Chapt8Fig14b.m

Angular Distribution of Scattering Rate

Thomas-Fermi Screening at $n = 10^{17}$ cm^{-3}

Semiconductor Physics Documentation

Abstract

This document analyzes `Chapt8Fig14b.m`, which plots the differential scattering rate as a function of scattering angle using Thomas-Fermi screening. Unlike Chapt8Fig14a (which integrates over angles), this code shows the angular distribution, revealing the forward-scattering peak characteristic of screened Coulomb scattering.

Contents

1 Theoretical Foundation

1.1 Differential Scattering Rate

Definition 1 (Angular Differential Rate). *The differential scattering rate per unit angle is:*

$$\frac{d(1/\tau)}{d\theta} = \frac{2\pi m^*}{\hbar^3 k^3} n_i \left(\frac{e^2}{4\pi\varepsilon_0\varepsilon_r} \right)^2 \frac{\sin\theta}{(q^2 + q_{TF}^2)^2} \tag{1}$$

where $q = 2k\sin(\theta/2)$*.*

1.2 Forward Scattering Peak

Theorem 1 (Small-Angle Dominance). *At small angles* ($\theta \to 0$)*:*

$$q = 2k\sin(\theta/2) \approx k\theta \to 0 \tag{2}$$

Without screening, the rate diverges as $1/q^4 \propto 1/\theta^4$*.*

With Thomas-Fermi screening:

$$\frac{d(1/\tau)}{d\theta} \propto \frac{\sin\theta}{(k^2\theta^2 + q_{TF}^2)^2} \tag{3}$$

The screening regularizes the divergence, producing a finite peak at small θ*.*

1.3 Transformation to η Variable

Using $\eta = \sin(\theta/2)$:

$$\frac{d(1/\tau)}{d\theta} = \frac{d(1/\tau)}{d\eta} \cdot \frac{d\eta}{d\theta} = \frac{d(1/\tau)}{d\eta} \cdot \frac{\cos(\theta/2)}{2} \tag{4}$$

2 Line-by-Line Code Analysis

2.1 Initialization and Constants

```
clear;clf;
n=1e17;
n=n*1e6;
```

$$n = 10^{17}\ \text{cm}^{-3} = 10^{23}\ \text{m}^{-3} \tag{5}$$

Carrier/impurity density.

```
m0=9.109382e-31;
echarge=1.6021764e-19;
hbar=1.05457159e-34;
epsilon0=8.8541878e-12;
hbar=1.054592e-34;
hbar3=hbar^3;
```

Physical constants (note: $\hbar$ *is defined twice, second value used).*

```
m=0.07*m0;
epsilonr0=13.2;
epsilon=epsilon0*epsilonr0;
```

$$m^* = 0.07m_0, \quad \varepsilon = \varepsilon_0\varepsilon_r \tag{6}$$

GaAs material parameters.

2.2 Fermi Wavevector

```
kF=(3*(pi^2)*n)^(1/3)
```

$$k_F = (3\pi^2 \times 10^{23})^{1/3} \approx 1.43 \times 10^8 \text{ m}^{-1} \tag{7}$$

Fermi wavevector.

2.3 Energy Loop

```
E=-0.1;
for j=1:1:2
E=E+0.2
    k=sqrt(2*m*E*echarge)/hbar;
k3=k^3;
```

$$E \in \{0.1, 0.3\} \text{ eV}, \quad k = \frac{\sqrt{2m^* E}}{\hbar} \tag{8}$$

Two electron energies for comparison.

2.4 Thomas-Fermi Wavevector

```
  qTF=sqrt(kF*m*echarge^2/(epsilon*(pi^2)*(hbar^2)));
```

$$q_{\text{TF}} = \sqrt{\frac{k_F m^* e^2}{\varepsilon \pi^2 \hbar^2}} \tag{9}$$

Thomas-Fermi screening wavevector.

2.5 Angular Grid

```
theta=[pi/180:pi/180:pi];
```

$$\theta \in [1, 180] \text{ with } 1^\circ \text{ steps} \tag{10}$$

Coarser grid than Exercise7 (1° vs 0.1°).

2.6 Momentum Transfer and Variables

```
        q=2*k*sin(theta/2);
    eta=sin(theta/2);
    deta=pi*cos(theta/2)./2/180;
    eta3=eta.^3;
```

$$q = 2k\sin(\theta/2), \quad \eta = \sin(\theta/2), \quad d\eta = \frac{\pi\cos(\theta/2)}{360} \tag{11}$$

Momentum transfer and angular variables.

2.7 Thomas-Fermi Dielectric Function

```
    TFepsilon=epsilon*(1+qTF^2./q.^2);
```

$$\varepsilon_{\text{TF}}(q) = \varepsilon\left(1 + \frac{q_{\text{TF}}^2}{q^2}\right) \tag{12}$$

Full dielectric function with screening.

2.8 Differential Scattering Rate

```
TFrate =2*pi*m/hbar3/k3*n*(echarge^2/4/pi)^2.*deta./TFepsilon.^2./
    eta3;
```

$$\frac{d(1/\tau)}{d\theta} = \frac{2\pi m^*}{\hbar^3 k^3} n \left(\frac{e^2}{4\pi}\right)^2 \frac{d\eta}{\varepsilon_{\mathrm{TF}}^2 \eta^3} \tag{13}$$

Differential scattering rate formula.

2.9 Plotting

```
plot(theta*180/pi, TFrate,'b');
axis([0 140 0 0.1e12]);
```

$$\text{Plot range: } \theta \in [0, 140], \quad \frac{1}{\tau} \in [0, 10^{11}] \text{ s}^{-1}/\text{degree} \tag{14}$$

Blue curve shows angular distribution of scattering.

3 Physical Interpretation

3.1 Forward Scattering Peak

The plot shows:

- Peak at small angles (forward scattering dominates)
- Rapid decay with increasing angle
- Higher energy electrons have broader angular distribution

3.2 Effect of Screening

Without screening ($q_{\mathrm{TF}} = 0$):

$$\frac{d(1/\tau)}{d\theta} \propto \frac{1}{\sin^4(\theta/2)} \to \infty \text{ as } \theta \to 0 \tag{15}$$

With Thomas-Fermi screening:

$$\frac{d(1/\tau)}{d\theta} \propto \frac{1}{(\sin^2(\theta/2) + q_{\mathrm{TF}}^2/4k^2)^2} \tag{16}$$

The screening cuts off the divergence at $\theta \sim q_{\mathrm{TF}}/k$.

3.3 Energy Dependence

For $E = 0.1$ eV vs $E = 0.3$ eV:

- Higher energy $\Rightarrow$ larger k
- Larger k $\Rightarrow$ smaller peak angle (screening less effective)
- Overall rate lower at higher energy ($\propto 1/k^3$)

Code	Density	Integration	Output
Chapt8Fig14a	$10^{17}, 10^{18}$	Over θ	$1/\tau$ vs E
Chapt8Fig14b	10^{17}	None	$d(1/\tau)/d\theta$ vs θ
Chapt8Exercise7a/b	$10^{18}, 10^{14}$	None	$d(1/\tau)/d\theta$ vs θ

Table 1: Comparison of Chapter 8 scattering codes

4 Comparison with Other Codes

5 Summary

This code plots the angular distribution of the ionized impurity scattering rate, showing the characteristic forward-scattering peak that is regularized by Thomas-Fermi screening. The peak width is determined by the ratio q_{TF}/k, and the amplitude scales inversely with k^3. This complements Chapt8Fig14a, which integrates over angles to give the total scattering rate.