IITM-CS2800-2025 : Design and Analysis of Algorithms

Practice Problem Sheet #5 Released On : Mar 6, 2 PM

1. Consider two sets A and B, each having n integers in the range from 0 to $10n$. We wish to compute the Cartesian sum of A and B, defined as:

$$C = \{x + y \mid x \in A \text{ and } y \in B\}$$

Note that the integers in C range from 0 to $2n$. We want to find the elements of C and the number of times each element of C is realized as a sum of elements in A and B. Design and $O(n \log n)$ algorithm for the problem.

2. Given a polnyomial $A(x) = \sum_{j=0}^{n-1} a_j x^j$, define $A^{rev}(x) = \sum_{j=0}^{n-1} a_{n-1-j} x^j$. Show how to dervice a point-value representation for $A^{rev}(x)$ from a point-value representation of $A(x)$, assuming that none of the points are zeroes.

3. What is the running time of BFS if we represent its input graph by an adjacency matrix and modify the algorithm to handle this form of input?

4. Give an example of a directed graph $G(V, E)$ and a source vertex $s \in V$, and a set of tree edges such that each vertex $v \in V$, the unique simple path in the graph (V, E_π) from s to v is a shortest path in G, yet the set of edges E_π cannot be produced by running BFS on G, no matter how the edges are ordered in the adjacency list.

5. There are two types of professional wrestlers: "babyfaces" ("good guys") and "heels" ("bad guys"). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.

6. The diameter of a tree $T(V, E)$ is defined to be the $\max_{u,v \in V} \delta(u, v)$. That is, the largest of all shortest-path distances in the tree.

   (a) Suppose that a and b are the endpoints of the path in the tree which achieve the diameter, and without loss of generality assume that a and b are the unique pair which do so. Let s be any vertex in T. Prove that the result of a single BFS will return either a or b (or both) as the vertex whose distance from s is greatest.

   (b) Use the above to give an efficient algorithm to compute the diameter of a tree, and analyze the running time of your algorithm.