Rappel sur quelques notions de probabilités-statistiques

Variables aléatoires discrètes

- DÉFINITIONS

  - Moyenne $$\mu = \sum_i X_i \, P(X_i)$$

  - Variance

$$\sigma^2 = \sum_i (X_i - \mu)^2 \, P(X_i)$$

  - Espérance mathématique et variance

$$E(X) = \mu$$
$$V(X) = \sigma^2 = E(X^2) - \mu^2$$

Distributions discrètes

➢ Loi Uniforme

tous les événements ont la même probabilité d'occurrence. Si n est le nombre de valeurs différentes prises par la variable aléatoire,

$$\forall i, \quad P(X = x_i) = \frac{1}{n}$$

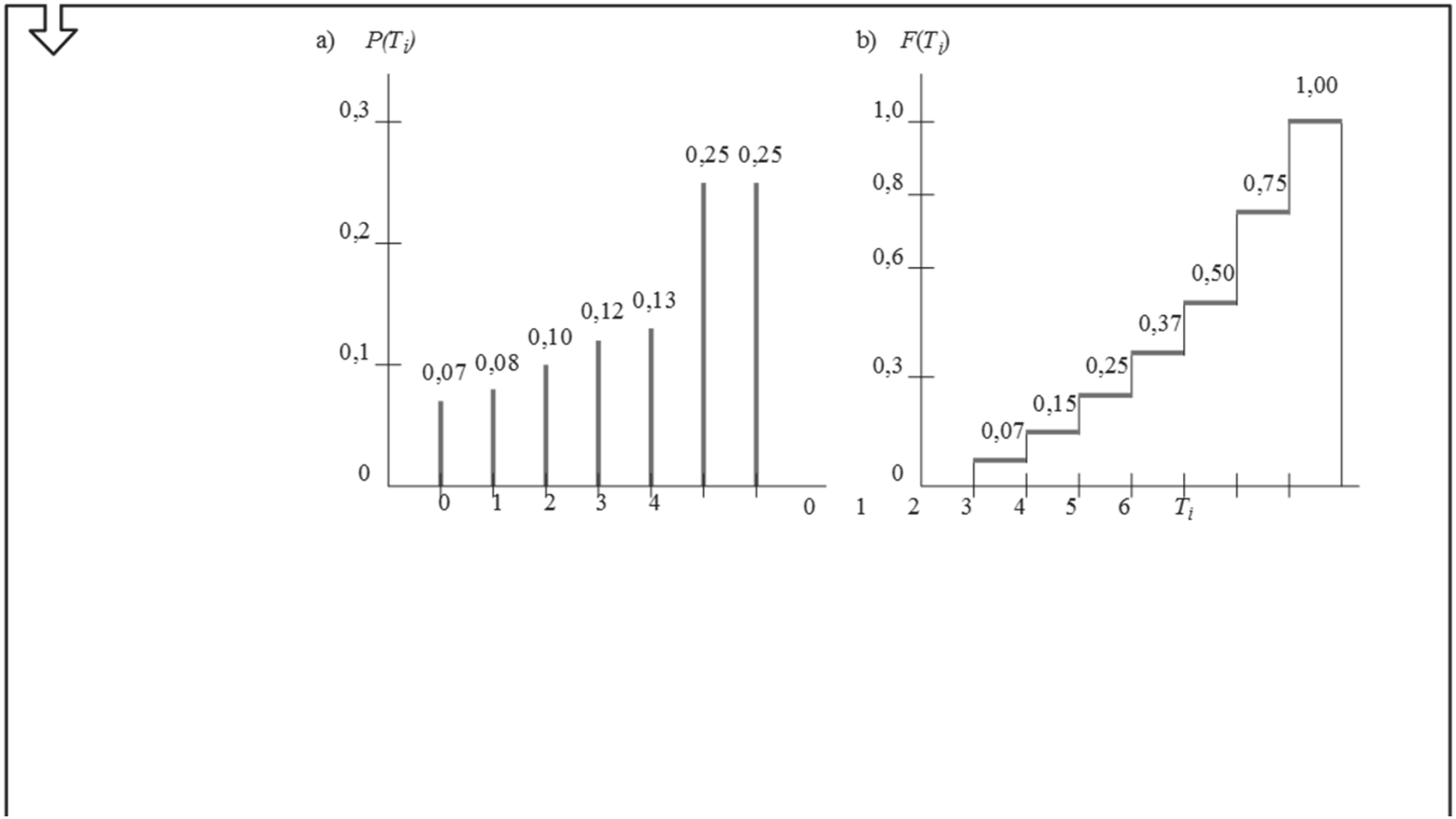
$$E(X) = \mu = \frac{n+1}{2}$$

$$V(X) = \sigma^2 = \frac{n^2 - 1}{12}$$

La distribution des probabilités et la distribution cumulative

(1) Nombre de traitements quotidiens T_i	(2) Probabilité $P(T_i)$	(3) Probabilité cumulative $F(T_i)$
0	0,07	**0,07**
1	0,08	**0,15**
2	0,10	**0,25**
3	0,12	**0,37**
4	0,13	**0,50**
5	0,25	**0,75**
6	**0,25**	**1,00**

La distribution des probabilités (a) *P*(*Ti*) e t l a distribution cumulative (b) *F*(*Ti*): exemple



Distributions continues

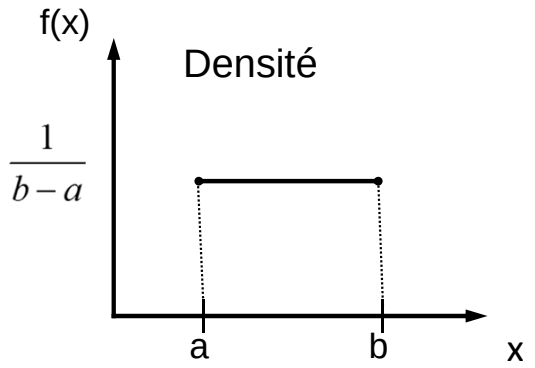
➢ Loi uniforme

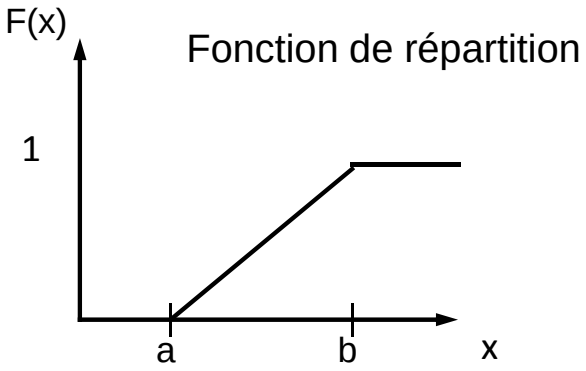
$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{si } a \le x \le b \\ 0 & \text{autrement} \end{cases}$$

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & \text{si } a \le x \le b \\ 1 & x > b \end{cases}$$

$$E(X) = \mu = \frac{a+b}{2}$$

$$V(X) = \sigma^2 = \frac{(b-a)^2}{12}$$





Distributions continues

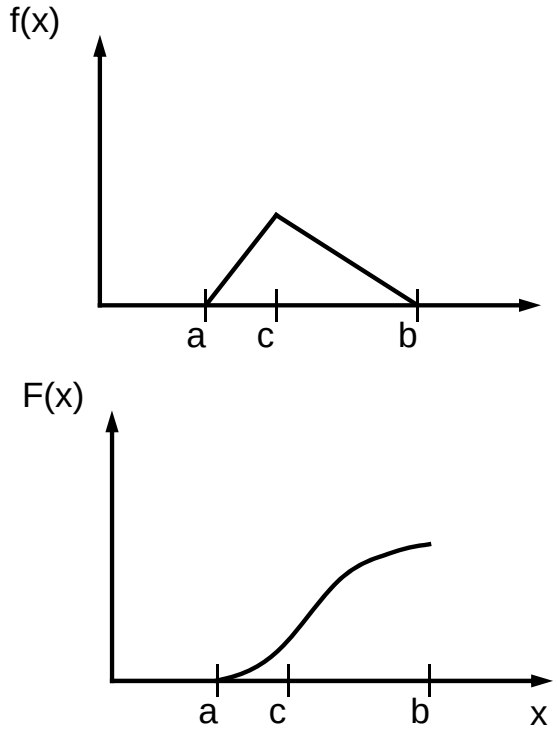
DISTRIBUTION TRIANGULAIRE

$$f(x) = \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)} & \text{si } a \le x \le c \\ \dfrac{2(b-x)}{(b-a)(b-c)} & \text{si } c \le x \le b \\ 0 \quad \text{autrement} \end{cases}$$

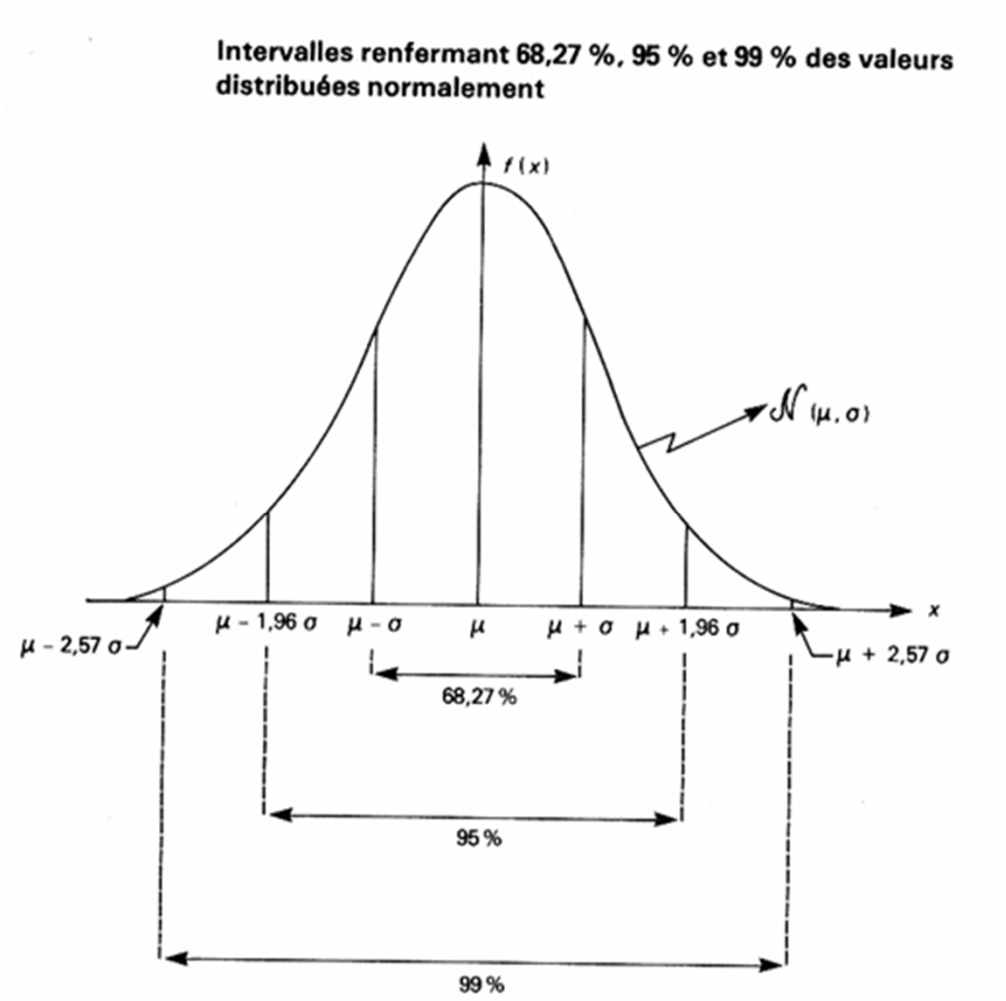
$$F(x) = \begin{cases} 0 & \text{si } x < a \\ \dfrac{(x-a)^2}{(b-a)(c-a)} & \text{si } a \le x \le c \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-c)} & \text{si } c \le x \le b \\ 1 & \text{si } x > b \end{cases}$$

$$\mu = \frac{1}{3}(a+b+c)$$

$$\sigma^2 = \frac{a^2+b^2+c^2-ab-ac-bc}{18}$$



La loi normale: N(μ,σ)



Espérance : $E(x) = \mu$

Variance : $Var(x) = \sigma^2$

Symétrique par rapport à μ

La loi normale: N(μ,σ)

Loi normale centrée réduite: moyenne = 0, écart type = 1.

$$Z = \frac{X - \mu}{\sigma}$$

$$-\infty < Z < +\infty$$

La loi normale: N(μ,σ)

Exemple: N(5,0.5), P(4 < x < 6.5)?

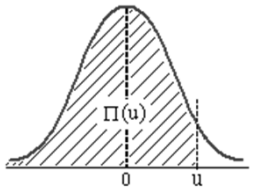
$$Z_1 = \frac{4-5}{0.5} = -2 \qquad Z_2 = \frac{6.5-5}{0.5} = 3$$

P (-2 < Z < 3)?

Réponse : calcul de P(0 < Z < 3) et de P(-2 < Z < 0) et addition des deux termes.

P(0 < Z < 3) = Φ(3) - Φ(0) = 0.999 - 0.5 = 0.499
P(-2 < Z < 0) = Φ(2) - Φ(0) = 0.977 - 0.5 = 0.477
P(-2 < Z < 3) = 0.499 + 0.477 = 0.976

Fonction de répartition P de la loi normale centrée réduite. Probabilité de trouver une valeur inférieure à u . P (-u) = 1 - P (u)



u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997