Problem 5

In [ ]:
```
import torch
from torch import tensor

p = tensor(18/37)
q = tensor(0.55)
K = 600
N = 3000
```

We sample $K = 600$ games regardless of end conditions for approximation.
Let $X = (X_1, \cdots, X_600)$ be result of games. Since each trial has binomial pdf and X is joint random variable of independent samples, we have

$$\mathbb{E}(X) = \int_{\{0,1\}^{600}} \sum_i X_i f(X) dX, \qquad \text{where} \quad f(X) = f_p(X) = \prod_i p^{X_i}(1-p)^{1-X_i}.$$

As suggested in problem, we use importance sampling with $q = 0.55$. Thus we must multiply $f_p(X)/f_q(X)$ to the result.
The result is the following.

In [ ]:
```
sample = torch.bernoulli(q*torch.ones((N,K)))

gain = 2*sample.sum(dim=1) - 500
weight = ((p**sample * (1-p)**(1-sample)) / (q**sample * (1-q)**(1-sample))).prod(

print(((gain>=200)*weight).mean())
```

```
tensor(1.2997e-06)
```