2

2.1

2.2

2.3

2.4

2.5

2.1

- ❑ $f(x)=0$ 方程的根 $f(x)$ 零点

  - ➢ $f(x)$ n

    $$f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0 \quad (a_n\neq 0)$$

    $$f(x)=0 \quad n \quad \text{代数方程}$$

  - ➢ $f(x)$

    $$f(x)=2+sinx-e^x$$

    $$f(x)=0 \quad \text{超越方程}$$

2.1

❑

$$\boldsymbol{f}(\boldsymbol{x}) = 0$$

$\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ $(x_i \in [a_i, b_i])$, $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_n(\boldsymbol{x}))^{\mathrm{T}} \in \mathbf{R}^n$.

$$f(x) = 0, \qquad x \in [a, b]$$

❑ $f(x)$

$$f(x) \in C([a, b]), \qquad f(a)f(b) < 0$$

$f(x) = 0$ $[a, b]$

$[a, b]$ (1) **有根区间** $f(x) = 0$

$[a, b]$ $[a, b]$ (1) **隔离区间**

2.1

- $f(x) = 0$
  -
    -
    -
  -
    - Matlab
    -

2.1

- ：

  $f(x)$

- $[a,b]$ $a_0 = a$ h $h = \frac{b-a}{n}$
  “ ” $x_k = a + kh(k = 0,1,2,\cdots,n)$
  $f(x_k)$ $f(x_{k-1}) \cdot f(x_k) \leq 0$， x^* x_{k-1} x_k
  $[x_{k-1}, x_k]$

- $f(x)=0$

2.2

❑

➢ $f(x)$ $[a,b]$ $f(a) \cdot f(b) < 0$

$f(x) = 0$ $[a,b]$

$[a,b]$ $f(x) = 0$ $[a,b]$

x^*

❑

➢ $f(x)$

➢ $a_0 = a, b_0 = b$

2.2

❑

➢ $[a_0,\ b_0]$ $[a_0,\ b_0]$ $x_0 =$

$\frac{1}{2}(a_0 + b_0)$ $f(x_o)$ $f(x_o) = 0$,

$x^* = \frac{1}{2}(a_0 + b_0)$ $f(x_o)$ $f(a_0)$

$f(b_0)$

❖ $f(a_0) \cdot f(x_o) < 0$, $[a_0, x_0]$ $a_1 = a_0$

$b_1 = x_0 = \frac{a_0+b_0}{2}$

❖ $f(x_o) \cdot f(b_0) < 0$, $[x_0, b_0]$ $a_1 = x_o =$

$\frac{a_0+b_0}{2}$ $b_1 = b_0$

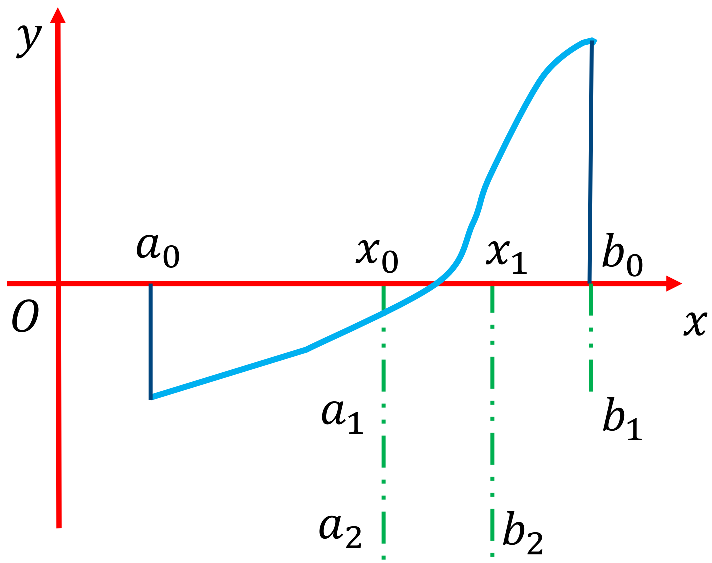
2.2

❑

➢ $[a_1, b_1]$ $[a_0, b_0]$

$[a_1, b_1]$

$$[a_0, b_0] \supset [a_1, b_1] \supset [a_2, b_2] \supset \cdots \supset [a_k, b_k] \supset \cdots$$



2.2

❑

➢ k

$[a_k, b_k]$

$$b_k - a_k = \frac{1}{2^k}(b-a)$$

➢ $[a,b]$ $(k \to \infty)$

x^*

➢ $[a,b]$

➢ $[a_k, b_k]$ $x_k = \frac{1}{2}(a_k + b_k)$ x^*

x^*

$x_0, x_1, x_2, \cdots x_k, \cdots$

2.2

❑

➢

$$|x^* - x_k| \le \frac{1}{2}(b_k - a_k) = \frac{1}{2^{k+1}}(b - a)$$

➢ $\varepsilon > 0$

$$\left|\frac{1}{2^{k+1}}(b - a)\right| < \varepsilon$$

$$\left|\frac{1}{2^k}\right| < \frac{2\varepsilon}{b - a} \quad 2^k > \frac{b - a}{2\varepsilon}$$

$k > \frac{\ln(b-a) - \ln 2\varepsilon}{\ln 2}$

$$|x^* - x_k| < \varepsilon$$

x_k

2.2

例2.1 $f(x) = 2 + \sin x - e^x \quad x \in [0,2]$

10^{-2}

$f'(x)$ $f(x)$

$f(0) > 0 \quad f(2) < 0 \quad f(x)$

$\frac{2-0}{2^{k+1}} \leq 10^{-2} \quad k \geq 7 \quad k = 7$

2.2

n	a_n	b_n	x_n	$f(x_n)$
0	0	2	1	0.123189
1	1	2	1.5	-1.48419
2	1	1.5	1.25	-0.54136
3	1	1.25	1.125	-0.17795
4	1	1.125	1.0625	-0.02002
5	1	1.0625	1.03125	0.053372
6	1.03125	1.0625	1.046875	0.017129
7	1.046875	1.0625	1.054688	-0.00133

$x^* \approx 1.05$

2.2

例2.2 $x^3 - x - 1 = 0 \quad x \in [1,2]$

10^{-3}

$f(x) = x^3 - x - 1 \quad f'(x) = 3x^2 - 1 \qquad f(x)$

$f(1) = -1 < 0 \quad f(2) = 5 > 0 \qquad f(x)$

$|x^* - x_n| \leq \frac{2-1}{2^{n+1}} < 10^{-3} \qquad n \geq \frac{3}{\lg 2} - 1$

$n = 9$

2.2

n	a_n	b_n	x_n	$f(x_n)$
0	1	2	1.5	+
1	1	1.5	1.25	-
2	1.25	1.5	1.375	+
3	1.25	1.375	1.3125	-
4	1.3125	1.375	1.3438	+
5	1.3125	1.3438	1.3281	+
6	1.3125	1.3281	1.3203	-
7	1.3203	1.3281	1.3242	-
8	1.3242	1.3281	1.3262	+
9	1.3242	1.3262	1.3252	+

$x_9 = 1.325$

$$x^3 - x - 1 = 0 \quad x \in [1,2]$$

$$x = \sqrt[3]{\frac{1}{2} + \sqrt{\frac{23}{108}}} + \sqrt[3]{\frac{1}{2} - \sqrt{\frac{23}{108}}} = 1.324717958\cdots$$

2.2

-
  -
  - $f(x)$ $f(x)$
  - $1/2$
-
  -
  -
  -

2.3

- $f(x)$ x^* $f(x) = 0$

  x_k k $f(x)$ x_k

$$f(x^*) \approx f(x_k) + f'(x_k)(x^* - x_k)$$

$$0 = f(x^*) \approx f(x_k) + f'(x_k)(x^* - x_k)$$

➢

$$y = f'(x_0)(x - x_0) + y_0$$

2.3

- ❑ $f'(x_k) \neq 0$， $x^* \approx x_k - \dfrac{f(x_k)}{f'(x_k)}$

$$x_{k+1}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \quad (k = 0,1,2,\cdots)$$

牛顿（Newton）迭代公式

- ➢ $\varphi(x) = x - \dfrac{f(x)}{f'(x)}$ $\varphi(x) = x - \dfrac{f(x)}{f'(x)}$ $f(x) = 0$

2.3

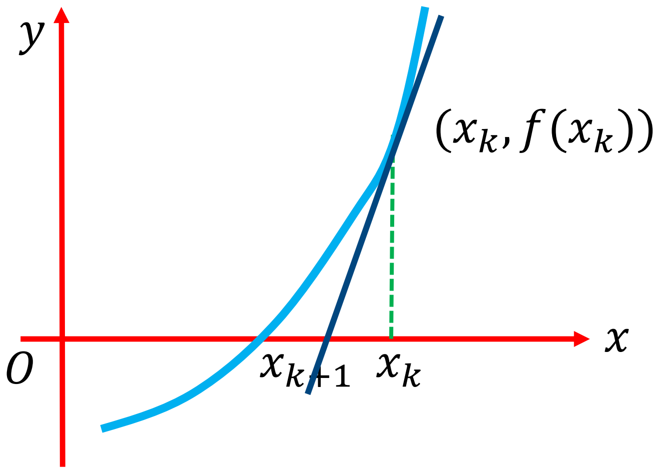
- $f(x) = 0$ x^* $y = f(x)$ x
- x^* x_k $y = f(x)$ $\left(x_k, f(x_k)\right)$ $f(x)$

$$f(x) - f(x_k) = f'(x_k)(x - x_k),$$ x

x^* x_{k+1} $0 - f(x_k) = f'(x_k)(x_{k+1} - x_k)$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

-



2.3

❑

定理2.2 $f(x) = 0$ $[a, b]$

- $f(a)f(b) < 0$
- $f''(x)$ $[a, b]$
- $x \in [a, b]$ $f'(x) \neq 0$
- $x_0 \in [a, b]$ $f(x_0)f''(x_0) > 0$

$\{x_k\}$ $f(x) = 0$ $[a, b]$ x^*

) x f

) x

$\varphi(x) = x - \frac{f(x)}{f'(x)}$ $\varphi'(x) = 1 - \frac{[f'(x)]^2 - f(x)f''(x)}{}$

2.3

$$f(x_0) > 0 \qquad f''(x_0) > 0$$

$$x_{k+1} - x^* = \varphi(x_k) - x^* = x_k - \frac{f(x_k)}{f'(x_k)} - x^*$$

$$= (x_k - x^*) - \frac{f'(\xi_k)}{f'(x_k)}(x_k - x^*)$$

$$= (x_k - x^*)(1 - \frac{f'(\xi_k)}{f'(x_k)})$$

ξ_k x_k x^*

$f'(x)$ $\frac{f'(\xi)}{f'(x_k)} > 0$ 1

$f'(x) > 0$ $f(x_0) > 0 = f(x^*)$ $x_0 > x^*$ $x_0 > \xi_0$ $f''(x) >$
0 $f'(x_0) > f'(\xi_0)$ $\frac{f'(\xi_0)}{f'(x_0)} < 1$ $x_1 - x^*$ $x_0 - x^*$

$x_0 > x_1 > x^*$ $x_k > x_{k+1} > x^*$

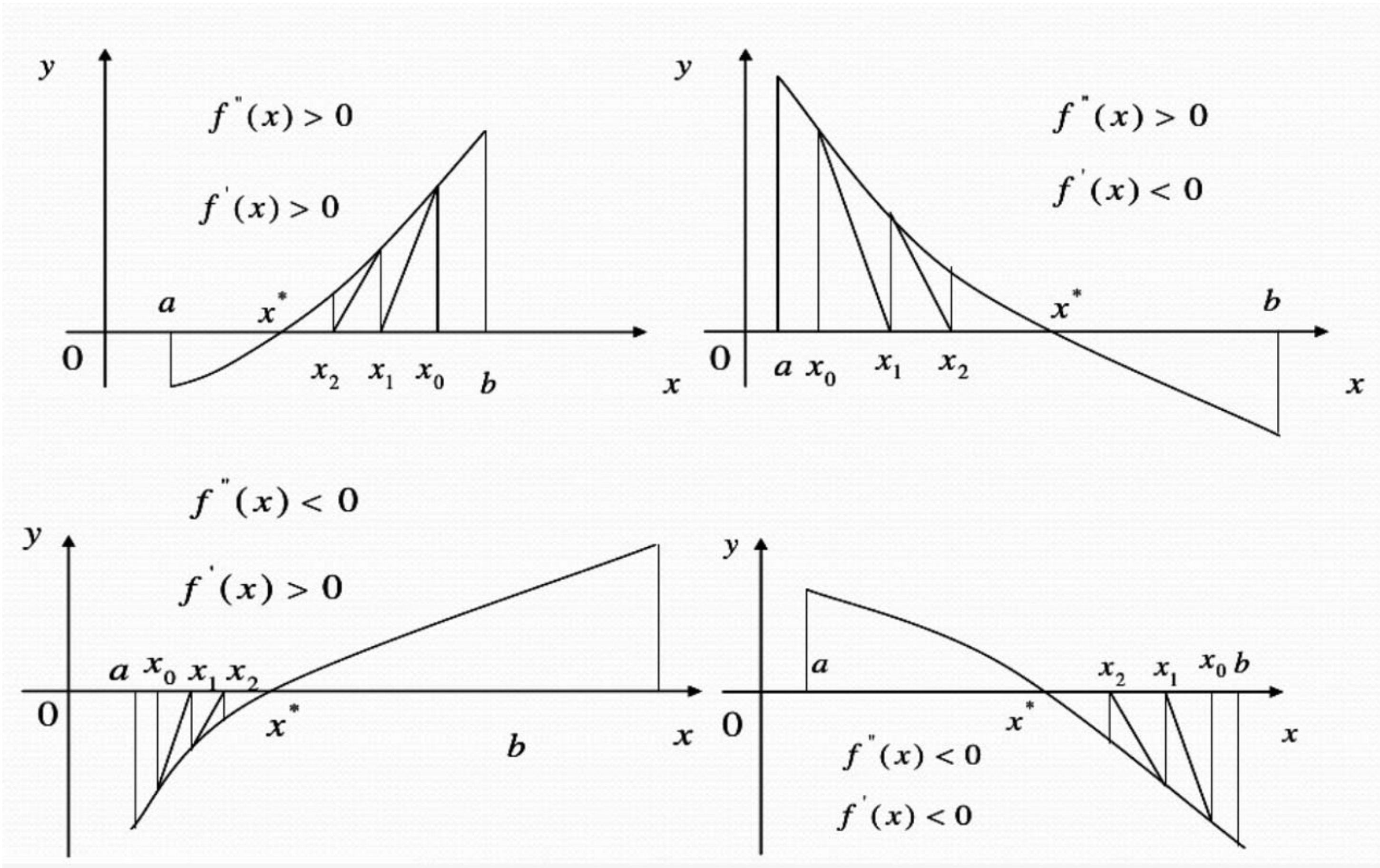
$f'(x) < 0$

x' $x' = \varphi(x')$

$f(x') = 0$ $x' = x^*$

2.3

❑ 4


y
f''(x) > 0
f'(x) > 0
a
x*
0
x2 x1 x0
b
x
y
f''(x) > 0
f'(x) < 0
x*
b
0
a x0 x1 x2
x
f''(x) < 0
y
f'(x) > 0
a x0 x1 x2
0
x*
b
x
y
a
x2 x1 x0 b
0
x*
x
f''(x) < 0
f'(x) < 0

2.3

例 2.5 $f(x) = 2 + si\,n\,x - e^x$ $x =$ 2

$f(1) > 0$ $f(2) < 0$ $f'(x) = \cos x - e^x$ $f''(x) = -\sin x - e^x$

$x = 2$ $f'(2) < 0$ $f''(2) < 0$， $f(2) \cdot f''(2) > 0$

$x_0 = 2$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{2+\sin x_k - e^{x_k}}{\cos x_k - e^{x_k}}$$

$$x_1 = x_0 - \frac{2+\sin x_0 - e^{x_0}}{\cos x_0 - e^{x_0}} = 2 - \frac{2+\sin 2 - e^2}{\cos 2 - e^2} \approx 1.4260$$

$$x_2 = x_1 - \frac{2+\sin x_1 - e^{x_1}}{\cos x_1 - e^{x_1}} \approx 1.1342$$

$$x_3 = x_2 - \frac{2+\sin x_2 - e^{x_2}}{\cos x_2 - e^{x_2}} \approx 1.0588$$

$$x_4 = x_3 - \frac{2+\sin x_3 - e^{x_3}}{\cos x_3 - e^{x_3}} \approx 1.0541$$

$$x_5 = x_4 - \frac{2+\sin x_4 - e^{x_4}}{\cos x_4 - e^{x_4}} \approx 1.0541$$

2.3

例2.6 $f(x)=x^3-x-1=0 \quad x=2$

$f(1)<0 \quad f(2)>0 \quad f'(x)=3x^2-1 \quad f''(x)=6x \quad x=2 \quad f'(2)>0 \quad f''(2)=12>0 \quad f(2)>0 \quad f(2)\cdot f''(2)>0$

$x_0=2$

$$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}=x_k-\frac{x_k^3-x_k-1}{3x_k^2-1}$$

$$x_1=x_0-\frac{x_0^3-x_0-1}{3x_0^2-1}=2-\frac{2^3-2-1}{3\times(2)^2-1}\approx 1.545455$$

$$x_2=x_1-\frac{x_1^3-x_1-1}{3x_1^2-1}\approx 1.359615$$

$$x_3=x_2-\frac{x_2^3-x_2-1}{3x_2^2-1}\approx 1.325802$$

$$x_4=x_3-\frac{x_3^3-x_3-1}{3x_3^2-1}\approx 1.32472$$

$$x_5=x_4-\frac{x_4^3-x_4-1}{3x_4^2-1}\approx 1.32472$$

2.3

2.4

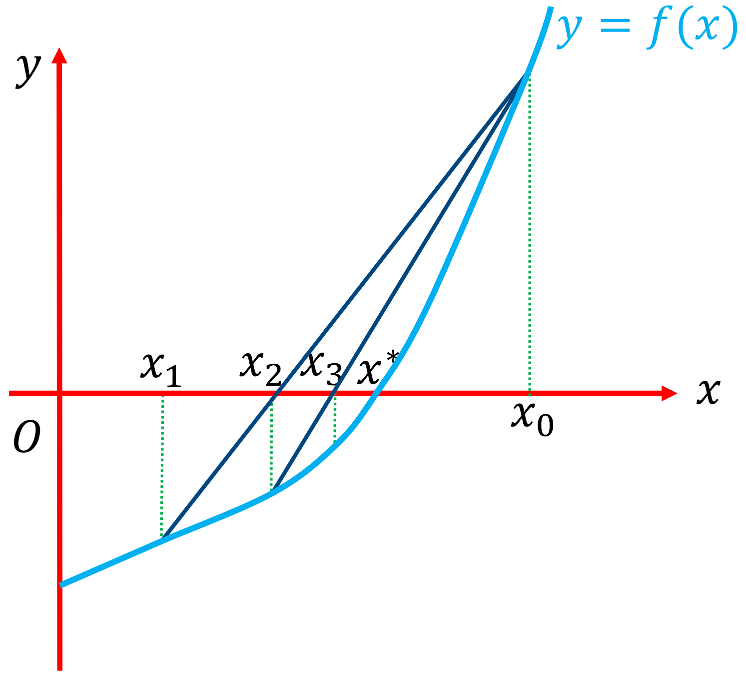
❑

$$f'(x_k) \approx \frac{f(x_k) - f(x_0)}{x_k - x_0}$$

-

$$x_{k+1} = x_k - \frac{(x_k - x_0)}{f(x_k) - f(x_0)} f(x_k)$$

- x_{k+1} x_k
-
-
-



2.4

❑

➢ $S(x^*,\delta)$ x_0, x_1, $(x_0, f(x_0))$ $(x_1, f(x_1))$

$$l_1:\quad y = f(x_1) + \frac{f(x_1)-f(x_0)}{x_1-x_0}(x - x_1)$$

x

$$x_2 = x_1 - \frac{x_1-x_0}{f(x_1)-f(x_0)} f(x_1)$$

➢ $(x_1, f(x_1))$ $(x_2, f(x_2))$

$$l_2:\quad y = f(x_2) + \frac{f(x_2)-f(x_1)}{x_2-x_1}(x - x_2)$$

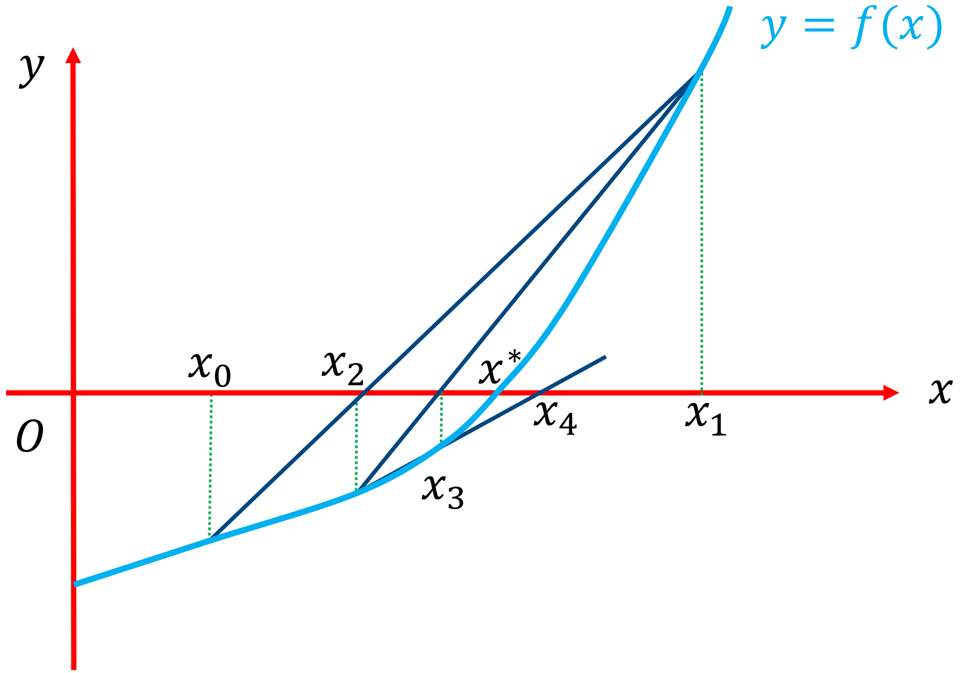
x

$$x_3 = x_2 - \frac{x_2-x_1}{f(x_2)-f(x_1)} f(x_2)$$

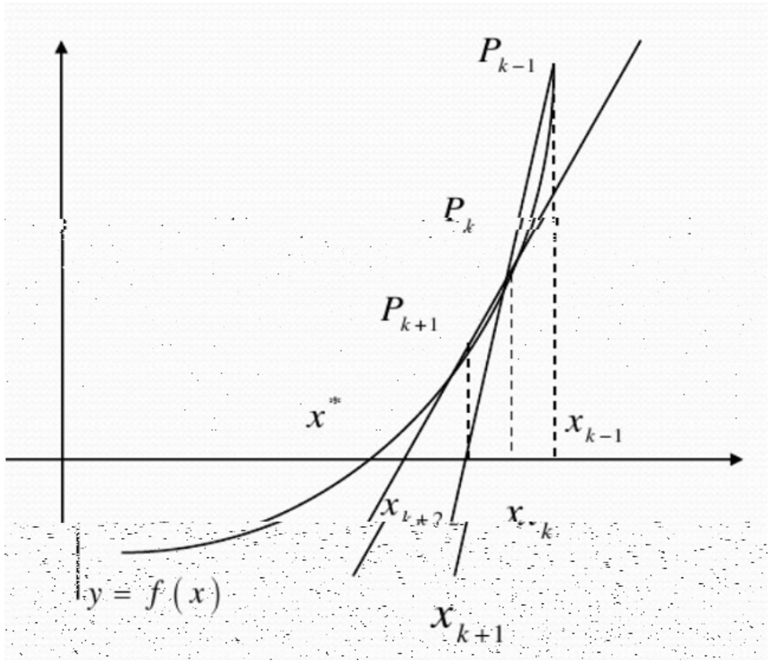
2.4

- x^* $x_0, x_1, x_2, \cdots, x_k, \cdots,$

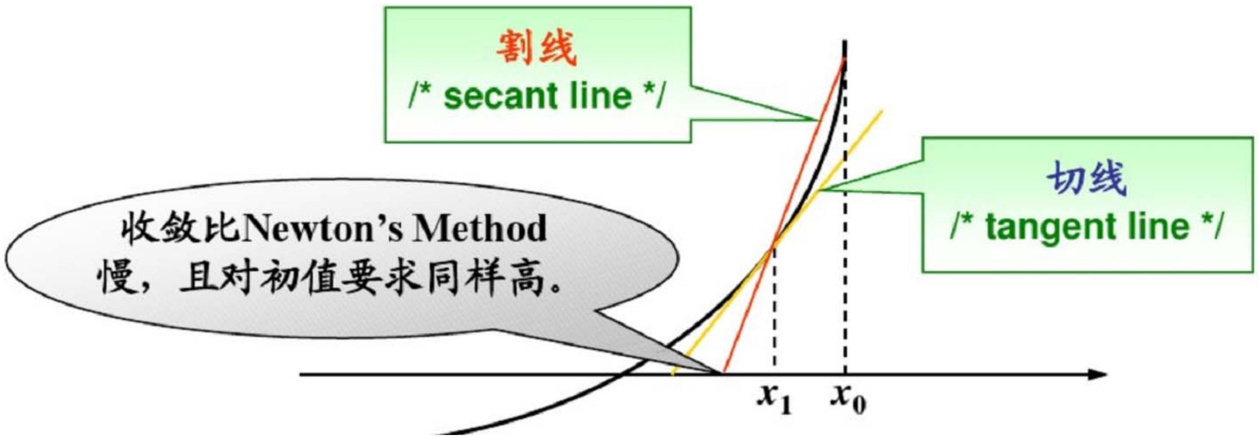
$$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} f(x_k),$$
$$k = 1,2,\cdots$$



2.4



P_{k-1}
P_k
P_{k+1}
x^*
x_{k-1}
x_k
x_{k+2}
x_{k+1}
$y = f(x)$



割线
/* secant line */
切线
/* tangent line */
收敛比Newton's Method
慢，且对初值要求同样高。
x_1
x_0

2.4

例2.3 $f(x) = 2 + \sin x - e^x \quad x \in [0,2]$

$f(0) = 1 > 0 \quad f(2) = -4.4798 < 0 \quad f(x)$

$f(x) = 0 \quad x \in [0,2] \qquad x_0 = 2, x_1 = 0,$

k	x_k	k	x_k	k	x_k
0	2	6	0.9919	0	1.0531
1	0	7	1.0226	1	1.0536
2	0.3650	8	1.0383	2	1.0539
3	0.6427	9	1.0462	3	1.0540
4	0.8256	10	1.0502	4	1.0541
5	0.9332	11	1.0522	5	1.0541

2.4

例2.4 $x^3 - x - 1 = 0 \quad x \in [1,2]$.

$f(0) = -1 < 0 \quad f(2) = 5 > 0 \quad f(x)$

$f(x) = 0 \quad x \in [1,2]$

$x_0 = 2, x_1 = 1.5,$

k	x_k	k	x_k
0	2	7	1.3256
1	1.5	8	1.3251
2	1.3939	9	1.3249
3	1.3532	10	1.3248
4	1.3367	11	1.3247
5	1.3298	12	1.3247
6	1.3269	13	1.3247

2.5

❑ or

例2.7 $f(x) = x^3 - x - 1 = 0$

$f(x)$ [1,2] $f(1) = -1 < 0$, $f(2) =$
$5 > 0$, $f(x) = 0$ [1,2]

$f(x) = 0$

$x = \varphi_1(x) = \sqrt[3]{x+1}$, $x = \varphi_2(x) = x^3 - 1$,

$x_0 = 1.5$, $\varphi_1(x)$ $x_1 = 1.35721$

2.5

k	x_k	*k*	x_k	*k*	x_k
0	1.5	3	1.32588	6	1.32473
1	1.3521	4	1.32492	7	1.32472
2	1.33086	5	1.32476	8	1.32472

$x_7 = x_8$ x_8

迭代法

2.7 $f(x) = 0$ $x = \varphi_2(x) = x^3 - 1$

$$x_{k+1} = {x_k}^3 - 1$$

$x_0 = 1.5$ $x_1 = 2.375$ $x_2 = 12.3965$ $x_3 = 1904.01 \cdots$

发散的

2.5

❑

$$f(x) = 0$$

$$x = \varphi(x)$$

$\varphi(x)$

$$x_{k+1} = \varphi(x_k), \mathrm{k} = 0,1,2 \cdots$$

x_0 $\{x_k\}_{k=0}^{\infty}$

$\lim_{k \to \infty} x_{k+1} = x^*$ $\varphi(x)$

$$x^* = \lim_{k \to \infty} x_{k+1} = \lim_{k \to \infty} \varphi(x_k) = \varphi\left(\lim_{k \to \infty} x_k\right) = \varphi(x^*)$$

x^* $x = \varphi(x)$ x^* $f(x) =$
0

2.5

- x^* $\varphi(x)$ **不动点** $x_{k+1} = \varphi(x_k), k = 0,1,2 \cdots$ **收敛的** **不动点迭代法**

- x_{k+1} x_k $x_{k+1} = \varphi(x_k), k = 0,1,2 \cdots$ **单步迭代法** x_k k

- $\{x_k\}$ $x_{k+1} = \varphi(x_k), k = 0,1,2 \cdots$ **发散的**

❑

- **如何选取迭代函数$\varphi(x)$，使迭代公式$x_{k+1} = \varphi(x_k)$收敛。**

2.5

定理2.2 $\varphi(x)$

- $\varphi(x)$ $[a,b]$ (a,b)
- $x \in [a,b]$ $\varphi(x) \in [a,b]$
- $L(0 < L < 1)$ $[a,b]$ $|\varphi'(x)| \le L < 1$

- $\varphi(x)$ $[a,b]$ x^*
- $x_0 \in [a,b]$, $x_{k+1} = \varphi(x_k)$ $\{x_k\}_{k=1}^{\infty} \in [a,b]$ $\lim_{k\to\infty} x_k = x^*$
-

$$|x_k - x^*| \le \frac{L}{1-L}|x_k - x_{k-1}|$$

$$|x_k - x^*| \le \frac{L^k}{1-L}|x_1 - x_0|$$

2.5

证明：

$x = \varphi(x)$ $[a, b]$ $\varphi(x)$ $[a, b]$

$\varphi_1(x) = x - \varphi(x)$ $[a, b]$ $\varphi(x) \in [a, b]$

$\varphi_1(a) = a - \varphi(a) \le 0,$ $\varphi_1(b) = b - \varphi(b) \ge 0$ $x^* \in [a, b]$

$$\varphi_1(x^*) = 0$$

$x^* = \varphi(x^*)$

$x = \varphi(x)$ [a,b] x^*

x^{**} $|\varphi'(x) \le L < 1|$

$$|x^* - x^{**}| = |\varphi(x^*) - \varphi(x^{**})| = |\varphi'(\xi)|\,|x^* - x^{**}| \le L|x^* - x^{**}|$$

ξ x^* x^{**}

$x^* = x^{**}$

2.5

证明:

$x_0 \in [a, b]$ $\qquad\qquad$ $x_k \in [a, b]$

$$x^* - x_{k+1} = \varphi(x^*) - \varphi(x_k) = \varphi'(\xi)(x^* - x_k)$$

$\xi \quad x^* \quad x_k$

$$|x^* - x_{k+1}| \leq L|x^* - x_k|(k = 0,1,2\cdots)$$

$$0 \leq |x^* - x_{k+1}| \leq L^k|x^* - x_0|$$

$0 < L < 1,\qquad k \to \infty \qquad L^k \to 0$

$$\lim_{k\to\infty}|x^* - x_k| = 0$$

$$\lim_{k\to\infty} x_k = x^*$$

2.5

证明：

$$|x^* - x_{k+1}| \le L|x^* - x_k| \; (k = 0,1,2\cdots)$$
$$|x_{k+1} - x_k| \le L|x_k - x_{k-1}| \;\; (k = 0,1,2\cdots)$$

$$\begin{aligned}|x_{k+1} - x_k| &= |(x^* - x_k) - (x^* - x_{k+1})| \\ &\ge |x^* - x_k| - |x^* - x_{k+1}| \\ &\ge |x^* - x_k| - L|x^* - x_k| = (1 - L)|x^* - x_k|\end{aligned}$$

$$|x^* - x_k| \le \frac{1}{1-L}|x_{k+1} - x_k| \le \frac{1}{1-L}|x_k - x_{k-1}|$$

$$|x^* - x_k| \le \frac{1}{1-L}|x_{k+1} - x_k| \le \frac{L}{1-L}|x_k - x_{k-1}| \le \frac{L^2}{1-L}|x_{k-1} - x_{k-2}| \le \cdots \le \frac{L^k}{1-L}|x_1 - x_0|$$

2.5

定义2.1 x^* $\varphi(x)$ x^* $N(x^*,\delta): |x-x^*| \le \delta$ $x_0 \in$ $N(x^*,\delta)$ $x_{k+1}=\varphi(x_k), k=0,1,2\cdots$ $\{x_k\}_{k=0}^{\infty} \subset N(x^*,\delta)$, $\lim\limits_{k\to\infty} x_k = x^*$ $x_{k+1} =$ $\varphi(x_k), k=0,1,2\cdots$

定理2.3 x^* $\varphi(x)$ $\varphi'(x)$ x^* $|\varphi'(x)|<1$

$x_{k+1}=\varphi(x_k), k=0,1,2\cdots$

2.5

证明:

$\varphi'(x)$ x^* $|\varphi'(x^*)| < 1$

x^* $N(x^*, \delta)$ $L(0 \le L < 1)$ $\forall x \in N(x^*, \delta)$

$|\varphi'(x)| \le L$。

$x \in N(x^*, \delta)$

$$|\varphi(x) - x^*| = |\varphi(x) - \varphi(x^*)| = |\varphi'(\xi)||x - x^*| \le L|x - x^*| < \delta$$

ξ x x^*

$\varphi(x) \in N(x^*, \delta)$

2.5

定义2.2 $\{x_k\}_{k=0}^{\infty}$ x^* $e_k = |x_k -$
$x^*|$ $p \geq 1$ $c \neq 0$

$$\lim_{k\to\infty} \frac{e_{k+1}}{e_k^p} = c$$

$\{x_k\}_{k=0}^{\infty}$ p $p = 1$ $p > 1$,
$p = 2$
$k \to \infty$ e_{k+1} e_k p p
$0 < |c| \leq 1.$

定理2.4 x^* $\varphi(x)$
$p \geq 2$, $\varphi^{(p)}(x)$ x^*
$\varphi^{(n)}(x^*) = 0$ $n = 1,2,\cdots,p-1$ $\varphi^{(p)}(x) \neq 0$
$x_{k+1} = \varphi(x_k), k = 0,1,2\cdots$ p

2.5

例2.8 $a>0, x_0>0,$ $x_{k+1}=x_k(x_k^2+3a)/(3x_k^2+a)$

$\sqrt{a}$ 3 $\lim\limits_{k\to\infty}\left(\sqrt{a}-x_{k+1}\right)/(\sqrt{a}-x_k)^3$.

证明:

$a>0, x_0>0$ $x_k>0(k=1,2,\cdots)$ $\{x_k\}$ $x^*,$

$x^*=x^*(x^{*2}+3a)/(3x^{*2}+a),$ $x^*=0, x^*=\pm\sqrt{a}.$ $x^*=\sqrt{a}$

$\lim\limits_{k\to\infty} x_k=\sqrt{a}$

$\varphi(x)=x(x^2+3a)/(3x^2+a)$

$$\varphi'(x)=\frac{(3x^2+3a)(3x^2+a)-x(x^2+3a)6x}{(3x^2+a)^2}=\frac{3(x^2-a)^2}{(3x^2+a)^2}$$

$\forall x>0, |\varphi'(x)|<1,$

$$\lim_{k\to\infty}\frac{\sqrt{a}-x_{k+1}}{(\sqrt{a}-x_k)^3}=\lim_{k\to\infty}\frac{\sqrt{a}-(x_k^3+3ax_k)/(3x_k^2+a)}{(\sqrt{a}-x_k)^3}$$

$$=\lim_{k\to\infty}\frac{(\sqrt{a}-x_k)^3}{(\sqrt{a}-x_k)^3(3x_k^2+a)}=\lim_{k\to\infty}\frac{1}{3x_k^2+a}=\frac{1}{4a}$$

3

2.5

❑

定理2.5 x^* $f(x) = 0$ $f“(x)$ x^*

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ $x_{k+1} = \varphi(x_k)$ $\varphi(x) = x - \frac{f(x)}{f\prime(x)}$

x^* $f(x) = 0$ $f'(x^*) \neq 0$ $x^* = \varphi(x^*)$ x^* $\varphi(x)$

$\varphi(x)$

$$\varphi'(x) = 1 - \frac{[f'(x)]^2 - f(x)f''(x)}{[f'(x)]^2} = \frac{f(x)f''(x)}{[f'(x)]^2}$$

$f'(x^*) \neq 0$ $\varphi'(x^*) = 0$ 2.2 $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ $f(x^*)$ x

2.5

$$0 = f(x^*) = f(x_k) + f'(x_k)(x^* - x_k) + \frac{f''(\xi_k)}{2}(x^* - x_k)^2$$

ξ_k x_k x^* $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$f(x_k) - f'(x_k)x_k = -f'(x_k)x_{k+1}$

$$0 = f'(x_k)(x^* - x_{k+1}) + \frac{f''(\xi_k)}{2}(x^* - x_k)^2$$

$$\frac{e_{k+1}}{e_k^2} = \frac{f''(\xi_k)}{2f'(x_k)} \qquad (e_k = |x_k - x^*|)$$

$k \to \infty$ $x_k \to x^*$ $\xi_k \to x^*$

$$\lim_{k\to\infty} \frac{e_{k+1}}{e_k^2} = \frac{f''(x^*)}{2f'(x^*)} = \mathrm{c}$$

$\mathrm{c} \neq 0$ $\mathrm{c} = 0$

- P66-67
  - 3.3 3.16 3.18
  - 3.16 C++
    - class