} + 2xy] + 2y\frac{dy}{dx} = 0 \implies (x^2 - y)\frac{dy}{dx} + 2xy = 0.$$

Problem 5:

$y = c_1e^{3x} + c_2xe^{3x} \implies y' = 3c_1e^{3x} + c_2e^{3x}(3x+1)$ en $y'' = 9c_1e^{3x} + c_2e^{3x}(9x+6)$.

Now, $y'' - 6y' + 9y = 9c_1e^{3x} + c_2e^{3x}(9x+6) - 18c_1e^{3x} - 6c_2e^{3x}(3x+1) + 9c_1e^{3x} + 9c_2xe^{3x} = 0$.

Problem 6:

$xy'' + 2y' = 0$. With $y = x^m$ we have $xm(m-1)x^{m-2} + 2mx^{m-1} = (m^2+m)x^{m-1} = 0 \implies m = 0$ or -1.

Problem 7:

Substitute the initial condition $y(-1) = 2$ in the solution: $2 = 1/(1 + ce^{-(-1)})$, so that $c = -1/(2e)$.

A solution to the given initial value problem is therefore $y = \dfrac{1}{1 - \frac{1}{2}e^{-(x+1)}}$.

Problem 8:

$x = c_1\cos t + c_2\sin t \implies x' = -c_1\sin t + c_2\cos t$

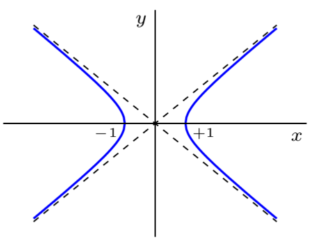
$$\left.\begin{array}{rlcr} x(\pi/4) = \sqrt{2}: & c_1\left(\frac{1}{\sqrt{2}}\right) + c_2\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2} & \implies & c_1 + c_2 = 2 \\ x'(\pi/4) = 2\sqrt{2}: & -c_1\left(\frac{1}{\sqrt{2}}\right) + c_2\left(\frac{1}{\sqrt{2}}\right) = 2\sqrt{2} & \implies & -c_1 + c_2 = 4 \end{array}\right\} \quad c_1 = -1, \quad c_2 = 3$$

$$\implies x = -\cos t + 3\sin t.$$

Problem 9:

(a) $3x^2 - y^2 = c \implies \dfrac{d}{dx}(3x^2 - y^2) = \dfrac{d}{dx}(c) \implies 6x - 2y\dfrac{dy}{dx} = 0 \implies y\dfrac{dy}{dx} = 3x$.

(b)



Explicit solutions: $y = +\sqrt{3x^2 - 3}$ and $y = -\sqrt{3x^2 - 3}$.
Interval of definition for each: $I = (-\infty, -1) \cup (1, \infty)$ [note: $x \neq \pm 1$].

(c) The explicit solution $y = +\sqrt{3x^2 - 3}$ satisfies $y(-2) = 3$.

(d) Yes, $y = \sqrt{3}\,x$ or $y = -\sqrt{3}\,x$.
