Floyd-Warshall Algorithm

$d(u,v;k)$ = shortest path from u to v
that only passes through vertices
numbered $1 \ldots r$

$$d(u,v;0) = \begin{cases} \infty & \text{if } (u,v) \notin E \\ w(u,v) & \text{o/w} \end{cases}$$

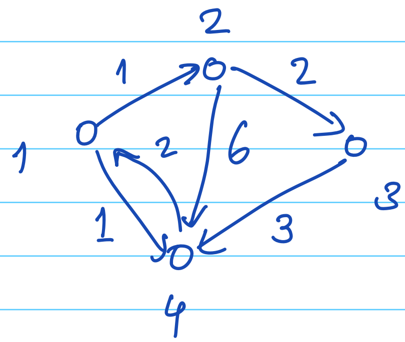
$$d(u,v,k) = \min \left\{ \begin{array}{l} d(u,v,k-1) \\ d(u,k,k-1) + d(k,v,k-1) \end{array} \right\}$$

- Numbering can be arbitrary

$$\left. \begin{array}{l} \text{for all vertices } k \\ \quad \text{for all vertices } u \\ \quad\quad \text{for all vertices } v \end{array} \right\} O(V^3)$$

if $d(u,v) > d(u,k) + d(k,v)$
$\quad d(u,v) = d(u,k) + d(k,v)$

APSP & Matrix multiplication



$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 1 & \infty & 1 \\
2 & \infty & 0 & 2 & 6 \\
3 & \infty & \infty & 0 & 3 \\
4 & 2 & \infty & \infty & 0
\end{array}
$$

$$D(u,v) = \min_{(w,v)\in E} \{ D(u,v),\ D(u,w) + wt(w,v) \}$$

$$D^2 = D * D = \begin{bmatrix} 0 & 1 & \infty & 1 \\ \infty & 0 & 2 & 6 \\ \infty & \infty & 0 & 3 \\ 2 & \infty & \infty & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & \infty & 1 \\ \infty & 0 & 2 & 6 \\ \infty & \infty & 0 & 3 \\ 2 & \infty & \infty & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 3 & 1 \\ 8 & 0 & 2 & 6 \\ 5 & \infty & 0 & 3 \\ 2 & 3 & \infty & 0 \end{bmatrix}$$

$$D^3 = D^2 * D \quad \begin{bmatrix} 0 & 1 & 3 & 1 \\ 8 & 0 & 2 & 6 \\ 5 & \infty & 0 & 3 \\ 2 & 3 & \infty & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & \infty & 1 \\ \infty & 0 & 2 & 6 \\ \infty & \infty & 0 & 3 \\ 2 & \infty & \infty & 0 \end{bmatrix}$$