Fabry-Pérot Optical Resonator: Short Cavity / VCSEL Regime

Detailed Analysis of `Chapt9Fig6b.m`

Generated Documentation

Abstract

This document provides comprehensive theoretical foundations and line-by-line code analysis for `Chapt9Fig6b.m`, which calculates the intensity spectrum of a short-cavity Fabry-Pérot resonator ($L = 3$ μm) with high reflectivity mirrors ($R_1 = R_2 = 0.95$). This configuration is representative of vertical-cavity surface-emitting lasers (VCSELs), demonstrating large free spectral range and true single-mode operation capability.

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1 Theoretical Foundation

1.1 Axioms of Short-Cavity Resonators

Axiom 1 (Mode Spacing Scaling)**.** *Free spectral range is inversely proportional to cavity length:*

$$\Delta\nu_{FSR} = \frac{c}{2n_r L} \propto \frac{1}{L} \tag{1}$$

Short cavities have widely spaced modes.

Axiom 2 (Single-Mode Condition)**.** *Single longitudinal mode operation occurs when the gain bandwidth is narrower than the FSR:*

$$\Delta\nu_{gain} < \Delta\nu_{FSR} \tag{2}$$

Axiom 3 (High-Q Requirement)**.** *Short cavities require high reflectivity to maintain sufficient photon lifetime:*

$$\tau_p = \frac{n_r L}{c(1 - \sqrt{R_1 R_2})} \tag{3}$$

1.2 Fundamental Definitions

Definition 1 (VCSEL Cavity)**.** *A vertical-cavity surface-emitting laser has:*

- *Cavity length $L \sim \lambda$ to few λ (typically 1–5 μm)*
- *Distributed Bragg Reflector (DBR) mirrors with $R > 0.99$*
- *Vertical emission perpendicular to substrate*

Definition 2 (Effective Cavity Length)**.** *Including DBR penetration depth:*

$$L_{eff} = L_{active} + 2L_{DBR} \tag{4}$$

where $L_{DBR} \approx \lambda/(4\Delta n)$ is the DBR penetration.

Definition 3 (Mode Number for Short Cavity)**.**

$$m = \frac{2n_r L}{\lambda} \tag{5}$$

For $L = 3$ μm, $n_r = 3.3$, $\lambda = 1.31$ μm: $m \approx 15$.

1.3 Core Theorems

Theorem 1 (High-Finesse Airy Function)**.** *For $R \to 1$, the Airy function approaches a series of narrow Lorentzians:*

$$I(\nu) \approx I_{max} \sum_m \frac{(\delta\nu/2)^2}{(\nu - \nu_m)^2 + (\delta\nu/2)^2} \tag{6}$$

where $\delta\nu = \Delta\nu_{FSR}/\mathcal{F}$ and $\nu_m = m \cdot \Delta\nu_{FSR}$.

Theorem 2 (Finesse for High Reflectivity)**.** *When $R \to 1$:*

$$\mathcal{F} = \frac{\pi\sqrt{R}}{1 - R} \approx \frac{\pi}{1 - R} \tag{7}$$

For $R = 0.9025$ ($R_1 = R_2 = 0.95$): $\mathcal{F} = \pi/0.0975 = 32.2$.

Theorem 3 (Threshold Condition for Short Cavity)**.** *The threshold gain must compensate mirror loss over the short cavity:*

$$\Gamma g_{th} L = \ln \frac{1}{\sqrt{R_1 R_2}} + \alpha_i L \tag{8}$$

For $R = 0.95$ and $L = 3$ μm:

$$\Gamma g_{th} = \frac{\ln(1/0.95)}{3 \times 10^{-4}} + \alpha_i \approx 171 + \alpha_i \ cm^{-1} \tag{9}$$

Theorem 4 (Single-Mode Stability)**.** *Single-mode operation is ensured when:*

$$\Delta\nu_{FSR} > \Delta\nu_{gain} \tag{10}$$

For short cavity with $\Delta\nu_{FSR} = 15$ THz and typical gain bandwidth 30 THz, the nearest mode is significantly detuned from gain peak.

2 Line-by-Line Code Analysis

2.1 Initialization and Parameters

```
%Chapt9Fig6b
%Fabry-Perot optical resonator
clear;
clf;
err=0.00001;
c=3e8;                    %speed of light in vacuum
nr=3.3;                   %effective refractive index
lambda0=1310e-9;          %center emission wavelength
Lc=3e-6;                  %Cavity length
```

Short cavity parameters:

$$c = 3 \times 10^8 \text{ m/s} \tag{11}$$
$$n_r = 3.3 \tag{12}$$
$$\lambda_0 = 1310 \text{ nm} \tag{13}$$
$$L_c = 3 \ \mu\text{m} \tag{14}$$

Cavity length reduced by factor of 100 compared to edge-emitters.

```
r1=0.95;                  %Reflectivity of mirror1
r2=0.95;                  %Reflectivity of mirror2
r=r1*r2;                  %Optical loss per round-trip
```

High-reflectivity symmetric mirrors:

$$R_1 = R_2 = 0.95 \quad \text{(DBR mirrors)} \tag{15}$$
$$R = R_1 R_2 = 0.9025 \tag{16}$$

Typical of multi-layer DBR stacks with 20–30 periods.

2.2 Finesse and Spectral Parameters

```
F=pi*r^0.5/(1-r)            %Optical Finesse
Fconst=(2*F/pi)^2;
Imax=1/((1-r)^2);           %Peak intensity normalized to I0
```

High finesse:

$$\mathcal{F} = \frac{\pi\sqrt{0.9025}}{1 - 0.9025} = \frac{\pi \times 0.95}{0.0975} = \frac{2.98}{0.0975} = 30.6 \tag{17}$$

Coefficient of finesse:

$$F_{const} = \left(\frac{2\mathcal{F}}{\pi}\right)^2 = \frac{4R}{(1-R)^2} = \frac{3.61}{0.0095} = 380 \tag{18}$$

Peak intensity enhancement:

$$I_{max} = \frac{1}{(1-R)^2} = \frac{1}{0.0095} = 105 \tag{19}$$

Enormous enhancement—photon density 100× higher inside cavity.

```
f0=c*1e-12/lambda0;         %center frequency in THz
deltaf=c*1e-12/(2*Lc*nr)    %FSR in THz
deltawavelength=lambda0^2/(2*Lc*nr)  %FSR in m
```

Large free spectral range:

$$f_0 = \frac{3 \times 10^8}{1.31 \times 10^{-6}} = 229 \text{ THz} \tag{20}$$

$$\Delta f_{FSR} = \frac{c}{2n_r L} = \frac{3 \times 10^8}{2 \times 3.3 \times 3 \times 10^{-6}} = 15.2 \text{ THz} \tag{21}$$

Mode spacing in wavelength:

$$\Delta\lambda_{FSR} = \frac{\lambda_0^2}{2n_r L} = \frac{(1.31)^2 \times 10^{-12}}{19.8 \times 10^{-6}} = 87 \text{ nm} \tag{22}$$

Mode linewidth:

$$\delta\nu = \frac{\Delta\nu_{FSR}}{\mathcal{F}} = \frac{15.2 \text{ THz}}{30.6} = 0.50 \text{ THz} = 500 \text{ GHz} \tag{23}$$

2.3 Extended Frequency Scan

```
for i=[1:1:10000]
    Frequency(i)=(f0+i*0.003);
    Fwavelength(i)=c/Frequency(i);
    x=sin(pi*Frequency(i)/deltaf);
    x2=x*x;
    Intensity(i)=Imax/(1+(Fconst*(x2)));
end
```

Key differences from previous codes:

- 10,000 points (vs 1,000)
- Step size 0.003 THz (vs 0.001 THz)
- Total span: 30 THz

This covers approximately two FSR periods to show multiple resonances.

Airy function with high F_{const}:

$$I(\nu) = \frac{105}{1 + 380\sin^2\left(\frac{\pi\nu}{15.2\ \text{THz}}\right)} \tag{24}$$

At anti-resonance:

$$I_{min} = \frac{105}{1 + 380} = 0.28 \tag{25}$$

Contrast ratio: $I_{max}/I_{min} = 105/0.28 = 375$.

2.4 Plotting

```
figure(1);
plot(Frequency,Intensity);
%axis([f0,f0+1,0,5]);
hold on;
xlabel('Frequency, \nu(THz)'),ylabel('Intensity');
ttl=sprintf('Chapt9Fig6b, r1=%4.2f, r2=%4.2f, nr=%4.2f, f0=%7.2e THz, Lc=%7.2e m',r1,r2,nr,f0,Lc)
title(ttl);

figure(2);
plot(Fwavelength*10^-12,Intensity);
xlabel('Wavelength, \lambda (m)'),ylabel('Intensity');
title(ttl);
```

Note: The axis command is commented out to show full range.

3 Comparison: Edge-Emitter vs VCSEL

Parameter	Fig6a1 (Edge, Fresnel)	Fig6a2 (Edge, Coated)	Fig6b (VCSEL)
L_c	300 μm	300 μm	3 μm
R_1, R_2	0.29, 0.29	0.4, 0.8	0.95, 0.95
R	0.082	0.32	0.9025
$\mathcal{F}$	0.98	2.61	30.6
I_{max}	1.19	2.16	105
$\Delta\nu_{FSR}$	152 GHz	152 GHz	15.2 THz
$\delta\nu$	155 GHz	58 GHz	500 GHz
Mode number m	1511	1511	15

4 Physical Interpretation

4.1 VCSEL Mode Structure

With only $m \approx 15$ longitudinal modes possible:

- Mode spacing 15 THz $\approx$ 87 nm in wavelength
- Only one mode typically within gain bandwidth
- Inherent single longitudinal mode operation

4.2 Gain-Mode Alignment

The condition for lasing:

$$g(\lambda_m) > g_{th} \tag{26}$$

With 87 nm mode spacing, only the mode nearest gain peak lases.

4.3 Threshold Considerations

Despite high R, threshold gain is high due to short path:

$$g_{th} = \frac{1}{\Gamma L} \ln \frac{1}{R} = \frac{1}{0.03 \times 3 \times 10^{-4}} \ln(1.11) = \frac{0.104}{9 \times 10^{-6}} = 11,600 \text{ cm}^{-1} \tag{27}$$

(assuming $\Gamma = 0.03$ for thin QW active region)

This requires quantum well active regions with high material gain.

4.4 Photon Lifetime

$$\tau_p = \frac{n_r}{c \cdot \alpha_m} = \frac{n_r L}{c \cdot \ln(1/R)} = \frac{3.3 \times 3 \times 10^{-6}}{3 \times 10^8 \times 0.103} = 0.32 \text{ ps} \tag{28}$$

Very short photon lifetime leads to:

- High modulation bandwidth potential
- Reduced relaxation oscillation effects
- Fast switching capability

4.5 Intensity Enhancement

The factor $I_{max} = 105$ means:

- Photon density inside cavity 100× incident
- Compensates for short gain length
- Essential for achieving lasing in thin active region

5 Numerical Methods

5.1 Resolution Requirements

With $\delta\nu = 500$ GHz and step size 3 GHz:

$$\text{Points per FWHM} = \frac{500}{3} \approx 167 \tag{29}$$

Excellent resolution of resonance peaks.

5.2 Span Coverage

Total scan: 10,000 × 0.003 = 30 THz, covering:

$$\frac{30 \text{ THz}}{15.2 \text{ THz}} \approx 2 \text{ FSR periods} \tag{30}$$

Shows two complete resonance peaks.

5.3 Numerical Precision

With $F_{const} = 380$, near resonance:

$$I \approx \frac{105}{1 + 380 \times (\pi\delta\nu/\Delta\nu_{FSR})^2} \tag{31}$$

Standard double precision handles this without issues.

6 Summary

This code simulates a short-cavity, high-finesse Fabry-Pérot resonator representative of VCSELs:

Key parameters:

- Cavity length $L = 3$ μm (100× shorter than edge-emitter)
- High reflectivity $R = 0.95$ (DBR mirrors)
- Finesse $\mathcal{F} = 30.6$ (12× higher than coated edge-emitter)

Resulting characteristics:

- Large FSR = 15 THz (87 nm)—ensures single-mode operation
- Few longitudinal modes ($m \approx 15$)
- High intensity enhancement (105×)—compensates short gain path
- Sharp resonances with contrast ratio 375:1

VCSEL advantages demonstrated:

- Inherent single longitudinal mode selection
- High modulation bandwidth from short τ_p
- Circular beam from vertical geometry (not shown in this 1D model)

This simulation captures the essential physics that makes VCSELs fundamentally different from edge-emitting lasers.