

# Capítulo 10

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## 1 Exercícios 12.1

1.

$$\begin{aligned} & \int \frac{1}{x} dx \\ &= \ln(x) + k \end{aligned}$$

2.

$$\begin{aligned} & \int 3 dx \\ &= 3 \int 1 dx \\ &= 3x + k \end{aligned}$$

3.

$$\begin{aligned} & \int x^5 dx \\ &= \frac{x^6}{6} + k \end{aligned}$$

4.

$$\begin{aligned} & \int x dx \\ &= \frac{x^2}{2} + k \end{aligned}$$

5.

$$\begin{aligned} & \int \sqrt{x} dx \\ &= \frac{2x^{\frac{3}{2}}}{3} + k \end{aligned}$$

6.

$$\begin{aligned}\int \sqrt[3]{(x^2)} \, dx \\ = \frac{5x^{\frac{7}{3}}}{7} + k\end{aligned}$$

7.

$$\begin{aligned}\int x^{-4} \, dx \\ = \int \frac{1}{x^4} \, dx \\ = -\frac{1}{3x^3} + k\end{aligned}$$

8.

$$\begin{aligned}\int \frac{1}{x^3} \, dx \\ = \frac{1}{2x^2} + k\end{aligned}$$

9.

$$\begin{aligned}\int \frac{x + x^2}{x^2} \, dx \\ = \int \frac{x + 1}{x} \, dx \\ = \int \left(\frac{1}{x} + 1\right) \, dx \\ = \int \frac{1}{x} \, dx + \int 1 \, dx \\ = \ln(x) + x + k\end{aligned}$$

10.

$$\begin{aligned}\int \left(\frac{1}{x} + \frac{1}{x^2}\right) \, dx \\ = \int \frac{1}{x} \, dx + \int \frac{1}{x^2} \, dx \\ = \ln(x) - \frac{1}{x} + k\end{aligned}$$

11.

$$\begin{aligned}& \int (x^2 + \frac{3}{x}) dx \\&= \int x^2 dx + 3 \int \frac{1}{x} dx \\&= 3\ln(x) + \frac{x^3}{3} + k\end{aligned}$$

12.

$$\begin{aligned}& \int \frac{x+1}{x} dx \\&= \int \frac{1}{x} dx + \int 1 dx \\&= \ln(x) + x + k\end{aligned}$$

13.

$$\begin{aligned}& \int (e^x + 4) dx \\&= \int e^x dx + 4 \int 1 dx \\&= e^x + 4x + k\end{aligned}$$

14.

$$\begin{aligned}& \int e^{5x} dx \\& \mathbf{u = 5x} \\&= \frac{1}{5} \int e^u du \\&= \frac{e^u}{5} \\&= \frac{e^{5x}}{5} + k\end{aligned}$$

15.

$$\begin{aligned}
 & \int e^{-2x} dx \\
 & \mathbf{u = -2x} \\
 & = -\frac{1}{2} \int e^u du \\
 & \quad = \frac{e^u}{2} \\
 & = \frac{e^{-2x}}{2} + k
 \end{aligned}$$

16.

$$\begin{aligned}
 & \int (e^{2x} + e^{-x}) dx \\
 & = \int e^{2x} dx + \int e^{-x} dx \\
 & = \frac{1}{2} \int e^u du + \int e^u du \\
 & = \frac{e^{2x}}{2} - e^{-x} + k
 \end{aligned}$$

17.

$$\begin{aligned}
 & \int \left( \frac{1}{x} + \frac{1}{e^x} \right) dx \\
 & = \int e^{-x} dx + \int \frac{1}{x} dx \\
 & \quad \mathbf{u = -x} \\
 & = \int e^u du + \int \frac{1}{x} dx \\
 & \quad = e^u + \ln(x) \\
 & = \ln(x) - e^{-x} + k
 \end{aligned}$$

18.

$$\begin{aligned}
 & \int (e^{4x} + \frac{1}{x^2}) dx \\
 &= \int e^{4x} dx + \int \frac{1}{x^2} dx \\
 & \quad \mathbf{u} = 4\mathbf{x} \\
 &= \frac{1}{4} \int e^u du + (-\frac{1}{x}) \\
 & \quad = \frac{e^u}{4} - \frac{1}{x} \\
 &= \frac{e^{4x}}{4} - \frac{1}{x} + k
 \end{aligned}$$

19.

$$\begin{aligned}
 & \int \frac{x^5 + x + 1}{x^2} dx \\
 &= \int (x^3 + \frac{1}{x} + \frac{1}{x^2}) dx \\
 &= \int x^3 dx + \int \frac{1}{x} dx + \int \frac{1}{x^2} dx \\
 &= \ln(x) + \frac{x^4}{4} - \frac{1}{x} + k
 \end{aligned}$$

20.

$$\begin{aligned}
 & \int (\frac{3}{x} + \frac{2}{x^3}) dx \\
 &= 3 \int \frac{1}{x} dx + 2 \int \frac{1}{x^3} dx \\
 &= 3\ln(x) - \frac{1}{x^2} + k
 \end{aligned}$$

21.

$$\begin{aligned}
 & \int e^{\sqrt{(2x)}} dx \\
 &= \int e^{\sqrt{2x}\sqrt{(x)}} dx \\
 & \quad \mathbf{u} = \sqrt{(x)} \\
 &= 2 \int ue^{\sqrt{(2u)}} du \\
 &= (\sqrt{2}\sqrt{(x)} - 1)e^{\sqrt{2}\sqrt{(x)}} + k
 \end{aligned}$$

**22.**

$$\begin{aligned} & \int_0^1 e^{2x} dx \\ &= \int e^{2x} dx \\ & \mathbf{u = 2x} \\ &= \frac{1}{2} \int e^u du \\ &= \frac{e^2}{2} \\ &= \frac{e^{2x}}{2} + k = \frac{e^2 - 1}{2} \end{aligned}$$

**23.**

$$\begin{aligned} & \int_1^2 x + \frac{1}{x} dx \\ &= \int x dx + \int \frac{1}{x} dx \\ &= \ln(x) + \frac{x^2}{2} + k = \ln(2) + \frac{3}{2} \end{aligned}$$

**24.**

$$\begin{aligned} & \int_{-1}^1 e^{-x} dx \\ & \mathbf{u = -x} \\ &= \int e^u du \\ &= -e^u \\ &= e^{-x} + k = e - e^{-1} \end{aligned}$$

**25.**

$$\begin{aligned} & \int_0^1 \frac{1}{1+x^2} dx \\ &= \arctan(x) + k = \frac{\pi}{4} \end{aligned}$$

26.

$$\begin{aligned} & \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx \\ &= \arcsen(x) + k = \frac{\pi}{6} \end{aligned}$$

27.

$$\begin{aligned} & \int_1^2 \frac{x^3+1}{x} dx \\ &= \int (x^2 + \frac{1}{x}) dx \\ &= \int x^2 dx + \int \frac{1}{x} dx \\ &= \ln(x) + \frac{x^3}{3} + k = \ln(2) + \frac{7}{3} \end{aligned}$$

28.

$$\begin{aligned} & \int \sen(x) dx \\ &= -\cos(x) + k \end{aligned}$$

29.

$$\begin{aligned} & \int \sen(2x) dx \\ & \mathbf{u = 2x} \\ &= \frac{1}{2} \int \sen(u) du \\ &= -\frac{\cos(u)}{2} \\ &= -\frac{\cos(2x)}{2} + k \end{aligned}$$

30.

$$\begin{aligned} & \int \cos(5x) dx \\ & \mathbf{u = 5x} \\ &= \frac{1}{5} \int \cos(u) du \\ &= \frac{1}{5} (\sen(u)) \\ &= \frac{\sen(5x)}{5} + k \end{aligned}$$

31.

$$\begin{aligned} & \int \cos \sqrt{3} t \, dt \\ &= \cos \sqrt{3} \int t \, dt \\ &= \frac{\cos(\sqrt{3}) t^2}{2} + k \end{aligned}$$

32.

$$\begin{aligned} & \int \left( \frac{1}{2} - \frac{1}{2} \cos(2x) \right) dx \\ &= \int \left( \frac{1}{2} - \frac{\cos(2x)}{2} \right) dx \\ &= \frac{1}{2} \int 1 \, dx - \frac{1}{2} \int \cos(2x) \, dx \\ &= \frac{x}{2} - \frac{\sin(2x)}{4} + k \\ &= \frac{\sin(2x) - 2x}{4} \end{aligned}$$

33.

$$\begin{aligned} & \int \left( 2 + \frac{1}{3} \sin 2x \right) dx \\ &= \int \left( \frac{\sin 2x}{3} + 2 \right) dx \\ &= \frac{1}{3} \int \sin 2x \, dx + 2 \int 1 \, dx \\ &= 2x - \frac{\cos(2x)}{6} + k \\ &= \frac{\sin(2x) - 2x}{4} \end{aligned}$$



34.

$$\begin{aligned}
 & \int_0^{\frac{\pi}{3}} \text{sen} 2x \, dx \\
 &= \int \text{sen} 2x \, dx \\
 & \quad \mathbf{u = 2x} \\
 &= \frac{1}{2} \int \text{sen} \, du \\
 &= \frac{1}{2} (-\cos(u)) \\
 &= -\frac{\cos(u)}{2} \\
 &= -\frac{\cos(2x)}{2} + k = \frac{3}{4}
 \end{aligned}$$

35.

$$\begin{aligned}
 & \int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \cos\left(\frac{x}{2}\right) \, dx \\
 & \quad \mathbf{u = \frac{x}{2}} \\
 &= 2 \int \cos(u) \, du \\
 &= 2(\text{sen}(u)) \\
 &= 2\text{sen}\left(\frac{x}{2}\right) + k = 2^{\frac{3}{2}}
 \end{aligned}$$

36.

$$\begin{aligned}
 & \int \cos^2 2x \, dx \\
 & \quad \mathbf{u = 2x} \\
 &= \frac{1}{2} \int \cos^2(u) \, du \\
 &= \frac{\cos(u)\text{sen}(u)}{2} + \frac{1}{2} \int 1 \, du \\
 &= \frac{\cos(u)\text{sen}(u)}{2} + \frac{u}{2} \\
 &= \frac{\cos(2x)\text{sen}(2x)}{4} + \frac{x}{2} + k \\
 &= \frac{\cos(4x) + 4x}{8} + k
 \end{aligned}$$

37.

$$\begin{aligned}
 & \int \cos^2 5x \, dx \\
 & \mathbf{u} = 5\mathbf{x} \\
 & = \frac{1}{5} \int \cos^2(u) \, du \\
 & = \frac{\cos(u)\sin(u)}{10} + \frac{u}{10} \\
 & = \frac{\cos(5x)\sin(5x)}{10} + \frac{x}{2} + k \\
 & = \frac{\cos(10x) + 10x}{20} + k
 \end{aligned}$$

38.

$$\begin{aligned}
 & \int \sin^2 3x \, dx \\
 & \mathbf{u} = 3\mathbf{x} \\
 & = \frac{1}{3} \int \sin^2(u) \, du \\
 & = \frac{\cos(u)\sin(u)}{6} + \frac{u}{6} \\
 & = \frac{\cos(3x)\sin(3x)}{6} + \frac{x}{6} + k \\
 & = \frac{\sin(6x) - 6x}{12} + k
 \end{aligned}$$

39.

$$\begin{aligned}
 & \int \cos^2\left(\frac{x}{2}\right) \, dx \\
 & \mathbf{u} = \frac{x}{2} \\
 & = 2 \int \cos^2(u) \, du \\
 & = \frac{\cos(u)\sin(u)}{2} + \frac{1}{2} \int 1 \, du \\
 & = \frac{\cos(u)\sin(u)}{2} + \frac{u}{2} \\
 & = \frac{x}{2} + \cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) + k \\
 & = \frac{\sin(x) + x}{2} + k
 \end{aligned}$$

39.

$$\begin{aligned}
 & \int \cos^2\left(\frac{x}{2}\right) dx \\
 & \mathbf{u} = \frac{x}{2} \\
 & = 2 \int \cos^2(u) du \\
 & = \frac{\cos(u)\sin(u)}{2} + \frac{1}{2} \int 1 du \\
 & = \frac{\cos(u)\sin(u)}{2} + \frac{u}{2} \\
 & = \frac{x}{2} + \cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) + k \\
 & = \frac{\sin(x) + x}{2} + k
 \end{aligned}$$

40.

$$\begin{aligned}
 & \int (\sin x + \cos x)^2 dx \\
 & = \int (x^2 + 2\cos(x) + \cos^2(x)) dx \\
 & = \int x^2 dx + 2 \int \cos(x) dx + \int \cos^2(x) dx \\
 & = \frac{x^3}{3} + 2\sin(x) + \frac{x}{2} + k
 \end{aligned}$$

41.

$$\begin{aligned}
 & \int \tan x dx \\
 & = \int \frac{\sin(x)}{\cos(x)} dx \\
 & \mathbf{u} = \cos(x) \\
 & = - \int \frac{1}{u} du \\
 & = -\ln(u) \\
 & = -\ln(\cos(x)) + k
 \end{aligned}$$

42.

$$\begin{aligned}
 & \int \sec^2 x dx \\
 & = \tan(x) + k
 \end{aligned}$$

43.

$$\begin{aligned}
 & \int tg^2x \, dx \\
 &= \int (\sec^2(x) - 1) \, dx \\
 &= \int \sec^2(x) \, dx - \int 1 \, dx \\
 &= tg(x) - x + k
 \end{aligned}$$

44.

$$\begin{aligned}
 & \int 5^x + e^{-x} \, dx \\
 &= \int 5^x \, dx + \int e^{-x} \, dx \\
 &= \frac{5^x}{\ln(5)} - e^{-x} + k
 \end{aligned}$$

45.

$$\begin{aligned}
 & \int (x + \sec^2 3x) \, dx \\
 &= \int \sec^2(3x) \, dx + \int x \, dx \\
 & \quad \mathbf{u = 3x} \\
 & \frac{1}{3} \int \sec(u) \, du + \left(\frac{x^2}{2}\right) \\
 & \quad \frac{tg(u)}{3} + \left(\frac{x^2}{2}\right) \\
 & \quad \frac{tg(3x)}{3} + \frac{x^2}{2} + k
 \end{aligned}$$

46.

$$\begin{aligned}
 & \int (\cos^3 x \sin x) \, dx \\
 & \quad \mathbf{u = \cos(x)} \\
 & \quad - \int u^3 \, du \\
 & \quad \quad - \frac{u^4}{4} \\
 & \quad - \frac{\cos^4(x)}{4} + k
 \end{aligned}$$

47.

$$\begin{aligned}
 & \int \frac{2}{x+3} dx \\
 \mathbf{u} &= \mathbf{x} + \mathbf{3} \\
 &= 2 \int \frac{1}{u} du \\
 &= 2\ln(u) \\
 &= 2\ln(x+3) + k
 \end{aligned}$$

48.

$$\begin{aligned}
 & \int \frac{5}{4x+3} dx \\
 \mathbf{u} &= \mathbf{4x} + \mathbf{3} \\
 &= \frac{5}{4} \int \frac{1}{u} du \\
 &= \frac{5}{4} \ln(u) \\
 &= \frac{5}{4} \ln(4x+3) \\
 &= \frac{5\ln(4x+3)}{4} + k
 \end{aligned}$$

49.

$$\begin{aligned}
 & \int \frac{3x}{5+6x^2} dx \\
 \mathbf{u} &= \mathbf{6x^2} + \mathbf{5} \\
 &= \frac{1}{4} \int \frac{1}{u} du \\
 &= \frac{1}{4} \ln(u) \\
 &= \frac{\ln(u)}{4} \\
 &= \frac{\ln(6x^2+5)}{4} + k
 \end{aligned}$$

50.

$$\begin{aligned} & \int \frac{1}{(x-1)^3} dx \\ & \mathbf{u = x - 1} \\ & = \int \frac{1}{u^3} du \\ & = -\frac{1}{2u^2} \\ & = -\frac{1}{2(x-1)^2} + k \end{aligned}$$