# Capítulo 10

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## 1 Exercícios 12.1

**1.**

$$\int \frac{1}{x}\, dx$$
$$= ln(x) + k$$

**2.**

$$\int 3\, dx$$
$$= 3 \int 1\, dx$$
$$= 3x + k$$

**3.**

$$\int x^5\, dx$$
$$= \frac{x^6}{6} + k$$

**4.**

$$\int x\, dx$$
$$= \frac{x^2}{2} + k$$

**5.**

$$\int \sqrt(x)\, dx$$
$$= \frac{2x^{\frac{3}{2}}}{3} + k$$

**6.**

$$\int \sqrt[3]{(x^2)}\, dx$$
$$= \frac{5x^{\frac{7}{3}}}{7} + k$$

**7.**

$$\int x^{-4}\, dx$$
$$= \int \frac{1}{x^4}\, dx$$
$$= -\frac{1}{3x^3} + k$$

**8.**

$$\int \frac{1}{x^3}\, dx$$
$$= \frac{1}{2x^2} + k$$

**9.**

$$\int \frac{x + x^2}{x^2}\, dx$$
$$= \int \frac{x+1}{x}\, dx$$
$$= \int (\frac{1}{x} + 1)\, dx$$
$$= \int \frac{1}{x}\, dx + \int 1\, dx$$
$$= ln(x) + x + k$$

**10.**

$$\int (\frac{1}{x} + \frac{1}{x^2})\, dx$$
$$= \int \frac{1}{x}\, dx + \int \frac{1}{x^2}\, dx$$
$$= ln(x) - \frac{1}{x} + k$$

**11.**

$$\int (x^2 + \frac{3}{x})\, dx$$
$$= \int x^2\, dx + 3 \int \frac{1}{x}\, dx$$
$$= 3ln(x) + \frac{x^3}{3} + k$$

**12.**

$$\int \frac{x+1}{x}\, dx$$
$$= \int \frac{1}{x}\, dx + \int 1\, dx$$
$$= ln(x) + x + k$$

**13.**

$$\int (e^x + 4)\, dx$$
$$= \int e^x\, dx + 4 \int 1\, dx$$
$$= e^x + 4x + k$$

**14.**

$$\int e^{5x}\, dx$$
$$\mathbf{u = 5x}$$
$$= \frac{1}{5} \int e^u\, du$$
$$= \frac{e^u}{5}$$
$$= \frac{e^{5x}}{5} + k$$

**15.**

$$\int e^{-2x}\,dx$$

$$\textbf{u = -2x}$$

$$= -\frac{1}{2}\int e^u\,du$$

$$= \frac{e^u}{2}$$

$$= \frac{e^{-2x}}{2} + k$$

**16.**

$$\int (e^{2x} + e^{-x})\,dx$$

$$= \int e^{2x}\,dx + \int e^{-x}\,dx$$

$$= \frac{1}{2}\int e^u\,du + \int e^u\,du$$

$$= \frac{e^{2x}}{2} - e^{-x} + k$$

**17.**

$$\int (\frac{1}{x} + \frac{1}{e^x})\,dx$$

$$= \int e^{-x}\,dx + \int \frac{1}{x}\,dx$$

$$\textbf{u = -x}$$

$$= \int e^u\,du + \int \frac{1}{x}\,dx$$

$$= e^u + ln(x)$$

$$= ln(x) - e^{-x} + k$$

**18.**

$$\int (e^{4x} + \frac{1}{x^2})\, dx$$
$$= \int e^{4x}\, dx + \int \frac{1}{x^2}\, dx$$
$$\mathbf{u = 4x}$$
$$= \frac{1}{4} \int e^u\, du + (-\frac{1}{x})$$
$$= \frac{e^u}{4} - \frac{1}{x}$$
$$= \frac{e^{4x}}{4} - \frac{1}{x} + k$$

**19.**

$$\int \frac{x^5 + x + 1}{x^2}\, dx$$
$$= \int (x^3 + \frac{1}{x} + \frac{1}{x^2})\, dx$$
$$= \int x^3\, dx + \int \frac{1}{x}\, dx + \int \frac{1}{x^2}\, dx$$
$$= ln(x) + \frac{x^4}{4} - \frac{1}{x} + k$$

**20.**

$$\int (\frac{3}{x} + \frac{2}{x^3})\, dx$$
$$= 3 \int \frac{1}{x}\, dx + 2 \int \frac{1}{x^3}\, dx$$
$$= 3ln(x) - \frac{1}{x^2} + k$$

**21.**

$$\int e^{\sqrt{(}2x)}\, dx$$
$$= \int e^{\sqrt{2}x\sqrt{(}x)}\, dx$$
$$\mathbf{u =} \sqrt{(}x)$$
$$= 2 \int u e^{\sqrt{(}2u)}\, du$$
$$= (\sqrt{2}\sqrt{(}x) - 1)e^{\sqrt{2}\sqrt{(}x)} + k$$

**22.**

$$\int_0^1 e^{2x}\,dx$$
$$= \int e^{2x}\,dx$$
$$\mathbf{u = 2x}$$
$$= \frac{1}{2}\int e^u\,du$$
$$= \frac{e^2}{2}$$
$$= \frac{e^{2x}}{2} + k = \frac{e^2 - 1}{2}$$

**23.**

$$\int_1^2 x + \frac{1}{x}\,dx$$
$$= \int x\,dx + \int \frac{1}{x}\,dx$$
$$= ln(x) + \frac{x^2}{2} + k = ln(2) + \frac{3}{2}$$

**24.**

$$\int_{-1}^1 e^{-x}\,dx$$
$$\mathbf{u = \text{-}x}$$
$$= \int e^u\,du$$
$$= -e^u$$
$$= e^{-x} + k = e - e^{-1}$$

**25.**

$$\int_0^1 \frac{1}{1+x^2}\,dx$$
$$= arctan(x) + k = \frac{\pi}{4}$$

**26.**

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{(1-x^2)}}\,dx$$
$$= arcsen(x) + k = \frac{\pi}{6}$$

**27.**

$$\int_1^2 \frac{x^3+1}{x}\,dx$$
$$= \int (x^2 + \frac{1}{x})\,dx$$
$$= \int x^2\,dx + \int \frac{1}{x}\,dx$$
$$= ln(x) + \frac{x^3}{3} + k = ln(2) + \frac{7}{3}$$

**28.**

$$\int sen(x)\,dx$$
$$= -cos(x) + k$$

**29.**

$$\int sen(2x)\,dx$$
$$\mathbf{u = 2x}$$
$$= \frac{1}{2}\int sen(u)\,du$$
$$= -\frac{cos(u)}{2}$$
$$= -\frac{cos(2x)}{2} + k$$

**30.**

$$\int cos(5x)\,dx$$
$$\mathbf{u = 5x}$$
$$= \frac{1}{5}\int cos(u)\,du$$
$$= \frac{1}{5}(sen(u))$$
$$= \frac{sen(5x)}{5} + k$$

**31.**

$$\int cos\sqrt(3)t\,dt$$
$$= cos\sqrt(3)\int t\,dt$$
$$= \frac{cos(\sqrt(3))t^2}{2} + k$$

**32.**

$$\int (\frac{1}{2} - \frac{1}{2}cos(2x))\,dx$$
$$= \int (\frac{1}{2} - \frac{cos(2x)}{2})\,dx$$
$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int cos(2x)\,dx$$
$$= \frac{x}{2} - \frac{(2x)}{4} + k$$
$$= \frac{sen(2x) - 2x}{4}$$

**33.**

$$\int (2 + \frac{1}{3}sen2x)\,dx$$
$$= \int (\frac{sen2x}{3} + 2)\,dx$$
$$= \frac{1}{3}\int sen2x\,dx + 2\int 1\,dx$$
$$= 2x - \frac{cos(2x)}{6} + k$$
$$= \frac{sen(2x) - 2x}{4}$$

**34.**

$$\int_0^{\frac{\pi}{3}} sen2x\,dx$$
$$= \int sen2x\,dx$$
$$\mathbf{u = 2x}$$
$$= \frac{1}{2}\int sen\,du$$
$$= \frac{1}{2}(-cos(u))$$
$$= -\frac{cos(u)}{2}$$
$$= -\frac{cos(2x)}{2} + k = \frac{3}{4}$$

**35.**

$$\int_{\frac{\pi}{2}}^{\frac{-\pi}{2}} cos(\frac{X}{2})\,dx$$
$$\mathbf{u = }\frac{x}{2}$$
$$= 2\int cos(u)\,du$$
$$= 2(sen(u))$$
$$= 2sen(\frac{x}{2}) + k == 2^{\frac{3}{2}}$$

**36.**

$$\int cos^2 2x\,dx$$
$$\mathbf{u = 2x}$$
$$= \frac{1}{2}\int cos^2(u)\,du$$
$$= \frac{cos(u)sen(u)}{2} + \frac{1}{2}\int 1\,du$$
$$= \frac{cos(u)sen(u)}{2} + \frac{u}{2}$$
$$= \frac{cos(2x)sen(2x)}{4} + \frac{x}{2} + k$$
$$= \frac{cos(4x) + 4x}{8} + k$$

**37.**

$$\int cos^2 5x\,dx$$

$$\mathbf{u = 5x}$$

$$= \frac{1}{5}\int cos^2(u)\,du$$

$$= \frac{cos(u)sen(u)}{10} + \frac{u}{10}$$

$$= \frac{cos(5x)sen(5x)}{10} + \frac{x}{2} + k$$

$$= \frac{cos(10x) + 10x}{20} + k$$

**38.**

$$\int sen^2 3x\,dx$$

$$\mathbf{u = 3x}$$

$$= \frac{1}{3}\int sen^2(u)\,du$$

$$= \frac{cos(u)sen(u)}{6} + \frac{u}{6}$$

$$= \frac{cos(3x)sen(3x)}{6} + \frac{x}{6} + k$$

$$= \frac{sen(6x) - 6x}{12} + k$$

**39.**

$$\int cos^2(\frac{x}{2})\,dx$$

$$\mathbf{u = }\frac{x}{2}$$

$$= 2\int cos^2(u)\,du$$

$$= \frac{cos(u)sen(u)}{2} + \frac{1}{2}\int 1\,du$$

$$= \frac{cos(u)sen(u)}{2} + \frac{u}{2}$$

$$= \frac{x}{2} + cos(\frac{x}{2}) + sen(\frac{x}{2}) + k$$

$$= \frac{sen(x) + x}{2} + k$$

**39.**

$$\int cos^2(\frac{x}{2})\,dx$$
$$\mathbf{u = \frac{x}{2}}$$
$$= 2\int cos^2(u)\,du$$
$$= \frac{cos(u)sen(u)}{2} + \frac{1}{2}\int 1\,du$$
$$= \frac{cos(u)sen(u)}{2} + \frac{u}{2}$$
$$= \frac{x}{2} + cos(\frac{x}{2}) + sen(\frac{x}{2}) + k$$
$$= \frac{sen(x) + x}{2} + k$$

**40.**

$$\int (senx + cosx)^2\,dx$$
$$= \int (^2(x) + 2cos(x) + cos^2(x))\,dx$$
$$= \int^2 (x)\,dx + 2\int cos(x)(x)\,dx + \int cos^2(x)\,dx$$
$$=^2 (x) + x + k$$

**41.**

$$\int tgx\,dx$$
$$= \int \frac{sen(x)}{cos(x)}\,dx$$
$$\mathbf{u = cos(x)}$$
$$= -\int \frac{1}{u}\,du$$
$$= -ln(u)$$
$$= -ln(cos(x)) + k$$

**42.**

$$\int sec^2x\,dx$$
$$= tg(x) + k$$

**43.**

$$\int tg^2x\,dx$$
$$= \int (sec^2(x) - 1)\,dx$$
$$= \int sec^2(x)\,dx - \int 1\,dx$$
$$= tg(x) - x + k$$

**44.**

$$\int 5^x + e^{-x}\,dx$$
$$= \int 5^x\,dx + \int e^{-x}\,dx$$
$$= \frac{5^x}{ln(5)} - e^{-x} + k$$

**45.**

$$\int (x + sec^2 3x)\,dx$$
$$= \int sec^2(3x)\,dx + \int x\,dx$$
$$\mathbf{u = 3x}$$
$$\frac{1}{3}\int sec(u)\,du + (\frac{x^2}{2})$$
$$\frac{tg(u)}{3} + (\frac{x^2}{2})$$
$$\frac{tg(3x)}{3} + \frac{x^2}{2} + k$$

**46.**

$$\int (cos^3 x senx)\,dx$$
$$\mathbf{u = cos(x)}$$
$$-\int u^3\,du$$
$$-\frac{u^4}{4}$$
$$-\frac{cos^4(x)}{4} + k$$

**47.**

$$\int \frac{2}{x+3}\,dx$$
$$\mathbf{u = x + 3}$$
$$= 2\int \frac{1}{u}\,du$$
$$= 2ln(u)$$
$$= 2ln(x+3) + k$$

**48.**

$$\int \frac{5}{4x+3}\,dx$$
$$\mathbf{u = 4x + 3}$$
$$= \frac{5}{4}\int \frac{1}{u}\,du$$
$$= \frac{5}{4}ln(u)$$
$$= \frac{5}{4}ln(4x+3)$$
$$= \frac{5ln(4x+3)}{4} + k$$

**49.**

$$\int \frac{3x}{5+6x^2}\,dx$$
$$\mathbf{u =} 6x^2 + 5$$
$$= \frac{1}{4}\int \frac{1}{u}\,du$$
$$= \frac{1}{4}ln(u)$$
$$= \frac{ln(u)}{4}$$
$$= \frac{ln(6x^2+5)}{4} + k$$

**50.**

$$\int \frac{1}{(x-1)^3}\,dx$$

$$\mathbf{u = x\text{ - }1}$$

$$= \int \frac{1}{u^3}\,du$$

$$= -\frac{1}{2u^2}$$

$$= -\frac{1}{2(x-1)^2} + k$$