GAMBITOR
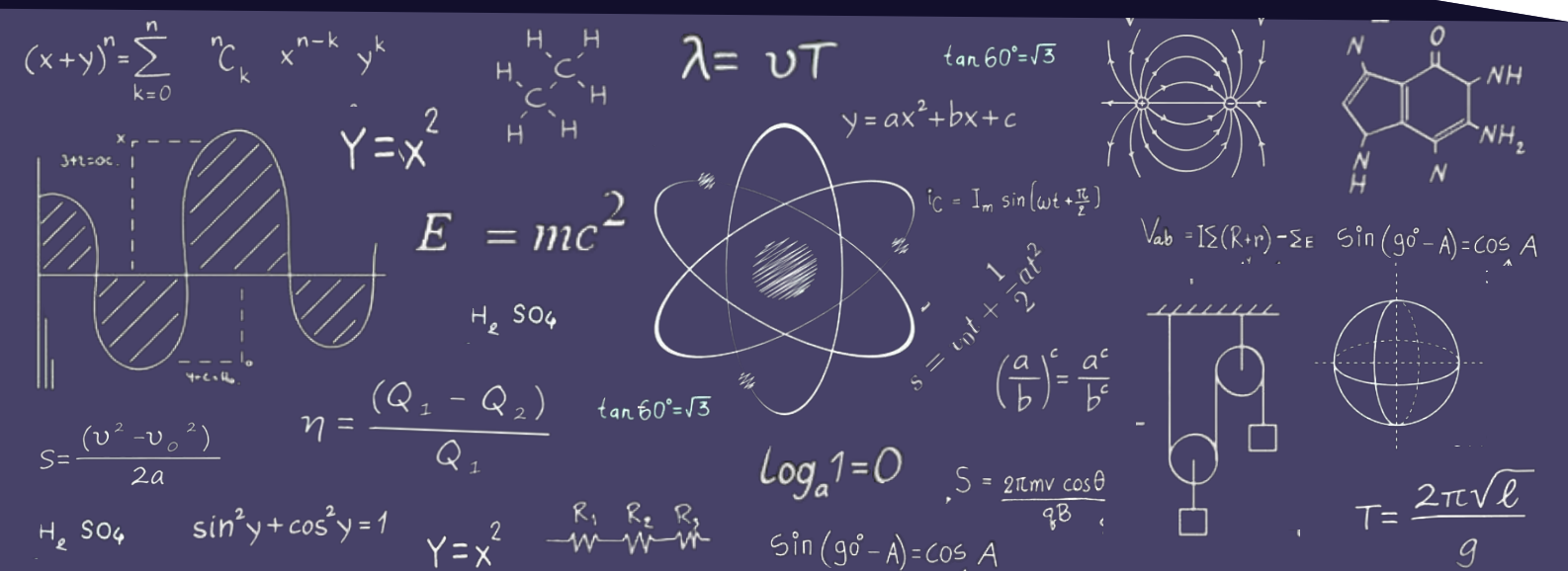
APOLLOX

Details

Name _____

Registration No. _____

Invigilator's Sign _____

Q1. Let f be a polynomial with integer coefficients. Define
$$a_1 = f(0), \quad a_2 = f(a_1) = f(f(0))$$
where, $\quad a_n = f(a_n-1)$ for $n > 2$.

If there exists a natural number $k > 2$ such that $a_k = 0$, then prove that either $a_1 = 0$ or $a_2 = 0$.

14 Marks

Q2. A massive disc rotates about a vertical axis at an angular velocity Ω. A smaller disc of mass m and radius r, whose axis is strictly vertical, is lowered on the first disc (given figure). The distance between the axes of the discs is d ($d > r$), and the coefficient of friction between them is μ.

Determine the steady-state angular velocity ω of the smaller disc. What moment of force M must be applied to the axis of the larger disc to maintain its velocity of rotation constant? The radius of the larger disc is $R > d + r$. The friction at the axes of the discs should be neglected.

14 Marks
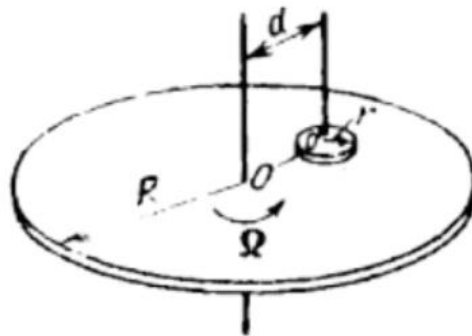
Q3. Let $r_1, r_2 \ldots \ldots r_9$, be the distinct complex roots of the polynomial $P(x) = x^9 - 9$.
Let $K = (r_i + r_j)$, that is, the product of all numbers of the form $r_i + r_j$, where i and j are integer for which $1 < i < j < 10$.
The value of $K^2/9^7$ will be....

14 Marks

Q4. In a the Pokemon world, the velocity distribution function of a gas molecules is given by
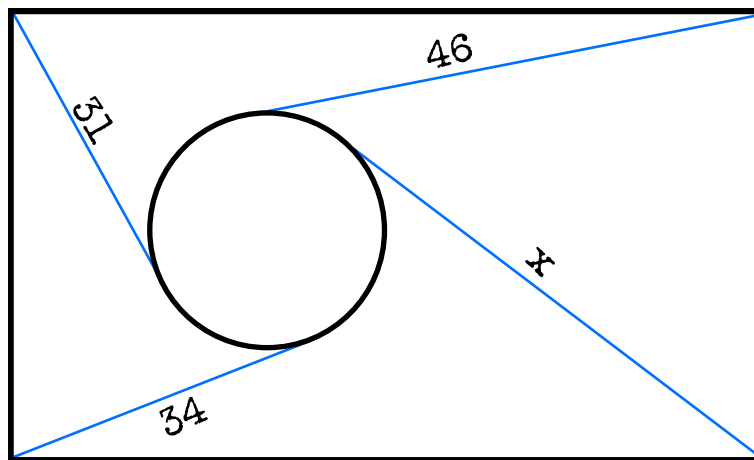
$$f(v) = 8v^2 \left(\frac{m}{2\pi k_b T} \right)^{\frac{5}{2}} e^{-\left(\frac{mv^2}{2k_b T} \right)}$$

Compute the most probable and the average speed.

14 Marks

Q 5. Find the value of 'x' if the given blue lines are tangent to the circle.

14 Marks

———————— *End of the paper* ————————