# Machine Learning in Imaging

BME 590L
Roarke Horstmeyer

Lecture 18: Physical layers with coherent fields

**Announcements**

1. Graded project proposals.
   - If you received an email from me, then you have an opportunity to re-write
   - re-submit via email by the end of this week (Friday at midnight)
   - If you didn't receive an email, then you got 100% and can go ahead with the project

2. Survey results
   - Thank you for the feedback! Only have 10 responses so far…
   - Anonymous, so not graded & won't count as a quiz, but I'll redistribute quiz 15% into other assignments…

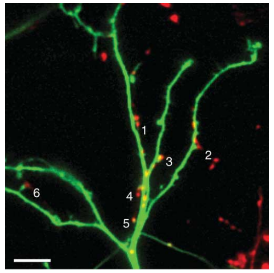
3. Homework #4 will be available this Thursday March 21 and is due two weeks from that date on April 4th

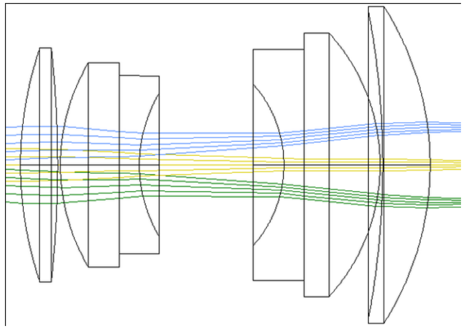
4. The final homework #5 will be available Thursday April 4th and will be due two weeks from that date on April 18

5. Final project presentations will take place at end of April (TBD)

# Summary of two models for image formation

- **Interpretation #1: Radiation** (***Incoherent***)
- Model: Rays



- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

# Mathematical model of for incoherent image formation

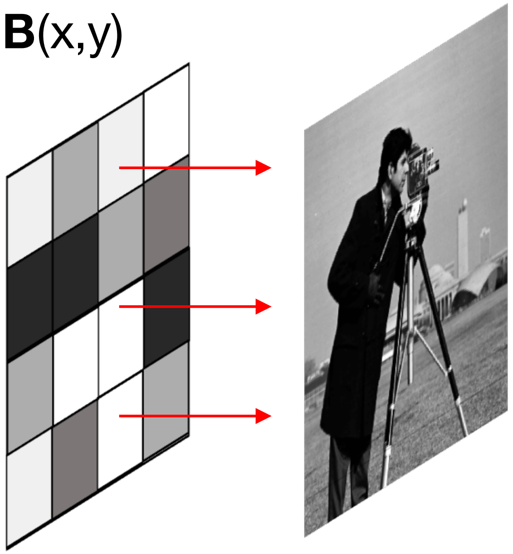
- All quantities are real, and non-negative

Object absorption:

$\mathbf{S_0}(x,y)$

Illumination brightness:

$\mathbf{B}(x,y)$



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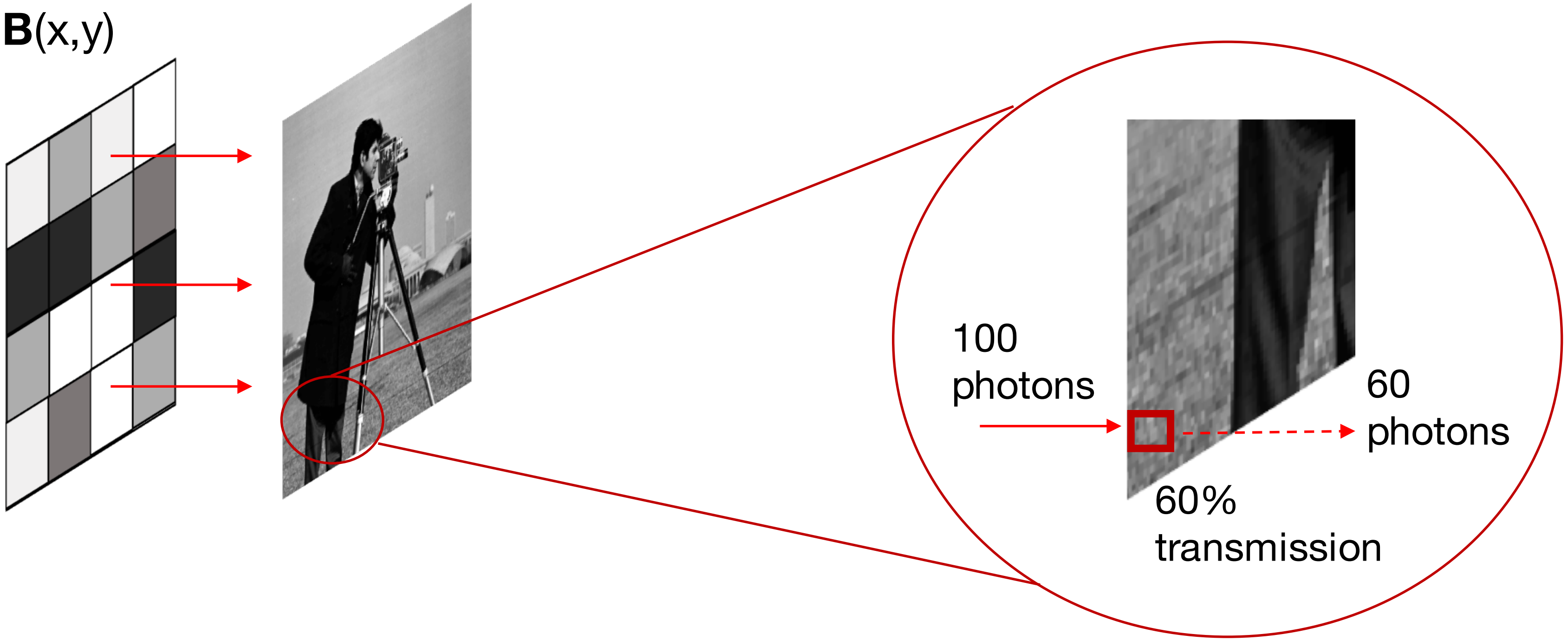

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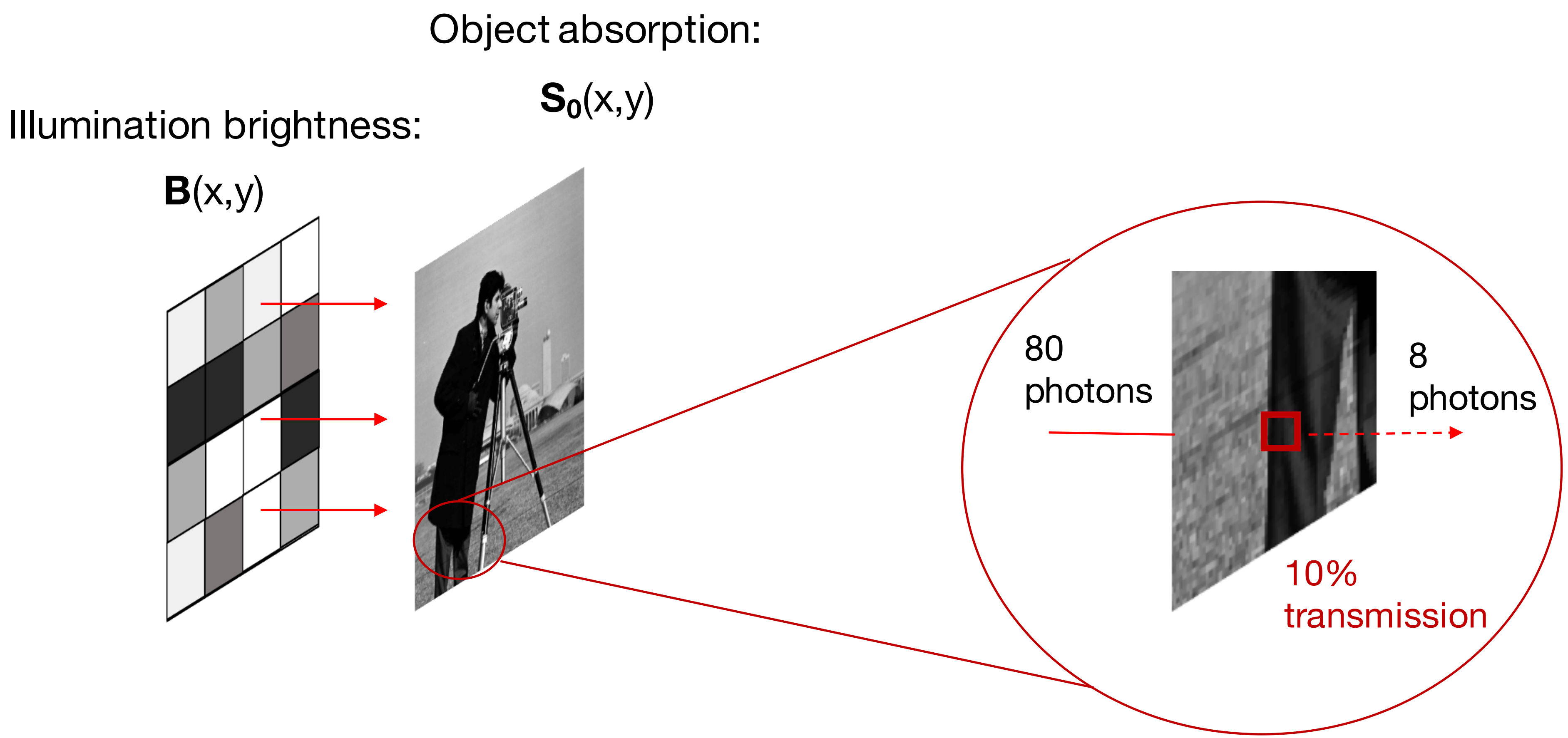

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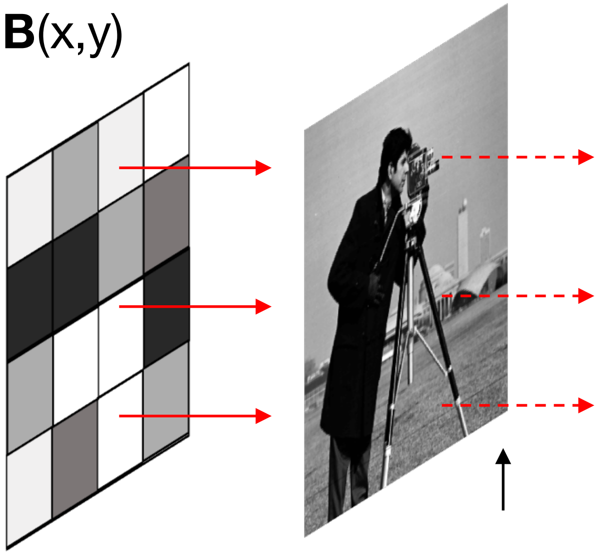

$\mathbf{B}\ \mathbf{S_0}$

multiplication

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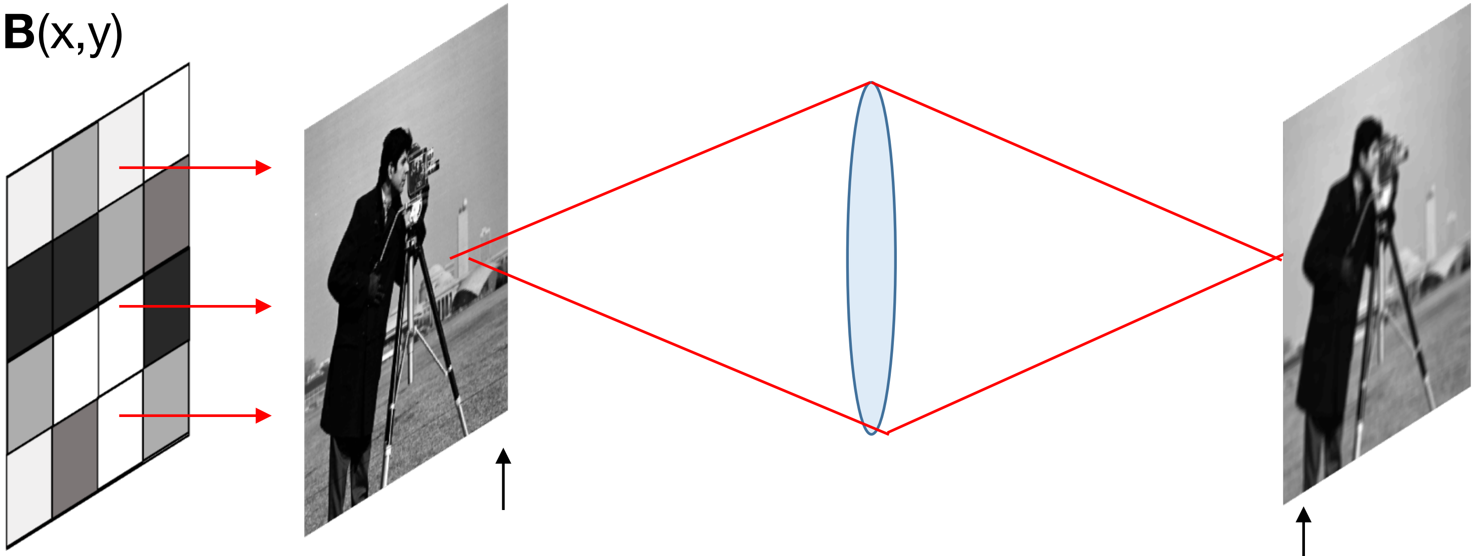

$\mathbf{B\ S_0}$

multiplication

$(\mathbf{B\ S_0}) * \mathbf{h}$

convolution

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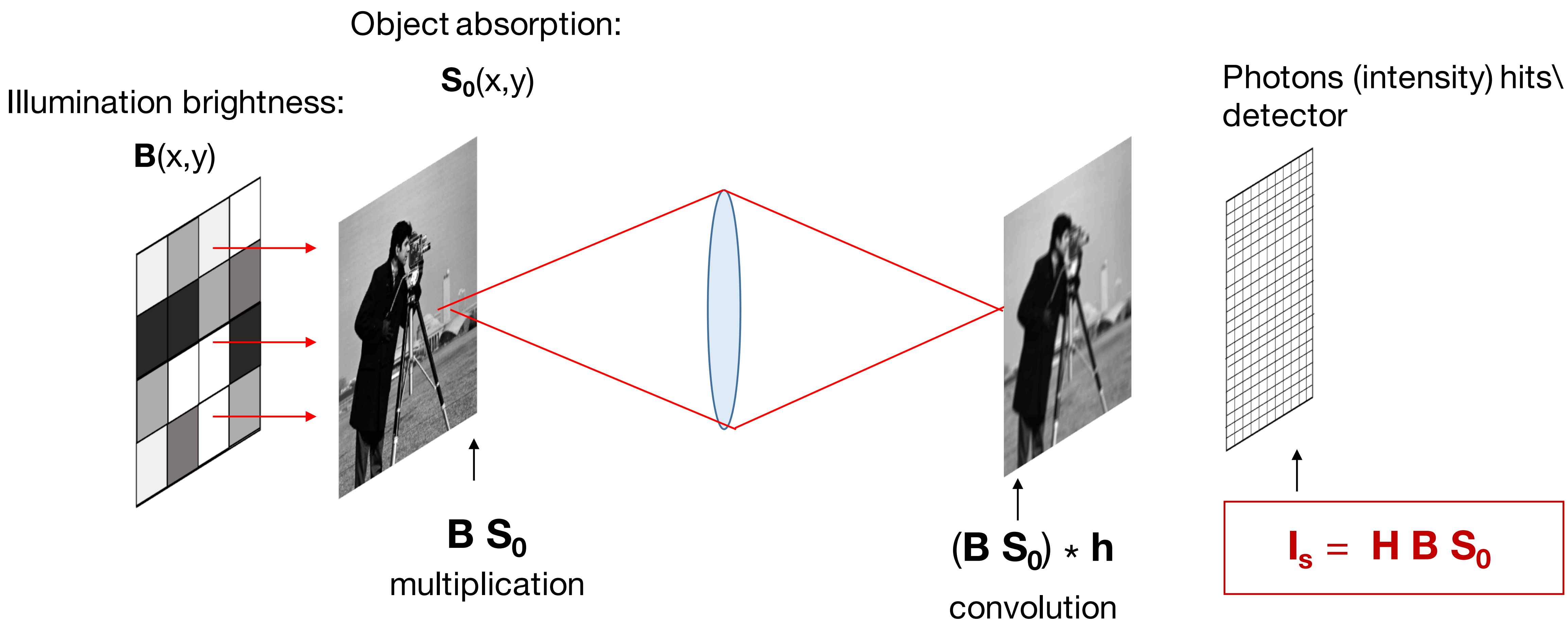
- All quantities are real, and non-negative



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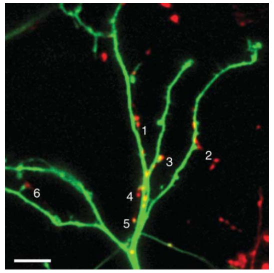

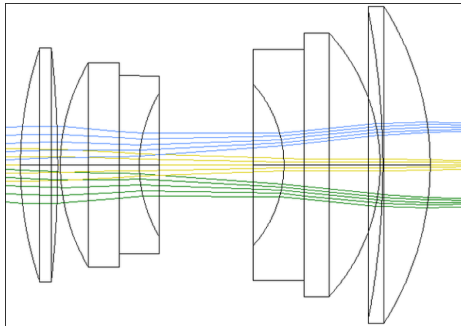

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$$\mathbf{I_s = H\ B\ S_0}$$

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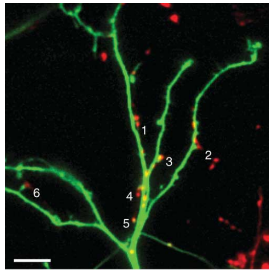



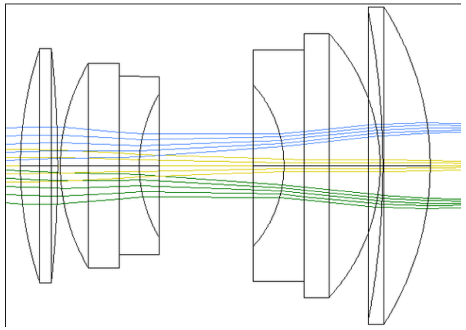

- Real, non-negative
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- **Interpretation #2: Electromagnetic wave (*Coherent*)**
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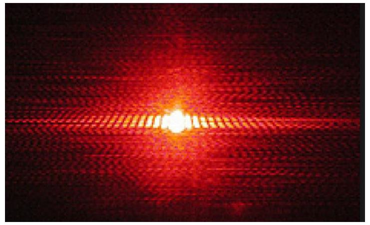
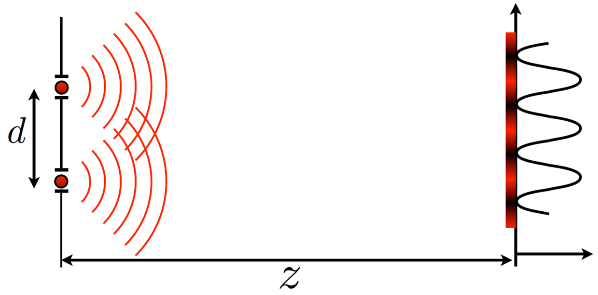






- Complex field
- Models Interference

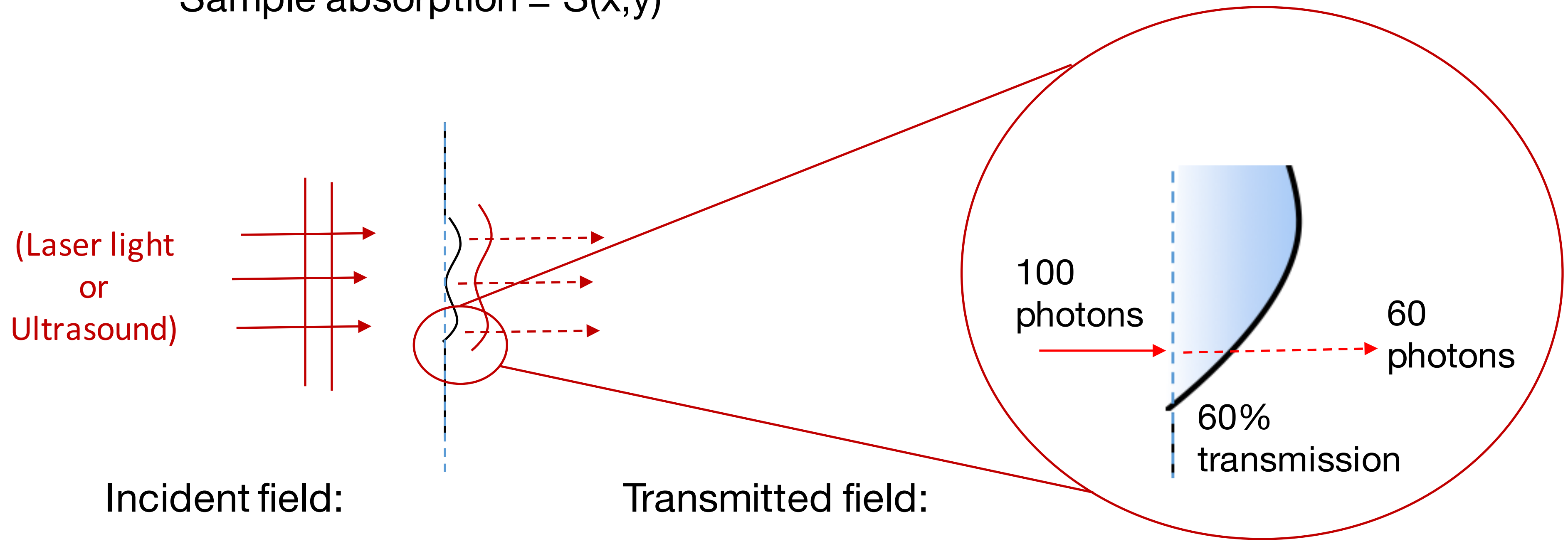
$E_{tot} = E_1 + E_2$

# Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Point #1: Amplitudes behave just like before




$A_t(x,y) = A_i(x,y)\, S(x,y)$

Incident field:

$C(x,y) = A_i(x,y)$

Transmitted field:

$U(x,y) = A_t(x,y) = ?$

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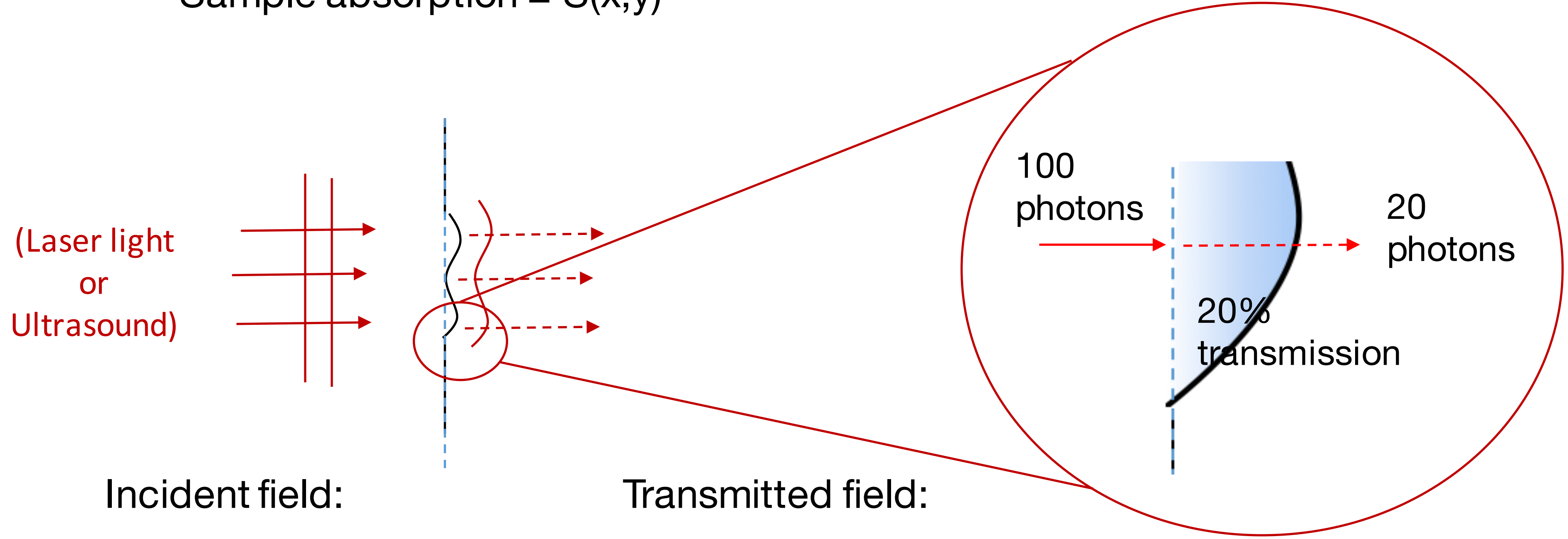


$$\boxed{A_t(x,y) = A_i(x,y)\, S(x,y)}$$

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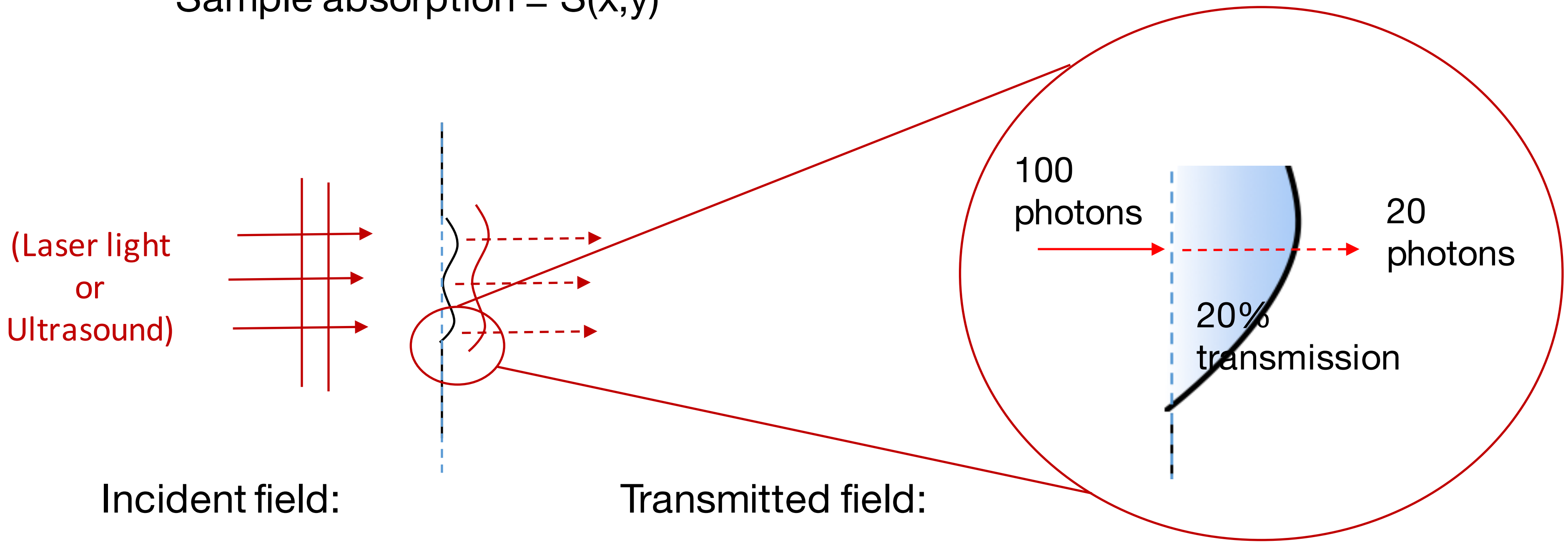


$A_t(x,y) = A_i(x,y)\ S(x,y)$

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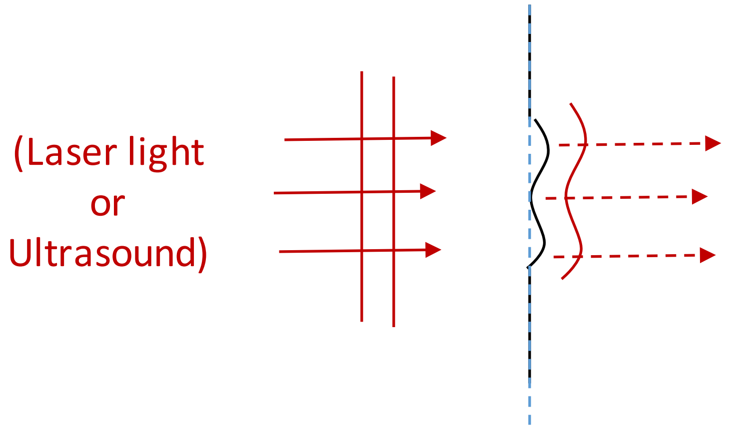
# Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Sample phase delay = $\exp[ik\varphi(x,y)]$

New: complex phase delay
- Needed to represent wave
- Represents wave delay across space




Incident field:

$C(x,y) = A_i(x,y)\exp[ik\varphi_i(x,y)]$

Transmitted field:

$U(x,y) = A_i(x,y)\,S(x,y)\exp[ik\varphi_t(x,y)]$

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Sample absorption = S(x,y)

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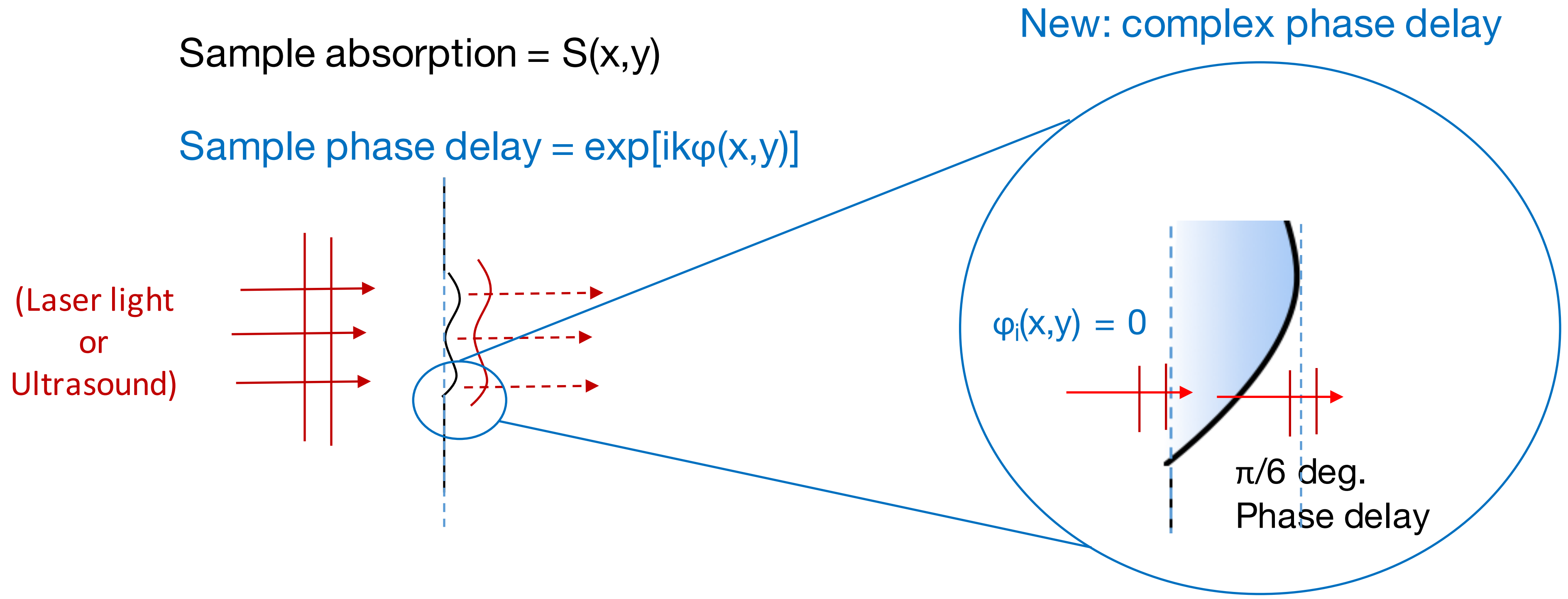

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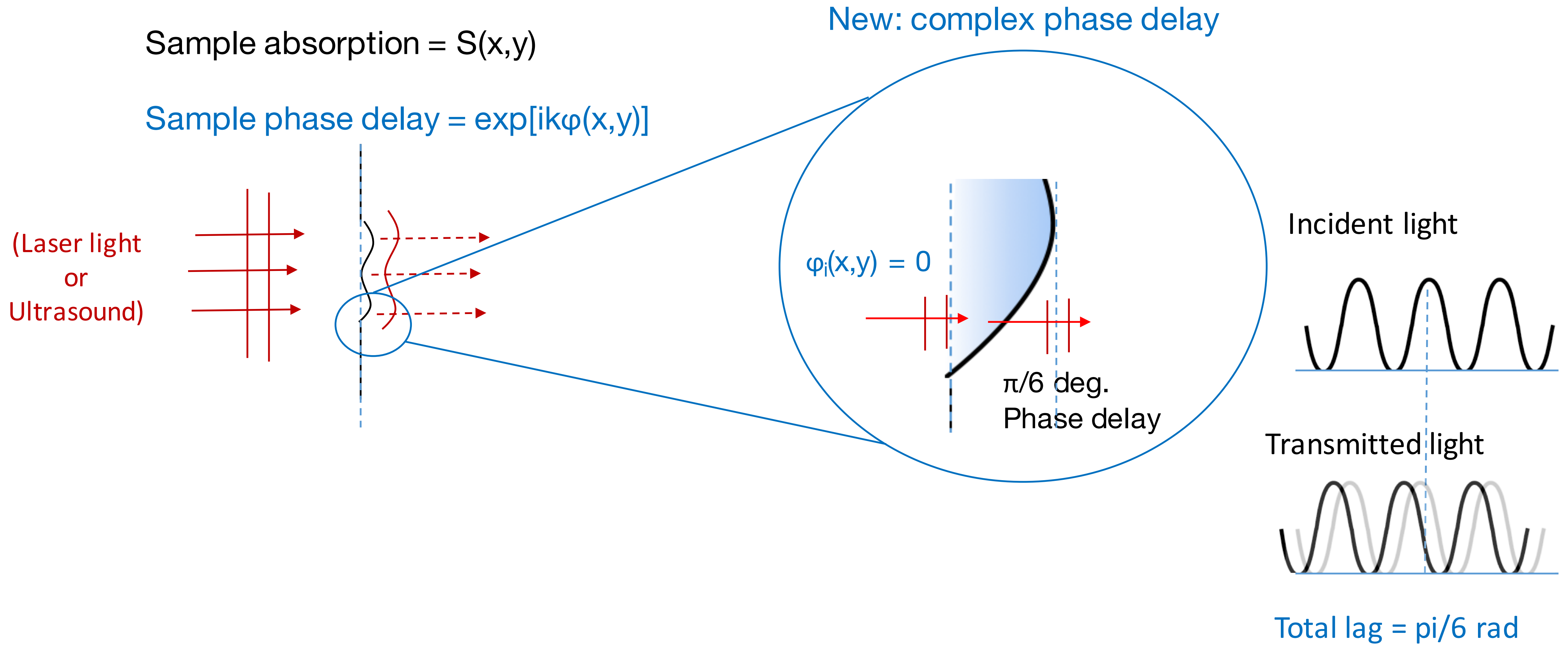

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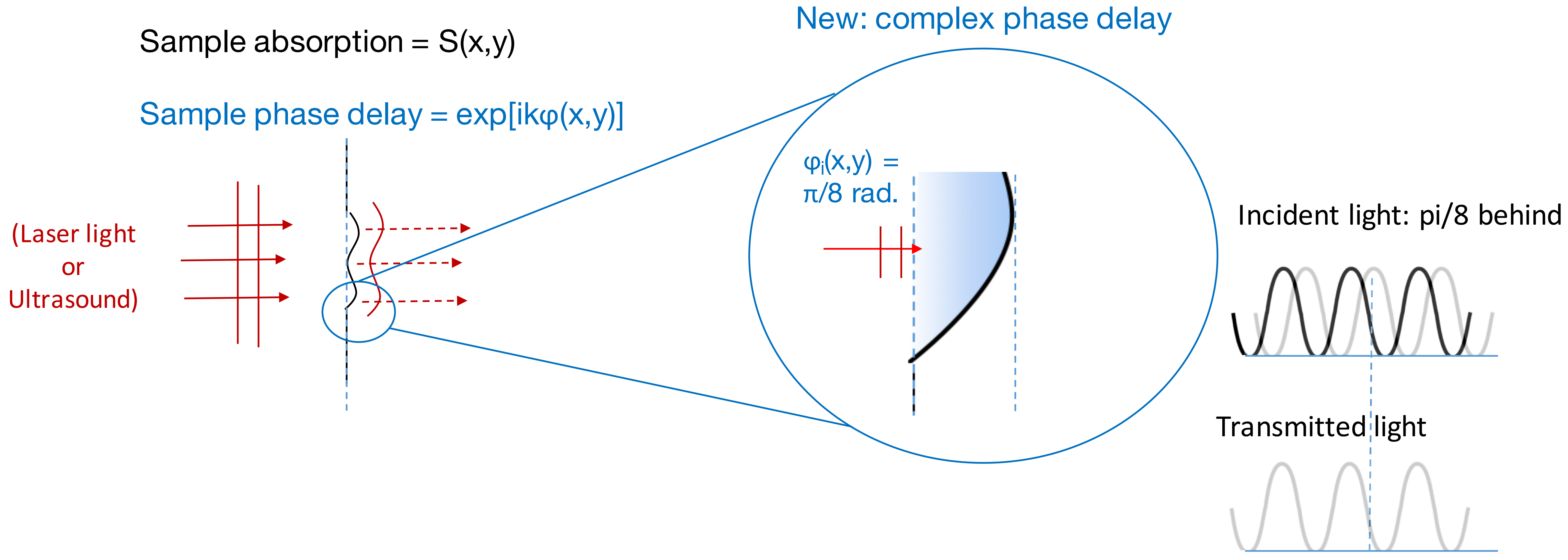

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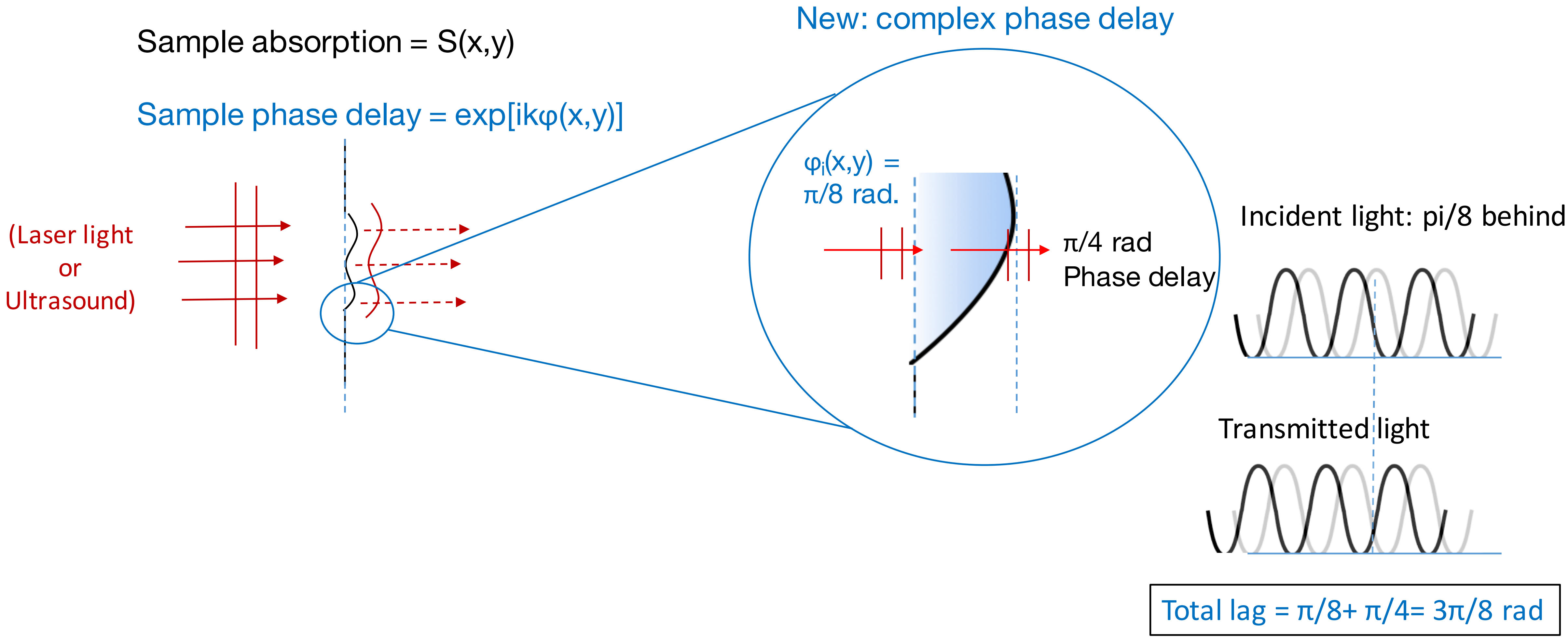

# Mathematical model of for coherent image formation

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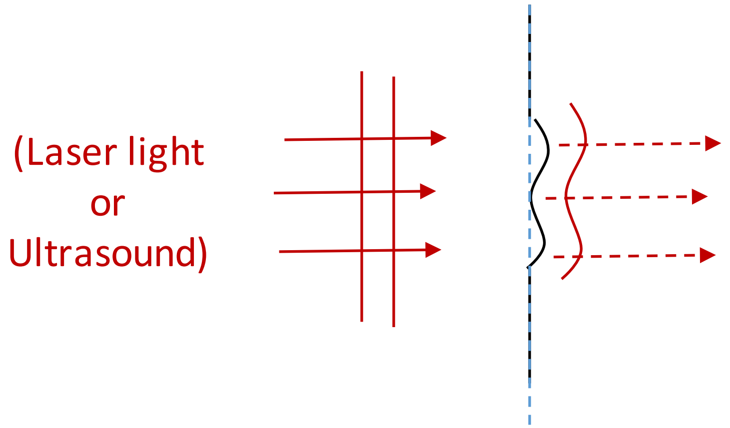
Sample absorption = $S(x,y)$

Sample phase delay = $\exp[ik\varphi(x,y)]$

Output phase is sum of phase delays, product of phasors




$$\varphi_t(x,y) = \varphi(x,y) + \varphi_i(x,y)$$

$$\exp[ik\varphi_t(x,y)] = \exp[ik\varphi_i(x,y)]\ \exp[ik\varphi(x,y)]$$

Incident field:

$C(x,y) = A_i(x,y)\ \exp[ik\varphi_i(x,y)]$

Transmitted field:

$U(x,y) = A_i(x,y)\ S(x,y)\ \exp[ik\varphi_i(x,y)]\ \exp[ik\varphi(x,y)]$

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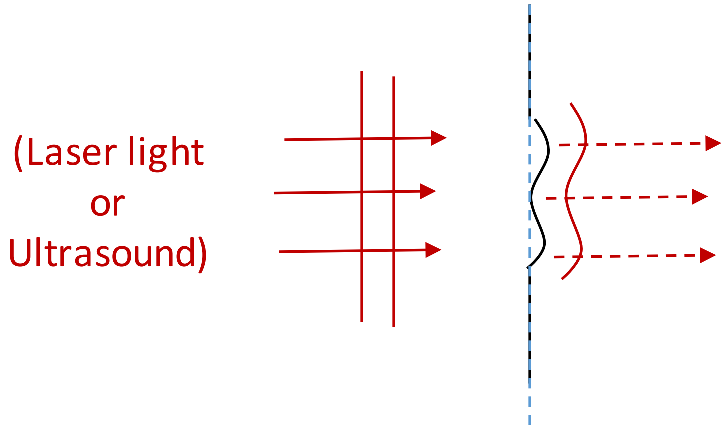



Incident field:

$$C(x,y) = A_i(x,y)\exp[ik\varphi_i(x,y)]$$

Transmitted field:

$$U(x,y) = A_i(x,y)\, S(x,y)\exp[ik\varphi_i(x,y)]\exp[ik\varphi(x,y)]$$

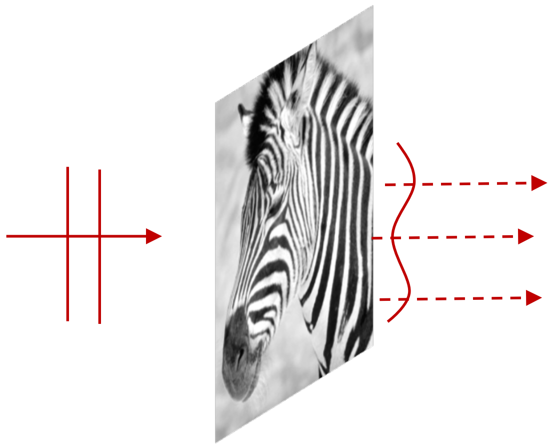
Conclusion:

Transmitted field = incident field x complex sample :

$$U(x,y) = C(x,y)\, S(x,y)\exp[ik\varphi(x,y)]$$

## Model of image formation for wave optics (coherent light):

Discrete sample function s(x,y) (complex)



Illumination field C(x,y)

Transmitted field $s_c(x,y) = C(x,y)\ s(x,y)$

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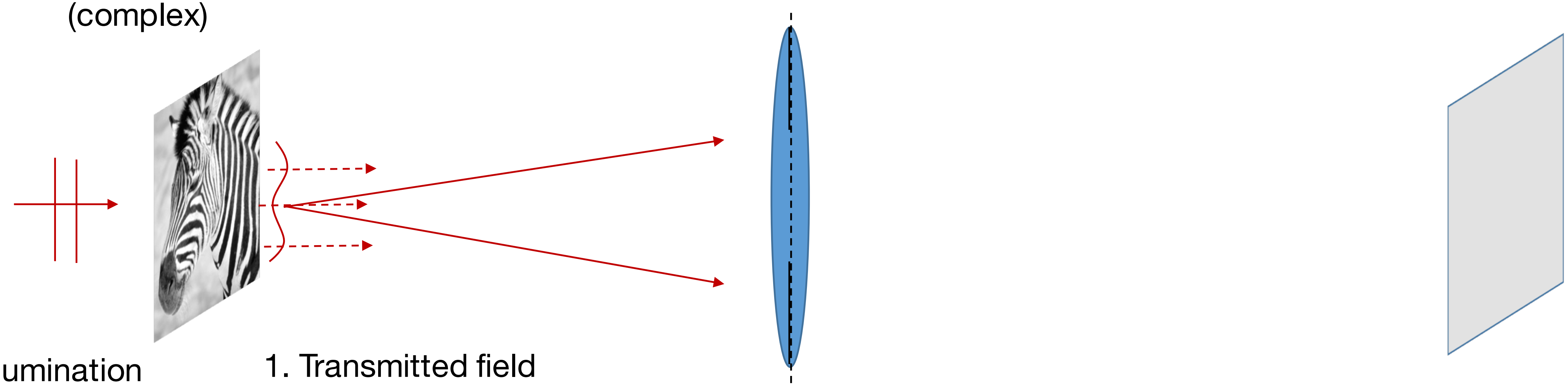

Illumination field C(x,y)

1. Transmitted field $s_c(x,y) = C(x,y)\ s(x,y)$

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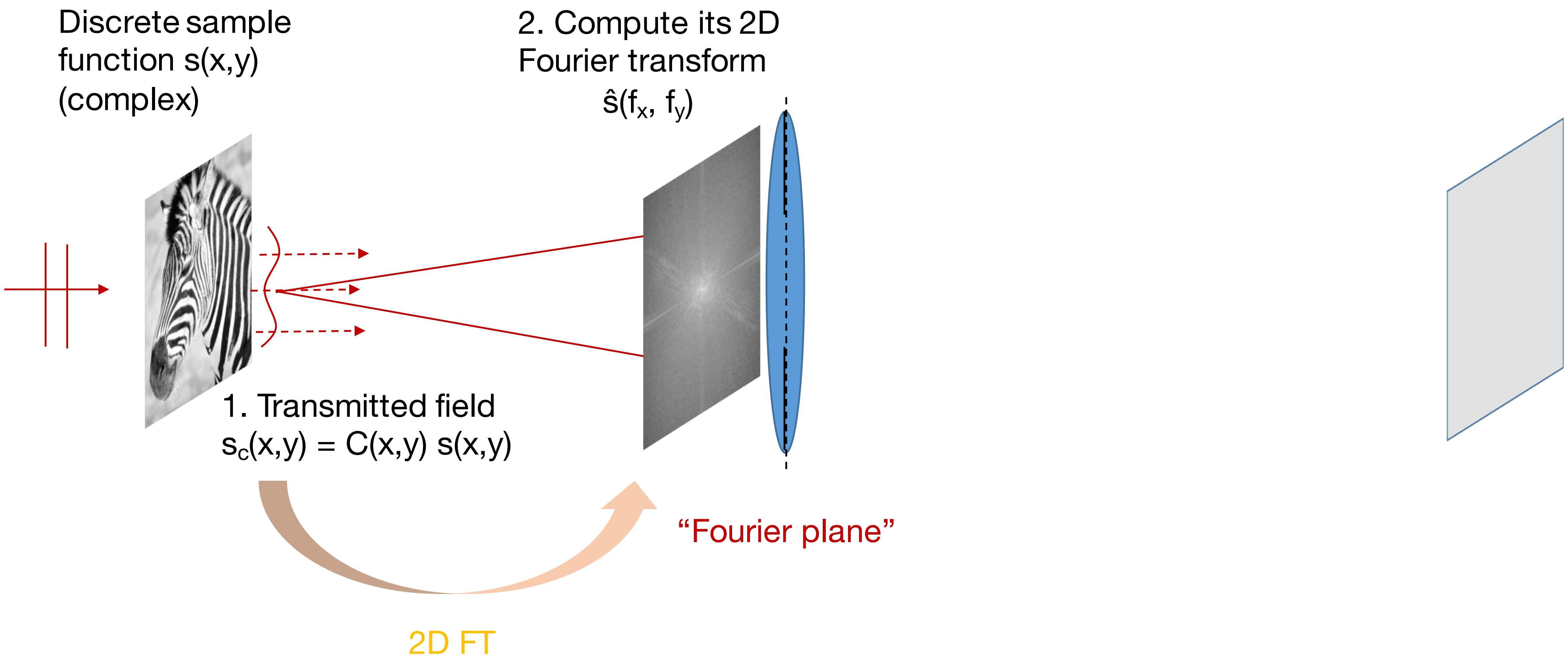
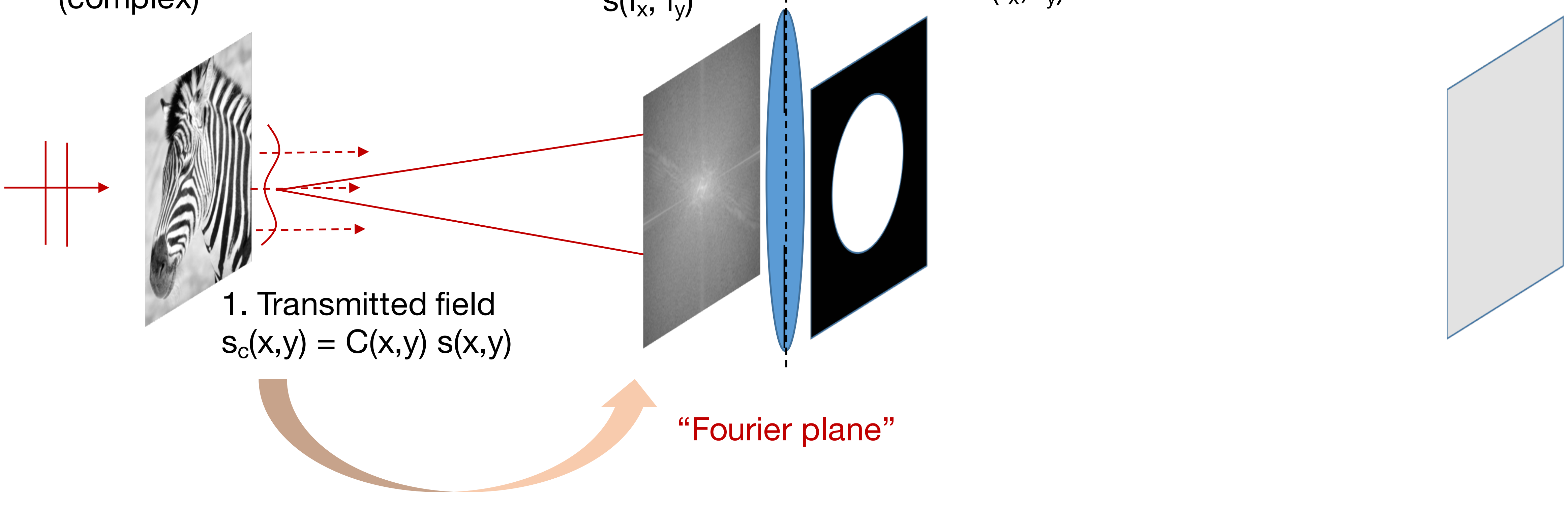


## Model of image formation for wave optics (coherent light):

Discrete sample function s(x,y) (complex)

2. Compute its 2D Fourier transform $\hat{s}(f_x, f_y)$

3. Multiply by “aperture” function $A(f_x, f_y)$



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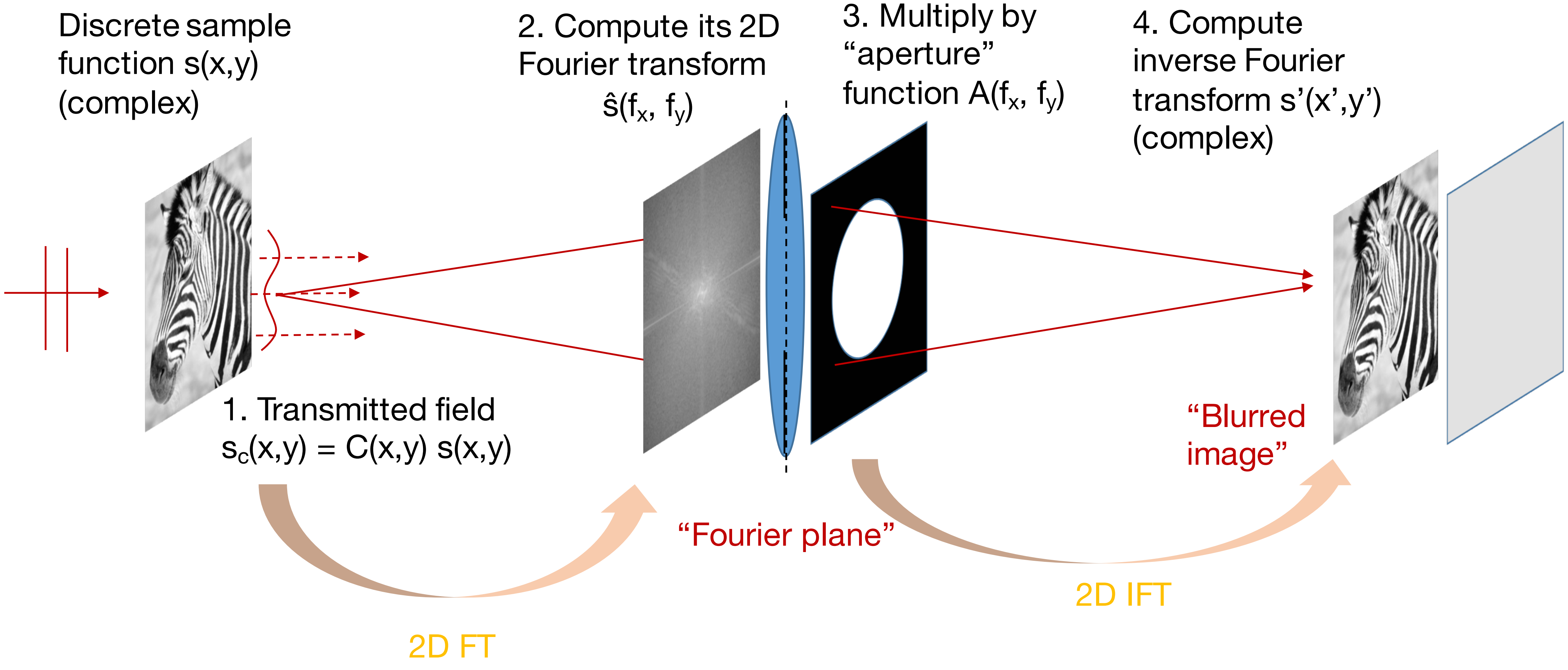

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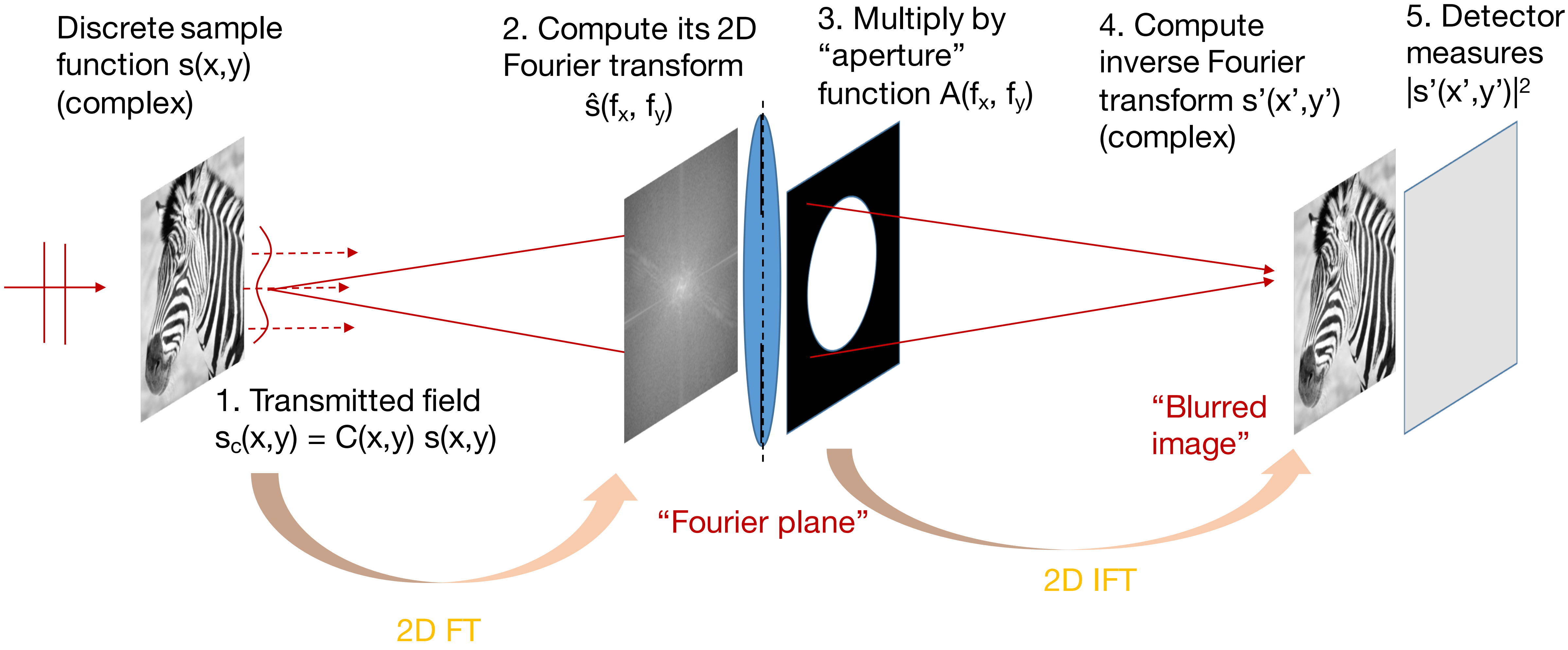

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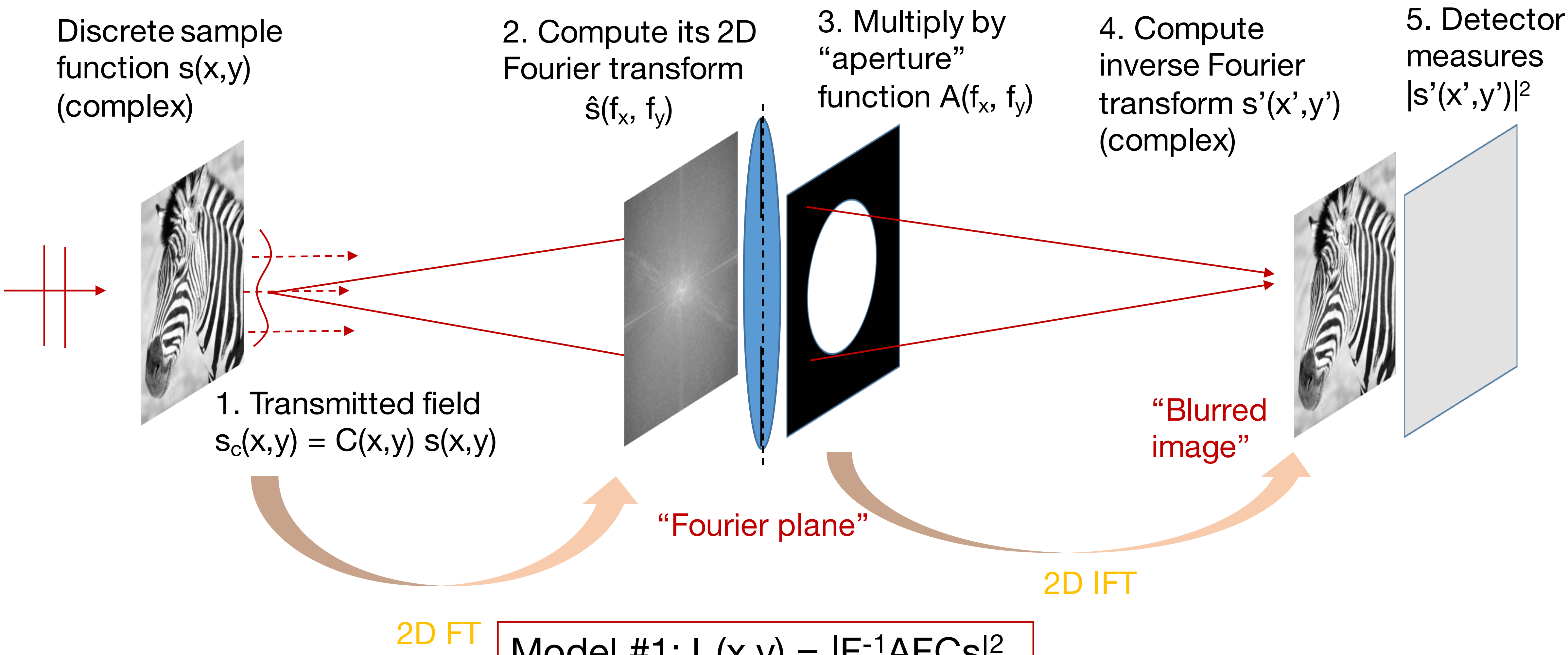



$$\boxed{\text{Model \#1: } I_c(x,y) = |F^{-1}AFCs|^2}$$

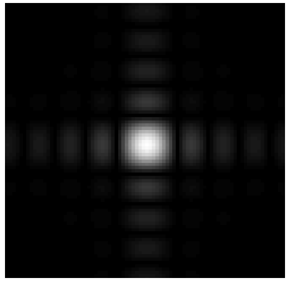
## Can also model this using the Convolution Theorem

Aperture function (lens shape)

$A(f_x, f_y)$

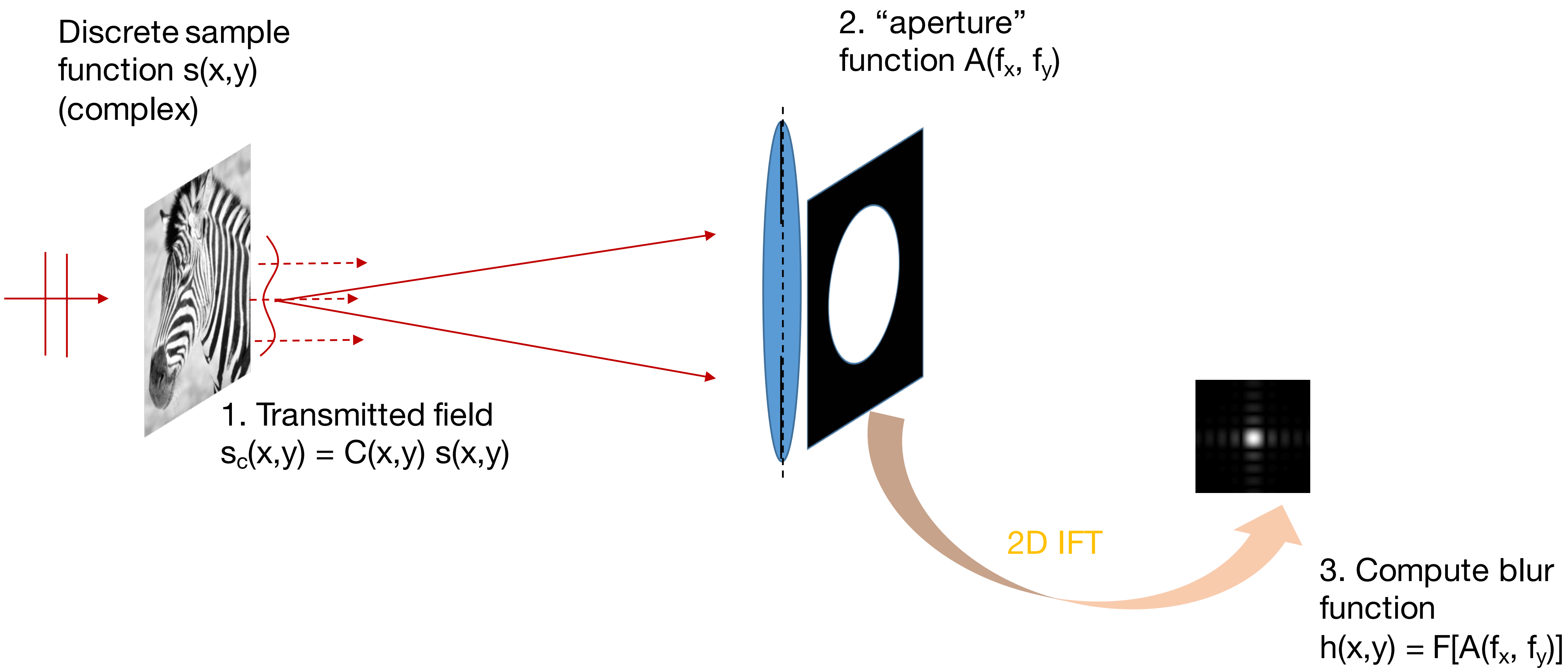
Camera blur function (IFT of lens shape)

$h(x, y)$



2D IFT

# Model of image formation for wave optics (coherent light):



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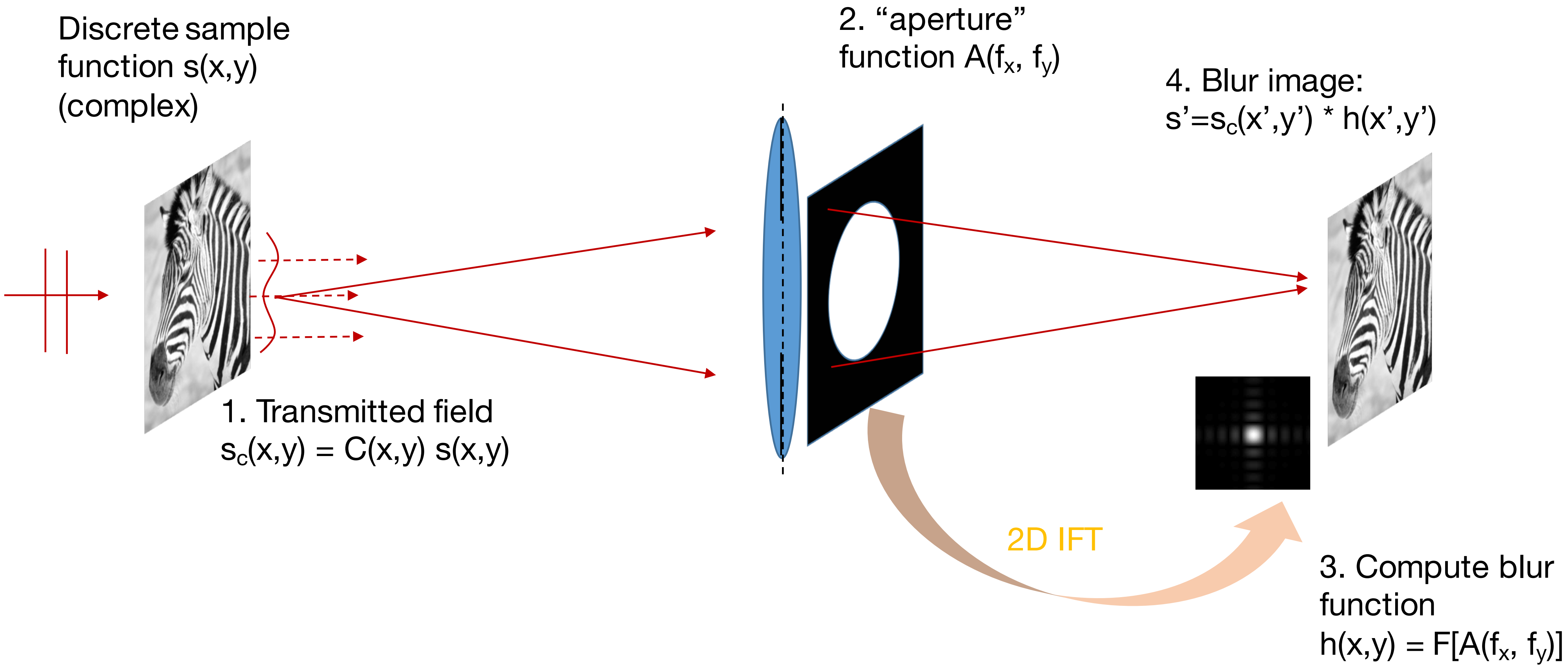

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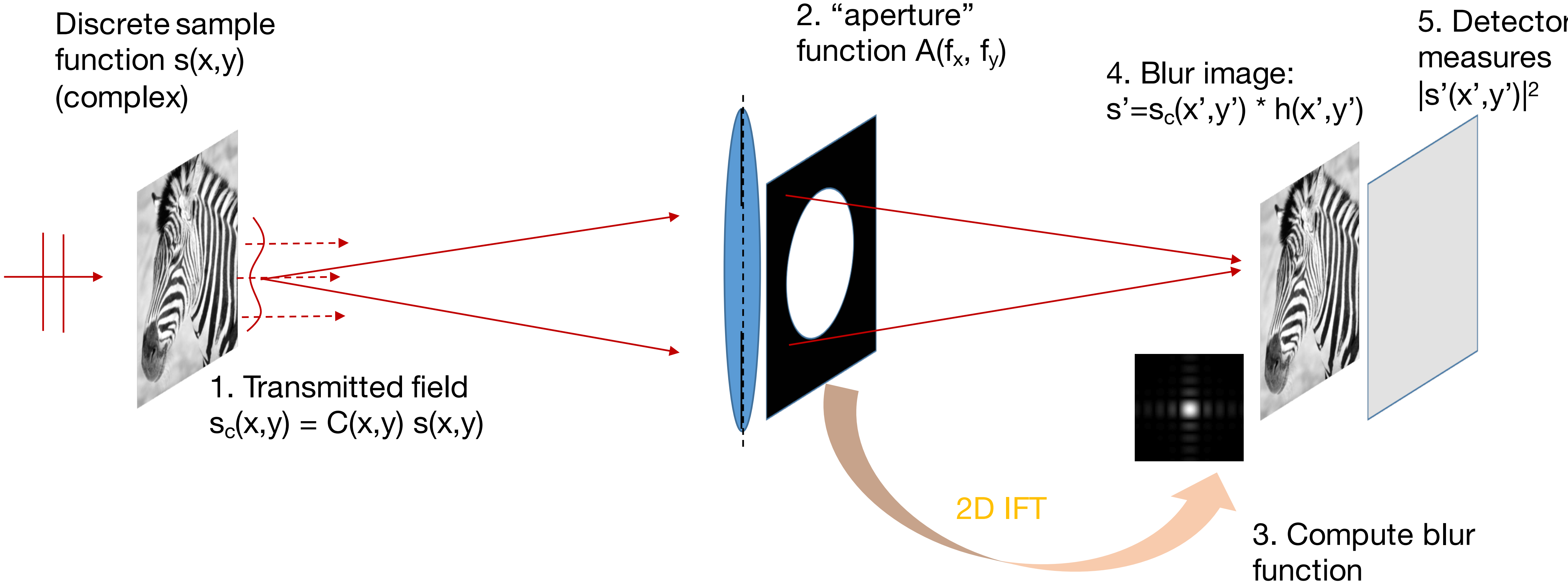


$$\boxed{\text{Model \#2: } I_c(x,y) = |h \ast Cs|^2}$$

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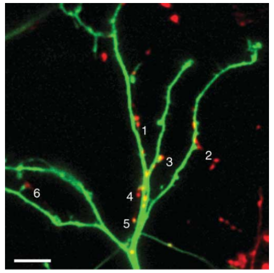
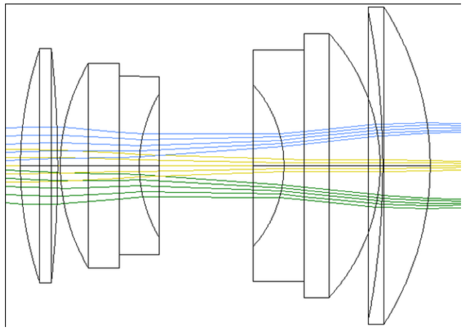


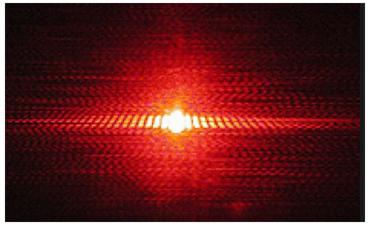


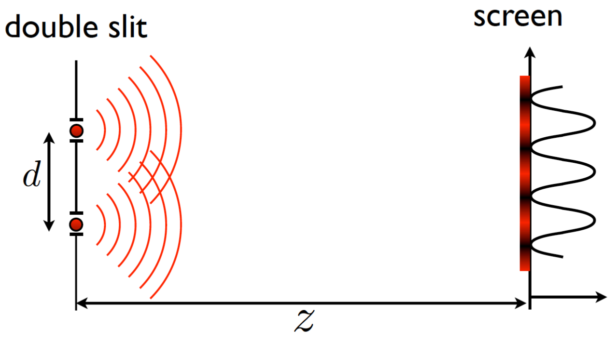
- Real, non-negative

$$\boxed{I_s = H\,B\,S_0}$$

- Sample absorption **S**
- Illumination brightness **B**
- Blur in **H**

- **Interpretation #2: Electromagnetic wave (*Coherent*)**
- Model: Waves






- Complex field

$$\boxed{I_C = |\,H\,C\,S_C\,|^2}$$

- Sample abs./phase **S**
- Illumination wave **B**
- Blur in **H**

## Coherent image formation equation as CNN operations

$$\boxed{I_C = D\,|\,H\,C\,S_C\,|^2}$$

CNN layer

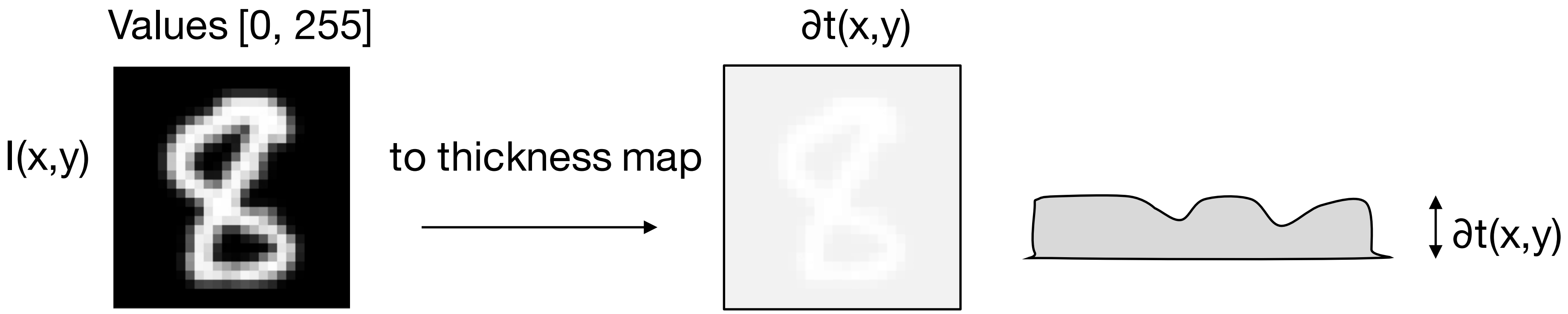
| | |
|---|---|
| Step 1: Multiply with weights | (Step 1: Normalization) |
| Step 2: Convolution | Step 2: Convolution |
| Step 3: Absolute value square (non-linearity) | Step 3: Non-linearity |
| Step 4: Down-sampling by detector | Step 4: Down-sampling by max pooling |

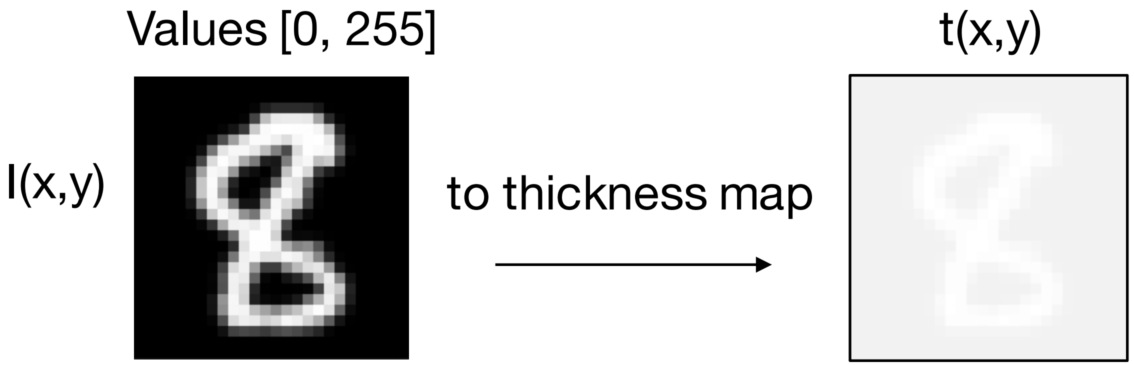
## Example #1: Optimizing coherent illumination pattern for improved classification

<u>Example future situation</u>: Hacking has brought online banking to a halt. We now rely on a special form of physical check that is made of visibly transparent plastic. To write the amount in, you press down with a pen-like instrument, and then the check is read out by shining a particular pattern of laser light onto it, and then imaging it with a lens.

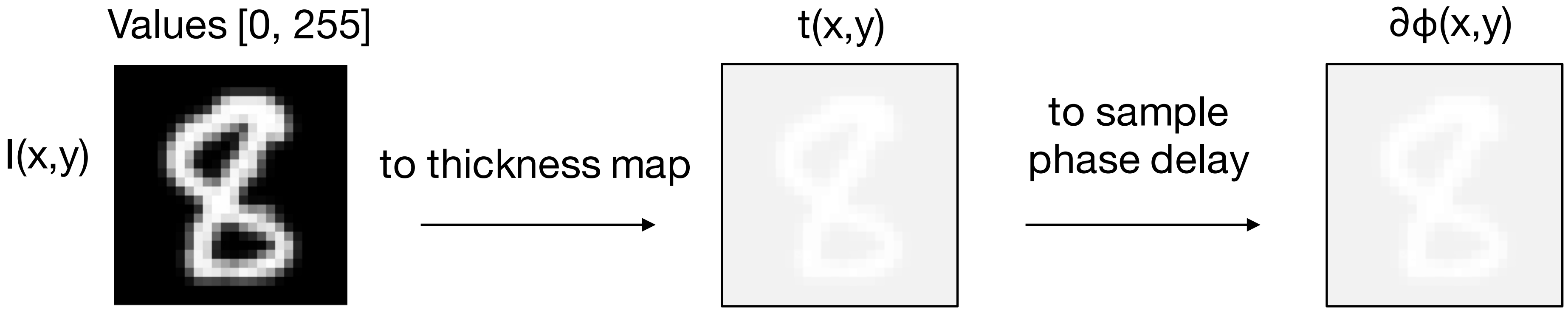
<u>Question</u>: What type of illumination should you use to maximize the classification accuracy of the numbers on the check?

### Step 1: Transform MNIST image data set into transparent plastic sheets with varying thickness

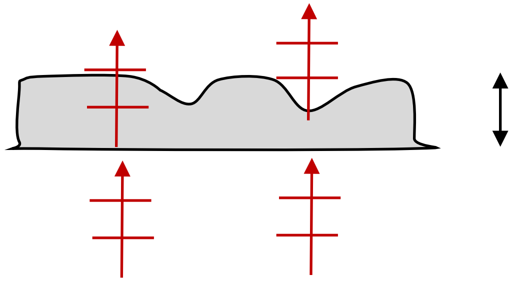





1. Normalize intensity map to 1
2. Define thickness map at some reasonable amount (100 μm max change)

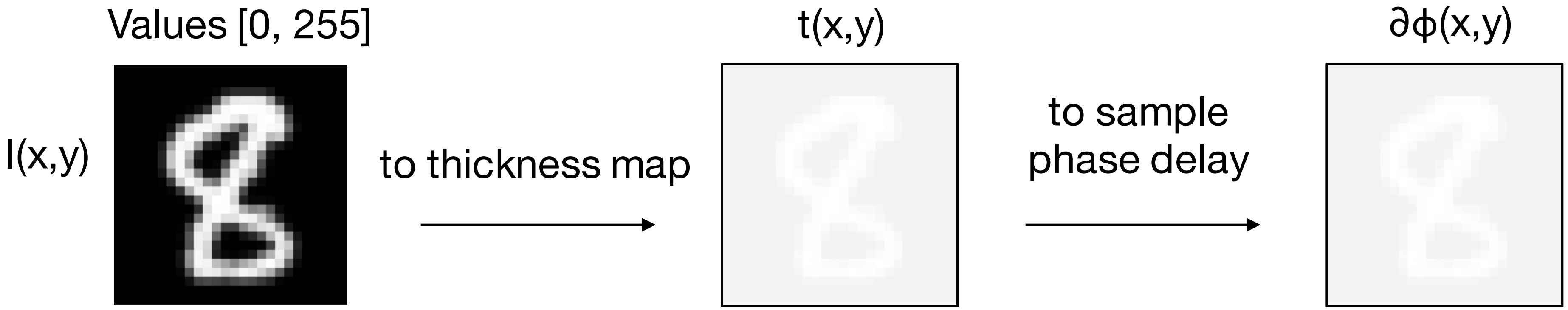



1. Normalize intensity map to 1
2. Define thickness map at some reasonable amount (100 μm max change)
3. Convert thickness map into optical phase delay:



$$\boxed{\partial\phi(x,y) = \exp[\, j\, n\, t(x,y) / \lambda \,]}$$

nt = Optical path length



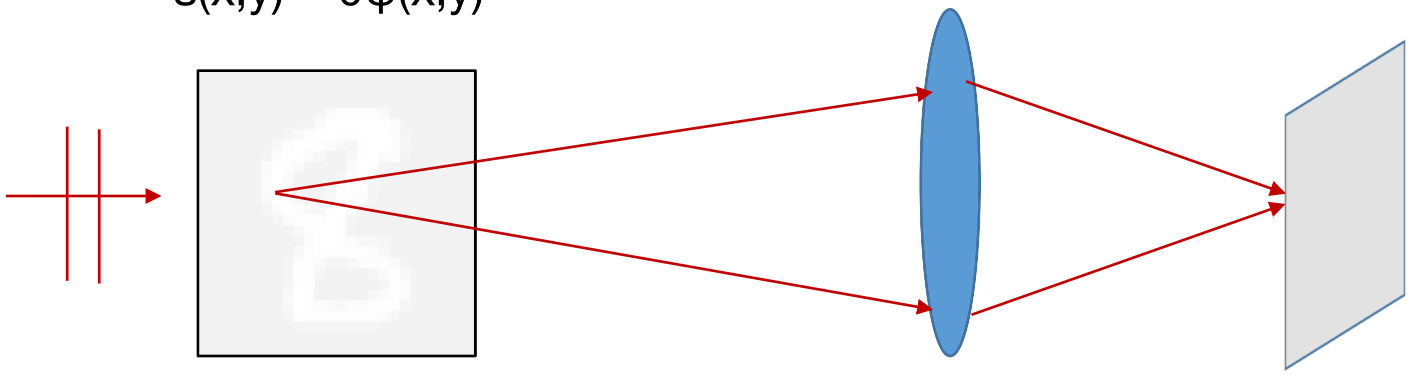

1. Normalize intensity map to 1
2. Define thickness map at some reasonable amount (100 μm max change)
3. Convert thickness map into optical phase delay:

```
n = 1
wavelength = 0.5e-3
mnist_raw_images = tf.placeholder(tf.float32, [image_size, None])
thickness_map = mnist_raw_images/np.amax(mnist_raw_images)
mnist_phase_delay_real = cos(thickness_map * n/wavelength)
mnist_phase_delay_imag = sin(thickness_map * n/wavelength)
mnist_phase_delay = tf.complex(mnist_phase_delay_real,mnist_phase_delay_imag)
```

## Example #1: Optimizing coherent illumination pattern for improved classification

Coherent image Model: $I_c(x,y) = |h * Cs|^2$

$s(x,y) = \partial\phi(x,y)$



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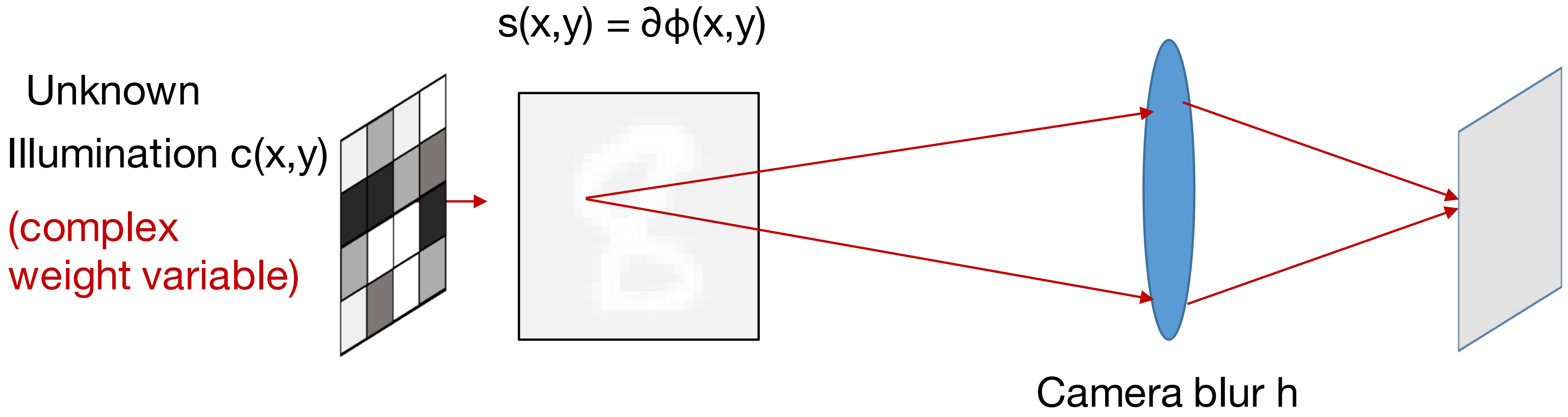
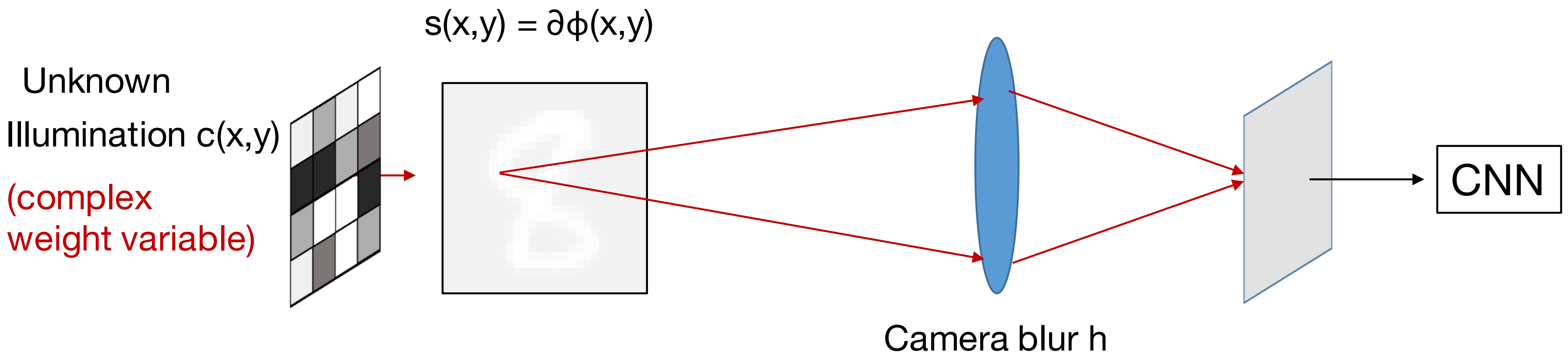


## Example #1: Optimizing coherent illumination pattern for improved classification

Coherent image Model: $I_c(x,y) = |h * Cs|^2$



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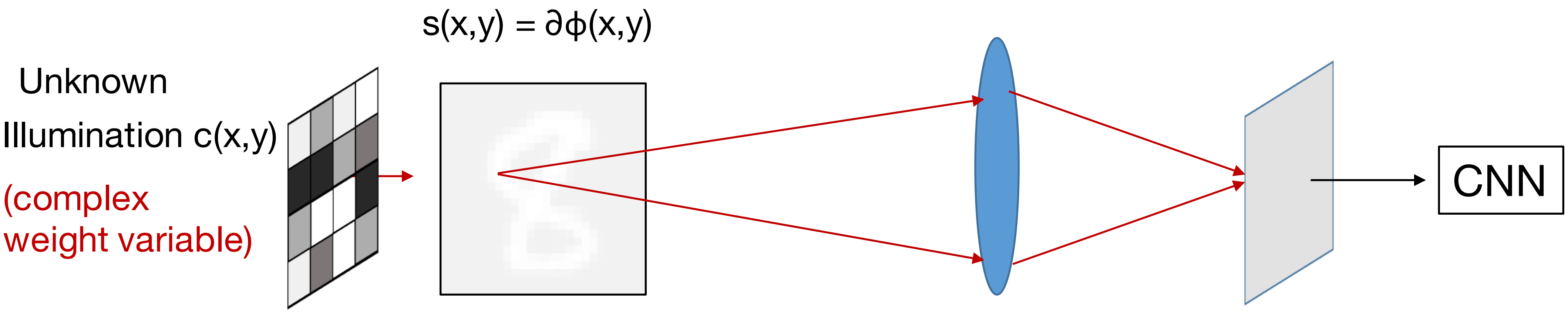


```
mnist_phase_delay = tf.reshape(mnist_phase_delay, [-1, image_size, image_size])
C0_real = tf.Variable([image_size, image_size])
C0_imag = tf.Variable([image_size, image_size])
C0_complex = tf.complex(C0_real, C0_imag)
x_C_complex = tf.mul(mnist_phase_delay, C0_complex)
image_complex = conv2d(x_C_complex, camera_blur)
detected_image = tf.complex_abs(image_complex)
```

detected_image then enters standard CNN classification pipeline

## Example #2: Optimizing aperture shape for improved digit classification

Example future situation: Hacking has brought online banking to a halt. We now rely on a special form of physical check that is made of visibly transparent plastic. To write the amount in, you press down with a pen-like instrument, and then the check is read out by shining a particular pattern of laser light onto it, and then imaging it with a lens.

Question #2: What type of aperture shape should you use to maximize classification accuracy?

## Example #2: Optimizing aperture shape for improved digit classification

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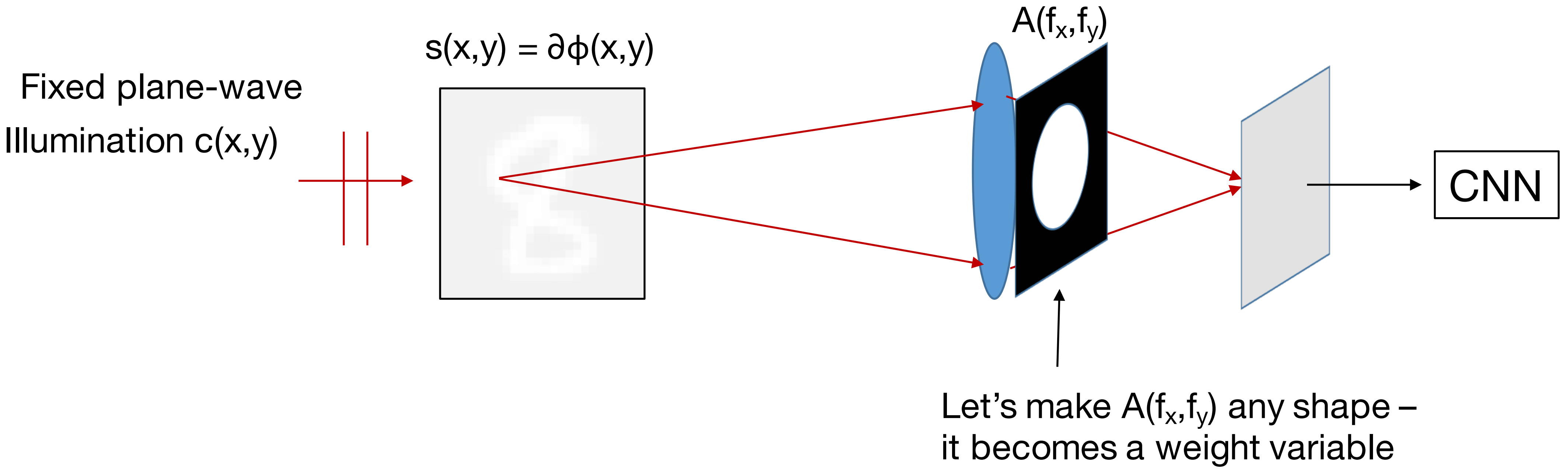
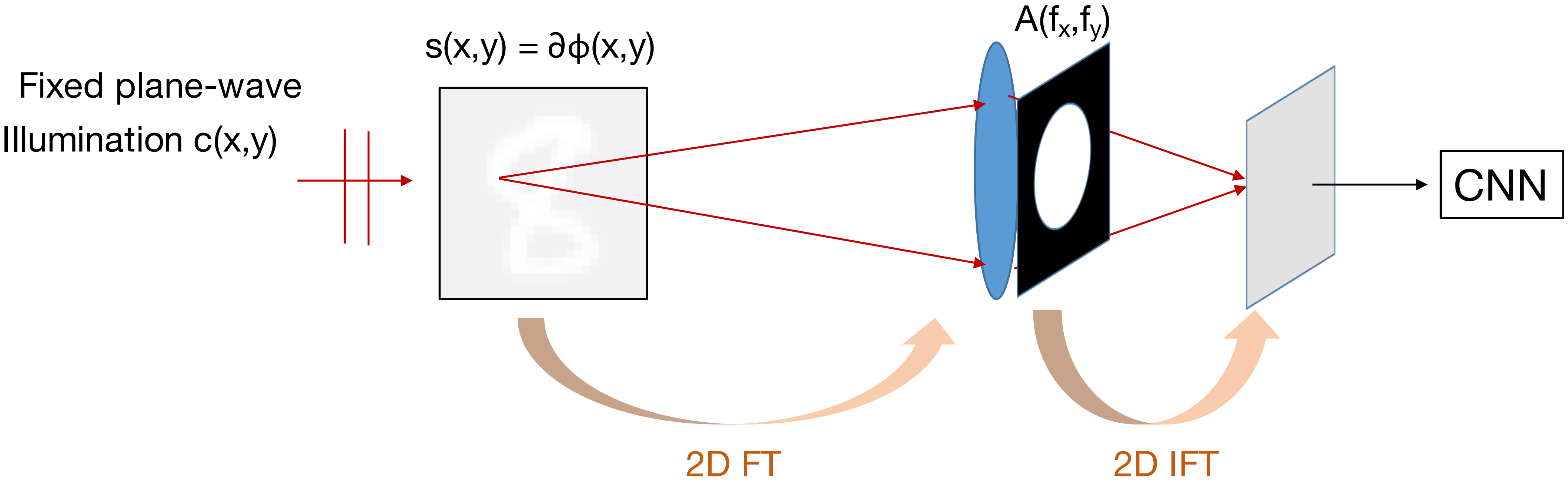
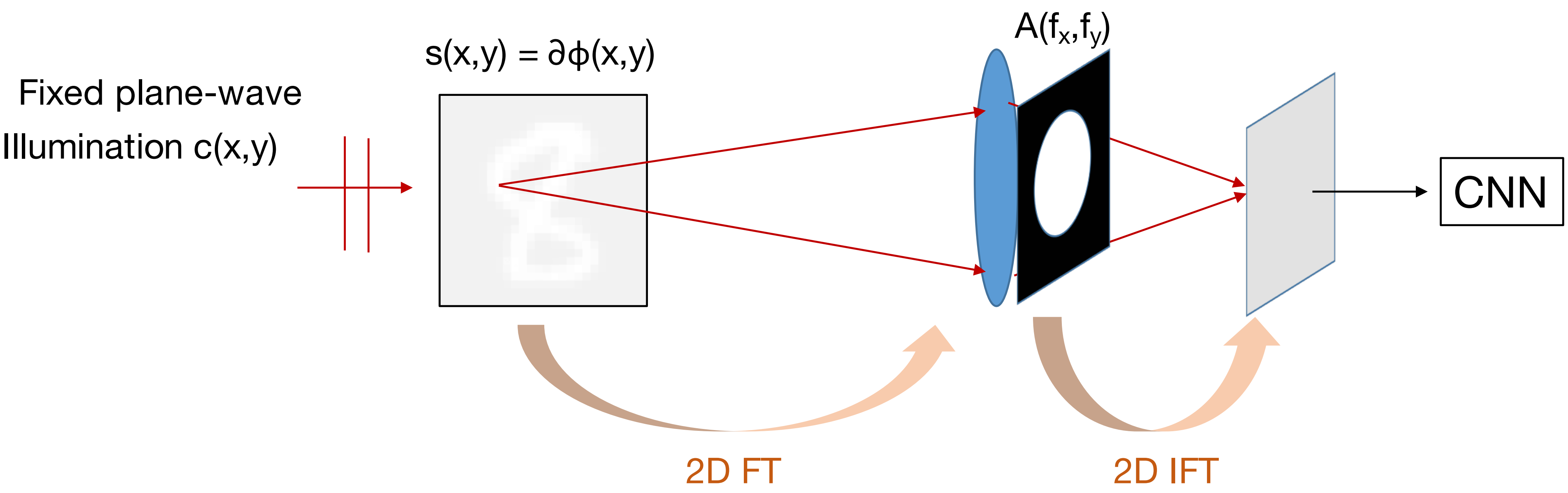


## Example #2: Optimizing aperture shape for improved digit classification



# Example #2: Optimizing aperture shape for improved digit classification




```
mnist_phase_delay = tf.reshape(mnist_phase_delay, [-1, image_size, image_size])
C0 = np.ones(image_size, image_size)
C0 = tf.constant(C0)
x_C_complex = tf.mul(mnist_phase_delay, C0)
fx_C_complex = tf.fft2d(x_C_complex)
ap_filter = tf.Variable([image_size, image_size])
filtered_x_C =  tf.mul(fx_C_complex, ap_filter)
image_complex = tf.ifft2d(filtered_x_C)
detected_image = tf.complex_abs(image_complex)
```