



Indian Institute of Technology Roorkee

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presents



GAMBITOR

METIOX

Duration : 2 hrs

Max. Marks : 100

Details

Name: _____

Registration No: _____

Invigilator Signature : _____

Instructions

- Read and attempt all the questions carefully.
- You are advised to not get stuck in one question. Time is limited and hence, manage it accordingly.
- No negative marking will be awarded.
- Show all the steps carefully. Step marking will be awarded, if steps are valid and correct.
- Clearly mention all assumptions (if any).
- Use of scientific calculator is allowed.
- If anyone is found using unfair means, strict actions will be taken against them.

All the Best!!!

This time, Sherlock Holmes was not presented with a murder case. Interestingly, in an attempt to test him, his friend A left him a series of problems to solve. Watson is busy today and thus, Holmes needs your help to show his friend A who the boss really is.

The problems are given below. Assist him in solving them!

Ques 1

10 Marks

Three friends worked on a project for 30 days. In the course of work, all of them remained absent for a few days. The first friend remained absent for 10 days more than the second friend. Also, the third friend did one third of the total work. How many days more than the third friend was the first one absent?

Ques 2

10 Marks

Holmes is given a bag containing 6561 dimensionally identical marbles. Exactly one marble is heavier than the remaining 6560 marbles. He also has a conventional weighing machine and an infinite number of empty bags. Help him find the minimum number of weighing attempts required to find the heavier marble.

Note: Both side of weighing machine is empty initially.

Ques 3

10 Marks

The friend A is also a mathematical genius. In an attempt to test Holmes' mathematical mettle, he left behind this problem. Can you help him?

Let N be a positive integer and P is a special integer whose all digits are the same. Given that, $N + 2N + \dots + 9N = P$ so find the sum of all digits of P.

Ques 4

10 Marks

Trigonometry is not really the detective's forte and thus he is completely relying upon you to find this answer.

Simplify the given expression as simple as possible:

$$\frac{\cos^8 x - \sin^8 x}{(1 - 2\sin^2 x \cos^2 x)(\sin x + \cos x)}$$

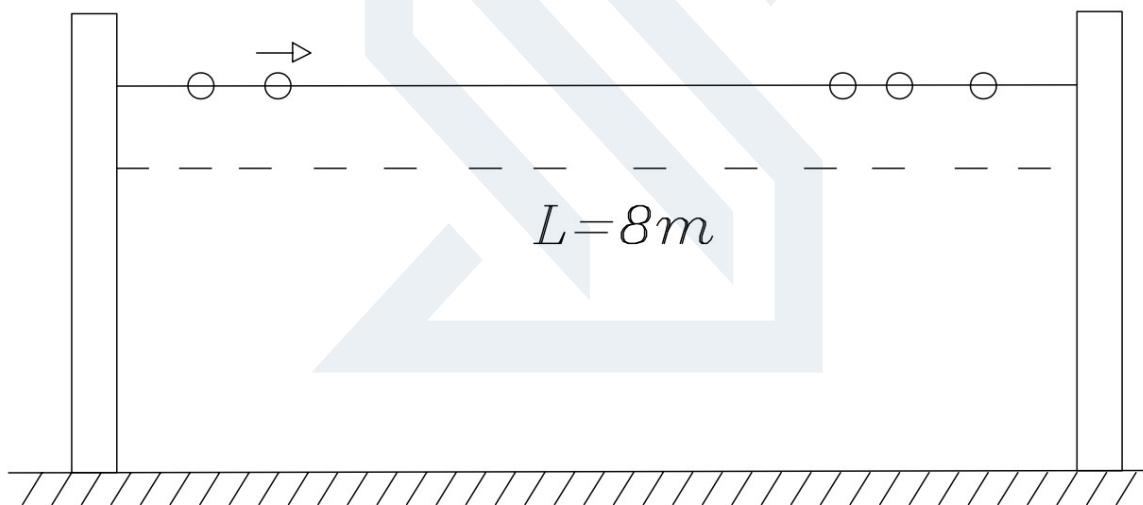
Ques 5

15 Marks

Holmes' friend A is astonished that he got through the halfway point so easily with your help. So this time, he presents you a puzzle from his favorite subject physics! The puzzle is given below:

100000 identical beads, each of mass 2 kg and radius 10mm are strung on a thin smooth rigid horizontal rod of length 8m and are at rest at random positions. The rod is mounted between two rigid supports. If one of the beads is now given speed v , the average force experienced by each support after a long time is (assume all collisions are elastic).

Hint: Force = Change in momentum/Total time or $(\Delta p / \Delta t)$



Ques 6

15 Marks

Friend A couldn't ever think that Holmes can solve his favorite physics puzzle so this time he tries his best that Holmes can't solve his puzzle and comes up with a mathematics puzzle!. Show The Friend A that no less than a mathematical genius than him!

A was so sure that Holmes can't solve this puzzle even with your help that he provided Holmes with some hints too! Who thought that it was one of his biggest mistakes!

Let a, b, c, \dots, n terms then

Athematic Mean (AM) is defined as $\frac{a + b + c + \dots}{n}$

Geometric Mean (GM) is defined as $\sqrt[n]{abc\dots}$

Harmonic Mean (HM) is defined as $\frac{n}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \dots}$

There is a relation between AM, GM & HM: $AM \geq GM \geq HM$.

Show that $a^4b^6d^{-8} + 2c^{\frac{5}{2}}b^{-3}d^3 + 2da^{-2} + 3c^{\frac{-5}{3}} \geq 8$

Ques 7

15 Marks

Friend A's vanity is totally destroyed as soon as Holmes cracked the mathematics puzzle with your help. He finally apologizes to you and Holmes for creating so much trouble. Holmes got to know that his friend A was trying to solve the Gambitor'23 Pinnacle Round test paper and was stuck midway. That's why he was asking you and Holmes for help in the form of puzzles. Now, you got to know that Holmes' friend needs genuine help, you would not lag behind in helping him. As a result here is a geometrical question that Friend A didn't know how to solve and asks you for help.

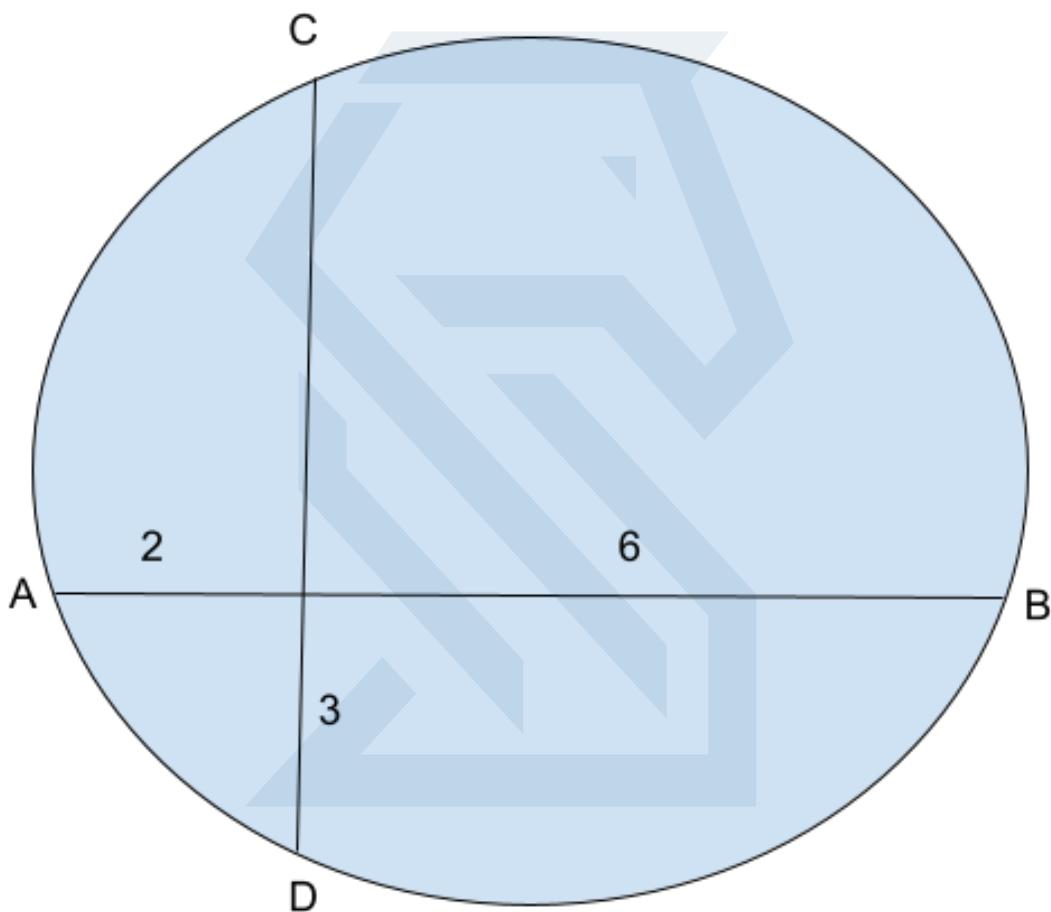
The question is:

Let AC be a line segment in the plane and B a point between A and C. Construct isosceles triangle ΔPAB and ΔQBC on one side of the segment AC such that $\angle APB = \angle BQC = 120^\circ$ and an isosceles triangle ΔRAC on the other side of AC such that $\angle ARC = 120^\circ$. Show that ΔPQR is an equilateral triangle.

Ques 8**15 Marks**

Friend A couldn't control his tears as he is on the verge to solve the complete Gambitor'23 Pinnacle Round test paper. But soon he noticed that only a little time is left and he still needs to solve one more question to complete the test paper. Hence he bid GoodBye to Holmes and directly asks you for help as he knows that you wouldn't deny helping him. Come on! Help him to solve the complete Gambitor'23 Pinnacle Round!

Find the radius of the circle.



End of the Paper
