Version 2

set of items $1, 2, \ldots, n$ — infinite copies

Capacity W

find numbers $k_1, k_2, \ldots, k_n$ s.t

$$\sum_{i=1}^{n} k_i w_i \leq W \quad \& \quad \sum_{i=1}^{n} k_i v_i \text{ is maximized}$$

$KS(i, w)$ = max value when choosing items from $\{1, \ldots, i\}$ with max capacity w

$$KS(i,w) = \max_{k_i \in \{0, \frac{w}{w_i}\}} \left\{ KS(i-1, w - k_i w_i) + k_i v_i \right\}$$

$\hookrightarrow O(nW^2)$ - time algorithm

Better recurrence

$KS(w)$ = max value attained when choosing items with total capacity w

$$KS(w) = \max_{i \in [n]} \left\{ KS(w - w_i) + v_i \right\}$$

$\hookrightarrow O(nW)$ - time algorithm

What are the base cases?

DP on trees- Independent set

Given rooted tree $T(V,E)$ find the maximum independent set V'

$V' \subseteq V$ is an independent set if

$\forall\, u,v \in V'$, $(u,v) \notin E$

Recursive structure: In an independent set, either the root is present or absent

$MIS^+(v)$: max independent set in the subtree rooted at v that contains v

$MIS^-(v)$: size of the max independent set in the subtree rooted a v that excludes v

$$MIS(v) = \max\{MIS^+(v), MIS^-(v)\}$$

$$MIS^+(v) = 1 + \sum_{(v,w)\in E} MIS^-(w)$$

$$MIS^-(v) = \sum_{(v,w)\in E} MIS(w)$$

Perform postorder traversal computing MIS

What is the dependency graph? $\rightarrow$ tree T

What are the base cases? $\rightarrow$ $MIS(v)$: v is a leaf