



Imputation of Incomplete Multilevel Data with mice

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Abstract

This is a tutorial paper on imputing incomplete multilevel data with **mice**. Footnotes in the current version show work in progress/under construction. The last section is not part of the manuscript, but purely for reminders. We aim to submit at JSS, so there is no word count limit (“There is no page limit, nor a limit on the number of figures or tables”).

Keywords: missing data, multilevel, clustering, **mice**, R.

1. Introduction

In many contemporary data analysis efforts, some form of hierarchical or clustered data structures are recorded. In the simplest case, such a structure entails the nesting of units within clusters (e.g., students within school classes). More complex clustered structures may occur when there are multiple hierarchical levels (e.g., patients within hospitals within regions or countries), or when the clustering is non-nested (e.g., data from diverse settings and populations within large databases). The clustered structure of multilevel data should be taken into account when developing analysis models: 1) for the simple reason that groups of observations share some common variance, and 2) because ignoring multilevel structures can be harmful to the statistical inferences and introduce bias in estimators (Hox, Moerbeek, and van de Schoot 2017). There are many names for models that take clustering into account. Some popular examples are ‘multilevel models’, ‘hierarchical models’, ‘mixed effect models’

Table 1: Concepts in multilevel modeling

Concept	Details
ICC	The variability due to clustering is often measured by means of the intraclass coefficient (ICC). The ICC can be seen as the percentage of variance that can be attributed to the cluster-level, where a high ICC would indicate that a lot of variability is due to the cluster structure.
Random intercept	Multilevel models typically accommodate for variability by including a separate group mean for each cluster. In addition to random intercepts, multilevel models can also include random effects and heterogeneous residual error variances across clusters [see e.g. @gelm06, @hox17 and @jong21].

and ‘random effect models’. Table 1 provides an overview of some key concepts in multilevel modeling.

1.1. Missingness in multilevel data

The process of analyzing multilevel data is further complicated when not all data entries are observed. Just as with single level data, missingness may occur at the unit level. But with multiple levels of data comes the potential for clustered missingness. Therefore, incomplete multilevel data can be categorized into two general patterns: systematic missingness and sporadic missingness (Resche-Rigon, White, Bartlett, Peters, and Thompson 2013). Systematic missingness implies that one or more variables are never observed in a certain cluster. With sporadic missingness there may be observed data for some but not all units in a cluster (Van Buuren 2018; Jolani 2018). We have visualized this difference in Figure 1, which shows an $n \times p$ set $\mathbf{X} = X_1, \dots, X_p$, with n units distributed over N clusters and p variables. Column X_1 is completely observed, column X_2 is systematically missing in cluster 2, and column X_3 is sporadically missing. To analyze these incomplete data, we have to take the nature of the missingness and the cluster structure into account. For example, the sporadic missingness in X_3 could be easily amended if this would be a cluster-level variable (and thus constant within clusters). We could then just extrapolate the true (but missing) value of X_3 for unit 1 from unit 2, and the value for unit 4 from unit 3. If X_3 would instead be a unit-level variable (which may vary within clusters), we could not just recover the unobserved ‘truth’, but would need to use some kind of missing data method, or discard the incomplete units altogether (i.e., complete case analysis). Complete case analysis can however introduce bias in statistical inferences and lowers statistical power. Further, with the systematic missingness in X_2 , it would be impossible to fit a multilevel model without accommodating the missingness in some way. Complete case analysis in that case would mean excluding the entire cluster from the analyses. The wrong choice of missing data handling method can thus be extremely harmful to the inferences.

Since excluding observations is not a desirable workflow, the missingness in multilevel data should be accommodated before or within the analysis of scientific interest. In this paper, we focus on the former approach: imputing (i.e., filling in) the missing data with plausible values, whereafter the completed data may be analyzed as if it were completely observed. Imputation separates the missing data problem from the scientific problem, which makes the

	cluster	X_1	X_2	X_3	...	X_p
1	1			NA		
2	1					
3	2		NA			
4	2		NA	NA		
...						
n	N					

Figure 1: Missingness in multilevel data

missing data strategy very generic and popular. If each missing value is replaced multiple times, the resulting inferences may validly convey the uncertainty due to missingness (c.f. ?). The R package **mice** has become the de-facto standard for imputation by chained equations, which iteratively solves the missingness on a variable-by-variable basis. **mice** is known to yield valid inferences under many different missing data circumstances (Van Buuren 2018). In this paper, we will discuss how to use **mice** in the context of multilevel data.

1.2. Aim of this paper

This paper serves as a tutorial for imputing incomplete multilevel data with **mice**. We provide practical guidelines and code snippets for different missing data situations, including missing not at random (MNAR) mechanisms (where the probability to be missing depends on unrecorded information, making the missingness non-ignorable, Rubin 1976; Meng 1994). For reasons of brevity, we focus on imputation by chained equations with **mice** exclusively¹. Other useful resources for the analysis of incomplete multilevel data include the R packages **mitml**, **miceadds**, and **mdmb**, and empirical work by Audigier, White, Jolani, Debray, Quartagno, Carpenter, van Buuren, and Resche-Rigon (2018) and Grund, Lüdtke, and Robitzsch (2018). Please note that this tutorial paper assumes a basic level of knowledge on multilevel models.² Assumed knowledge also includes the use of the ‘piping operator’, `%>%`, adopted from the **magrittr** package, and the **lme4** notation for multilevel models.³

We illustrate how to impute incomplete multilevel data by means of three case studies:

¹Note that the alternative, joint modeling imputation for multilevel data or **jomo** Quartagno, Grund, and Carpenter (2019), has been implemented in **mice** as well but is outside the scope of this tutorial.

²Note to self: We’re providing an overview of implementations. It’s up-to the reader to decide which multilevel strategy suits their data. We won’t go into detail for the different methods (and equations). This paper is just a software tutorial, so we’ll keep it practical.

³TODO: Add environment info, seed and version number(s) somewhere!

- **popmis** from the **mice** package (simulated data on perceived popularity, $n = 2,000$ pupils across $N = 100$ schools, [van Buuren and Groothuis-Oudshoorn 2021](#));
- **hiv** from the **GJRM** package (simulated data on HIV diagnoses, $n = 6,416$ patients across $N = 9$ regions, [Radice 2021](#));
- **impact** from the **metamisc** package (empirical data on traumatic brain injuries, $n = 11,022$ patients across $N = 15$ studies, [Debray and de Jong 2021](#)).

For each of these datasets, we will discuss the nature of the missingness, choose one or more imputation models and evaluate the imputed data, but we will also highlight one specific aspect of the imputation workflow. With the **popmis** data, we show how (and how not) to develop an imputation model. With the **hiv** data we focus on extending the imputation model to include Heckman-type selection-inclusion methods. With the **impact** data we provide an example of multivariate missingness in real-world data. Together, this should give enough scaffolding for applied researchers who are faced with incomplete multilevel data.⁴

Set-up the R environment and load the necessary packages:

```
R> set.seed(2022)
R> library(mice)
R> library(ggmice)
R> library(ggplot2)
R> library(dplyr)
R> library(lme4)
R> library(mitml)
```

REVIEW UNTIL THIS POINT SVP

2. Case Study I: How (not) to impute

In this section we'll go over the different steps involved with imputing incomplete multilevel data. The data we're using is the **popmis** dataset from the **mice** package. This is a simulated dataset with pupils ($n = 2000$) clustered within schools ($N = 100$). In this tutorial we'll use the following variables:

- **school**, school identification number (clustering variable);
- **popular**, pupil popularity (self-rating between 0 and 10; unit-level);
- **sex**, pupil sex (0=boy, 1=girl; unit-level);
- **texp**, teacher experience (in years; cluster-level).

⁴TODO: Add notation paragraph or 'translation table' linking multilevel equations to **lme4** formulas. Use betas instead of gamma's and mu's. Add interpretation of values in predictormatrix (-2 for the cluster variable, 2 for random effects). Add ICC and congeniality here as well. And make missingness mechanism table as well.

The analysis model corresponding to this dataset is multilevel regression with random intercepts, random slopes and a cross-level interaction. The outcome variable is `popular`, which is predicted from the unit-level variable `sex` and the cluster-level variable `texp`. The regression equation⁵ and `lme4` notation for this model are

$$\text{popular}_{ij} = \gamma_{00} + \gamma_{10} \text{sex}_{ij} + \gamma_{01} \text{texp}_j + \gamma_{11} \text{texp}_j \times \text{sex}_{ij} + u_{0j} + u_{1j} \text{sex}_{ij} + e_{ij}$$

```
popular ~ 1 + sex + texp + sex:texp + (1 + sex | school)
```

Since the data is simulated and the missingness is induced, we can compare our inferences after imputation to the true complete data. The data is created in such a way that the clustering variable `school` explains quite some variance in the outcome variable `popular`. We express this using the intraclass correlation, $\text{ICC} = 0.58$. We'll evaluate the ICC after each missing data strategy, and compare the estimated fixed effects:

	Estimate with 95% CI
1 (Intercept)	3.314 [2.998, 3.629]
2 sex	1.330 [1.069, 1.590]
3 texp	0.110 [0.090, 0.130]
4 sex:texp	-0.034 [-0.051, -0.017]

Incomplete data

Load the data into the environment and select the relevant variables:

```
R> popmis <- popmis[, c("school", "popular", "sex", "texp")]
```

Plot the missing data pattern:

```
R> plot_pattern(popmis)
```

The missingness is univariate and sporadic, which is illustrated in the missing data pattern in Figure 2. The ICC in the incomplete data is 0.56. This tells us that the multilevel structure of the data should probably be taken into account. If we don't, we'll may end up with incorrect imputations, biasing the effect of the clusters towards zero.

Plot the correlations in the incomplete data:

```
R> plot_corr(popmis)
```

⁵add the 'level notation' (Bryk and Raudenbush, 1992) and/or matrix notation ('linear mixed effects model'; Laird and Ware, 1982) too?

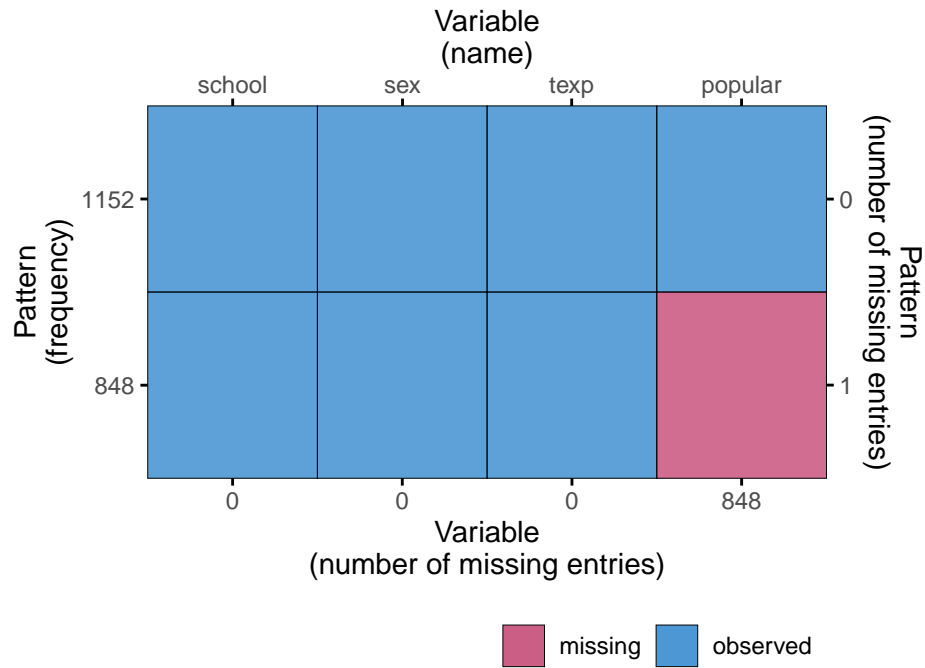
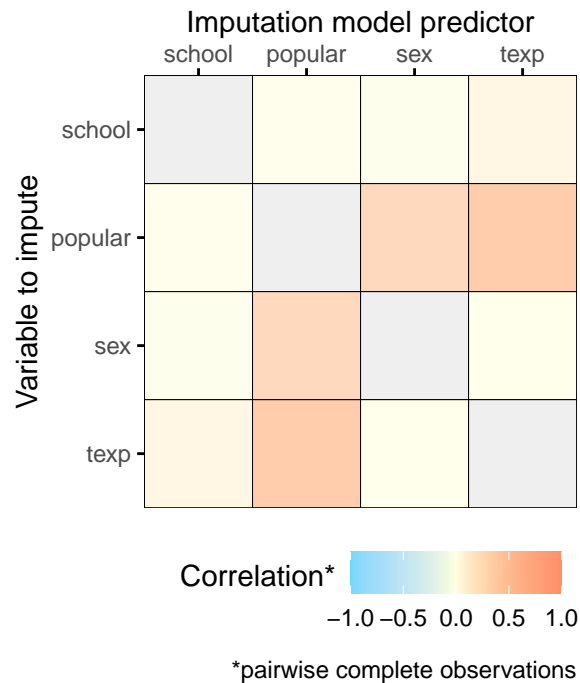


Figure 2: Missing data pattern in the popularity data

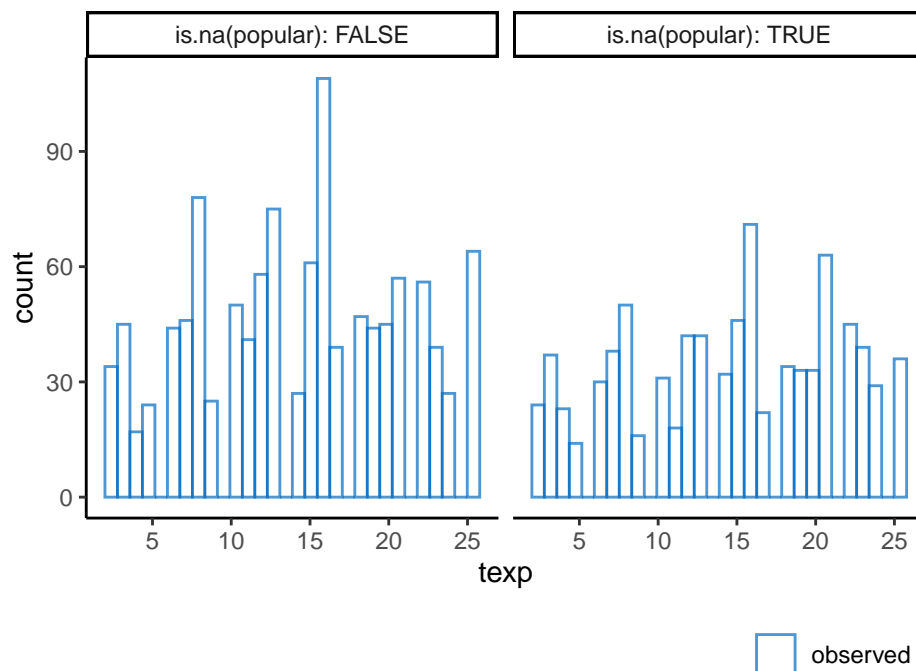


To develop the best imputation model for the incomplete variable `popular`, we need to know whether the missingness depends on the observed values of other variables. We'll highlight one other variable to illustrate, but ideally one would inspect all relations. The questions we'll

ask are: ‘Does the missing data of pupil popularity (`popular`) depend on observed teacher popularity (`texp`)?’. This can be evaluated statistically, but visual inspection usually suffices. We’ll make a histogram of `texp` separately for the pupils with known popularity and missing popularity.

Plot the histogram for teacher experience conditional on the missingness indicator of `popular`:

```
R> ggmmice(popmis, aes(texp)) +  
+   geom_histogram(fill = "white") +  
+   facet_grid(. ~ is.na(popular), labeller = label_both)
```



TODO: think about what is a meaningful rule of thumb to signal that the user should be worried?

This shows us that there are no apparent differences in the distribution of `texp` depending on the missingness indicator of `popular` ($t = -0.873$, $p = 0.383$).

Complete case analysis (not recommended)

Complete case analysis ignores the observations with missingness altogether, which lowers statistical power and may even introduce bias in MCAR situations.

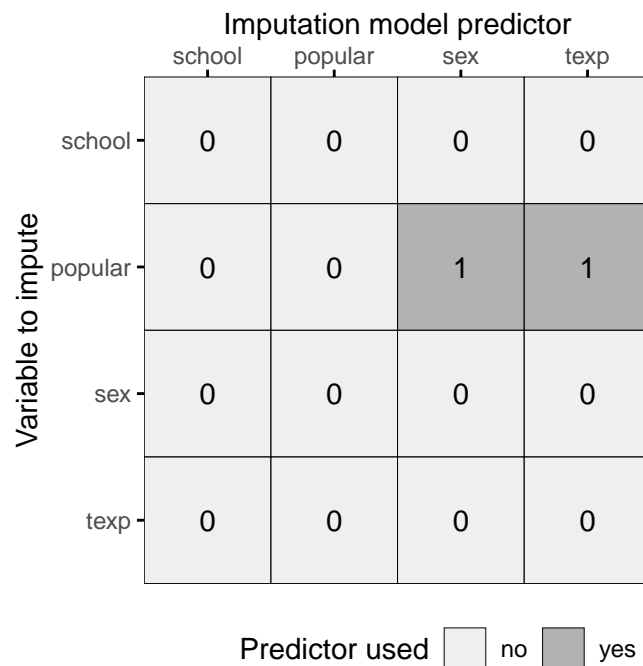
Imputation ignoring the cluster variable (not recommended)

The first imputation model that we’ll use is likely to be invalid. We do not use the cluster identifier `school` as imputation model predictor. With this model, we ignore the multilevel structure of the data, despite the high ICC. This assumes exchangeability between units. We include it purely to illustrate the effects of ignoring the clustering in our imputation effort.

We'll use the default imputation methods in `mice()` (predictive mean matching to impute the continuous variables and logistic regression to impute binary variables).

Create a methods vector and predictor matrix for `popular`, and make sure `school` is not included as predictor:

```
R> meth <- make.method(popmis) # methods vector
R> pred <- quickpred(popmis)   # predictor matrix
R> plot_pred(pred)
```



Impute the data, ignoring the cluster structure:

```
R> imp_ignored <- mice(popmis, maxit = 1, pred = pred, print = FALSE)
```

TODO: remove the broom.mixed output, use mitml only

Analyze the imputations:

```
R> fit_ignored <- imp_ignored %>%
+   with(lme4::lmer(popular ~ 1 + sex + taxp + sex:taxp + (1 + sex | school)))
R> testEstimates(as.mitml.result(fit_ignored), var.comp = TRUE)
```

Call:

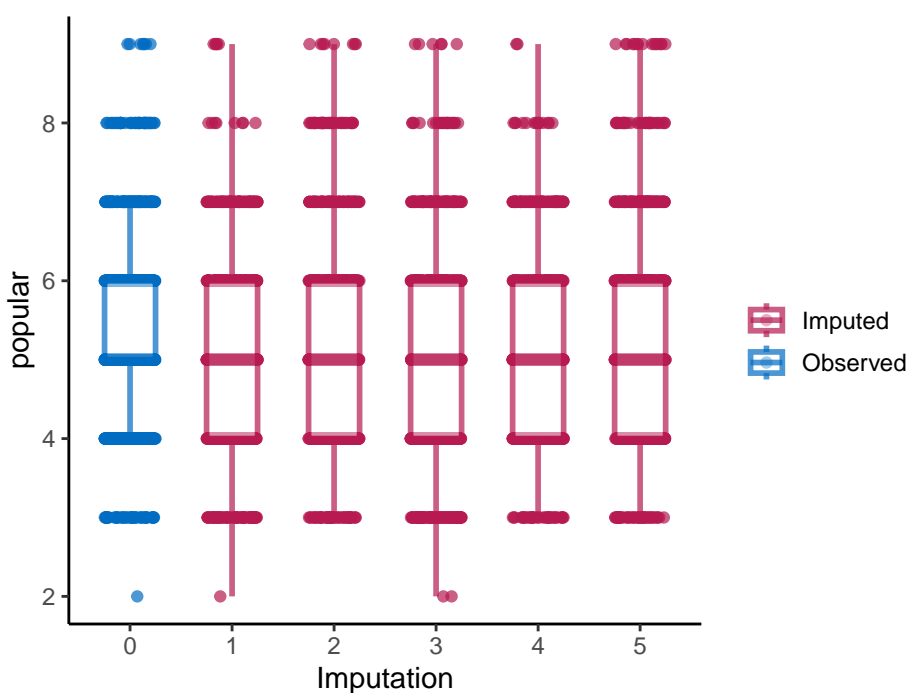
```
testEstimates(model = as.mitml.result(fit_ignored), var.comp = TRUE)
```


Final parameter estimates and inferences obtained from 5 imputed data sets.

	Estimate	Std.Error	t.value	df	P(> t)	RIV	FMI
(Intercept)	3.354	0.245	13.704	6.831	0.000	3.260	0.813
sex	1.247	0.332	3.759	6.154	0.009	4.160	0.849
texp	0.110	0.017	6.320	6.084	0.001	4.286	0.852
sex:texp	-0.027	0.022	-1.225	5.914	0.267	4.631	0.862

	Estimate
Intercept~~Intercept school	0.177
sex~~sex school	0.232
Intercept~~sex school	-0.043
Residual~~Residual	0.670
ICC school	0.209

Unadjusted hypothesis test as appropriate in larger samples.



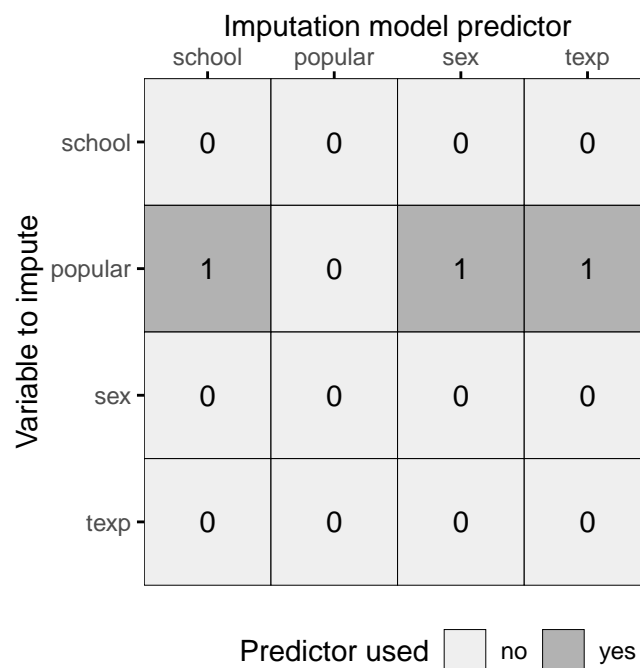
Imputation with the cluster variable as predictor (not recommended)

We'll now use `school` as a predictor to impute all other variables. This is still not recommended practice, since it only works under certain circumstances and results may be biased (Drechsler 2015; Enders, Mistler, and Keller 2016). But at least, it includes some multilevel aspect. This method is also called 'fixed cluster imputation', and uses N-1 indicator variables representing allocation of N clusters as a fixed factor in the model (Reiter, Raghunathan, and Kinney 2006; Enders et al. 2016). Colloquially, this is 'multilevel imputation for dummies'.

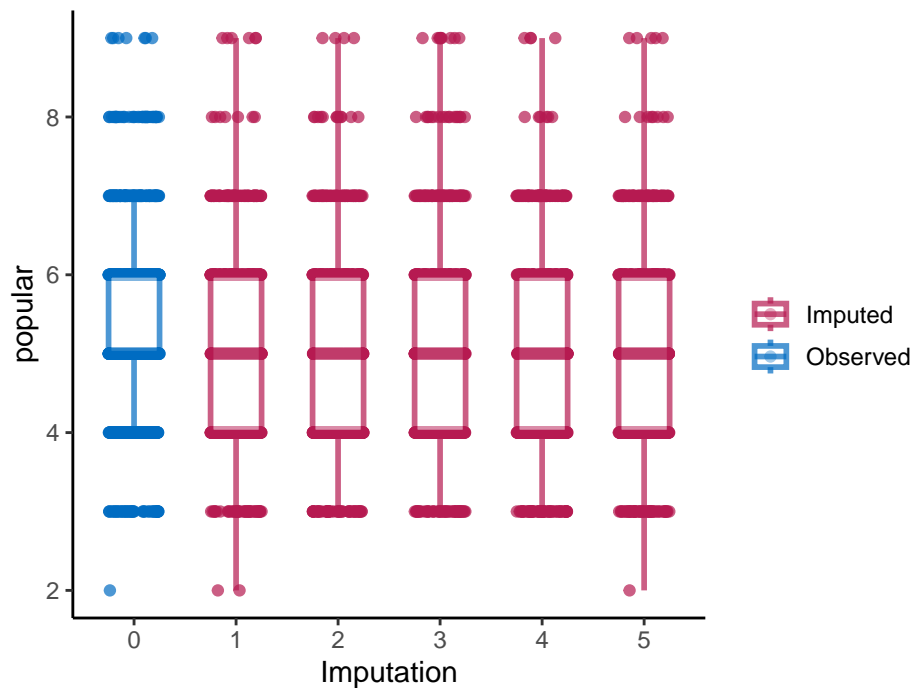
Add: doesn't work with syst missing (only sporadically). There's some pro's and con's. May not differ much if the number of clusters is low.

The more the random effects are of interest, the more you need ml models.

```
R> # adjust the predictor matrix
R> pred["popular", "school"] <- 1
R> plot_pred(pred)
```



```
R> # impute the data, cluster as predictor
R> imp_predictor <- mice(popmis, maxit = 1, pred = pred, print = FALSE)
```



Now, we can clearly see that the imputed values of `texp` are higher than the observed values, which is in line with right-tailed MAR.

The ICCs are way more in line with the ICCs in the incomplete data. But this is a quick and dirty way of imputing multilevel data. We should be using a multilevel model.

Imputation with random effects

With `2l.norm` we impute the outcome with a multilevel model assuming random slopes for each variable in the imputation model and homogeneous within-cluster variance.

“Van Buuren (2011) considered the homoscedastic linear mixed model as invalid for imputing incomplete predictors, and investigated only the `2l.norm` method, which allows for heterogeneous error variances” (Van Buuren 2018).

```
R> # adjust the predictor matrix
R> pred["popular", ] <- c(school = -2, popular = 0, sex = 2, texp = 2)
R> plot_pred(pred)
```

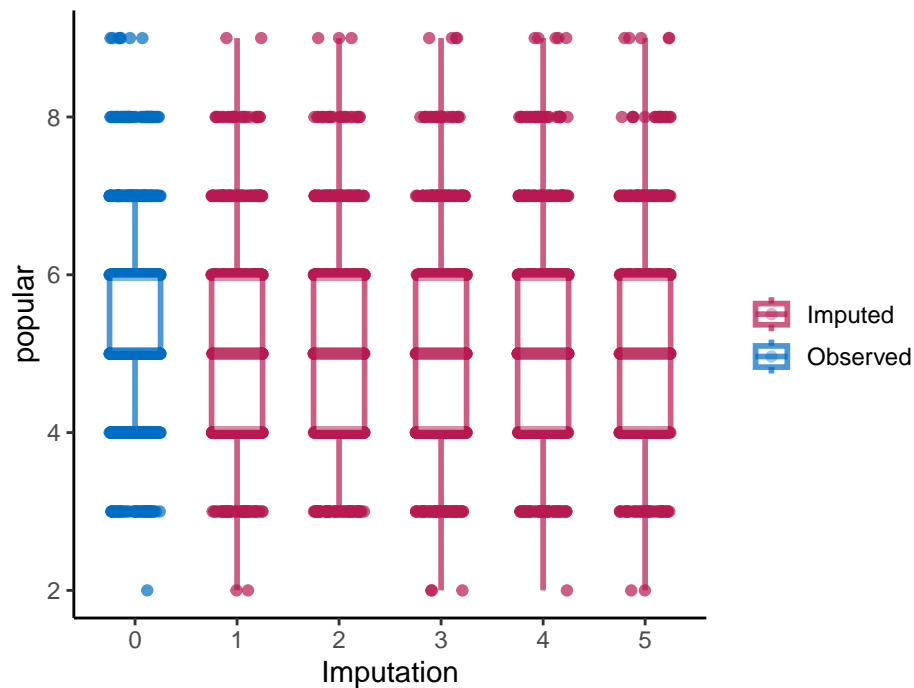
		Imputation model predictor			
		school	popular	sex	texp
Variable to impute	school	0	0	0	0
	popular	-2	0	2	2
	sex	0	0	0	0
	texp	0	0	0	0

Predictor used ☐ no ☒ yes

```

R> meth <- make.method(popmis)
R> meth["popular"] <- "2l.pmm"
R> imp_pmm_2l <-
+   mice(
+     popmis %>% mutate(school = as.integer(school)),
+     pred = pred,
+     meth = meth,
+     maxit = 1,
+     print = FALSE
+   )

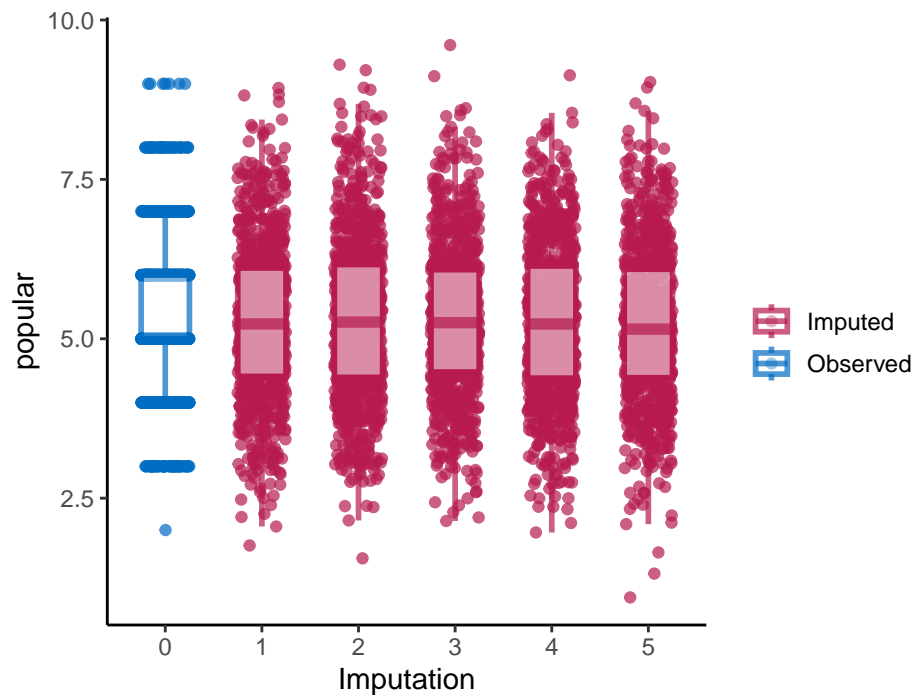
```



Imputation with random effects and heterogeneity

This method assumes random slopes for each variable in the imputation model. In contrast to `2l.norm` this method allows a cluster-specific residual error variance.

```
R> pred["popular", ] <- c(-2, 2, 1, 2)
R> meth <- c("", "2l.pan", "", "")
R> imp_pan_2l <-
+   mice(
+     popmis %>% mutate(school = as.integer(school)),
+     pred = pred,
+     meth = meth,
+     maxit = 1,
+     print = FALSE
+   )
```

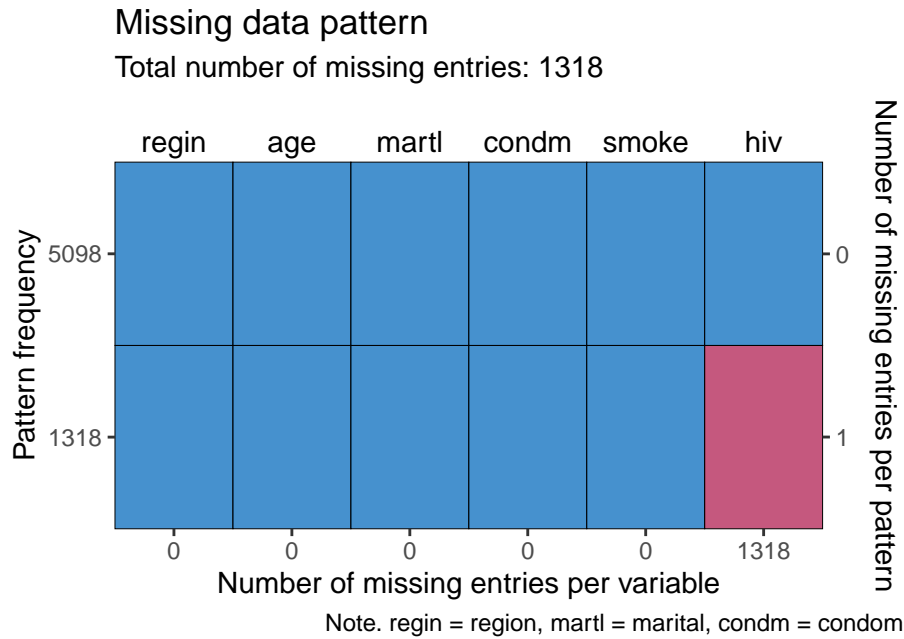


3. How to handle non-random selection (Case study II: HIV)

Data are simulated and included in the `GJRM` package. We will use the following variables:

- `region` Cluster variable,
- `hiv` HIV diagnosis (0=no, 1=yes),
- `age` Age of the patient,
- `marital` Marital status,
- `condom` Condom use during last intercourse,
- `smoke` Smoker (levels; inclusion restriction variable).

The imputation of these data is based on the toy example from [IPDMA Heckman Github repo](#).



From the missing data pattern we see that we can set `maxit` to 1, since there is only one variable with missingness.

The inclusion restriction variable should be a predictor of the the actual value of the variable of interest, but not of missingness indicator for the variable of interest. In this case, the data were simulated to adhere to this requirement. Namely, $\beta_{smoke} = -0.064$, 95% CI [-0.256, 0.126] for the analysis model (`formula = hiv ~ .`), and $\beta_{smoke} = -0.265$, 95% CI [-0.422, -0.11] for the selection model (`formula = is.na(hiv) ~ .`). This means the assumptions for the Heckman-type selection model are met.

4. How to handle multivariate missingness (Case study III: IMPACT)

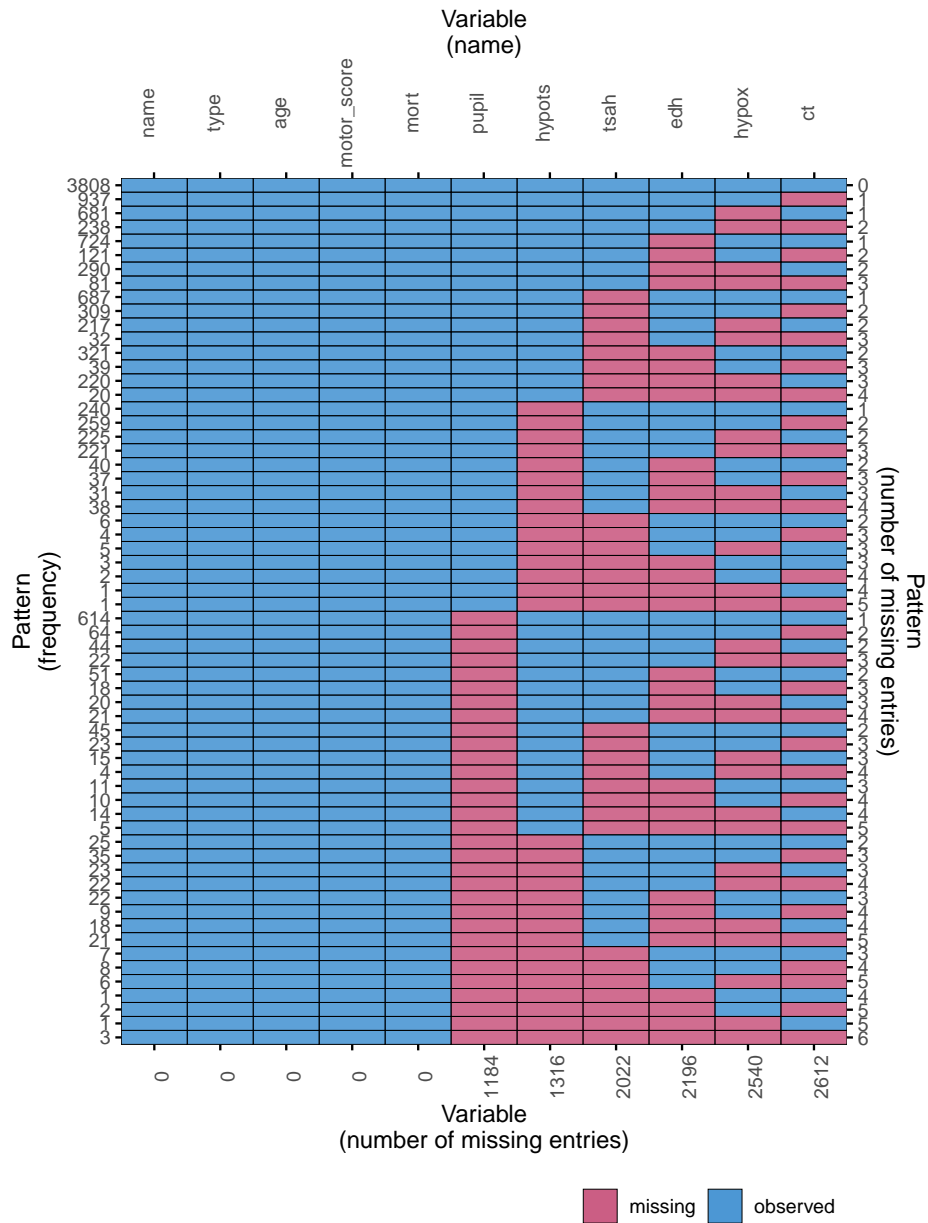
`impact` is traumatic brain injury data with patients, $n = 11022$, clustered in studies, $N = 15$. With the following 11 variables:

- `name` Name of the study,
- `type` Type of study (RCT: randomized controlled trial, OBS: observational cohort),
- `age` Age of the patient,
- `motor_score` Glasgow Coma Scale motor score,
- `pupil` Pupillary reactivity,
- `ct` Marshall Computerized Tomography classification,
- `hypox` Hypoxia (0=no, 1=yes),
- `hypots` Hypotension (0=no, 1=yes),
- `tsah` Traumatic subarachnoid hemorrhage (0=no, 1=yes),
- `edh` Epidural hematoma (0=no, 1=yes),
- `mort` 6-month mortality (0=alive, 1=dead).

The analysis model for this dataset is a prediction model with `mort` as the outcome.⁶ The data is already imputed (Steyerberg et al, 2008), so we've induced missingness again based on the missingness in the original data.

$$\text{mort}_{ij} = \gamma_{00} + \gamma_{01} \text{ type}_j + \gamma_{10} \text{ age}_{ij} + \gamma_{10} \text{ moter_score}_{ij} + \gamma_{10} \text{ pupil}_{ij} + \gamma_{10} \text{ ct}_{ij} + u_{0j} + u_{1j} \text{ age}_{ij} + u_{1j} \text{ moter_score}_{ij}$$

```
glmer(mort ~ 1 + type + age + motor_score + pupil + ct + (1 | name))
```



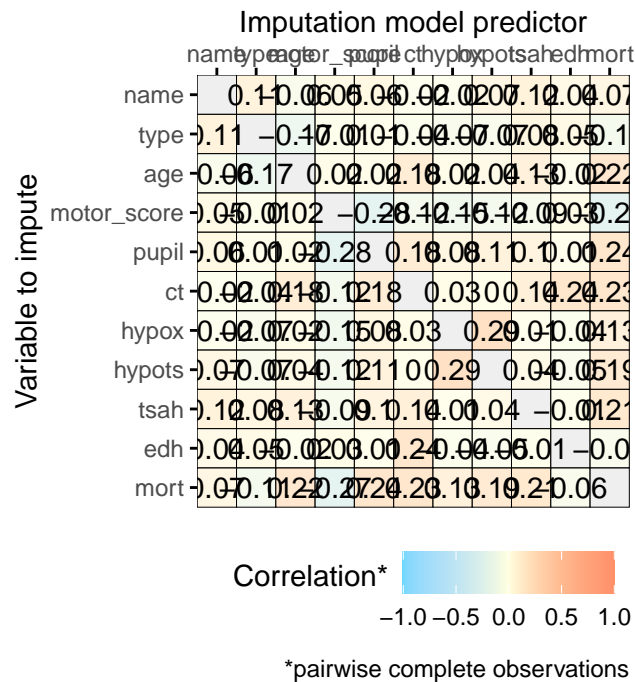
⁶Look at analysis model, maybe copy from GREAT data example e.g., adjusted prognostic effect of `ct` on unfortunate outcomes, we just want to know the adjusted odds ratio for `ct`. Add something about systematically missing data here.


```
# A tibble: 9 x 7
```

	effect	group	term	estimate	std.error	statistic	p.value
	<chr>	<chr>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>
1	fixed	<NA>	(Intercept)	-2.66	0.194	-13.7	1.11e-42
2	fixed	<NA>	typeRCT	-0.412	0.172	-2.40	1.64e- 2
3	fixed	<NA>	age	0.0298	0.00190	15.7	2.56e-55
4	fixed	<NA>	as.factor(motor_score)2	-0.631	0.0823	-7.66	1.82e-14
5	fixed	<NA>	as.factor(motor_score)3	-0.994	0.0845	-11.8	5.61e-32
6	fixed	<NA>	as.factor(motor_score)4	-1.54	0.0868	-17.7	2.88e-70
7	fixed	<NA>	pupil	0.438	0.0397	11.0	3.01e-28
8	fixed	<NA>	ct	0.430	0.0343	12.5	5.34e-36
9	ran_pars	name	sd__(Intercept)	0.257	NA	NA	NA

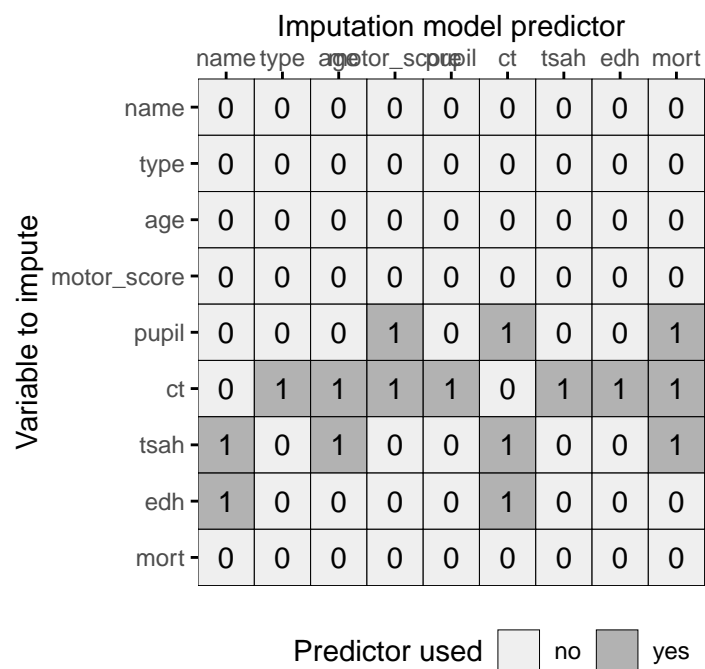
We should impute the variables `ct` and `pupil` and any auxiliary variables we might want to use to impute these incomplete analysis model variables. We can evaluate which variables may be useful auxiliaries by plotting the pairwise complete correlations:

```
R> plot_corr(impact_NA, label = T)
```



This shows us that `hypox` and `hypot` would not be useful auxiliary variables for imputing `ct`. Depending on the minimum required correlation, `tsah` could be useful, while `edh` has the strongest correlation with `ct` out of all the variables in the data and should definitely be included in the imputation model. For the imputation of `pupil`, none of the potential auxiliary variables has a very strong relation, but `hypots` could be used. We conclude that we can exclude `hypox` from the data since this is neither an analysis model variable, nor an auxiliary variable for imputation.

```
R> impact_NA <- select(impact_NA, -c(hypox, hypots))
R> pred <- quickpred(impact_NA)
R> plot_pred(pred)
```



```
R> pred[pred == 1] <- 2
R> pred["mort", ] <- 2
R> pred[, "mort"] <- 2
R> pred[c("name", "type", "age", "motor_score", "mort"), ] <- 0
R> pred[, "name"] <- -2
R> diag(pred) <- 0
R> plot_pred(pred)
```

Imputation model predictor

	name	type	age	motor_score	pupil	ct	tsah	edh	mort
name	0	0	0	0	0	0	0	0	0
type	-2	0	0	0	0	0	0	0	0
age	-2	0	0	0	0	0	0	0	0
motor_score	-2	0	0	0	0	0	0	0	0
pupil	-2	0	0	2	0	2	0	0	2
ct	-2	2	2	2	2	0	2	2	2
tsah	-2	0	2	0	0	2	0	0	2
edh	-2	0	0	0	0	2	0	0	2
mort	-2	0	0	0	0	0	0	0	0

Variable to impute

Predictor used no yes

```
R> meth <- make.method(impact_NA)
R> meth[meth != ""] <- "2l.pmm"
```

```
R> imp <- mice(impact_NA, method = meth, predictorMatrix = pred, m = 2, maxit = 1)
R> # imp_impact <- impact_NA %>%
R> #   mutate(name = as.integer(name), motor_score = as.numeric(motor_score)) %>%
R> #   mice::mice(., m = 2, maxit = 1, method = meth, predictorMatrix = pred)
R> # look at jomo for categorical variables?
R> # semi-cont with jomo is not ideal (schafer, '97) because you need 2-step approach
R> # pmm is better (more efficient) because it will still look for donors (maybe outside c
R> # make assumptions of these methods explicit!
```

```
R> fit <- imp_impact_pmm %>%
+   with(glmer(mort ~ type + age + as.factor(motor_score) + pupil + ct + (1 | name), fami
R> tidy(pool(fit))
R> as.mitml.result(fit)
R> # testEstimates(as.mitml.result(fit))
```

Compare estimates to complete data

```
R> fit <- glmer(mort ~ type + age + as.factor(motor_score) + pupil + ct + (1 | name), fami
R> tidy(fit)
```

```
# A tibble: 11 x 7
  effect    group term          estimate std.error statistic    p.value
  <chr>     <chr> <chr>          <dbl>     <dbl>     <dbl>     <dbl>
```

1	fixed	<NA>	(Intercept)	-2.29	0.171	-13.4	4.94e- 41
2	fixed	<NA>	typeRCT	-0.338	0.172	-1.96	5.01e- 2
3	fixed	<NA>	age	0.0322	0.00160	20.2	2.13e- 90
4	fixed	<NA>	as.factor(motor_score~	-0.593	0.0698	-8.50	1.95e- 17
5	fixed	<NA>	as.factor(motor_score~	-0.904	0.0713	-12.7	8.43e- 37
6	fixed	<NA>	as.factor(motor_score~	-1.29	0.0732	-17.6	1.67e- 69
7	fixed	<NA>	pupilNone	1.32	0.0616	21.4	2.09e-101
8	fixed	<NA>	pupilOne	0.626	0.0711	8.80	1.36e- 18
9	fixed	<NA>	ctIII	0.810	0.0700	11.6	5.12e- 31
10	fixed	<NA>	ctIV/V	0.832	0.0593	14.0	9.47e- 45
11	ran_pars	name	sd__(Intercept)	0.277	NA	NA	NA

5. Discussion

- JOMO in **mice** -> on the side for now
- Additional levels of clustering
- More complex data types: timeseries and polynomial relationship in the clustering.

6. Think about

- Adding some kind of help function to mice that suggests a suitable predictor matrix to the user, given a certain analysis model.
- Adding a `multilevel_ampute()` wrapper function in mice.
- Exporting `mids` objects to other packages like `lme4` or `coxme`?
- Adding a ICC=0 dataset to show that even if there is no clustering it doesn't hurt.
- Show use case for deductive imputation for cluster level variables?
- env dump in repo

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