Week 12: Simulations and applications:

For simulation I am going to recommend we use Excel

- Faster
- Easier to understand

I am going to explain simulation by going trough the examples

Buta Bookshop :

Buta Bookshop must order calendars in September for 2021. The cost to print a calendar is R15 and it sells for R45. All calendars which have not been sold after the first quarter of 2021 will be bought back by the printer for R10. Buta believes that sales of the calendars follows the discrete distribution below (based on market research of course.)

Vraag Demand	Waarskynlikheid Probability
100	0.2
150	0.2
200	0.3
250	0.25
300	0.05

How many calendars should he order if he can only order in multiples of 50 between 100 and 300?

Our goal here to determine how many calendars to order for max profit. We do this by simulating a number of random variables that will be represented as demand.

A	B	C	D	E	F	G	H	I	J
Buta bookshop									
Demand	P(X=x)					Purchasing policy			Average annual income
100	0.2					order qty:			
150	0.2								
200	0.3					Sales price		45	R/u
250	0.25					Print cost		15	R/u
300	0.05					Salvage price		10	R/u

This is what we are given. We want to make a simulation where we have a random variable (0-1) that will represent the demand for a year. There we make upper- and lowerbounds (catagories) for the probabilities.

Buta bookshop									
Demand	P(X=x)	LB_i	UB_i	Demand		Purchasing policy			Average annual income
100	0.2	0	0.2	100		order qty:			
150	0.2	0.2	0.4	150					
200	0.3	0.4	0.7	200		Sales price		45	R/u
250	0.25	0.7	0.95	250		Print cost		15	R/u
300	0.05	0.95	1	300		Salvage price		10	R/u

So now if we have a random value x = 0.427
We know to the years demand wil be 200 since it's LB is 0.4 and UP Is 0.7 (0.4 <= 0.427 <=0.7).
If we have the order quantity of 200 we will sell all our calendars for Max profit.

Buta bookshop									
Demand	P(X=x)	LB_i	UB_i	Demand		Purchasing policy			Average annual income
100	0.2	0	0.2	100		order qty:	200		4981.71
150	0.2	0.2	0.4	150					
200	0.3	0.4	0.7	200		Sales price		45	R/u
250	0.25	0.7	0.95	250		Print cost		15	R/u
300	0.05	0.95	1	300		Salvage price		10	R/u
Year	Random	Demand	Sold_i	Surplus_i	Net Inc_i				
1	0.42714	200	200	0	6000				

But if we have less quantity than the demand we will sell all our Calenders , but it would be inefficient since we could have sold more

A	B	C	D	E	F	G	H	I	J
Buta bookshop									
Demand	P(X=x)	LB_i	UB_i	Demand		Purchasing policy			Average annual income
100	0.2	0	0.2	100		order qty:	50		1500
150	0.2	0.2	0.4	150					
200	0.3	0.4	0.7	200		Sales price		45	R/u
250	0.25	0.7	0.95	250		Print cost		15	R/u
300	0.05	0.95	1	300		Salvage price		10	R/u
Year	Random	Demand	Sold_i	Surplus_i	Net Inc_i				
1	0.427	200	50	0	1500				

This is why we simulate .. to see which order quantity will give us the Most value, but only for a large number of random variables

A	B	C	D	E	F	G	H	I	J	K	L	M
Buta bookshop												
Demand	P(X=x)	LB_i	UB_i	Demand		Purchasing policy			Average annual income			
100	0.2	0	0.2	100		order qty:	250		5281			
150	0.2	0.2	0.4	150								
200	0.3	0.4	0.7	200		Sales price		45	R/u			
250	0.25	0.7	0.95	250		Print cost		15	R/u			
300	0.05	0.95	1	300		Salvage price		10	R/u			
Year	Random	Demand	Sold_i	Surplus_i	Net Inc_i				Average annual income			
1	0.09485	100	100	150	2250			Purchasing Qty				
2	0.08153	100	100	150	2250			100	150	200	250	300
3	0.26699	150	150	100	4000		5281	3000	4164	4999	5281	5129
4	0.0905	100	100	150	2250							
5	0.60341	200	200	50	5750							
6	0.25559	150	150	100	4000							
7	0.55096	200	200	50	5750							
8	0.79916	250	250	0	7500							
9	0.74233	250	250	0	7500							
10	0.87102	250	250	0	7500							
11	0.25175	150	150	100	4000							
12	0.97837	300	250	0	7500							
13	0.53987	200	200	50	5750							
14	0.75776	250	250	0	7500							
15	0.29575	150	150	100	4000							
16	0.98901	300	250	0	7500							
17	0.37304	150	150	100	4000							
18	0.72829	250	250	0	7500							
19	0.04643	100	100	150	2250							
20	0.73446	250	250	0	7500							
21	0.04075	100	100	150	2250							
22	0.52605	200	200	50	5750							
23	0.8602	250	250	0	7500							
24	0.58706	200	200	50	5750							
25	0.76482	250	250	0	7500							
26	0.22282	150	150	100	4000							
27	0.57138	200	200	50	5750							
28	0.36781	150	150	100	4000							
29	0.80664	250	250	0	7500							
30	0.17964	100	100	150	2250							
31	0.74046	250	250	0	7500							

Excel formulas you need to know :

Random n
0.990995

1. To generate a random value we use : =RAND()

Demand	P(X=x)	LB_i	UB_i	Demand	
100	0.2	0	0.2	100	
150	0.2	0.2	0.4	150	
200	0.3	0.4	0.7	200	
250	0.25	0.7	0.95	250	
300	0.05	0.95	1	300	

Year	Random n	Demand_i	Sold_i	Surplus_i	Net Inc_i
1	0.887944	=+VLOOKUP(B12,$C$4:$E$8,3,1)			
2	0.589943	200	200	0	600

2. to get the demand from our random variable :
=VLOOKUP("the value we have and want to search(random variable)",
"the array we want to search",
"the column number we want to return (demand)",
"TRUE finds partial matches(1)")

If you don't know how to use absolute refrence (its where we Lock the cell's position) it's F4. We use it for the array above.

0	0.2	100		order qty:	200
0.2	0.4	150			
0.4	0.7	200		Sales price	
0.7	0.95	250		Print cost	
0.95	1	300		Salvage price	

Demand_i	Sold_i	Surplus_i	Net Inc_i
250	=MIN(C12,$H$4)		6000

3. To calculate the amount sold we use : =min("value 1", "value 2")

UB_i	Demand		Purchasing policy			Averag
0.2	100		order qty:	200		5021.
0.4	150					
0.7	200		Sales price		45	R/u
0.95	250		Print cost		15	R/u
1	300		Salvage price		10	R/u
Sold_i	Surplus_i	Net Inc_i				Averag
200	0	=+D12*$I$6+E12*$I$8-$H$4*$I$7				Qty

To get the profit made we say profit = sold*salesprice + surplus*salvage price - order quantity*print cost

nd		Purchasing policy			Average annual income	
100		order qty:	200		=AVERAGE(F12:F1011)	
150						
200		Sales price		45	R/u	
250		Print cost		15	R/u	
300		Salvage price		10	R/u	
s_i	Net Inc_i				Average annual income	
0	6000			Purchasing Qty		
0	6000			100	150	200
0	6000		5021.75	3000	4186.75	5021.75
0	6000					
0	6000					
0	6000					
0	6000					
100	2500					
0	6000					
100	2500					
100	2500					
100	2500					
100	2500					
0	6000					
100	2500					

4. we then use =average(“column”) to get the average of the column

Now looking at Queue Simulations :

M/M/1 Queue simulation in ExcelFile

Interarrival times of vessels at a quay are exponential distributed

with 11 hours between arrivals and it takes on average 10 hours to load

a vessel. Vessels are loaded one by one and vessels waiting to be

loaded fall in a single line for service. Determine whether the time in

the system and the time in the queue of this system converges to the

result one would expect in a M/M/1-system.

Given above we have λ = 1/11 = 0.09 = arrival rate of vessels per hour;
μ = 1/10 = 0.1 = service rate of vessel per hour;

We want to determine if the caculated time in the system and time in the queue are the same for the simulated time in system and queue

For a M/M/1 we know : $W = \frac{\lambda}{\lambda(\mu - \lambda)} = \frac{1}{\mu - \lambda}$ = 1 /(1/10-1/11) = 110

$$W_q = \frac{\lambda^2}{\lambda\mu(\mu - \lambda)} = \frac{\lambda}{\mu(\mu - \lambda)}$$ = 1/11 / 1/10*(1/10 – 1/11) = 100

Now we need to calculate the arrival time and service time with Random variables :

First make a random variable with excel using RAND() function for both arrival- and service time. Then we use the Inverse trans-formation method on them both to get the random arrival and service time.

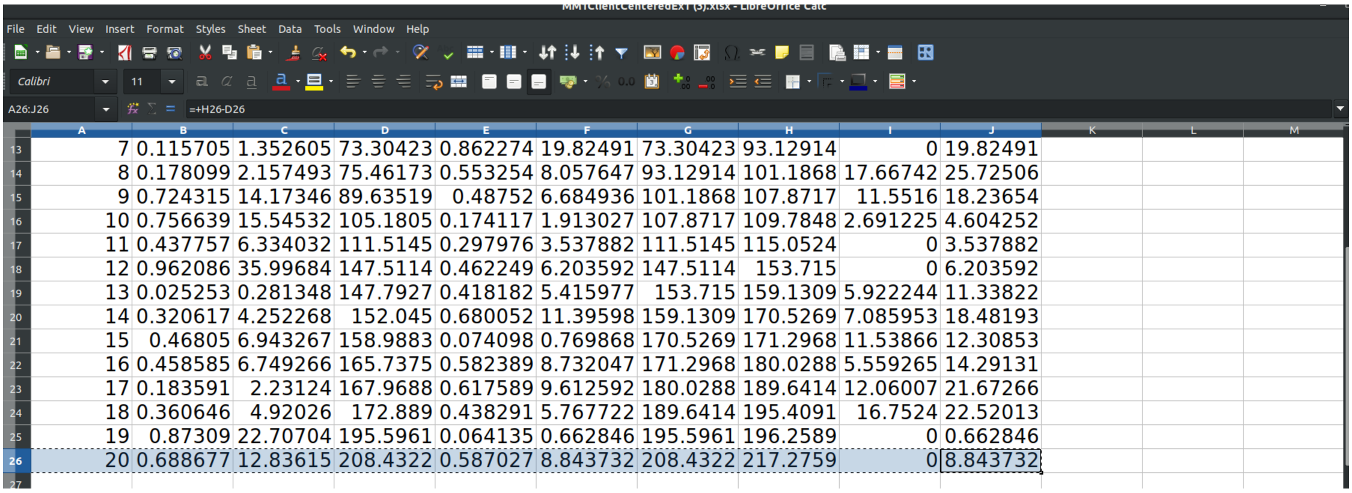
Vehicle number	random variable	Interarrival time	actual AT	random variable	Interservice time	Service start	Service ended	TIQ	TIS
1	0.07019	0.8005299	0.80053	0.183349	2.02542936	0.80053	2.825959	0	2.025429

-(1/λ)*LN(1-Random variable)

-(1/μ)*LN(1-Random variable)

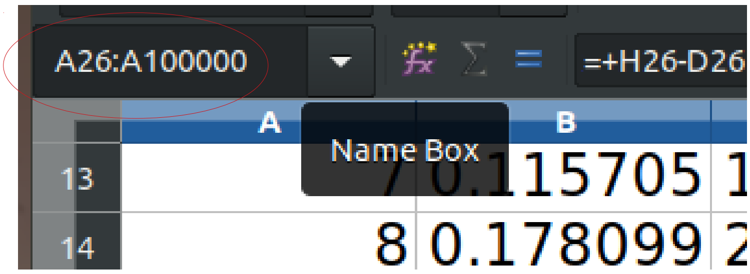
Now we need to do the simulation for a lot of random variables :

1. copy line

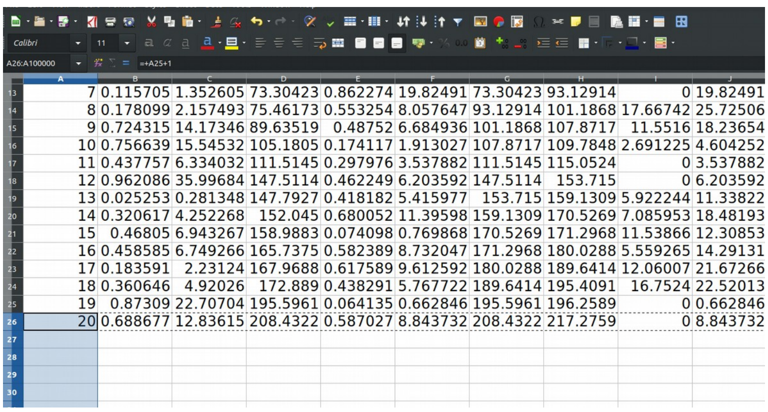


	A	B	C	D	E	F	G	H	I	J
13	7	0.115705	1.352605	73.30423	0.862274	19.82491	73.30423	93.12914	0	19.82491
14	8	0.178099	2.157493	75.46173	0.553254	8.057647	93.12914	101.1868	17.66742	25.72506
15	9	0.724315	14.17346	89.63519	0.48752	6.684936	101.1868	107.8717	11.5516	18.23654
16	10	0.756639	15.54532	105.1805	0.174117	1.913027	107.8717	109.7848	2.691225	4.604252
17	11	0.437757	6.334032	111.5145	0.297976	3.537882	111.5145	115.0524	0	3.537882
18	12	0.962086	35.99684	147.5114	0.462249	6.203592	147.5114	153.715	0	6.203592
19	13	0.025253	0.281348	147.7927	0.418182	5.415977	153.715	159.1309	5.922244	11.33822
20	14	0.320617	4.252268	152.045	0.680052	11.39598	159.1309	170.5269	7.085953	18.48193
21	15	0.46805	6.943267	158.9883	0.074098	0.769868	170.5269	171.2968	11.53866	12.30853
22	16	0.458585	6.749266	165.7375	0.582389	8.732047	171.2968	180.0288	5.559265	14.29131
23	17	0.183591	2.23124	167.9688	0.617589	9.612592	180.0288	189.6414	12.06007	21.67266
24	18	0.360646	4.92026	172.889	0.438291	5.767722	189.6414	195.4091	16.7524	22.52013
25	19	0.87309	22.70704	195.5961	0.064135	0.662846	195.5961	196.2589	0	0.662846
26	20	0.688677	12.83615	208.4322	0.587027	8.843732	208.4322	217.2759	0	8.843732

2.enter row you want to copy to

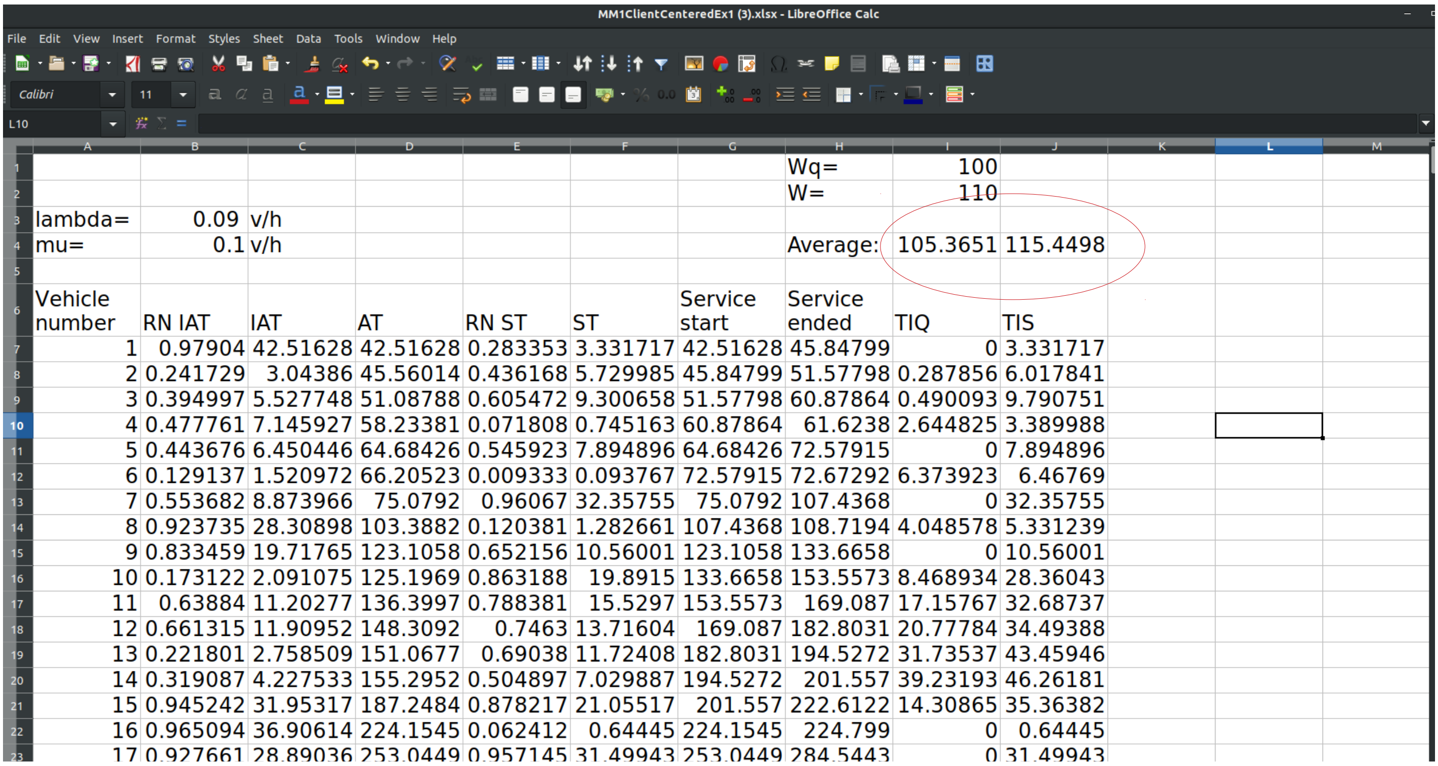


3. Enter



	A	B	C	D	E	F	G	H	I	J
13	7	0.115705	1.352605	73.30423	0.862274	19.82491	73.30423	93.12914	0	19.82491
14	8	0.178099	2.157493	75.46173	0.553254	8.057647	93.12914	101.1868	17.66742	25.72506
15	9	0.724315	14.17346	89.63519	0.48752	6.684936	101.1868	107.8717	11.5516	18.23654
16	10	0.756639	15.54532	105.1805	0.174117	1.913027	107.8717	109.7848	2.691225	4.604252
17	11	0.437757	6.334032	111.5145	0.297976	3.537882	111.5145	115.0524	0	3.537882
18	12	0.962086	35.99684	147.5114	0.462249	6.203592	147.5114	153.715	0	6.203592
19	13	0.025253	0.281348	147.7927	0.418182	5.415977	153.715	159.1309	5.922244	11.33822
20	14	0.320617	4.252268	152.045	0.680052	11.39598	159.1309	170.5269	7.085953	18.48193
21	15	0.46805	6.943267	158.9883	0.074098	0.769868	170.5269	171.2968	11.53866	12.30853
22	16	0.458585	6.749266	165.7375	0.582389	8.732047	171.2968	180.0288	5.559265	14.29131
23	17	0.183591	2.23124	167.9688	0.617589	9.612592	180.0288	189.6414	12.06007	21.67266
24	18	0.360646	4.92026	172.889	0.438291	5.767722	189.6414	195.4091	16.7524	22.52013
25	19	0.87309	22.70704	195.5961	0.064135	0.662846	195.5961	196.2589	0	0.662846
26	20	0.688677	12.83615	208.4322	0.587027	8.843732	208.4322	217.2759	0	8.843732

4.Paste :



	A	B	C	D	E	F	G	H	I	J
1								Wq=	100	
2								W=	110	
3	lambda=	0.09	v/h							
4	mu=	0.1	v/h					Average:	105.3651	115.4498
5										
6	Vehicle number	RN IAT	IAT	AT	RN ST	ST	Service start	Service ended	TIQ	TIS
7	1	0.97904	42.51628	42.51628	0.283353	3.331717	42.51628	45.84799	0	3.331717
8	2	0.241729	3.04386	45.56014	0.436168	5.729985	45.84799	51.57798	0.287856	6.017841
9	3	0.394997	5.527748	51.08788	0.605472	9.300658	51.57798	60.87864	0.490093	9.790751
10	4	0.477761	7.145927	58.23381	0.071808	0.745163	60.87864	61.6238	2.644825	3.389988
11	5	0.443676	6.450446	64.68426	0.545923	7.894896	64.68426	72.57915	0	7.894896
12	6	0.129137	1.520972	66.20523	0.009333	0.093767	72.57915	72.67292	6.373923	6.46769
13	7	0.553682	8.873966	75.0792	0.96067	32.35755	75.0792	107.4368	0	32.35755
14	8	0.923735	28.30898	103.3882	0.120381	1.282661	107.4368	108.7194	4.048578	5.331239
15	9	0.833459	19.71765	123.1058	0.652156	10.56001	123.1058	133.6658	0	10.56001
16	10	0.173122	2.091075	125.1969	0.863188	19.8915	133.6658	153.5573	8.468934	28.36043
17	11	0.63884	11.20277	136.3997	0.788381	15.5297	153.5573	169.087	17.15767	32.68737
18	12	0.661315	11.90952	148.3092	0.7463	13.71604	169.087	182.8031	20.77784	34.49388
19	13	0.221801	2.758509	151.0677	0.69038	11.72408	182.8031	194.5272	31.73537	43.45946
20	14	0.319087	4.227533	155.2952	0.504897	7.029887	194.5272	201.557	39.23193	46.26181
21	15	0.945242	31.95317	187.2484	0.878217	21.05517	201.557	222.6122	14.30865	35.36382
22	16	0.965094	36.90614	224.1545	0.062412	0.64445	224.1545	224.799	0	0.64445
23	17	0.927661	28.89036	253.0449	0.957145	31.49943	253.0449	284.5443	0	31.49943

Now you can also see the average W and Wq is close to the Calculated one.