4

4.1

4.2

4.3

4.4

4.5

4.6

4.7

4.8

4.1

4.1

- ➢
- ➢ $\left(x_i \quad f(x_i)\right), i = 0,1,2,\ldots n$

  $p(x)$

  $p(x)$

  $p(x_i) = f(x_i)$ $p(x)$ $f(x)$

4.1

❑

定义4.1 $= (\ )$ $[\ ,\ ]$ $[\ ,\ ]$ $(\ )$

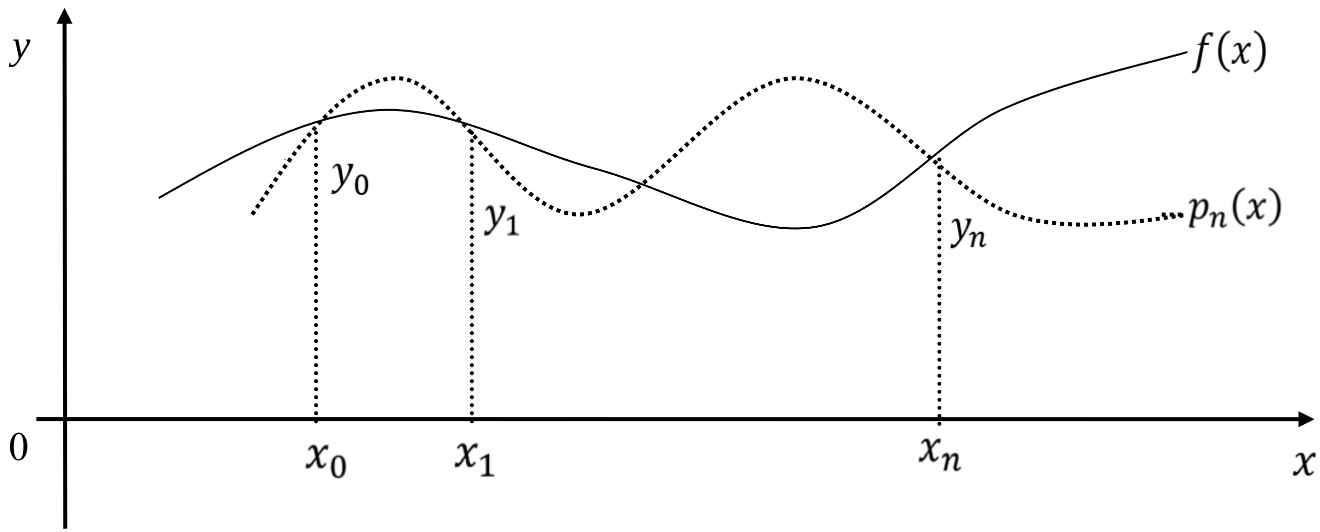
$[\ ,\ ]$ $+\ 1$ $\leq\ <\ <\ <\ \leq$ $=$

$(\ ),\ = 0,1,2,\dots$

n $(\ ) = + + + + ,$

$$(\ ) = ,(\ = 0,1,2,\dots\ )$$

$(\ )$ $(\ )$ $[\ ,\ ]$ $\{\ \}$



4.1

4.1

-
  -
  - $\left(x_{ш} \quad f(x_{ш})\right)$

    $p(x)$ $\quad$ $p(x_{ш}) = f(x_{ш})$
  - $f(x)$ $\quad$ $p(x)$

4.1

- $f(x)$ $p(x)$

$$||p-f||_{\text{ا}} = \max_{\text{ربطبذ}} |p(x)-f(x)|$$

$p(x)$ $f(x)$

- $p_{\text{ص}}(x)$

$\lim\limits_{\text{ص}\to\text{ا}} ||p_{\text{ص}}(x)-f(x)||_{\text{ا}} = 0$ $\{p_{\text{ص}}(x)\}$

$[a,b]$ $f(x)$ $\{p_{\text{ص}}(x)\}$ $f(x)$

4.1

❑

➢

$$||p-f||_{\text{ب}}^{\text{ب}} = \int_{\text{ذ}}^{\text{ر}} (p(x)-f(x))^{\text{ب}}$$

$p(x)$ $f(x)$

➢ $p_{\text{ص}}(x)$

$$\lim_{\text{ص}\to\text{ا}} ||p_{\text{ص}}(x)-f(x)||_{\text{ب}}^{\text{ب}} = 0$$

$\{p_{\text{ص}}(x)\}$ $[a,b]$ $f(x)$

$\{p_{\text{ص}}(x)\}$ $f(x)$

4.1

-

  -

$$\|p - f\|_2^2 = \sum_{i=1}^{n} (p(x_i) - f(x_i))^2$$

$p(x)$ $f(x)$

$p(x)$ $f(x)$

4.2

- n
  - $+\ 1$ $(\ ,\ ),\ =0,1,\ ,$ n $(\ )$

    $(\ )\ =\ \ (\ =0,1,2,\ ,\ )$
  -
    $$(\ )\ =\ (\ )$$

    $(\ )$

    $$(\ )\ =\ \begin{cases}1 & = \\ 0 & \neq\end{cases}$$
  - $(\ )$

    $$(\ )\ =\ (\ -\ )$$

4.2

- $l_k(x_k) = 1$,

$$l_k(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)} = \prod_{\substack{j=0\\ j\neq k}}^{n} \frac{x-x_j}{x_k-x_j}$$

- n $L_n(x)$

$$L_n(x) = \sum_{k=0}^{n}\left(\prod_{\substack{j=0\\ j\neq k}}^{n} \frac{x-x_j}{x_k-x_j}\right) y_k$$

拉格朗日(Lagrange)插值多项式

$l_k(x)$ $L_n(x)$

$$L_n(x_j) = \sum_{k=0}^{n} l_k(x_j)\, y_k = y_j$$

$L_n(x)$

4.2

例4.1 = 1,2,3,4,5 ( ) = 5,14,28,3,1, 4

(3.2)

:

$(\ )$

$$= \frac{(\ -2)(\ -3)(\ -4)(\ -5)}{(1-2)(1-3)(1-4)(1-5)} \times 5 + \frac{(\ -1)(\ -3)(\ -4)(\ -5)}{(2-1)(2-3)(2-4)(2-5)}$$

$$\times 14 + \frac{(\ -1)(\ -2)(\ -4)(\ -5)}{(3-1)(3-2)(3-4)(3-5)} \times 28$$

$$+ \frac{(\ -1)(\ -2)(\ -3)(\ -5)}{(4-1)(4-2)(4-3)(4-5)} \times 3 + \frac{(\ -1)(\ -2)(\ -3)(\ -4)}{(5-1)(5-2)(5-3)(5-4)}$$

$$\times 1 = \frac{53}{12} \ - \frac{172}{3} \ + \frac{2413}{12} \ - 300 \ + 15$$

$$(3.2) \approx \ (3.2) = -301.4842667$$

4.2

- -
  - (x_0, y_0) $\left(x_1, y_1\right)$, $f(x_0) = y_0$, $f(x_1) = y_1$
  - (x_0, y_0) (x_1, y_1)

$$L_1(x) = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$$

$L_1(x_0) = y_0$, $L_1(x_1) = y_1$

$L_1(x_1)$

4.2

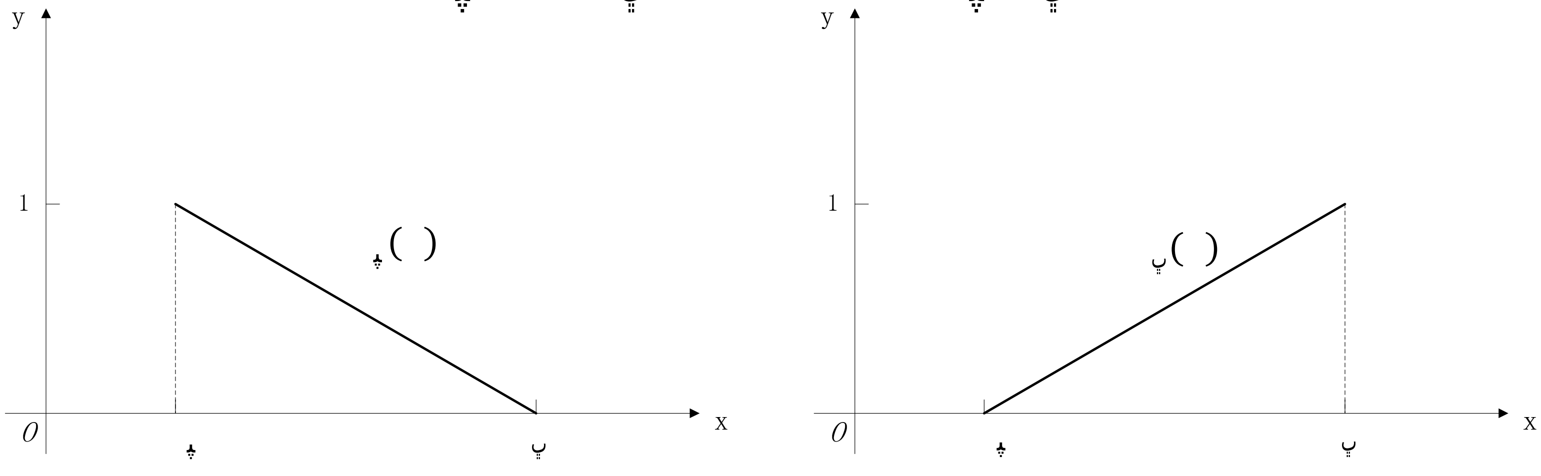
$$l_k(x) = \frac{x - x_{k+1}}{x_k - x_{k+1}}, \quad l_{k+1}(x) = \frac{x - x_k}{x_{k+1} - x_k}$$

$$L_1(x) = y_k l_k(x) + y_{k+1} l_{k+1}(x)$$

$l_k(x)$ $l_{k+1}(x)$

$$l_k(x_k) = 1, \quad l_k(x_{k+1}) = 0$$
$$l_{k+1}(x_k) = 0, \quad l_{k+1}(x_{k+1}) = 1$$

$l_k(x)$, $l_{k+1}(x)$ x_k, x_{k+1}



(a) (b)

4.2

例4.2 $\sqrt{25}=5$ $\sqrt{36}=6$ $y=\sqrt{31}$

$$x_0=25,\ y_0=5,\ x_1=36,\ y_1=6,$$

$$L_1(x)=\frac{x-36}{25-36}\times 5+\frac{x-25}{36-25}\times 6$$

$x=31,$

$$y=\sqrt{31}\approx\frac{31-36}{25-36}\times 5+\frac{31-25}{36-25}\times 6=5.54545$$

2

- $L_1(x)$ $l_0(x)$ $l_1(x)$ y_0, y_1

4.2

- -
  - $(x_0, y_0), (x_1, y_1), (x_2, y_2)$

$$f(x_i) = y_i, i = 0,1,2$$

  - $L_2(x)$ $L_2(x)$,

$$L_2(x_i) = y_i \quad (i = 0,1,2)$$

$$L_2(x)$$

$$L_2(x) = y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x)$$

$l_0(x), l_1(x), l_2(x)$

4.2

- -

  -

$$_3(\ _1) = \ _1,\quad _3(\ _2) = \ _2,\quad _3(\ _3) = \ _3$$

$$_1(\ ),\quad _2(\ ),\quad _3(\ )$$

$$_3(\ _1) = \ _1,\quad _3(\ _2) = \ _2,\quad _3(\ _3) = \ _3$$

$$_1(\ ),\quad _2(\ ),\quad _3(\ )$$

$$\begin{aligned} _1(\ _1) &= 1,\ _1(\ _2) = 0,\ _1(\ _3) = 0 \\ _2(\ _1) &= 0,\ _2(\ _2) = 1,\ _2(\ _3) = 0 \\ _3(\ _1) &= 0,\ _3(\ _2) = 0,\ _3(\ _3) = 1 \end{aligned}$$

4.2

- -

➢ ﭙ( )

پ, پ ﭙ( )

ﭙ( ) = (− پ)(− پ)

ﭙ(ﭙ) = 1

= 1 / ((ﭙ − پ)(ﭙ − پ))

ﭙ( ) = (ط ﭡ ط)(ط ﭡ ط) / (ط ﭡ ط)(ط ﭡ ط)

پ( ) = (ط ﭡ ط)(ط ﭡ ط) / (ط ﭡ ط)(ط ﭡ ط)   پ( ) = (ط ﭡ ط)(ط ﭡ ط) / (ط ﭡ ط)(ط ﭡ ط)

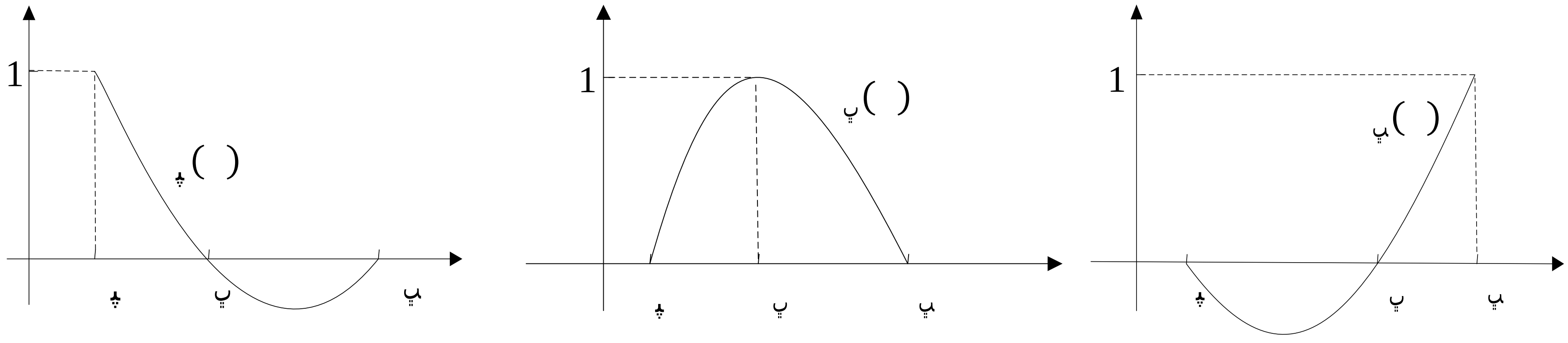
4.2

- -

  - $L_2(x)$

$$L_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

  - $L_2(x_0) = y_0$, $L_2(x_1) = y_1$, $L_2(x_2) = y_2$,
    $l_0(x)$, $l_1(x)$, $l_2(x)$ x_0, x_1, x_2



(a) (b) (c)

4.2

例4.3 $\sqrt{25} = 5, \sqrt{36} = 6, \sqrt{49} = 7,\quad = \sqrt{31}$

$= 25, \quad = 5, \quad = 36, \quad = 6, \quad = 49, \quad = 7,$

$(\quad)$

$$= \frac{(\quad - 36)(\quad - 49)}{(25 - 36)(25 - 49)} \times 5 + \frac{(\quad - 25)(\quad - 49)}{(36 - 25)(36 - 49)} \times 6$$

$$+ \frac{(\quad - 25)(\quad - 36)}{(49 - 25)(49 - 36)} \times 7$$

$= 31 \quad , \qquad = \sqrt{31} \approx \frac{(\quad)(\quad)}{(\quad)(\quad)} \times 5 + \frac{(\quad)(\quad)}{(\quad)(\quad)} \times 6 + \frac{(\quad)(\quad)}{(\quad)(\quad)} \times 7 = 5.5629$

2

4.2

❑

➢ ( ) [,] $( ),$ $( ) = ( ) -$
$( )$ $( )$

定理4.1 $( )$ $[,]$ $+ 1$
$( )$ $[,]$ $( )$ $\leq \; < \; < \; <$
$\leq$, , $( ) =$
$(= 0,1,2, \; , \;)$ $( ),$ $[,]$

$$( ) = ( ) - ( )$$

$$= \frac{( )}{(+ 1)!} (-) = \frac{( )}{(+ 1)!} ( )$$

$$< \; < \; , \; = \prod (-)$$

4.2

- $|f^{(n+1)}(x)| \le \max_{a\le x\le b}\left|f^{(n+1)}(x)\right| = M_{n+1}$

$$|R_n(x)| \le \frac{M_{n+1}}{(n+1)!}\left|\prod_{i=0}^{n}(x-x_i)\right|$$

- $n = 1$ $\quad |R_1(x)| \le \frac{M_2}{2!}\left|\prod_{i=0}^{1}(x-x_i)\right| \le \frac{M_2}{8}(x_1-x_0)^2$
- $n = 2$ $\quad |R_2(x)| \le \frac{M_3}{3!}\left|\prod_{i=0}^{2}(x-x_i)\right|$

4.2

例4.4 $\sqrt{25}=5, \sqrt{36}=6, \sqrt{49}=7,$

$$y=f(x)=\sqrt{x},$$

$$(\ )=\frac{1}{2}\quad,\qquad (\ )=-\frac{1}{4}\quad,\qquad (\ )=\frac{3}{8}\quad (\qquad [25,49])$$

$$= \max \left|-\frac{1}{4}\right| \le \frac{1}{500},\qquad = \max\left|\frac{3}{8}\right| \le \frac{3}{8}\times\frac{1}{3125},$$

➢ $|\ (\ )| \le \left| \times \times (31-25)\times(31-36)\right| = 0.03 < 0.05,$

$\sqrt{31}$ 5.54545 2

➢ $|\ (\ )| \le \left| \times \times (31-25)\times(31-36)\times(31-49)\right| =$

$0.0108 < 0.05,$ $\sqrt{31}$ 5.5629 2

4.2

-

$$\omega_{n+1}(x) = (x - x_0)(x - x_1)\cdots(x - x_n),$$

$$\omega'_{n+1}(x_k) = (x_k - x_0)(x_k - x_1)\cdots(x_k - x_{k-1})(x_k - x_{k+1})\cdots(x_k - x_n)$$

$l_k(x)$

$$l_k(x) = \frac{\omega_{n+1}(x)}{(x - x_k)\,\omega'_{n+1}(x_k)}$$

$$L_n(x) = \sum_{k=0}^{n} \frac{\omega_{n+1}(x)}{(x - x_k)\,\omega'_{n+1}(x_k)}\, y_k$$

$R_n(x)$

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\,\omega_{n+1}(x)$$

4.2

❑

ص( ), ( )

ص( ) ( )

( ) + 1 پتص( )

ص( ) ( ) ص( ) ( )

( )

پتص( ) = 0 ص( ) = 0 ( ) ≡ ص ( )

( ) + 1

( ) ≡ 1

∑ ( ) ≡ 1

4.2

-

$$(\quad) =$$

$$(\quad) \qquad |\ (\quad)|$$

$$|\prod (\quad - \quad)|$$

$$(\quad)$$

4.2

( )

| پتص( )| پتک

3

پ، پ، پ، پ، پ

( ) پ( ) پ، پ

( ) پ( )

( ) − پ( ) = (پ) / 2 (− پ)(− پ)

( ) − پ( ) = (پ) / 2 (− پ)(− پ)

4.2

( ) $[a,b]$ (ې) ≈

(ې)

$$\frac{(\ ) - \ ې(\ )}{(\ ) - \overline{}ې(\ )} \approx \frac{-\ ې}{-\ ې}$$

$$|\ (\ ) - \ ې(\ )| \approx \left|\frac{-\ ې}{-\ ې}(\ ې(\ ) - \overline{}ې(\ ))\right|$$

$$\leq \left|\frac{-\ ې}{-\ ې}\right|\ |\ ې(\ ) - \overline{}ې(\ )|$$

$\to \infty$ ص( ) ( )

4.2

例4.5 $\sqrt{25}=5,\ \sqrt{36}=6,\ \sqrt{49}=7$ $\sqrt{31}$

$$=25,\quad =5,\quad =36,\quad =6,\quad =49,\quad =7,$$

$$(\ )= \times 5 + \times 7$$

$$(31)= \times 5 + \times 7 = 5.5$$

$$(31)= \times 5 + \times 6 = 5.5454$$

$$|\ (31) - (31)| \le |\ | \times |5.5454 - 5.5| = 0.01746 \le 0.05$$

$\sqrt{31}$ 5.545 2

4.2

-

$$f(x) = \frac{1}{1+25x^2} \quad (-1 \le x \le 1)$$

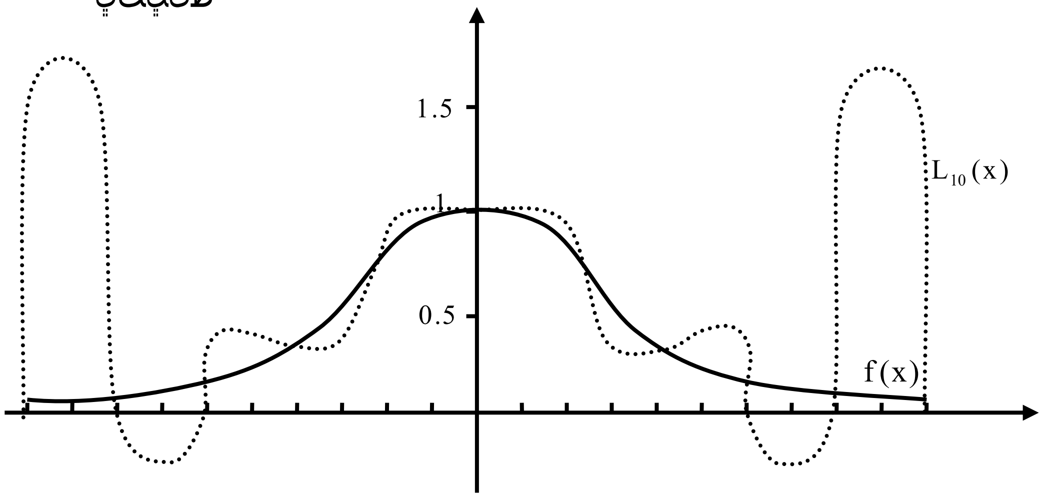
$$x_k = -1 + kh \quad \left(h = \frac{1}{5},\ k = 0,1,\ldots,10\right)$$

$$L_n(x) = \sum_{k=0}^{n} \left(\prod_{\substack{j=0 \\ j\neq k}}^{n} \frac{x - x_j}{x_k - x_j} \right) \frac{1}{1+25x_k^2} \quad (n = 10)$$

4.2

❑

- ➢ پ( ) $(\;) = \dfrac{پ}{پﺗپﻧﻄ}$



- ➢ $|\;| \le 3.63/5$ ص( )

$$\lim_{ص \to} \;ص(\;) = (\;)$$

- ➢ $|\;| \ge 3.63/5$ {ص( )}
- ➢

4.2

( )

[ ]

( )

( )

4.2

❑

定义4.2

( ) n+1 ( ) = (=

0,1,2, ,) ( ) [,] ( )

❖ ( ) [,] [,] (= 0,1,2, ,)

❖ ( ) = (= 0,1,2, ,)

❖ ( ) [,]

( ) [,]

➢ [,](=

0,1,2, ,) (,) (,)

4.2

➢ ﭗ( ) [ﮎﺗﭗ, ﮎ](= 1,2, ,)

ﭗ( ) = (ﻁﺗﻁ)/(ﻁ ﺭﺗﻁ) ﮎﺗﭗ + (ﻁﺗﻁ ﺭ)/(ﻁﺗﻁ ﺭ) ﮎ = ﮎﺗﭗ( ) ﮎﺗﭗ + ﮎ( ) ﮎ (ﮎﺗﭗ ≤ ≤ ﮎ)

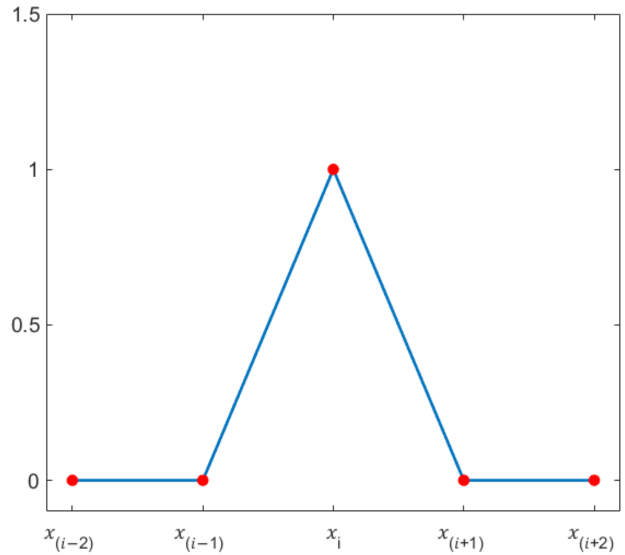
➢ ﭗ( ) [ﮎ, ﮎﺜﭗ] (= 0,1,2, , − 1)

ﭗ( ) = (ﻁﺗﻁ ﺮ)/(ﻁﺗﻁ ﺮ) ﮎ + (ﻁﺗﻁ)/(ﻁ ﺮﺗﻁ) ﮎﺜﭗ = ﮎ( ) ﮎ + ﮎﺜﭗ( ) ﮎﺜﭗ (ﮎ ≤ ≤ ﮎﺜﭗ)

➢ ﭗ( ) [,]

ﭗ( ) = ∑ ﮎ( ) ﮎ

ﮎ( ) = { (− ﮎﺗﭗ)/(ﮎ − ﮎﺗﭗ) ﮎﺗﭗ ≤ ≤ ﮎ (= 0) ; (− ﮎﺜﭗ)/(ﮎ − ﮎﺜﭗ) ﮎ < ≤ ﮎﺜﭗ (= , = 0) ; 0 [,] − [ﮎﺗﭗ, ﮎﺜﭗ] }



4.2

- ➢ $(\ )$ 0 0
- ➢
- ➢

定理4.2 $(\ )$ $(\ ) = (=$
$0,1, \ , \ )$ $(\ )$ $[\ , \ ]$ $''(\ )$ $[\ , \ ]$ $(\ )$
$[\ , \ ]$ $(\ , \ )(\ = 0,1,2, \ , \ )$

$$| \ (\ )| = | \ (\ ) - \ (\ )| \le \frac{}{8}$$

$$= \max | \ - \ |, M = \max | \ ''(\ )|$$

4.2

例4.4 $f(x)=\dfrac{1}{1+25x^2}$ $[-1,1]$ $n=10$

$L_1(x)$ $\qquad$ $L_1(x)$ $f(x)$

: $h=\dfrac{b-(-a)}{n}=0.2$ $\quad x_i=-1+ih=-1+0.2i\ (0\le i\le 10)$

$[x_i,\ x_{i+1}]\ (i=0,1,2,\dots,9)$

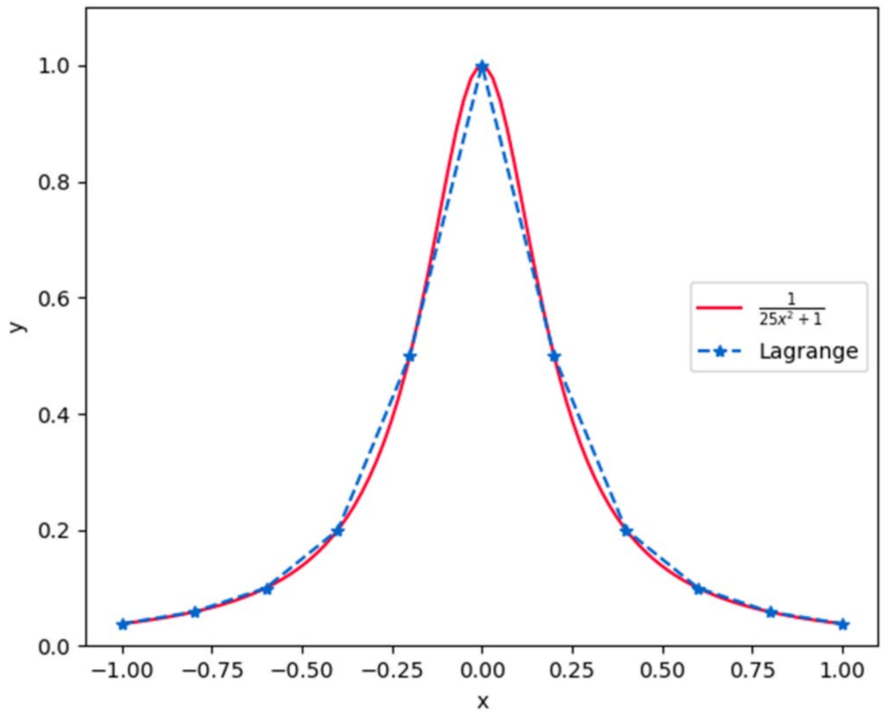
$$L_1^{(i)}(x)=\frac{x-x_{i+1}}{x_i-x_{i+1}}y_i+\frac{x-x_i}{x_{i+1}-x_i}y_{i+1}$$

$$=5\cdot\frac{x_{i+1}-x}{25x_i^2+1}+5\cdot\frac{x-x_i}{25x_{i+1}^2+1}\quad(i=0,1,2,\dots,9)$$

4.2

[ﮭ, ﮭﭤﭖ](= 0,1, , 9)

	±0.1	±0.3	±0.5	±0.7	±0.9
( )	0.8000	0.3077	0.1379	0.0755	0.0471
ﭖ( )	0.7500	0.3500	0.1500	0.0794	0.0486
	0.0500	0.0423	0.0121	0.0039	0.0016



4.2

$$(\ ) = \frac{\ }{\ }$$

$$(\ ) = \frac{-50}{(25\ \ + 1)}$$

$$(\ ) = \frac{3750\ \ - 50}{(25\ \ + 1)}$$

$$'''(\ ) = \frac{15000\ \ (1 - 25\ \ )}{(25\ \ + 1)}$$

$$'''(\ ) = 0 \qquad ''(\ ) \qquad 0 \quad \pm 0.2$$

$$\{|\ ''(\ )|\} = \{|\ (0)|, |\ (\pm 0.2)|, |\ (\pm 1)|\} = 2$$

$$|\ (\ )| \leq \frac{1}{8} \times 2 = 0.25 \quad (x \quad [-1,1])$$

4.2

( )

4.3

4.3

- ( ) ( ) ( )

( )

( )= + (−) + (−)(−)

( ) = =

( ) = =

( ) = =

- + 1),),...,), ( ) =

+ (−) + (−)(−) + + (−) (−)

( ) = (= 0,1, ,)

(= 0,1, ,) (=

0,1,2, ,)

4.3

定义4.3

- ( )　　ۑ، پ، ، ص

  (ۑ), (پ), , (ص)

- (ۑ، پ) = $\dfrac{\text{س(ط،) - س(طں)}}{\text{ط، - طں}}$　( )　ۑ، پ

- (ۑ، پ، ں) = $\dfrac{\text{س(ط،،ط۽) - س(طں،ط،)}}{\text{ط۽ - طں}}$　( )　ۑ، پ ، ں

- (ۑ، پ، ⋯ ، ص) = $\dfrac{\text{س(ط،،ط۽،...،ط) - س(طں،ط،،...،ط ،)}}{\text{ط - طں}}$　( )　ۑ،

  پ ، ⋯ ، ص

- (ڛ)　( )　ڛ

4.3

: (پ) , (پ), … , (ش)

(پ , پ , … , ص) = ∑ش سة پ س(ط) / ∏ ى(ط ة ط)

(پ , پ , … , ش)

(, پ , پ , … , ش) (, پ , پ , … , ش , پتش)

$(\quad - 1)$

(, پ , پ , … , ش , پتش) = س(ط,طى,…,ط) ة س(طى,طر,…,ط ـ ر) / ط ة ط ـ ر

= پتش 0

(− پتش) $(\quad - 1)$

( ) ص[,], س [,],(= 0,1, … ,)

(پ , پ , … , ص) = س( )(و) / ص! ((,))

(Rolle)

4.3

❑

➢ [,]

( ) = (ٻ) + (, ٻ)(− ٻ)

(, ٻ) = (ٻ, ٻ) + (, ٻ, ٻ)(− ٻ)

(, ٻ, , صۃٻ)= (ٻ, ٻ, , ص) + (, ٻ, , ص)(− ص)

➢

( ) = (ٻ) + (ٻ, ٻ)(− ٻ) + (ٻ, ٻ, ٻ)(− ٻ)(− ٻ) + +

(ٻ, ٻ, , ص)(− ٻ)(− ٻ) (− ٻۃص) + (, ٻ, , ص) ٻتص( )

= ص( ) + ص( )

4.3

- ➢

ص( ) = (ﻬ) + (ﻬ, پ)(− ﻬ) + + (ﻬ, پ, , ص)(−
ﻬ)(− پ) (− پ ﺗ ص)

ص( ) = ( ) − ص( ) = (, ﻬ, , ص) پﺘص( )

پﺘص( ) = (− ﻬ)(− پ) (− ص)

- ➢ پﺘص(ﺳ) = 0(= 0,1,2, ,) ص(ﺳ) = 0

ص(ﺳ) = (ﺳ)(= 0,1, ,)

- ➢ ( ) ( )

4.3

❑

➢

پثص( )

= ص( ) + (− پ)(− پ) (− ص) (پ, پ, , پثص)

➢ ( )

(پ), (پ, پ), , (پ, پ, , ص)

➢

4.3

❑

	x_k	$f(x_k)$	一阶差商	二阶差商	三阶差商	
0	x_0	$f(x_0)$				
1	x_1	$f(x_1)$	$f[x_0, x_1]$			
2	x_2	$f(x_2)$	$f[x_1, x_2]$	$f[x_0, x_1, x_2]$		
3	x_3	$f(x_3)$	$f[x_2, x_3]$	$f[x_1, x_2, x_3]$	$f[x_0, x_1, x_2, x_3]$	
4	x_4	$f(x_4)$	$f[x_3, x_4]$	$f[x_2, x_3, x_4]$	$f[x_1, x_2, x_3, x_4]$	

4.3

例4.5 $\sqrt{25}=5$ $\sqrt{36}=6$ $\sqrt{49}=7$

$\sqrt{31}$

x	$\sqrt{x}$	一阶差商	二阶差商
25	5		
36	6	0.090909	
49	7	0.076923	-0.00058275

4.3

$$\sqrt{31} \approx N_{\text{پ}}(31) = 5 + 0.090909 \times (31 - 25) = 5.545454$$

$$\sqrt{31} \approx N_{\text{پ}}(31)$$

$$= N_{\text{پ}}(31) + (-0.00058275) \times (31 - 25) \times (31 - 36)$$

$$= 5.548367$$

4.4

- Hermite

  - ڊ, پ

(ڊ) = ڊ, (پ) = پ

(ڊ)= ڊ, (پ)= پ

( ),

(ڊ) = ڊ (پ) = پ

(ڊ)= ڊ, (پ)= پ

( )

4.4

➢

$$H(x)= h_{\text{ﭘ}}(x)y_{\text{ﭘ}} + h_{\text{پ}}(x)y_{\text{پ}} + H_{\text{ﭘ}}(x)m_{\text{ﭘ}} + H_{\text{پ}}(x)m_{\text{پ}}$$

➢ $h_{\text{ﭘ}}(x)$ $h_{\text{پ}}(x)$ $H_{\text{ﭘ}}(x)$ $H_{\text{پ}}(x)$

	ﭘ	پ	ﭘ	پ
$_{\text{ﭘ}}(\ )$	1	0	0	0
$_{\text{پ}}(\ )$	0	1	0	0
$_{\text{ﭘ}}(\ )$	0	0	1	0
$_{\text{پ}}(\ )$	0	0	0	1

4.4

- $(\ )\quad (\ )= \quad (\ ) = 0, \quad (\ ) \quad (\ - \ )$,
  $(\ )$

$$(\ ) = (\ + \ (\ - \ ))\left(\frac{-}{-}\right)$$

- $(\ ) = 1 \quad = 1 \quad (\ )$
  $(\ ) = 0 \quad = \frac{\ }{\ }$

$$(\ ) = \left(1 + 2\frac{\ }{\ }\right)\left(\frac{\ }{\ }\right)$$

-

$$(\ ) = \left(1 + 2\frac{\ }{\ }\right)\left(\frac{\ }{\ }\right)$$

4.4

- $(\ )$ $(\ ) = (\ ) = 0$ $(\ ) = 0$ $(\ )$

  $(\ -\ )\ (\ -\ )$ $(\ )$

$$(\ ) = (\ -\ )\left(\frac{\ }{\ }\right)$$

$(\ ) = 1$ $= 1$

$$(\ ) = (\ -\ )\left(\frac{\ }{\ }\right)$$

- ,

$$(\ ) = (\ -\ )\left(\frac{\ }{\ }\right)$$

4 $(\ )$ $(\ )$ $(\ )$ $(\ )$ $(\ )$

4.4

例4.6 (0) =5 (1) =12 (0) = 8, (1)=36 Hermite

( )

1

$$( ) = \left(1 + 2\frac{\;}{\;}\right)\left(\frac{\;}{\;}\right)^{\;} = (1+2\;)(1-\;)^{\;}$$

$$( ) = \left(1 + 2\frac{\;}{\;}\right)\left(\frac{\;}{\;}\right)^{\;} = (3-2\;)\;$$

$$( ) = (\; - 0)\left(\frac{\;}{\;}\right)^{\;} = \;(1-\;)^{\;}$$

$$( ) = (\; - 1)\left(\frac{\;}{\;}\right)^{\;} = (\; - 1)\;$$

$$( ) = (1+2\;)(1-\;)^{\;} \times 5 + (3-2\;)\; \times 12 + \;(1-\;)^{\;} \times 8 + (\; - 1)\; \times 36$$

$$= (5 + 18\;)(1-\;)^{\;} + 12\;$$

$$= 30\; - 31\; + 8\; + 5$$

4.4

2

$$H(x)= a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$H'(x)=a_1 + 2a_2 x + 3a_3 x^2$$

$$\left\{\begin{array}{c} 5 = H(0)= a_0 \\ 8 = H'(0)=a_1 \\ 12 = H(1)= a_0 + a_1 + a_2 + a_3 \\ 36 = H'(1)=a_1 + 2a_2 + 3a_3 \end{array}\right. \rightarrow \left\{\begin{array}{c} a_0 = 5 \\ a_1 = 8 \\ a_2 = -31 \\ a_3 = 30 \end{array}\right.$$

$$H(x)= 5+8x - 31x^2 + 30x^3$$

4.4

定理4.3

$H(x)$ x_0, x_1 Hermite

$f(x) \in C^4[a,b]$, $f^{(4)}(x)$ $[a,b]$ $[a,b]$

x_0, x_1 $x \in [a,b]$,

ξ x

$$R(x) = f(x) - H(x) = \frac{f^{(4)}(\xi)}{4!}(x - x_0)^2 (x - x_1)^2$$

4.5

定义4.4 Spline

➢ $[a, b]$ $a \le x <$

$x < \cdots < x \le b$ $S(x)$

❖ ( ) [,], ( ) [,]

❖ ( ) [,](= 0,1,2, ,)

❖ ( )

( ) = ( ) (= 0,1,2, ,)

$S(x)$ $[a, b]$

4.5

- [,] 4

  [,] n ( ) $4n$

- 1 ( ) $(n\text{-}1)$

$$(\ -0) = (\ +0)$$
$$'(\ -0) = '(\ +0)$$
$$''(\ -0) = ''(\ +0)$$

3(− 1) 3 (+ 1)

(4 − 2) ( )

- $'(\ ) = ' ,\quad (\ ) =$

  $''(\ ) = '' ,\quad (\ ) = \ ,\qquad ''(\ ) =$

  $(\ ) = 0$

  $f(x)$ $(\ ) = (\ )$

  $(\ ) = (\ ) = (\ ),\quad (\ +0) = (\ -0),\quad (\ +0) =$

  $(\ -0)$ ( )

4.5

例 4.7 ( ) (−5) = 20, (0) = − 2, (5) = 5, [−5,5] g( ) S( )

[−5,5] [−5,0] [0,5]

$$(\ ) = \begin{cases} (\ ) = \ + \ + \ + \ , & [-5,0] \\ (\ ) = \ + \ + \ + \ , & [0,5] \end{cases}$$

$$(-5) = 20, \quad (0) = -2, \quad (0) = -2, \quad (5) = 5$$

$$\begin{cases} -125 \ + 25 \ - 5 \ + \ = 20 \\ = -2 \\ = -2 \\ 125 \ + 25 \ + 5 \ + \ = 5 \end{cases}$$

4.5

= 0 ′( ), ″( ) ﭘ′(0) = پ′(0), ﭘ″(0) =

پ″(0),

ﭘ = پ

ﭘ = پ

ﭘ (−5) = 0, پ (5) = 0

−30 ﭘ + 2 ﭘ = 0

30 پ + 2 پ = 0

ﭘ = − پ = 0.058, ﭘ = پ = 0.87, ﭘ = پ = −1.5, ﭘ = پ = −2

( ) = ﭘ( ) = 0.058 ﯾ + 0.87 ﻬ − 1.5 − 2, [−5,0]

پ( ) = −0.058 ﯾ + 0.87 ﻬ − 1.5 − 2, [0,5]

4.5

- ( )

$$( ) = \quad (\; = 0,1,2, \quad , \;)$$

- ( )

$[\;,\;]$, $\; = \; - \;$

$( )$

$$= (1 + 2 \frac{-}{\;})(\frac{-}{-})^{} \; + (1 + 2 \frac{-}{-})(\frac{-}{\;})^{} \; + (\; - \;)(\frac{-}{-})^{} \; + (\; - \;)(\frac{-}{\;})^{} \;$$

$$= \frac{[\; + 2(\; - \;)](\; - \;)^{}}{} \; + \frac{[\; - 2(\; - \;)](\; - \;)^{}}{} \; + \frac{(\; - \;)(\; - \;)^{}}{} \; + \frac{(\; - \;)(\; - \;)^{}}{} \;$$

4.5

❑

( )

$$( ) = \frac{\quad}{\quad} + \frac{\quad}{\quad}$$

$$+ \frac{\quad}{\quad}(-) \quad (\quad [\; , \;])$$

$$\lim ( ) = -\frac{4}{\quad} - \frac{2}{\quad} + \frac{6}{\quad}(-)$$

i-1 i i i+1 ( ) [,]

$$\lim ( ) = \frac{4}{\quad} + \frac{2}{\quad} - \frac{6}{\quad}(-)$$

4.5

❑

$$\lim \quad (\ ) = \lim \quad (\ )$$

$$\frac{}{} \quad + 2\left(\frac{}{} + \frac{}{}\right) \quad + \frac{}{}$$

$$= 3\left(\frac{\quad - \quad}{} + \frac{\quad - \quad}{}\right)\left(\ = 1,2, \quad , \quad - 1\right)$$

$$\frac{}{} + \frac{}{} \qquad \frac{}{}$$

$$\lambda \quad + 2 \quad + \quad = \quad \left(\ = 1,2, \quad , \quad - 1\right)$$

$$\lambda = \frac{}{\quad + \quad} \qquad = \frac{}{\quad + \quad}$$

$$= 3\left(\quad \frac{\quad - \quad}{} + \lambda \frac{\quad - \quad}{}\right)$$

4.5

-

  - $1,2,\ ,\ -1$ $(\ -1)$ $(\ +1)$

    $(\ )$

  - $(\ = 1,2,\ ,\ )$

4.5

❑

➢ $S(x)$, $m_{\text{ټ}} = f_{\text{ټ}}, m_{\text{ص}} = f_{\text{ص}},$ n-1

$$\begin{pmatrix} 2 & \text{پ} & & & \\ \lambda_{\text{پ}} & 2 & \text{پ} & & \\ & & \ddots & & \\ & & \lambda_{\text{پ ت ص}} & 2 & \text{پ ت ص} \\ & & & \lambda_{\text{پ ت ص}} & 2 \end{pmatrix} \begin{pmatrix} \text{پ} \\ \text{پ} \\ \vdots \\ \text{پ ت ص} \\ \text{پ ت ص} \end{pmatrix} = \begin{pmatrix} \text{پ} - \lambda_{\text{پ}} \ \text{ټ} \\ \text{پ} \\ \vdots \\ \text{پ ت ص} \\ \text{پ ت ص} - \text{پ ت ص} \ \text{ص} \end{pmatrix}$$

4.5

-

  - $S(\quad)$

4.5

❑

$$2m_{\text{ڼ}} + m_{\text{پ}} = 3\,\frac{\text{ط ، ﺗ ط ہ}}{\text{س ہ}} \text{-} \frac{\text{س ہ}}{\text{پ}} f_{\text{ڼ}}$$

$$i = n-1, x = x_{\text{ص}},$$

$$S\ \ (x_{\text{ص}})$$

$$= \frac{2}{h_{\text{ص ﺗ پ}}} m_{\text{ص ﺗ پ}} + \frac{4}{h_{\text{ص ﺗ پ}}} m_{\text{ص}} - \frac{6}{h_{\text{ص ﺗ پ}}^{\text{پ}}} (y_{\text{ص}} - y_{\text{ص ﺗ پ}}) = f_{\text{ص}}$$

$$m_{\text{ص ﺗ پ}} + 2m_{\text{ص}} = 3\,\frac{\text{ط ﺗ ط ،}}{\text{س ،}} - \frac{\text{س ،}}{\text{پ}} f_{\text{ص}}$$

4.5

- ➢ $n+1$

$$\begin{pmatrix} 2 & 1 & & & \\ \lambda & 2 & & & \\ & \lambda & 2 & & \\ & & \ddots & & \\ & & \lambda & 2 & \\ & & & 1 & 2 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \\ \\ \\ \end{pmatrix}$$

$$= 3 \qquad\qquad = 3$$

- ➢

4.5

例4.8 $\quad = 2, \quad = -2$

(2.1)

	0	1	2	3
	1	1.4	2.6	3
( )	0.2	1.8	3.4	3.2

$$= 0.4 \qquad = 1.2 \qquad = 0.4$$

$$\lambda = \frac{\quad}{\quad} \qquad \lambda = 0.75 \qquad \lambda = 0.25$$

$$= \frac{\quad}{\quad} \qquad = 0.25 \qquad = 0.75$$

$$= 3\left(\frac{\quad}{\quad} + \lambda \frac{\quad}{\quad} \right) \qquad = 10 \qquad = -0.125$$

4.5

$$\left\{\begin{array}{l} 0.75\quad + 2\quad + 0.25\quad = 10 \\ 0.25\quad + 2\quad + 0.75\quad = -0.125 \\ \quad = \quad = 2 \\ \quad = \quad = -2 \end{array}\right. \rightarrow \begin{array}{l} = 4.23 \\ = 0.16 \end{array}$$

$[1, 1.4]$:

$$(\ ) = 32.6875 \quad - 105.8875 \quad + 113.9125 \quad - 40.5125$$

$[1.4, 2.6]$:

$$(\ ) = 1.1974 \quad - 18.2251 \quad + 54.0392 \quad - 43.2942$$

$[2.6, 3]$:

$$(\ ) = -5.25 \quad + 2.15 \quad + 150.3 \quad - 296.4$$

$$(2.1) \approx \ (2.1) = 0.9045$$

4.5

-

  -

  -

  - “ ”

4.6

❑

定义4.5 $[a,b]$ $\rho(\ )$ $[a,b]$

ذ ر ρ( ) ش = 1,2, ( ) $[a,b]$

ذ ر ρ( ) ( ) =0

( ) ≡ 0 ρ( ) $[a,b]$

❑

- ρ( ) = 1(−1 ≤ ≤ 1)
- ρ( ) = پ / √(پ ز ط ـ) (−1 ≤ ≤ 1)
- ρ( ) = ز ط (0 < < +∞)
- ρ( ) = ز ط ـ (0 < < +∞)

4.6

❑

定义4.6 ( ), ( ) [*a*,*b*], ρ( ) [*a*,*b*]

(,) = ρ( ) ( ) ( )

( ), ( ) [*a*,*b*]

❑

定义4.7 { }(= 0,1,2, ,), { } (= 0,1,2, ,),

(,) = ( ) ( )

( ), ( ) { }(= 0,1,2, ,)

4.6

- ❑

  - $(f,g)=(g,f)$
  - $(c_1 f+c_2 g,h)=c_1 (f,h)+c_2(g,h)$
  - $(f,f)\geq 0,$ $f\equiv 0$ $(f,f)=0$

4.6

定义4.8 ( ), ( ) [a,b], ρ( ) [a,b]

(,) = ρ( ) ( ) ( ) = 0

( ), ( ) [a,b] ρ( )

定义4.9 { }(= 0,1,2, ,), { }(= 0,1,2, ,),

(,) = ( ) ( ) = 0

( ), ( ) { }(= 0,1,2, ,) (= 0,1,2, ,)

4.6

❑

定义4.10 $\{\varphi_0(x), \varphi_1(x), \cdots, \varphi_n(x), \cdots\}$ $[a,b]$

$$(\varphi_k, \varphi_j) = \int_a^b \rho(x)\varphi_k(x)\varphi_j(x)\,dx = \begin{cases} 0 & k \neq j \\ A_k > 0 & k = j \end{cases} \quad (k, j = 0,1,2,\cdots)$$

$\{\varphi_k(x)\}$ $[a,b]$ $\rho(x)$

❑

定义4.11 $\{\varphi_0(x), \varphi_1(x), \cdots, \varphi_n(x), \cdots\}$ $[a,b]$

$\{x_i\}$ $(i = 0,1,2,\cdots,m)$, $\{\omega_i\}$ $(i = 0,1,2,\cdots,m)$,

$$(\varphi_k, \varphi_j) = \sum_{i=0}^{m} \omega_i \varphi_k(x_i)\varphi_j(x_i) = \begin{cases} 0 & k \neq j \\ A_k > 0 & k = j \end{cases} \quad (k, j = 0,1,2,\cdots)$$

$\{\varphi_k(x)\}$ $\{x_i\}(i = 0,1,2,\cdots,m)$ $\omega_i(i = 0,1,2,\cdots,m)$

4.6

例4.9 $\{1, \quad , \quad , \quad 2 \ , \quad 2 \ , \quad \}$ $[-\pi, \pi]$

$\rho(\ ) \equiv 1$

$$(1,1) = \qquad = 2\pi$$

$$(\sin \quad , \sin \quad) = \quad \sin \quad \sin \qquad = \begin{matrix} \pi & = \\ 0 & \neq \end{matrix} \quad (\ , \quad = 1,2, \quad)$$

$$(\cos \quad , \cos \quad) = \quad \cos \quad \cos \qquad = \begin{matrix} \pi & = \\ 0 & \neq \end{matrix} \quad (\ , \quad = 1,2, \quad)$$

$$(\cos \quad , \sin \quad) = \quad \cos \quad \sin \qquad = 0(\ , \quad = 0,1,2, \quad)$$

4.6

□

定义4.12 $_{ص}(\ )$ $_{ص} \neq 0$ $\{ _{ص}(\ )\}_{ﺌ}^{ﺍ}$

$$(\ _{ﺷ},\ _{ﺷ}) = \int_{ﺫ}^{ﺭ} \rho(\ )\ _{ﺷ}(\ )\ _{ﺷ}(\ ) \quad = \begin{cases} 0 & \neq \\ _{ﺷ} > 0 & = \end{cases}$$

$\{ _{ص}(\ )\}_{ﺌ}^{ﺍ}$ $[a,b]$ $\rho(\ )$ n

□

定义4.13 $_{ص}(\ )$ $_{ص} \neq 0$ $\{ _{ص}(\ )\}_{ﺌ}^{ﺍ}$

$$(\ _{ﺷ},\ _{ﺷ}) = \sum_{ﺳ ﺓ ﺌ}^{ﺷ}\ _{ﺳ}\ _{ﺷ}(\ _{ﺳ})\ _{ﺷ}(\ _{ﺳ}) = \begin{cases} 0 & \neq \\ _{ﺷ} > 0 & = \end{cases} \quad (\ ,\ \ = 0,1,2,\ \ )$$

$\{ _{ص}(\ )\}_{ﺌ}^{ﺍ}$ $\{ _{ﺳ}\}$ $(\ \ = 0,1,2,\ \ ,\ \ )$ $_{ﺳ}$

$(\ \ = 0,,1,2,\ \ ,\ \ )$ n

4.6

- cos( )= ص( ) [-1,1]

ص( ) = cos(arccos) (= 0,1,)

- = cos ص( ) = cos (0 ≤ ≤)

cos cos

ص( )

4.6

❑

➢

$$\begin{cases} \cos 2\alpha = 2\cos^{2}\alpha - 1 \\ \cos 3\alpha = 4\cos^{3}\alpha - 3\cos\alpha \\ \cos 4\alpha = 8\cos^{4}\alpha - 8\cos^{2}\alpha + 1 \\ \cos 5\alpha = 16\cos^{5}\alpha - 20\cos^{3}\alpha + 5\cos\alpha \end{cases}$$

➢

$$2 \quad = \quad (-1) \quad (2 \quad) \quad , (\quad ; \quad)$$

$$2 \quad = (2 \quad) + \quad (-1) \quad (2 \quad)$$

4.6

- n

$$= 2^{n-1}(\quad)^{n} - \quad(\quad)^{n-2} + \quad(\quad)^{n-4} + \cdots$$

$[0,\ ]$ 1 -1

$$[-1,1],\quad [0,\ ]$$

- $=$

$$= \qquad [-1,1] \qquad [0,\ ]$$

$$\cos(\qquad)$$

$$= 2^{n-1}[\cos(\qquad)]^{n} - \quad[\cos(\qquad)]^{n-2}$$

$$+ \quad[\cos(\qquad)]^{n-4} + \cdots$$

$$= 2^{n-1} \quad ^{n} - \quad ^{n-2} + \quad ^{n-4} + \cdots$$

4.6

❑

$$(\;,\;) = \frac{(\;)(\;)}{\sqrt{\;}} = \begin{cases} 0, & \neq \\ , & = \quad \neq 0 \\ , & = \quad = 0 \end{cases}$$

$$= \qquad , \qquad = -$$

$$(\;,\;) = \frac{(\;)\;(\;)}{\sqrt{1 - \;}} = \cos \qquad \sin \qquad = \begin{cases} 0, & \neq \\ \frac{}{2}, & = \quad \neq 0 \\ , & = \quad = 0 \end{cases}$$

4.6

❑

پتص( ) = 2 ص( ) − ص ة پ( ) (= 1,2,)

پ( ) = 1 پ( ) = = پتص( ) =

cos(+ 1) cos(+ 1) + cos(− 1) =

2 cos cos

پ( ) = 2 پ − 1

پ( ) = 4 پ − 3

پ( ) = 8 پ − 8 پ + 1

ن( ) = 16 ن − 20 پ + 5

ن( ) = 32 ن − 48 پ + 18 پ − 1

4.6

$$_{\text{ص}}(-\quad) = (-1)^{\text{ص}}\ _{\text{ص}}(\quad)$$

$_{\text{ص}}(\quad)$ (-1,1)

$$_{\text{ش}} = \cos\frac{2\quad - 1}{2}\qquad (\quad = 1,2,\quad ,\quad)$$

[-1,1] +1

$$_{\text{ش}} = \cos -\qquad (\quad = 0,1,2,\quad ,\quad)$$

$_{\text{ص}}(\quad)$ ص $2^{\text{ص ت پ}}(\quad \geq 1)$

4.7

定义 4.14

$f(x)$

4.7

- (Bernstein)

$$B_n(f,x)=\sum_{k=0}^{n} f\left(\frac{k}{n}\right)P_k(x)$$

$$P_k(x)=\binom{n}{k}x^k(1-x)^{n-k},\ \binom{n}{k}=\frac{n(n-1)\cdots(n-k+1)}{k!},$$

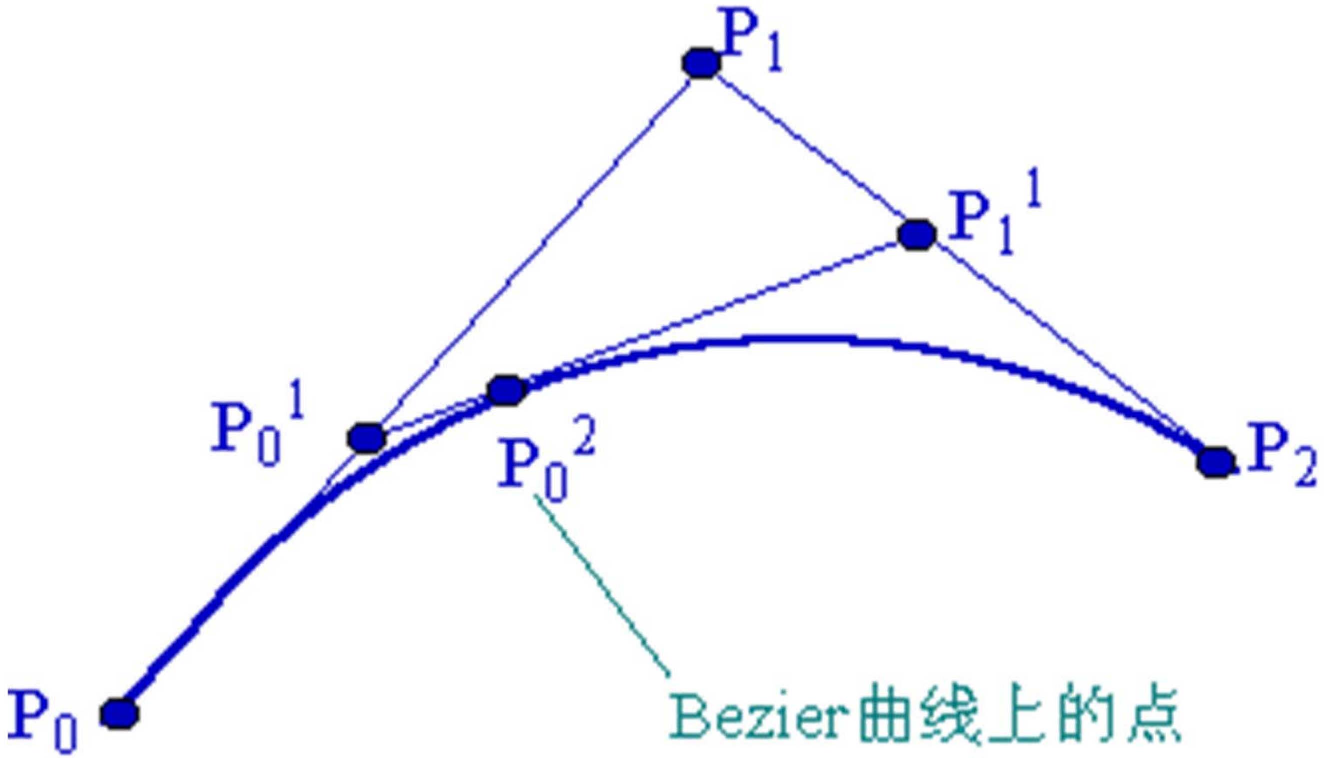
$\lim_{n\to\infty} B_n(f,x)=f(x)$ [0,1]

- n

$$p_n(x) \qquad \max_{a\le x\le b}|p_n(x)-f(x)|$$

4.7

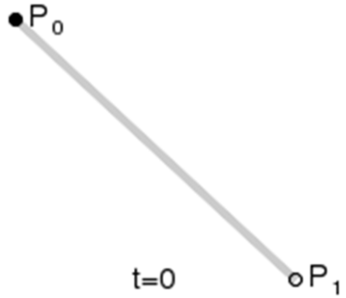
- $$B_n(t)=\sum_{i=0}^{n}\binom{n}{i}t^i(1-t)^{n-i}P_i$$



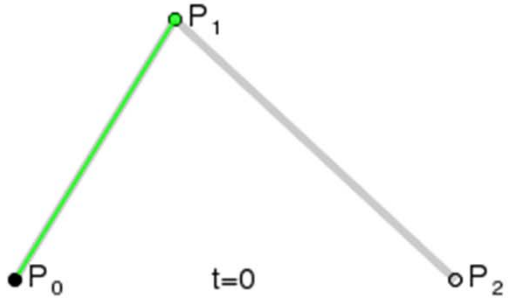
$$B_2(t)=(1-t)^2P_0+2t(1-t)P_1+t^2P_2$$

4.7

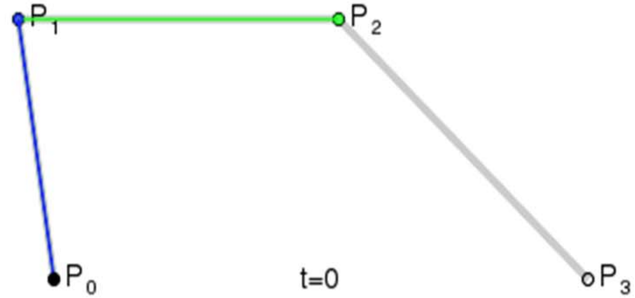
$$B_n(t)=\sum_{i=0}^{n}\binom{n}{i}t^{i}(1-t)^{n-i}P_i$$



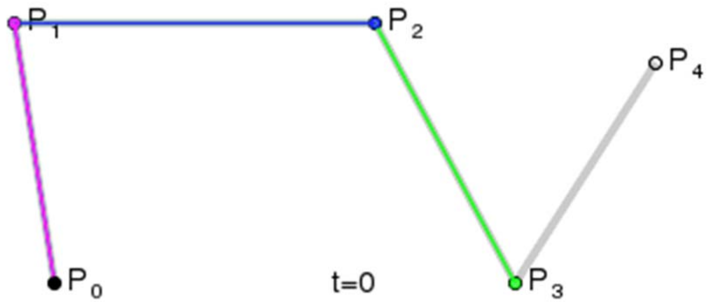
一阶



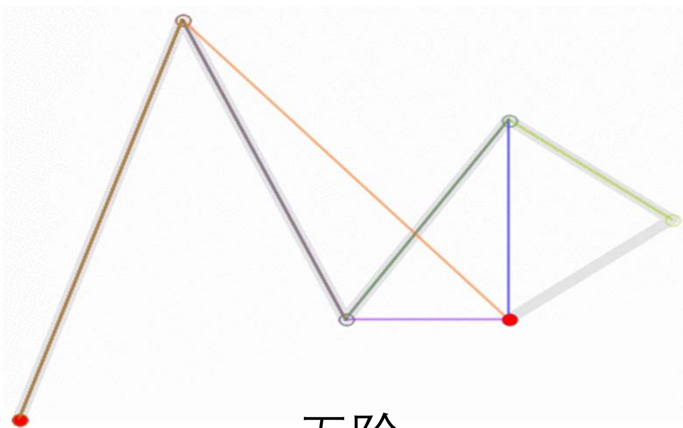
二阶



三阶



四阶



五阶

4.7

定义4.15

ص = { } ( ) [,] ( ) ص

|| − ||ˌ = max | ( ) − ( )| = (≥ 0)
ذبطبر

( ) ( )

ؤ [,], | (ؤ) − (ؤ)| = , ؤ ( ) ( )

(ؤ) − (ؤ) = (ؤ) − (ؤ) = −

定义4.16

( ) [,] ص( ) ص,

|| − ص||ˌ = min|| − ||ˌ
ص ح

ص( ) ( ) [,]

4.7

定理4.5

( ) [,] ص( ) ص

|| − ص||ا = min || − ||ا
ص ح

定理4.6

ص( ) ( ) [,]

$[a,b]$ + 2

+ 2 ≤ پ < پ < < پتص ≤

ص(ش) − (ش) = (−1)ش || − ص||ا (7.3.3)

= ±1 = 1,2, , + 2,

{ ش}پتص پ

4.7

例4.10 $f(x)$ $[a, b]$ f (x) $[a, b]$

$f(x)$ $p_{پ}(x) =$

$a_{ټ} + a_{پ}x.$

➢ ( ) [,] ( ) [,] 4.6

≤ پ < ٻ < ٻ ≤

پ(ش) − (ش) = (−1)^ش ‖ پ( ) − ( )‖ ι

= 1,2,3, = ±1

4.7

- $(\quad) \neq 0$ $(\quad)$ $(\quad) - \quad (\quad) =$

$(\quad) - \quad = 0$ $[\quad,\quad]$ $(\quad)$ $(\quad)$

$= \quad , \quad =$

$(\quad) - (\quad) = (\quad) - (\quad) = -[\ (\quad) - (\quad)]$

$$= \frac{(\quad) - (\quad)}{-}$$

$$= \frac{1}{2}[\ (\quad) - (\quad)] - \frac{+}{2} \cdot \frac{(\quad) - (\quad)}{-}$$

$(\quad) =$

4.7

❑

- $(\ )$ $-1(\ \geq 1)$
- $(\ ) = \frac{}{}\ (\ )$ 1

定理4.7

$1 \quad n$

$(\ )$

7.3.5 $(\ ) = 5$ [-1,1] $(\ )$

$$(\ ) = \frac{}{}(4 \ - 3\ ) = 5 \ - \frac{}{} \qquad = \frac{}{}$$

4.8

定义4.17

ﭙ( )　پ( )　ص( )　[,]　ﭙ　پ

ص

Φ = { ( )　| ( ) = ﭙ ﭙ( ) + + ص ص( )}

span{ ﭙ, پ, , ص}, Φ پ( ) پ( ) ص( )

پ( ) پ( ) ص( )

定理4.8 ﭙ( ), پ( ), , ص( ) [,] ﭙ( ), پ( ), , ص( )

det ص ≠ 0

ص = [(ﭙ, ﭙ) (ﭙ, پ) (ﭙ, ص) ; (پ, ﭙ) (پ, پ) (پ, ص) ; (ص, ﭙ) (ص, پ) (ص, ص)]

(ﺳ, ﺷ) ﺳ( ) ﺷ( )

4.8

- $\varphi_j(x) = x^j$ $[0,1]$ $\rho(x) = 1$ $f(x) \in C[0,1]$,

  $\mathrm{span}\{1, x, \cdots, x^n\}$ n

$$P_n(x) = a_0 + a_1 x + \cdots + a_n x^n$$

- $(\varphi_i, \varphi_j) = \int_0^1 x^{i+j} dx = \frac{1}{i+j+1}$, $\boldsymbol{G}_n$ $\boldsymbol{H}_n$

$$\boldsymbol{H}_n = \begin{bmatrix} 1 & 1/2 & \cdots & 1/(n+1) \\ 1/2 & 1/3 & \cdots & 1/(n+2) \\ \vdots & & \ddots & \vdots \\ 1/(n+1) & 1/(n+2) & \cdots & 1/(2n+1) \end{bmatrix} = (h_{ij})$$

$(h_{ij}) = 1/(i+j+1)$ $\boldsymbol{H}_n$

4.8

定义4.18

( ) [,] ( ) Φ

ر ( )[( ) − ( )]ٻ = min ر ( )[( ) − ( )]ٻ
ذ ح(ط) ؤ ذ

( ) ( ) Φ

- Φ = ص = span{1,x, , ص} ( ) ( )

n

4.8

定理4.9

( ) [,] ( ) Φ ( )

( ) ( )

$$(\text{ﭗ}, \text{پ}, \ , \text{ص}) = \text{ر} \ \text{ذ} \ ( ) \left[\text{ص} \ \text{ش} \ \text{ش}( ) - ( ) \ \text{ش ﮗ ﭗ} \right]^{\text{پ}}$$

ﭗ, پ, , ص 0

$$\frac{}{\text{ش}} = 2 \ \text{ر} \ \text{ذ} \ ( ) \left[\text{ص} \ \text{ش} \ \text{ش}( ) - ( ) \ \text{ش ﮗ ﭗ} \right] \text{ش}( ) \quad (\ = 0,1, \ , \ )$$

$$\text{ص} \ (\ \text{ش}( ), \ \text{ش}( )) \ \text{ش} = \left(( ), \ \text{ش}( ) \right) \quad (\ = 0,1, \ , \ ) \ \text{ش ﮗ ﭗ}$$

➢ ﭗ, پ, , ص

4.8

پ( ), پ( ), , ص( )

ش = ش (= 0,1, ,)

= ﭖ ﭖ( ) + پ پ( ) + + ص ص( )

- ( )

ر

= ( )[( ) − ( )]پ − ر ( )[( ) − ( )]پ

ذ ذ

ر ر

= ( )[( ) − ( )]پ + ( )[( ) − ( )][( ) − ( )]

ذ ذ

( ) ش(= 0,1, ,) $\sum$ ص ش ﺓ ﭖ(ش( ), ش( )) ش =

(( ), ش( ))

ر

( )[( ) − ( )] ش( ) = 0

ذ

ر

= ( )[( ) − ( )]پ ≥ 0

( ) ذ ( )

4.8

❑

➢ $_{ش}(\quad) = \quad {}^{ش}(\quad = 0,1,\quad ,\quad)$ $[0,1]$ $(\quad) = 1$ $(\quad)$ $[0,1]$

$= \quad _{ص} = \mathrm{span}\{1,\quad ,\quad ,\quad {}^{ص}\}$

$$_{ص}(\quad) = \quad _{ﭖ} + \quad _{پ} + \quad + \quad _{ص} \quad {}^{ص}$$

➢

$$(\quad _{ش},\quad _{ش}) = \int_{ﭖ}^{پ} \quad {}^{ششتش} \quad = \frac{1}{\quad + \quad + 1} (\quad ,\quad = 0,1,\quad ,\quad)$$

$$(\quad ,\quad _{ش}) = \int_{ﭖ}^{پ} \quad (\quad) \quad {}^{ش} \quad = \quad _{ش}(\quad = 0,1,\quad ,\quad)$$

➢ $_{ص}$

$$_{ص} = \begin{bmatrix} 1 & 1/2 & & 1/(\quad + 1) \\ 1/2 & 1/3 & & 1/(\quad + 2) \\ & & & \\ 1/(\quad + 1) & 1/(\quad + 2) & & 1/(2\quad + 1) \end{bmatrix} = (\quad _{شس})$$

4.8

❑

➢ = (ﭙ, پ, , ص)خ = (ﭙ, پ, , ص)خ

ص =

ش = ش(= 0,1, ,)

ص( )

4.8

例4.11 $(\ \ ) = \sqrt{3\ \ + 2\ \ + 5}$ $[0,1]$ $(\ \ )$

$$(\ \ ) = \ \ + \ \ $$

$$= \sqrt{3\ \ + 2\ \ + 5} \approx 2.632$$

$$= \sqrt{3\ \ + 2\ \ + 5} \approx 1.394$$

$$\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \begin{bmatrix} \ \\ \ \end{bmatrix} = \begin{bmatrix} 2.632 \\ 1.394 \end{bmatrix}$$

$$= 2.164 \qquad = 0.936$$

$$(\ \ ) = 2.164 + 0.936$$

4.8

- Φ
  - $(\ )\quad [\ ,\ ]\quad = \mathrm{span}\{\ ,\ ,\ \}$
    $\neq \quad (\ ,\ ) = 0 \quad (\ ,\ ) > 0$
    $=$
    $\dfrac{(\ ,\ )}{(\ ,\ )}\ (\ = 0,1,\ )\quad (\ )$

$$(\ ) = \sum \frac{(\ ,\ )}{\|\ \|^{2}}\ (\ )$$

(Fourier)

4.8

❑

➢

➢ (,)(= 0,1,2,)

( )

( ) (,)(= 0,1,2,)

= ( ) − (=

0,1,2,)

4.8

❑

- $\|\delta\|_1 = \sum_{i=1}^{n} |\delta_i|$
- $\|\delta\|_2^2 = \sum_{i=1}^{n} \delta_i^2$
- $\|\delta\|_\infty = \max_{1 \le i \le n} |\delta_i|$
- 2-

  2-

4.8

❑

➢ $(\quad , \quad)(\quad = 0,1,2, \quad)$ $=$

$\{ \quad (\quad), \quad (\quad), \quad (\quad)\}$ $(\quad)$ 2-

$$\| \quad \| = \quad = \quad (\quad (\quad) - \quad)$$

➢ $(\quad), \quad (\quad), \quad (\quad)$

$(\quad) \quad \{ \quad (\quad)\} \quad (\quad = 0,1, \quad)$

$(\quad) = \quad (\quad) + \quad (\quad) + \quad (\quad) \quad (\quad < \quad)$

4.8

- 2- $\|\ \|$

$$(\ ,\ ,\ ) = \sum \left(\sum (\ ) - \ \right)$$

$$(\ ,\ ,\ )$$

$$\frac{}{} = 0 \qquad (\ = 0,1,2,\ )$$

-

$$\sum \left(\sum (\ ) - \ \right) (\ ) = 0 \qquad (\ = 0,1,2,\ )$$

4.8

-
  -

$$(\varphi_k,\varphi_j)=\sum_{i=0}^{m}\varphi_k(x_i)\varphi_j(x_i)\qquad (f,\varphi_k)=\sum_{i=0}^{m} f_i\varphi_k(x_i)=d_k$$

  -

$$\sum_{j=0}^{n}(\varphi_k,\varphi_j)a_j=d_k\qquad (k=0,1,2,\ldots,n)$$

$$\begin{bmatrix}(\varphi_0,\varphi_0) & (\varphi_0,\varphi_1) & \cdots & (\varphi_0,\varphi_n)\\(\varphi_1,\varphi_0) & (\varphi_1,\varphi_1) & \cdots & (\varphi_1,\varphi_n)\\ \vdots & \vdots & & \vdots\\(\varphi_n,\varphi_0) & (\varphi_n,\varphi_1) & \cdots & (\varphi_n,\varphi_n)\end{bmatrix}\begin{bmatrix}a_0\\a_1\\ \vdots\\a_n\end{bmatrix}=\begin{bmatrix}d_0\\d_1\\ \vdots\\d_n\end{bmatrix}$$

4.8

- $= \{\ (\ ),\ (\ ),\ \ (\ )\} = \{1,\ ,\ ,\ \}$

$$(\ ) = \ +\ \ +\ $$

$$\begin{bmatrix} 1 & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

- “ ”

4.8

例4.12

	-1	-2	0	1	3
	2	-1	0	2	3

$$= \quad + \quad + $$

-1 -2 0 1 3

$$(\ ) = 1, \quad (\ ) = \quad , \quad (\ ) = $$

$$= \begin{pmatrix} 1 & & \\ & & \\ & & \end{pmatrix} = \begin{pmatrix} 5 & 1 & 15 \\ 1 & 15 & 19 \\ 15 & 19 & 99 \end{pmatrix}$$

$$((\ ,1),(\ ,\ ),(\ ,\ ))= (6,11,27)$$

4.8

$$\begin{pmatrix} 5 & 1 & 15 \\ 1 & 15 & 19 \\ 15 & 19 & 99 \end{pmatrix} \begin{pmatrix} \text{ټ} \\ \text{پ} \\ \text{پہ} \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ 27 \end{pmatrix}$$

$$\text{ټ} = 1.19$$
$$\text{پ} = 0.71$$
$$\text{پہ} = -0.04$$

$$(\ \ ) = 1.19 + 0.71 \ \ - 0.04 \ \text{پہ}$$