Linear-Space Data Structure for Range Mode with Logarithmic Query Time

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Abstract. The abstract should briefly summarize the contents of the paper in 150–250 words.

Keywords: Data structure · Logarithmic Time · Range queries · Mode · Linear-Space.

1 Introduction

2 Data Structure Construction

Data Structure Precomputation.

Additional Definitions.

For an array A, an element x and an interval $[i, j]$, we will define $A.cnt_x(i, j)$, as the number of occurrences of element x in the range $A[i : j]$. In terms of the fundamental **rank**\ **select** operations: $A.cnt_x(i, j) = \mathbf{rank}_x(A, j) - \mathbf{rank}_x(A, i - 1)$

For any block i , let $L(i)$ be its left endpoint, and $R(i)$ be its right endpoint. For any pair of blocks i, j s.t. $i \leq j$, let $\phi(i, j)$ be the frequency of the range mode in the range $A[L(i) : R(j)]$, and let $\mu(i, j)$ be the range mode value, calculated by the precomputation algorithm, for the range $A[L(i) : R(j)]$.

We define the function ϕ' the following way:

$$\phi'(i, j) = \begin{cases} \phi(i, j) - \phi(i, j - 1), & j > i \\ \phi(i, i), & \end{cases}$$

We define the function ϕ'' the following way:

$$\phi''(i, j) = \begin{cases} \phi'(i, j) - \phi'(i - 1, j), & i > 1 \\ \phi'(i, j), & i = 1 \end{cases}$$

Lemma 1. *For any pair of blocks i, j s.t. $j \geq i$, the following holds:*

$$\phi'(i, j) \geq 0$$

Proof.

Lemma 2. *For any pair of blocks i, j s.t. $j \geq i$, the following holds:*

$$\phi''(i,j) \geq 0$$

Proof.

Lemma 3. *For any pair of blocks i, j s.t. $j \geq i$, the following holds:*

$$\phi'(i,j) = cnt_{\mu(i,j)}(L(j), R(j))$$

Proof.

Lemma 4. *For any pair of blocks i, j s.t. $j \geq i$, the following holds:*

$$\phi''(i,j) \leq cnt_{\mu(i,j)}(L(j), R(j))$$

Proof.

Lemma 5. *For any pair of blocks i, j s.t. $j \geq i$, if $\phi''(i,j) > 0$, then for any $1 \leq k < i$, the following holds:*

$$\text{If } \phi''(k,j) > 0, \text{ then } \mu(k,j) \neq \mu(i,j)$$

Proof.

Theorem 1.

$$\sum_{i=1}^{s} \sum_{j=i}^{s} \phi''(i,j) \leq n$$

Proof.

Theorem 2.

$$\phi(i,j) = \sum_{t=i}^{j} \sum_{k=1}^{i} \phi''(k,t)$$

Proof.

Now, we will define the array S the following way:

Definition 1. *The array S consists of elements in the alphabet $\{1, 2, \ldots, s+1\}$, and obeys the following properties:*

Property 1. *The array S contains s elements with value $s+1$. The k-th element with value $s+1$ corresponds to the right border of the k-th blocks.*

Property 2. *For any $1 \leq i \leq n$, for any $1 \leq k \leq i$, in the interval of S between the $i-1$-th element with value $s+1$ and the i-th element with value $s+1$ there will be $\phi''(k,i)$ elements with value k.*

Lemma 6. *For any pair of blocks i, j, s.t. $i \leq j$, let $p_1 = \boldsymbol{select}_{s+1}(S, j-1)$ and $p_2 = \boldsymbol{select}_{s+1}(S, j)$. Then, the following holds:*

$$\phi''(i, j) = S.cnt_i(p_1, p_2)$$

Proof.

Lemma 7. *For any pair of blocks i, j, s.t. $i \leq j$, let $p_1 = \boldsymbol{select}_{s+1}(S, i-1)$ and $p_2 = \boldsymbol{select}_{s+1}(S, j)$. Then, the following holds:*

$$\phi(i, j) = \sum_{k=1}^{i} S.cnt_k(p_1, p_2)$$

Proof.

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