The CAPM Formula

$$R_i = R_f + \beta_i \times (R_m - R_f)$$

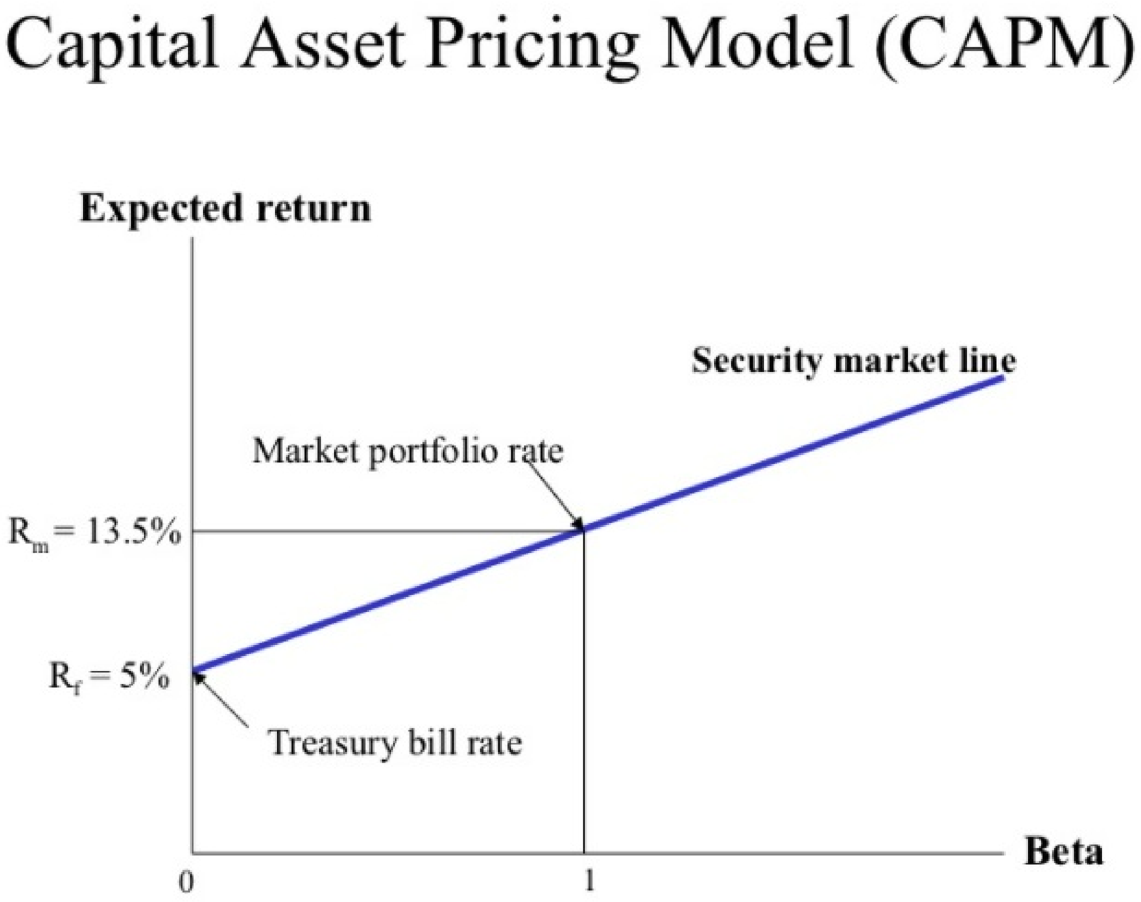
Where:

- R_i: expected return on the investment i
- R_f: risk-free rate of return
- β_i: beta of the investment
- R_m: expected return of the market

Beta is calculated as:

$$\beta_i = Cov(R_i, R_m)/Var(R_m)$$

A nice visualisation of CAPM is as follow:



In [1]:

```
import pandas as pd
import pandas_datareader.data as web
import yfinance as yf
import numpy as np
import datetime as dt
import matplotlib.pyplot as plt
import seaborn as sns
import getFamaFrenchFactors as gff
import statsmodels.api as sm
```

1. Download stock prices and Fama-French 3 factors using Python codes.

In [2]:

```python
# To obtain the data of the five stocks, we will use the pandas_datareader.data to obtain the various stocks from yahoo

# we will acquire the stock data over the past five years starting from 2018,7,31 down to 2022,7,31
start = dt.datetime(2021,1,30)
end = dt.datetime(2022,10,31)

# Define the selected tickers in a list and use it to fetch the stock data from Yahoo Finance.
tickers = ['GOOG','AAPL', 'TSLA', 'MSFT', 'CAT']
```

In [3]:

```python
#Extract the adjusted close prices from the stock data
#df = web.get_data_yahoo(tickers, start, end)
df = yf.download(tickers, start, end )
df.head()
```

```
[*********************100%***********************]  5 of 5 completed
```

Out[3]:

	Adj Close					Close				
	AAPL	CAT	GOOG	MSFT	TSLA	AAPL	CAT	GOOG	MSFT	TSLA
Date										
2021-02-01	132.548828	177.725113	95.067497	235.451660	279.936676	134.139999	184.720001	95.067497	239.649994	279.9366
2021-02-02	133.388748	185.210495	96.375504	235.314117	290.929993	134.990005	192.500000	96.375504	239.509995	290.9299
2021-02-03	132.351212	184.200272	103.503502	238.742981	284.896667	133.940002	191.449997	103.503502	243.000000	284.8966
2021-02-04	135.760284	184.392670	103.118500	237.770309	283.329987	137.389999	191.649994	103.118500	242.009995	283.3299
2021-02-05	135.339676	185.691559	104.900002	237.956970	284.076660	136.759995	193.000000	104.900002	242.199997	284.0766

5 rows × 30 columns

In [4]:

```python
df.shape
```

Out[4]:

```
(441, 30)
```

In [5]:

```python
df = df['Adj Close']
df.head()
```

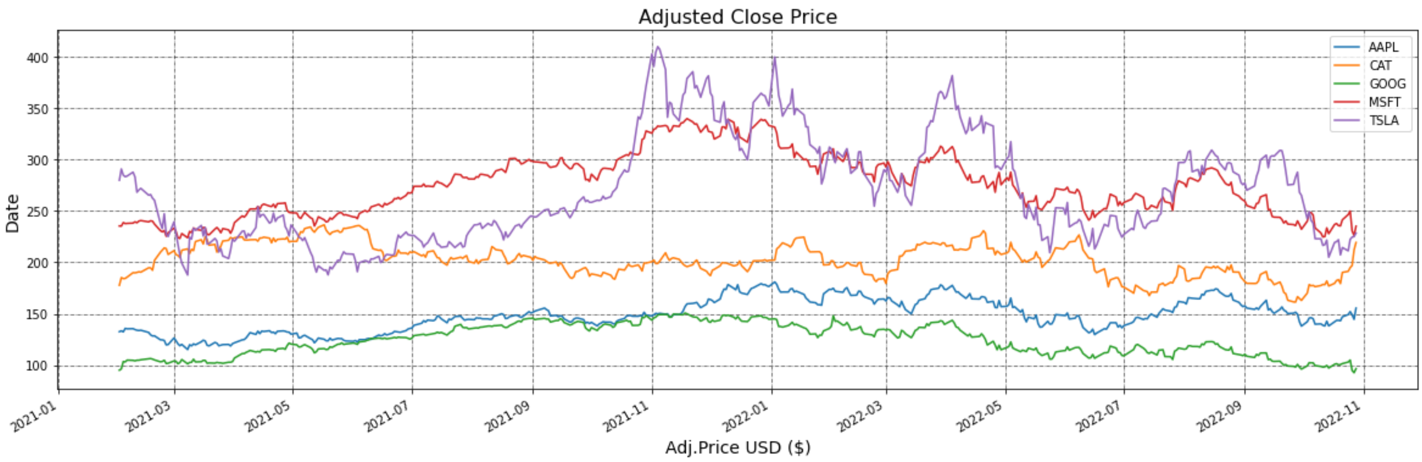
Out[5]:

	AAPL	CAT	GOOG	MSFT	TSLA
Date					
2021-02-01	132.548828	177.725113	95.067497	235.451660	279.936676

Date	AAPL	CAT	GOOG	MSFT	TSLA
2021-02-02	133.888748	185.210495	96.875504	235.314117	290.929993
2021-02-03	132.351212	184.200272	103.503502	238.742981	284.896667
2021-02-04	135.760284	184.392670	103.118500	237.770309	283.329987
2021-02-05	135.339676	185.691559	104.900002	237.956970	284.076660

In [6]:

```
df.plot(figsize=(20, 6))

# Define the label for the title of the figure
plt.title("Adjusted Close Price", fontsize=16)

# Define the labels for x-axis and y-axis
plt.ylabel('Date', fontsize=14)
plt.xlabel('Adj.Price USD ($)', fontsize=14)

# Plot the grid lines
plt.grid(which="major", color='k', linestyle='-.', linewidth=0.5)

# Show the legend
plt.legend()

plt.show()
```



In [7]:

```
#We'll Use the pct_change function to calculate the percentage difference of monthly price compared with the previous month.
#The percentage difference is used as the monthly rate of stock return
returns = df.pct_change()[1:]
returns
```

Out[7]:

Date	AAPL	CAT	GOOG	MSFT	TSLA
2021-02-02	0.006337	0.042118	0.013759	-0.000584	0.039271
2021-02-03	-0.007778	-0.005454	0.073961	0.014571	-0.020738
2021-02-04	0.025758	0.001045	-0.003720	-0.004074	-0.005499
2021-02-05	-0.003098	0.007044	0.017276	0.000785	0.002635
2021-02-08	0.001097	0.023057	-0.002426	0.001115	0.013130
...	...	...	...	...	...
2022-10-24	0.014803	0.005678	0.014683	0.021188	-0.014876
2022-10-25	0.019338	0.018505	0.019035	0.013792	0.052876
2022-10-26	-0.019627	0.010881	-0.096350	-0.077156	0.009981
2022-10-27	-0.030465	0.077071	-0.023413	-0.019756	0.002003

	AAPL	CAT	GOOG	MSFT	TSLA
Date					
2022-10-28	0.075553	0.033940	0.042981	0.040220	0.015238

440 rows × 5 columns

In [8]:

```
# we will resample our acquired stock data to convert them into monthly prices.
# we use the dropna function to remove any row with null values.
returns_monthly = returns.resample('M').last().dropna()
returns_monthly.head()
```

Out[8]:

	AAPL	CAT	GOOG	MSFT	TSLA
Date					
2021-02-28	0.002232	-0.026779	0.002708	0.014804	-0.009850
2021-03-31	0.018766	-0.005191	0.006368	0.016908	0.050832
2021-04-30	-0.015133	0.002813	-0.008136	-0.001307	0.047917
2021-05-31	-0.005348	-0.000829	0.003767	0.001484	-0.008925
2021-06-30	0.004621	0.011762	-0.005575	-0.001842	-0.001557

In [9]:

```
returns_monthly.shape
```

Out[9]:

```
(21, 5)
```

Fama-French 3 factors model

Developed in 1992 by University of Chicago professors Eugene Fama and Kenneth French, it is based on the observation that value shares tend to outperform growth shares and small-cap shares tend to outperform large-cap shares. Jumping off those observations the two economists developed their three-factor model as an expansion of the Capital Asset Pricing model (CAPM). Rather than just gauge market risk as the CAPM does, the Fama-French Three Factor model adds value risk and size risk to the calculation.

The Fama-French Three Factor model calculates an investment's likely rate of return based on three elements:

overall market risk, the degree to which small companies outperform large companies and the degree to which high-value companies outperform low-value companies. Fama-French 3 factors formula:

$$R_i = R_f + \beta_i \times (R_m - R_f)$$

Where:

- R_i: expected return on the investment i
- R_f: risk-free rate of return
- β_i: beta of the investment
- R_m: expected return of the market

Fama-French regression

The regression equation can be written as:

$$R_i = R_f + \beta_i \times (R_m - R_f)$$

Where:

- R_i: expected return on the investment i
- R_f: risk-free rate of return
- β_i: beta of the investment
- R_m: expected return of the market

In [10]:

```
#downloading fama french data
ff = web.DataReader('F-F_Research_Data_Factors', 'famafrench', start, end)[0]
ff.head(2)
```

Out[10]:

	Mkt-RF	SMB	HML	RF
Date				
2021-01	-0.03	7.34	2.96	0.01
2021-02	2.78	2.06	7.18	0.00

In [11]:

```
ff = ff[1:]
ff.shape
```

Out[11]:

```
(21, 4)
```

In [12]:

```
# printing the shape of the stocks and fama french 3 factor model.
# from the data, we have equal row and columns from both stocks and fama french 3 factor
print(returns_monthly.shape)
print(ff.shape)
```

```
(21, 5)
(21, 4)
```

In [13]:

```
# getting same index for monthly stocks and fama french 3 factor
returns_monthly.index = ff.index
returns_monthly.head()
```

Out[13]:

	AAPL	CAT	GOOG	MSFT	TSLA
Date					
2021-02	0.002232	-0.026779	0.002708	0.014804	-0.009850
2021-03	0.018766	-0.005191	0.006368	0.016908	0.050832
2021-04	-0.015133	0.002813	-0.008136	-0.001307	0.047917
2021-05	-0.005348	-0.000829	0.003767	0.001484	-0.008925
2021-06	0.004621	0.011762	-0.005575	-0.001842	-0.001557

2. Merge/combine the datasets

Note: The expression "Mkt-RF" refers to the market premium. Market premium is the difference between the expected return on a market portfolio and the risk-free rate.

In [14]:

```
# Merging stocks return and fama french 3 factor on Date
merge = pd.merge(returns_monthly, ff, on = 'Date')
merge
```

Out[14]:

Date	AAPL	CAT	GOOG	MSFT	TSLA	Mkt-RF	SMB	HML	RF
2021-02	0.002232	-0.026779	0.002708	0.014804	-0.009850	2.78	2.06	7.18	0.00
2021-03	0.018766	-0.005191	0.006368	0.016908	0.050832	3.08	-2.37	7.40	0.00
2021-04	-0.015133	0.002813	-0.008136	-0.001307	0.047917	4.93	-3.19	-0.94	0.00
2021-05	-0.005348	-0.000829	0.003767	0.001484	-0.008925	0.29	-0.25	7.08	0.00
2021-06	0.004621	0.011762	-0.005575	-0.001842	-0.001557	2.75	1.70	-7.82	0.00
2021-07	0.001511	-0.027333	-0.009664	-0.005550	0.014542	1.27	-3.99	-1.76	0.00
2021-08	-0.008425	-0.002743	-0.000052	-0.005633	0.006581	2.91	-0.43	-0.16	0.00
2021-09	-0.009312	-0.029818	-0.009333	-0.007324	-0.007462	-4.37	0.72	5.08	0.00
2021-10	-0.018155	-0.000392	0.014655	0.022414	0.034316	6.65	-2.35	-0.48	0.00
2021-11	0.031578	-0.013118	-0.025063	-0.017943	0.006834	-1.55	-1.32	-0.44	0.00
2021-12	-0.003535	0.003203	-0.009061	-0.008841	-0.012669	3.10	-1.66	3.28	0.01
2022-01	0.026126	0.001988	0.018073	0.008824	0.106776	-6.25	-5.94	12.75	0.00
2022-02	0.001638	0.002780	0.002762	0.004978	0.074777	-2.29	2.23	3.04	0.00
2022-03	-0.017776	-0.001165	-0.020996	-0.017683	-0.014982	3.05	-1.60	-1.80	0.01
2022-04	-0.036605	-0.008944	-0.037224	-0.041812	-0.007692	-9.46	-1.41	6.19	0.01
2022-05	-0.005346	-0.005941	0.010993	-0.005014	-0.001804	-0.34	-1.85	8.41	0.03
2022-06	-0.018028	-0.025725	-0.025691	-0.013179	-0.017579	-8.43	2.09	-5.97	0.06
2022-07	0.032793	0.055419	0.017890	0.015665	0.057850	9.57	2.81	-4.10	0.08
2022-08	-0.010635	-0.011929	-0.006915	-0.005704	-0.007526	-3.77	1.39	0.31	0.19
2022-09	-0.030039	-0.010970	-0.019778	-0.019368	-0.011036	-9.35	-0.82	0.03	0.19
2022-10	0.075553	0.033940	0.042981	0.040220	0.015238	7.83	0.10	8.06	0.23

In [15]:

```
# Diving Fama french library by 100 to be on the same scale with the stocks return
merge[['Mkt-RF','SMB','HML','RF']] = merge[['Mkt-RF','SMB','HML','RF']] / 100
# rename column:
merge.rename(columns = {'Mkt-RF': 'Mkt_RF'}, inplace = True)

merge
```

Out[15]:

Date	AAPL	CAT	GOOG	MSFT	TSLA	Mkt_RF	SMB	HML	RF
2021-02	0.002232	-0.026779	0.002708	0.014804	-0.009850	0.0278	0.0206	0.0718	0.0000
2021-03	0.018766	-0.005191	0.006368	0.016908	0.050832	0.0308	-0.0237	0.0740	0.0000
2021-04	-0.015133	0.002813	-0.008136	-0.001307	0.047917	0.0493	-0.0319	-0.0094	0.0000
2021-05	-0.005348	-0.000829	0.003767	0.001484	-0.008925	0.0029	-0.0025	0.0708	0.0000
2021-06	0.004621	0.011762	-0.005575	-0.001842	-0.001557	0.0275	0.0170	-0.0782	0.0000

Date	AAPL	CAT	GOOG	MSFT	TSLA	Mkt-RF	SMB	HML	RF
2021-07	0.001511	-0.027333	-0.009664	-0.005550	0.014542	0.0127	-0.0399	-0.0176	0.0000
2021-08	-0.008425	-0.002743	-0.000052	-0.005633	0.006581	0.0291	-0.0043	-0.0016	0.0000
2021-09	-0.009312	-0.029818	-0.009333	-0.007324	-0.007462	-0.0437	0.0072	0.0508	0.0000
2021-10	-0.018155	-0.000392	0.014655	0.022414	0.034316	0.0665	-0.0235	-0.0048	0.0000
2021-11	0.031578	-0.013118	-0.025063	-0.017943	0.006834	-0.0155	-0.0132	-0.0044	0.0000
2021-12	-0.003535	0.003203	-0.009061	-0.008841	-0.012669	0.0310	-0.0166	0.0328	0.0001
2022-01	0.026126	0.001988	0.018073	0.008824	0.106776	-0.0625	-0.0594	0.1275	0.0000
2022-02	0.001638	0.002780	0.002762	0.004978	0.074777	-0.0229	0.0223	0.0304	0.0000
2022-03	-0.017776	-0.001165	-0.020996	-0.017683	-0.014982	0.0305	-0.0160	-0.0180	0.0001
2022-04	-0.036605	-0.008944	-0.037224	-0.041812	-0.007692	-0.0946	-0.0141	0.0619	0.0001
2022-05	-0.005346	-0.005941	0.010993	-0.005014	-0.001804	-0.0034	-0.0185	0.0841	0.0003
2022-06	-0.018028	-0.025725	-0.025691	-0.013179	-0.017579	-0.0843	0.0209	-0.0597	0.0006
2022-07	0.032793	0.055419	0.017890	0.015665	0.057850	0.0957	0.0281	-0.0410	0.0008
2022-08	-0.010635	-0.011929	-0.006915	-0.005704	-0.007526	-0.0377	0.0139	0.0031	0.0019
2022-09	-0.030039	-0.010970	-0.019778	-0.019368	-0.011036	-0.0935	-0.0082	0.0003	0.0019
2022-10	0.075553	0.033940	0.042981	0.040220	0.015238	0.0783	0.0010	0.0806	0.0023

3. Produce and discuss the covarience matrix and summary statistics of stock returns during the sample period.

covarience is a mathemimatical concept that is mostly use in statictics. it determines how amount two random variables varies or move together. it is the direction or relationship between two asset prices

In [16]:

```
# Getting Coveriance matrix for the stocks return
cov_matrix = returns_monthly.cov()
cov_matrix
```

Out[16]:

	AAPL	CAT	GOOG	MSFT	TSLA
AAPL	0.000631	0.000274	0.000321	0.000304	0.000346
CAT	0.000274	0.000391	0.000211	0.000165	0.000279
GOOG	0.000321	0.000211	0.000333	0.000295	0.000312
MSFT	0.000304	0.000165	0.000295	0.000307	0.000282
TSLA	0.000346	0.000279	0.000312	0.000282	0.001169

Variance is define as how a set of observation defers from one another. note the square root of varience is volatility

In [17]:

```
# Getting the variance for the stock returns
returns_monthly.var()
```

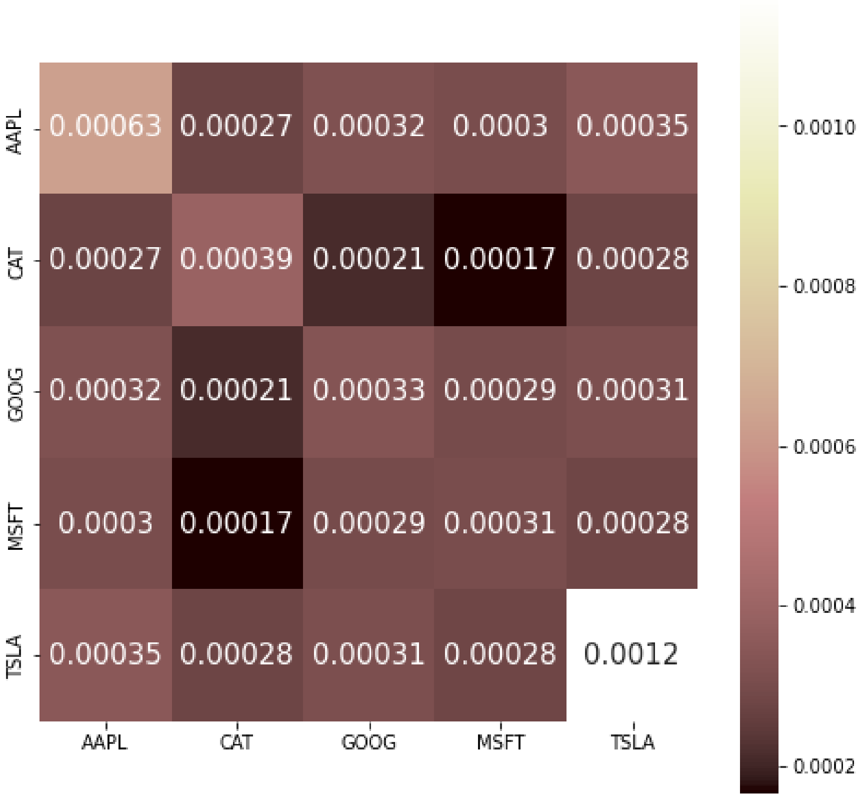
Out[17]:

```
AAPL    0.000631
CAT     0.000391
GOOG    0.000333
MSFT    0.000307
TSLA    0.001169
dtype: float64
```

Coveriance Map

In [18]:

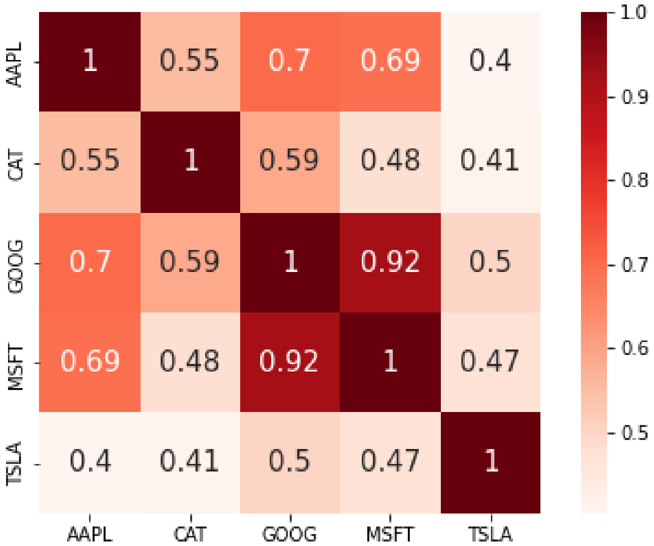
```
plot = plt.figure(figsize=(8,8))
sns.heatmap(returns_monthly.cov(), cmap='pink', cbar=True, square=True, annot=True, annot
_kws={'size':15})
plt.show()
```



Correlation Map

In [19]:

```
plot = plt.figure(figsize=(8,5))
sns.heatmap(returns_monthly.corr(), cmap='Reds', cbar=True, square=True, annot=True, anno
t_kws={'size':15})
plt.show()
```



Statistical Summary

In [20]:

```
# Statistical S
```

```
# Statistical Summary
returns_monthly.describe()
```

Out[20]:

	AAPL	CAT	GOOG	MSFT	TSLA
count	21.000000	21.000000	21.000000	21.000000	21.000000
mean	0.000785	-0.002808	-0.002728	-0.001233	0.014980
std	0.025116	0.019783	0.018244	0.017510	0.034196
min	-0.036605	-0.029818	-0.037224	-0.041812	-0.017579
25%	-0.015133	-0.011929	-0.009664	-0.008841	-0.008925
50%	-0.005346	-0.002743	-0.005575	-0.005014	-0.001557
75%	0.004621	0.002780	0.006368	0.008824	0.034316
max	0.075553	0.055419	0.042981	0.040220	0.106776

4. Consider the collection of stocks in your data sample as a portfolio. Perform portfolio optimization to show: (1) portfolio with the lowest risk, (2) portfolio with the highest return and (3) portfolio with the highest sharpe ratio.

$$\beta_i = Cov(R_i, R_m)/Var(R_m)$$

$$R_i = R_f + \beta_i \times (R_m - R_f)$$

Where:

- R_i: expected return on the investment i
- R_f: risk-free rate of return
- β_i: beta of the investment
- R_m: expected return of the market

In [127]:

```
#portfolio Optimization

portfolio_returns = []
portfolio_risks = []
sharp_ratios = []
portfolio_weights = []

RF = 0.01

number_of_portfolios = 4500

for portfolio in range(number_of_portfolios):

    # Generate random portfolio weights
    weights = np.random.random_sample(len(tickers))

    # all the weight did not round up to 1 and for us to do this we
    # need to summarize the whole weight and divide the indiviadual by the total
    weights = np.round((weights / np.sum(weights)), 3)
    portfolio_weights.append(weights)
```

```
    #calculate annual return
    annual_return = np.sum(returns.mean() * weights) * 252
    portfolio_returns.append(annual_return)

    #matrix covariance & portfolio risk calculation
    matrix_covariance = returns.cov() * 252
    portfolio_variance = np.dot(weights.T, np.dot(matrix_covariance, weights))
    portfolio_standard_deviation = np.sqrt(portfolio_variance)
    portfolio_risks.append(portfolio_standard_deviation)

    #Sharp ratio
    sharp_ratio = (annual_return - RF / portfolio_standard_deviation)
    sharp_ratios.append(sharp_ratio)

portfolio_returns = np.array(portfolio_returns)
portfolio_risks = np.array(portfolio_risks)
sharp_ratios = np.array(sharp_ratios)

portfolio_metrics = [portfolio_returns, portfolio_risks,sharp_ratios, portfolio_weights]

portfolios_df = pd.DataFrame(portfolio_metrics).T
portfolios_df.columns = ['Return', 'Risk', 'Sharp','Weights']

lowest_risk = portfolios_df.iloc[portfolios_df['Risk']
                                .astype(float).idxmin()]
highest_return = portfolios_df.iloc[portfolios_df['Return']
                                .astype(float).idxmin()]

highest_sharpe_ratio  = portfolios_df.iloc[portfolios_df['Sharp']
                                .astype(float).idxmin()]
print("matrix_covariance:")
print(matrix_covariance)
print("......")
print("portfolio_variance:",portfolio_variance)
print("......")
print("portfolio_standard_deviation:",portfolio_standard_deviation)
print("......")
print("weights:",weights )
```

```
matrix_covariance:
          AAPL       CAT      GOOG      MSFT      TSLA
AAPL  0.087998  0.025332  0.065109  0.064787  0.103391
CAT   0.025332  0.092643  0.027599  0.021993  0.041433
GOOG  0.065109  0.027599  0.097354  0.070634  0.087013
MSFT  0.064787  0.021993  0.070634  0.079864  0.087935
TSLA  0.103391  0.041433  0.087013  0.087935  0.356306
......
portfolio_variance: 0.092095737623719
......
portfolio_standard_deviation: 0.303472795524935
......
weights: [0.458 0.112 0.186 0.003 0.242]
```

Portfolio with the highest Sharpe ratio

Sharpe ratio:

- **Represents both the risk and return**
- **Developed by Nobel laureate William F. Sharpe and is used to help investors understand the return of an investment compared to its risk.**
  The Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. The formula used to calculate Sharpe-ratio is given below:

$$R_i = R_f + \beta_i \times (R_m - R_f)$$

Where:

- R_i: expected return on the investment i

- R_f: risk-free rate of return
- $β_i$: beta of the investment
- R_m: expected return of the market

In [22]:

```
#lowest_risk
print('lowest_risk')
print(lowest_risk)
print(tickers)
print("")

#highest_return
print('highest_return')
print(highest_return)
print(tickers)
print("")

#highest_sharpe_ratio
print('highest_sharpe_ratio')
print(highest_sharpe_ratio)
print(tickers)
print("")
```

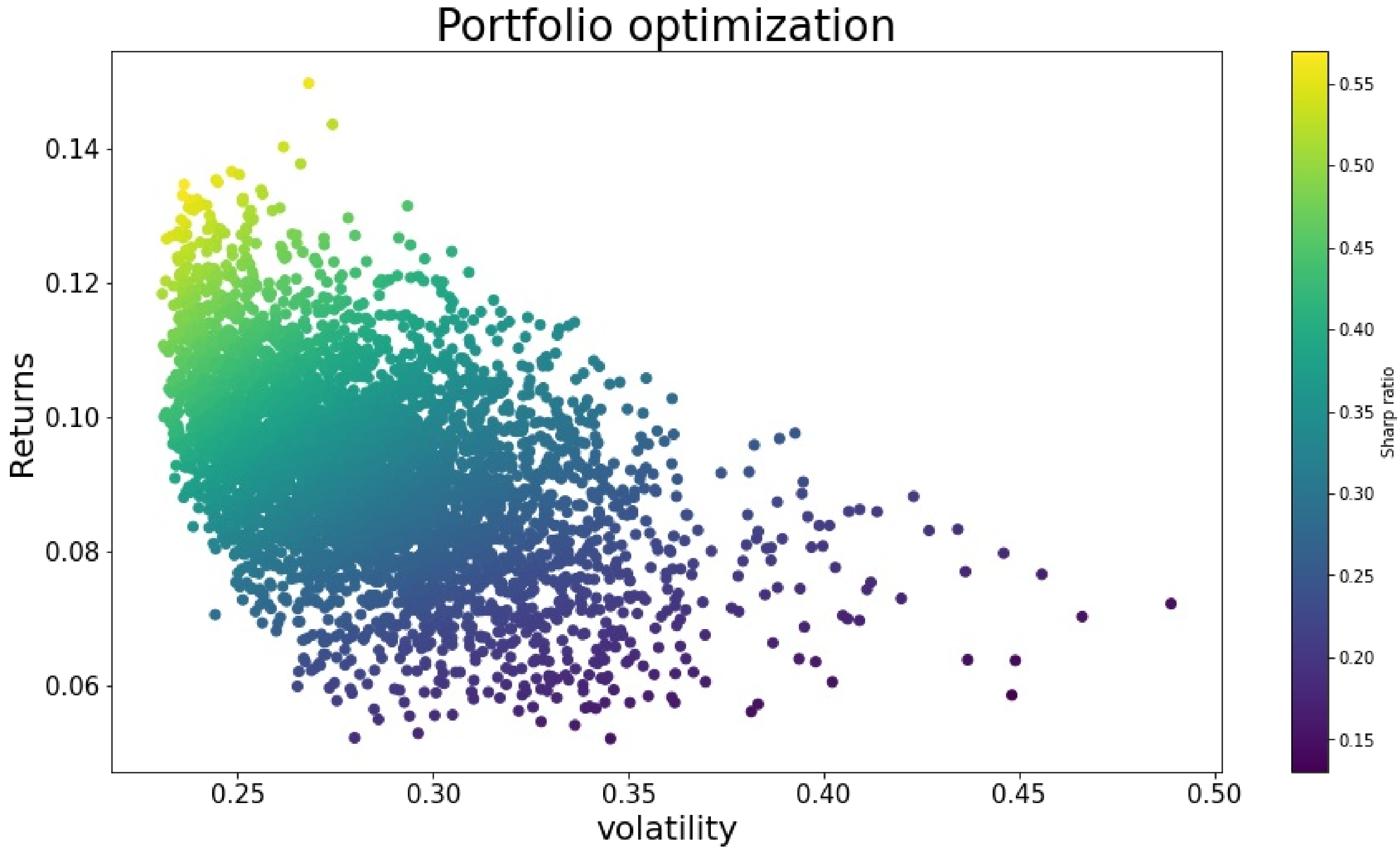
```
lowest_risk
Return                                      0.118328
Risk                                        0.230673
Sharp                                       0.074977
Weights    [0.297, 0.393, 0.017, 0.287, 0.006]
Name: 1584, dtype: object
['GOOG', 'AAPL', 'TSLA', 'MSFT', 'CAT']

highest_return
Return                                    0.051961
Risk                                      0.345385
Sharp                                     0.023008
Weights    [0.018, 0.015, 0.05, 0.547, 0.37]
Name: 790, dtype: object
['GOOG', 'AAPL', 'TSLA', 'MSFT', 'CAT']

highest_sharpe_ratio
Return                                      0.052081
Risk                                        0.279917
Sharp                                       0.016357
Weights    [0.039, 0.011, 0.337, 0.549, 0.064]
Name: 2590, dtype: object
['GOOG', 'AAPL', 'TSLA', 'MSFT', 'CAT']
```

Visualization

In [23]:

```
plt.figure(figsize = (15,8))
plt.scatter(portfolio_risks, portfolio_returns,
            c = portfolio_returns / portfolio_risks)

plt.title('Portfolio optimization', fontsize =26)
plt.xlabel('volatility', fontsize = 20)
plt.ylabel('Returns', fontsize = 20)
plt.xticks(fontsize = 15)
plt.yticks(fontsize = 15)
plt.colorbar(label = 'Sharp ratio')
```

```
plt.show()
```



5. Estimate the empirical asset pricing models for a single stock. Visualise the results i.e., plot the data points and the estimated regression line on a figure(s) and discuss your results.

$$\beta_i = Cov(R_i, R_m)/Var(R_m)$$

$$R_i = R_f + \beta_i \times (R_m - R_f)$$

Where:

- R_i: expected return on the investment i
- R_f: risk-free rate of return
- β_i: beta of the investment
- R_m: expected return of the market

Note that, except for alpha, this is the equation for CAPM - that is, the beta you get from Sharpe's derivation of equilibrium prices is essentially the same beta you get from doing a least-squares regression against the data.

Beta is the slope of this line. Alpha, the vertical intercept, tells you how much better/worse the investment did than CAPM predicted.

In [24]:

```
# generate Google excess return:
merge['GOOG_Rf'] = merge['GOOG'] - merge['RF']

merge
```

Out[24]:

Date	AAPL	CAT	GOOG	MSFT	TSLA	Mkt_RF	SMB	HML	RF	GOOG_Rf
2021-02	0.002232	-0.026779	0.002708	0.014804	-0.009850	0.0278	0.0206	0.0718	0.0000	0.002708
2021-03	0.018766	-0.005191	0.006368	0.016908	0.050832	0.0308	-0.0237	0.0740	0.0000	0.006368
2021-04	-0.015133	0.002813	-0.008136	-0.001307	0.047917	0.0493	-0.0319	-0.0094	0.0000	-0.008136
2021-05	-0.005348	-0.000829	0.003767	0.001484	-0.008925	0.0029	-0.0025	0.0708	0.0000	0.003767
2021-06	0.004621	0.011762	-0.005575	-0.001842	-0.001557	0.0275	0.0170	-0.0782	0.0000	-0.005575
2021-07	0.001511	-0.027333	-0.009664	-0.005550	0.014542	0.0127	-0.0399	-0.0176	0.0000	-0.009664
2021-08	-0.008425	-0.002743	-0.000052	-0.005633	0.006581	0.0291	-0.0043	-0.0016	0.0000	-0.000052
2021-09	-0.009312	-0.029818	-0.009333	-0.007324	-0.007462	-0.0437	0.0072	0.0508	0.0000	-0.009333
2021-10	-0.018155	-0.000392	0.014655	0.022414	0.034316	0.0665	-0.0235	-0.0048	0.0000	0.014655
2021-11	0.031578	-0.013118	-0.025063	-0.017943	0.006834	-0.0155	-0.0132	-0.0044	0.0000	-0.025063
2021-12	-0.003535	0.003203	-0.009061	-0.008841	-0.012669	0.0310	-0.0166	0.0328	0.0001	-0.009161
2022-01	0.026126	0.001988	0.018073	0.008824	0.106776	-0.0625	-0.0594	0.1275	0.0000	0.018073
2022-02	0.001638	0.002780	0.002762	0.004978	0.074777	-0.0229	0.0223	0.0304	0.0000	0.002762
2022-03	-0.017776	-0.001165	-0.020996	-0.017683	-0.014982	0.0305	-0.0160	-0.0180	0.0001	-0.021096
2022-04	-0.036605	-0.008944	-0.037224	-0.041812	-0.007692	-0.0946	-0.0141	0.0619	0.0001	-0.037324
2022-05	-0.005346	-0.005941	0.010993	-0.005014	-0.001804	-0.0034	-0.0185	0.0841	0.0003	0.010693
2022-06	-0.018028	-0.025725	-0.025691	-0.013179	-0.017579	-0.0843	0.0209	-0.0597	0.0006	-0.026291
2022-07	0.032793	0.055419	0.017890	0.015665	0.057850	0.0957	0.0281	-0.0410	0.0008	0.017090
2022-08	-0.010635	-0.011929	-0.006915	-0.005704	-0.007526	-0.0377	0.0139	0.0031	0.0019	-0.008815
2022-09	-0.030039	-0.010970	-0.019778	-0.019368	-0.011036	-0.0935	-0.0082	0.0003	0.0019	-0.021678
2022-10	0.075553	0.033940	0.042981	0.040220	0.015238	0.0783	0.0010	0.0806	0.0023	0.040681

Note: Beta is a measure of the volatility of a security in comparison to the market as a whole. This is important to understand what beta tells us:

If beta = 1.0, the security (e.g. stock) price is perfectly correlated with the market. If beta < 1.0, the security is less volatile than the market If beta > 1.0, the security is more volatile than the market.

The beta can be estimated using a linear regression approach. We can reshape the CAPM formula and obtain a linear equation as below:

Ra = Rf + B(Mkt - Rf) + C

y = BX + C

Where

```
y = Ra - Rf

B = Beta of the stock

X = Mkt - Rf (Market Return)

C = constant
```

GOOG-Rf

We'll be Using the Python Stats package to estimate the beta and also we will make use of the stats OLS

function to build a linear model by fetching the y and C parameters to the function. Next, display the regression summary.

CAMP Regression

In [41]:

```
# construct independent variable:
GOOG_Rf_Xx = merge['Mkt_RF']

# construct dependent variable:
GOOG_Rf_yy = merge['GOOG_Rf']

# add constant:
GOOG_Rf_X11 = sm.add_constant(GOOG_Rf_Xx)

# run regression:
GOOG_model1 = sm.OLS(GOOG_Rf_yy, GOOG_Rf_X11).fit()

# output results:
print(GOOG_model1.summary())

#GOOG_intercept and GOOG_beta
GOOG_intercept1, GOOG_beta= GOOG_model1.params

# predict GOOG_predicted
GOOG_predicted1 = GOOG_model1._results.predict(GOOG_Rf_X11)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                GOOG_Rf   R-squared:                       0.379
Model:                            OLS   Adj. R-squared:                  0.346
Method:                 Least Squares   F-statistic:                     11.59
Date:                Sun, 18 Dec 2022   Prob (F-statistic):            0.00298
Time:                        12:48:09   Log-Likelihood:                 59.989
No. Observations:                  21   AIC:                            -116.0
Df Residuals:                      19   BIC:                            -113.9
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0033      0.003     -1.049      0.308      -0.010       0.003
Mkt_RF         0.2026      0.060      3.404      0.003       0.078       0.327
==============================================================================
Omnibus:                        3.009   Durbin-Watson:                   1.902
Prob(Omnibus):                  0.222   Jarque-Bera (JB):                1.605
Skew:                           0.656   Prob(JB):                        0.448
Kurtosis:                       3.334   Cond. No.                         18.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

From the summary result, we can look for the coefficient of the “Mkt-RF” and use it as our beta value. The result above shows that the beta is 1.0931

With the risk-free rate, the rate of market return, and the beta ready, we can now proceed to estimate the expected return on Google stock by applying the formula

In [42]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()

expected_monthly_return = rf + GOOG_beta * market_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.000617
Expected yearly return: 0.007407
```

Fama French Regression

In [62]:

```
# construct independent variable:
GOOG_Rf_X = merge[['Mkt_RF','SMB','HML']]

# construct dependent variable:
GOOG_Rf_y = merge['GOOG_Rf']

# add constant:
GOOG_Rf_X1 = sm.add_constant(GOOG_Rf_X )

# run regression:
GOOG_model = sm.OLS(GOOG_Rf_y, GOOG_Rf_X1).fit()

# output results:
print(GOOG_model.summary())

#GOOG_intercept and GOOG_beta
GOOG_intercept, GOOG_beta1, GOOG_beta2, GOOG_beta3 = GOOG_model.params

# predict GOOG_predicted
GOOG_predicted = GOOG_model._results.predict(GOOG_Rf_X1)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                GOOG_Rf   R-squared:                       0.645
Model:                            OLS   Adj. R-squared:                  0.583
Method:                 Least Squares   F-statistic:                     10.32
Date:                Sun, 18 Dec 2022   Prob (F-statistic):           0.000421
Time:                        12:59:03   Log-Likelihood:                 65.879
No. Observations:                  21   AIC:                            -123.8
Df Residuals:                      17   BIC:                            -119.6
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0067      0.003     -2.405      0.028      -0.013      -0.001
Mkt_RF         0.2266      0.048      4.717      0.000       0.125       0.328
SMB            0.1120      0.124      0.904      0.379      -0.149       0.374
HML            0.1902      0.054      3.548      0.002       0.077       0.303
==============================================================================
Omnibus:                        0.430   Durbin-Watson:                   1.532
Prob(Omnibus):                  0.806   Jarque-Bera (JB):                0.548
Skew:                          -0.119   Prob(JB):                        0.760
Kurtosis:                       2.245   Cond. No.                         49.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [63]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()
size_premium = merge['SMB'].mean()
value_premium = merge['HML'].mean()

expected_monthly_return = rf + GOOG_beta1 * market_premium + GOOG_beta2 * size_premium +
GOOG_beta3 * value_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.004000
Expected yearly return: 0.047995
```

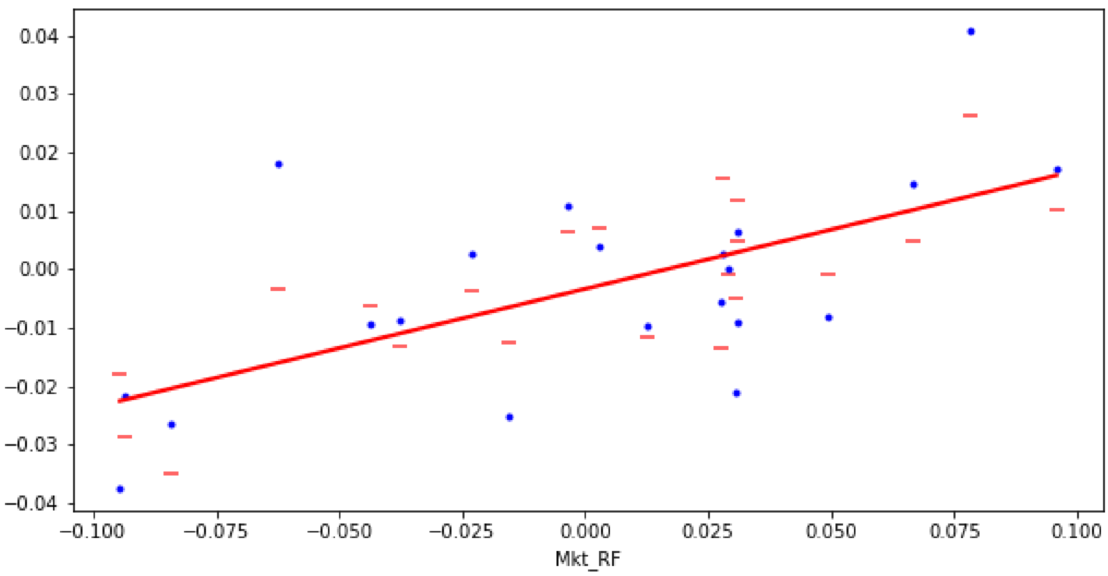
Visualise the regression result

In [87]:

```
plt.figure(figsize=(10,5))
plt.plot(merge['Mkt_RF'], GOOG_Rf_y, 'b.')
sns.regplot(x=GOOG_Rf_Xx, y= GOOG_predicted, data=merge, ci=None, scatter_kws={"s": 80},
marker= '_', color='red')
```

Out[87]:

```
<AxesSubplot:xlabel='Mkt_RF'>
```



6. Estimate the empirical asset pricing models for the entire sample (portfolio), visualise the results i.e., plot the data points and the estimated regression line on figures and discuss your results.

In [56]:

```
# generate AAple excess return:
merge['AAPl_Rf'] = merge['AAPL'] - merge['RF']

# generate Microdoft excess return:
merge['MSFT_Rf'] = merge['MSFT'] - merge['RF']

# generate Tesla excess return:
merge['TSLA_Rf'] = merge['TSLA'] - merge['RF']
```

```
# generate CAT excess return:
merge['CAT_Rf'] = merge['CAT'] - merge['RF']

merge
```

Out[56]:

Date	AAPL	CAT	GOOG	MSFT	TSLA	Mkt_RF	SMB	HML	RF	GOOG_Rf	AAPl_Rf	MSFT_Rf	TSLA
2021-02	0.002232	-0.026779	0.002708	0.014804	-0.009850	0.0278	0.0206	0.0718	0.0000	0.002708	0.002232	0.014804	0.00
2021-03	0.018766	-0.005191	0.006368	0.016908	0.050832	0.0308	-0.0237	0.0740	0.0000	0.006368	0.018766	0.016908	0.05
2021-04	-0.015133	0.002813	-0.008136	-0.001307	0.047917	0.0493	-0.0319	-0.0094	0.0000	-0.008136	-0.015133	-0.001307	0.04
2021-05	-0.005348	-0.000829	0.003767	0.001484	-0.008925	0.0029	-0.0025	0.0708	0.0000	0.003767	-0.005348	0.001484	0.00
2021-06	0.004621	0.011762	-0.005575	-0.001842	-0.001557	0.0275	0.0170	-0.0782	0.0000	-0.005575	0.004621	-0.001842	0.00
2021-07	0.001511	-0.027333	-0.009664	-0.005550	0.014542	0.0127	-0.0399	-0.0176	0.0000	-0.009664	0.001511	-0.005550	0.01
2021-08	-0.008425	-0.002743	-0.000052	-0.005633	0.006581	0.0291	-0.0043	-0.0016	0.0000	-0.000052	-0.008425	-0.005633	0.00
2021-09	-0.009312	-0.029818	-0.009333	-0.007324	-0.007462	-0.0437	0.0072	0.0508	0.0000	-0.009333	-0.009312	-0.007324	0.00
2021-10	-0.018155	-0.000392	0.014655	0.022414	0.034316	0.0665	-0.0235	-0.0048	0.0000	0.014655	-0.018155	0.022414	0.03
2021-11	0.031578	-0.013118	-0.025063	-0.017943	0.006834	-0.0155	-0.0132	-0.0044	0.0000	-0.025063	0.031578	-0.017943	0.00
2021-12	-0.003535	0.003203	-0.009061	-0.008841	-0.012669	0.0310	-0.0166	0.0328	0.0001	-0.009161	-0.003635	-0.008941	0.01
2022-01	0.026126	0.001988	0.018073	0.008824	0.106776	-0.0625	-0.0594	0.1275	0.0000	0.018073	0.026126	0.008824	0.10
2022-02	0.001638	0.002780	0.002762	0.004978	0.074777	-0.0229	0.0223	0.0304	0.0000	0.002762	0.001638	0.004978	0.07
2022-03	-0.017776	-0.001165	-0.020996	-0.017683	-0.014982	0.0305	-0.0160	-0.0180	0.0001	-0.021096	-0.017876	-0.017783	0.01
2022-04	-0.036605	-0.008944	-0.037224	-0.041812	-0.007692	-0.0946	-0.0141	0.0619	0.0001	-0.037324	-0.036705	-0.041912	0.00
2022-05	-0.005346	-0.005941	0.010993	-0.005014	-0.001804	-0.0034	-0.0185	0.0841	0.0003	0.010693	-0.005646	-0.005314	0.00
2022-06	-0.018028	-0.025725	-0.025691	-0.013179	-0.017579	-0.0843	0.0209	-0.0597	0.0006	-0.026291	-0.018628	-0.013779	0.01
2022-07	0.032793	0.055419	0.017890	0.015665	0.057850	0.0957	0.0281	-0.0410	0.0008	0.017090	0.031993	0.014865	0.05
2022-08	-0.010635	-0.011929	-0.006915	-0.005704	-0.007526	-0.0377	0.0139	0.0031	0.0019	-0.008815	-0.012535	-0.007604	0.00
2022-09	-0.030039	-0.010970	-0.019778	-0.019368	-0.011036	-0.0935	-0.0082	0.0003	0.0019	-0.021678	-0.031939	-0.021268	0.01
2022-10	0.075553	0.033940	0.042981	0.040220	0.015238	0.0783	0.0010	0.0806	0.0023	0.040681	0.073253	0.037920	0.01

CAT STOCK

CAMP

In [88]:

```
# construct independent variable:
CAT_Rf_Xx = merge['Mkt_RF']

# construct dependent variable:
CAT_Rf_yy = merge['CAT_Rf']

# add constant:
CAT_Rf_X11 = sm.add_constant(CAT_Rf_Xx)

# run regression:
CAT_model1 = sm.OLS(CAT_Rf_yy, CAT_Rf_X11).fit()

# output results:
print(CAT_model1.summary())

#GOOG_intercept and GOOG_beta
CAT_intercept1, CAT_beta = CAT_model1.params

# predict GOOG_predicted
CAT_predicted1 = CAT_model1._results.predict(CAT_Rf_X11)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 CAT_Rf   R-squared:                       0.370
Model:                            OLS   Adj. R-squared:                  0.337
Method:                 Least Squares   F-statistic:                     11.15
Date:                Sun, 18 Dec 2022   Prob (F-statistic):            0.00345
Time:                        13:14:23   Log-Likelihood:                 58.157
No. Observations:                  21   AIC:                            -112.3
Df Residuals:                      19   BIC:                            -110.2
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0034      0.003     -0.989      0.335      -0.011       0.004
Mkt_RF         0.2169      0.065      3.339      0.003       0.081       0.353
==============================================================================
Omnibus:                        1.140   Durbin-Watson:                   1.527
Prob(Omnibus):                  0.566   Jarque-Bera (JB):                0.305
Skew:                           0.268   Prob(JB):                        0.859
Kurtosis:                       3.249   Cond. No.                         18.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [71]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()

expected_monthly_return = rf + CAT_beta * market_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.000634
Expected yearly return: 0.007603
```

Fama French 3 Model

In [89]:

```
# construct independent variable:
CAT_Rf_X = merge[['Mkt_RF','SMB','HML']]

# construct dependent variable:
CAT_Rf_y = merge['CAT_Rf']

# add constant:
CAT_Rf_X1 = sm.add_constant(CAT_Rf_X  )

# run regression:
CAT_model = sm.OLS(CAT_Rf_y, CAT_Rf_X1).fit()

# output results:
print(CAT_model.summary())

#GOOG_intercept and GOOG_beta
CAT_intercept, CAT_beta1, CAT_beta2, CAT_beta3 = CAT_model.params

# predict GOOG_predicted
CAT_predicted = CAT_model._results.predict(CAT_Rf_X1)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 CAT_Rf   R-squared:                       0.388
Model:                            OLS   Adj. R-squared:                  0.280
Method:                 Least Squares   F-statistic:                     3.595
Date:                Sun, 18 Dec 2022   Prob (F-statistic):             0.0354
Time:                        13:14:43   Log-Likelihood:                 58.467
No. Observations:                  21   AIC:                            -108.9
Df Residuals:                      17   BIC:                            -104.8
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0031      0.004     -0.784      0.444      -0.012       0.005
Mkt_RF         0.2173      0.068      3.177      0.006       0.073       0.362
SMB            0.1255      0.176      0.711      0.486      -0.247       0.498
HML            0.0241      0.076      0.316      0.756      -0.137       0.185
==============================================================================
Omnibus:                        1.010   Durbin-Watson:                   1.534
Prob(Omnibus):                  0.604   Jarque-Bera (JB):                0.130
Skew:                           0.065   Prob(JB):                        0.937
Kurtosis:                       3.362   Cond. No.                         49.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [90]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()
size_premium = merge['SMB'].mean()
value_premium = merge['HML'].mean()

expected_monthly_return = rf + CAT_beta1 * market_premium + CAT_beta2 * size_premium + C
AT_beta3 * value_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.000312
```

Expected yearly return: 0.003749

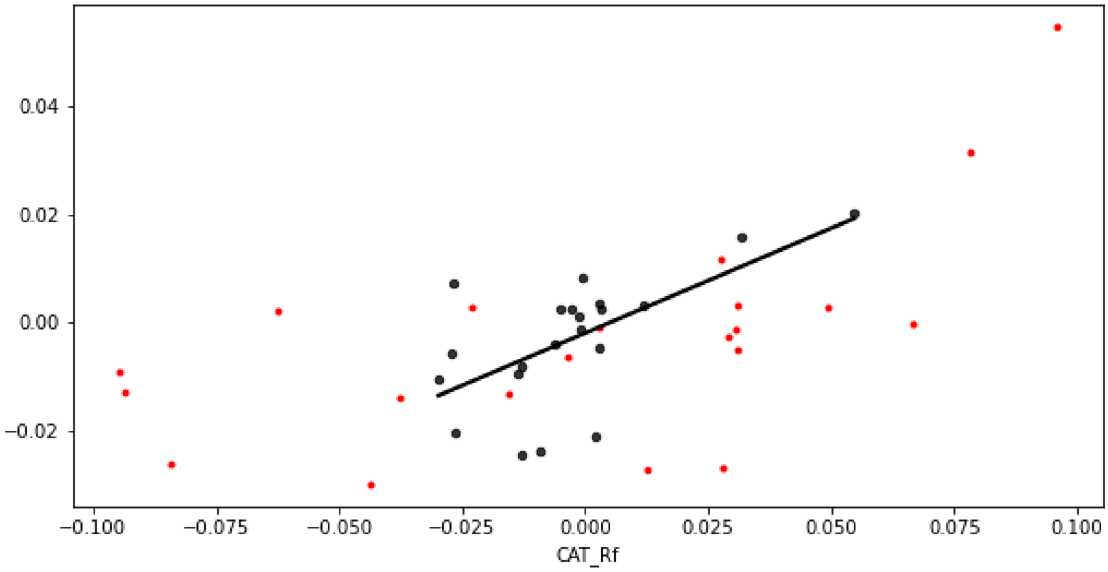
Visualise the regression result

In [92]:

```
plt.figure(figsize=(10,5))
plt.plot(merge['Mkt_RF'], CAT_Rf_y, 'r.')
sns.regplot(x=CAT_Rf_y, y= CAT_predicted, data=merge, ci=None, scatter_kws={"s": 80}, ma
rker=".", color='black')
```

Out[92]:

<AxesSubplot:xlabel='CAT_Rf'>



TSLA STOCK

CAMP

In [77]:

```
# construct independent variable:
TSLA_Rf_Xx = merge['Mkt_RF']

# construct dependent variable:
TSLA_Rf_yy = merge['TSLA_Rf']

# add constant:
TSLA_Rf_X11 = sm.add_constant(TSLA_Rf_Xx)

# run regression:
TSLA_model1 = sm.OLS(TSLA_Rf_yy, TSLA_Rf_X11).fit()

# output results:
print(TSLA_model1.summary())

#GOOG_intercept and GOOG_beta
TSLA_intercept1, TSLA_beta = TSLA_model1.params

# predict GOOG_predicted
TSLA_predicted = TSLA_model1._results.predict(TSLA_Rf_X11)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                TSLA_Rf   R-squared:                       0.036
Model:                            OLS   Adj. R-squared:                 -0.014
Method:                 Least Squares   F-statistic:                    0.7160
Date:                Sun, 18 Dec 2022   Prob (F-statistic):              0.408
```

```
Time:                        13:09:48   Log-Likelihood:                 41.906
No. Observations:                  21   AIC:                            -79.81
Df Residuals:                      19   BIC:                            -77.72
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0145      0.008      1.915      0.071      -0.001       0.030
Mkt_RF         0.1192      0.141      0.846      0.408      -0.176       0.414
==============================================================================
Omnibus:                       13.822   Durbin-Watson:                   1.860
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               12.121
Skew:                           1.589   Prob(JB):                      0.00233
Kurtosis:                       4.937   Cond. No.                         18.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

In [78]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()

expected_monthly_return = rf + TSLA_beta * market_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.000522
Expected yearly return: 0.006263
```

Fama French 3 Model

In [79]:

```
# construct independent variable:
TSLA_Rf_X = merge[['Mkt_RF','SMB','HML']]

# construct dependent variable:
TSLA_Rf_y = merge['TSLA_Rf']

# add constant:
TSLA_Rf_X1 = sm.add_constant(TSLA_Rf_X   )

# run regression:
TSLA_model = sm.OLS(TSLA_Rf_y, TSLA_Rf_X1).fit()

# output results:
print(TSLA_model.summary())

#GOOG_intercept and GOOG_beta
TSLA_intercept, TSLA_beta1, TSLA_beta2, TSLA_beta3 = TSLA_model.params

# predict GOOG_predicted
TSLA_predicted = TSLA_model._results.predict(TSLA_Rf_X1)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                TSLA_Rf   R-squared:                       0.190
Model:                            OLS   Adj. R-squared:                  0.047
Method:                 Least Squares   F-statistic:                     1.326
Date:                Sun, 18 Dec 2022   Prob (F-statistic):              0.299
Time:                        13:09:55   Log-Likelihood:                 43.726
No. Observations:                  21   AIC:                            -79.45
Df Residuals:                      17   BIC:                            -75.27
Df Model:                           3
Covariance Type:            nonrobust
```

```
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0089      0.008      1.112      0.281      -0.008       0.026
Mkt_RF         0.1463      0.138      1.061      0.304      -0.145       0.437
SMB           -0.4273      0.356     -1.200      0.246      -1.178       0.324
HML            0.1217      0.154      0.790      0.440      -0.203       0.446
==============================================================================
Omnibus:                        7.548   Durbin-Watson:                   1.901
Prob(Omnibus):                  0.023   Jarque-Bera (JB):                5.442
Skew:                           1.209   Prob(JB):                       0.0658
Kurtosis:                       3.610   Cond. No.                         49.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [80]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()
size_premium = merge['SMB'].mean()
value_premium = merge['HML'].mean()

expected_monthly_return = rf + TSLA_beta1 * market_premium + TSLA_beta2 * size_premium +
TSLA_beta3 * value_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

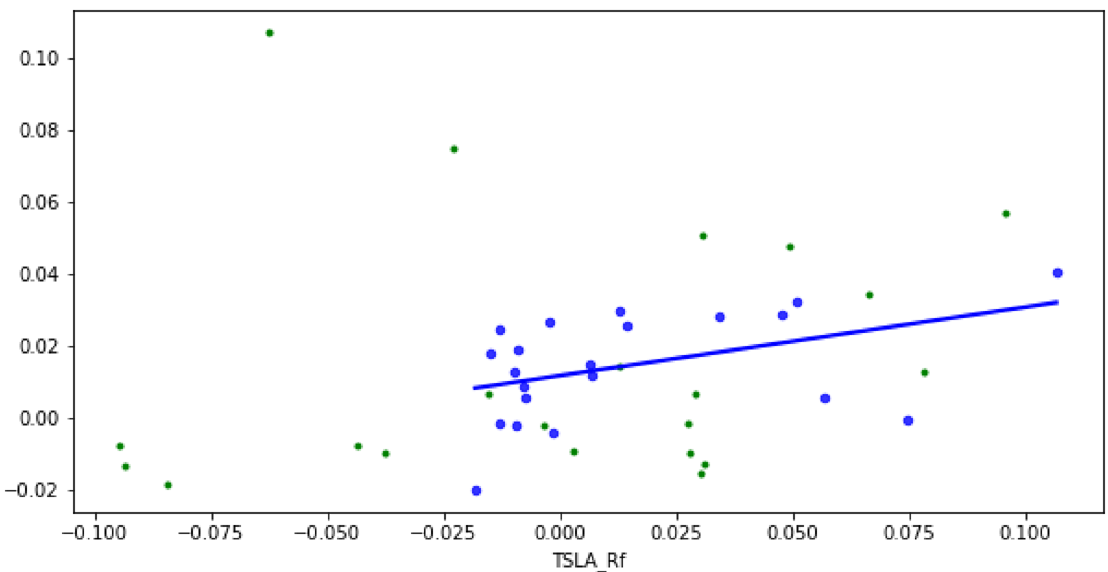
```
Expected monthly return: 0.006045
Expected yearly return: 0.072537
```

In [81]:

```
plt.figure(figsize=(10,5))
plt.plot(merge['Mkt_RF'], TSLA_Rf_y, 'g.')
sns.regplot(x=TSLA_Rf_y, y= TSLA_predicted, data=merge, ci=None, scatter_kws={"s": 80},
marker=".", color='blue')
```

Out[81]:

```
<AxesSubplot:xlabel='TSLA_Rf'>
```



MSFT STOCK

MSFT STOCK

CAMP

In [82]:

```
# construct independent variable:
MSFT_Rf_Xx = merge['Mkt_RF']

# construct dependent variable:
MSFT_Rf_yy = merge['MSFT_Rf']

# add constant:
MSFT_Rf_X11 = sm.add_constant(MSFT_Rf_Xx)

# run regression:
MSFT_model1 = sm.OLS(MSFT_Rf_yy, MSFT_Rf_X11).fit()

# output results:
print(MSFT_model1.summary())

#GOOG_intercept and GOOG_beta
MSFT_intercept1, MSFT_beta = MSFT_model1.params

# predict MSFT_predicted
MSFT_predicted = MSFT_model1._results.predict(MSFT_Rf_X11)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                MSFT_Rf   R-squared:                       0.467
Model:                            OLS   Adj. R-squared:                  0.439
Method:                 Least Squares   F-statistic:                     16.64
Date:                Sun, 18 Dec 2022   Prob (F-statistic):           0.000640
Time:                        13:12:56   Log-Likelihood:                 62.421
No. Observations:                  21   AIC:                            -120.8
Df Residuals:                      19   BIC:                            -118.8
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0019      0.003     -0.657      0.519      -0.008       0.004
Mkt_RF         0.2162      0.053      4.079      0.001       0.105       0.327
==============================================================================
Omnibus:                        0.233   Durbin-Watson:                   1.639
Prob(Omnibus):                  0.890   Jarque-Bera (JB):                0.420
Skew:                           0.147   Prob(JB):                        0.810
Kurtosis:                       2.372   Cond. No.                         18.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [83]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()

expected_monthly_return = rf + MSFT_beta * market_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.000633
Expected yearly return: 0.007594
```

Fama French 3 Model

In [84]:

```
# construct independent variable:
MSFT_Rf_X = merge[['Mkt_RF','SMB','HML']]

# construct dependent variable:
MSFT_Rf_y = merge['MSFT_Rf']

# add constant:
MSFT_Rf_X1 = sm.add_constant(MSFT_Rf_X  )

# run regression:
MSFT_model = sm.OLS(MSFT_Rf_y, MSFT_Rf_X1).fit()

# output results:
print(MSFT_model.summary())

#GOOG_intercept and GOOG_beta
MSFT_intercept, MSFT_beta1, MSFT_beta2, MSFT_beta3 = MSFT_model.params

# predict MSFT_predicted
MSFT_predicted = MSFT_model._results.predict(MSFT_Rf_X1)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                MSFT_Rf   R-squared:                       0.617
Model:                            OLS   Adj. R-squared:                  0.549
Method:                 Least Squares   F-statistic:                     9.120
Date:                Sun, 18 Dec 2022   Prob (F-statistic):           0.000801
Time:                        13:12:59   Log-Likelihood:                 65.888
No. Observations:                  21   AIC:                            -123.8
Df Residuals:                      17   BIC:                            -119.6
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0039      0.003     -1.402      0.179      -0.010       0.002
Mkt_RF         0.2322      0.048      4.836      0.000       0.131       0.334
SMB            0.1401      0.124      1.131      0.274      -0.121       0.401
HML            0.1378      0.054      2.572      0.020       0.025       0.251
==============================================================================
Omnibus:                        0.306   Durbin-Watson:                   1.769
Prob(Omnibus):                  0.858   Jarque-Bera (JB):                0.446
Skew:                          -0.222   Prob(JB):                        0.800
Kurtosis:                       2.441   Cond. No.                         49.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [85]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()
size_premium = merge['SMB'].mean()
value_premium = merge['HML'].mean()

expected_monthly_return = rf + MSFT_beta1 * market_premium + MSFT_beta2 * size_premium +
MSFT_beta3 * value_premium
expected_yearly_return = expected_monthly_return * 12
```

```
print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

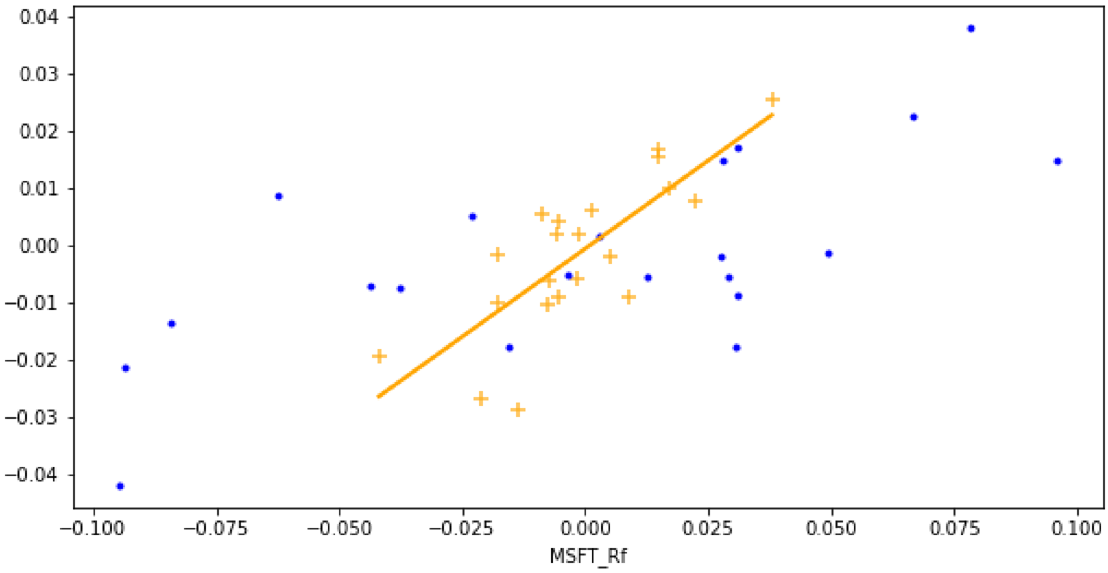
```
Expected monthly return: 0.002687
Expected yearly return: 0.032249
```

In [86]:

```
plt.figure(figsize=(10,5))
plt.plot(merge['Mkt_RF'], MSFT_Rf_y, 'b.')
sns.regplot(x=MSFT_Rf_y, y= MSFT_predicted, data=merge, ci=None, scatter_kws={"s": 80},
marker="+", color='orange')
```

Out[86]:

```
<AxesSubplot:xlabel='MSFT_Rf'>
```



AAPL STOCK

CAMP

In [95]:

```
# construct independent variable:
AAPl_Rf_Xx = merge['Mkt_RF']

# construct dependent variable:
AAPl_Rf_yy = merge['AAPl_Rf']

# add constant:
AAPl_Rf_X11 = sm.add_constant(AAPl_Rf_Xx)

# run regression:
AAPl_model1 = sm.OLS(AAPl_Rf_yy, AAPl_Rf_X11).fit()

# output results:
print(AAPl_model1.summary())

#AAPl_intercept and AAPl_beta
AAPl_intercept1, AAPl_beta = AAPl_model1.params

# predict AAPl_predicted
AAPl_predicted1 = AAPl_model1._results.predict(AAPl_Rf_X11)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                AAPl_Rf   R-squared:                       0.242
Model:                            OLS   Adj. R-squared:                  0.202
Method:                 Least Squares   F-statistic:                     6.067
Date:                Sun, 18 Dec 2022   Prob (F-statistic):             0.0235
```

```
Time:                        13:20:00   Log-Likelihood:                 51.149
No. Observations:                  21   AIC:                            -98.30
Df Residuals:                      19   BIC:                            -96.21
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0001      0.005      0.030      0.977      -0.010       0.010
Mkt_RF         0.2233      0.091      2.463      0.023       0.034       0.413
==============================================================================
Omnibus:                        6.266   Durbin-Watson:                   2.133
Prob(Omnibus):                  0.044   Jarque-Bera (JB):                4.038
Skew:                           1.007   Prob(JB):                        0.133
Kurtosis:                       3.747   Cond. No.                         18.7
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [97]:

```python
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()

expected_monthly_return = rf + AAPl_beta * market_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.000641
Expected yearly return: 0.007692
```

Fama French 3 Model

In [111]:

```python
# construct independent variable:
AAPl_Rf_X = merge[['Mkt_RF','SMB','HML']]

# construct dependent variable:
AAPl_Rf_y = merge['AAPl_Rf']

# add constant:
AAPl_Rf_X1 = sm.add_constant(AAPl_Rf_X  )

# run regression:
AAPl_model = sm.OLS(AAPl_Rf_y, AAPl_Rf_X1).fit()

# output results:
print(AAPl_model.summary())

#AAPl_intercept and AAPl_beta
AAPl_intercept, AAPl_beta1, AAPl_beta2, AAPl_beta3 = AAPl_model.params

# predict MSFT_predicted
AAPl_predicted = AAPl_model._results.predict(AAPl_Rf_X1)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                AAPl_Rf   R-squared:                       0.367
Model:                            OLS   Adj. R-squared:                  0.255
Method:                 Least Squares   F-statistic:                     3.282
Date:                Sun, 18 Dec 2022   Prob (F-statistic):             0.0464
Time:                        13:22:23   Log-Likelihood:                 53.037
```

```
No. Observations:                  21   AIC:                            -98.07
Df Residuals:                      17   BIC:                            -93.90
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0028      0.005     -0.538      0.598      -0.014       0.008
Mkt_RF         0.2451      0.089      2.768      0.013       0.058       0.432
SMB            0.1511      0.228      0.661      0.517      -0.331       0.633
HML            0.1808      0.099      1.830      0.085      -0.028       0.389
==============================================================================
Omnibus:                        3.102   Durbin-Watson:                   2.086
Prob(Omnibus):                  0.212   Jarque-Bera (JB):                2.373
Skew:                           0.811   Prob(JB):                        0.305
Kurtosis:                       2.718   Cond. No.                         49.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specifie
d.
```

```
C:\Users\PC\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142: FutureWarning: I
n a future version of pandas all arguments of concat except for the argument 'objs' will
be keyword-only
  x = pd.concat(x[::order], 1)
```

In [112]:

```
rf = merge['RF'].mean()
market_premium = merge['Mkt_RF'].mean()
size_premium = merge['SMB'].mean()
value_premium = merge['HML'].mean()

expected_monthly_return = rf + AAPl_beta1 * market_premium + AAPl_beta2 * size_premium +
AAPl_beta3 * value_premium
expected_yearly_return = expected_monthly_return * 12

print("Expected monthly return: %f"%(expected_monthly_return))
print("Expected yearly return: %f"%(expected_yearly_return))
```

```
Expected monthly return: 0.003556
Expected yearly return: 0.042673
```

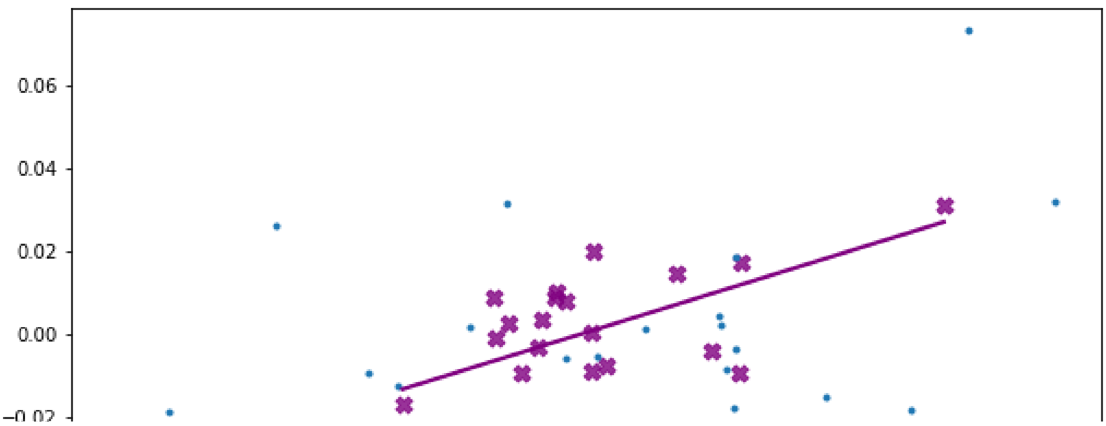
Visualization Regression

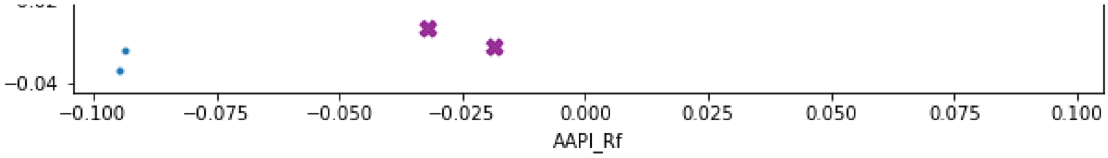
In [113]:

```
plt.figure(figsize=(10,5))
plt.plot(merge['Mkt_RF'], AAPl_Rf_y, '.')
sns.regplot(x=AAPl_Rf_y, y= AAPl_predicted, data=merge, ci=None, scatter_kws={"s": 80},
marker="X", color='purple')
```

Out[113]:

<AxesSubplot:xlabel='AAPl_Rf'>



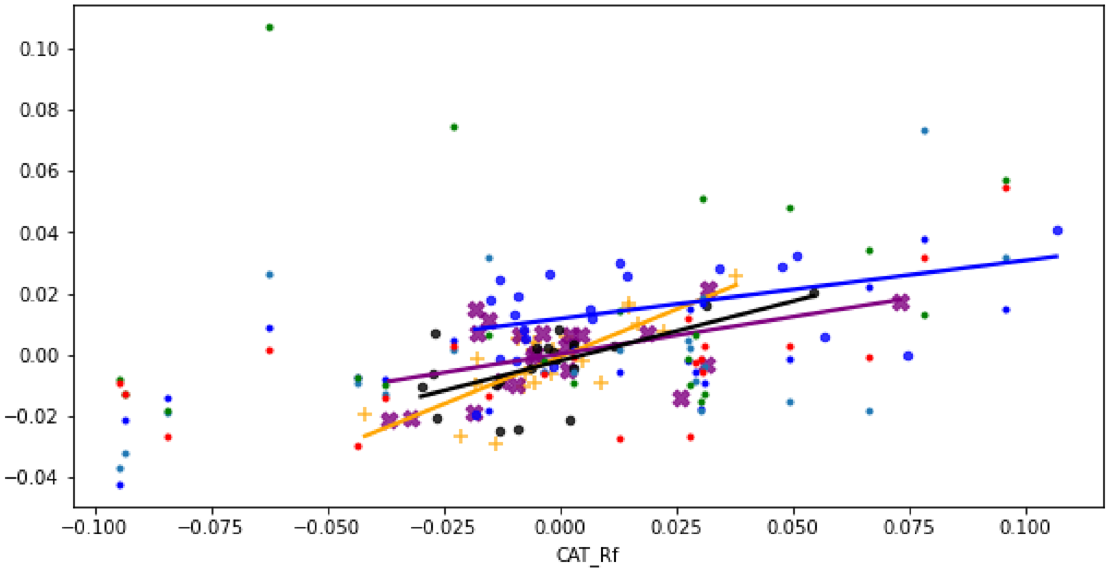


In [114]:

```
plt.figure(figsize=(10,5))
plt.plot(merge['Mkt_RF'], MSFT_Rf_yy, 'b.')
sns.regplot(x=MSFT_Rf_yy, y= MSFT_predicted, data=merge, ci=None, scatter_kws={"s": 80},
marker="+", color='orange')

plt.plot(merge['Mkt_RF'], AAPl_Rf_yy, '.')
sns.regplot(x=AAPl_Rf_yy, y= AAPl_predicted1, data=merge, ci=None, scatter_kws={"s": 80}
, marker="X", color='purple')

plt.plot(merge['Mkt_RF'], TSLA_Rf_yy, 'g.')
sns.regplot(x=TSLA_Rf_yy, y= TSLA_predicted, data=merge, ci=None, scatter_kws={"s": 80},
marker=".", color='blue')

plt.plot(merge['Mkt_RF'], CAT_Rf_yy, 'r.')
sns.regplot(x=CAT_Rf_yy, y= CAT_predicted, data=merge, ci=None, scatter_kws={"s": 80}, m
arker=".", color='black')
```

Out[114]:

```
<AxesSubplot:xlabel='CAT_Rf'>
```



7. Conduct relevant hypothesis tests for the validity of the asset pricing models for a single stocks and for the whole data sample and discuss your results

$$R_i = R_f + \beta_i \times (R_m - R_f)$$

Where:

- R_i: expected return on the investment i
- R_f: risk-free rate of return
- β_i: beta of the investment
- R_m: expected return of the market

SINGLE STOCK HYPOTHESIS TESTING

In [115]:

```
#Hypothesis test on GOOG stock
#Declaration of GOOG hypothesis t-test value
GOOG_model_hypothesis = 'Mkt_RF=0'

#perform t-test
GOOG_model.t_test(GOOG_model_hypothesis).summary()
```

Out[115]:

Test for Constraints

	coef	std err	t	P>|t|	[0.025	0.975]
c0	0.2266	0.048	4.717	0.000	0.125	0.328

In [116]:

```
#F_test on GOOG stock
GOOG_hypothesis02 = 'Mkt_RF=SMB=HML=0'
GOOG_model.f_test(GOOG_hypothesis02)
```

Out[116]:

```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[10.31786057]]), p=0.00042117635547313166, df_denom=17, df_num=3>
```

Multiple stock hypothesis testing

In [117]:

```
#Hypothesis test on AAPL stock return

AAPl_hypothesis = 'Mkt_RF=0'

#perform t-test on AAPL stock return
AAPl_model.t_test(AAPl_hypothesis).summary()
```

Out[117]:

Test for Constraints

	coef	std err	t	P>|t|	[0.025	0.975]
c0	0.2451	0.089	2.768	0.013	0.058	0.432

In [118]:

```
#F_test on AAPL stock return
AAPl_hypothesis02 = 'Mkt_RF=SMB=HML=0'
AAPl_model.f_test(AAPl_hypothesis02)
```

Out[118]:

```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[3.28178378]]), p=0.046399857636994515, df_denom=17, df_num=3>
```

In [119]:

```
#Hypothesis test on MSFT stock return
MSFT_hypothesis = 'Mkt_RF=0'

#perform t-test on MSFT return
MSFT_model.t_test(MSFT_hypothesis).summary()
```

Out[119]:

Test for Constraints

	coef	std err	t	P>|t|	[0.025	0.975]
c0	0.2322	0.048	4.836	0.000	0.131	0.334

In [120]:

```
#F_test on MSFT stock return
MSFT_hypothesis02 = 'Mkt_RF=SMB=HML=0'
MSFT_model.f_test(MSFT_hypothesis02)
```

Out[120]:

```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[9.12004395]]), p=0.0008010175464139585, df_denom=17, df_num=3>
```

In [121]:

```
#Hypothesis test on TSLA stock return
TSLA_hypothesis = 'Mkt_RF=0'

#perform t-test on TSLA stock return
TSLA_model.t_test(TSLA_hypothesis).summary()
```

Out[121]:

Test for Constraints

	coef	std err	t	P>|t|	[0.025	0.975]
c0	0.1463	0.138	1.061	0.304	-0.145	0.437

In [122]:

```
#F_test on TSLA stock return
TSLA_hypothesis02 = 'Mkt_RF=SMB=HML=0'
TSLA_model.f_test(TSLA_hypothesis02)
```

Out[122]:

```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[1.32616661]]), p=0.2986860925839583, df_denom=17, df_num=3>
```

In [123]:

```
#Hypothesis test on CAT stock return
CAT_hypothesis = 'Mkt_RF=0'

#perform t-test CAT stock return
CAT_model.t_test(CAT_hypothesis).summary()
```

Out[123]:

Test for Constraints

	coef	std err	t	P>|t|	[0.025	0.975]
c0	0.2173	0.068	3.177	0.006	0.073	0.362

In [124]:

```
#F_test on CAT stock return
CAT_hypothesis02 = 'Mkt_RF=SMB=HML=0'
CAT_model.f_test(CAT_hypothesis02)
```

Out[124]:

```
<class 'statsmodels.stats.contrast.ContrastResults'>
<F test: F=array([[3.59488548]]), p=0.03540682665342358, df_denom=17, df_num=3>
```

In [ ]: