Chapt9Exercise2.m

Peak Gain and Total Spontaneous Emission

Carrier Density Dependence for Laser Design

Semiconductor Physics Documentation

Abstract

This document analyzes `Chapt9Exercise2.m`, which calculates the peak optical gain and total spontaneous emission as functions of carrier density. These relationships are fundamental for semiconductor laser design, determining threshold current and efficiency.

Contents

1 Theoretical Foundation

1.1 Gain-Carrier Density Relationship

Theorem 1 (Empirical Gain Model). *The peak gain can be approximated by a logarithmic or linear relationship:*

$$g_{\max} \approx g_0 \ln\left(\frac{n}{n_0}\right) \quad \text{or} \quad g_{\max} \approx a(n - n_0) \tag{1}$$

where n_0 is the transparency carrier density and g_0 (or a) is the gain coefficient.

Definition 1 (Transparency Carrier Density). *At transparency ($g = 0$), the quasi-Fermi level separation equals the bandgap:*

$$\Delta\mu = \mu_e + \mu_h = E_g \tag{2}$$

(in the notation where energies are measured from band edges)

1.2 Total Spontaneous Emission

Definition 2 (Integrated Spontaneous Emission). *The total spontaneous emission rate is:*

$$R_{sp} = \int_0^\infty r_{sp}(E)\, dE = \int_0^\infty C\sqrt{E} \cdot f_e(E) \cdot f_h(E)\, dE \tag{3}$$

Theorem 2 (Carrier Lifetime). *The radiative recombination rate defines the spontaneous lifetime:*

$$R_{sp} = \frac{n}{\tau_{sp}} = Bn^2 \tag{4}$$

where B is the radiative recombination coefficient (for non-degenerate case).

1.3 Quasi-Fermi Level Separation

Theorem 3 (Transparency Condition). *For population inversion to exist at the bandgap energy:*

$$\mu_e + \mu_h > E_g \quad \Rightarrow \quad \Delta\mu > 0 \tag{5}$$

(measuring chemical potentials from band edges)

2 Line-by-Line Code Analysis

2.1 Physical Constants and Material Parameters

```
clf
    echarge=1.6021764e-19;
    hbar=1.05457159e-34;
    c = 2.99792458e8;
    kB=8.61734e-5;
    epsilon0=8.8541878e-12;
```

Fundamental constants.

```
    m0=9.109382e-31;
    me=0.07*m0;
    mhh=0.5*m0;
    mr=1/(1/me+1/mhh);
    rerr=1e-3;
```

$$m_e = 0.07m_0, \quad m_{hh} = 0.5m_0, \quad m_r = \frac{m_e m_{hh}}{m_e + m_{hh}} \tag{6}$$

GaAs effective masses.

```
nr=3.3;
Eg=1.4
```

$$n_r = 3.3, \quad E_g = 1.4 \text{ eV} \tag{7}$$

GaAs optical and electronic properties.

2.2 Carrier Density Range

```
n1=1e18;
ncarrier1=n1*1e6;
n2=1e19;
ncarrier2=n2*1e6;
```

$$n \in [10^{18}, 10^{19}] \text{ cm}^{-3} \tag{8}$$

Carrier density range spanning typical laser operation.

2.3 Temperature

```
kelvin=300.0;
kBT=kB*kelvin;
beta=1/kBT;
```

$$T = 300 \text{ K}, \quad k_B T \approx 25.9 \text{ meV} \tag{9}$$

Room temperature operation.

2.4 Gain Constant

```
const=2.64e4;
```

$$C = 2.64 \times 10^4 \text{ cm}^{-1} \tag{10}$$

Calibrated to give realistic gain values.

2.5 Main Loop Over Carrier Density

```
deltacarrier=(ncarrier2-ncarrier1)/100;
for k=1:100
    ncarrier(k)=ncarrier1+(k-1)*deltacarrier;
```

$$n_k = 10^{18} + (k-1) \times 9 \times 10^{22} \text{ m}^{-3} \tag{11}$$

100 carrier density points from 10^{18} to 10^{19} cm^{-3}.

2.6 Chemical Potential Calculation

```
    muhh(k)=mu(mhh,ncarrier(k),kelvin,rerr);
mue(k) =mu(me, ncarrier(k),kelvin,rerr);
deltamu(k)=mue(k)+muhh(k);
```

$$\mu_h(n_k), \quad \mu_e(n_k), \quad \Delta\mu(n_k) = \mu_e + \mu_h \tag{12}$$

Calculate quasi-Fermi levels and their sum for each carrier density.

2.7 Energy Integration Loop

```
deltae=0.001;
rspon=0;
peakgain(k)=-10000;
rspontotal(k)=0;
```

Initialize with $\Delta E = 1$ meV, large negative initial peak gain.

```
for j=1:500
Energy(j)=j*deltae;
Ehh=Energy(j)/(1+mhh/me);
Ee=Energy(j)/(1+me/mhh);
fhh=fermi(beta,Ehh,muhh(k));
fe=fermi(beta,Ee,mue(k));
```

$$E_j \in [1, 500] \text{ meV}, \quad E_h, E_e, f_h, f_e \tag{13}$$

Loop over energies and compute Fermi functions.

2.8 Gain and Spontaneous Emission Computation

```
gain(j)=const*(Energy(j)^0.5)*(fhh+fe-1);
rspon=rspon+(const*(Energy(j)^0.5))*(fe*fhh)*deltae;
if(gain(j) > peakgain(k))
   peakgain(k)=gain(j);
end
```

$$g(E) = C\sqrt{E}(f_e + f_h - 1) \tag{14}$$

$$R_{sp} = \sum_j C\sqrt{E_j} \cdot f_e \cdot f_h \cdot \Delta E \tag{15}$$

Track maximum gain and integrate spontaneous emission.

```
rspontotal(k)=rspon;
```

Store total spontaneous emission for this carrier density.

2.9 Plotting Results

```
figure(1)
plot(ncarrier./1e6, deltamu);
xlabel('Carrier concentration, n (cm^{-3})');
ylabel('Difference in chemical potential,  \Delta\mu (eV)');
```

Plot 1: $\Delta\mu$ vs carrier density.

```
figure(2)
plot(ncarrier./1e6, peakgain);
xlabel('Carrier concentration, n (cm^{-3})');
ylabel('Peak optical gain, g (cm^{-1})');
```

Plot 2: Peak gain vs carrier density.

```
figure(3)
plot(ncarrier./1e6, rspontotal);
xlabel('Carrier concentration, n (cm^{-3})');
ylabel('Total spontaneous emission, rsp _{total} (arb.)');
```

Plot 3: Total spontaneous emission vs carrier density.

3 Numerical Method: Peak Finding and Integration

3.1 Peak Gain Determination

The code uses a simple maximum search:

$$g_{\max} = \max_{E}\{g(E)\} \tag{16}$$

A more sophisticated approach would use gradient-based search or interpolation to find the true maximum between grid points.

3.2 Trapezoidal Integration for R_{sp}

The spontaneous emission is integrated using a Riemann sum:

$$R_{sp} \approx \sum_{j=1}^{500} r_{sp}(E_j) \cdot \Delta E \tag{17}$$

This is equivalent to the trapezoidal rule for uniformly spaced points.

4 Physical Interpretation

4.1 $\Delta\mu$ vs Carrier Density

- $\Delta\mu$ increases with n
- At low n: $\Delta\mu < 0$ (absorption)
- At transparency: $\Delta\mu = 0$
- At high n: $\Delta\mu > 0$ (gain)
- Relationship is approximately logarithmic in n

4.2 Peak Gain vs Carrier Density

- Below transparency: $g_{\max} < 0$ (net absorption)
- Above transparency: $g_{\max}$ increases approximately linearly with $n - n_0$
- Typical values: $g_{\max} \sim 100$–1000 cm^{-1} for laser operation

4.3 Total Spontaneous Emission

- Increases faster than linearly with n (due to Fermi function overlap)
- Represents fundamental loss mechanism in lasers
- Determines below-threshold LED emission

5 Design Implications

5.1 Threshold Condition

At laser threshold:

$$g_{\max} = \alpha_i + \alpha_m \tag{18}$$

where α_i is internal loss and α_m is mirror loss.

From the $g_{\max}(n)$ curve, one can determine the threshold carrier density n_{th}.

5.2 Threshold Current

$$J_{th} = \frac{e n_{th} d}{\tau_{sp}(n_{th})} \tag{19}$$

where d is the active layer thickness.

6 Summary

This code generates the key curves for semiconductor laser design:

1. $\Delta\mu(n)$: Determines transparency condition
2. $g_{\max}(n)$: Determines threshold carrier density
3. $R_{sp}(n)$: Determines below-threshold emission and carrier lifetime

These relationships are essential for optimizing laser structures and predicting device performance.