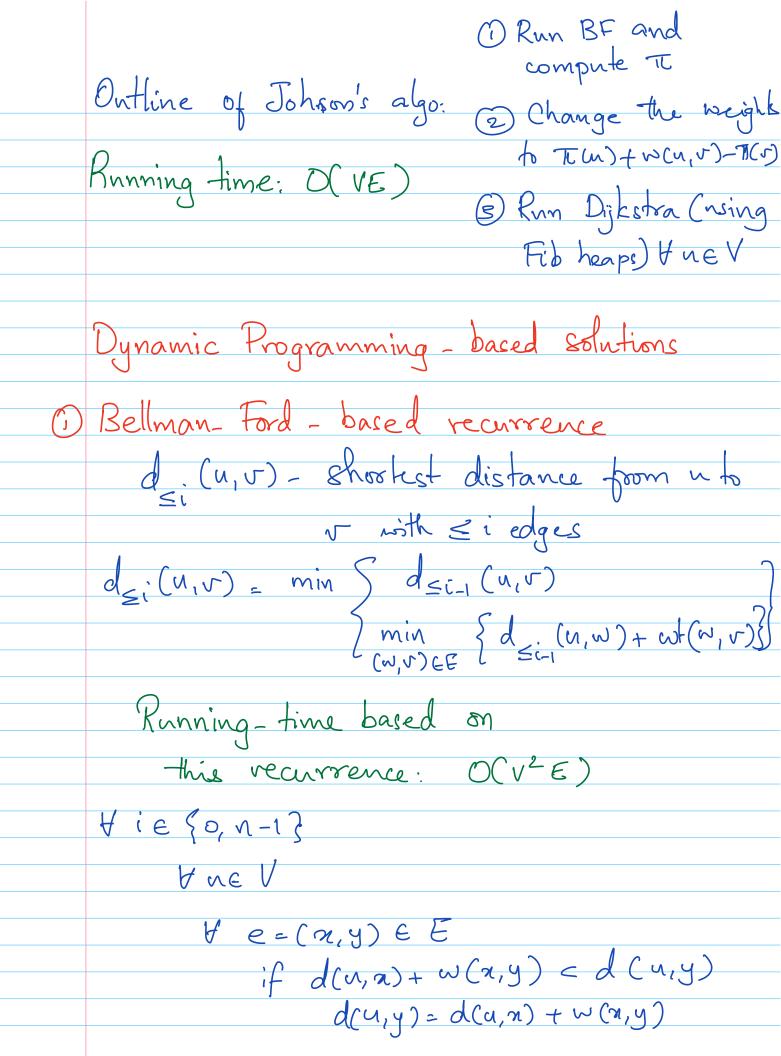
All Pairs Shortest Paths (APSP)
Johnson's algorithm
- Repeat - The SSSP from each vertex in
The graph.
- If - The edge weights are non-negative
* O(VElogV) time using
O(V3log V) = Binary heaps
* O(VE + Vlog V) time using
O(V3) = Fibonacci heaps
,
- If the edge weights are negative
$\mathcal{L} = \mathcal{L} + $
* O(V ² E) - repeated Bellman-Ford (5) O(V ⁴) - time algorithm
(D(1/4) time alonithus
- O(V) - THE AGONMA
_
Convert the edge weight as that they
Convert the edge weights so that they are non-negative a perform Dijkstra

Observation: Let T: V-)R be any function Define w cu,v)= T(u)+w(n,v)-T(v) Jhen G(v, E, w') has the same shortest pathe as in G(v, E, w) $W(u \sim r) = \sum_{i=0}^{\infty} \pi(u_i) + W(u_i, u_{i+1}) - \pi(u_i)$ = T(u0)+ W(n-2v) - T(uk) Thus all the path tengths with w' are just T(n)-T(v) + original lengths Goal: Construct TC: V -> R s.t + (u,v)EE TL(n)+W(u,v)-TL(v) >0 Construct a new vertex & & add edges Csuu) Y NEV Let d(s,u) be the shortest path from s to u (Say oftained by BF) Then $\pi(u) = d(s, n)$ $\pi(n) + w(u, v) - \pi(v) \neq 0$ because BF halfed



@ Fischer-Meyer recurrence C Divide - 4 - conquer) d(u,v,i) - shortest path from u to v with < 2 edges $d(u,v,i) = \min_{w \in V} \left\{ d(u,w,i-1) + d(w,v,i-1) \right\}$ Base case: d(u,v,o) = So if (u,v) & E \w(u,v) 0/w d(u,u,o) = 0 ∀ i ∈ §0,1,.., log v 3 for the V

for the V V3/09 V if dcu,v) > dcu,w) + dcw,v) d(u,v) = d(u,w) + d(w,v)