Chapt9Exercise1.m

Optical Gain and Spontaneous Emission in Semiconductors

Direct-Gap Semiconductor Laser Theory

Semiconductor Physics Documentation

Abstract

This document provides a comprehensive analysis of `Chapt9Exercise1.m`, which calculates the optical gain spectrum and spontaneous emission rate as functions of photon energy for various carrier densities. The theory covers the fundamental physics of population inversion and stimulated emission in direct-gap semiconductors.

Contents

1 Theoretical Foundation

1.1 Fundamental Axioms of Light-Matter Interaction

Axiom 1 (Einstein Relations). *For a two-level system, the rates of absorption, stimulated emission, and spontaneous emission are related by:*

$$B_{12} = B_{21}, \quad A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21} \tag{1}$$

where B coefficients describe stimulated processes and A describes spontaneous emission.

Axiom 2 (Fermi's Golden Rule for Optical Transitions). *The optical transition rate is:*

$$W = \frac{2\pi}{\hbar} |M_{cv}|^2 \rho_r(E) \tag{2}$$

where M_{cv} is the optical matrix element and ρ_r is the reduced density of states.

1.2 Joint Density of States

Definition 1 (Reduced Density of States). *For parabolic bands with electron mass m_e and hole mass m_{hh}, the reduced mass is:*

$$m_r = \frac{m_e m_{hh}}{m_e + m_{hh}} \tag{3}$$

The joint density of states is:

$$\rho_r(\hbar\omega - E_g) = \frac{1}{2\pi^2} \left(\frac{2m_r}{\hbar^2}\right)^{3/2} \sqrt{\hbar\omega - E_g} \tag{4}$$

1.3 Optical Gain Theory

Theorem 1 (Gain Formula). *The optical gain (or absorption) coefficient is:*

$$g(\hbar\omega) = C \cdot \sqrt{\hbar\omega - E_g} \cdot (f_e + f_h - 1) \tag{5}$$

where:

- *C is a material constant*
- *f_e, f_h are electron and hole Fermi functions*
- *E_g is the bandgap energy*

Definition 2 (Bernard-Duraffourg Condition). *For optical gain ($g > 0$), we require:*

$$f_e + f_h > 1 \tag{6}$$

This is equivalent to population inversion.

Theorem 2 (Population Inversion Criterion). *The condition $f_e + f_h > 1$ is satisfied when:*

$$\hbar\omega < \mu_e + \mu_h = \Delta\mu \tag{7}$$

where μ_e and μ_h are the quasi-Fermi levels for electrons and holes.

1.4 Spontaneous Emission

Theorem 3 (Spontaneous Emission Rate). *The spontaneous emission spectrum is:*

$$r_{sp}(\hbar\omega) = C \cdot \sqrt{\hbar\omega - E_g} \cdot f_e \cdot f_h \tag{8}$$

1.5 Energy Distribution in Bands

For a photon of energy $\hbar\omega$ above the bandgap:

$$E_e = \frac{\hbar\omega - E_g}{1 + m_e/m_{hh}} = \frac{m_{hh}}{m_e + m_{hh}}(\hbar\omega - E_g) \tag{9}$$

$$E_h = \frac{\hbar\omega - E_g}{1 + m_{hh}/m_e} = \frac{m_e}{m_e + m_{hh}}(\hbar\omega - E_g) \tag{10}$$

2 Line-by-Line Code Analysis

2.1 Physical Constants

```
clf;
   echarge=1.6021764e-19;
   hbar=1.05457159e-34;
   c = 2.99792458e8;
   kB=8.61734e-5;
   epsilon0=8.8541878e-12;
```

$$e, \hbar, c, k_B, \varepsilon_0 \tag{11}$$

Fundamental physical constants.

2.2 Material Parameters

```
   m0=9.109382e-31;
   me=0.07*m0;
   mhh=0.5*m0;
   mr=1/(1/me+1/mhh);
   rerr=1e-3;
```

$$m_e = 0.07m_0, \quad m_{hh} = 0.5m_0, \quad m_r = \frac{m_e m_{hh}}{m_e + m_{hh}} \tag{12}$$

Effective masses for GaAs: light electron mass, heavy hole mass.

```
   nr=3.3;
   Eg=1.4
```

$$n_r = 3.3, \quad E_g = 1.4 \text{ eV} \tag{13}$$

Refractive index and bandgap of GaAs.

2.3 Temperature Parameters

```
   kelvin=300.0;
   kBT=kB*kelvin;
   beta=1/kBT;
```

$$T = 300 \text{ K}, \quad k_B T = 25.9 \text{ meV}, \quad \beta = 1/(k_B T) \tag{14}$$

Room temperature thermal parameters.

2.4 Carrier Density Loop

```
  for k=1:1:10
     n=k*1.e18;
       ncarrier=n*1e6;
```

$$n \in \{1, 2, \ldots, 10\} \times 10^{18} \text{ cm}^{-3} \tag{15}$$

Loop over 10 carrier densities from 10^{18} *to* 10^{19} *cm*$^{-3}$.

2.5 Chemical Potentials

```
  muhh=mu(mhh,ncarrier,kelvin,rerr)
  mue=mu(me,ncarrier,kelvin,rerr)
  deltamu=mue+muhh
```

$$\mu_h = \mu(m_{hh}, n, T), \quad \mu_e = \mu(m_e, n, T), \quad \Delta\mu = \mu_e + \mu_h \tag{16}$$

Calculate quasi-Fermi levels using external function mu.m.

2.6 Material Constant

```
     const=2.64e4;
```

$$C = 2.64 \times 10^4 \text{ cm}^{-1} \tag{17}$$

Constant calibrated to give ~ 330 *cm*$^{-1}$ *gain at* $n = 2 \times 10^{18}$ *cm*$^{-3}$.

2.7 Energy Loop

```
     deltae=0.001;
       for j=1:300
       Energy(j)=j*deltae;
```

$$\Delta E = 1 \text{ meV}, \quad E_j = j \times 1 \text{ meV for } j = 1, \ldots, 300 \tag{18}$$

Energy above bandgap, from 1 meV to 300 meV.

2.8 Energy Distribution

```
       Ehh=(Energy(j))/(1+mhh/me);
       Ee=(Energy(j))/(1+me/mhh);
```

$$E_h = \frac{E}{1 + m_{hh}/m_e}, \quad E_e = \frac{E}{1 + m_e/m_{hh}} \tag{19}$$

Partition energy between electron and hole according to effective masses.

2.9 Fermi Functions

```
       fhh=fermi(beta,Ehh,muhh);
       fe=fermi(beta,Ee,mue);
```

$$f_h = \frac{1}{e^{\beta(E_h - \mu_h)} + 1}, \quad f_e = \frac{1}{e^{\beta(E_e - \mu_e)} + 1} \tag{20}$$

Fermi-Dirac occupation probabilities.

2.10 Gain and Spontaneous Emission

```
        gain(j)=const*(Energy(j)^0.5)*(fe+fhh-1);
        rspon(j)=(const)*(Energy(j)^0.5)*(fe*fhh);
```

$$g(E) = C\sqrt{E}(f_e + f_h - 1) \tag{21}$$

$$r_{sp}(E) = C\sqrt{E} \cdot f_e \cdot f_h \tag{22}$$

Core physics: gain requires population inversion, spontaneous emission proportional to product.

2.11 Plotting

```
    figure(1)
    hold on;
    plot(Energy+Eg, gain);
    xlabel('Photon energy, h\omega (eV)');
    ylabel('Optical gain, g (cm^{-1})');
```

Plot gain vs photon energy (Energy + Eg gives absolute photon energy).

```
    figure(2)
    hold on;
    plot(Energy+Eg,rspon,'r');
    xlabel('Photon energy, h\omega (eV)');
    ylabel('Spontaneous emission, rsp (arb.)');
```

Plot spontaneous emission spectrum.

3 Physical Interpretation

3.1 Gain Spectrum Features

1. **Transparency point**: $g = 0$ when $f_e + f_h = 1$
2. **Peak gain**: Occurs near $\hbar\omega \approx E_g + k_B T$
3. **High-energy cutoff**: $g \to 0$ as $\hbar\omega \to \Delta\mu + E_g$
4. **Low-energy cutoff**: $g \to 0$ as $\hbar\omega \to E_g$ (no states)

3.2 Carrier Density Dependence

- Higher $n \Rightarrow$ larger μ_e, μ_h
- Larger $\Delta\mu \Rightarrow$ wider gain spectrum
- Peak gain increases approximately linearly with n

3.3 Spontaneous Emission Spectrum

- Peaks at higher energy than gain peak
- Broader than gain spectrum (no sharp cutoff)
- Area under curve gives total spontaneous emission rate

4 The mu Function: Chemical Potential Calculation

The external function `mu(m, n, T, rerr)` solves:

$$n = N_c \mathcal{F}_{1/2}\left(\frac{\mu}{k_B T}\right) \tag{23}$$

where $\mathcal{F}_{1/2}$ is the Fermi-Dirac integral of order 1/2.

Typical approach:

1. Initial guess: Non-degenerate approximation $\mu_0 = k_B T \ln(n/N_c)$
2. Newton-Raphson iteration
3. Convergence when relative change $< rerr$

5 Summary

This code demonstrates the fundamental physics of semiconductor optical gain:

- Gain requires population inversion ($f_e + f_h > 1$)
- The gain spectrum has a characteristic shape determined by the joint density of states and Fermi functions
- Higher carrier injection increases both peak gain and spectral width
- Spontaneous emission is always present and represents the minimum loss mechanism