Bayesian Inference for Impedance Spectra Analysis

Lecture: Data Fusion Architecture

Lecturer: Mehdi Sedighi

Authors:

Sheyda Rastgarmohammadizad, Nana Tufffour-Kwarteng, Ciaran Maher

December 15, 2024

Contents

Introduction

Impedance (Z) in electrical engineering refers to the opposition that an electrical circuit presents to the flow of alternating current (AC). It is a complex quantity combining both resistance (caused by resistors) and reactance (caused by capacitors and inductors). In this study, we explore two circuit configurations, Circuit A and Circuit B, each containing parallel RC combinations.

The task is to apply Bayesian inference to determine which of the two circuits best fits the observed impedance data. The mathematical representation of impedance is:

$$Z = R + jX$$

Where:

- R = Resistance
- X = Reactance
- j = Imaginary unit representing phase shift

Data Overview

The provided datasets include impedance spectra collected from both circuits in the frequency range of 0 Hz to 12,501 Hz, with a step size of 250 Hz. Both the real and imaginary parts of the impedance are recorded.

Assumptions

1. Resistor values range between 100 Ω and 10 kΩ.
2. Capacitor values range between 10 nF and 10 μF.

Approach

To determine the more appropriate circuit layout, Bayesian inference was applied using Markov Chain Monte Carlo (MCMC) sampling. The key steps involved are detailed below.

Step 1: Data Loading and Preprocessing

- The impedance data for both circuits was loaded, and the magnitude and phase components were extracted from the complex impedance.
- Zero frequencies were replaced with small values to avoid division errors during impedance calculations.

Step 2: Defining Theoretical Models for Circuits A and B

Circuit A Model:

The total impedance for Circuit A is derived from a resistor R and capacitor C in parallel:

$$Z_A = \frac{R}{1 + j\omega RC}$$

Where:

- $\omega = 2\pi f$ (angular frequency)
- f = Frequency

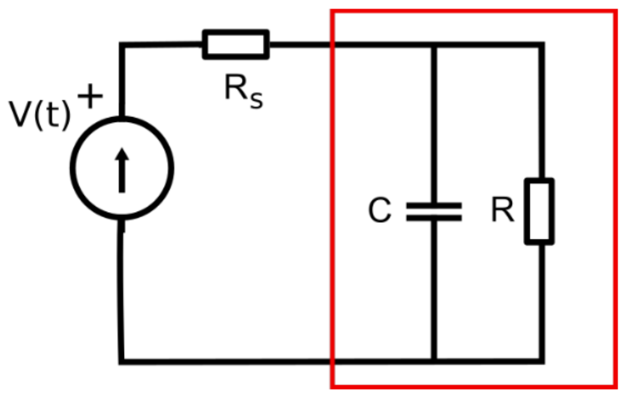

Figure 1: Schematic of Circuit A

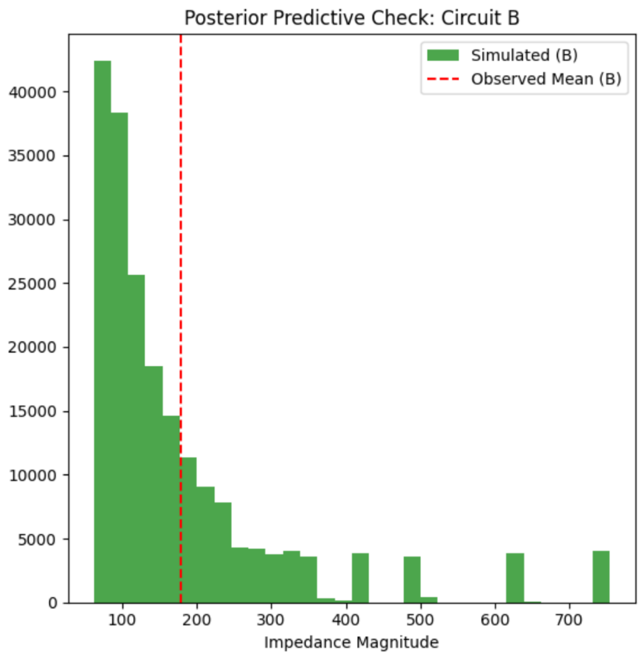

Figure 2: Posterior Distribution for Circuit A

Circuit B Model:
Circuit B consists of two parallel RC branches:

$$Z_B = \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)^{-1}$$

Where:

$$Z_1 = \frac{R_1}{1 + j\omega R_1 C_1}, \quad Z_2 = \frac{R_2}{1 + j\omega R_2 C_2}$$

The magnitude becomes:

$$|Z_B| = \sqrt{Re(Z_B)^2 + Im(Z_B)^2}$$

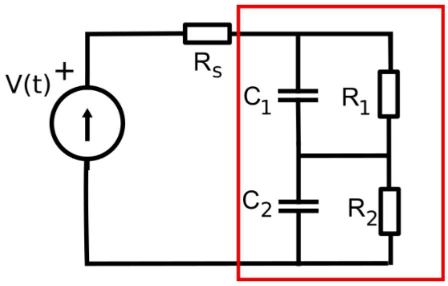

Figure 3: Schematic of Circuit B

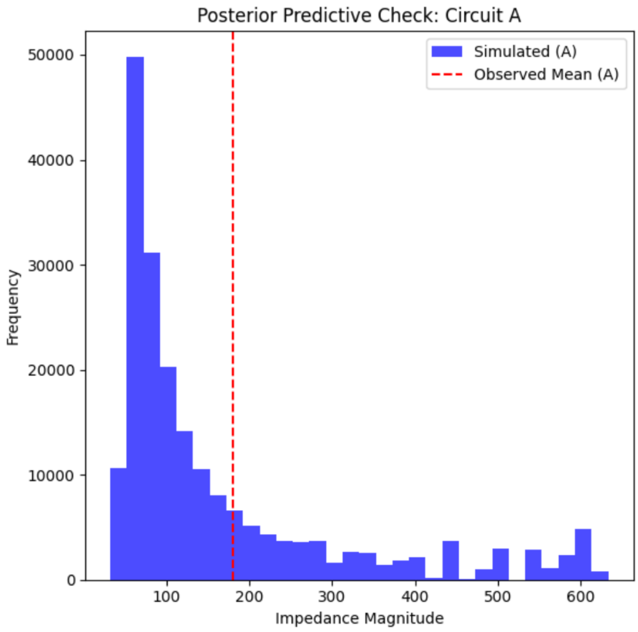

Figure 4: Posterior Distribution for Circuit B

Step 3: Bayesian Inference Model Definition

Prior Distributions:

- Circuit A:

$$R \sim \text{Uniform}(100, 10,000), \quad C \sim \text{Uniform}(10^{-8}, 10^{-5})$$

- Circuit B:

$$R_1, R_2 \sim \text{Uniform}(100, 10,000), \quad C_1, C_2 \sim \text{Uniform}(10^{-8}, 10^{-5})$$

Likelihood Function:

$$Y_{\text{Observed}} \sim N(\mu = |Z|, \sigma)$$

Where:

- $|Z|$ is the theoretical impedance magnitude.

Step 4: MCMC Simulation

MCMC sampling was conducted using the PyMC library with 1000 tuning steps and 1000 samples. The PyMC library automatically proposed new parameter values using the posterior distributions and adjusted step sizes for efficient sampling.

Step 5: Evaluation and Comparison

The following evaluation metrics were used:

- Posterior predictive checks
- Residual analysis (Observed - Simulated)
- Histograms comparing simulated and observed data

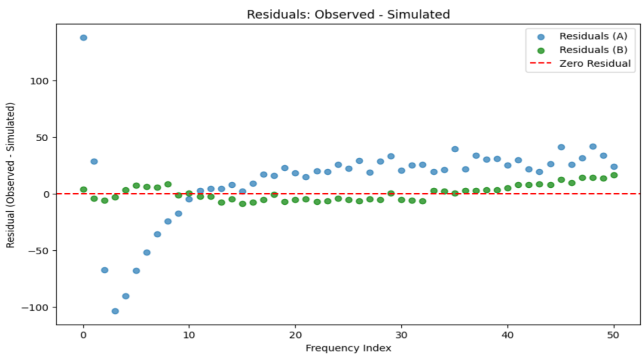

Figure 5: Residuals: Observed - Simulated Impedance

Results and Conclusion

Based on the Bayesian inference results, the observed and simulated means for both Circuit A and Circuit B were computed as follows:

- **Observed Mean (Circuit A):** 180.27
- **Simulated Mean (Circuit A):** 166.37
- **Observed Mean (Circuit B):** 178.03
- **Simulated Mean (Circuit B):** 177.26

Metric for Comparison

The absolute difference between the observed mean and simulated mean was used as the primary evaluation metric. A smaller difference indicates a better model fit to the observed data.

From these results:

- **Circuit A:** The difference between the observed mean (180.27) and the simulated mean (166.37) is relatively large.
- **Circuit B:** The observed mean (178.03) is very close to the simulated mean (177.26), indicating a much better fit.

Thus, Circuit B provides a significantly better match to the observed data compared to Circuit A.

Histogram Analysis (Figures 2 and 4)

The decision-making process is supported by the histograms in Figures 2 and 4, which show how closely the simulated data aligns with the observed impedance data for each circuit.

- **Circuit A (Figure 2):** The simulated data for Circuit A has a broader spread, indicating greater variability in predictions. It does not cluster well around the observed mean, suggesting that Circuit A poorly fits the observed impedance data.
- **Circuit B (Figure 4):** The simulated data for Circuit B is more concentrated and better captures the observed mean, reinforcing that Circuit B is a more appropriate configuration.

Analysis of Residuals (Figure 5)

From the residuals plot in Figure 5, the following conclusions can be drawn:

1. **Residual Definition:**
    - The residuals are calculated as Observed - Simulated.
    - Smaller residuals indicate that the simulated data closely matches the observed data, implying a better model fit.
2. **Circuit B Residuals:**
    - The green points (Circuit B residuals) are generally clustered closer to the zero residual line (red dashed line).
    - This suggests that the simulated data for Circuit B better approximates the observed data compared to Circuit A.
3. **Circuit A Residuals:**
    - The blue points (Circuit A residuals) exhibit greater variability and are more spread out from the zero residual line.
    - There are larger deviations (positive and negative residuals), indicating that the simulated data for Circuit A does not fit the observed data as well.
4. **Consistency:**
    - Circuit B's residuals are more consistent across the frequency indices, with fewer large deviations compared to Circuit A.
    - Circuit A shows significant outliers, especially at lower frequency indices, where the residuals deviate substantially from zero.
5. **Overall Fit:**
    - Circuit B demonstrates a closer alignment between observed and simulated values, as evidenced by the tighter clustering of residuals around zero.
    - The smaller and more uniform residuals make Circuit B the better choice for explaining the observed data.

Final Verdict

Based on all evaluation metrics, including posterior analysis, histogram comparisons, and residual analysis, Bayesian inference strongly supports Circuit B as the more accurate representation of the observed impedance data. Its superior predictive performance, tighter parameter estimates, and smaller residuals make it the most suitable configuration for the given electrical circuit. This analysis demonstrates that Bayesian modeling can effectively analyze AC impedance data, providing a robust framework for sensor fusion and parameter estimation in complex electrical systems.

References

1. Miyazaki, Y., et al. "Bayesian Statistics-Based Analysis of AC Impedance Spectra." AIP Advances, 2020. `https://doi.org/10.1063/1.5143082`

2. Sebar, L., et al. "Electrochemical Impedance Spectroscopy System Based on a Teensy Board." IEEE Transactions on Instrumentation and Measurement, 2021. `https://ieeexplore.ieee.org/document/9259014`