

# Team Note of wa on test 287

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## 1 Data Structures For Range Query

### 1.1 Sparse Table

Usage: RMQ l r: min(lift[l][len], lift[r-(1<<len)+1][len])

Time Complexity:  $\mathcal{O}(N) - \mathcal{O}(1)$

```
int k = ceil(log2(n));
vector<vector<int>> lift(n, vector<int>(k));
for (int i=0; i<n; ++i)
    lift[i][0] = lcp[i];
for (int i=1; i<k; ++i) {
    for (int j=0; j<=n-(1<<i); ++j)
        lift[j][i] = min(lift[j][i-1], lift[j+(1<<(i-1))][i-1]);
}
vector<int> bits(n+1);
for (int i=2; i<=n; ++i) {
    bits[i] = bits[i-1];
    while (1 << bits[i] < i)
        bits[i]++;
    bits[i]--;
}
```

### 1.2 Merge Sort Tree

Time Complexity:  $\mathcal{O}(N \log N) - \mathcal{O}(\log^2 N)$

```
struct mst {
    int n;
    vector<vector<int>> tree;
    void init(vector<int> &arr) {
        n = 1 << (int)ceil(log2(arr.size()));
        tree.resize(n*2);
        for (int i=0; i<arr.size(); ++i)
            tree[n+i].push_back(arr[i]);
        for (int i=n-1; i>0; --i) {
            tree[i].resize(tree[i*2].size() + tree[i*2+1].size());
            merge(tree[i*2].begin(), tree[i*2].end(),
                tree[i*2+1].begin(), tree[i*2+1].end(), tree[i].begin());
        }
    }
    int sum(int l, int r, int k) {
        int ret = 0;
        for (l+=n, r+=n; l<=r; l/=2, r/=2) {
            if (l%2)
                ret += upper_bound(tree[l].begin(), tree[l].end(), k) -
                    tree[l].begin(), l++;
            if (r%2 == 0)
                ret += upper_bound(tree[r].begin(), tree[r].end(), k) -
                    tree[r].begin(), r--;
        }
        return ret;
    }
};
```

### 1.3 Persistence Segment Tree

Time Complexity:  $\mathcal{O}(\log^2 N)$

```
struct pst {
    struct node {
        int cnt = 0;
        array<int, 2> go{};
    };
    vector<node> tree;
    vector<int> roots;
    pst() {
        roots.push_back(1);
    }
};
```

```

tree.resize(1 << 18);
for (int i = 1; i < (1 << 17); ++i) {
    tree[i].go[0] = i * 2;
    tree[i].go[1] = i * 2 + 1;
}
}
int insert(int x, int prev) {
    int curr = tree.size();
    roots.push_back(curr);
    tree.emplace_back();
    for (int i = 16; i >= 0; --i) {
        const int next = (x >> i) & 1;
        tree[curr].go[next] = tree.size();
        tree.emplace_back();
        tree[curr].go[!next] = tree[prev].go[!next];
        tree[curr].cnt = tree[prev].cnt + 1;
        curr = tree[curr].go[next];
        prev = tree[prev].go[next];
    }
    tree[curr].cnt = tree[prev].cnt + 1;
    return roots.back();
}
int query(int u, int v, int lca, int lca_par, int k) {
    int ret = 0;
    for (int i = 16; i >= 0; --i) {
        const int cnt = tree[tree[u].go[0]].cnt +
            tree[tree[v].go[0]].cnt -
                tree[tree[lca].go[0]].cnt -
                tree[tree[lca_par].go[0]].cnt;

        if (cnt >= k) {
            u = tree[u].go[0];
            v = tree[v].go[0];
            lca = tree[lca].go[0];
            lca_par = tree[lca_par].go[0];
        } else {
            k -= cnt;
            u = tree[u].go[1];
            v = tree[v].go[1];
            lca = tree[lca].go[1];
            lca_par = tree[lca_par].go[1];
        }
    }
}

```

```

        ret += 1 << i;
    }
}
return ret;
}
};

```

## 1.4 Segment Tree Beats

**Usage:** Note the potential function

**Time Complexity:**  $\mathcal{O}(\log^2 N)$

```

struct seg {
    vector<node> tree;
    void push(int x, int s, int e) {
        tree[x].x += tree[x].l;
        tree[x].o += tree[x].l;
        tree[x].a += tree[x].l;
        if (s != e) {
            tree[x*2].l += tree[x].l;
            tree[x*2+1].l += tree[x].l;
        }
        tree[x].l = 0;
    }
    void init(int x, int s, int e, const vector<int> &a) {
        if (s == e)
            tree[x].x = tree[x].o = tree[x].a = a[s];
        else {
            const int m = (s+e) / 2;
            init(x*2, s, m, a);
            init(x*2+1, m+1, e, a);
            tree[x] = tree[x*2] + tree[x*2+1];
        }
    }
    void off(int x, int s, int e, int l, int r, int v) {
        push(x, s, e);
        if (e < l || r < s || (tree[x].o & v) == 0)
            return;
        if (l <= s && e <= r && !(v & (tree[x].a ^ tree[x].o))) {
            tree[x].l -= v & tree[x].o;
            push(x, s, e);
        }
    }
}

```

```

    } else {
        const int m = (s+e) / 2;
        off(x*2, s, m, l, r, v);
        off(x*2+1, m+1, e, l, r, v);
        tree[x] = tree[x*2] + tree[x*2+1];
    }
}

void on(int x, int s, int e, int l, int r, int v) {
    push(x, s, e);
    if (e < l || r < s || (tree[x].a & v) == v)
        return;
    if (l <= s && e <= r && !(v & (tree[x].a ^ tree[x].o))) {
        tree[x].l += v & ~tree[x].o;
        push(x, s, e);
    } else {
        const int m = (s+e) / 2;
        on(x*2, s, m, l, r, v);
        on(x*2+1, m+1, e, l, r, v);
        tree[x] = tree[x*2] + tree[x*2+1];
    }
}

int sum(int x, int s, int e, int l, int r) {
    push(x, s, e);
    if (e < l || r < s)
        return 0;
    if (l <= s && e <= r)
        return tree[x].x;
    const int m = (s+e) / 2;
    return max(sum(x*2, s, m, l, r), sum(x*2+1, m+1, e, l, r));
}
};

```

## 1.5 Fenwick RMQ

**Time Complexity:** Fast  $\mathcal{O}(\log N)$

```

struct fenwick {
    static constexpr pii INF = {1e9 + 7, -(1e9 + 7)};
    vector<pii> tree1, tree2;
    const vector<int> &arr;
    static pii op(pii l, pii r) {

```

```

        return {min(l.first, r.first), max(l.second, r.second)};
    }
    fenwick(const vector<int> &a) : arr(a) {
        const int n = a.size();
        tree1.resize(n + 1, INF);
        tree2.resize(n + 1, INF);
        for (int i = 0; i < n; ++i)
            update(i, a[i]);
    }
    void update(int x, int v) {
        for (int i = x + 1; i < tree1.size(); i += i & -i)
            tree1[i] = op(tree1[i], {v, v});
        for (int i = x + 1; i > 0; i -= i & -i)
            tree2[i] = op(tree2[i], {v, v});
    }
    pii query(int l, int r) {
        pii ret = INF;
        l++, r++;
        int i;
        for (i = r; i - (i & -i) >= l; i -= i & -i)
            ret = op(tree1[i], ret);
        for (i = l; i + (i & -i) <= r; i += i & -i)
            ret = op(tree2[i], ret);
        ret = op({arr[i - 1], arr[i - 1]}, ret);
        return ret;
    }
};

```

## 2 Graph & Flow

### 2.1 BipartiteMatching

**Usage:** Run dfs for all left nodes. The count of return value true equal to count of max possible matches.

**Time Complexity:**  $\mathcal{O}(VE)$

```

vector<int> from(n, -1);
vector<bool> visited(n, false);
bool dfs(vector<vector<int>>& adjList, vector<bool>& visited,
vector<int>& from, int curNode) {

```

```

    for (int nextNode : adjList[curNode]) {
        if (from[nextNode] == -1) {
            from[nextNode] = curNode;
            return true;
        }
    }
    for (int nextNode : adjList[curNode]) {
        if (visited[nextNode]) continue;
        visited[nextNode] = true;
        if (dfs(adjList, visited, from, from[nextNode])) {
            from[nextNode] = curNode;
            return true;
        }
    }
    return false;
}

```

## 2.2 Hopcroft-Karp & König's

**Usage:** Dinic's variant. Maximum Matching = Minimum Vertex Cover = S - Maximum Independence Set

**Time Complexity:**  $\mathcal{O}(VE)$

```

vector<vector<ll>> e(sz);
vector<int> l, r;
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m; ++j) {
        if (s[i][j] == '*')
            continue;
        const int x = idx[{i, j}];
        ((i + j) % 2 ? l : r).push_back(x);
        for (int k = 0; k < 2; ++k) {
            const int ni = "12"[k] - '1' + i;
            const int nj = "21"[k] - '1' + j;
            if (ni >= n || nj >= m || s[ni][nj] == '*')
                continue;
            const int next = idx[{ni, nj}];
            e[x].push_back(next);
            e[next].push_back(x);
        }
    }
}

```

```

}
int flow = 0;
vector<int> match(sz, -1);
while (true) {
    vector<int> level(sz, -1);
    queue<int> q;
    for (int x : l) {
        if (match[x] == -1) {
            level[x] = 0;
            q.push(x);
        }
    }
    while (!q.empty()) {
        const int x = q.front();
        q.pop();

        for (int next : e[x]) {
            if (match[next] != -1 && level[match[next]] == -1) {
                level[match[next]] = level[x] + 1;
                q.push(match[next]);
            }
        }
    }
    if (level.empty() || *max_element(level.begin(), level.end()) == -1)
        break;
    function<bool(int)> dfs = [&](int x) {
        for (int next : e[x]) {
            if (match[next] == -1 ||
                (level[match[next]] == level[x] + 1 && dfs(match[next])))
            {
                match[next] = x;
                match[x] = next;
                return true;
            }
        }
        return false;
    };
    int total = 0;
    for (int x : l) {

```

```

    if (level[x] == 0)
        total += dfs(x);
}
if (total == 0)
    break;
flow += total;
}
set<int> alt;
function<void(int, bool)> dfs = [&](int x, bool left) {
    if (alt.contains(x))
        return;
    alt.insert(x);
    for (int next : e[x]) {
        if ((next != match[x]) && left)
            dfs(next, false);
        if ((next == match[x]) && !left)
            dfs(next, true);
    }
};
for (int x : l) {
    if (match[x] == -1)
        dfs(x, true);
}
int test = 0;
for (int i : l) {
    if (alt.contains(i)) {
        auto &[y, x] = pos[i];
        s[y][x] = 'C';
        test++;
    }
}
for (int i : r) {
    if (!alt.contains(i)) {
        auto &[y, x] = pos[i];
        s[y][x] = 'C';
        test++;
    }
}
}

```

## 2.3 Max Flow

Time Complexity:  $\mathcal{O}(VE^2)$

```

while (true) {
    vector<int> prev(n, -1);
    queue<int> q;
    q.push(start);
    while (!q.empty() && prev[end] == -1) {
        const int x = q.front();
        q.pop();
        for (int next : e[x]) {
            if (cap[x][next] - flow[x][next] > 0 && prev[next] ==
                -1) {
                prev[next] = x;
                q.push(next);
            }
        }
    }
    if (prev[end] == -1)
        break;
    int bot = 1e9+7;
    for (int i=end; i!=start; i=prev[i])
        bot = min(bot, cap[prev[i]][i] - flow[prev[i]][i]);
    for (int i=end; i!=start; i=prev[i]) {
        flow[prev[i]][i] += bot;
        flow[i][prev[i]] -= bot;
    }
}

```

## 2.4 Min Cost Max Flow

Time Complexity:  $\mathcal{O}(VEf)$

```

void mcmf(){
    int cp = 0;
    while(cp<2){
        int prev[MN],dist[MN],inq[MN]={0};
        queue <int> Q;
        fill(prev, prev+MN, -1);
        fill(dist, dist+MN, INF);
    }
}

```

```

dist[S] = 0; inq[S] = 1;
Q.push(S);
while(!Q.empty()){
    int cur= Q.front();
    Q.pop();
    inq[cur] = 0;
    for(int nxt: adj[cur]){
        if(cap[cur][nxt] - flow[cur][nxt] > 0 &&
            dist[nxt] > dist[cur]+cst[cur][nxt]){
            dist[nxt] = dist[cur] + cst[cur][nxt];
            prev[nxt] = cur;
            if(!inq[nxt]){
                Q.push(nxt);
                inq[nxt] = 1;
            }
        }
    }
}
if(prev[E]==-1) break;
int tmp = INF;
for(int i=E;i!=S;i=prev[i])
    tmp = min(tmp, cap[prev[i]][i]-flow[prev[i]][i]);

for(int i=E;i!=S;i=prev[i]){
    ans += tmp * cst[prev[i]][i];
    flow[prev[i]][i] += tmp;
    flow[i][prev[i]] -= tmp;
}
cp++;
}
}

```

## 2.5 Dinic's

Time Complexity:  $\mathcal{O}(V^2E)$

```

while (true) {
    vector<int> level(n * 2 + 2, -1);
    queue<int> q;
    level[st] = 0;
    q.push(st);

```

```

while (!q.empty()) {
    const int x = q.front();
    q.pop();
    for (int next : e[x]) {
        if (level[next] == -1 && cap[x][next] - flow[x][next] > 0) {
            level[next] = level[x] + 1;
            q.push(next);
        }
    }
}
if (level[dt] == -1)
    break;
vector<int> vis(n * 2 + 1);
function<int(int, int)> dfs = [&](int x, int total) {
    if (x == dt)
        return total;
    for (int &i = vis[x]; i < e[x].size(); ++i) {
        const int next = e[x][i];
        if (level[next] == level[x] + 1 && cap[x][next] -
            flow[x][next] > 0) {
            const int res = dfs(next, min(total, cap[x][next] -
                flow[x][next]));
            if (res > 0) {
                flow[x][next] += res;
                flow[next][x] -= res;
                return res;
            }
        }
    }
    return 0;
};
while (true) {
    const int res = dfs(st, 1e9 + 7);
    if (res == 0)
        break;
    ans += res;
}
}

```

## 2.6 Strongly Connected Component

Time Complexity:  $\mathcal{O}(N)$

```
int idx = 0, scnt = 0;
vector<int> scc(n, -1), vis(n, -1), st;
function<int (int)> dfs = [&] (int x) {
    int ret = vis[x] = idx++;
    st.push_back(x);
    for (int next : e[x]) {
        if (vis[next] == -1)
            ret = min(ret, dfs(next));
        else if (scc[next] == -1)
            ret = min(ret, vis[next]);
    }
    if (ret == vis[x]) {
        while (!st.empty()) {
            const int t = st.back();
            st.pop_back();
            scc[t] = scnt;
            if (t == x)
                break;
        }
        scnt++;
    }
    return ret;
};
```

## 2.7 Biconnected Component

Time Complexity:  $\mathcal{O}(N)$

```
int idx = 0;
vector<int> vis(n, -1);
vector<pii> st;
vector<vector<pii>> bcc;
vector<bool> cut(n); // articulation point
function<int (int, int)> dfs = [&] (int x, int p) {
    int ret = vis[x] = idx++;
    int child = 0;
    for (int next : e[x]) {
```

```
        if (next == p)
            continue;
        if (vis[next] < vis[x])
            st.emplace_back(x, next);
        if (vis[next] != -1)
            ret = min(ret, vis[next]);
        else {
            int res = dfs(next, x);
            ret = min(ret, res);
            child++;
            if (vis[x] <= res) {
                if (p != -1)
                    cut[x] = true;
                bcc.emplace_back();
                while (st.back() != pii{x, next}) {
                    bcc.back().push_back(st.back());
                    st.pop_back();
                }
                bcc.back().push_back(st.back());
                st.pop_back();
            } // vis[x] < res to find bridges
        }
    }
    if (p == -1 && child > 1)
        cut[x] = true;
    return ret;
};
```

## 2.8 Lowest Common Ancestor

Usage: Query with the sparse table

Time Complexity:  $\mathcal{O}(N \log N) - \mathcal{O}(\log N)$

```
for (int i=1; i<16; ++i) {
    for (int j=0; j<n; ++j)
        par[j][i] = par[par[j][i-1]][i-1];
}
```

## 2.9 Heavy-Light Decomposition

Usage: Query with the ETT number and it's root node



**Time Complexity:**  $\mathcal{O}(N) - \mathcal{O}(\log N)$

```
vector<int> par(n), ett(n), root(n), depth(n), sz(n);
function<void (int)> dfs1 = [&] (int x) {
    sz[x] = 1;
    for (int &next : e[x]) {
        if (next == par[x])
            continue;
        depth[next] = depth[x]+1;
        par[next] = x;
        dfs1(next);
        sz[x] += sz[next];
        if (e[x][0] == par[x] || sz[e[x][0]] < sz[next])
            swap(e[x][0], next);
    }
};
int idx = 1;
function<void (int)> dfs2 = [&] (int x) {
    ett[x] = idx++;
    for (int next : e[x]) {
        if (next == par[x])
            continue;
        root[next] = next == e[x][0] ? root[x] : next;
        dfs2(next);
    }
};
```

## 2.10 Centroid Decomposition

**Usage:** cent[x] is the parent in centroid tree

**Time Complexity:**  $\mathcal{O}(N \log N)$

```
vector<int> sz(n);
vector<bool> fin(n);
function<int (int, int)> get_size = [&] (int x, int p) {
    sz[x] = 1;
    for (int next : e[x]) {
        if (!fin[next] && next != p)
            sz[x] += get_size(next, x);
    }
    return sz[x];
};
```

```
};
function<int (int, int, int)> get_cent = [&] (int x, int p, int all)
{
    for (int next : e[x]) {
        if (!fin[next] && next != p && sz[next]*2 > all)
            return get_cent(next, x, all);
    }
    return x;
};
vector<int> cent(n, -1);
function<void (int, int)> get_cent_tree = [&] (int x, int p) {
    get_size(x, p);
    x = get_cent(x, p, sz[x]);
    fin[x] = true;
    cent[x] = p;
    function<void (int, int, int, bool)> dfs = [&] (int x, int p,
    int d, bool test) {
        if (test) // update answer
        else // update state
        for (int next : e[x]) {
            if (!fin[next] && next != p)
                dfs(next, x, d, test);
        }
    };
    for (int next : e[x]) {
        if (!fin[next]) {
            dfs(next, x, init, true);
            dfs(next, x, init+curr, false);
        }
    }
    for (int next : e[x]) {
        if (!fin[next] && next != p)
            get_cent_tree(next, x);
    }
};
get_cent_tree(0, -1);
```

### 3 Geometry

#### 3.1 Counter Clockwise

**Usage:** It returns  $\{-1, 0, 1\}$  - the ccw of  $b - a$  and  $c - b$

**Time Complexity:**  $\mathcal{O}(1)$

```
auto ccw = [] (const pii &a, const pii &b, const pii &c) {
    pii x = { b.first - a.first, b.second - a.second };
    pii y = { c.first - b.first, c.second - b.second };
    ll ret = 1LL * x.first * y.second - 1LL * x.second * y.first;
    return ret == 0 ? 0 : (ret > 0 ? 1 : -1);
};
```

#### 3.2 Line intersection

**Usage:** Check the intersection of  $(x_1, x_2)$  and  $(y_1, y_2)$ . It requires an additional condition when they are parallel

**Time Complexity:**  $\mathcal{O}(1)$

```
ccw(x1, x2, y1) != ccw(x1, x1, y2) && ccw(y1, y2, x1) != ccw(y1, y2, x2)
```

#### 3.3 Graham Scan

**Time Complexity:**  $\mathcal{O}(N \log N)$

```
struct point {
    int x, y, p, q;
    point() { x = y = p = q = 0; }
    bool operator < (const point& other) {
        if (1LL * other.p * q != 1LL * p * other.q)
            return 1LL * other.p * q < 1LL * p * other.q;
        else if (y != other.y)
            return y < other.y;
        else
            return x < other.x;
    }
};

swap(points[0], *min_element(points.begin(), points.end()));
for (int i=1; i<points.size(); ++i) {
    points[i].p = points[i].x - points[0].x;
```

```
    points[i].q = points[i].y - points[0].y;
}
sort(points.begin()+1, points.end());
vector<int> hull;
for (int i=0; i<points.size(); ++i) {
    while (hull.size() >= 2 && ccw(points[hull[hull.size()-2]],
        points[hull.back()], points[i]) < 1)
        hull.pop_back();
    hull.push_back(i);
}
```

#### 3.4 Monotone Chain

**Usage:** Get the upper and lower hull of the convex hull

**Time Complexity:**  $\mathcal{O}(N \log N)$

```
pair<vector<pii>, vector<pii>> getConvexHull(vector<pii> pt){
    sort(pt.begin(), pt.end());
    vector<pii> uh, dh;
    int un=0, dn=0; // for easy coding
    for (auto &tmp : pt) {
        while(un >= 2 && ccw(uh[un-2], uh[un-1], tmp))
            uh.pop_back(), --un;
        uh.push_back(tmp); ++un;
    }
    reverse(pt.begin(), pt.end());
    for (auto &tmp : pt) {
        while(dn >= 2 && ccw(dh[dn-2], dh[dn-1], tmp))
            dh.pop_back(), --dn;
        dh.push_back(tmp); ++dn;
    }
    return {uh, dh};
} // ref: https://namnamseo.tistory.com
```

#### 3.5 Rotating Calipers

**Usage:** Get the maximum distance of the convex hull

**Time Complexity:**  $\mathcal{O}(N)$

```
auto ccw4 = [&] (point& a1, point& a2, point& b1, point& b2) {
    return 1LL * (a2.x - a1.x) * (b2.y - b1.y) > 1LL * (a2.y - a1.y)
        * (b2.x - b1.x);
};
```

```

};
auto dist = [] (point& a, point& b) {
    return 1LL * (a.x - b.x) * (a.x - b.x) + 1LL * (a.y - b.y) *
        (a.y - b.y);
};
ll maxi = 0;
for (int i=0, j=1; i<hull.size(); ) {
    maxi = max(maxi, dist(hull[i], hull[j]));
    if (j < hull.size()-1 && ccw4(hull[i], hull[i+1], hull[j],
        hull[j+1]))
        j++;
    else
        i++;
}

```

## 4 Fast Fourier Transform

### 4.1 Fast Fourier Transform

Usage: FFT and multiply polynomials

Time Complexity:  $\mathcal{O}(N \log N)$

```

using cd = complex<double>;
void fft(vector<cd> &f, cd w) {
    int n = f.size();
    if (n == 1)
        return;
    vector<cd> odd(n/2), even(n/2);
    for (int i=0; i<n; ++i)
        (i%2 ? odd : even)[i/2] = f[i];
    fft(odd, w*w);
    fft(even, w*w);
    cd x(1, 0);
    for (int i=0; i<n/2; ++i) {
        f[i] = even[i] + x * odd[i];
        f[i+n/2] = even[i] - x * odd[i];
        x *= w; // get through power to better accuracy
    }
}
vector<cd> mult(vector<cd> a, vector<cd> b) {

```

```

int n;
for (n=1; n<a.size() || n<b.size(); n*=2);
n *= 2;
vector<cd> ret(n);
a.resize(n);
b.resize(n);
static constexpr double PI = 3.1415926535897932384;
cd w(cos(PI*2/n), sin(PI*2/n));
fft(a, w);
fft(b, w);
for (int i=0; i<n; ++i)
    ret[i] = a[i] * b[i];
fft(ret, cd(1, 0)/w);
for (int i=0; i<n; ++i) {
    ret[i] /= cd(n, 0);
    ret[i] = cd(round(ret[i].real()), round(ret[i].imag()));
}
return ret;
}

```

### 4.2 Number Theoretic Transform

Usage: FFT with integer - to get better accuracy

Time Complexity:  $\mathcal{O}(N \log N)$

```

// w is the root of mod e.g. 3/998244353 and 5/1012924417
void ntt(vector<ll> &f, const ll w, const ll mod) {
    const int n = f.size();
    if (n == 1)
        return;
    vector<ll> odd(n/2), even(n/2);
    for (int i=0; i<n; ++i)
        (i%2 ? odd : even)[i/2] = f[i];
    ntt(odd, w*w%mod, mod);
    ntt(even, w*w%mod, mod);
    ll x = 1;
    for (int i=0; i<n/2; ++i) {
        f[i] = (even[i] + x * odd[i] % mod) % mod;
        f[i+n/2] = (even[i] - x * odd[i] % mod + mod) % mod;
        x = x*w%mod;
    }
}

```

```

    }
}

```

### 4.3 Fast Walsh Hadamard Transform

**Usage:** XOR convolution

**Time Complexity:**  $\mathcal{O}(N \log N)$

```

void fwht(vector<ll> &f) {
    const int n = f.size();
    if (n == 1)
        return;
    vector<ll> odd(n/2), even(n/2);
    for (int i=0; i<n; ++i)
        (i&1 ? odd : even)[i/2] = f[i];
    fwht(odd);
    fwht(even);
    for (int i=0; i<n/2; ++i) {
        f[i*2] = even[i] + odd[i];
        f[i*2+1] = even[i] - odd[i];
    }
}

```

## 5 String

### 5.1 Knuth-Morris-Pratt

**Time Complexity:**  $\mathcal{O}(N)$

```

vector<int> fail(m);
for (int i=1, j=0; i<m; ++i) {
    while (j > 0 && p[i] != p[j])
        j = fail[j-1];
    if (p[i] == p[j])
        fail[i] = ++j;
}
vector<int> ans;
for (int i=0, j=0; i<n; ++i) {
    while (j > 0 && t[i] != p[j])
        j = fail[j-1];
}

```

```

    if (t[i] == p[j]) {
        if (j == m-1) {
            ans.push_back(i-j);
            j = fail[j];
        } else
            j++;
    }
}

```

### 5.2 Rabin-Karp

**Usage:** The Rabin fingerprint for const-length hashing

**Time Complexity:**  $\mathcal{O}(N)$

```

ull hash, p;
vector<ull> ht;
for (int i=0; i<=l-mid; ++i) {
    if (i == 0) {
        hash = s[0];
        p = 1;
        for (int j=1; j<mid; ++j) {
            hash = hash * pi + s[j];
            p = p * pi; // pi is the prime e.g. 13
        }
    } else
        hash = (hash - p * s[i-1]) * pi + s[i+mid-1];
    ht.push_back(hash);
}

```

### 5.3 Manacher

**Usage:** Longest radius of palindrome substring

**Time Complexity:**  $\mathcal{O}(N)$

```

vector<int> man(m);
int r = 0, p = 0;
for (int i=0; i<m; ++i) {
    if (i <= r)
        man[i] = min(man[p*2 - i], r - i);
    while (i-man[i] > 0 && i+man[i] < m-1 && v[i-man[i]-1] ==
        v[i+man[i]+1])

```

```

    man[i]++;
    if (r < i + man[i]) {
        r = i + man[i];
        p = i;
    }
}

```

## 5.4 Suffix Array and LCP Array

Time Complexity:  $\mathcal{O}(N \log N) - \mathcal{O}(N)$

```

const int m = max(255, n)+1;
vector<int> sa(n), ord(n*2), nrd(n*2);
for (int i=0; i<n; ++i) {
    sa[i] = i;
    ord[i] = s[i];
}
for (int d=1; d<n; d*=2) {
    auto cmp = [&] (int i, int j) {
        if (ord[i] == ord[j])
            return ord[i+d] < ord[j+d];
        return ord[i] < ord[j];
    };
    vector<int> cnt(m), tmp(n);
    for (int i=0; i<n; ++i)
        cnt[ord[i+d]]++;
    for (int i=0; i+1<m; ++i)
        cnt[i+1] += cnt[i];
    for (int i=n-1; i>=0; --i)
        tmp[--cnt[ord[i+d]]] = i;
    fill(cnt.begin(), cnt.end(), 0);
    for (int i=0; i<n; ++i)
        cnt[ord[i]]++;
    for (int i=0; i+1<m; ++i)
        cnt[i+1] += cnt[i];
    for (int i=n-1; i>=0; --i)
        sa[--cnt[ord[tmp[i]]]] = tmp[i];
    nrd[sa[0]] = 1;
    for (int i=1; i<n; ++i)
        nrd[sa[i]] = nrd[sa[i-1]] + cmp(sa[i-1], sa[i]);
    swap(ord, nrd);
}

```

```

}
vector<int> inv(n), lcp(n);
for (int i=0; i<n; ++i)
    inv[sa[i]] = i;
for (int i=0, k=0; i<n; ++i) {
    if (inv[i] == 0)
        continue;
    for (int j=sa[inv[i]-1]; s[i+k]==s[j+k]; ++k);
    lcp[inv[i]] = k ? k-- : 0;
}

```

## 5.5 Aho-Corasick

Time Complexity:  $\mathcal{O}(N + \sum M)$

```

struct trie {
    array<trie *, 3> go;
    trie *fail;
    int output, idx;
    trie() {
        fill(go.begin(), go.end(), nullptr);
        fail = nullptr;
        output = idx = 0;
    }
    ~trie() {
        for (auto &x : go)
            delete x;
    }
    void insert(const string &input, int i) {
        if (i == input.size())
            output++;
        else {
            const int x = input[i] - 'A';
            if (!go[x])
                go[x] = new trie();
            go[x]->insert(input, i+1);
        }
    }
};

queue<trie*> q; // make fail links; requires root->insert before
root->fail = root;

```

```

q.push(root);
while (!q.empty()) {
    trie *curr = q.front();
    q.pop();
    for (int i=0; i<26; ++i) {
        trie *next = curr->go[i];
        if (!next)
            continue;
        if (curr == root)
            next->fail = root;
        else {
            trie *dest = curr->fail;
            while (dest != root && !dest->go[i])
                dest = dest->fail;
            if (dest->go[i])
                dest = dest->go[i];
            next->fail = dest;
        }
        if (next->fail->output)
            next->output = true;
        q.push(next);
    }
}
trie *curr = root; // start query
bool found = false;
for (char c : s) {
    c -= 'a';
    while (curr != root && !curr->go[c])
        curr = curr->fail;
    if (curr->go[c])
        curr = curr->go[c];
    if (curr->output) {
        found = true;
        break;
    }
}
}

```

## 6 Offline Query

### 6.1 Mo's

Usage: sort by  $(L\sqrt{L}, R)$

Time Complexity:  $\mathcal{O}(Q \log Q + N\sqrt{N})$

```

sort(q.begin(), q.end(), [&] (const auto &a, const auto &b) {
    if (get<0>(a)/rt != get<0>(b)/rt)
        return get<0>(a)/rt < get<0>(b)/rt;
    return get<1>(a) < get<1>(b);
});
int res = 0, s = get<0>(q[0]), e = get<1>(q[0]);
vector<int> count(1e6), result(m);
for (int i=s; i<=e; ++i)
    res += count[a[i]]++ == 0;
result[get<2>(q[0])] = res;
for (int i=1; i<m; ++i) {
    while (get<0>(q[i]) < s)
        res += count[a[--s]]++ == 0;
    while (get<1>(q[i]) > e)
        res += count[a[++e]]++ == 0;
    while (get<0>(q[i]) > s)
        res -= --count[a[s++]] == 0;
    while (get<1>(q[i]) < e)
        res -= --count[a[e--]] == 0;
    result[get<2>(q[i])] = res;
}

```

### 6.2 Parallel Binary Search

Time Complexity:  $\mathcal{O}(N \log N)$

```

vector<int> lo(q, -1), hi(q, m), answer(q);
while (true) {
    int fin = 0;
    vector<vector<int>> mids(m);
    for (int i=0; i<q; ++i) {
        if (lo[i] + 1 < hi[i])
            mids[(lo[i] + hi[i])/2].push_back(i);
        else

```

```

        fin++;
    }
    if (fin == q)
        break;
    ufind uf;
    uf.init(n+1);
    for (int i=0; i<m; ++i) {
        const auto &[eig, a, b] = edges[i];
        uf.merge(a, b);
        for (int x : mids[i]) {
            if (uf.find(qs[x].first) == uf.find(qs[x].second)) {
                hi[x] = i;
                answer[x] = -uf.par[uf.find(qs[x].first)];
            } else
                lo[x] = i;
        }
    }
}

```

## 7 DP Optimization

### 7.1 Convex Hull Trick w/ Stack

Usage:  $dp[i] = \min(dp[j] + b[j] * a[i]), b[j] \geq b[j+1]$

Time Complexity:  $\mathcal{O}(N \log N) - \mathcal{O}(N)$  where  $a[i] \leq a[i+1]$

```

struct lin {
    ll a, b;
    double s;
    ll f(ll x) { return a*x + b; }
};

inline double cross(const lin &x, const lin &y) {
    return 1.0 * (x.b - y.b) / (y.a - x.a);
}

vector<ll> dp(n);
vector<lin> st;
for (int i=1; i<n; ++i) {
    lin curr = { b[i-1], dp[i-1], 0 };
    while (!st.empty()) {
        curr.s = cross(st.back(), curr);
    }
}

```

```

        if (st.back().s < curr.s)
            break;
        st.pop_back();
    }
    st.push_back(curr);
    int x = -1;
    for (int y = st.size(); y > 0; y /= 2) {
        while (x+y < st.size() && st[x+y].s < a[i])
            x += y;
    }
    dp[i] = s[x].f(a[i]);
}
while (x+1 < st.size() && st[x+1].s < a[i]) ++x; //  $\mathcal{O}(N)$  case

```

### 7.2 Convex Hull Trick w/ Li-Chao Tree

Usage:  $\text{update}(l, r, 0, \{a, b\})$

Time Complexity:  $\mathcal{O}(N \log N)$

```

static constexpr ll INF = 2e18;
struct lin {
    ll a, b;
    ll f(ll x) { return a*x + b; }
};

struct lichao {
    struct node {
        int l, r;
        lin line;
    };
    vector<node> tree;
    void init() { tree.push_back({-1, -1, { 0, -INF }}); }
    void update(ll s, ll e, int n, const lin &line) {
        lin hi = tree[n].line;
        lin lo = line;
        if (hi.f(s) < lo.f(s))
            swap(lo, hi);
        if (hi.f(e) >= lo.f(e)) {
            tree[n].line = hi;
            return;
        }
        const ll m = s + e >> 1;
    }
}

```

```

    if (hi.f(m) > lo.f(m)) {
        tree[n].line = hi;
        if (tree[n].r == -1) {
            tree[n].r = tree.size();
            tree.push_back({-1, -1, { 0, -INF }});
        }
        update(m+1, e, tree[n].r, lo);
    } else {
        tree[n].line = lo;
        if (tree[n].l == -1) {
            tree[n].l = tree.size();
            tree.push_back({-1, -1, { 0, -INF }});
        }
        update(s, m, tree[n].l, hi);
    }
}
ll query(ll s, ll e, int n, ll x) {
    if (n == -1)
        return -INF;
    const ll m = s + e >> 1;
    if (x <= m)
        return max(tree[n].line.f(x), query(s, m, tree[n].l, x));
    else
        return max(tree[n].line.f(x), query(m+1, e, tree[n].r, x));
}
};

```

### 7.3 Divide and Conquer Optimization

Usage:  $dp[t][i] = \min(dp[t-1][j] + c[j][i])$ ,  $c$  is Monge

Time Complexity:  $\mathcal{O}(KN \log N)$

```

vector<vector<ll>> dp(n, vector<ll>(t));
function<void (int, int, int, int, int)> dnc = [&] (int l, int r,
int s, int e, int u) {
    if (l > r)
        return;
    const int mid = (l + r) / 2;
    int opt;
    for (int i=s; i<=min(e, mid); ++i) {
        ll x = sum[i][mid] + C;
    }
}

```

```

        if (i && u)
            x += dp[i-1][u-1];
        if (x >= dp[mid][u]) {
            dp[mid][u] = x;
            opt = i;
        }
    }
    dnc(l, mid-1, s, opt, u);
    dnc(mid+1, r, opt, e, u);
};
for (int i=0; i<t; ++i)
    dnc(0, n-1, 0, n-1, i);

```

### 7.4 Monotone Queue Optimization

Usage:  $dp[i] = \min(dp[j] + c[j][i])$ ,  $c$  is Monge, find cross

Time Complexity:  $\mathcal{O}(N \log N)$

```

auto cross = [&] (ll p, ll q) {
    ll lo = min(p, q) - 1, hi = n + 1;
    while (lo + 1 < hi) {
        const ll mid = (lo + hi) / 2;
        if (f(p, mid) < f(q, mid)) lo = mid;
        else hi = mid;
    }
    return hi;
};
deque<pll> st;
for (int i = 1; i <= n; ++i) {
    pll curr{i - 1, 0};
    while (!st.empty() &&
        (curr.second = cross(st.back().first, i - 1)) <=
        st.back().second)
        st.pop_back();
    st.push_back(curr);
    while (st.size() > 1 && st[1].second <= i) st.pop_front();
    dp[i] = f(st[0].first, i);
}

```



## 7.5 Aliens Trick

**Usage:**  $dp[t][i] = \min(dp[t-1][j] + c[j+1][i])$ ,  $c$  is Monge, find lambda w/ half bs

**Time Complexity:**  $\mathcal{O}(N \log N)$

```
ll lo = 0, hi = 1e15;
while (lo + 1 < hi) {
    const ll mid = (lo + hi) / 2;
    auto [dp, cnt] = dec(mid); // the best DP[N][K] and its K value
    if (cnt < k) hi = mid;
    else lo = mid;
}
cout << (dec(lo).first - lo * k) / 2;
```

## 7.6 Knuth Optimization

**Usage:**  $dp[i] = \min(dp[i][k] + dp[k][j]) + c[i][j]$ , Monge, Monotonic

**Time Complexity:**  $\mathcal{O}(N^2)$

```
vector<vector<int>> dp(n, vector<int>(n)), opt(n, vector<int>(n));
for (int i=0; i<n; ++i)
    opt[i][i] = i;
for (int j=1; j<n; ++j) {
    for (int s=0; s<n-j; ++s) {
        int e = s+j;
        dp[s][e] = 1e9+7;
        for (int o=opt[s][e-1]; o<min(opt[s+1][e+1], e); ++o) {
            if (dp[s][e] > dp[s][o] + dp[o+1][e]) {
                dp[s][e] = dp[s][o] + dp[o+1][e];
                opt[s][e] = o;
            }
        }
        dp[s][e] += sum[e+1] - sum[s];
    }
}
```

## 7.7 Slope Trick

**Usage:** Use priority queue, convex condition

**Time Complexity:**  $\mathcal{O}(N \log N)$

```
pq.push(A[0]);
for (int i=1; i<N; ++i) {
    pq.push(A[i] - i);
    pq.push(A[i] - i);
    pq.pop();
    A[i] = pq.top();
}
```

## 7.8 Sum Over Subsets

**Usage:**  $dp[mask] = \sum(A[i])$ ,  $i$  is in mask

**Time Complexity:**  $\mathcal{O}(N2^N)$

```
for (int i=0; i<(1<<n); i++)
    f[i] = a[i];
for (int j=0; j<n; j++)
    for(int i=0; i<(1<<N); i++)
        if (i & (1<<j))
            f[i] += f[i ^ (1<<j)];
```

# 8 Number Theory

## 8.1 Modular Operator

**Usage:** For Fermat's little theorem and Pollard rho

**Time Complexity:**  $\mathcal{O}(\log N)$

```
using ull = unsigned long long;
ull modmul(ull a, ull b, ull n) { return ((unsigned __int128)a * b) % n; }
ull modmul(ull a, ull b, ull n) { // if __int128 isn't available
    if (b == 0)
        return 0;
    if (b == 1)
        return a;
    ull t = modmul(a, b/2, n);
    t = (t+t)%n;
    if (b % 2)
        t = (t+a)%n;
    return t;
}
```

```

}
ull modpow(ull a, ull d, ull n) {
    if (d == 0)
        return 1;
    ull r = modpow(a, d/2, n);
    r = modmul(r, r, n);
    if (d % 2)
        r = modmul(r, a, n);
    return r;
}
ull gcd(ull a, ull b) { return b ? gcd(b, a%b) : a; }

```

## 8.2 Modular Inverse in $\mathcal{O}(N)$

**Usage:** Get inverse of factorial

**Time Complexity:**  $\mathcal{O}(N) - \mathcal{O}(1)$

```

const int mod = 1e9+7;
vector<int> fact(n+1), inv(n+1), factinv(n+1);
fact[0] = fact[1] = inv[1] = factinv[0] = factinv[1] = 1;
for (int i=2; i<=n; ++i) {
    fact[i] = 1LL * fact[i-1] * i % mod;
    inv[i] = mod - 1LL * mod/i * inv[mod%i] % mod;
    factinv[i] = 1LL * factinv[i-1] * inv[i] % mod;
}

```

## 8.3 Extended Euclidean

**Usage:** get a and b as arguments and return the solution  $(x, y)$  of equation  $ax + by = \gcd(a, b)$ .

**Time Complexity:**  $\mathcal{O}(\log a + \log b)$

```

pair<ll, ll> extGCD(ll a, ll b) {
    if (b != 0) {
        auto tmp = extGCD(b, a % b);
        return {tmp.second, tmp.first - (a / b) * tmp.second};
    } else return {1ll, 0ll};
}

```

## 8.4 Miller-Rabin

**Usage:** Fast prime test for big integers

**Time Complexity:**  $\mathcal{O}(k \log N)$

```

bool is_prime(ull n) {
    const ull as[7] = {2, 325, 9375, 28178, 450775, 9780504,
1795265022};
    // const ull as[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,
37}; // easier to remember
    auto miller_rabin = [] (ull n, ull a) {
        ull d = n-1, temp;
        while (d % 2 == 0) {
            d /= 2;
            temp = modpow(a, d, n);
            if (temp == n-1)
                return true;
        }
        return temp == 1;
    };
    for (ull a : as) {
        if (a >= n)
            break;
        if (!miller_rabin(n, a))
            return false;
    }
    return true;
}

```

## 8.5 Chinese Remainder Theorem

**Usage:** Solution for the system of linear congruence

**Time Complexity:**  $\mathcal{O}(\log N)$

```

w1 = modpow(mod2, mod1-2, mod1);
w2 = modpow(mod1, mod2-2, mod2);
ll ans = ((__int128)mod2 * w1 * f1[i] + (__int128)mod1 * w2 * f2[i])
% (mod1*mod2);

```

## 8.6 Pollard Rho

**Usage:** Factoring large numbers fast

**Time Complexity:**  $\mathcal{O}(N^{1/4})$

```

void pollard_rho(ull n, vector<ull> &factors) {
    if (n == 1)
        return;
    if (n % 2 == 0) {
        factors.push_back(2);
        pollard_rho(n/2, factors);
        return;
    }
    if (is_prime(n)) {
        factors.push_back(n);
        return;
    }
    ull x, y, c = 1, g = 1;
    auto f = [&] (ull x) { return (modmul(x, x, n) + c) % n; };
    y = x = 2;
    while (g == 1 || g == n) {
        if (g == n) {
            c = rand() % 123;
            y = x = rand() % (n-2) + 2;
        }
        x = f(x);
        y = f(f(y));
        g = gcd(n, y>x ? y-x : x-y);
    }
    pollard_rho(g, factors);
    pollard_rho(n / g, factors);
}

```

## 9 ETC

### 9.1 Gaussian Elimination

Time Complexity:  $\mathcal{O}(\log N)$

```

struct basis {
    const static int n = 30; // log2(1e9)
    array<int, n> data{};
    void insert(int x) {
        for (int i=0; i<n; ++i)
            if (data[i] && (x >> (n-1-i) & 1)) x ^= data[i];
    }
}

```

```

int y;
for (y=0; y<n; ++y)
    if (!data[y] && (x >> (n-1-y) & 1)) break;
if (y < n) {
    for (int i=0; i<n; ++i)
        if (data[i] >> (n-1-y) & 1) data[i] ^= x;
    data[y] = x;
}
}
basis operator+(const basis &other) {
    basis ret{};
    for (int x : data) ret.insert(x);
    for (int x : other.data) ret.insert(x);
    return ret;
}
};

```

### 9.2 Ternary Search

Time Complexity:  $\mathcal{O}(\log N)$

```

int l = 0, r = T;
while (l+2 < r) {
    int p = (2*l+r)/3, q = (l+2*r)/3;
    ll pd = calc(p, N, stars), qd = calc(q, N, stars);
    if (pd <= qd)
        r = q-1;
    else
        l = p+1;
} // check l..r

```

### 9.3 Erasable Heap

Time Complexity:  $\mathcal{O}(\log N)$

```

priority_queue<int> pq, eraser;
pq.push(value), eraser.push(value);
while (!pq.empty() && !eraser.empty() && pq.top() == eraser.top())
    pq.pop(), eraser.pop();

```

## 9.4 Randomized Meldable Heap

**Usage:** Min-heap  $H$  is declared as `Heap<T> H`. You can use `push`, `size`, `empty`, `top`, `pop` as `std::priority_queue`. Use `H.meld(G)` to meld contents from  $G$  to  $H$ .

**Time Complexity:**  $\mathcal{O}(\log n)$

```
namespace Meldable {
mt19937 gen(0x94949);
template<typename T>
struct Node {
    Node *l, *r;
    T v;
    Node(T x): l(0), r(0), v(x){}
};
template<typename T>
Node<T>* Meld(Node<T>* A, Node<T>* B) {
    if(!A) return B; if(!B) return A;
    if(B->v < A->v) swap(A, B);
    if(gen()&1) A->l = Meld(A->l, B);
    else A->r = Meld(A->r, B);
    return A;
}
template<typename T>
struct Heap {
    Node<T> *r; int s;
    Heap(): r(0), s(0){}
    void push(T x) {
        r = Meld(new Node<T>(x), r);
        ++s;
    }
    int size(){ return s; }
    bool empty(){ return s == 0; }
    T top(){ return r->v; }
    void pop() {
        Node<T>* p = r;
        r = Meld(r->l, r->r);
        delete p;
        --s;
    }
    void Meld(Heap x) {
        s += x->s;
```

```
        r = Meld(r, x->r);
    }
};
}
```

## 9.5 Berlekamp-Massey

**Usage:** `get_nth({1, 1, 2, 3, 5}, n)`

```
const int mod = 998244353;
using lint = long long;
lint ipow(lint x, lint p){
    lint ret = 1, piv = x;
    while(p){
        if(p & 1) ret = ret * piv % mod;
        piv = piv * piv % mod;
        p >>= 1;
    }
    return ret;
}
vector<int> berlekamp_massey(vector<int> x){
    vector<int> ls, cur;
    int lf, ld;
    for(int i=0; i<x.size(); i++){
        lint t = 0;
        for(int j=0; j<cur.size(); j++){
            t = (t + 1ll * x[i-j-1] * cur[j]) % mod;
        }
        if((t - x[i]) % mod == 0) continue;
        if(cur.empty()){
            cur.resize(i+1);
            lf = i;
            ld = (t - x[i]) % mod;
            continue;
        }
        lint k = -(x[i] - t) * ipow(ld, mod - 2) % mod;
        vector<int> c(i-lf-1);
        c.push_back(k);
        for(auto &j : ls) c.push_back(-j * k % mod);
        if(c.size() < cur.size()) c.resize(cur.size());
        for(int j=0; j<cur.size(); j++){
```

```

    c[j] = (c[j] + cur[j]) % mod;
}
if(i-lf+(int)ls.size()>=(int)cur.size()){
    tie(ls, lf, ld) = make_tuple(cur, i, (t - x[i]) % mod);
}
cur = c;
}
for(auto &i : cur) i = (i % mod + mod) % mod;
return cur;
}
int get_nth(vector<int> rec, vector<int> dp, lint n){
    int m = rec.size();
    vector<int> s(m), t(m);
    s[0] = 1;
    if(m != 1) t[1] = 1;
    else t[0] = rec[0];
    auto mul = [&rec](vector<int> v, vector<int> w){
        int m = v.size();
        vector<int> t(2 * m);
        for(int j=0; j<m; j++){
            for(int k=0; k<m; k++){
                t[j+k] += 111 * v[j] * w[k] % mod;
                if(t[j+k] >= mod) t[j+k] -= mod;
            }
        }
        for(int j=2*m-1; j>=m; j--){
            for(int k=1; k<=m; k++){
                t[j-k] += 111 * t[j] * rec[k-1] % mod;
                if(t[j-k] >= mod) t[j-k] -= mod;
            }
        }
        t.resize(m);
        return t;
    };
    while(n){
        if(n & 1) s = mul(s, t);
        t = mul(t, t);
        n >>= 1;
    }
    lint ret = 0;

```

```

    for(int i=0; i<m; i++) ret += 111 * s[i] * dp[i] % mod;
    return ret % mod;
}
int guess_nth_term(vector<int> x, lint n){
    if(n < x.size()) return x[n];
    vector<int> v = berlekamp_massey(x);
    if(v.empty()) return 0;
    return get_nth(v, x, n);
}
struct elem{int x, y, v;}; // A_(x, y) <- v, 0-based. no duplicate
please..
vector<int> get_min_poly(int n, vector<elem> M){
    // smallest poly P such that A^i = sum_{j < i} {A^j \times P_j}
    vector<int> rnd1, rnd2;
    mt19937 rng(0x14004);
    auto randint = [&rng](int lb, int ub){
        return uniform_int_distribution<int>(lb, ub)(rng);
    };
    for(int i=0; i<n; i++){
        rnd1.push_back(randint(1, mod - 1));
        rnd2.push_back(randint(1, mod - 1));
    }
    vector<int> gobs;
    for(int i=0; i<2*n+2; i++){
        int tmp = 0;
        for(int j=0; j<n; j++){
            tmp += 111 * rnd2[j] * rnd1[j] % mod;
            if(tmp >= mod) tmp -= mod;
        }
        gobs.push_back(tmp);
    }
    vector<int> nxt(n);
    for(auto &i : M){
        nxt[i.x] += 111 * i.v * rnd1[i.y] % mod;
        if(nxt[i.x] >= mod) nxt[i.x] -= mod;
    }
    rnd1 = nxt;
}
auto sol = berlekamp_massey(gobs);
reverse(sol.begin(), sol.end());
return sol;

```

```

}
lint det(int n, vector<elem> M){
    vector<int> rnd;
    mt19937 rng(0x14004);
    auto randint = [&rng](int lb, int ub){
        return uniform_int_distribution<int>(lb, ub)(rng);
    };
    for(int i=0; i<n; i++) rnd.push_back(randint(1, mod - 1));
    for(auto &i : M){
        i.v = 111 * i.v * rnd[i.y] % mod;
    }
    auto sol = get_min_poly(n, M)[0];
    if(n % 2 == 0) sol = mod - sol;
    for(auto &i : rnd) sol = 111 * sol * ipow(i, mod - 2) % mod;
    return sol;
}

```

## 9.6 Splay Tree w/ Lazy

```

struct splay_tree {
    struct node {
        node *l, *r, *p;
        ll key, sum, lazy, cnt;
        bool inv;
        node(ll value) {
            l = r = p = nullptr;
            cnt = 1;
            key = value;
            sum = value;
            lazy = 0;
            inv = false;
        }
    } *tree;
    void push(node *x) {
        x->key += x->lazy;
        if (x->inv) swap(x->l, x->r);
        if (x->l) {
            x->l->lazy += x->lazy;
            x->l->sum += x->lazy * x->l->cnt;
            x->l->inv ^= x->inv;
        }
    }
}

```

```

}
if (x->r) {
    x->r->lazy += x->lazy;
    x->r->sum += x->lazy * x->r->cnt;
    x->r->inv ^= x->inv;
}
x->lazy = 0;
x->inv = false;
}
void rotate(node *x) {
    auto p = x->p;
    node *tmp;
    push(p), push(x);
    if (x == p->l) {
        p->l = tmp = x->r;
        x->r = p;
    } else {
        p->r = tmp = x->l;
        x->l = p;
    }
    x->p = p->p;
    p->p = x;
    if (tmp) tmp->p = p;
    (x->p ? (x->p->l == p ? x->p->l : x->p->r) : tree) = x;
    update(p), update(x);
}
void splay(node *x) {
    while (x->p) {
        auto p = x->p;
        auto g = p->p;
        if (g) rotate((x == p->l) == (p == g->l) ? p : x);
        rotate(x);
    }
}
void update(node *x) {
    x->cnt = 1;
    x->sum = x->key;
    if (x->l) {
        x->cnt += x->l->cnt;
        x->sum += x->l->sum;
    }
}

```

```

    }
    if (x->r) {
        x->cnt += x->r->cnt;
        x->sum += x->r->sum;
    }
}
void init(int n) {
    node *x;
    tree = x = new node(0);
    tree->cnt = n;
    for (int i = 1; i < n; ++i) {
        x->r = new node(0);
        x->r->p = x;
        x = x->r;
        x->cnt = n - i;
    }
}
void add(int i, ll v) {
    find_kth(i);
    tree->sum += v;
    tree->key += v;
}
void add(int l, int r, ll v) {
    interval(l, r);
    auto x = tree->r->l;
    x->sum += v * x->cnt;
    x->lazy += v;
}
void interval(int l, int r) {
    find_kth(l - 1);
    auto x = tree;
    tree = x->r;
    tree->p = nullptr;
    find_kth(r - l + 1);
    x->r = tree;
    tree->p = x;
    tree = x;
}
ll sum(int l, int r) {
    interval(l, r);

```

```

    return tree->r->l->sum;
}
void reverse(int l, int r) {
    interval(l, r);
    tree->r->l->inv ^= true;
}
void insert(ll key) {
    auto x = new node(key);
    if (!tree) {
        tree = x;
        return;
    }
    auto p = tree;
    node **t;
    while (true) {
        if (key == p->key) return;
        if (key < p->key) {
            if (!p->l) {
                t = &p->l;
                break;
            }
            p = p->l;
        } else {
            if (!p->r) {
                t = &p->r;
                break;
            }
            p = p->r;
        }
    }
    *t = x;
    x->p = p;
    splay(x);
}
bool find(int key) {
    if (!tree) return false;
    auto p = tree;
    while (p) {
        push(p);
        if (key == p->key) break;

```

```

    if (key < p->key) {
        if (!p->l) break;
        p = p->l;
    } else {
        if (!p->r) break;
        p = p->r;
    }
}
splay(p);
return key == p->key;
}

void erase(ll key) {
    if (!find(key)) return;
    auto p = tree;
    if (p->l) {
        if (p->r) {
            tree = p->l;
            tree->p = nullptr;
            auto x = tree;
            while (x->r)
                x = x->r;
            x->r = p->r;
            p->r->p = x;
            splay(x);
            delete p;
            return;
        }
        tree = p->l;
        tree->p = nullptr;
        delete p;
        return;
    }
    if (p->r) {
        tree = p->r;
        tree->p = nullptr;
        delete p;
        return;
    }
    delete p;
    tree = nullptr;
}

```

```

}
void find_kth(int k) {
    auto x = tree;
    while (x) {
        push(x);
        while (x->l && x->l->cnt > k) {
            x = x->l;
            push(x);
        }
        if (x->l) k -= x->l->cnt;
        if (!k--) break;
        x = x->r;
    }
    splay(x);
}
};

```

## 9.7 Useful Stuff

- Catalan Number  
 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900  
 $C_n = \text{binomial}(n * 2, n) / (n + 1)$ ;  
 - 길이가  $2n$ 인 올바른 괄호 수식의 수  
 -  $n + 1$ 개의 리프를 가진 풀 바이너리 트리의 수  
 -  $n + 2$ 각형을  $n$ 개의 삼각형으로 나누는 방법의 수
- Burnside's Lemma  
 경우의 수를 세는데, 특정 transform operation(회전, 반사, ...) 해서 같은 경우들은 하나로 친다. 전체 경우의 수는? 각 operation마다 이 operation을 했을 때 변하지 않는 경우의 수를 센다 (단, “아무것도 하지 않는다” 라는 operation도 있어야 함!)  
 전체 경우의 수를 더한 후, operation의 수로 나눈다. (답이 맞다면 항상 나누어 떨어져야 한다)
- 알고리즘 게임
  - Nim Game의 해법 : 각 더미의 돌의 개수를 모두 XOR했을 때 0 이 아니면 첫번째, 0 이면 두번째 플레이어가 승리.
  - Grundy Number : 어떤 상황의 Grundy Number는, 가능한 다음 상황들의 Grundy Number를 모두 모은 다음, 그 집합에 포함 되지 않는 가장 작은 수가 현재 state의 Grundy Number가 된다. 만약 다음 state가 독립된 여러개의 state 들로 나눌 경우, 각각의 state의 Grundy Number의 XOR 합을 생각한다.
  - Subtraction Game : 한 번에  $k$  개까지의 돌만 가져갈 수 있는 경우, 각 더미의



돌의 개수를  $k + 1$ 로 나눈 나머지를 XOR 합하여 판단한다.

- Index-k Nim : 한 번에 최대  $k$ 개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을  $k + 1$ 로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.

- Pick's Theorem  
격자점으로 구성된 simple polygon이 주어짐.  $I$ 는 polygon 내부의 격자점 수,  $B$ 는 polygon 선분 위 격자점 수,  $A$ 는 polygon의 넓이라고 할 때, 다음과 같은 식이 성립한다.  $A = I + B/2 - 1$
- 가장 가까운 두 점 : 분할정복으로 가까운 6개의 점만 확인
- 홀의 결혼 정리 : 이분그래프(L-R)에서, 모든 L을 매칭하는 필요충분 조건 = L에서 임의의 부분집합  $S$ 를 골랐을 때, 반드시  $(S$ 의 크기)  $\leq$  ( $S$ 와 연결되어있는 모든 R의 크기)이다.
- 오일러 정리 :  $V - E + f(\text{면})$ 가 일정
- 소수 : 10 007 , 10 009 , 10 111 , 31 567 , 70 001 , 1 000 003 , 1 000 033 , 4 000 037 , 99 999 989 , 999 999 937 , 1 000 000 007 , 1 000 000 009 , 9 999 999 967 , 99 999 999 977
- 소수 개수 : ( $1e5$  이하 : 9592), ( $1e7$  이하 : 664 579) , ( $1e9$  이하 : 50 847 534)
- $10^{15}$  이하의 정수 범위의 나눗셈 한번은 오차가 없다.
- $N$ 의 약수의 개수 =  $O(N^{1/3})$ ,  $N$ 의 약수의 합 =  $O(N \log \log N)$
- $\phi(mn) = \phi(m)\phi(n)$ ,  $\phi(pr^n) = pr^n - pr^{n-1}$ ,  $a^{\phi(n)} \equiv 1 \pmod{n}$  if coprime
- Euler's phi  $\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$
- Lucas' Theorem  $\binom{m}{n} \equiv \prod \binom{m_i}{n_i} \pmod{p}$   $m_i, n_i$ 는  $p^i$ 의 계수

## 9.8 Template

```
#include <bits/stdc++.h>
using namespace std;
using ll = long long;
using pii = pair<int, int>;
int main() {
    ios::sync_with_stdio(false);
```

```
    cin.tie(nullptr);
    int t; cin >> t;
    while (t--) solve();
    return 0;
}
// precision
cout.precision(16);
cout << fixed;
// gcc bit operator
__builtin_popcount(bits) // popcountll for ll
__builtin_clz(bits) // left
__builtin_ctz(bits) // right
// random number generator
random_device rd;
mt19937 mt;
uniform_int_distribution<> half(0, 1);
cout << half(mt);
// 128MB = int * 33,554,432
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

## 9.9 자주 쓰이는 문제 접근법

- 비슷한 문제를 풀어본 적이 있던가?
- 단순한 방법에서 시작할 수 있을까? (brute force)
- 내가 문제를 푸는 과정을 수식화할 수 있을까? (예제를 직접 해결해보면서)
- 문제를 단순화할 수 없을까?

- 그림으로 그려볼 수 있을까?
- 수식으로 표현할 수 있을까?
- 문제를 분해할 수 있을까?
- 뒤에서부터 생각해서 문제를 풀 수 있을까?
- 순서를 강제할 수 있을까?
- 특정 형태의 답만을 고려할 수 있을까? (정규화)
- 특수 조건을 꼭 활용
- 여사건으로 생각하기
- 게임이론 - 거울 전략 혹은 DP 연계
- 겁먹지 말고 경우 나누어 생각
- 해법에서 역순으로 가능한가?
- 딱 맞는 시간복잡도에 집착하지 말자
- 문제에 의미있는 작은 상수 이용
- 스물투라지나 트라이 같은 트릭 생각
- 잘못된 방법으로 파고들지 말고 버리자

## 9.10 DP 최적화 접근

- 1사분면에 단조 감소하는 점들의 집합 P, 3사분면에 단조 감소하는 점들의 집합 Q에 대해, P의 점 하나와 Q의 점 하나를 이용하여 만들 수 있는 직사각형의 넓이는 Monge (ICPC WF Money For Nothing)
- l.r의 값들을 sum이나 min은 Monge
- 식 정리해서 일차(CHT) 혹은 비슷한(MQ) 함수를 발견, 구현 힘들면 Li-Chao
- $a \leq b \leq c \leq d$ 에서  $A[a, c] + A[b, d] \leq A[a, d] + A[b, c]$
- Monge 성질을 보이기 어려우면  $N^2$  나이브 짜서 opt의 단조성을 확인하고 찍맞
- 식이 간단하거나 변수가 독립적이면 DP 테이블을 세그 위에 올려서 해결
- 침착하게 점화식부터 세우고 Monge인지 판별
- Monge에 집착하지 말고 단조성이나 볼록성만 보여도 됨
- 못풀 것 같으면 버리자

## 9.11 Fast I/O

```
#pragma GCC optimize("O3")
#pragma GCC optimize("Ofast")
#pragma GCC optimize("unroll-loops")

inline int readChar();
template<class T = int> inline T readInt();
template<class T> inline void writeInt(T x, char end = 0);
inline void writeChar(int x);
inline void writeWord(const char *s);
static const int buf_size = 1 << 18;
inline int getChar(){
    #ifndef LOCAL
        static char buf[buf_size];
        static int len = 0, pos = 0;
        if(pos == len) pos = 0, len = fread(buf, 1, buf_size, stdin);
        if(pos == len) return -1;
        return buf[pos++];
    #endif
}
inline int readChar(){
    #ifndef LOCAL
        int c = getChar();
        while(c <= 32) c = getChar();
        return c;
    #else
        char c; cin >> c; return c;
    #endif
}
template <class T>
inline T readInt(){
    #ifndef LOCAL
        int s = 1, c = readChar();
        T x = 0;
        if(c == '-') s = -1, c = getChar();
        while('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
        return s == 1 ? x : -x;
    #else
        T x; cin >> x; return x;
    #endif
}
```

```
    #endif
}
static int write_pos = 0;
static char write_buf[buf_size];
inline void writeChar(int x){
    if(write_pos == buf_size) fwrite(write_buf, 1, buf_size,
        stdout), write_pos = 0;
    write_buf[write_pos++] = x;
}
template <class T>
inline void writeInt(T x, char end){
    if(x < 0) writeChar('-'), x = -x;
    char s[24]; int n = 0;
    while(x || !n) s[n++] = '0' + x % 10, x /= 10;
    while(n--) writeChar(s[n]);
    if(end) writeChar(end);
}
inline void writeWord(const char *s){
    while(*s) writeChar(*s++);
}
struct Flusher{
    ~Flusher(){ if(write_pos) fwrite(write_buf, 1, write_pos,
        stdout), write_pos = 0; }
}flusher;
```