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OLA 2: Proofs, Induction

Exercise-1 P is an odd integer $\longleftrightarrow$ $3n+5$ is even

Counter-example :- $n = (1/3)$

$$3n+5 = 3(1/3)+5$$
$$= 6$$

$1/3$ is not an odd integer. Therefore we have counter argument.

Exercise-2 Exhaustive Proof

$2 \leq n \leq 4$, $n^2 \geq 2^n$

If $n = 2$, $n^2 = (2)^2 = 4$
$2^n = 2^2 = 4$
$\therefore$ for $n = 2$, $n^2 = 2^n$

If $n = 3$, $n^2 = (3)^2 = 9$
and, $2^n = 2^3 = 8$

as $9 > 8$, so $n^2 > 2^n$

n	n^2	2^n	$n^2 \geq 2^n$
2	4	4	yes
3	9	8	yes
4	16	16	yes

If $n = 4$, $n^2 = (4)^2 = 16$
and $2^n = 2^4 = 16$

$\therefore$ for $n = 4$, $n^2 = 2^n$

$\therefore$ $n^2 > 2^n$ and $n^2 = 2^n$

$\therefore$ $\boxed{n^2 \geq 2^n}$

Therefore we can say that for $2 \leq n \leq 4$, $n^2 \geq 2^n$.

Excercise - 3

Given :-

The Sum of even integer and odd integer is odd.
Let x be an even integer and y be odd integer

$x = 2n \qquad y = (2m+1)$

1. $x = 2n$ (hyp)
2. $y = 2m+1$ (hyp)
3. $x+y = 2n + 2m+1$ (sub)
4. $x+y = 2(n+m)+1$ (algebra)

$\therefore$ $x+y$ is an odd integer

Excercise - 4

If $x^2 + 2x - 3 = 0$, then $x \neq 2$

1. $x^2 + 2x - 3 = 0$ (hyp)
2. $x = 2$ (hyp)
3. $2^2 + 2(2) - 3 = 0$ (sub)
4. $4 + 4 - 3 = 0$ (contradiction)

$\therefore$ $x^2 + 2x - 3 \neq 0$, when $x = 2$

Exercise 5 $2+4+6+\ldots+2n = n(n+1)$

1: $LHS = P(1) = 2$
$RHS = 1(1+1) = 2$
Verification: LHS = RHS for $P(1)$

2 inductive Hypothesis $P(K)$
We assume: $2+4+6+\ldots+2k = K(K+1)$

3 Equation for $P(K+1)$
$P(K+1): 2+4+6+\ldots+2(K+1) = (K+1)((K+1+1))$

4 Prove $P(K+1)$ is true given 2
b - (Hypothesis) $= 2+4+6+\ldots+2k = k(k+1)$
$p(k+1) = 2+4+6\ldots+2(k+1) = (k+1)(k+2)$

Add $2(k+1)$ to both sides:
$(2+4+6+\ldots+2k) + 2(K+1) = K(k+1)+2(k+1)$

$2(1+2+3+\ldots+k)$

$= 2\left(\frac{k(k+1)}{2}\right) : K(k+1)+2(k+1) = (k+1)(k+2)$

$= (K+1)(K+2) = RHS$

Hence, LHS = RHS for $P(K+1)$ given $P(k)$

$2+4+6+\ldots+2(K+1) = (K+1)(K+2)$

Exercise 6: (1) $2+6+18+\dots+2\times 3^{n-1}=3^n-1$

P(1): Left side: 2
Right side: $3^1-1=2$, thus LHS = RHS for P(1)

(1) - Inductive Hypothesis: $2+6+18+\dots+2\times 3^{k-1}=3^k-1$

- Induction step P(k+1):

(2) $2+6+18+\dots+2\times 3^{(k+1)-1}=3^{(k+1)}-1$ (1, algebra)

if

$2+6+18+\dots+2\times 3^{k}=3\times 3^k-1$ (2, algebra)

then: $= 3^k-1+2\times 3^k$

$= 3\times 3^k-1$

$= 3^1\times 3^k-1$

$= \boxed{3^{k+1}-1}$

$\therefore 2+6+18+\dots+2\times 3^{k-1}=3^k-1$ (3, algebra)

Part (2) (2) $1\times 2^1+2\times 2^2+3\times 2^3+\dots+n\times 2^n=(n-1)\times 2^{n+1}+2$

P(1): L.H.S $=1\times 2=2$ R.H.S $=(1-1)\times 2^{1+1}+2=0+2=2$

L.H.S = R.H.S

Inductive Hypothesis

$P(k) = 1\times 2^1+2\times 2^2+3\times 2^3+\dots+k\times 2^k=(k-1)\times 2^{k+1}+2$

Induction step

$P(k+1)=1\times 2^1+2\times 2^2+3\times 2^3+\dots+(k+1)\times 2^{k+1}=((k+1)-1)\times 2^{(k+1)+1}+2$

$= 1\times 2^1+2\times 2^2+3\times 2^3+\dots+k\times 2^k+(k+1)\times 2^{k+1}=k\times 2^{k+2}+2$

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Adding $(k+1)(2^{k+1})$

$$1\times 2^1+2\times 2^2+3\times 2^3+\ldots+k\times 2^k+(k+1)\times 2^{k+1}=(k-1)*2^{k+1}+2+(k+1)(2^{k+1})$$

$$= (k-1)\times 2^{k+1}+2+(k+1)(2^{k+1})$$

$$= k*2^{k+1}-2^{k+1}+2+k*2^{k+1}+2^{k+1}$$

$$= k*2^{k+1}+2+k*2^{k+1}$$

$$= 2*k2^{k+1}+2$$

$$= k(2^{k+1}*2^1)+2$$

$$= \boxed{k*2^{k+2}+2}$$

$$\therefore\ 1\times 2^1+2\times 2^2+3\times 2^3+\ldots+k*2^k+(k+1)(2^{k+1})=k*2^{k+2}+2$$