Notes

(Dated: September 9, 2014)

I. MISC

Coupling a $U(1)$ gauge field to a charge N matter field reduces the gauge symetry to Z_N gauge field (for $N > 1$).

II. DIMER MODEL

A. Correlation Functions

Here we check the implementation of our numerical methods. First we validate that the dimer pair correlations in a fully packed dimer model decay as $1/r^2$. Second we validate that the monomer pair (defect) correlations in a fully packed dimer model decay as $1/r^{1/2}$ ($1/r^{1/3}$ in the fully packed loop model). In fig. 1 we show a histogram of the correlation function

$$C_{dimer}(0, r) = \langle s_0 s_r \rangle - \langle s_0 \rangle \langle s_r \rangle \tag{1}$$

for a lattice size of 64×64 The disconnected piece of the correlation function is

$$\langle s_0 \rangle \langle s_r \rangle = \frac{1}{4}\frac{1}{4} = \frac{1}{16}. \tag{2}$$

The connected component is calculated numerically, in this case, s_0 is the value of the link at the right of the $(0, 0)$ vertex and s_r are only chosen from the same vertical column of links. In fig. 2 we show a log log plot of the first half of the data shown in fig. 1. Using a weighted fit we find the slope of the line to be 2.3437 ± 0.0007.

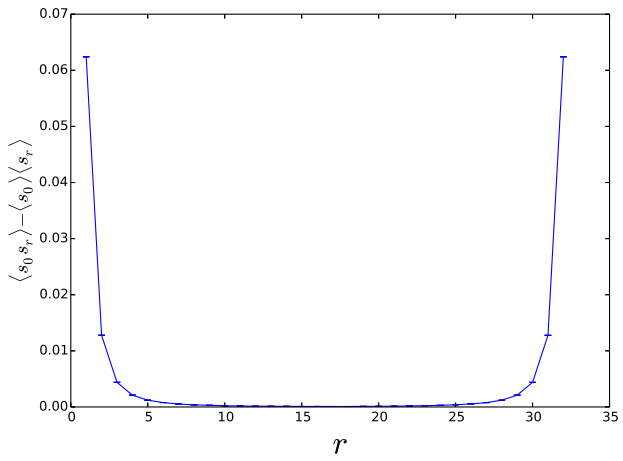

FIG. 1. lattice size: 64×64. Number of bins: 80,000. Each bin averaged over 50,000 configurations. Each configuration used was spaced by 4 random walks.

III. STAR DIMER

Very preliminary results of the star star correlation function are shown in fig. 3.

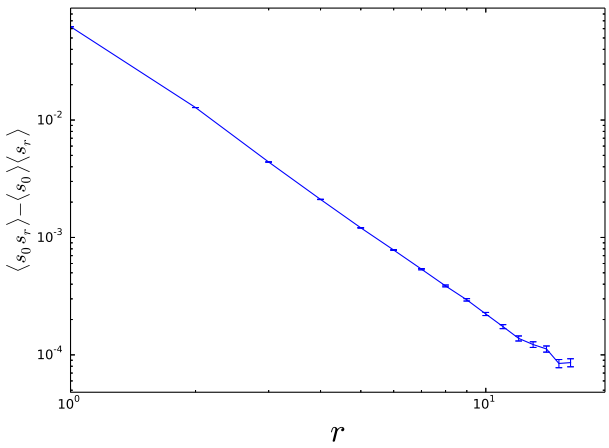

FIG. 2. lattice size: 64×64. Number of bins: 80,000. Each bin averaged over 50,000 configurations. Each configuration used was spaced by 4 random walks.

$$C_{star}(0, r) = \langle star_0 star_r \rangle - \langle star_0 \rangle \langle star_r \rangle \tag{3}$$

The disconnected piece has been numerically measured for a couple of different lattice sizes shown in table I.

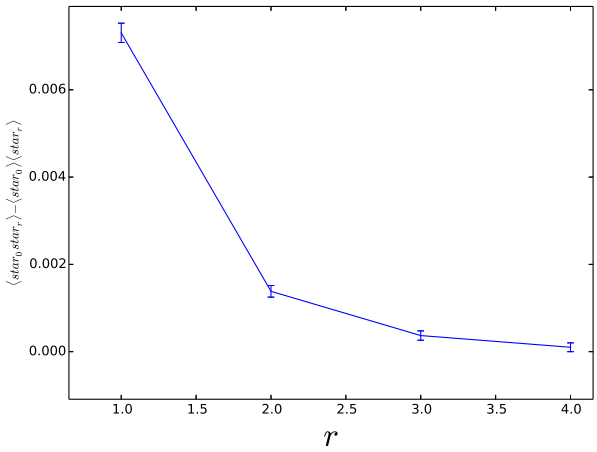

FIG. 3. Star star correlation function (just along y direction)

A. Star Pair Creation and Annihilation

Creation and annihilation of a horizontal pair of stars is shown in fig. 6. Creation and annihilation of a vertical pair of stars can be is simply what is shown in fig. 6 rotated by $\pi/2$.

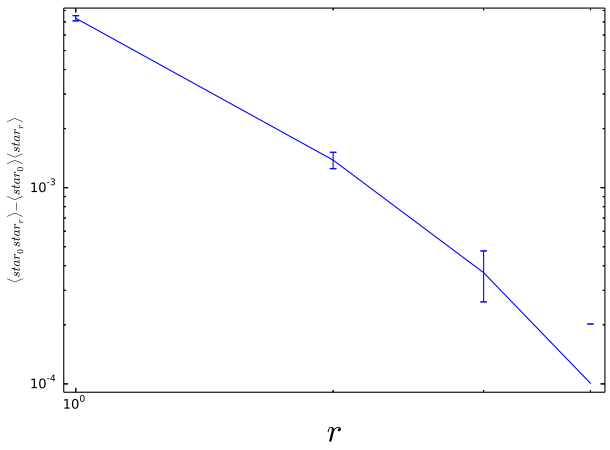

FIG. 4. Star star correlation function (just along y direction) log log plot.

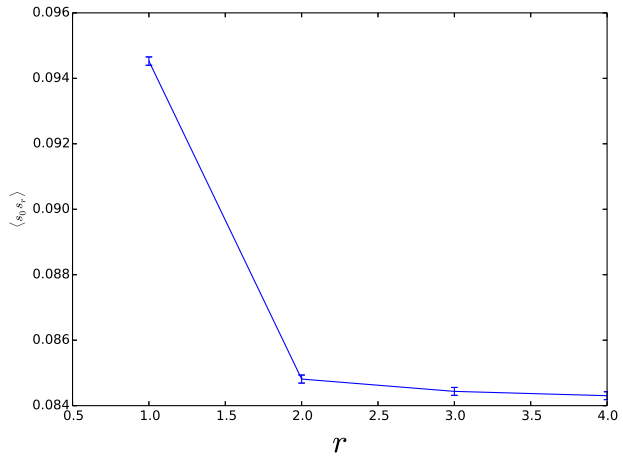

FIG. 5. Dimer dimer correlation function in the star dimer model

B. Star Horizontal/Vertical Translation

Horizontal translations of a star is shown in fig. 7. The vertical translations can be obtained by rotating fig. 7 by $\pi/2$. Horizontal translations must be preformed such that the star remains on its original sub-lattice, thus the minimal translation is by two vertices.

C. Star Diagonal Translation

A star can remain on its original sub-lattice through a diagonal translation as well. We show a star moving to the lower right diagonal in fig. 8.

D. Equilibration Time

We show three order parameters as a function of Monte Carlo time, t_{mc}:

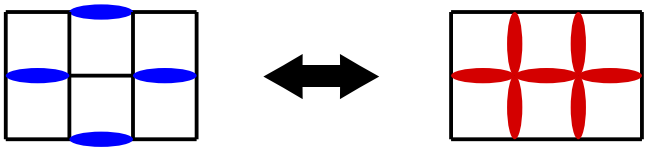

FIG. 6. The creation and annihilation of a pair of stars.

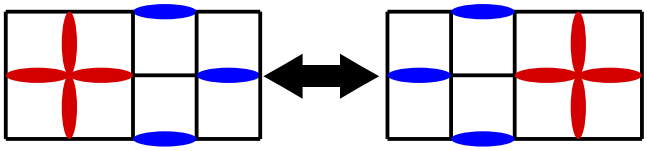

FIG. 7. Horizontal translation of a star.

- The number of stars.
- The difference between the number of vertical dimers and horizontal dimers.
- The number of flippable plaquetts.

in fig. 9. We define t_{mc} to be $2lh$ local updates, where l is the length of the system and h is the height of the system in number of vertices. This lattice size used was 64×64. Each data point shown is the average over 100 different simulations. The error bars are the standard deviations of the counts used in taking the mean. It appears as if the equilibration time is around $400t_{mc}$.

E. Star Concentrations

The following statistics are given for different lattice sizes but each set of data is found using 100,000 bins, each an average over 10 measurements, each measurement taken after $1 \times t_{mc}$. We have not included the first 400 t_{mc} so that the averages are found in an equilibrated system.

TABLE I. Some Counting

lattice size	16×16	32×32	64×64
$\langle \#stars \rangle$		55.65171	222.51807
$\sigma_{\#stars}$		8.61922	17.23504
$\langle star_{i,j} \rangle$		0.05435 ± 0.00842	0.05432 ± 0.00421
$\langle star_{i,j} \rangle \langle star_{i,j} \rangle$		0.00295 ± 0.00007	$0.00295 \pm .00002$

F. Old Moves and Rules

Creating a pair of stars in the staggered state is shown in fig. 10. We notice the creation of a star produces two flippable plaquettes between the two stars.
In fig. 11 we show the rightmost star move right in the same configuration. In any single star must remain on the sublattice it was created on, so the center of the star moves by two vertices.

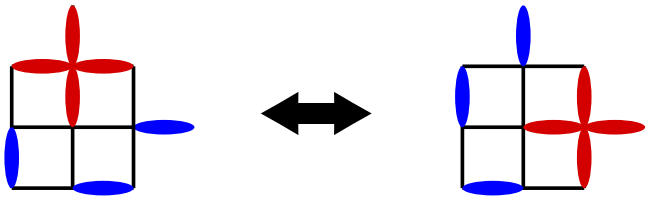

FIG. 8. Diagonal translation to the lower right diagonal.

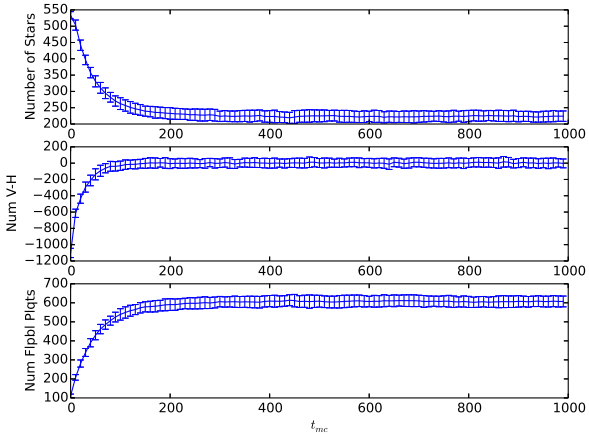

FIG. 9. From the top down: The number of stars on the lattice, the difference between the number of vertical dimers and horizontal dimers, and the number of flippable plaquetts as a function of Monte Carlo time t_{mc}.

We notice that the horizontal propagation of a star creates two boundaries of flippable plaquettes across the top and bottom of the star pair.

Still figuring out the possible ways to move stars vertically. In six steps we show it is possible to implement a specific type of local vertical move.

- Step 1 (fig. 12): create a pair of stars.
- Step 2 (fig. 13): Move rightmost star right four times creating a horizontal set of flippable plaquettes.
- Step 3 (fig. 14): Flip some of the plaquettes.
- Step 4 (fig. 15): With several vertical pairs of dimers we can flip on plaquette to make a horizontal pair that was not part of the original set of horizontal dimers after the star move. The x and y positions of this plaquette are marked in fig. 15 by red dots along the x and y axes. If all of the dimers were left as vertical or horizontal the move would not be possible.
- Step 5 (fig. 16): Create another pair of stars.
- Step 6 (fig. 17): Move the rightmost star of the new pair up. When this is done the dimers must be rearranged to satisfy the packing conditions and conserve the number of dimers. Right now this is the only configuration I see for this move that can satisfy these conditions.

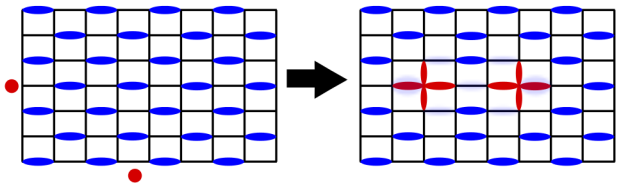

FIG. 10. The creation of a pair of stars in the fully packed staggered configuration. The red dots on the x and y axes show the dimer about which the stars are created.

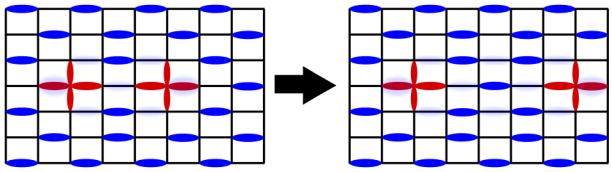

FIG. 11. The move of the rightmost star in fig. 10

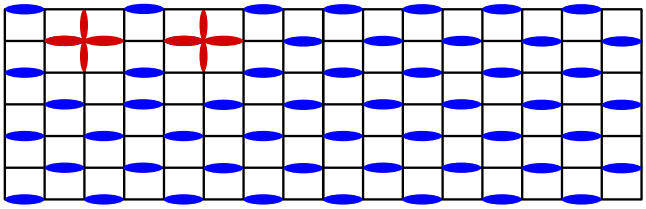

FIG. 12. Step 1 in the vertical move example. Create a pair of stars.

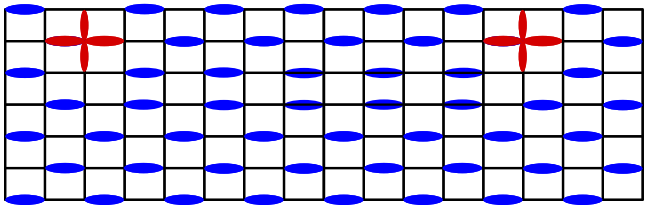

FIG. 13. Step 2 in the vertical move example. Move the rightmost star right four times creating a horizontal set of flippable plaquettes.

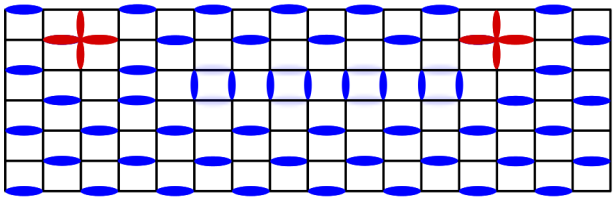

FIG. 14. Step 3 in the vertical move example. Flip some of the plaquettes.

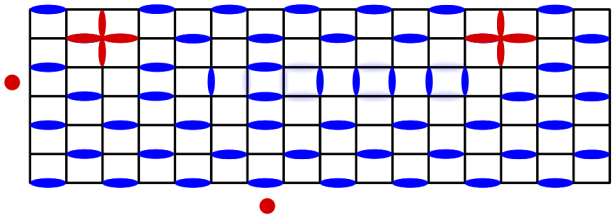

FIG. 15. Step 4 in the vertical move example. Flip the plaquette at the x and y coordinates marked by the red dots.

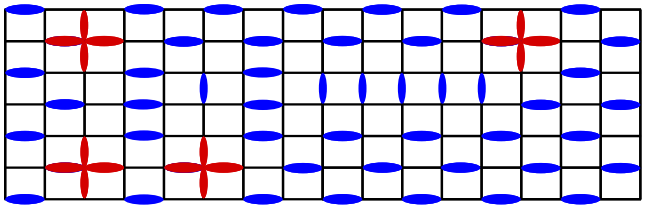

FIG. 16. Step 5 in the vertical move example. Create another pair of stars.

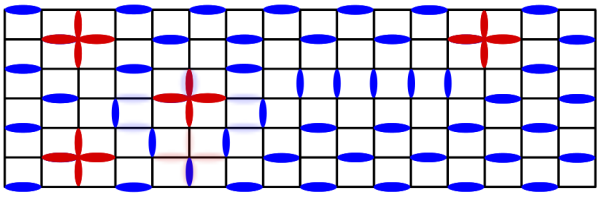

FIG. 17. Step 6 in the vertical move example. Move the rightmost star of the new pair up. Rearrange the dimers around it to satisfy packing and conservation rules.