Light-Current Characteristics of Semiconductor Lasers

Detailed Analysis of `Chapt9Exercise5.m`

Generated Documentation

Abstract

This document provides comprehensive theoretical foundations and line-by-line code analysis for `Chapt9Exercise5.m`, which computes the steady-state light output versus injection current (L-I curve) for a semiconductor laser diode. The code uses fourth-order Runge-Kutta integration to solve rate equations at each current level, extracting the steady-state response to construct the characteristic L-I curve that defines laser threshold and slope efficiency.

Contents

1 Theoretical Foundation

1.1 Axioms of L-I Characteristics

Axiom 1 (Steady-State Equilibrium). *At each injection current level, the laser reaches a steady state where:*

$$\frac{dn}{dt} = 0, \quad \frac{dS}{dt} = 0 \tag{1}$$

Axiom 2 (Threshold Transition). *Below threshold ($I < I_{th}$), the laser operates as an LED with spontaneous emission. Above threshold ($I > I_{th}$), stimulated emission dominates and output power increases linearly with current.*

Axiom 3 (Carrier Clamping). *Above threshold, the carrier density clamps at the threshold value:*

$$n \approx n_{th} = n_0 + \frac{\kappa}{\Gamma g_0 v_g / n_g} \quad \text{for } I > I_{th} \tag{2}$$

1.2 Fundamental Definitions

Definition 1 (Threshold Current). *The threshold current is the injection level required to achieve transparency plus compensate all optical losses:*

$$I_{th} = \frac{eV}{\tau_n(n_{th})} \cdot n_{th} = eV(An_{th} + Bn_{th}^2 + Cn_{th}^3) \tag{3}$$

Definition 2 (Differential Quantum Efficiency). *The external differential quantum efficiency relates photon output rate to injected electron rate:*

$$\eta_d = \eta_i \cdot \frac{\alpha_m}{\alpha_m + \alpha_i} \tag{4}$$

where η_i is the internal quantum efficiency.

Definition 3 (Slope Efficiency). *The slope of the L-I curve above threshold:*

$$\frac{dP}{dI} = \eta_d \cdot \frac{h\nu}{e} = \eta_d \cdot \frac{hc}{e\lambda} \tag{5}$$

with units of W/A.

1.3 Core Theorems

Theorem 1 (Steady-State Photon Density). *Above threshold, the steady-state photon density is:*

$$S_0 = \frac{I - I_{th}}{eV \cdot \kappa} \cdot \frac{1}{1 + \varepsilon S_0/(g - \kappa)} \tag{6}$$

For small gain compression:

$$S_0 \approx \frac{\Gamma(I - I_{th})}{eV \cdot \kappa} \tag{7}$$

Proof. Setting $dS/dt = 0$ in the rate equation:

$$(g - \kappa)S_0 + \beta R_{sp} = 0 \tag{8}$$

Above threshold, $g \approx \kappa$ (gain clamping), so the excess current $(I - I_{th})$ goes entirely into stimulated emission:

$$\frac{I - I_{th}}{eV} = g \cdot S_0 \approx \kappa S_0 \tag{9}$$

□

Theorem 2 (L-I Curve Shape). *The complete L-I characteristic is:*

$$P(I) = \begin{cases} \beta B n^2 V \cdot \frac{hc}{\lambda} \cdot \frac{\alpha_m}{\alpha_m + \alpha_i} & I < I_{th} \\ \\ \eta_d \frac{hc}{e\lambda}(I - I_{th}) & I > I_{th} \end{cases} \tag{10}$$

Theorem 3 (Carrier Density vs Current). *The carrier density behavior:*

$$n(I) = \begin{cases} \text{increases with } I & I < I_{th} \\ n_{th} \approx \text{constant} & I > I_{th} \end{cases} \tag{11}$$

This "clamping" is a signature of lasing action.

2 Line-by-Line Code Analysis

2.1 Header and Constants

```
%Chapt9Exercise5.m
%  solves single mode laser rate equations
%  using 4th order Runge-Kutta method and plots
%  light output, L, and carrier density, n, vs current, I
```

Program header describing L-I curve computation.

```
%  gain=g=gamma*gslope*(n-n0)*(1-epsi*s)
%  fcn=n'=(I/ev)-(n/tau_n)-(g*s)
%  fcs=s'=(g-K)*s+beta*Rsp
```

Rate equations:

$$g = \Gamma g_0 (n - n_0)(1 - \varepsilon S) \tag{12}$$

$$\frac{dn}{dt} = \frac{I}{eV} - \frac{n}{\tau_n} - gS \tag{13}$$

$$\frac{dS}{dt} = (g - \kappa)S + \beta R_{sp} \tag{14}$$

```
clear;
clf;
% declare constants
hconstjs=6.6262e-34;       %Planck's constant (J s)
echargec=1.6021e-19;       %electron charge (C)
vlightcm=2.997925e10;      %velocity of light (cm s-1)
pi2=2*pi;                  %value of 2pi

ngplot=100;                %number of points plotted
nsteps=2000;               %steps in time integration
```

Physical constants and simulation parameters:

$$h = 6.6262 \times 10^{-34} \text{ J}\cdot\text{s} \tag{15}$$

$$e = 1.6021 \times 10^{-19} \text{ C} \tag{16}$$

$$c = 2.997925 \times 10^{10} \text{ cm/s} \tag{17}$$

The L-I curve has 100 points; each uses 2000 RK4 steps to reach steady state.

2.2 Parameter File Reading

```
fp = fopen('datainLI.txt','r');
for i=1:1:20
    x(i) = fscanf(fp, '%e', 1);
end
fclose(fp);

ngroup     = x(1);         %refractive index
clength    = x(2);         %cavity length (cm)
thick      = x(3);         %thickness (cm)
width      = x(4);         %width (cm)
```

Cavity parameters from file: Reading standardized input file for device geometry.

```
tincrement = x(5);         %time increment (s)
initialn   = x(6);         %initial carrier density (cm-3)

Anr        = x(7);         %non-radiative rate (s-1)
Bcons      = x(8);         %radiative coefficient (cm3 s-1)
Ccons      = x(9);         %Auger coefficient (cm6 s-1)
```

Recombination parameters: The ABC model for carrier lifetime:

$$\frac{1}{\tau_n} = A + Bn + Cn^2 \tag{18}$$

```
n0density  = x(10);        %transparency density (cm-3)
gslope     = x(11);        %gain-slope (cm2 s-1)
epsi       = x(12);        %gain compression (cm3)
beta       = x(13);        %spontaneous emission factor
gamma_cons = x(14);        %confinement factor
wavelength = x(15);        %emission wavelength (um)
```

Gain parameters:

$$g = \Gamma g_0(n - n_0)(1 - \varepsilon S) \tag{19}$$

```
mirrone    = x(16);        %reflectivity mirror 1
mirrtwo    = x(17);        %reflectivity mirror 2
alfa_i     = x(18);        %internal loss (cm-1)

Imin       = x(19);        %start current value
Imax       = x(20);        %end current value
```

Optical losses and current range:

$$\alpha_m = \frac{1}{2L} \ln \frac{1}{R_1 R_2} \tag{20}$$

Current swept from I_{min} to I_{max}.

2.3 Initialization

```
sdensity = 0.0;
ndensity = initialn * 1.0e-21;

alfa_m   = log( 1.0/(mirrone * mirrtwo) ) / ( 2.0 * clength );
alfasum  = alfa_m + alfa_i;
```

Initial conditions: Start with empty cavity and calculate total optical loss.

2.4 Unit Rescaling

```
echarge     = echargec*1.0e9*1.0e3;     % rescaled charge
vlight      = vlightcm/1.0e9/1.0e-7;  % nm/ns

tincrement = tincrement * 1.0e9;
gslope      = gslope * 1.0e14;
n0density   = n0density * 1.0e-21;
```

Unit conversions: Time in ns, length in nm, current in mA for numerical stability.

```
clength     = clength / 1.0e-7;
width       = width / 1.0e-7;
thick       = thick / 1.0e-7;

Anr         = Anr / 1.0e9;
Bcons       = Bcons / (1.0e9*1.0e-21);
Ccons       = Ccons / (1.0e9*1.0e-42);
```

Length dimensions converted to nm; recombination coefficients rescaled.

```
epsi        = epsi / 1.0e-21;
gslope      = gslope * vlight / ngroup;
alfasum     = alfasum * 1.0e-7;
kappa       = alfasum * vlight / ngroup;
volume      = clength * width * thick;
```

Key quantities:

$$\kappa = \frac{\alpha_{tot} v_g}{n_g} \quad \text{(photon decay rate)} \tag{21}$$

$$V = L \times w \times d \quad \text{(active volume)} \tag{22}$$

2.5 Power Output Scaling

```
tmp   = hconstjs * (vlightcm * 1.0e-2)/(wavelength * 1.0e-6);
tmp   = tmp * alfa_m * vlightcm;
tmp   = tmp * 1000.0*volume/ngroup;
lightout = tmp*(1.0-mirrone)/(2.0-mirrone-mirrtwo);
```

Photon to power conversion:

$$P = \frac{hc}{\lambda} \cdot \alpha_m v_g S V \cdot \frac{1 - R_1}{(1 - R_1) + (1 - R_2)} \tag{23}$$

Output power from facet 1 proportional to its transmission.

2.6 Data Array

```
data(1)=gamma_cons;
data(2)=gslope;
data(3)=n0density;
data(4)=epsi;
data(5)=kappa;
data(6)=Bcons;
data(7)=Anr;
data(8)=Ccons;
data(9)=volume;
data(10)=beta;
```

Parameters passed to Runge-Kutta integrator.

2.7 Main L-I Curve Loop

```
photonold=0;
current=0;
dcurrent=(Imax-Imin)/ngplot;

for j=1:ngplot
   current=current+dcurrent;

   for i = 1:nsteps;
       [sdensity, ndensity]=runge4(current, sdensity, ndensity, tincrement, data);
       carrier = ndensity * 1.0e21/1.0e18;
       photon = (lightout*sdensity);
       photonold=photon;
   end;
   currentI(j)=current;
   photonL(j)=photon;
   carrierN(j)=carrier;
end
```

Algorithm structure:

1. Sweep current from I_{min} to I_{max} in 100 steps
2. At each current, run 2000 RK4 iterations to reach steady state
3. Record final (steady-state) photon density and carrier density
4. Use previous steady state as initial condition for next current (warm start)

Warm start advantage: By using the steady state from I_j as initial condition for I_{j+1}, convergence is faster for small ΔI.

2.8 Plotting

```
figure(1);
plot(currentI,photonL,'r-');
title('Chapt9Exercise5, light output as function of injection current')
xlabel('Current, I (mA)');
ylabel('Light, L (mW)');
```

L-I curve (linear scale): Shows threshold kink and linear slope above threshold.

```
figure(2);
plot(currentI,log(photonL),'r-');
xlabel('Current, I (mA)');
ylabel('log(light), log(L) (mW)');
```

L-I curve (log scale): Reveals spontaneous emission below threshold.

```
figure(3);
plot(currentI,carrierN,'b');
title('Chapt9Exercise5, carrier density as function of injection current')
xlabel('Current, I (mA)');
ylabel('Carrier density, n (cm^{-3})');
```

n-I curve: Shows carrier clamping above threshold.

3 Numerical Methods

3.1 Steady-State Extraction

The code uses time-domain integration to find steady state rather than solving algebraic equations directly. This approach:

- Handles the nonlinear gain compression term naturally
- Avoids issues with multiple solutions
- Captures hysteresis if present (though not in this simple model)

3.2 Convergence Criterion

The implicit criterion is that 2000 time steps (at typically 1 ps each = 2 ns) is sufficient for transients to decay. For typical parameters:

- Carrier lifetime $\tau_n \sim 3$ ns
- Relaxation oscillation period ~ 0.2 ns
- 2000 steps covers several τ_n, ensuring convergence

3.3 Warm Start Strategy

By not resetting initial conditions between current points:

$$(S_0, n_0)_{I_{j+1}} = (S_{ss}, n_{ss})_{I_j} \tag{24}$$

This dramatically reduces required steps since the system starts near the new equilibrium.

4 Physical Interpretation

4.1 Below Threshold ($I < I_{th}$)

The device operates as an LED:

- Carrier density increases with current: $n \propto I$
- Spontaneous emission: $P \propto Bn^2 \propto I^2$
- Gain < loss, so no amplification

4.2 At Threshold ($I = I_{th}$)

The transition point:

- $g(n_{th}) = \kappa$
- Round-trip gain equals unity
- Coherent oscillation begins

4.3 Above Threshold ($I > I_{th}$)

Laser operation:

- Carrier density clamps: $n \approx n_{th}$
- Additional current goes to photons: $\Delta P \propto (I - I_{th})$
- Slope efficiency: $\eta_d \cdot hc/(e\lambda) \approx 0.3 - 0.5$ W/A

4.4 Characteristic Features

The L-I curve reveals:

1. **Threshold current**: The kink position
2. **Slope efficiency**: Derivative above threshold
3. **Internal losses**: From $\eta_d = \alpha_m/(\alpha_m + \alpha_i)$
4. **Carrier clamping**: Constant n above threshold

5 Summary

This code computes the fundamental L-I characteristic of a semiconductor laser by:

- Sweeping injection current over a specified range
- Using RK4 time integration to find steady-state at each current
- Employing warm start for computational efficiency
- Extracting both light output and carrier density vs current

The resulting L-I curve demonstrates:

- Sharp threshold transition
- Linear power increase above threshold
- Carrier density clamping
- Logarithmic plot reveals sub-threshold spontaneous emission

The external parameter file approach allows easy variation of device parameters to study their effects on threshold current and slope efficiency.