



Indian Institute of Technology Roorkee

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presents



APOLLOX

Duration : 2 hrs

Max. Marks : 100

Details

Name: _____

Registration No: _____

Invigilator Signature : _____

Instructions

- Read and attempt all the questions carefully.
- You are advised to not get stuck in one question. Time is limited and hence, manage it accordingly.
- No negative marking will be awarded.
- Show all the steps carefully. Step marking will be awarded, if steps are valid and correct.
- Clearly mention all assumptions (if any).
- Use of scientific calculator is allowed.
- If anyone is found using unfair means, strict actions will be taken against them.

All the Best!!!

Ques 1

15 Marks

Due to the recession, the CEO of Meta wishes to fire A number of high-paid employees and hire B number of low-paid employees. The CEO is a mathematics enthusiast, so she decided that A and B should be:

$$A = \sum_{1 \leq \alpha < \beta \leq 200} (\alpha + \beta), \text{ when } (\alpha + \beta) \text{ is even, and}$$

$$B = \sum_{1 \leq \alpha < \beta \leq 200} (\alpha + \beta), \text{ when } (\alpha + \beta) \text{ is odd}$$

Find the value of (B-A).

Ques 2

10 Marks

Once upon a time, there was a young scientist named Parvati who loved to conduct experiments in her backyard. One day, she decided to play with soap bubbles and see what happened when she gave them an electric charge.

She took a soap solution and blew a soap bubble of radius "r". The bubble was floating in the air, looking beautiful and fragile. Parvati then connected an ideal battery to the ends of a wire and placed them on either side of the bubble. Moments later, she noticed that the bubble started to glow with a faint electric light, and its surface became charged.

Curious to know what would happen next, Parvati decided to measure the radius of the bubble after giving it an electric charge. She observed that the bubble's new radius "R" was related to its initial radius "r", the surface tension "T", and the potential "V" applied to the wire.

Using her scientific knowledge, Parvati got an equation that related the new radius R to the initial radius r, where P_0 is the atmospheric pressure, the surface tension T, and the potential V. The equation was:

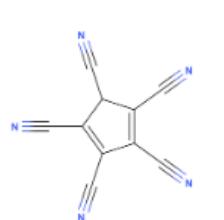
$$P_o(R^3 - r^3) + 4T(R^2 - r^2) - \frac{\epsilon_o(V^2 R)}{2} = 0$$

Unfortunately, Parvati lost the paper on which the proof was written. Now, as a fellow physics enthusiast, Parvati asks for your help in helping to prove the above equation.

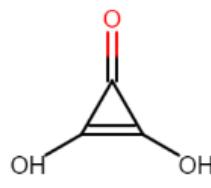
Ques 3

10 Marks

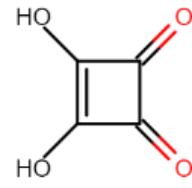
(a) Arrange the following compounds in order of increasing pK_a values, giving the reason for your answer in brief as well



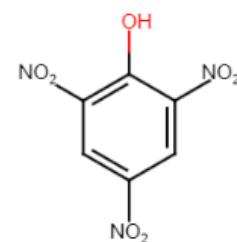
(A)



(B)

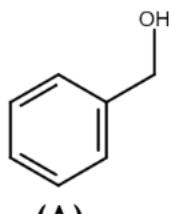


(C)

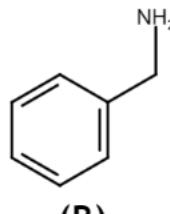


(D)

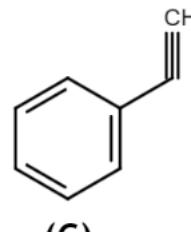
(b) Find the stability order of the conjugate bases of the given compounds.



(A)



(B)



(C)

Ques 4

15 Marks

Let A, B, C, and D lie on a line such that $AB = BC = CD = 1$. Points A and C are also joined by a semicircle with AC as the diameter, and P is a variable point on this semicircle such that $\angle PBD = \theta$, $0 \leq \theta \leq \pi$.

Let R is the region bounded by the arc AP, the straight line PD, and the line AD.

(a) The maximum possible area of region R is:

a) $\frac{(2\pi - \sqrt{3})}{12}$

b) $\frac{(2\pi + 3\sqrt{3})}{6}$

c) $\frac{(2\pi + 6\sqrt{3})}{12}$

d) $\frac{(2\pi + 3\sqrt{3})}{12}$

(b) Let 'L' is the perimeter of region 'R', then L is equal to:

a) $3 - \theta + \sqrt{5 - 4 \cos \theta}$

b) $3 + \pi + \sqrt{5 - 4 \cos \theta}$

c) $3 + \pi - \theta + \sqrt{5 - 4 \cos \theta}$

d) $3 + \pi + \theta + \sqrt{5 + 4 \cos \theta}$

(c) The non-negative difference between the greatest and least values of L is:

a) $3 - \sqrt{3} + \frac{\pi}{3}$

b) $-3 + \sqrt{3} + \frac{\pi}{3}$

c) $-3 + \sqrt{3} + \frac{2\pi}{3}$

d) $\pi - 2$

Ques 5

15 Marks

Let, $z = \prod_{i=0}^{88} \cos((i+1)^\circ)$ $y = \prod_{i=0}^{21} \cos((4i+2)^\circ)$

And, $z = \frac{y}{x}$

Find the nearest integer to $\frac{2}{7} \log_2(x)$

10 Marks

Ques 6

10 L of an ideal gas underwent an isochoric process, which resulted in a four-fold increase in pressure; the gas was then isobarically cooled, resulting in a reduction in volume to half. The initial pressure of the gas is 100 kPa.

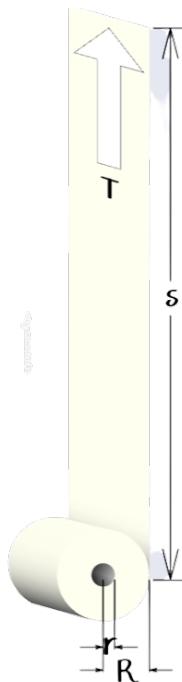
The Poisson constant of the gas, which identifies it as a monatomic ideal gas, i.e., $k = 5/3$

Find the overall amount of heat produced overall, the change in internal energy of the gas, and the work done by external forces during the described process.

15 Marks

A roll of toilet paper is held by the first piece and allowed to unfurl, as shown in the figure. The roll has an outer radius $R = 6.0 \text{ cm}$, an inner radius $r = 1.8 \text{ cm}$, a mass $m = 200 \text{ g}$, and falls a distance $s = 3.0 \text{ m}$. Assuming the outer diameter of the roll does not change significantly during the fall, determine

1. the tension in the sheets
2. the translational acceleration of the roll
3. the angular acceleration of the roll
4. the final translation speed of roll
5. the final angular speed of the roll



Ques 8

10 Marks

A solution contains a mixture of Ag^+ (0.1 M) and Hg^{2+} (0.1 M), which are to be separated by selective precipitation. Calculate the maximum concentration of iodide ion at which one of them gets precipitated almost completely. What percentage of that metal ion is precipitated?

$$K_{\text{sp}} (\text{AgI}) = 8.5 \times 10^{-17}$$

$$K_{\text{sp}} (\text{Hg}_2\text{I}_2) = 2.5 \times 10^{-26}$$

End of the Paper
