

Supplementary file for “Automated Design of Hybrid Optimization Algorithms”

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This supplementary file contains the supplementary material to the paper titled “Automated Design of Hybrid Optimization Algorithms”. The file is organized as follows. In Section S-I, we present the description of hybridization factors and more details about the generalized procedure of hybridization, which will help readers understand the proposed template of hybrid OAs better. In Section S-II, more details about the assortive mating method and the vertical cultural transmission strategy in MFEAs are presented. Section III provides more details about the computational experiment. The CEC2017 performance evaluation method is provided in Section S-III.A, and more details about the design choices in the experiment are presented in Section S-III.B. Besides, more detailed experimental results are shown in Section S-III.C.

S-I. DETAILED DESCRIPTIONS FOR THE PROPOSED METHODOLOGY

A. Brief Descriptions of Hybridization Factors

Here, we will briefly present the elements (i.e., hybridization factors) of a hybridization strategy involved in the taxonomy we proposed previously [1]. Five hybridization factors can be used to distinguish diverse hybridization strategies, including the relationship between parent OAs (PR), hybridization level (HL), operating order (OO), type of information transfer (TIT), and type of transferred information (TTI).

Firstly, three kinds of PRs are mainly considered in this taxonomy: collaboration, embedding, and assistance. The HL of the parent OAs, which highly depends on their respective operating levels (OLs), refers to the OL at which they are hybridized, indicating the spatial assignment of parent OAs. The OO indicates the temporal assignment of parent OAs, and three OO types are concerned: no order, serial and parallel. The TIT indicates the interconnection structure of parent OAs, concerning the information flow between them. Taking each parent OA as an ‘individual (particle)’, we can equate the TIT with the population topology of parent OAs. As a crucial design element of hybrid OAs, the TTI is the most complicated factor among the aforementioned hybridization factors. It is not only related to specific OAs but also to the type of problems to be solved. Common TTIs include current solutions, a group of elite solutions, the so-far-best solution, solution components, and control parameters. Besides, the transferred information may be a combination of the TTIs mentioned earlier. More detailed descriptions are available in [1]. For brevity, some concise notations are employed to denote the candidate classes concerning each hybridization factor as follows.

- 1) Parent relationship (PR)
Collaboration: $\langle C \rangle$, Embedding: $\langle E \rangle$, Assistance: $\langle A \rangle$.
- 2) Hybridization level (HL)
Population level: $\langle P \rangle$, Sub-population level: $\langle S \rangle$, Individual level: $\langle I \rangle$, Component level: $\langle C \rangle$.
- 3) Operating order (OO)
Sequential order: $\langle S \rangle$, Parallel order: $\langle P \rangle$, No order: $\langle N \rangle$.
- 4) Type of information transfer (TIT)
Simplex TIT: $\langle S \rangle$, Duplex TIT: $\langle D \rangle$.
- 5) Type of transferred information (TTI)
Solutions: $\langle S \rangle$, Fitness information: $\langle F \rangle$, Solution components: $\langle S_c \rangle$, Auxiliaries: $\langle A \rangle$, Control parameters: $\langle C_p \rangle$, Algorithm-induced in-betweens: $\langle A_i \rangle$.

B. Generalized Procedure for Hybridization

More details about the generalized procedure for hybridization are presented here to provide a better insight into the proposed template of hybrid OAs. As mentioned in the main article, the HL of a hybridization strategy is largely dependent on the OLs of parent OAs. Thus, we provide a unified description of different types of OLs which take various forms of realization to support the cross-level hybridization of parent OAs (see **Algorithm S-I**).

Algorithm S-I The procedure for determining the operation objects of a parent OA (*OperObj()*).

Input: $PF, D, N_P, OL_1, r_{sp}, P_{ind}, P_{comp}, flag_{cros}, recd_{OL}$.
Output: U_{ob} .
 $OL_{real} \leftarrow recd_{OL}(OL_1)$;
switch (OL_{real})
case 1:
 $U_{ob} \leftarrow I_{N_P \times D}$; $\{/* I$ denotes a matrix in which every element is equal to 1. $*/\}$
case 2:
 $U_{ob} \leftarrow Divide_subp(OL_1, r_{sp}, D, N_P, PF)$; $\{/*$ The procedure of the division of sub-populations $*/\}$
case 3:
 if $flag_{cros} == 0$ **then** $\{/*$ Homogeneous Level $*/\}$
 for $i = 1$ **to** N_P **do**
 Generate a random number $rd \in [0, 1]$;
 if $rd \geq P_{ind}$ **then**
 $U_{ob}(i, :) \leftarrow I_{1 \times D}$;
 end if
 end for
 else $\{/*$ Cross Level $*/\}$
 $U_{ob} \leftarrow Select_ind(OL_1, D, N_P, PF)$; $\{/*$ The procedure of the selection of individuals $*/\}$
 end if
case 4:
 $U_{ob} \leftarrow Select_comp(OL_1, D, N_P, P_{comp})$; $\{/*$ The procedure of the selection of components $*/\}$
end switch

Denote by $U_{ob} = [u_{ob}(i, j)]_{N_P \times D}$ the operation objects of a parent OA. Since the essence of the difference among

diverse OLs mainly stems from their operation objects, U_{ob} is treated as the representation for the OLs of a parent OA. The operation objects of an OA can be obtained by **Algorithm S-I** according to its OL. Some parameters are adopted to determine the operation objects for different OLs, and their values will be set according to the diverse realization forms of each OL. Specifically, r_{sp} indicates the ratios of individuals in different sub-populations. P_{ind} and P_{comp} denote the probability of an individual and a component being selected in various realization forms, respectively. $flag_{cros}$ indicates whether the parent OAs work at different OLs. If so, $flag_{cros} = 1$; otherwise, $flag_{cros} = 0$.

Algorithm S-II The generalized procedure of hybridization.

Input: $D, N_P, OL, OO, r_{sp}, p_{comp}, p_{ind}, recd_{OL}, recd_{OO}$
Output: the best solution found for the given problem
Initialize the population Pop with N_P individuals.
Initialize the algorithm-specific and common control parameters for each OA.
 $PF \leftarrow FitFunc(Pop), OO_{real} \leftarrow recd_{OO}(OO).$
 $OL_{real} \leftarrow recd_{OL}(OL).$
while The termination criterion has not been met **do**
 if $OO_{real} == 0$ **then** *{/* Serial OO */}*
 Determine which OA should be enabled according to the alternation mechanism of OO , and then update PE accordingly;
 if $PE(1) == 1$ **then** *{/* Parent 1 is enabled */}*
 Update $U1$ using **Algorithm S-I** with $OL(1).$
 $X1 \leftarrow SearchOA1(Pop, U1);$ *{/* The search procedure of parent OA1 */}*
 $PF1 \leftarrow FitFunc(X1);$ *{/* The fitness function */}*
 $[Pop, PF] \leftarrow Selector(Pop, X1, PF, PF1);$ *{/* The procedure of survivor selection */}*
 end if
 if $PE(2) == 1$ **then** *{/* Parent 2 is enabled */}*
 if $OL(1) == OL(2)$ **then** *{/* Homogeneous Level */}*
 if $OL_{real}(2) < 3$ **then** *{/* P-level or S-level */}*
 $U2 \leftarrow U1;$
 else *{/* I-level or C-level */}*
 $U2 \leftarrow \neg U1;$
 end if
 else *{/* Cross Level */}*
 Update $U2$ using **Algorithm S-I** with $OL(2).$
 end if
 $X2 \leftarrow SearchOA2(Pop, U2);$ *{/* The search procedure of parent OA2 */}*
 $PF2 \leftarrow FitFunc(X2);$
 $[Pop, PF] \leftarrow Selector(Pop, X2, PF, PF2);$
 end if
 else *{/* Parallel OO */}*
 if $OL(1) == OL(2)$ **then** *{/* Homogeneous Level */}*
 Update $U1$ using **Algorithm S-I** with $OL(1).$
 if $OL_{real}(2) == 2$ **then** *{/* S-level */}*
 $U2 \leftarrow \neg U1;$
 else *{/* P-level or I-level */}*
 $U2 \leftarrow U1;$
 end if
 else *{/* Cross Level */}*
 Update $U1$ using **Algorithm S-I** with $OL(1).$
 Update $U2$ using **Algorithm S-I** with $OL(2).$
 end if
 $X1 \leftarrow SearchOA1(Pop, U1), PF1 \leftarrow FitFunc(X1);$
 $X2 \leftarrow SearchOA2(Pop, U2), PF2 \leftarrow FitFunc(X2);$
 $[Pop, PF] \leftarrow Selector(X1, X2, PF1, PF2);$
 end if
 Update the algorithm-specific and common control parameters of each OA based on the information in the newly generated population as well as the history.
end while
return The best solution found yet.

Denote by $X1 = [x1_{i,j}]_{N_P \times D}$ and $X2 = [x2_{i,j}]_{N_P \times D}$ the populations generated by the parent OAs, respectively. Besides, $U1 = [u1_{i,j}]_{N_P \times D}$ and $U2 = [u2_{i,j}]_{N_P \times D}$ define the operation objects of the parent OAs, respectively. More details about the generalized procedure of hybrid OAs are presented in **Algorithm S-II**. It is worth pointing out that various components concerning the generation of new solutions for the candidate parent OAs have been integrated into the proposed template of hybrid OAs. The most essential one is the *generative* operator that can produce new solutions. Besides, the *adjustment* operators, which will update the algorithm-specific and common control parameters, are also contained in the proposed template.

More specifically, at the end of each iteration, the algorithm-specific and common control parameters for each OA will be updated based on the information from the newly generated population and the history (if needed). The algorithm-specific parameters could be associated with every individual (e.g., the velocity of a particle in the particle swarm optimization (PSO) algorithm). In this situation, the velocities of all the particles (i.e., individuals) will be updated no matter which individuals have been evolved by PSO. Besides, the probability model adopted in the estimation of distribution algorithm (EDA) and the ant colony optimization (ACO) algorithm will be updated based on the information from the whole population, regardless of their own operation objects. The reason behind this is that all the types of information will be shared among the parent OAs in the proposed template.

Besides, in the original version of some candidate OAs, several control parameters will be adjusted dynamically during the iterative process. For these parameters, all the information available at the end of each iteration can be used to accomplish the update. For example, the population size will be adjusted adaptively in L-SHADE. In this template, when the L-SHADE participates in the hybridization, the population size (N_P) will be updated and the population will be reconstructed accordingly at the end of each iteration. Other parameters will be updated in a similar way once the corresponding OA has been involved in the hybridization.

S-II. MORE DETAILS ABOUT THE PROPOSED MFEA-MOPHOA

In MFEAs, new offspring can be obtained through assortative mating. Particularly, a criterion is used to determine which operator will be executed in the so-called assortative mating process [2]. It declares that individuals are prone to reproduce descendants with those sharing the same cultural background (i.e., skill factor). In MFEAs, the skill factor is regarded as a computational expression of an individual's cultural bias. Therefore, if two parent individuals picked out from the mating pool share the same skill factor, then they will go through the crossover process directly. On the contrary, once they vary in skill factors, they will turn to crossover with a predefined probability (i.e., random mating probability rpm); otherwise, they will undergo the mutation process to generate new offspring individuals.

Besides, the skill factor of a new offspring will be determined by the vertical cultural transmission strategy in MFEAs.

Vertical cultural transmission is treated as an inheritance pattern in which the biological genetic process can be realized in parallel and the phenotype of an offspring is directly affected by those of its parents. To achieve such a memetic phenomenon, a selective imitation strategy is employed in MFEAs to assign skill factors for the generated offspring individuals. If an offspring is reproduced by a single parent, then it will inherit its parent's skill factor directly. Otherwise, it will inherit the skill factor from either of its parents randomly. Besides, objective values of other unevaluated tasks will be set as a very large number.

S-III. SUPPLEMENTARY DETAILS ABOUT THE COMPUTATIONAL EXPERIMENT

A. Performance Evaluation Method of a Hybrid OA

The CEC2017 performance evaluation method adopted in the computational experiment for each OA is based on a score of 100 which integrates the following two scores:

- 50% summation of error values for all dimensions as follows:

$$SE = \omega_1 \times \sum_{i=1}^{30} ef_{10D} + \omega_2 \times \sum_{i=1}^{30} ef_{30D} + \omega_3 \times \sum_{i=1}^{30} ef_{50D}, \quad (S-1)$$

$$score1 = (1 - \frac{SE - SE_{min}}{SE}) \times 50, \quad (S-2)$$

where ef is the error value for each function and SE_{min} is the minimal sum of errors from all the OAs.

- 50% summation of ranks for each problem in each dimension as follows:

$$SR = \omega_1 \times \sum_{i=1}^{30} rank_{10D} + \omega_2 \times \sum_{i=1}^{30} rank_{30D} + \omega_3 \times \sum_{i=1}^{30} rank_{50D}, \quad (S-3)$$

$$score2 = (1 - \frac{SR - SR_{min}}{SR}) \times 50, \quad (S-4)$$

where SR_{min} is the minimal sum of ranks from all the OAs.

Hence, the score of a certain OA can be expressed as $Score = score1 + score2$. Moreover, higher dimensions are assigned with higher weights in this performance evaluation method and the weight vector adopted in this comparative experiment is set as $\omega = [0.22, 0.33, 0.45]$.

B. Design Choices in the Experiment

The parameters of the involved OAs are set as recommended in the corresponding literature, and the setting of some key parameters is listed in Table S-I. More details are available in the corresponding work.

TABLE S-I
PARAMETER SETTING OF CANDIDATE PARENT OAS.

Algorithm	Parameter setting
GA [3]	$p_c = 0.95, p_m = 0.05, k_{tour} = 2;$
PSO [3]	$cc = [2, 2], w(gen) = 0.9 - 0.5 * gen/MaxGen,$ $v_{max} = 40, v_{min} = -40;$
ACO [3]	$nSample = 0.8, q_{ACO} = 0.5, zeta = 1;$
DE family [4]	$CR = 0.9, F(gen) = 1 - 0.5 * gen/MaxGen, F1 = F, F2 = F;$
ABC [3]	$N_{ABC} = 20;$
EDA [3]	$k_e = 0.3;$
SA [3]	$r_{SA} = 0.99, \sigma = 40;$
SHADE [5]	$r_{archive} = 1.4, freq_{inti} = 0.5, N_{mem} = 5, p_b = 0.4,$ $p_s = 0.5, r_{pbest} = 0.11, LP = 20;$
L-SHADE [6]	$r_{archive} = 1.4, freq_{inti} = 0.5, N_{mem} = 5, p_b = 0.4,$ $p_s = 0.5, r_{pbest} = 0.11, LP = 20, N_{P_{min}} = 6;$

C. More Details of the Numerical Results

More detailed results about the comparison of instances of each dimension between the automatically obtained portfolio with the top-three CEC2017 competition winner methods are presented in Tables S-II, S-III, and S-IV, respectively. The comparison results provide the mean and standard deviation of the error between the objective values of each method and the optimal value, which are gathered from 51 independent runs of each method for every instance.

TABLE S-II
COMPARISON RESULTS ON THE INSTANCES WITH $D = 10$.

No.		EBOWithCMAR	jSO	LSHADE-cnEpSin	The Portfolio
F1	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F2	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F3	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F4	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F5	mean	0 (-)	1.7558 (-)	1.6851 (-)	2.0359
	std	0	0.7526	0.7534	0.8897
F6	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F7	mean	10.5531 (-)	11.7916 (-)	11.9796 (\approx)	12.0517
	std	0.1731	0.6008	0.4799	0.6552
F8	mean	0 (-)	1.9509 (-)	1.7969 (\approx)	2.4002
	std	0	0.7362	0.7714	1.2295
F9	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F10	mean	37.2101 (-)	35.8974 (-)	43.0254 (-)	60.8898
	std	53.3595	54.9303	55.742	78.7252
F11	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F12	mean	90.1549 (+)	2.6621 (+)	101.28 (+)	2.5940
	std	73.6373	16.6166	73.0329	13.3777
F13	mean	2.1716 (-)	2.9644 (\approx)	3.657 (-)	3.9772
	std	2.5004	2.3302	2.6566	2.0515
F14	mean	0.0605 (+)	0.0585 (+)	0.078 (+)	0.0004
	std	0.2338	0.2341	0.2702	0.0023
F15	mean	0.1090 (\approx)	0.2208 (\approx)	0.3239 (+)	0.1825
	std	0.1727	0.1985	0.2162	0.1913
F16	mean	0.417 (+)	0.5688 (+)	0.5372 (+)	0.1045
	std	0.1965	0.2618	0.2934	0.1258
F17	mean	0.1472 (+)	0.5023 (+)	0.3072 (+)	0.1049
	std	0.2008	0.3446	0.3815	0.1453
F18	mean	0.7 (\approx)	0.308 (\approx)	3.8592 (+)	0.2870
	std	2.7403	0.1932	7.6265	0.2127
F19	mean	0.015 (\approx)	0.0107 (-)	0.0447 (+)	0.016
	std	0.0186	0.0124	0.2088	0.0093
F20	mean	0.1469 (+)	0.3428 (+)	0.2571 (+)	0
	std	0.1558	0.1275	0.2311	0
F21	mean	114.0207 (+)	132.3776 (+)	146.3637 (+)	100.2059
	std	34.8462	47.8889	51.6667	1.0187
F22	mean	98.4647 (\approx)	100 (\approx)	100.0136 (\approx)	53.4743
	std	10.8565	0	0.068	46.2398
F23	mean	300.1747 (-)	301.2056 (+)	302.0022 (+)	300.8027
	std	0.7002	1.5741	1.6424	31.8619
F24	mean	166.2054 (+)	296.5982 (+)	315.8258 (+)	100.0000
	std	98.7333	78.5413	54.5119	0.0000
F25	mean	412.3489 (+)	405.9629 (+)	425.5561 (+)	200.0706
	std	20.9607	17.3058	22.3592	111.0748
F26	mean	265.4001 (+)	300 (+)	300 (+)	94.1414
	std	46.9666	0	0	100.8200
F27	mean	391.5744 (+)	389.3875 (+)	389.4981 (+)	374.8269
	std	2.3735	0.2231	1.9636	1.3875
F28	mean	307.1384 (+)	339.0758 (+)	384.8847 (+)	301.0721
	std	71.0908	95.5958	118.8246	0.4526
F29	mean	231.1875 (\approx)	234.196 (+)	228.4069 (-)	230.5998
	std	3.7306	2.9268	1.7219	2.5609
F30	mean	406.6838 (\approx)	394.5193 (-)	17618.431 (\approx)	394.7059
	std	17.6238	0.0445	86129.791	0.0947

TABLE S-III
COMPARISON RESULTS ON THE INSTANCES WITH $D = 30$.

No.		EBOwithCMAR	jSO	LSHADE-cnEpSin	The Portfolio
F1	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F2	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F3	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F4	mean	56.4521 (+)	58.6705 (+)	42.2816 (+)	23.4583
	std	11.0363	0.7703	3.0697	2.0772
F5	mean	2.7761 (-)	8.5568 (-)	12.2506 (-)	14.6317
	std	1.7270	2.0773	2.343	4.4152
F6	mean	0 (\approx)	0 (\approx)	0 (+)	0
	std	0	0	0	0
F7	mean	33.4609 (-)	38.9268 (-)	43.2952 (\approx)	45.8267
	std	0.8286	1.445	2.1667	6.5235
F8	mean	2.0216 (-)	9.0918 (-)	12.9261 (-)	18.9042
	std	1.3043	1.8218	2.8641	4.9748
F9	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F10	mean	1408.0774 (-)	1527.6633 (-)	1388.4398 (-)	1801.7949
	std	212.734	274.431	210.4744	544.0973
F11	mean	4.4931 (-)	3.0375 (-)	13.5389 (\approx)	9.1248
	std	8.6818	2.6203	19.3841	16.3059
F12	mean	462.8859 (+)	170.3814 (\approx)	372.4478 (+)	217.9176
	std	260.2657	100.9340	200.532	143.9647
F13	mean	14.8926 (\approx)	14.8400 (\approx)	17.2576 (\approx)	15.5027
	std	6.1866	4.7836	10.2069	6.3348
F14	mean	21.8987 (+)	21.8345 (+)	21.5729 (+)	14.2710
	std	3.8012	1.2336	2.26	10.0671
F15	mean	3.6854 (\approx)	1.0879 (-)	3.24 (\approx)	3.3944
	std	2.1308	0.6845	1.9802	1.9984
F16	mean	42.6242 (+)	78.923 (+)	22.8842 (\approx)	13.6685
	std	56.3761	83.9339	30.7306	3.3557
F17	mean	29.7549 (+)	32.9248 (+)	28.6012 (+)	21.3023
	std	7.4224	7.9971	5.5593	10.1762
F18	mean	22.1353 (+)	20.4111 (\approx)	21.0854 (\approx)	20.7921
	std	1.082	2.8443	0.752	2.8532
F19	mean	8.0404 (+)	4.5031 (\approx)	5.8283 (+)	4.8134
	std	2.2554	1.7152	1.9247	1.7807
F20	mean	35.7234 (+)	29.3684 (+)	30.3457 (+)	8.5504
	std	7.4216	5.7971	7.3523	8.9040
F21	mean	198.9054 (-)	209.2889 (+)	212.052 (+)	205.5477
	std	20.0386	1.9361	2.5616	2.4207
F22	mean	100 (-)	100 (-)	100 (\approx)	100
	std	0	0	0	0
F23	mean	351.2183 (\approx)	350.7495 (\approx)	356.1519 (+)	350.4224
	std	3.4777	3.2667	3.7319	4.2825
F24	mean	418.1112 (-)	426.4564 (+)	428.4759 (+)	423.7052
	std	45.0096	2.442	2.9483	1.4609
F25	mean	386.5320 (\approx)	386.6986 (-)	386.6768 (-)	386.7089
	std	0.7490	0.0076	0.0089	0.015
F26	mean	537.333 (+)	920.2079 (+)	948.6114 (+)	278.4314
	std	302.8247	42.53	46.0273	41.5390
F27	mean	502.3951 (+)	497.3851 (+)	504.1608 (+)	467.6137
	std	3.9871	6.9327	6.6966	7.2765
F28	mean	308.3104 (-)	308.7296 (-)	315.2244 (+)	310.7548
	std	28.5162	29.9517	38.5917	32.9872
F29	mean	432.7956 (+)	433.6701 (+)	434.5624 (+)	427.5496
	std	11.2003	13.507	7.3625	13.7664
F30	mean	1987.855 (\approx)	1971.2418 (\approx)	1977.4487 (-)	1990.8716
	std	41.6975	18.7748	41.663	50.6452

Moreover, more details of the statistical results on the frequency of use for candidate types of each hybridization factor are shown in Figs. S-I~S-IV.

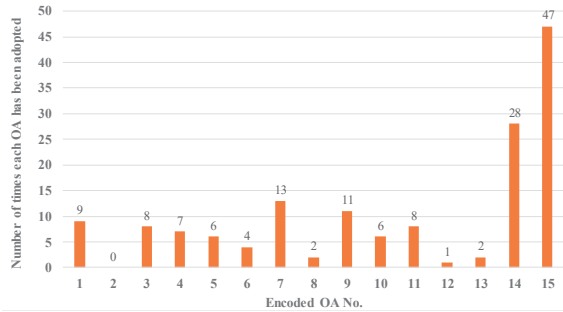


Fig. S-I. Statistical result about the frequency of use for candidate OAs.

Besides, the average objective values of the top four AutoHOAs automatically designed by the proposed methodology on the given instances for each dimension are shown in Table S-V, S-VI, and S-VII. The main processes of the top four AutoHOAs are presented in **Algorithms S-III-S-VI**.

REFERENCES

[1] B. Xin, Y. Wang, and B. Liu, *A unifying framework for swarm intelligence-based hybrid algorithms*. Stevenage, UK: The Institution of Engineering and Technology (IET), Sept. 2018, vol. 1, pp. 55–83.

TABLE S-IV
COMPARISON RESULTS ON THE INSTANCES WITH $D = 50$.

No.		EBOwithCMAR	jSO	LSHADE-cnEpSin	The Portfolio
F1	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F2	mean	0 (\approx)	0 (\approx)	1.5686 (+)	0
	std	0	0	1.9314	0
F3	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F4	mean	42.8636 (-)	56.2126 (+)	51.4012 (-)	46.345
	std	32.9029	48.2828	44.2623	37.3214
F5	mean	7.5846 (-)	16.4053 (-)	25.1664 (\approx)	25.0282
	std	2.3977	3.4279	6.4447	10.4899
F6	mean	0 (-)	0 (+)	0 (+)	0
	std	0	0	0	0
F7	mean	57.8843 (-)	66.4965 (+)	76.6392 (+)	59.6594
	std	1.5136	3.4386	6.0618	1.6885
F8	mean	7.9114 (-)	16.9623 (-)	26.3186 (\approx)	22.5374
	std	2.4448	3.1045	6.5917	10.1446
F9	mean	0 (\approx)	0 (\approx)	0 (\approx)	0
	std	0	0	0	0
F10	mean	3114.7365 (-)	3139.7576 (-)	3200.1057 (-)	3763.7686
	std	396.7750	363.545	339.7187	772.5387
F11	mean	26.3614 (-)	27.9386 (-)	21.3930 (-)	36.1352
	std	3.3297	3.2957	2.0902	5.3714
F12	mean	1938.7542 (+)	1680.5634 (+)	1475.3162 (-)	1445.3234
	std	825.678	517.772	364.7242	405.9710
F13	mean	41.4014 (\approx)	30.5989 (-)	69.4303 (+)	40.2753
	std	24.5301	21.0166	34.457	21.4614
F14	mean	31.2149 (-)	24.9637 (-)	26.5224 (-)	38.3004
	std	3.4827	1.8549	2.4924	6.5067
F15	mean	29.3582 (-)	23.8643 (-)	25.5962 (-)	36.526
	std	5.1452	2.4637	4.0567	8.0088
F16	mean	346.3637 (+)	450.5211 (+)	274.5338 (+)	128.4226
	std	144.3739	136.3953	99.6917	24.3010
F17	mean	274.7809 (+)	282.8672 (+)	207.056 (+)	60.0599
	std	85.499	85.2934	73.0642	55.6658
F18	mean	32.0337 (+)	24.2828 (-)	24.3318 (-)	26.1515
	std	5.9279	1.9976	2.1179	2.769
F19	mean	24.4784 (+)	14.1386 (\approx)	17.4062 (+)	14.196
	std	3.897	2.2400	2.4713	2.5395
F20	mean	147.2184 (+)	140.1016 (+)	114.1247 (+)	22.0964
	std	73.7127	76.6129	35.4831	4.7912
F21	mean	210.6179 (+)	219.1995 (+)	226.7608 (+)	207.6011
	std	4.017	3.7284	7.0598	3.2074
F22	mean	365.3677 (+)	1487.2365 (\approx)	1594.9612 (+)	100.0000
	std	915.2109	1735.82	1665.9416	0.0000
F23	mean	434.047 (+)	430.0837 (+)	439.2895 (+)	423.4662
	std	8.082	6.175	6.9001	5.7414
F24	mean	506.4619 (-)	507.45 (-)	512.8218 (-)	524.387
	std	3.8095	4.0866	5.5948	7.3085
F25	mean	488.6051 (+)	480.878 (-)	480.3372 (-)	481.6157
	std	24.4355	2.7722	1.0816	3.692
F26	mean	705.5736 (+)	1128.7789 (+)	1202.6435 (+)	300.0000
	std	402.3718	55.6132	118.6957	0.0000
F27	mean	522.3751 (+)	511.2716 (-)	525.4343 (+)	518.3083
	std	7.6784	10.9673	9.2143	6.2712
F28	mean	466.5108 (+)	459.8068 (+)	459.1279 (+)	457.8616
	std	17.7639	6.7724	11.9044	7.1579
F29	mean	347.3439 (+)	362.9353 (+)	352.891 (+)	303.3983
	std	19.4973	13.0271	9.7796	8.6259
F30	mean	618165.3965 (+)	601051.7451 (\approx)	657533.29 (+)	607104.8156
	std	35826.8689	29564.5785	72412.981	31899.5442

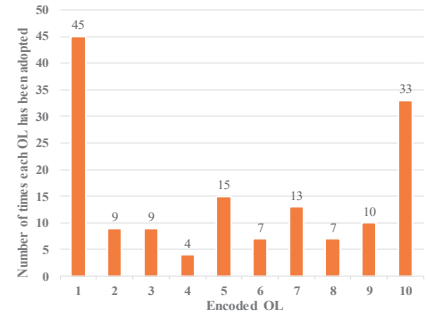


Fig. S-II. Statistical result about the frequency of use for encoded OLs.

[2] A. Gupta, Y.-S. Ong, L. Feng, and K. C. Tan, “Multiobjective multifactorial optimization in evolutionary multitasking,” *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1652–1665, 2017.

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[6] R. Tanabe and A. S. Fukunaga, “Improving the search performance of shade using linear population size reduction,” in *Proc. IEEE Cong. Evol. Comput.* IEEE, 2014, pp. 1658–1665.

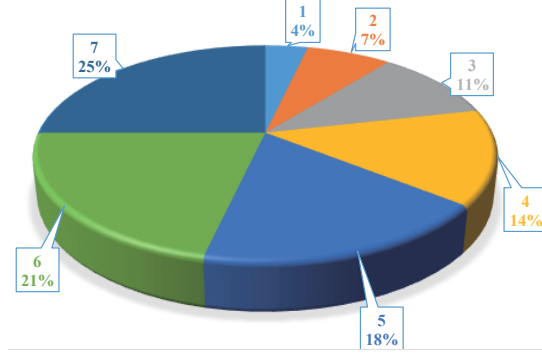


Fig. S-III. Statistical result about the frequency of use for encoded OOs.

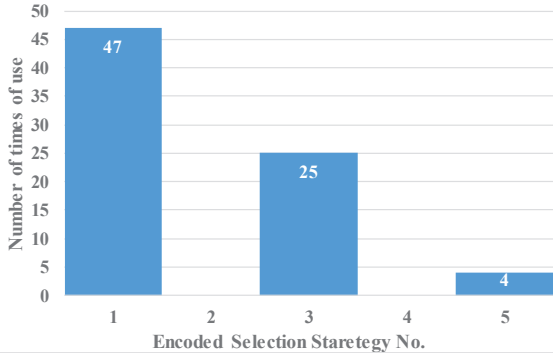


Fig. S-IV. Statistical result about the frequency of use for encoded selection strategies.

Algorithm S-III AutoHOA1.

```

Initialize the Population  $Pop$  with  $N_P$  solutions.
 $PF \leftarrow FitFunc(Pop)$ ;
while The termination criterion has not been met do
  for  $i = 1$  to  $N_P$  do
    if  $PF(i) \leq mean(PF)$  then
       $U1(i, :) \leftarrow I_{1 \times D}$ ;
    end if
  end for
   $U2 \leftarrow \neg U1$ ;
   $X1 \leftarrow DE/rand-to-best/1(Pop)$ ,  $X2 \leftarrow L-SHADE(Pop)$ ;
  for  $i = 1$  to  $N_P$  do
    for  $j = 1$  to  $D$  do
       $Pop1(i, j) \leftarrow X1(i, j) \times U1(i, j) + X2(i, j) \times U2(i, j)$ ;
    end for
  end for
   $PF1 \leftarrow FitFunc(Pop1)$ ;
   $[Pop, PF] \leftarrow TruncationSelector(Pop, Pop1, PF, PF1)$ .
  Update both the algorithm-specific and common control parameters of each OA.
  Update  $N_P$  and reconstruct the population  $Pop$ .
end while
return The best solution (individual) found yet.

```

TABLE S-V
PERFORMANCE OF REPRESENTATIVE AUTOHOAs W.R.T AVERAGE OBJECTIVE VALUE ON THE INSTANCES OF 10D.

No.	AutoHOA1	AutoHOA2	AutoHOA3	AutoHOA4
F1	0	0	0	0
F2	0	0	0	0
F3	0	0	0	0
F4	0	0	0	0
F5	5.989262	3.882291	4.896758	6.418459
F6	0	0	0	0
F7	16.61508	14.47728	14.86438	16.37179
F8	6.145333	4.252962	4.526088	6.867165
F9	0	0	0	0
F10	182.0132	91.332	118.5084	111.3439
F11	0.175581	0.058527	0.19509	0.136563
F12	10.50732	149.1955	166.384	142.3351
F13	2.793383	3.771148	3.973229	3.928654
F14	0.63397	0.117054	0.370671	0.799869
F15	0.177199	0.33924	0.250064	0.158711
F16	0.529385	1.573618	0.944503	0.906088
F17	1.099874	3.153739	0.969813	1.193477
F18	0.253057	5.531763	0.379878	0.217615
F19	0.017878	0.058408	0.014743	0.021726
F20	0.316757	1.250267	0.042847	0.520259
F21	146.0382	183.1276	164.8436	153.8734
F22	88.65585	100.0181	98.34681	98.28639
F23	306.8191	304.8798	299.6806	306.478
F24	298.2844	315.0144	329.3517	313.3955
F25	416.0268	426.8787	421.4008	422.2173
F26	300	300	300	300
F27	388.1659	388.749	386.5391	388.0493
F28	331.1929	416.6496	386.5649	406.0906
F29	229.8395	230.4331	231.4501	230.0301
F30	16010.26	28049.75	47544.58	113528

TABLE S-VI
PERFORMANCE OF REPRESENTATIVE AUTOHOAs W.R.T AVERAGE OBJECTIVE VALUE ON THE INSTANCES OF 30D.

No.	AutoHOA1	AutoHOA2	AutoHOA3	AutoHOA4
F1	0	0	0	0
F2	0	0	0	0
F3	0	0	0	0
F4	55.79055	56.46094	51.8293	52.85361
F5	17.9684	19.5285	21.81333	22.59141
F6	1.67E-07	0	8.72E-09	6.29E-07
F7	47.43944	50.66386	56.34136	53.6757
F8	17.59711	19.92278	24.07635	25.90794
F9	0	0	0	0
F10	1582.283	1692.355	1661.047	1811.252
F11	11.85798	20.64085	18.23403	14.32722
F12	522.8735	568.1258	682.4851	704.6856
F13	14.94828	14.31643	17.03204	17.1151
F14	23.56911	20.41298	24.79523	22.74546
F15	1.996245	4.353342	4.179875	3.948591
F16	73.38453	118.3548	53.58968	68.63058
F17	25.26385	29.7237	19.51562	24.78721
F18	21.15915	22.57456	24.2822	21.65712
F19	4.44995	6.213836	7.248483	5.376878
F20	30.04256	21.61874	17.22131	13.9851
F21	218.4571	218.7257	224.5501	224.0695
F22	100	100	100	100
F23	361.6901	365.2764	367.9637	370.51
F24	435.0101	435.6454	439.2885	441.3287
F25	386.6953	386.6987	386.6894	386.6906
F26	986.0758	1005.747	1020.336	1029.622
F27	503.1152	503.1952	502.7579	503.0513
F28	325.3663	324.9471	330.0297	316.0078
F29	426.7386	423.6565	428.9228	425.911
F30	1985.701	2006.67	2020.665	2006.346

TABLE S-VII

PERFORMANCE OF REPRESENTATIVE AUTOHOAs W.R.T AVERAGE OBJECTIVE VALUE ON THE INSTANCES OF 50D.

No.	AutoHOA1	AutoHOA2	AutoHOA3	AutoHOA4
F1	0.018906	0	0	0
F2	99.47059	0.039216	0	0
F3	6.8E-08	0	0	0
F4	70.70196	61.07463	53.21998	50.75583
F5	30.31038	30.92176	38.88916	38.12924
F6	2.69E-05	1.86E-06	1.01E-06	6.66E-05
F7	80.2019	79.87729	90.05456	89.43796
F8	32.63898	32.58002	39.28129	38.98513
F9	0	0	0	0
F10	4757.74	3421.025	3707.96	4147.744
F11	28.10998	27.34295	27.74718	27.6673
F12	1626.998	1481.529	1634.106	1577.791
F13	90.16885	75.78901	50.62798	45.04648
F14	27.73673	29.49836	31.21125	29.76614
F15	23.90136	26.68806	29.61262	25.39603
F16	326.9392	352.3431	385.7811	359.1915
F17	283.3277	290.9122	317.1407	298.2794
F18	25.91293	26.38503	27.55393	26.52937
F19	15.17111	18.40446	20.98745	17.24924
F20	209.1686	130.357	184.2463	193.5972
F21	231.3328	230.9378	238.5009	236.7207
F22	2208.361	2565.275	3286.679	2131.982
F23	449.8322	450.4261	455.8486	459.3755
F24	524.3088	519.3816	531.7824	529.5548
F25	481.5121	481.8015	481.9556	481.1097
F26	1303.026	1206.942	1327.389	1328.374
F27	528.1136	530.7248	527.7605	527.9893
F28	460.7643	464.5953	459.6857	461.9368
F29	366.1342	372.8953	347.7408	340.5835
F30	611350.6	598690.6	604677.9	610611.7

Algorithm S-IV AutoHOA2.

Initialize the Population Pop with N_P solutions.
 $PF \leftarrow FitFunc(Pop)$, $P_{ind} = 0.5$.
while The termination criterion has not been met **do**
 for $i = 1$ to N_P **do**
 if $rand \leq P_{ind}$ **then**
 $U1(i, :) \leftarrow I_{1 \times D}$;
 else
 $U1(i, :) \leftarrow O_{1 \times D}$;
 end if
 end for
 $U2 \leftarrow \neg U1$, $X1 \leftarrow L-SHADE(Pop)$, $X2 \leftarrow EDA(Pop)$;
 for $i = 1$ to N_P **do**
 for $j = 1$ to D **do**
 $Pop1(i, j) \leftarrow X1(i, j) \times U1(i, j) + X2(i, j) \times U2(i, j)$;
 end for
 end for
 $PF1 \leftarrow FitFunc(Pop1)$;
 for $i = 1$ to N_P **do**
 if $PF1(i) \leq PF(i)$ **then**
 $Pop(i, :) \leftarrow Pop1(i, :)$, $PF(i) \leftarrow PF1(i)$;
 end if
 end for
 Update P_{ind} according to the performance feedback of all the individuals and update both the algorithm-specific and common control parameters of each OA.
 Update the probability model of EDA based on the elite individuals in Pop . Then, update N_P and reconstruct the population.
end while
return The best solution (individual) found yet.

Algorithm S-V AutoHOA3.

Initialize the Population Pop with N_P solutions.
 $PF \leftarrow FitFunc(Pop)$, $U1(i, :) \leftarrow I_{N_P \times D}$, $F_b \leftarrow \min(PF)$.
while The termination criterion has not been met **do**
 $Pop1 \leftarrow SHADE(Pop)$,
 $PF1 \leftarrow FitFunc(Pop1)$;
 if $\min(PF \cup PF1) < F_b$ **then**
 $F_b \leftarrow \min(PF \cup PF1)$;
 else
 $X2 \leftarrow L-SHADE(Pop)$, $Pop1 \leftarrow Pop$;
 for $i = 1$ to N_P **do**
 for $j = 1$ to D **do**
 if $rand \leq P_{comp}$ **then**
 $Pop1(i, j) \leftarrow X2(i, j)$;
 end if
 end for
 end for
 $PF1 \leftarrow FitFunc(Pop1)$;
 end if
 for $i = 1$ to N_P **do**
 if $PF1(i) \leq PF(i)$ **then**
 $Pop(i, :) \leftarrow Pop1(i, :)$, $PF(i) \leftarrow PF1(i)$;
 end if
 end for
 Update both the algorithm-specific and common control parameters of each OA.
 Update N_P and reconstruct the population Pop .
end while
return The best solution (individual) found yet.

Algorithm S-VI AutoHOA4.

Initialize the Population Pop with N_P solutions.
 $PF \leftarrow FitFunc(Pop)$.
while The termination criterion has not been met **do**
 $X1 \leftarrow SHADE(Pop)$, $X2 \leftarrow DE/current-to-rand/I(Pop)$;
 $F1 \leftarrow FitFunc(X1)$, $F2 \leftarrow FitFunc(X2)$;
 for $i = 1$ to N_P **do**
 if $F1(i) < F2(i)$ **then**
 $Pop1(i, :) \leftarrow X1(i, :)$, $PF1(i) \leftarrow F1(i)$;
 else
 $Pop1(i, :) \leftarrow X2(i, :)$, $PF1(i) \leftarrow F2(i)$;
 end if
 if $PF1(i) \leq PF(i)$ **then**
 $Pop(i, :) \leftarrow Pop1(i, :)$, $PF(i) \leftarrow PF1(i)$;
 end if
 end for
 Update both the algorithm-specific and common control parameters of each OA.
end while
return The best solution (individual) found yet.
