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2018

计算机类本科规划教材
计算方法
（第2版）
李桂成 编著
电子工业出版社
http://www.phei.com.cn

普通高等学校信息与计算科学专业系列丛书
普通高等教育"十一五"国家级规划教材
最优化方法
（第2版）
孙文瑜 徐成贤 朱德通
高等教育出版社

2010

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2018

- Stephen Boyd Lieven Vandenberghe

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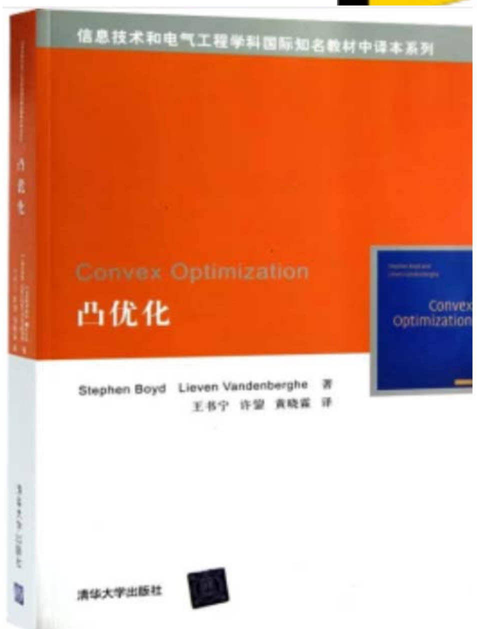

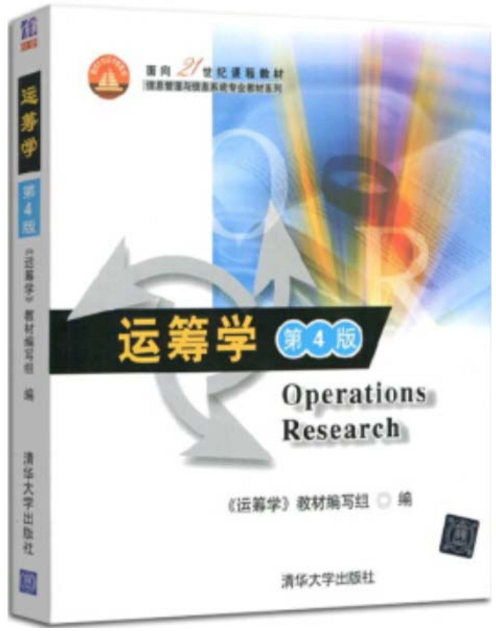

- Numerical Computation
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- Optimization and Operations Research
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- C Matlab

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1.1

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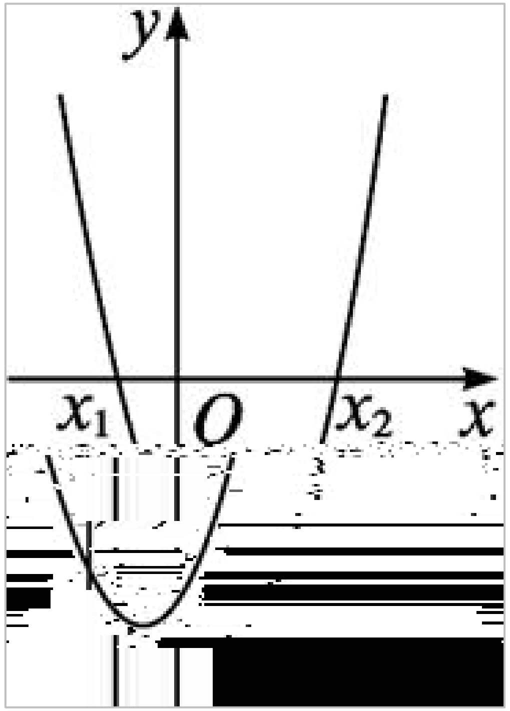
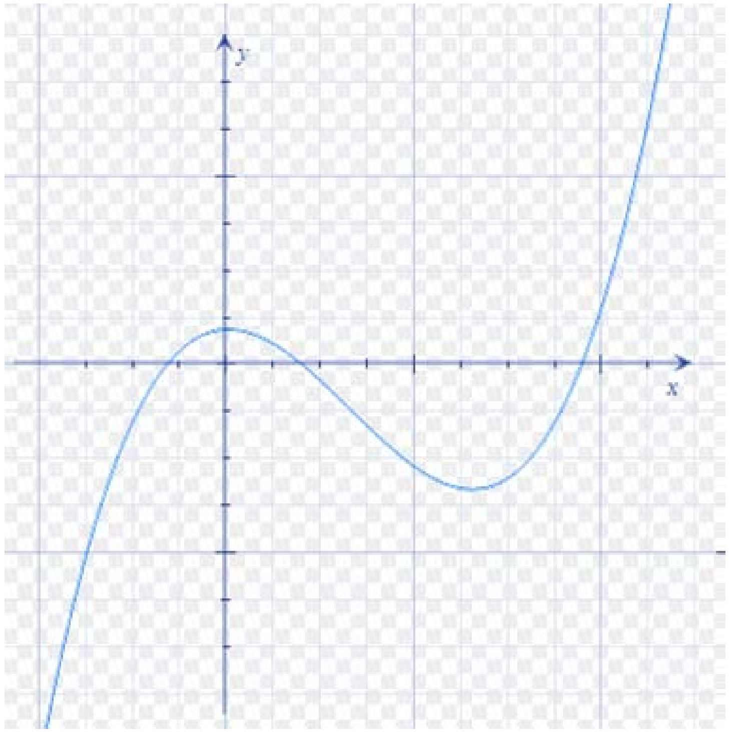
1.3

1.4

1.1

- Roots of Equation





$$\boxed{f(x) = 0\ ?}$$

1.1

❑ Roots of Equation

$f(x)$ $[a,b]$ $f(a) \cdot f(b) < 0$ $f(x) =$
0 $[a,b]$

- ➢ $f(x) = ux^2 + vx + r$

  $f(x) = 0$

$$x_{1,2} = \frac{-v \pm \sqrt{v^2 - 4ur}}{2u}$$

- ➢ $f(x) = x^3 + px + q$ $x = \alpha + \beta$

$$(\alpha + \beta)^3 + p(\alpha + \beta) + q = 0$$

$$\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 + p\alpha + p\beta + q = 0$$

1.1

$$(\alpha^3 + \beta^3 + q) + (\alpha + \beta)(3\alpha\beta + p) = 0$$

$x = \alpha + \beta$ 0 $\alpha^3 + \beta^3 + q = 0$

$$3\alpha\beta + p = 0$$

$\alpha\beta = -\frac{p}{3}$ $\alpha^3\beta^3 = -\frac{p^3}{27}$ $\alpha^3 + \beta^3 = -q$

α^3, β^3 $z^2 + qz - \frac{p^3}{27} = 0$

$$\alpha^3 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \quad \beta^3 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

α β

- $f(y) = y^3 - 1 = (y-1)(y^2 + y + 1)$ $f(y) = 0$

$$y_1 = 1 \quad y_2 = \frac{-1+i\sqrt{3}}{2} = w \quad y_3 = \frac{-1-i\sqrt{3}}{2} = w^2 \quad (i^2 = -1)$$

1.1

α β

$$\alpha_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \quad \alpha_2 = w\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \quad \alpha_3 = w^2\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

$$\beta_1 = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \quad \beta_2 = w\sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \quad \beta_3 = w^2\sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

$\alpha + \beta$ $\alpha\beta = -\frac{p}{3}$

$$\begin{cases} x_1 = \alpha_1 + \beta_1 \\ x_2 = \alpha_2 + \beta_3 \\ x_3 = \alpha_3 + \beta_2 \end{cases}$$

1.1

- $f(x) = ax^3 + \mathrm{b}x^2 + cx + d$ $f(x) = 0$

a $x = y - \frac{b}{3a}$

$$y^3 + py + q = 0$$

y_1, y_2, y_3 $x_i = y_i - \frac{b}{3a} (i = 1,2,3)$

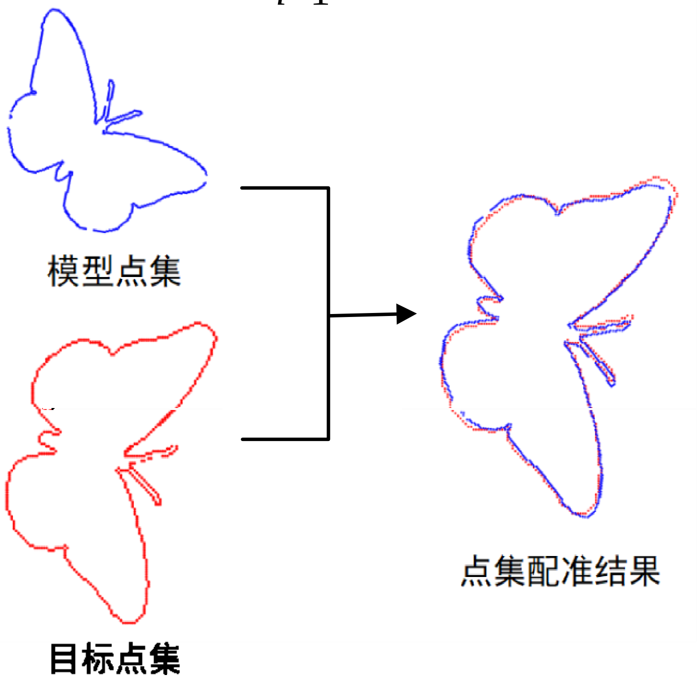
- $f(x)$

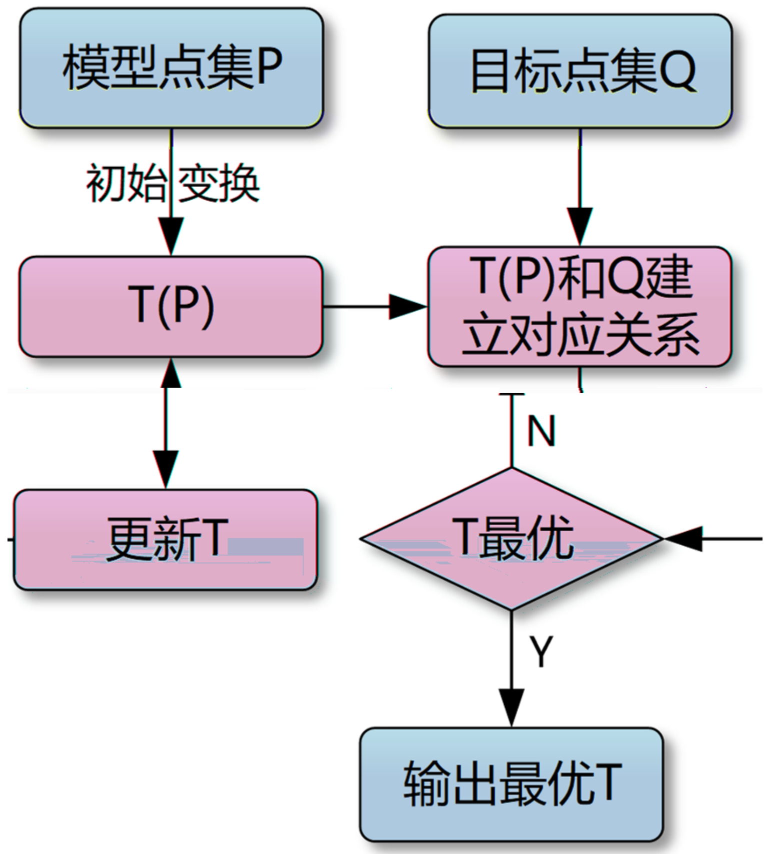
- $f(x)$ $f(x) = 0$

1.1

$$P=\{x_i\}_{i=1}^{N_x},\ Q=\{y_i\}_{i=1}^{N_y}$$

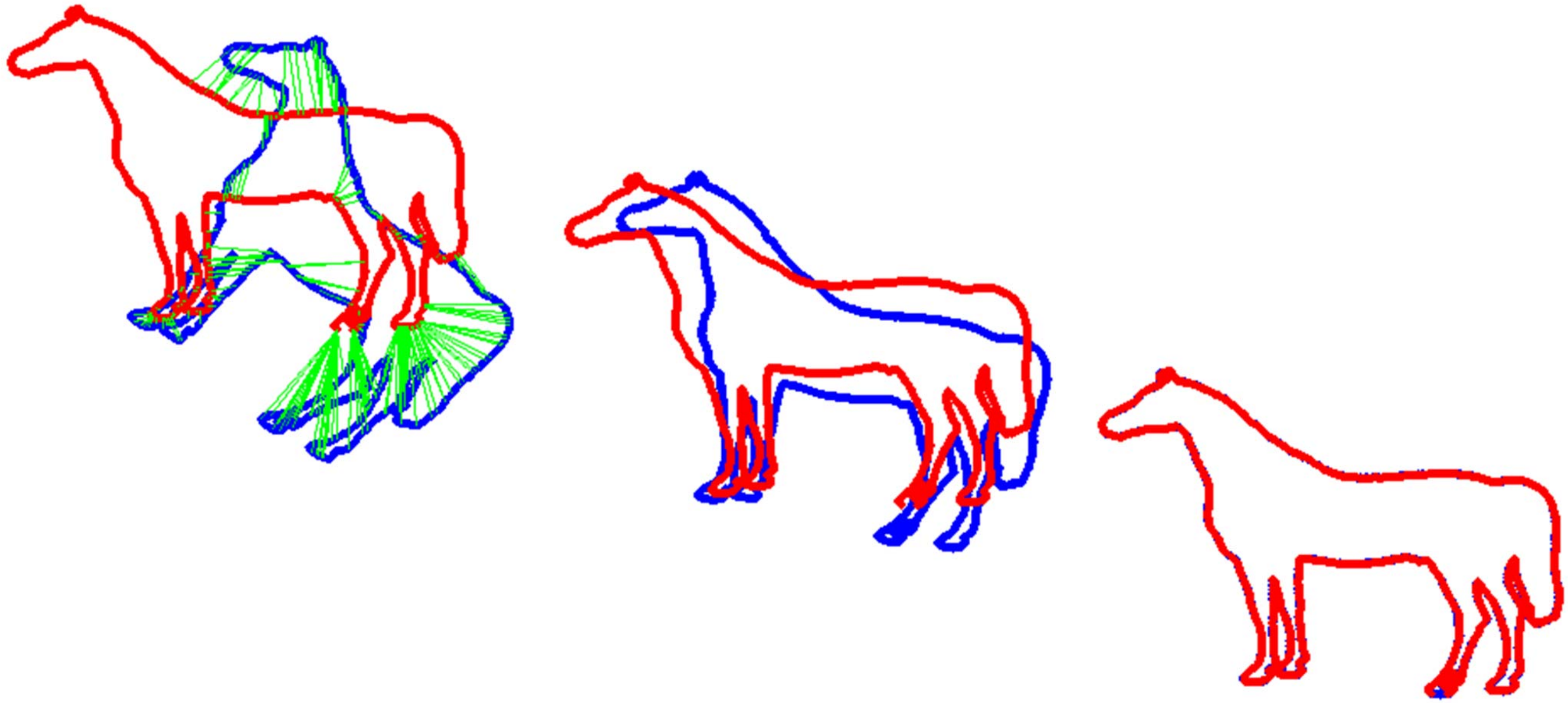
$$\min_{T,c(i)\in\{1,2,\ldots,N_y\}}\left(\sum_{i=1}^{N_x}\|T(\vec{x}_i)-\vec{y}_{c(i)}\|_2^2\right)$$





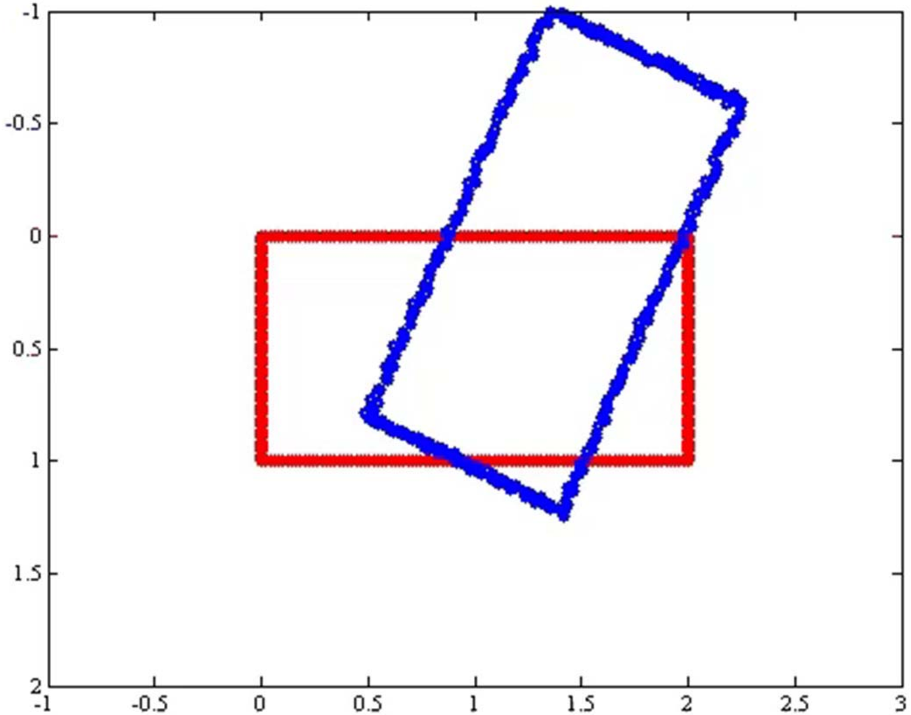
1.1

- Iterative Closest Point



1.1

❑ Iterative Closest Point



引例一和二说明

1.1

- Solution of Linear Equations System

n $\boldsymbol{A}x = b$

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{pmatrix}$$

- (Gramer) A det(A)≠0 $Ax = b$

$$x_1 = \frac{\det(\boldsymbol{A}_1)}{\det(\boldsymbol{A})} \quad x_2 = \frac{\det(\boldsymbol{A}_2)}{\det(\boldsymbol{A})} \quad \dots \quad x_n = \frac{\det(\boldsymbol{A}_n)}{\det(\boldsymbol{A})}$$

1.1

$$A_j = \begin{pmatrix} a_{11} & \cdots & a_{1(j-1)} & b_1 & a_{1(j+1)} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2(j-1)} & b_2 & a_{2(j+1)} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{n(j-1)} & b_n & a_{n(j+1)} & \cdots & a_{nn} \end{pmatrix} (j = 1,2,\ldots,n)$$

b $\boldsymbol{A}$ j n

$a_{1(j-1)}$ 1 j−1

➢ $\det(\boldsymbol{A}), \det(\boldsymbol{A}_1), \cdots, \det(\boldsymbol{A}_n)$, $n+1$ $n!$ $n-1$

引例三说明

1.2

❑

定义1.1 n $x \in \mathrm{R}^n$ $\|\cdot\|$

1) $\|x\| \geq 0, \|x\| = 0$ $x = 0$,

2) $\|\alpha x\| = |\alpha|\|x\|, \alpha \in \mathrm{R}$,

3) $\|x + y\| \leq \|x\| + \|y\|, \forall x, y \in \mathrm{R}^n$.

➢ $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}, \ x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathrm{R}^n$

➢

❖ 1- $\|x\|_1 = |x_1| + |x_2| + \cdots + |x_n|$

❖ 2- $\|x\|_2 = (|x_1|^2 + |x_2|^2 + \cdots + |x_n|^2)^{1/2}$

❖ ∞- $\|x\|_\infty = \max(|x_1|, |x_2|, \cdots, |x_n|)$

1.2

例1.1 $\boldsymbol{A}$ n $\|x\|_A = (x^T \boldsymbol{A} x)^{\frac{1}{2}}$

证明：(1) $\boldsymbol{A}$ $x = 0$ $\|x\|_A = 0$ $x \neq 0,$
$\|x\|_A = (x^{\mathrm{T}} \boldsymbol{A} x)^{\frac{1}{2}} > 0$

(2) $\alpha,$

$$\|\alpha x\|_A = \sqrt{\alpha x^{\mathrm{T}} \boldsymbol{A}(\alpha x)} = |\alpha| \sqrt{x^{\mathrm{T}} \boldsymbol{A} x} = |\alpha| \cdot \|x\|_A$$

(3) $\boldsymbol{A}$ $\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^T,\ \boldsymbol{L}$

$$\|x\|_A = (x^{\mathrm{T}} \boldsymbol{A} x)^{\frac{1}{2}} = (\, x^{\mathrm{T}}\, \boldsymbol{L}\boldsymbol{L}^{\mathrm{T}} x\,)^{\frac{1}{2}} = ((\boldsymbol{L}^{\mathrm{T}} x)^{\mathrm{T}} (\boldsymbol{L}^{\mathrm{T}} x))^{\frac{1}{2}} = \left\|\boldsymbol{L}^{\mathrm{T}} x\right\|_2$$

x y

$$\|x + y\|_A = \left\|\boldsymbol{L}^{\mathrm{T}}(x + y)\right\|_2 = \left\|\boldsymbol{L}^{\mathrm{T}} x + \boldsymbol{L}^{\mathrm{T}} y\right\|_2 \leq \left\|\boldsymbol{L}^{\mathrm{T}} x\right\|_2 + \left\|\boldsymbol{L}^{\mathrm{T}} y\right\|_2$$
$$= \|x\|_A + \|y\|_A$$

$$\|x\|_A = (x^{\mathrm{T}} \boldsymbol{A} x)^{\frac{1}{2}}$$

1.2

❑

定理 1.1 n $x \in \mathbf{R}^n$ $\mathbf{R}^n$

$\|x\|_\alpha$ $\|x\|_\beta$ $c_1, c_2 > 0$

x

$$c_1\|x\|_\beta \leq \|x\|_\alpha \leq c_2\|x\|_\beta$$

$$\|x\|_2 \leq \|x\|_1 \leq \sqrt{n}\|x\|_2$$

$$\|x\|_\infty \leq \|x\|_1 \leq n\|x\|_\infty$$

$$\|x\|_\infty \leq \|x\|_2 \leq \sqrt{n}\|x\|_\infty$$

x

1.2

❑

$\phi \quad D \subset \mathbf{R}^n \to \mathbf{R}^n$

$$x = \phi(x), x \in D \subset \mathbf{R}^n$$

映射ϕ的不动点

定义 1.3 $\mathbf{R}^n$ $\|\cdot\|$ $q \in [0,1)$，
$\phi \quad D \subset \mathbf{R}^n \to \mathbf{R}^n$ $D_0 \subset D$

$$\|\phi(x) - \phi(y)\| \leq q\|x - y\|, \forall\, x, y \in D_0$$

ϕ $D_0 \subset D$ q

定理 1.3 (Banach (巴拿赫)压缩映射原理) $\phi: D \subset \mathbf{R}^n \to \mathbf{R}^n$
$D_0 \subset \mathrm{D}$ $\phi(D_0) \subset D_0$, ϕ D_0 x^*

1.2

❑

定义1.4 n $\boldsymbol{A} \in \mathrm{R}^{n\times n}$ $\|\cdot\|$

1) $\|\boldsymbol{A}\| \geq 0, \|\boldsymbol{A}\| = 0$ $\boldsymbol{A} = 0,$

2) $\|\alpha\boldsymbol{A}\| = |\alpha|\|\boldsymbol{A}\|, \alpha \in \mathrm{R},$

3) $\|\boldsymbol{A} + \boldsymbol{B}\| \leq \|\boldsymbol{A}\| + \|\boldsymbol{B}\|, \forall \boldsymbol{A}, \boldsymbol{B} \in \mathrm{R}^{n\times n},$

4) $\|\boldsymbol{A} \cdot \boldsymbol{B}\| \leq \|\boldsymbol{A}\| \cdot \|\boldsymbol{B}\|, \forall \boldsymbol{A}, \boldsymbol{B} \in \mathrm{R}^{n\times n}.$

➢ $\|\boldsymbol{A}\| = \sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|,\ \boldsymbol{A} = (a_{ij}) \in \mathrm{R}^{n\times n}$

➢

❖ $\boldsymbol{A}$ $\|\boldsymbol{A}\|_\infty = \max\limits_{1\leq i\leq n}\sum_{j=1}^{n}|a_{ij}|$

❖ $\boldsymbol{A}$ $\|\boldsymbol{A}\|_1 = \max\limits_{1\leq j\leq n}\sum_{i=1}^{n}|a_{ij}|$

❖ $\boldsymbol{A}$ $\|\boldsymbol{A}\|_2 = \sqrt{\lambda_{max}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})}$

1.2

例1.2 $\boldsymbol{A}=\left(a_{ij}\right)_{n\times n}$ $\|\boldsymbol{A}\|=\sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}\right|$

证明：(1) $\|\boldsymbol{A}\|=\sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}\right|\geq 0$且$\|\boldsymbol{A}\|=0\Leftrightarrow\boldsymbol{A}=0$

(2) α

$$\|\alpha\boldsymbol{A}\|=\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\alpha a_{ij}\right|=|\alpha|\sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}\right|=|\alpha|\cdot\|\boldsymbol{A}\|$$

(3) $\|\boldsymbol{A}+\boldsymbol{B}\|=\sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}+b_{ij}\right|\leq$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}\right|+\sum_{i=1}^{n}\sum_{j=1}^{n}\left|b_{ij}\right|=\|\boldsymbol{A}\|+\|\boldsymbol{B}\|$$

(4) $\|\boldsymbol{AB}\|=\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\sum_{k=1}^{n}a_{ik}b_{kj}\right|\leq\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}|a_{ik}|\left|b_{kj}\right|$

$$\leq\left(\sum_{i=1}^{n}\sum_{k=1}^{n}|a_{ik}|\right)\left(\sum_{k=1}^{n}\sum_{j=1}^{n}\left|b_{kj}\right|\right)=\|\boldsymbol{A}\|\cdot\|\boldsymbol{B}\|$$

$\|\boldsymbol{A}\|$

1.2

$$\|\boldsymbol{A}x\|_p \le \|\boldsymbol{A}\|_p \cdot \|x\|_p \quad 或 \quad \frac{\|\boldsymbol{A}x\|_p}{\|x\|_p} \le \|\boldsymbol{A}\|_p \quad (p = 1,2,\infty)$$

❑

定义 1.5 $\|x\|_p (p = 1,2,\infty)$

$$f(A) = \|\boldsymbol{A}\|_p$$

$$\|\boldsymbol{A}\|_p = \max_{x \ne 0} \frac{\|\boldsymbol{A}x\|_p}{\|x\|_p} = \max_{\|x\|_p = 1} \|\boldsymbol{A}x\|_p$$

$\|\boldsymbol{A}\|_p$ $\|\boldsymbol{A}\|_p$

1.2

❑

定理1.4 n $\boldsymbol{A}$ $\mathbf{R}^{n\times n}$

$\|\boldsymbol{A}\|_\alpha$ $\|\boldsymbol{A}\|_\beta$ $c_1, c_2 > 0,$

n $\boldsymbol{A}$

$$c_1\|\boldsymbol{A}\|_\alpha \le \|\boldsymbol{A}\|_\beta \le c_2\|\boldsymbol{A}\|_\alpha$$

$$\frac{1}{\sqrt{n}}\|\boldsymbol{A}\|_2 \le \|\boldsymbol{A}\|_\infty \le \sqrt{n}\|\boldsymbol{A}\|_2$$

$$\frac{1}{n}\|\boldsymbol{A}\|_\infty \le \|\boldsymbol{A}\|_1 \le n\|\boldsymbol{A}\|_\infty$$

1.3

Errors

$$I_n = \int_0^1 \frac{x^n}{x+5} dx \quad (n = 1,2,\cdots,20)$$

$$I_n + 5I_{n-1} = \int_0^1 \frac{x^n}{x+5} dx + 5\int_0^1 \frac{x^{n-1}}{x+5} dx = \int_0^1 \frac{x^n + 5x^{n-1}}{x+5} dx$$

$$= \int_0^1 x^{n-1} dx = \frac{1}{n} x^n |_0^1 = \frac{1}{n}$$

$$I_n = \frac{1}{n} - 5I_{n-1} \qquad 1$$

$$I_0 = \int_0^1 \frac{1}{x+5} \mathrm{dx} = \ln(x+5)|_0^1 = \ln 6 - \ln 5 = \ln\frac{6}{5} \approx 0.182322$$

1.3

I_n

$I_n > 0$ $f(x)$ 0 1

I_n $n_1 > n_2$ $I_{n_1} < I_{n_2}$

$\lim\limits_{n\to\infty} I_n = 0$ $\lim\limits_{n\to\infty} \frac{x^n}{x+5} = 0$ $x \in [0,1]$

$\frac{1}{6n} < I_{n-1} < \frac{1}{5n}$ $(n > 1)$

1 $I_{\mathrm{n}-1} = \frac{1}{5n} - \frac{1}{5} I_n < \frac{1}{5n}$ I_n $I_n = \frac{1}{n} - 5I_{\mathrm{n}-1} < I_{\mathrm{n}-1}$, $I_{\mathrm{n}-1} > \frac{1}{6n}$, I_n.

1.3

方法A： 1 $I_n = \frac{1}{n} - 5I_{n-1}$ I_1 I_{20}

n	I_n	n	I_n	n	I_n	n	I_n
1	0.0883922	6	0.0243239	11	0.0173247	16	-10.1569
2	0.0580389	7	0.0212378	12	-0.00329022	17	50.8433
3	0.0431687	8	0.0188109	13	-0.0933742	18	-254.161
4	0.0343063	9	0.0170566	14	-0.395442	19	1270.86
5	0.0284686	10	0.0147169	15	2.04388	20	-6354.23

❑

➢ A I_0 6

I_{12} I_n

A

1.3

❑

➢ A 1 $I_n = \frac{1}{n} - 5I_{n-1}$ I_{n-1} I_n

I_{n-1} 5

方法B： I_n $I_{\mathrm{n}-1} \approx (\frac{1}{5n} + \frac{1}{6n})/2$

$$I_{20} \approx \frac{\frac{1}{6\times21} + \frac{1}{5\times21}}{2} = 0.00873016$$

$I_{\mathrm{n}-1} = \frac{1}{5n} - \frac{1}{5} I_n$ I_{20} I_1

1.3

n	I_n	n	I_n	n	I_n	n	$\mathrm{I_n}$
19	0.00825397	14	0.0112292	9	0.0169265	4	0.0343063
18	0.00887552	13	0.0120399	8	0.0188369	3	0.0431387
17	0.00933601	12	0.0129766	7	0.0212326	2	0.0580389
16	0.00989750	11	0.0140713	6	0.0243250	1	0.0883922
15	0.0105205	10	0.0153676	5	0.0284684	0	0.182322

- B I_{20} I_0 A
- B $I_{\mathrm{n}-1} = \frac{1}{5n} - \frac{1}{5} I_n$ I_n I_{n-1} I_n 1/5

1.3

误差

- truncation error
- round-off error

1.3

例1.4 $\int_0^1 e^{-x^2}dx$

e^{-x^2} Taylor

$$\int_0^1 e^{-x^2}dx = \int_0^1 \left(1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots\right) dx$$

$$= \underbrace{1 - \frac{1}{3} + \frac{1}{2!}\times\frac{1}{5} - \frac{1}{3!} + \frac{1}{7}}_{S} + \underbrace{\frac{1}{4!}\times\frac{1}{9} - \frac{1}{5!}\times\frac{1}{11}\cdots}_{E}$$

$$\int_0^1 e^{-x^2}dx \approx S$$

$$E = \frac{1}{4!}\times\frac{1}{9} - \frac{1}{5!}\times\frac{1}{11}\cdots$$

截断误差

1.3

例1.4 $\int_0^1 e^{-x^2}dx$

$$\int_0^1 e^{-x^2}dx \approx S = 1 - \frac{1}{3} + \frac{1}{2!} \times \frac{1}{5} - \frac{1}{3!} + \frac{1}{7}$$

$$= 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42}$$

$$\approx 1 - \underline{0.333} + 0.100 - \underline{0.024}$$

$$= 0.743$$

舍入误差

1.3

❑

➢ Absolute Error

定义1.6 x^* $\tilde{x}$

$E(\tilde{x}) = x^* - \tilde{x}$ $\tilde{x}$

❖ δ $E(\tilde{x}) = |x^* - \tilde{x}| \leq \delta$ δ

$\tilde{x}$

1.3

❑

➢ Relative Error

定义1.7 x^* $\tilde{x}$

$E_r(\tilde{x}) = \frac{x^*-\tilde{x}}{x^*} \times 100\% (x^* \neq 0)$ $\tilde{x}$

❖ δ_r $|E_r(\tilde{x})| = \left|\frac{x^*-\tilde{x}}{\tilde{x}}\right| \le \delta_r$ δ_r $\tilde{x}$

■

■

1.3

❑

➢ Significant Figures

定义1.8 $\tilde{x}$

$\tilde{x}$ n n $\tilde{x}$

定义1.9 $\tilde{x}$ x

1.3

❑

$$E_r(\tilde{x}) = \frac{E(\tilde{x})}{\tilde{x}}$$

$$\delta_r(\tilde{x}) = \frac{\delta(\tilde{x})}{|\tilde{x}|}$$

1.3

-

  -

$\tilde{x}$ x^* $f(x^*)$ $\tilde{x}$

$$f(x^*) \approx f(\tilde{x}) + f'(\tilde{x})(x^* - \tilde{x})$$

$$|f(x^*) - f(\tilde{x})| \le |f'(\tilde{x})| \cdot |x^* - \tilde{x}|$$

$f(x^*)$ $f(\tilde{x})$

$$\begin{cases} \delta f(\tilde{x}) \le |f'(\tilde{x})| \cdot \delta(\tilde{x}) \\ \delta_r f(\tilde{x}) = \left|\dfrac{\delta f(\tilde{x})}{f(\tilde{x})}\right| \le \left|\dfrac{f'(\tilde{x})}{f(\tilde{x})}\right| \cdot \delta(\tilde{x}) \end{cases}$$

$\delta(\tilde{x})$ $\tilde{x}$

1.3

例1.5 $\tilde{x} > 0, \tilde{x}$ ε_r, $\ln\tilde{x}$

解: $f(x) = \ln x$,

$$\begin{cases} \delta f(\tilde{x}) \le |f'(\tilde{x})| \cdot \delta(\tilde{x}) \\ \delta_r f(\tilde{x}) = \left|\dfrac{\delta f(\tilde{x})}{f(\tilde{x})}\right| \le \left|\dfrac{f'(\tilde{x})}{f(\tilde{x})}\right| \cdot \delta(\tilde{x}) \end{cases}$$

$$\delta\big(f(\tilde{x})\big) = |f(x^*) - f(\tilde{x})| \le |f'(\tilde{x})| \cdot \delta(\tilde{x}) = \frac{1}{|\tilde{x}|}|x^* - \tilde{x}| = \varepsilon_r,$$

$\ln\tilde{x}$ ε_r

1.3

❑

➢

$f(x^*, y^*)$ $(\tilde{x}, \tilde{y})$

$$f(x^*, y^*) \approx f(\tilde{x}, \tilde{y}) + \frac{\partial f(\tilde{x},\tilde{y})}{\partial x}(x^* - \tilde{x}) + \frac{\partial f(\tilde{x},\tilde{y})}{\partial y}(y^* - \tilde{y})$$

$$|f(x^*, y^*) - f(\tilde{x}, \tilde{y})| \le \left|\frac{\partial f(\tilde{x},\tilde{y})}{\partial x}\right| \cdot |x^* - \tilde{x}| + \left|\frac{\partial f(\tilde{x},\tilde{y})}{\partial y}\right| \cdot |y^* - \tilde{y}|$$

$f(x^*, y^*)$ $f(x, y)$

$$\begin{cases} \delta f(\tilde{x}, \tilde{y}) \le \left|\frac{\partial f(\tilde{x},\tilde{y})}{\partial x}\right| \cdot \delta(\tilde{x}) + \left|\frac{\partial f(\tilde{x},\tilde{y})}{\partial y}\right| \cdot \delta(\tilde{y}) \\ \delta_r|f(\tilde{x}, \tilde{y})| = \left|\frac{\delta f(\tilde{x},\tilde{y})}{f(\tilde{x},\tilde{y})}\right| \end{cases}$$

$\delta(\tilde{x})$ $\delta(\tilde{y})$ $\tilde{x}$ $\tilde{y}$

1.3

-
  - $f(x,y)=x\pm y$, $f(x,y)=x\cdot y$, $f(x,y)=x/y$, $\tilde{x}$ x^* $\tilde{y}$ y^*

$$\begin{cases} \delta(\tilde{x}\pm\tilde{y}) \le \delta(\tilde{x})+\delta(\tilde{y}) \\ \delta_r(\tilde{x}\pm\tilde{y}) \le \frac{\delta(\tilde{x})+\delta(\tilde{y})}{|\tilde{x}\pm\tilde{y}|} \end{cases}$$

$$\begin{cases} \delta(\tilde{x}\tilde{y}) \le |\tilde{y}|\delta(\tilde{x})+|\tilde{x}|\delta(\tilde{y}) \\ \delta_r(\tilde{x}\tilde{y}) \le \dfrac{\delta(\tilde{x})}{|\tilde{x}|}+\dfrac{\delta(\tilde{y})}{|\tilde{y}|} = \delta_r(\tilde{x})+\delta_r(\tilde{y}) \end{cases}$$

$$\begin{cases} \delta\left(\frac{\tilde{x}}{\tilde{y}}\right) \le \frac{1}{|\tilde{y}|}\delta(\tilde{x})+\left|\frac{\tilde{x}}{\tilde{y}^2}\right|\delta(\tilde{y}) \\ \delta_r\left(\frac{\tilde{x}}{\tilde{y}}\right) \le \frac{\delta(\tilde{x})}{|\tilde{x}|}+\frac{\delta(\tilde{y})}{|\tilde{y}|} = \delta_r(\tilde{x})+\delta_r(\tilde{y}) \end{cases}$$

1.3

❑

定义1.10

例1.6 $\{x_n\}$ $x_{n+1} = 8x_n + 6(n = 0,1,2\cdots)$. $x_0 = \sqrt{2} \approx 1.41$ x_{10} ?

解： $x_0 = \sqrt{2}$ $\tilde{x}_0 = 1.41,$

$\delta($ $)$

1.4

❑

$$\delta_r(\tilde{x}-\tilde{y}) \le \frac{\delta(\tilde{x}-\tilde{y})}{|\tilde{x}-\tilde{y}|}$$

$\tilde{x}$ $\tilde{y}$ $\tilde{x}-\tilde{y}$

❑

$z=\frac{x}{y}$

$$\delta\left(\frac{\tilde{x}}{\tilde{y}}\right) \le \frac{1}{|\tilde{y}|}\delta(\tilde{x})+\frac{|\tilde{x}|}{|\tilde{y}^2|}\delta(\tilde{y})$$

1.4

❑ “ ”

1

$x = 10^9 + 1$ $x = 0.1 \times 10^{10} + 0.0000000001 \times 10^{10}$。 8 $x = 0.1 \times 10^{10}$

例1.7： $x^2 - (10^9 + 1)x + 10^9 = 0$

$$(x-1)(x-10^9) = 0$$

$$x_1 = 10^9 \quad x_2 = 1$$

1.4

$$x_{1,2} = \frac{10^9+1 \pm \sqrt{(10^9+1)^2 - 4\times 10^9}}{2}$$

8

$10^9 + 1 \approx 10^9$ $\sqrt{(10^9 + 1)^2 - 4 \times 10^9} \approx 10^9$

$x_1 = 10^9$ $x_2 = 0$

“ ”

$$x_2 = \frac{10^9+1-\sqrt{(10^9+1)^2-4\times 10^9}}{2}$$

$$x_2 = \frac{2\times 10^9}{10^9+1+\sqrt{(10^9+1)^2-4\times 10^9}}$$

$$x_2 \approx \frac{2\times 10^9}{10^9+10^9} = 1$$

1.4

-

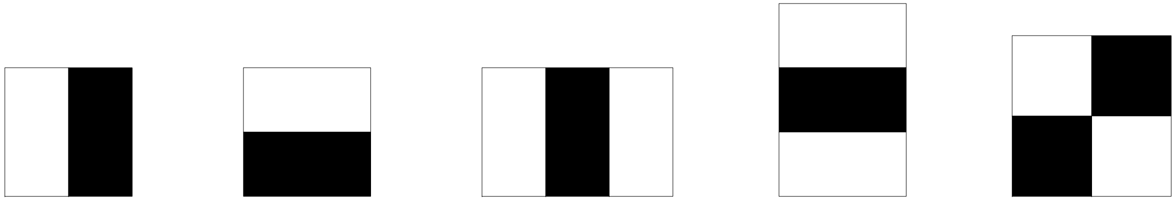
-

A

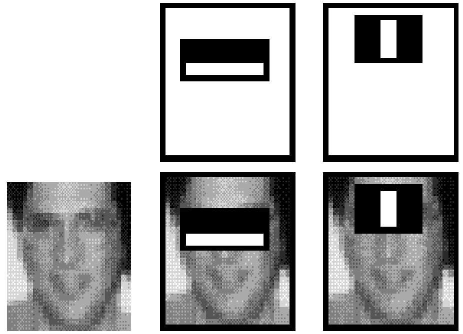
B

1.4

❑ Haar-like features

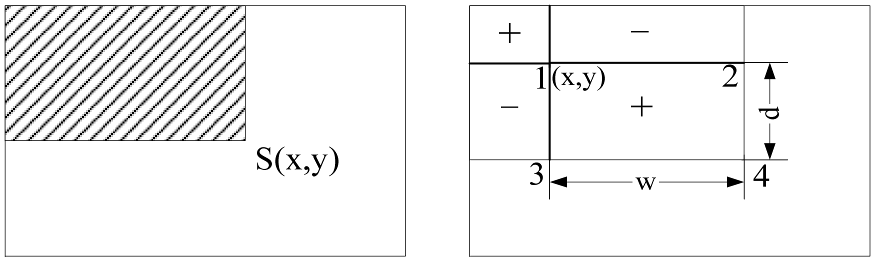


(a) (b) (c) (d) (e)



❑ Fast computation schedule

- ➢ Integral image
- ➢ Rectangle features is computed by integral image



(a) (b)

$$S_1 + S_4 - S_2 - S_3$$