JonesMatrices.py

Polarization Optics

Jones Calculus for Polarizers

ElecSus Library Documentation

Abstract

This document provides comprehensive documentation for `JonesMatrices.py`, which implements Jones matrices for polarization analysis. The module provides polarizer matrices in both linear (x-y) and circular (L-R) bases, essential for modeling polarization-sensitive atomic spectroscopy experiments.

Contents

1 Theoretical Foundation

1.1 Jones Vector Formalism

Axiom 1 (Polarization State Representration). *A fully polarized monochromatic light beam can be represented by a complex 2-component vector:*

$$\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} |E_x|e^{i\phi_x} \\ |E_y|e^{i\phi_y} \end{pmatrix} \tag{1}$$

in the linear basis, or equivalently in the circular basis.

Definition 1 (Linear Basis States). *In the x-y (horizontal-vertical) basis:*

$$\hat{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \hat{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

Definition 2 (Circular Basis States). *Left and right circular polarizations:*

$$\hat{L} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \hat{R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{3}$$

In the L-R basis representation:

$$\hat{L} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \hat{R} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

1.2 Jones Matrix Formalism

Theorem 1 (Optical Element Action). *A polarization-transforming optical element is represented by a 2×2 matrix:*

$$\vec{E}_{out} = M \cdot \vec{E}_{in} \tag{5}$$

Definition 3 (Polarizer Matrix). *An ideal polarizer transmitting polarization $\hat{p}$ has Jones matrix:*

$$P = \hat{p}\hat{p}^\dagger = |\hat{p}\rangle\langle\hat{p}| \tag{6}$$

This is a projection operator satisfying $P^2 = P$.

1.3 Basis Transformation

Theorem 2 (Basis Change Matrix). *The transformation from x-y to L-R basis:*

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \tag{7}$$

A Jones matrix transforms as: $M_{LR} = UM_{xy}U^\dagger$.

2 Line-by-Line Code Analysis

2.1 Module Imports

```
import numpy as np
```

NumPy for matrix operations.

2.2 Horizontal Polarizer

```
# Horizontal polarisers (P_x)
HorizPol_xy = np.matrix([[1,0],[0,0]])
HorizPol_lr = 1./2 * np.matrix([[1,1],[1,1]])
```

In the x-y basis:

$$P_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = |\hat{x}\rangle\langle\hat{x}| \tag{8}$$

In the L-R basis:

$$P_x = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{9}$$

Horizontal polarizer passes $\hat{x}$ polarization, blocks $\hat{y}$.

2.3 Vertical Polarizer

```
# Vertical polarisers (P_y)
VertPol_xy = np.matrix([[0,0],[0,1]])
VertPol_lr = 1./2 * np.matrix([[1,-1],[-1,1]])
```

In the x-y basis:

$$P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = |\hat{y}\rangle\langle\hat{y}| \tag{10}$$

2.4 +45 Linear Polarizer

```
# Linear polarisers at plus 45 degrees wrt x-axis
LPol_P45_xy = 1./2 * np.matrix([[1,1],[1,1]])
LPol_P45_lr = 1./2 * np.matrix([[1,-1.j],[1.j,1]])
```

The +45 polarization state:

$$\hat{d} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{11}$$

Jones matrix:

$$P_{+45} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{12}$$

2.5 −45 Linear Polarizer

```
# Linear polarisers at minus 45 degrees wrt x-axis
LPol_M45_xy = 1./2 * np.matrix([[1,-1],[-1,1]])
LPol_M45_lr = 1./2 * np.matrix([[1,1.j],[-1.j,1]])
```

The −45 polarization state:

$$\hat{a} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{13}$$

2.6 Circular Polarizers

```
# Left Circular polariser (in circular basis only)
CPol_L_lr = np.matrix([[1,0],[0,0]])
CPol_R_lr = np.matrix([[0,0],[0,1]])
```

In the L-R basis:

$$P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{14}$$

These are simple projection matrices in the circular basis.

3 Mathematical Properties

3.1 Projection Operator Properties

Theorem 3 (Idempotency)**.** *All polarizer matrices satisfy:*

$$P^2 = P \tag{15}$$

Physically: passing light through two identical polarizers is equivalent to one.

Theorem 4 (Orthogonality)**.** *Orthogonal polarizers satisfy:*

$$P_\perp \cdot P = 0 \tag{16}$$

Example: $P_x \cdot P_y = 0$ *(crossed polarizers block all light).*

3.2 Completeness

Theorem 5 (Resolution of Identity)**.** *Orthogonal polarizer pairs sum to identity:*

$$P_x + P_y = \nVdash, \quad P_L + P_R = \nVdash \tag{17}$$

4 Applications in Spectroscopy

4.1 Polarization Analysis

The Stokes parameters can be measured using these polarizers:

$$S_0 = I_x + I_y \tag{18}$$

$$S_1 = I_x - I_y \tag{19}$$

$$S_2 = I_{+45} - I_{-45} \tag{20}$$

$$S_3 = I_L - I_R \tag{21}$$

4.2 Faraday Rotation Measurement

For small rotation angle θ:

$$I_{out} = I_0 \sin^2\theta \approx I_0\theta^2 \tag{22}$$

when measured with crossed polarizers.

5 Summary

The `JonesMatrices.py` module provides:

Key features:

- Matrices provided in both linear and circular bases
- All matrices are 2×2 complex NumPy matrices
- Follows conventions from Auzinsh, Budker, and Rochester (2010)
- Used for polarization analysis in ElecSus spectral calculations

Polarizer	Variable (xy)	Variable (lr)
Horizontal (x)	`HorizPol_xy`	`HorizPol_lr`
Vertical (y)	`VertPol_xy`	`VertPol_lr`
+45	`LPol_P45_xy`	`LPol_P45_lr`
−45	`LPol_M45_xy`	`LPol_M45_lr`
Left circular	–	`CPol_L_lr`
Right circular	–	`CPol_R_lr`

Table 1: Available polarizer matrices