Notes

(Dated: September 23, 2014)

I. MISC

At the RK point the ground state is the short-range resonating valence bond state

$$|\Psi_{RVB}\rangle = \sum_C |C\rangle \tag{1}$$

where $\{C\}$ is the set of fully packed dimer configurations.

$$\|\Psi_{RVB}\|^2 = Z_{\text{classical dimer}} \tag{2}$$

i.e. the norm of Ψ_{RVB} is the partition function of the classical dimer model.
On the square lattice the dimer dimer correlation function at the RK point (coefficient in front of kinetic energy is equal to the coefficient in front of potential energy $t - v$) was solved exactly by Fisher and Stephenson in 1963. They showed that correlations decay as $1/r^2$. This algebraic decay means that ground state has no long range dimer order (not a crystal). It is not a liquid either. In a liquid correlations decay exponentially.
all above is from Fradkin.

$U(1)$ gauge theory describes low-energy strong local correlations. Coupling a $U(1)$ gauge field to a charge N matter field reduces the gauge symmetry to Z_N gauge field (for $N > 1$).

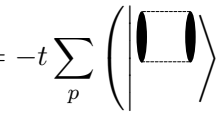
The structure of the quantum dimer models phase diagrams depends on if a lattice is bipartite or not; if it is bipartite it also depends on the coordination number. Most of the time the valence bond crystal is the ground state.

The Hamiltonian of the quantum dimer model is split in the resonance term and the diagonal term. The resonance term being the kinetic energy term and the diagonal term being the potential energy term diagonal term being the potential energy term.

$$H = H_{\text{res}} + H_{\text{diag}} \tag{3}$$

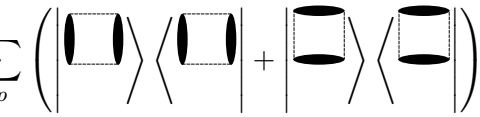
where

$$H_{\text{res}} = -t \sum_p \left(\left| \right\rangle \left\langle \right| + \text{h.c.} \right) \tag{4}$$



and

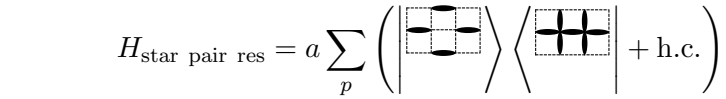
$$H_{\text{diag}} = v \sum_p \left(\left| \right\rangle \left\langle \right| + \left| \right\rangle \left\langle \right| \right) \tag{5}$$



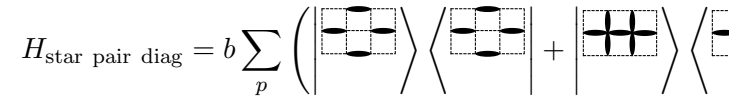
The sum over p is the sum over all plaqquettes on the lattice.

In the star dimer model we introduce the additional potential and kinetic energy terms.

$$H_{\text{star pair res}} = a \sum_p \left(\left| \right\rangle \left\langle \right| + \text{h.c.} \right) \tag{6}$$



$$H_{\text{star pair diag}} = b \sum_p \left(\left| \right\rangle \left\langle \right| + \left| \right\rangle \left\langle \right| \right) \tag{7}$$



II. DIMER MODEL

A. Correlation Functions

Here we check the implementation of our numerical methods. First we validate that the dimer pair correlations in a fully packed dimer model decay as $1/r^2$. Second we validate that the monomer pair (defect) correlations in a fully packed dimer model decay as $1/r^{1/2}$ ($1/r^{1/3}$ in the fully packed loop model). In fig. 1 we show a histogram of the correlation function

$$C_{dimer}(0, r) = \langle s_0 s_r \rangle - \langle s_0 \rangle \langle s_r \rangle \tag{8}$$

for a lattice size of 64×64 The disconnected piece of the correlation function is

$$\langle s_0 \rangle \langle s_r \rangle = \frac{1}{4}\frac{1}{4} = \frac{1}{16}. \tag{9}$$

The connected component is calculated numerically, in this case, s_0 is the value of the link at the right of the $(0, 0)$ vertex and s_r are only chosen from the same vertical column of links. In fig. 2 we show a log log plot of the relevent half of the data shown in fig. 1.

We now look at the same correlation function on a 128×128 lattice. Using a weighted fit of on points $[3, 13]$, shown in fig. 5 we find the slope of the line to be -2.4203 ± 0.0106.

Now the correlation function on a 256×256 lattice. Using a weighted fit of on points $[3, 14]$, shown in fig. 7 we find the slope of the line to be 2.3057 ± 0.0269.

B. Multiple Poin Choice Slopes

In these figs. **??** we plot the weighted fit slope for a variety of histogram bin selections. The histogram bins used in the fit are chosen in the following way:

- A cut off point in the histograms is chosen such that all the histogram bins beyond this point have errors that are too large to justify including them in a fit.
- All histogram bins until the cut off are used in a weighted fit. This is plotted as the first point with error bars that are the error of the fit.
- The first histogram bin is omitted from the fit.
- The second histogram bin is omitted from the fit.
- ...

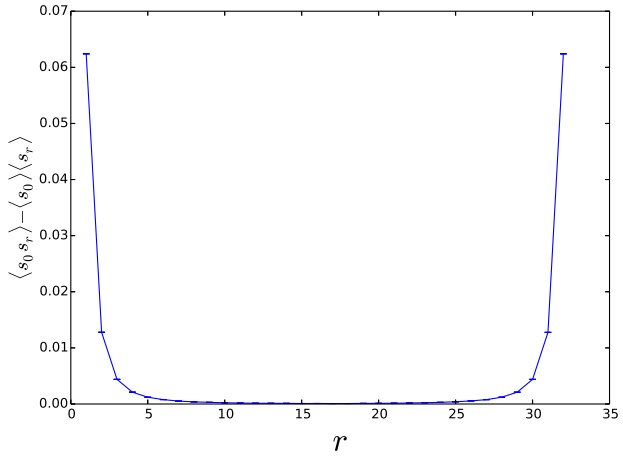


FIG. 1. lattice size: 64×64. Number of bins: 80,000. Each bin averaged over 50,000 configurations. Each configuration used was spaced by 4 random walks.

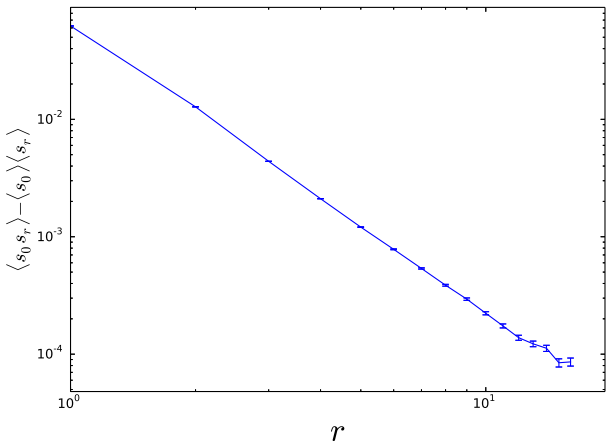


FIG. 2. lattice size: 64×64. Number of bins: 80,000. Each bin averaged over 50,000 configurations. Each configuration used was spaced by 4 random walks.

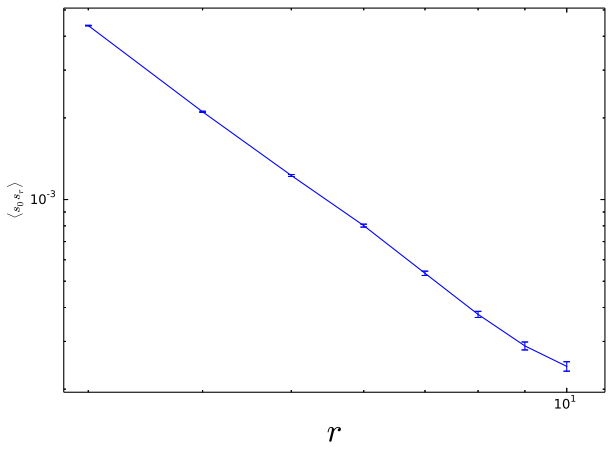


FIG. 3. The points chosen from fig 2 that are used to calculate the slope.

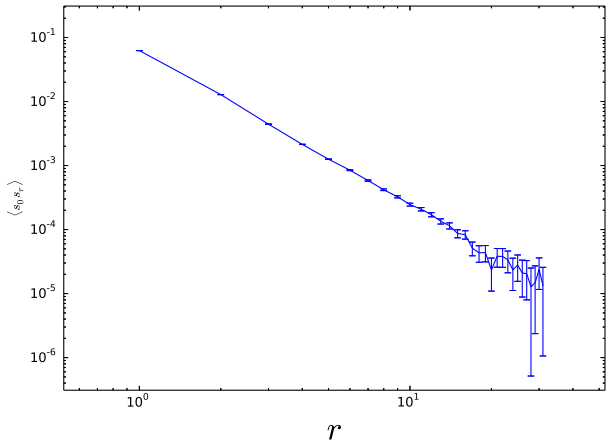


FIG. 4. The log log plot of the points used to calculate the sope of the dimer dimer correlation function on a 128×128 lattice. Number of bins: 9,000. Each bin is averaged over 500,000 measurments

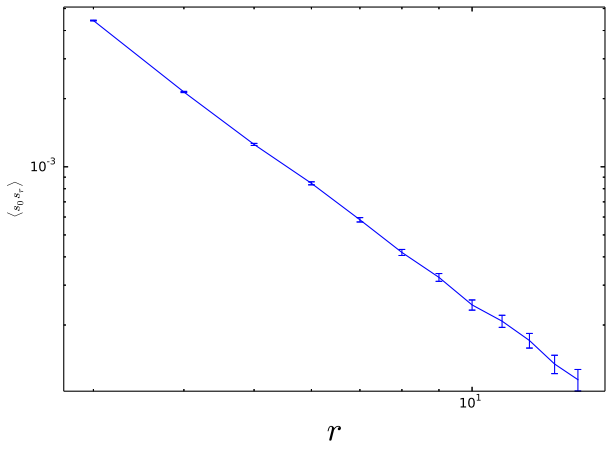


FIG. 5. The points used in the fit for the 128×128 lattice.

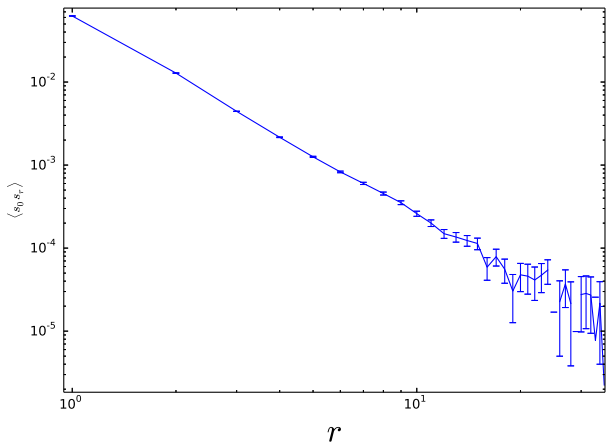


FIG. 6. The log log plot of the points used to calculate the sope of the dimer dimer correlation function on a 256×256 lattice. Number of bins: 9,000. Each bin is averaged over 500,000 measurments

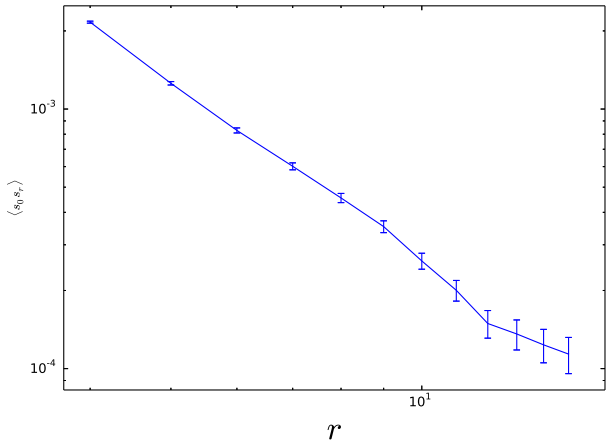


FIG. 7. The points used in the fit for the 256×256 lattice.

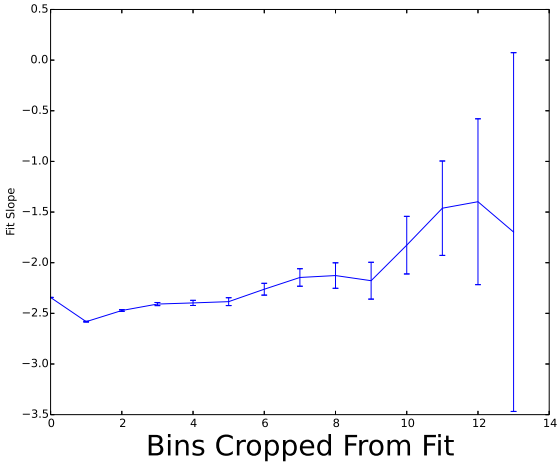


FIG. 8. Lattice size 64×64 x-axis: The number of histogram bins omitted from the fit. Omission begins with the bins corresponding to the smallest values of r and works its way up to a cut off point beyond which errors of the bin values are too large. y-axis: The slope calculated from a weighted fit with error bars found in the usual way from the fit

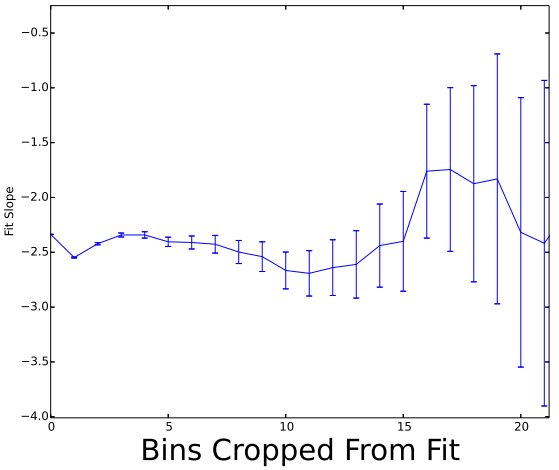


FIG. 9. Same as fig. 8 but for lattice size 128×128.

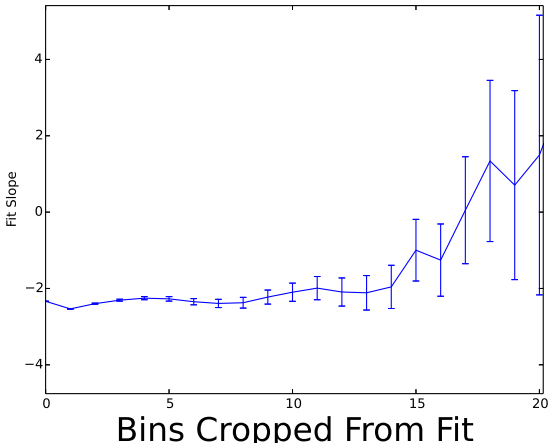


FIG. 10. Same as fig. 8 but for lattice size 256×256.

III. STAR DIMER

Very preliminary results of the star star correlation function are shown in fig. **??**.

$$C_{star}(0,r) = \langle star_0 star_r \rangle - \langle star_0 \rangle \langle star_r \rangle \tag{10}$$

The disconnected piece has been numerically measured for a couple of different lattice sizes shown in table I.

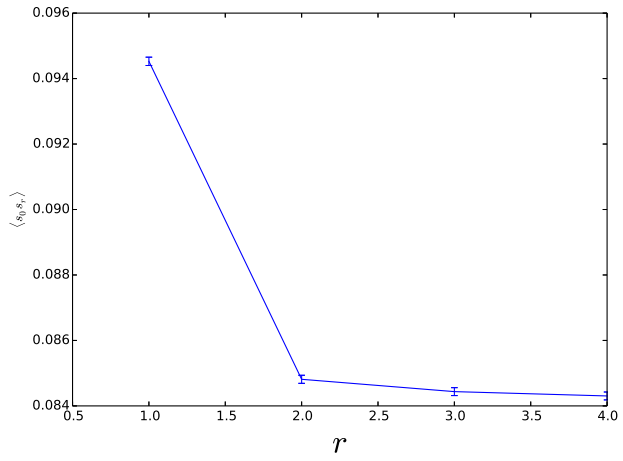


FIG. 11. Dimer dimer correlation function in the star dimer model. Lattice size is 32×32.

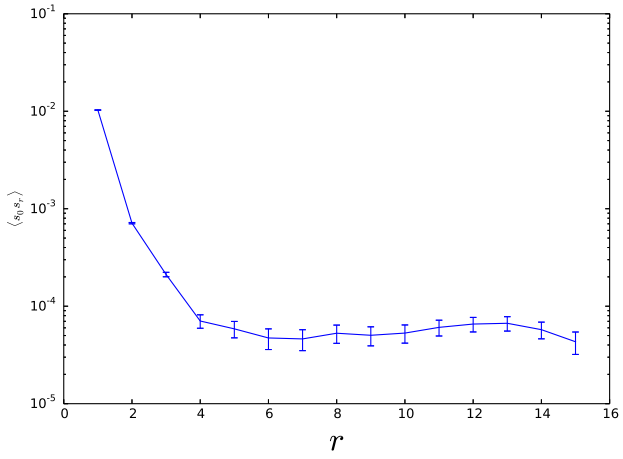


FIG. 12. Same data as in fig. 11 but a log plot (y-axis only).

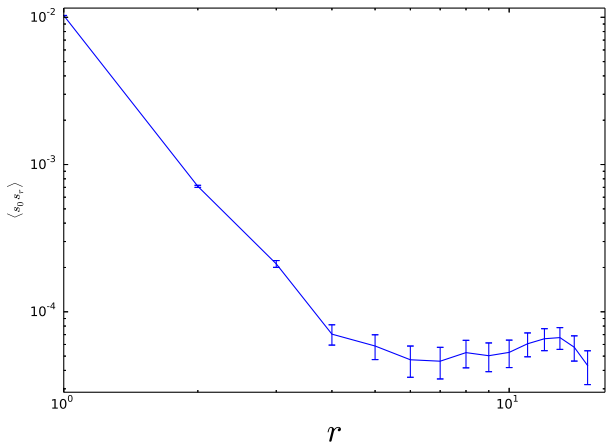


FIG. 13. Same data as in fig. 11 but a log log plot.

A. Star Pair Creation and Annihilation

Creation and annihilation of a horizontal pair of stars is shown in fig. 14. Creation and annihilation of a vertical pair of stars can be is simply what is shown in fig. 14 rotated by $\pi/2$.

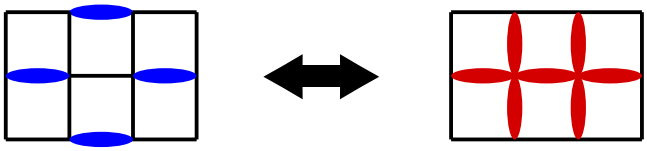


FIG. 14. The creation and annihilation of a pair of stars.

B. Star Horizontal/Vertical Translation

Horizontal translations of a star is shown in fig. 15. The vertical translations can be obtained by rotating fig. 15 by $\pi/2$. Horizontal translations must be preformed such that the star remains on its original sub-lattice, thus the minimal translation is by two vertices.

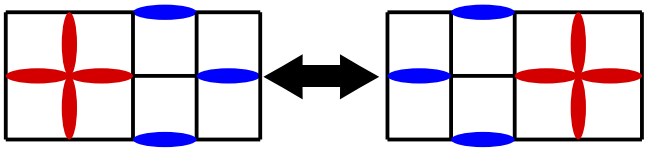


FIG. 15. Horizontal translation of a star.

C. Star Diagonal Translation

A star can remain on its original sub-lattice through a diagonal translation as well. We show a star moving to the lower right diagonal in fig. 16.

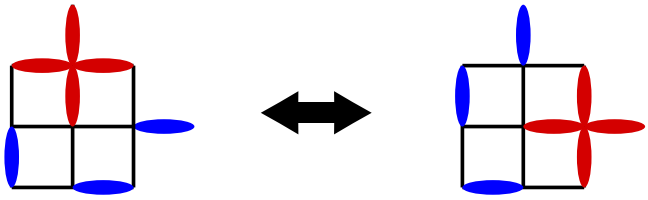


FIG. 16. Diagonal translation to the lower right diagonal.

D. Autocorrelation Time

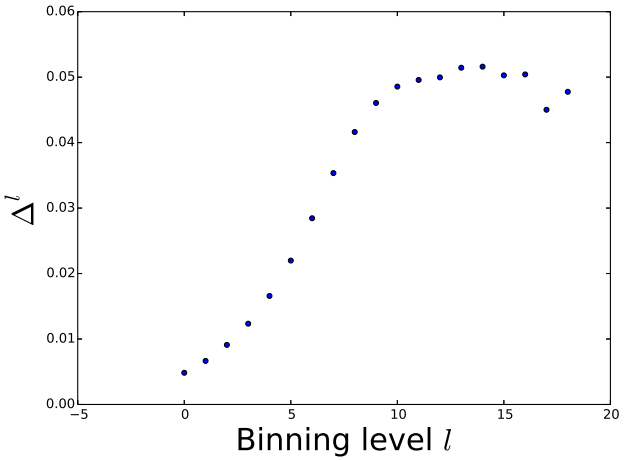


FIG. 17. Autocorrelation time for the number of stars on a 32×32 lattice. Binning was preformed on 4×10^6 measurements each spaced t_{mc}.

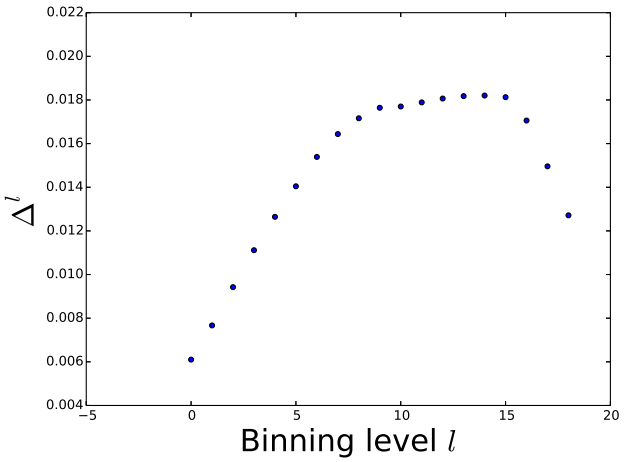


FIG. 18. Autocorrelation time for the number of horizontal dimers on a 32×32 lattice. Binning was preformed on 4×10^6 measurements each spaced t_{mc}.

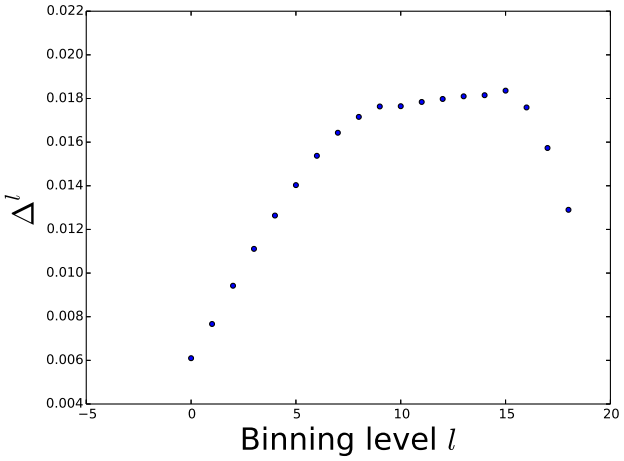


FIG. 19. Autocorrelation time for the number of vertical dimers on a 32×32 lattice. Binning was preformed on 4×10^6 measurements each spaced t_{mc}.

E. Equilibration Time

We show three order parameters as a function of Monte Carlo time, t_{mc}:

- The number of stars.
- The difference between the number of vertical dimers and horizontal dimers.
- The number of flippable plaquetts.

in fig. 20. We define t_{mc} to be $2lh$ local updates, where l is the length of the system and h is the height of the system in number of vertices. This lattice size used was 64×64. Each data point shown is the average over 100 different simulations. The error bars are the standard deviations of the counts used in taking the mean. It appears as if the equilibration time is around $400t_{mc}$.

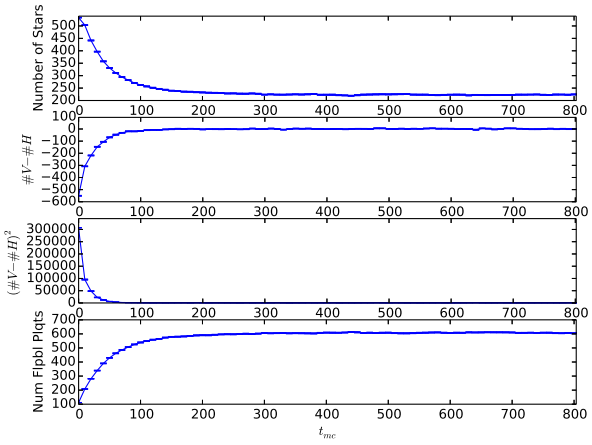


FIG. 20. From the top down: The number of stars on the lattice, the difference between the number of vertical dimers and horizontal dimers, and the number of flippable plaquetts as a function of Monte Carlo time t_{mc}.

F. Star Concentrations

The following statistics are given for different lattice sizes but each set of data is found using 100,000 bins, each an average over 10 measurements, each measurement taken after $1 \times t_{mc}$. We have not included the first 400 t_{mc} so that the averages are found in an equilibrated system.

TABLE I. Some Counting

lattice size	16×16	32×32	64×64
$\langle \# stars \rangle$		55.66655	222.51807
$\sigma_{\# stars}$		9.67847	17.23504
$\langle star_{i,j} \rangle$		0.05436	0.05432
$\langle star_{i,j} \rangle \langle star_{i,j} \rangle$		0.00296	0.00295
$\langle \# - \rangle$		256.02732	
$\sigma_{\# -}$		12.20027	
$\langle -_{i,j} \rangle$		0.25003	
$\langle star_{i,j} \rangle \langle star_{i,j} \rangle$		0.06251	
$\langle \# \| \rangle$		255.97332	
$\sigma_{\# \|}$		12.19720	
$\langle \|_{i,j} \rangle$		0.24997	
$\langle \|_{i,j} \rangle \langle \|_{i,j} \rangle$		0.06249	

G. Translational Symmetry Breaking

On an 8×8 lattice we find that there is always some residual asymmetry in the horizontal dimers (speculation:) due to the initial configuration of dimers. This appears to be an effect of small system size as the 16×16 lattice looks to be transnationally invariant.

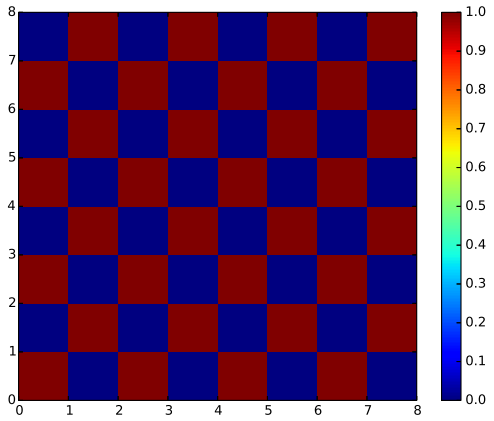


FIG. 21. Heat map of horizontal dimer placement in the initial staggered configuration of the lattice. Lattice size: 8×8

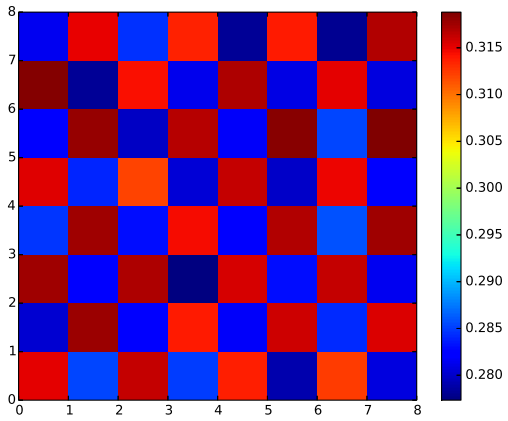


FIG. 22. Heat map of horizontal dimer placement on an 8×8 lattice. Each representing the occupation (red) and absence (blue) of a horizontal dimer is averaged over 50,000 configurations. Each configuration included in the average is separated by $100t_{mc}$. Time passed before collecting data was $5 \times 10^6 t_{mc}$.

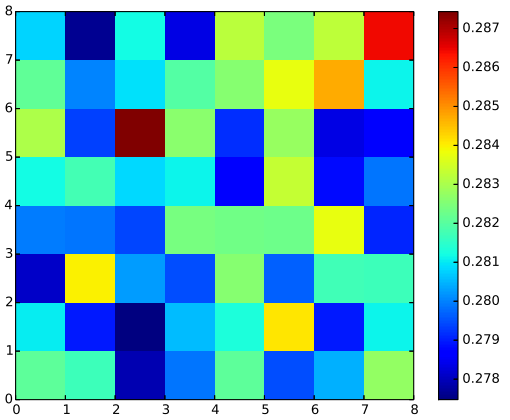


FIG. 23. Heat map of average vertical dimer occupations on an 8×8 lattice. Dimer occupations averaged over 50,000 configurations. Each configuration included in the average is separated by $100t_{mc}$. Time passed before collecting data was $5 \times 10^6 t_{mc}$.

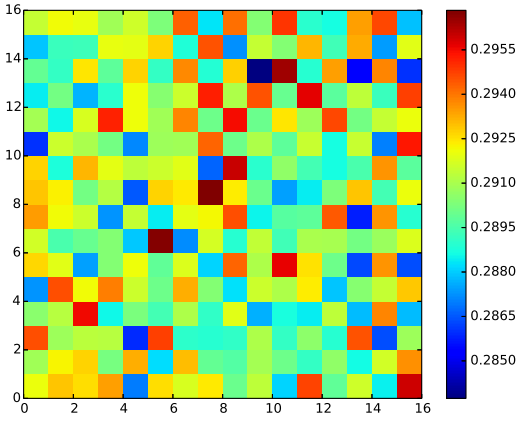


FIG. 24. Heat map of horizontal dimer occupations on an 16×16 lattice. Dimer occupations averaged over 50,000 configurations. Each configuration included in the average is separated by $100t_{mc}$. Time passed before collecting data was $5 \times 10^6 t_{mc}$.

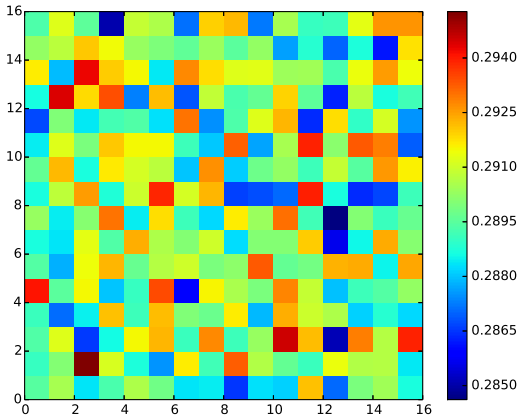


FIG. 25. Heat map of vertical dimer occupations on an 16×16 lattice. Dimer occupations averaged over 50,000 configurations. Each configuration included in the average is separated by $100t_{mc}$. $5 \times 10^6 t_{mc}$ passed before collecting data.

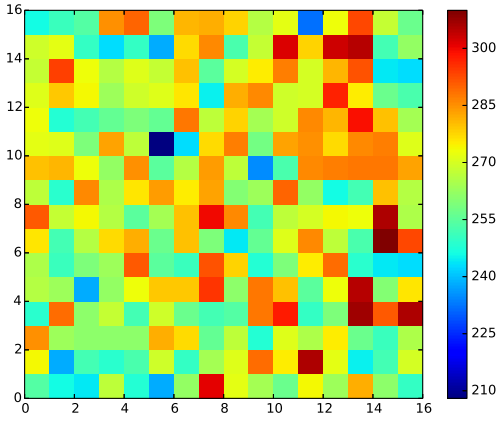


FIG. 26. Heat map of average star occupation on an 16×16 lattice. Dimer occupations averaged over 50,000 configurations. Each configuration included in the average is separated by $100t_{mc}$. $5 \times 10^6 t_{mc}$ passed before collecting data.

H. Old Moves and Rules

Creating a pair of stars in the staggered state is shown in fig. 27. We notice the creation of a star produces two flippable plaquettes between the two stars.

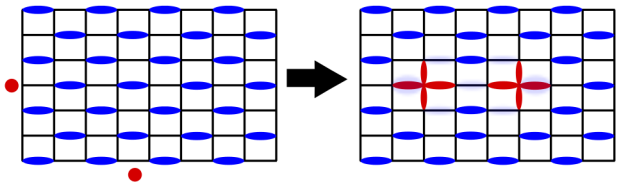


FIG. 27. The creation of a pair of stars in the fully packed staggered configuration. The red dots on the x and y axes show the dimer about which the stars are created.

In fig. 28 we show the rightmost star move right in the same configuration. In any single star must remain on the sublattice it was created on, so the center of the star moves by two vertices. We notice that the horizontal propagation of a star creates two boundaries of flippable plaquettes across the top and bottom of the star pair.

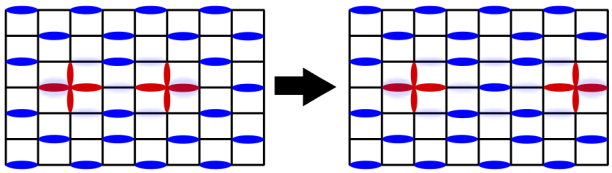


FIG. 28. The move of the rightmost star in fig. 27

Still figuring out the possible ways to move stars vertically. In six steps we show it is possible to implement a specific type of local vertical move.

- Step 1 (fig. 29): create a pair of stars.
- Step 2 (fig. 30): Move rightmost star right four times creating a horizontal set of flippable plaquettes.
- Step 3 (fig. 31): Flip some of the plaquettes.
- Step 4 (fig. 32): With several vertical pairs of dimers we can flip on plaquette to make a horizontal pair that was not part of the original set of horizontal dimers after the star move. The x and y positions of this plaquette are marked in fig. 32 by red dots along the x and y axes. If all of the dimers were left as vertical or horizontal the move would not be possible.
- Step 5 (fig. 33): Create another pair of stars.
- Step 6 (fig. 34): Move the rightmost star of the new pair up. When this is done the dimers must be rearranged to satisfy the packing conditions and conserve the number of dimers. Right now this is the only configuration I see for this move that can satisfy these conditions.

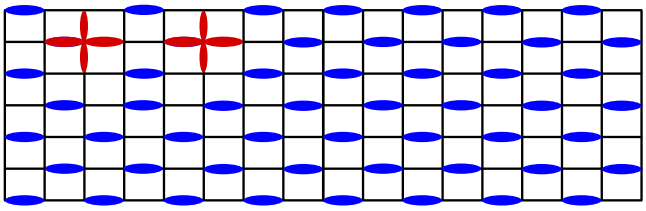


FIG. 29. Step 1 in the vertical move example. Create a pair of stars.

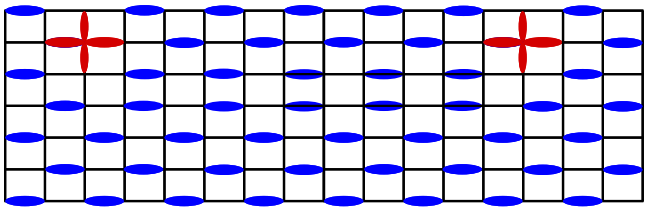


FIG. 30. Step 2 in the vertical move example. Move the rightmost star right four times creating a horizontal set of flippable plaquettes.

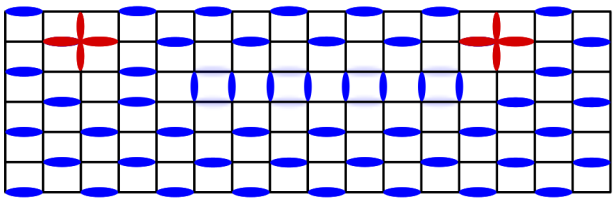


FIG. 31. Step 3 in the vertical move example. Flip some of the plaquettes.

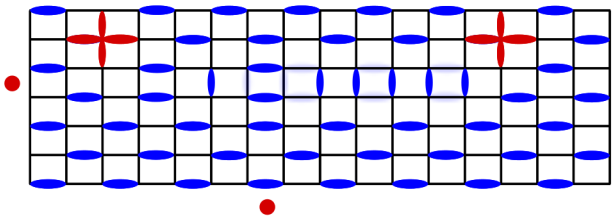


FIG. 32. Step 4 in the vertical move example. Flip the plaquette at the x and y coordinates marked by the red dots.

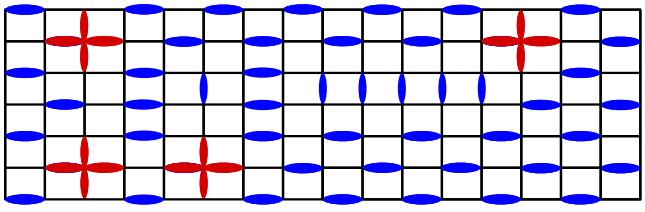


FIG. 33. Step 5 in the vertical move example. Create another pair of stars.

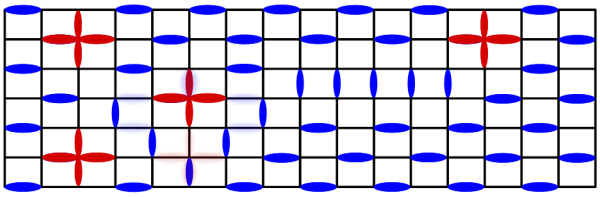


FIG. 34. Step 6 in the vertical move example. Move the rightmost star of the new pair up. Rearrange the dimers around it to satisfy packing and conservation rules.