



Indian Institute of Technology Roorkee

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presents



ATHENOX

Duration : 2 hrs

Max. Marks : 100

### Details

Name: \_\_\_\_\_

Registration No: \_\_\_\_\_

Invigilator Signature : \_\_\_\_\_

# Instructions

- Read and attempt all the questions carefully.
- You are advised to not get stuck in one question. Time is limited and hence, manage it accordingly.
- No negative marking will be awarded.
- Show all the steps carefully. Step marking will be awarded, if steps are valid and correct.
- Clearly mention all assumptions (if any).
- Use of scientific calculator is allowed.
- If anyone is found using unfair means, strict actions will be taken against them.

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**All the Best!!!**

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**15 Marks**

**Ques 1**

Let  $a, b, c$  be positive real numbers such that.

$$a^2 + b^2 + c^2 + (a + b + c)^2 \leq 4.$$

Prove that

$$\frac{(ab+1)}{(a+b)^2} + \frac{(bc+1)}{(b+c)^2} + \frac{(ca+1)}{(c+a)^2} \geq 3$$

**Ques 2**

**10 Marks**

Consider the following  $3 \times 2$  array formed by using the numbers 1, 2, 3, 4, 5, and 6 :

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = \begin{pmatrix} 1 & 6 \\ 2 & 5 \\ 3 & 4 \end{pmatrix}$$

Observe that all rows' sums are equal, but the sum of the squares is not the same for each row. Extend the above array to a  $3 \times k$  array  $(a_{1j})$   $3 \times k$  for a suitable  $k$ , adding more columns. Using the numbers 7, 8, 9,.....  $3k$  such that

$$\sum_{j=1}^k a_{1j} = \sum_{j=1}^k a_{2j} = \sum_{j=1}^k a_{3j} \text{ and } \sum_{j=1}^k (a_{1j})^2 = \sum_{j=1}^k (a_{2j})^2 = \sum_{j=1}^k (a_{3j})^2$$

**Ques 3**

**10 Marks**

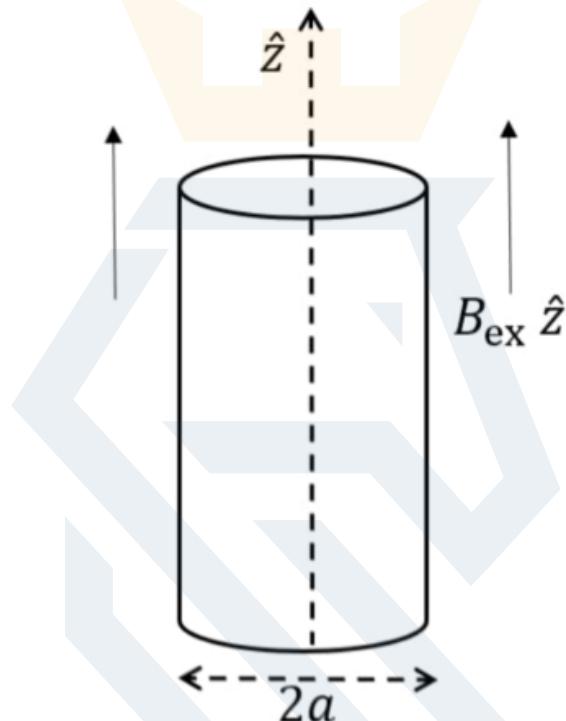
$$I = \int \sqrt[2]{x} \sqrt[3]{x} \sqrt[4]{x} \sqrt[5]{x} \dots dx$$

$$I = \frac{f(x)}{k} + c$$

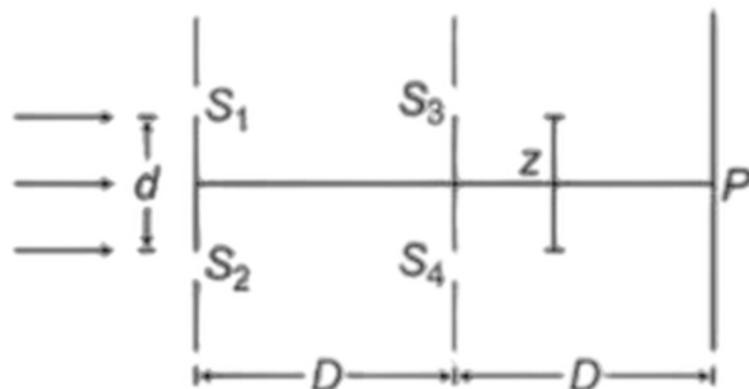
Find the nearest integer to  $\log_x [f(x)]$ .

**15 Marks****Ques 4**

An insulating uniformly charged cylindrical shell of radius lies with its axis along the  $z$ -axis. The shell's moment of inertia per unit length about the  $z$ -axis and the surface charge density are  $I$  and  $\sigma$ , respectively. The cylinder is placed in an external uniform magnetic field  $B_{ex}\hat{z}$ , and is initially at rest. Starting at  $t=0$ , the external magnetic field is slowly reduced to zero. What is the final angular velocity of the cylinder?

**15 Marks****Ques 5**

Consider the arrangement shown in the figure. By some mechanism, the separation between the slits  $S_3$  and  $S_4$  can be changed. The intensity is measured at point  $P$ , which is at the common perpendicular bisector of  $S_1S_2$  and  $S_3S_4$ . When  $z = D\lambda/2d$ , the intensity measured at  $P$  is  $I$ . Find the intensity when  $z$  is equal to  $D\lambda/d$ .

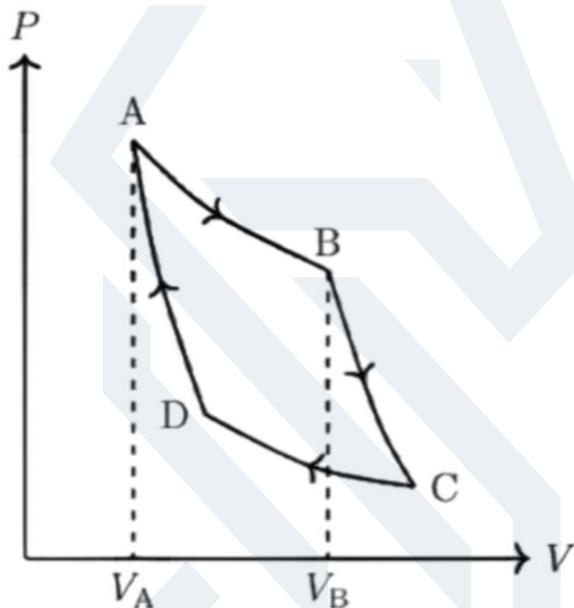


**10 Marks**

**Ques 6**

A certain gas obeys the equation of state  $U(S,V,N) = aS^7/V^4N^2$ , where  $a$  is a dimensional constant. Here  $U$  represents the internal energy of the gas,  $S$  is the entropy,  $V$  is the volume, and  $N$  is the fixed number of particles in the system.

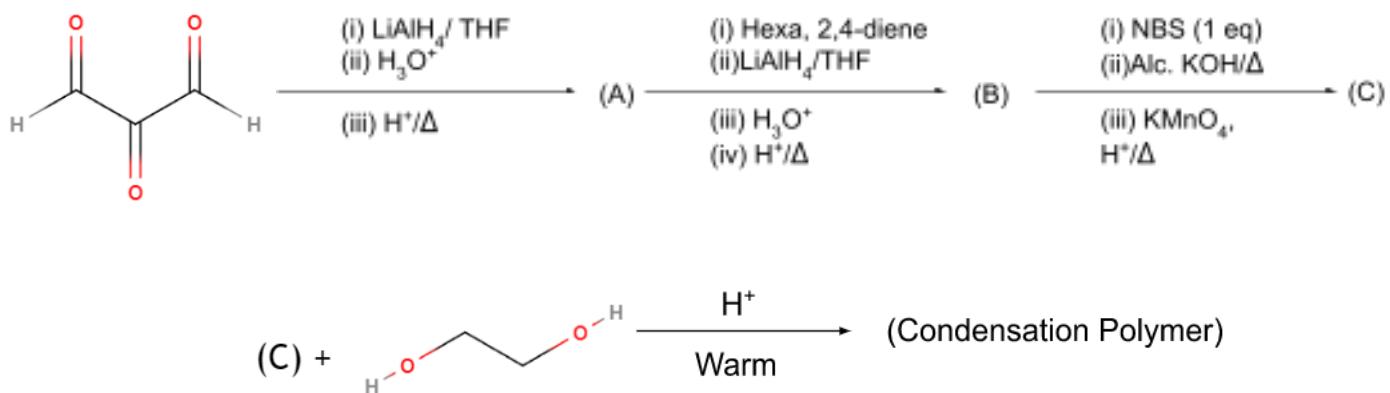
One mole of this gas executes a Carnot cycle ABCDA between reservoirs at temperatures  $T_1$  and  $T_2$  ( $T_1 > T_2$ ). Obtain the heat change in the process AB ( $Q_{AB}$ ) and work done by the system in the processes AB and BC ( $W_{AB}$ ,  $W_{BC}$ ) of the cycle. Express your answers only in terms of temperatures  $T_1$ ,  $T_2$ , volumes  $V_A$ ,  $V_B$ , and the other constants.



**15 Marks**

**Ques 7**

Find out (A), (B), (C), and the Condensation Polymer given in the above reactions.



### **Ques 8**

**10 Marks**



Given:  $K_f$  of  $\text{Zn}(\text{CN})_4^{2-} = 10^{16}$ ;  $K_f$  of  $\text{Cu}(\text{NH}_3)_4^{2+} = 10^{12}$ ;

$$E^\circ_{\text{Zn}|\text{Zn}^{2+}} = 0.76\text{V}; \quad E^\circ_{\text{Cu}^{2+}|\text{Cu}} = 0.34\text{V}, \quad 2.303\text{RT/F} = 0.06$$

The emf of the above cell is-

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**End of the Paper**

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