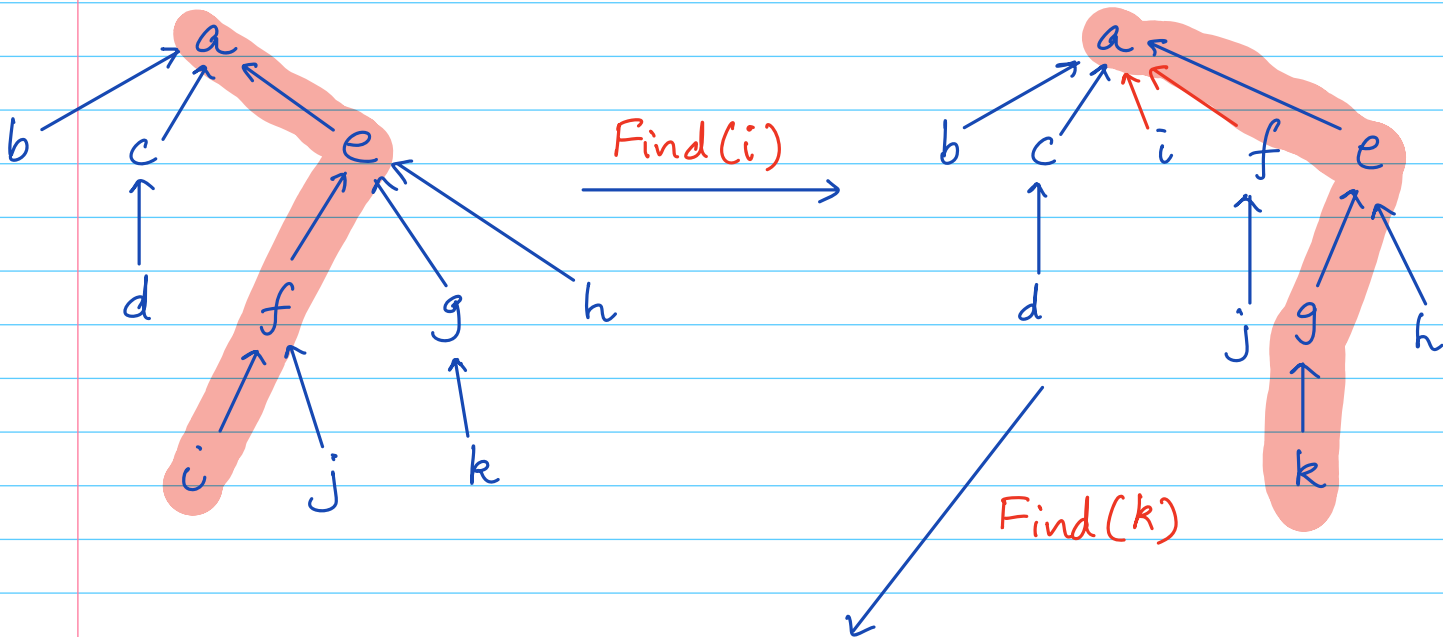


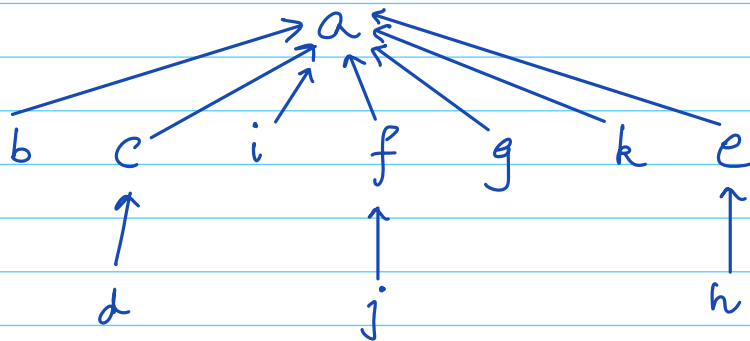
Union-by-rank with path compression

* Perform book-keeping during each find operation to reduce the depth of the tree.

- For each node in the path from u to the root, connect the node directly to the root.



```
Find(u)
  if  $u \neq p(u)$ 
     $p(u) = \text{Find}(p(u))$ 
  return  $p(u)$ 
```



Amortized analysis: Accounting method

* Maintaining Dynamic tables

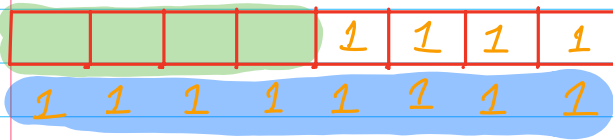
- How do we implement the vector class in STL efficiently
 - Lists of arbitrary sizes
 - $O(1)$ - access times
 - Linked lists does not allow $O(1)$ -access times
 - Resizing arrays seem expensive

Algorithmic idea:

- Initialize an array of some size m
- Whenever the array is full, create a new array of size $2m$, copy the elements into the new array, deleting the old array

Can take $O(n)$ in the worst-case for one insertion!

- Pay extra cost / operation
- Use the extra cost for a more expensive operation later



accounts for the
resizing operation

$$\text{Total cost} = 3n$$

For each item to be
inserted, pay a cost of 3

- 1 for the insertion
- 1 for a later copy
- 1 for copying another
elt