GREEDY ALGORITHMS

* Make locally optimal choices
* Prove that they lead to globally optimal solns.

Interval Scheduling

set $\mathcal{I} = \{ (s(i), f(i)) \mid 1 \le i \le n \}$

Goal: Find a largest set $\mathcal{I}' \subseteq \mathcal{I}$ of non-overlapping intervals

$(s(i), f(i))$ $(s(j), f(j))$

$f(i) < s(j)$ or $f(j) < s(i)$

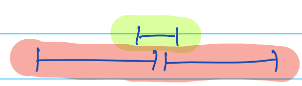
$I \in \mathcal{I}$

$I \notin$ soln: Opt among all intervals other than I

$I \in$ soln: opt among all intervals that don't overlap with I

$O(n^3)$-time algorithm

Greedy 1: Choose the interval with the shortest time $\min_i f(i) - s(i)$, add to $\mathcal{I}'$

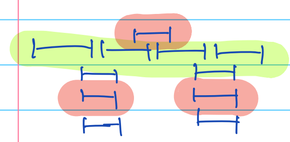


Greedy choice gives 1

OPT is 2

Ex: Prove that Greedy 1 always gives a value k s.t. $k \geq OPT/2$

Greedy 2: Choose the interval that intersects with fewest other intervals

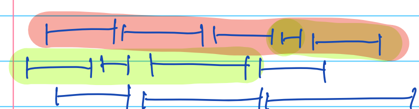


Greedy choice: 3

OPT: 4

Greedy 3: Choose the interval with the earliest finish time

* $\exists$ optimal solns. that are not obtained by greedy choices



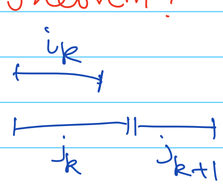
Theorem: Let $\mathcal{I}'$ be the set of intervals obtained by greedy choice 3. Let OPT be an optimal set of intervals.
Then $|\mathcal{I}'| = |OPT|$

$\mathcal{I}' = \{ i_1, i_2, .., i_k \}$ $\quad$ $OPT = \{ j_1, j_2, .., j_m \}$

<u>Claim:</u> $\forall r \leq k \quad f(i_r) \leq f(j_r)$ $\quad$ ↳ sorted in increasing finish times

Why does Claim $\Rightarrow$ Theorem?

if $k < m$

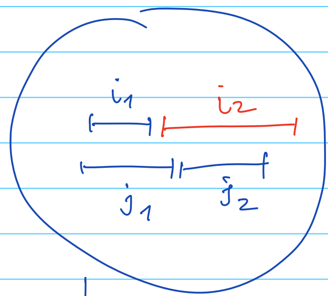


if such a j_{k+1} exists, then $\mathcal{I}'$ would have added j_{k+1} to it

contradicts maximality of $\mathcal{I}'$

Proof of claim: Induction: $f(i_1)$ is the smallest among all intervals

$\Rightarrow f(i_1) \leq f(j_1)$



j_2 does not overlap with i_1

$\Downarrow$

$f(i_2) \leq f(j_2)$

$\downarrow$

true $\forall\ i_x, j_x$