Fabry-Pérot Optical Resonator: Low Reflectivity Regime

Detailed Analysis of `Chapt9Fig6a1.m`

Generated Documentation

Abstract

This document provides comprehensive theoretical foundations and line-by-line code analysis for `Chapt9Fig6a1.m`, which calculates the intensity transmission spectrum of a Fabry-Pérot optical resonator with Fresnel reflectivity (low R). The code demonstrates the fundamental physics of optical cavities, including mode spacing, finesse, and the Airy function that governs resonance structure.

Contents

1 Theoretical Foundation

1.1 Axioms of Optical Resonators

Axiom 1 (Wave Superposition). *Electric fields from multiple reflections add coherently:*

$$E_{total} = \sum_{n=0}^{\infty} E_n e^{i\phi_n} \tag{1}$$

where the phase ϕ_n depends on optical path length.

Axiom 2 (Resonance Condition). *Constructive interference occurs when the round-trip phase is a multiple of 2π:*

$$2n_r L = m\lambda, \quad m \in \mathbb{Z} \tag{2}$$

Axiom 3 (Energy Conservation). *For a lossless cavity: $R + T = 1$, where R is reflectivity and T is transmissivity.*

1.2 Fundamental Definitions

Definition 1 (Fresnel Reflectivity). *At normal incidence on a dielectric interface:*

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \tag{3}$$

For a semiconductor-air interface ($n_r \approx 3.3$):

$$R = \left(\frac{1 - 3.3}{1 + 3.3}\right)^2 = \left(\frac{-2.3}{4.3}\right)^2 \approx 0.286 \tag{4}$$

Definition 2 (Free Spectral Range). *The frequency spacing between adjacent longitudinal modes:*

$$\Delta\nu_{FSR} = \frac{c}{2n_r L} \tag{5}$$

or in wavelength:

$$\Delta\lambda_{FSR} = \frac{\lambda^2}{2n_r L} \tag{6}$$

Definition 3 (Finesse). *The ratio of mode spacing to mode linewidth:*

$$\mathcal{F} = \frac{\Delta\nu_{FSR}}{\delta\nu} = \frac{\pi\sqrt{R_1 R_2}}{1 - R_1 R_2} \tag{7}$$

Definition 4 (Coefficient of Finesse). *An alternative measure used in the Airy function:*

$$F = \left(\frac{2\mathcal{F}}{\pi}\right)^2 = \frac{4R_1 R_2}{(1 - R_1 R_2)^2} \tag{8}$$

1.3 Core Theorems

Theorem 1 (Airy Function for Transmission). *The intensity transmission through a Fabry-Pérot cavity is:*

$$\frac{I_t}{I_0} = \frac{T_{max}}{1 + F\sin^2(\delta/2)} \tag{9}$$

where $\delta = 2\pi \cdot 2n_r L/\lambda = 4\pi n_r L\nu/c$ is the round-trip phase.

Proof. Consider successive reflections inside the cavity. The transmitted field is:

$$E_t = E_0 t_1 t_2 \sum_{n=0}^{\infty} (r_1 r_2)^n e^{in\delta} \tag{10}$$

The geometric series sums to:

$$E_t = \frac{E_0 t_1 t_2}{1 - r_1 r_2 e^{i\delta}} \tag{11}$$

Taking the intensity $I = |E|^2$:

$$I_t = \frac{I_0 T_1 T_2}{|1 - r_1 r_2 e^{i\delta}|^2} = \frac{I_0 T_1 T_2}{(1-R)^2 + 4R\sin^2(\delta/2)} \tag{12}$$

For $R_1 = R_2 = R$ and using $T = 1 - R$:

$$I_t = \frac{I_0}{1 + F\sin^2(\delta/2)}, \quad F = \frac{4R}{(1-R)^2} \tag{13}$$

□

Theorem 2 (Peak Intensity Enhancement). *At resonance* ($\sin(\delta/2) = 0$)*, the internal intensity is enhanced:*

$$\frac{I_{internal}}{I_0} = \frac{1}{(1-R)^2} = \frac{1}{(1-\sqrt{R_1 R_2})^2} \tag{14}$$

Theorem 3 (Mode Linewidth). *The full-width at half-maximum of each resonance:*

$$\delta\nu = \frac{\Delta\nu_{FSR}}{\mathcal{F}} = \frac{c(1-R)}{2\pi n_r L\sqrt{R}} \tag{15}$$

2 Line-by-Line Code Analysis

2.1 Initialization

```
%Chapt9Fig6a1
%Fabry-Perot optical resonator
clear;
clf;
err=0.00001;
```

Program header; `err` is defined but not used (likely for convergence checks in other versions).

2.2 Physical Parameters

```
c=3e8;                  %speed of light in vacuum
nr=3.3;                 %effective refractive index
lambda0=1310e-9;        %center emission wavelength
Lc=300e-6;              %cavity length
```

Cavity parameters:

$$c = 3 \times 10^8 \text{ m/s} \tag{16}$$

$$n_r = 3.3 \quad \text{(GaAs/InP range)} \tag{17}$$

$$\lambda_0 = 1310 \text{ nm} \tag{18}$$

$$L_c = 300\ \mu\text{m} \tag{19}$$

```
r1=((1-nr)/(1+nr))^2;      %reflectivity of mirror1
r2=((1-nr)/(1+nr))^2;      %reflectivity of mirror2
r=r1*r2;                   %round-trip reflectivity
```

Fresnel reflectivity calculation:

$$R_1 = R_2 = \left(\frac{1-n_r}{1+n_r}\right)^2 = \left(\frac{-2.3}{4.3}\right)^2 = 0.286 \tag{20}$$

Round-trip: $R = R_1 R_2 = 0.0819$.

Note: The code uses the full Fresnel formula but squares it, giving the intensity reflectivity.

2.3 Finesse and Spectral Parameters

```
F=pi*r^0.5/(1-r);          %optical Finesse
Fconst=(2*F/pi)^2;
Imax=1/((1-r)^2);          %peak intensity normalized to I0
```

Finesse calculation:

$$\mathcal{F} = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi\sqrt{0.0819}}{1-0.0819} = \frac{0.899}{0.918} = 0.98 \tag{21}$$

This is a **low finesse** cavity—broad resonances.

Coefficient of finesse:

$$F_{const} = \left(\frac{2\mathcal{F}}{\pi}\right)^2 = \frac{4R}{(1-R)^2} = \frac{0.328}{0.843} = 0.389 \tag{22}$$

Peak intensity enhancement:

$$I_{max} = \frac{1}{(1-R)^2} = \frac{1}{0.843} = 1.19 \tag{23}$$

Only 19% enhancement due to low finesse.

```
f0=c*1e-12/lambda0;        %center frequency in THz
deltaf=c*1e-12/(2*Lc*nr); %FSR in THz
deltawavelength=lambda0^2/(2*Lc*nr);  %FSR in m
```

Spectral quantities:

$$f_0 = \frac{c}{\lambda_0} = \frac{3\times10^8}{1.31\times10^{-6}} = 229 \text{ THz} \tag{24}$$

$$\Delta f_{FSR} = \frac{c}{2n_r L} = \frac{3\times10^8}{2\times3.3\times300\times10^{-6}} = 0.152 \text{ THz} = 152 \text{ GHz} \tag{25}$$

$$\Delta\lambda_{FSR} = \frac{\lambda_0^2}{2n_r L} = \frac{(1.31\times10^{-6})^2}{1.98\times10^{-3}} = 0.87 \text{ nm} \tag{26}$$

2.4 Main Computation Loop

```
for i=[1:1:1000]
    Frequency(i)=(f0+i*0.001);
    Fwavelength(i)=c/Frequency(i);
    x=sin(pi*Frequency(i)/deltaf);
    x2=x*x;
    Intensity(i)=Imax/(1+(Fconst*(x2)));
end
```

Frequency sweep: From f_0 to $f_0 + 1$ THz in 0.001 THz steps.

Airy function implementation:

$$I(\nu) = \frac{I_{max}}{1 + F_{const} \sin^2\left(\frac{\pi\nu}{\Delta\nu_{FSR}}\right)} \tag{27}$$

The argument $\pi\nu/\Delta\nu_{FSR}$ gives:

- At $\nu = m \cdot \Delta\nu_{FSR}$: $\sin = 0$, resonance peak
- At $\nu = (m + 1/2) \cdot \Delta\nu_{FSR}$: $\sin = \pm 1$, minimum

2.5 Plotting

```
figure(1);
plot(Frequency,Intensity);
axis([f0,f0+1,0,4]);
xlabel('Frequency, \nu(THz)'),ylabel('Intensity');
ttl=sprintf('Chapt9Fig6a1, r1=%4.2f, r2=%4.2f, nr=%4.2f, f0=%7.2e THz, 
    Lc=%7.2e m',r1,r2,nr,f0,Lc)
title(ttl);
```

Frequency-domain plot showing resonance peaks.

```
figure(2);
plot(Fwavelength*10^-12,Intensity);
xlabel('Wavelength, \lambda (m)'),ylabel('Intensity');
title(ttl);
```

Wavelength-domain plot (note: `10^-12` scaling issue in original code).

3 Numerical Methods

3.1 Direct Evaluation

The code uses direct evaluation of the Airy function rather than summing infinite series. This is numerically stable and efficient.

3.2 Frequency Grid

With 1000 points over 1 THz:

$$\Delta\nu_{sample} = 1 \text{ GHz} \tag{28}$$

Since $\Delta\nu_{FSR} = 152$ GHz, there are ~ 7 FSR periods in the plot, each sampled by ~ 150 points—adequate resolution.

3.3 Peak Resolution

For low finesse $\mathcal{F} \approx 1$, the linewidth is:

$$\delta\nu \approx \Delta\nu_{FSR} = 152 \text{ GHz} \tag{29}$$

The 1 GHz sampling easily resolves these broad peaks.

4 Physical Interpretation

4.1 Low Finesse Characteristics

With $R_1 = R_2 = 0.286$:

- **Finesse** $\mathcal{F} \approx 1$: Poor quality factor
- **Broad resonances**: Linewidth $\approx$ FSR
- **Weak enhancement**: Peak only 19% above baseline
- **Poor mode selectivity**: Overlapping resonances

4.2 Comparison to Laser Requirements

A practical laser requires:

- Higher reflectivity: $R > 0.9$ for $\mathcal{F} > 30$
- Sharp resonances: For single-mode selection
- Higher enhancement: For reduced threshold

This simulation represents a bare semiconductor facet without coatings.

4.3 Mode Spectrum

The allowed longitudinal modes satisfy:

$$\nu_m = m \cdot \Delta\nu_{FSR} = m \cdot \frac{c}{2n_r L} \tag{30}$$

Near $\lambda_0 = 1310$ nm:

$$m = \frac{2n_r L}{\lambda_0} = \frac{2 \times 3.3 \times 300}{1.31} = 1511 \tag{31}$$

Mode number ~ 1500, confirming the cavity supports many modes.

5 Summary

This code simulates a Fabry-Pérot resonator with:

- Fresnel reflectivity from semiconductor-air interface
- Low finesse ($\mathcal{F} \approx 1$) due to $R \approx 0.29$
- 300 μm cavity length typical of edge-emitting lasers
- FSR of 152 GHz (0.87 nm)

Key results:

- Broad, overlapping resonance peaks
- Minimal intensity enhancement
- Many longitudinal modes ($m \sim 1500$)
- Representative of uncoated laser facets

This serves as a baseline for comparison with high-reflectivity coated cavities.