Fabry-Pérot Optical Resonator: High Reflectivity Coated Mirrors

Detailed Analysis of `Chapt9Fig6a2.m`

Generated Documentation

Abstract

This document provides comprehensive theoretical foundations and line-by-line code analysis for `Chapt9Fig6a2.m`, which calculates the intensity spectrum of a Fabry-Pérot optical resonator with asymmetric high-reflectivity mirrors ($R_1 = 0.4$, $R_2 = 0.8$). The code demonstrates enhanced finesse and sharper resonance peaks compared to uncoated facets, illustrating the importance of mirror coatings in laser design.

Contents

1 Theoretical Foundation

1.1 Axioms of High-Finesse Cavities

Axiom 1 (Multiple Reflection Interference). *In a high-reflectivity cavity, many round trips contribute significantly to the interference pattern, leading to narrow resonances.*

Axiom 2 (Asymmetric Cavity Output). *For asymmetric mirrors ($R_1 \neq R_2$), output power is preferentially emitted from the lower-reflectivity facet.*

Axiom 3 (Photon Lifetime Enhancement). *Higher reflectivity increases the average number of round trips before photon escape:*

$$N_{rt} \approx \frac{1}{1 - R_1 R_2} \tag{1}$$

1.2 Fundamental Definitions

Definition 1 (Asymmetric Finesse). *For mirrors with different reflectivities:*

$$\mathcal{F} = \frac{\pi (R_1 R_2)^{1/4}}{1 - \sqrt{R_1 R_2}} \tag{2}$$

Definition 2 (Differential Output). *The fraction of power from each mirror:*

$$\frac{P_1}{P_{total}} = \frac{1 - R_1}{(1 - R_1) + (1 - R_2)}, \quad \frac{P_2}{P_{total}} = \frac{1 - R_2}{(1 - R_1) + (1 - R_2)} \tag{3}$$

Definition 3 (Quality Factor). *Relation to finesse:*

$$Q = \frac{\nu_0}{\delta\nu} = \frac{2 n_r L}{\lambda} \cdot \mathcal{F} = m \cdot \mathcal{F} \tag{4}$$

where m is the mode number.

1.3 Core Theorems

Theorem 1 (Finesse Enhancement). *Increasing reflectivity from $R = 0.3$ to $R = 0.56$ (geometric mean of 0.4 and 0.8):*

$$\frac{\mathcal{F}_{high}}{\mathcal{F}_{low}} = \frac{(1 - R_{low})\sqrt{R_{high}}}{(1 - R_{high})\sqrt{R_{low}}} \tag{5}$$

Theorem 2 (Cavity Loss Reduction). *The mirror loss for asymmetric cavity:*

$$\alpha_m = \frac{1}{2L} \ln \frac{1}{R_1 R_2} = \frac{1}{2L} \ln \frac{1}{0.32} = \frac{1.14}{2L} \tag{6}$$

Compare to Fresnel: $\ln(1/0.082)/2L = 2.5/2L$*—reduction by factor of 2.2.*

Theorem 3 (Threshold Reduction). *The threshold gain requirement:*

$$g_{th} = \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \tag{7}$$

Increasing R directly reduces g_{th} and hence I_{th}.

2 Line-by-Line Code Analysis

2.1 Initialization and Parameters

```
%Chapt9Fig6a2
%Fabry-Perot optical resonator
clear;
clf;
err=0.00001;
c=3e8;                  %speed of light in vacuum
nr=3.3;                 %effective refractive index
lambda0=1310e-9;        %center emission wavelength
Lc=300e-6;              %Cavity length
```

Same cavity geometry as Fig6a1: $L = 300\ \mu$m, $n_r = 3.3$, $\lambda_0 = 1310$ nm.

```
r1=0.4                  %Reflectivity of mirror1
r2=0.8                  %Reflectivity of mirror2
r=r1*r2;                %Optical loss per round-trip
```

High-reflectivity coated mirrors:

$$R_1 = 0.4 \quad \text{(output coupler)} \tag{8}$$

$$R_2 = 0.8 \quad \text{(high reflector)} \tag{9}$$

$$R = R_1 R_2 = 0.32 \tag{10}$$

The asymmetric design: HR coating on rear facet, partial reflector on output facet.

2.2 Finesse Calculation

```
F=pi*r^0.5/(1-r)        %Optical Finesse
Fconst=(2*F/pi)^2;
Imax=1/((1-r)^2);       %Peak intensity normalized to I0
```

Finesse:

$$\mathcal{F} = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi\sqrt{0.32}}{0.68} = \frac{1.78}{0.68} = 2.61 \tag{11}$$

Improvement: $\mathcal{F}_{high}/\mathcal{F}_{low} = 2.61/0.98 = 2.7\times$.

Coefficient of finesse:

$$F_{const} = \left(\frac{2\mathcal{F}}{\pi}\right)^2 = \frac{4R}{(1-R)^2} = \frac{1.28}{0.462} = 2.77 \tag{12}$$

Peak intensity enhancement:

$$I_{max} = \frac{1}{(1-R)^2} = \frac{1}{0.462} = 2.16 \tag{13}$$

Enhancement doubled compared to Fresnel case.

2.3 Spectral Parameters

```
f0=c*1e-12/lambda0;     %center frequency in THz
deltaf=c*1e-12/(2*Lc*nr)  %FSR in THz
deltawavelength=lambda0^2/(2*Lc*nr)  %FSR in m
```

Free spectral range: Same as Fig6a1 since geometry unchanged:

$$\Delta f_{FSR} = 0.152 \text{ THz} = 152 \text{ GHz} \tag{14}$$
$$\Delta\lambda_{FSR} = 0.87 \text{ nm} \tag{15}$$

Mode linewidth (new, narrower):

$$\delta\nu = \frac{\Delta\nu_{FSR}}{\mathcal{F}} = \frac{152}{2.61} = 58 \text{ GHz} \tag{16}$$

2.4 Airy Function Computation

```
for i=[1:1:1000]

   Frequency(i)=(f0+i*0.001);
   Fwavelength(i)=c/Frequency(i);
   x=sin(pi*Frequency(i)/deltaf);
   x2=x*x;
   Intensity(i)=Imax/(1+(Fconst*(x2)));

end
```

Identical algorithm to Fig6a1, but with different F_{const} and I_{max}:

$$I(\nu) = \frac{2.16}{1 + 2.77\sin^2\left(\frac{\pi\nu}{\Delta\nu_{FSR}}\right)} \tag{17}$$

At anti-resonance ($\sin^2 = 1$):

$$I_{min} = \frac{2.16}{1 + 2.77} = 0.57 \tag{18}$$

Contrast ratio: $I_{max}/I_{min} = 2.16/0.57 = 3.8$.

2.5 Plotting

```
figure(1);
plot(Frequency,Intensity);
axis([f0,f0+1,0,4]);
hold on;
xlabel('Frequency, \nu(THz)'),ylabel('Intensity');
ttl=sprintf('Chapt9Fig6a2, r1=%4.2f, r2=%4.2f, nr=%4.2f, f0=%7.2e THz, 
   Lc=%7.2e m',r1,r2,nr,f0,Lc)
title(ttl);

figure(2);
plot(Fwavelength*10^-12,Intensity);
xlabel('Wavelength, \lambda (m)'),ylabel('Intensity');
title(ttl);
```

Both frequency and wavelength domain plots with parameter annotation.

3 Comparison: Low vs High Reflectivity

Parameter	**Fig6a1 (Fresnel)**	**Fig6a2 (Coated)**
R_1	0.286	0.40
R_2	0.286	0.80
$R = R_1 R_2$	0.082	0.32
Finesse $\mathcal{F}$	0.98	2.61
F_{const}	0.39	2.77
I_{max}	1.19	2.16
Linewidth $\delta\nu$	155 GHz	58 GHz
Q factor ($m = 1500$)	1,470	3,915
Mirror loss α_m	83 cm^{-1}	38 cm^{-1}

4 Physical Interpretation

4.1 Improved Mode Selectivity

With higher finesse:

- Resonance peaks are sharper ($\delta\nu$ reduced by 2.7×)
- Better discrimination between adjacent modes
- Improved single-mode operation potential

4.2 Threshold Reduction

The mirror loss reduction from 83 to 38 cm^{-1} means:

$$\Delta g_{th} = 45 \text{ cm}^{-1} \tag{19}$$

For typical gain slope $g_0 \sim 2000$ cm^{-1}/(10^{18} cm^{-3}), this saves:

$$\Delta n_{th} = \frac{45}{2000} \times 10^{18} = 2.25 \times 10^{16} \text{ cm}^{-3} \tag{20}$$

4.3 Asymmetric Design Rationale

The choice $R_1 = 0.4 < R_2 = 0.8$:

- Most power exits through facet 1 (lower R)
- Rear facet (high R) acts as "back reflector"
- Photon makes more round trips before escaping
- Common in single-sided output devices

Power distribution:

$$\frac{P_1}{P_2} = \frac{1 - R_1}{1 - R_2} = \frac{0.6}{0.2} = 3 : 1 \tag{21}$$

75% of output from facet 1.

4.4 Still Moderate Finesse

Even with coatings, $\mathcal{F} = 2.6$ is still relatively low:

- Mode linewidth 58 GHz still significant
- Adjacent modes at 152 GHz spacing show overlap
- For true single-mode operation, need $\mathcal{F} > 10$ or additional mode selection

5 Numerical Methods

5.1 Resolution Adequacy

With 1 GHz sampling and 58 GHz linewidth:

$$\frac{\delta\nu}{\Delta\nu_{sample}} = 58 \tag{22}$$

Still adequate resolution (58 points across FWHM).

5.2 Contrast Visibility

The visibility of the interference pattern:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2.16 - 0.57}{2.16 + 0.57} = 0.58 \tag{23}$$

Improved from $V = 0.09$ in the Fresnel case.

6 Summary

This code simulates a Fabry-Pérot resonator with asymmetric coated mirrors:

- Output coupler: $R_1 = 0.4$
- High reflector: $R_2 = 0.8$
- Same 300 μm cavity as Fig6a1

Key improvements over uncoated cavity:

- Finesse increased 2.7× (0.98 → 2.61)
- Linewidth reduced 2.7× (155 → 58 GHz)
- Peak enhancement nearly doubled (1.19 → 2.16)
- Mirror loss reduced 2.2× (83 → 38 cm^{-1})

This represents a practical laser design with preferential output from one facet, though even higher reflectivities would further improve performance.