proj4 Fys4150 29.10.19

a)

$$Z = 2e^{8\beta J} + 2e^{-8\beta J} + 12$$

$$P_i = \frac{e^{-\beta E_i}}{Z}$$

$$E_i = -J\sum_{\langle kl \rangle} s_k s_l = -J\frac{1}{2}\sum_i \sum_{nn} s_i s_{nn}$$

$$= -\frac{J}{2}\Big[s_1(2s_2+2s_3) + s_2(2s_1+2s_4) + s_3(2s_4+2s_1) + s_4(2s_2+2s_3) \Big]$$

$$= -\frac{J}{2}\cdot\Big[2(s_4+s_1)(s_2+s_3) + 2(s_2+s_3)(s_4+s_1)\Big]$$

$$= -\frac{J}{2}\Big[4(s_4+s_1)(s_2+s_3)\Big]$$

$$\boxed{= -2J\big[(s_4+s_1)(s_2+s_3)\big]}$$

$$\langle E \rangle = \sum_i^N E_i P_i = \left(2(-8J)e^{8\beta J} + 2\cdot 8Je^{-8\beta J}\right)/Z$$

$$= \left(-16J\right)\left(e^{8\beta J} - e^{-8\beta J}\right)/Z$$

$$\langle M(T) \rangle = \sum_i^N |M_i| P_i(T)$$

$$= \left(2\cdot 4e^{-8\beta J} + 8\cdot 2e^{0}\right)\frac{1}{Z}$$

$$= 8\left(1-2e^{-8\beta J}\right)\frac{1}{Z}$$

$$\sigma_E^2 = \sum_i E_i^2 P_i - \langle E \rangle^2 = \frac{(2\cdot(-8J))^2 e^{8\beta J} + (2\cdot 8J)^2 e^{-8\beta J}}{Z} - \langle E \rangle^2$$

$$= \frac{(16J)^2\left(e^{8\beta J} + e^{-8\beta J}\right)}{Z} - \frac{\left[(-16J)\left(e^{8\beta J} - e^{-8\beta J}\right)\right]^2}{Z^2}$$

$$= 16^2 J^2\left[Z\left(e^{8\beta J} + e^{-8\beta J}\right) - e^{8\beta J} + e^{-8\beta J} - 2e^{0}\right]\frac{1}{Z^2}$$

$$\langle |M(T)| \rangle = \left|\sum_i^N M_i P_i\right| = \left|\frac{4e^{-8\beta J} + 8\cdot 2e^{0} + -4\cdot e^{-8\beta J}}{Z}\right| 2 = 0 \; ?$$

$$= \cancel{\frac{2}{Z}}$$

$$\sigma_{|M|}^2 = \langle |M(T)|^2 \rangle - \langle |M(T)| \rangle^2 = \left|\sum_i M_i^2 P_i\right| = \frac{\left|16e^{-8\beta J} + 16e^{-8\beta J} + 4e^{0} + 4e^{0}\right|}{Z}$$

$$= \frac{\left|32e^{-8\beta J} + 8\right|}{Z}$$

$$\chi = \frac{\sigma_{|M|}^2}{k_B T} \qquad C_V = \frac{\sigma_E^2}{k_B T^2}$$

$$\cancel{\sigma_{|M|}^2 = 2.}$$