Math Programming Exam 1 Take-Home

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1 Algebraic Formulation

1.1 Decision Variables

1. T_1L_i: Turns of Truck Type 1 (34 Ton Capacity) to Ft. Liberty for Week $i \in \{1, 2, \ldots, 4\}$
2. T_2L_i: Turns of Truck Type 2 (18 Ton Capacity) to Ft. Liberty for Week $i \in \{1, 2, \ldots, 4\}$
3. T_3L_i: Turns of Truck Type 3 (11 Ton Capacity) to Ft. Liberty for Week $i \in \{1, 2, \ldots, 4\}$
4. T_1J_i: Turns of Truck Type 1 (34 Ton Capacity) to Ft. Jackson for Week $i \in \{1, 2, \ldots, 4\}$
5. T_2J_i: Turns of Truck Type 2 (18 Ton Capacity) to Ft. Jackson for Week $i \in \{1, 2, \ldots, 4\}$
6. T_3J_i: Turns of Truck Type 3 (11 Ton Capacity) to Ft. Jackson for Week $i \in \{1, 2, \ldots, 4\}$
7. I_i: Tons of Supplies Held for the next Week for Week $i \in \{1, 2, ..., 4\}$

1.2 Objective

Minimize Cost. Objective Function: $z = \sum_{i=1}^{4} 360T_1L_i + 240T_2L_i + 195T_3L_i + 600T_1J_i + 390T_2J_i + 330T_3J_i$

1.3 Constraints

1. $T_1L_i + T_1J_i \leq 20 \ \forall \ i \in \{1, 2, \ldots, 4\}$: Total available turns per week of Truck Type 1
2. $T_2L_i + T_2J_i \leq 30 \ \forall \ i \in \{1, 2, \ldots, 4\}$: Total available turns per week of Truck Type 2
3. $T_3L_i + T_3J_i \leq 24 \ \forall \ i \in \{1, 2, \ldots, 4\}$: Total available turns per week of Truck Type 3
4. $34T_1L_1 + 18T_2L_1 + 11T_3L_1 \leq 700$: Supplies available from Ft. Liberty Week 1 (Tons)
5. $34T_1L_2 + 18T_2L_2 + 11T_3L_2 \leq 600$: Supplies available from Ft. Liberty Week 2 (Tons)
6. $34T_1L_3 + 18T_2L_3 + 11T_3L_3 \leq 500$: Supplies available from Ft. Liberty Week 3 (Tons)
7. $34T_1L_4 + 18T_2L_4 + 11T_3L_4 \leq 600$: Supplies available from Ft. Liberty Week 4 (Tons)
8. $34T_1J_1 + 18T_2J_1 + 11T_3J_1 \leq 600$: Supplies available from Ft. Jackson Week 1 (Tons)
9. $34T_1J_2 + 18T_2J_2 + 11T_3J_2 \leq 800$: Supplies available from Ft. Jackson Week 2 (Tons)
10. $34T_1J_3 + 18T_2J_3 + 11T_3J_3 \leq 400$: Supplies available from Ft. Jackson Week 3 (Tons)
11. $34T_1J_4 + 18T_2J_4 + 11T_3J_4 \leq 500$: Supplies available from Ft. Jackson Week 4 (Tons)
12. $34(T_1L_1 + T_1J_1) + 18(T_2L_1 + T_2J_1) + 11(T_3L_1 + T_3J_1) + I_0 - I_1 \geq 1000$: Supplies for Week 1 (Tons)

13. $34(T_1L_2 + T_1J_2) + 18(T_2L_2 + T_2J_2) + 11(T_3L_2 + T_3J_2) + I_1 - I_2 \geq 1200$: Supplies for Week 2 (Tons)
14. $34(T_1L_3 + T_1J_3) + 18(T_2L_3 + T_2J_3) + 11(T_3L_3 + T_3J_3) + I_2 - I_3 \geq 1200$: Supplies for Week 3 (Tons)
15. $34(T_1L_4 + T_1J_4) + 18(T_2L_4 + T_2J_4) + 11(T_3L_4 + T_3J_4) + I_3 - I_4 \geq 1000$: Supplies for Week 4 (Tons)
16. $I_i \leq 300 \ \forall \ i \in \{1, 2, \dots 4\}$: Maximum amount of Supplies that can be held over for the next week (Tons)
17. $T_jL_i, T_jJ_i, I_i \geq 0 \ \forall \ i \in \{1, 2, \dots, 4\}, j \in \{1, 2, 3\}$: Non-negativity

2 Report:

2.1 Minimum Cost Solution

The table below shows the truck allocation that will minimize the cost to supply these training events.

Truck Type	Truck Destination	Week 1	Week 2	Week 3	Week 4
34 Tons	Ft. Liberty	5	0	11	8
18 Tons	Ft. Liberty	27	29	7	18
11 Tons	Ft. Liberty	4	7	0	0
34 Tons	Ft. Jackson	15	20	9	12
18 Tons	Ft. Jackson	0	0	4	0
11 Tons	Ft. Jackson	0	1	2	0

Using this allocation, the total cost will be $66,375.

2.2 Notes and Recommendations

The 34 ton trucks provide our most economical method of moving supplies, while the 11 ton trucks are inefficient for the task. Each week, we use the maximum available turns with the 34 ton trucks, but in total we only use 14 turns of the 11 ton trucks. As a result, we could reduce cost by using only six of the 11-ton trucks, allowing the other six to be used for other missions. However, even through utilization of the 11-ton trucks will be minimal during the final two weeks, we do not recommend dropping lower than six so that we have some built-in margin for potential maintenance issues and so that we can transport supplies in and around the training area more easily.

Of our two sources of supplies, Ft. Liberty is significantly more economical to draw supplies from. Week to week, this plan will draw all but five tons of the total tonnage of supplies they have available to offer us. If more supplies were available from Ft. Liberty, or if they had a change in mission allowing for supplies to be reallocated to us from there, we could reduce costs further.

Week two into week three is the critical phase of this operation from a supply perspective. Both weeks have a higher supply requirement, while week three has our lowest supply availability for the forts. As a result, we must fully max out our on-site storage from week two into week three in order to meet the demands of week three. This leaves us with no flexibility or room for error, as that week we draw all available supplies and use the max we are able to carry over. At the same time, we have capacity unused both in supplies available and assets to transport them in weeks one and two. We recommend identifying another site to store additional supplies in the training area, perhaps by having earlier units take extra supplies forward to pre-stage them for follow-on units, so we can build in some safety margin to allow for Murphy.