Electric Field Lines

Electric Field due to moving charge

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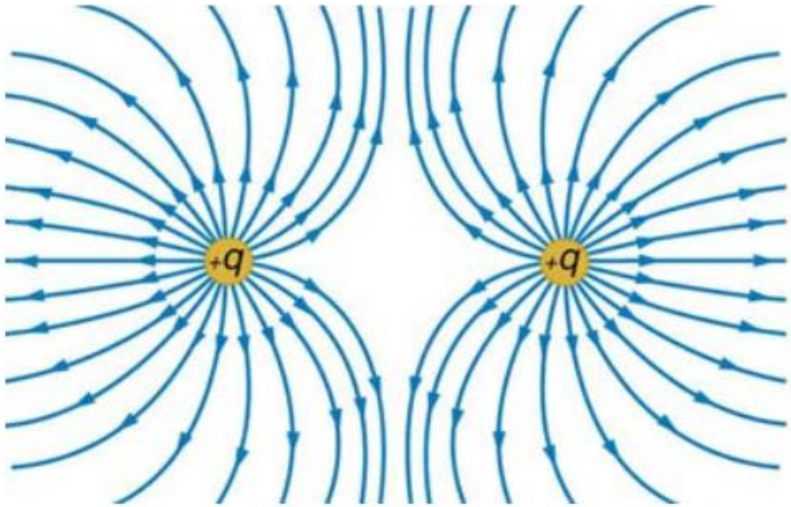
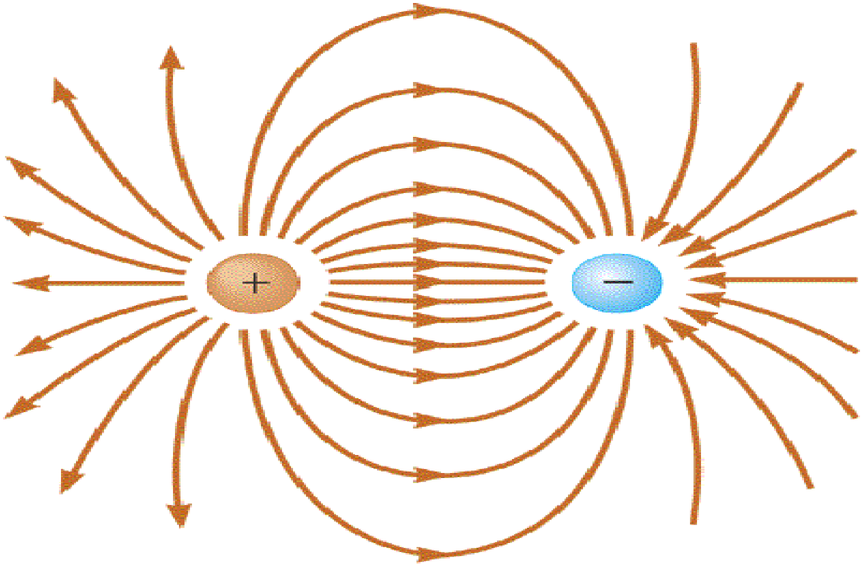
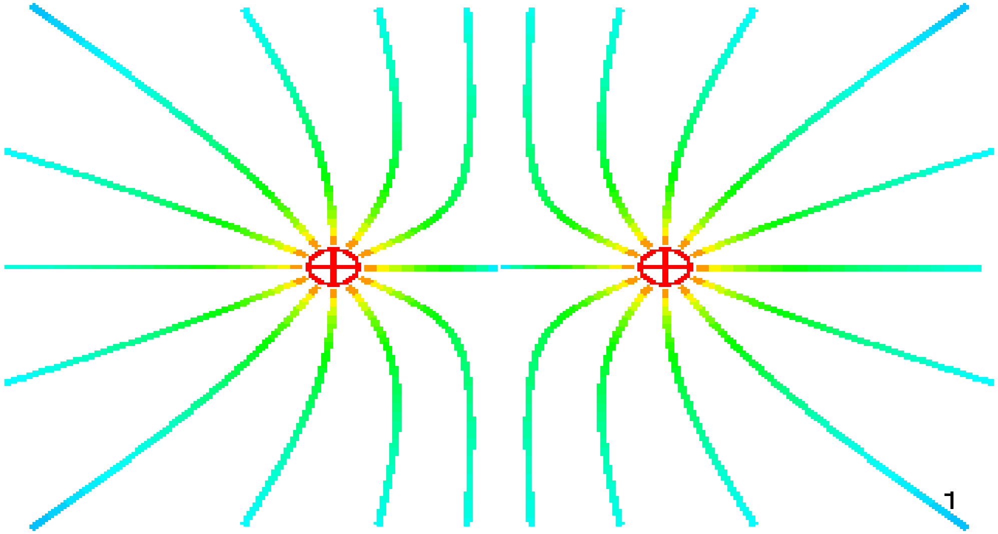
Electric field Lines (Direction)







Electric Field Lines

A convenient way of visualizing electric field patterns is to draw curved lines that are parallel to the electric field vector at any point in space.

These lines, called *electric field lines and first introduced by Faraday, are* related to the electric field in a region of space in the following manner:

- The electric field vector E is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.

- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.

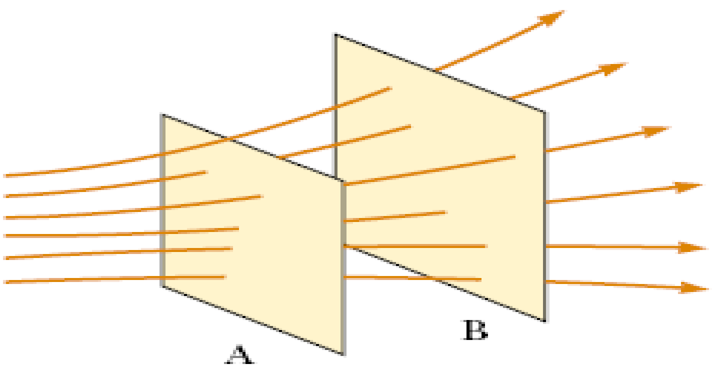


Figure 23.20 Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

Electric Field Lines of a point Charge

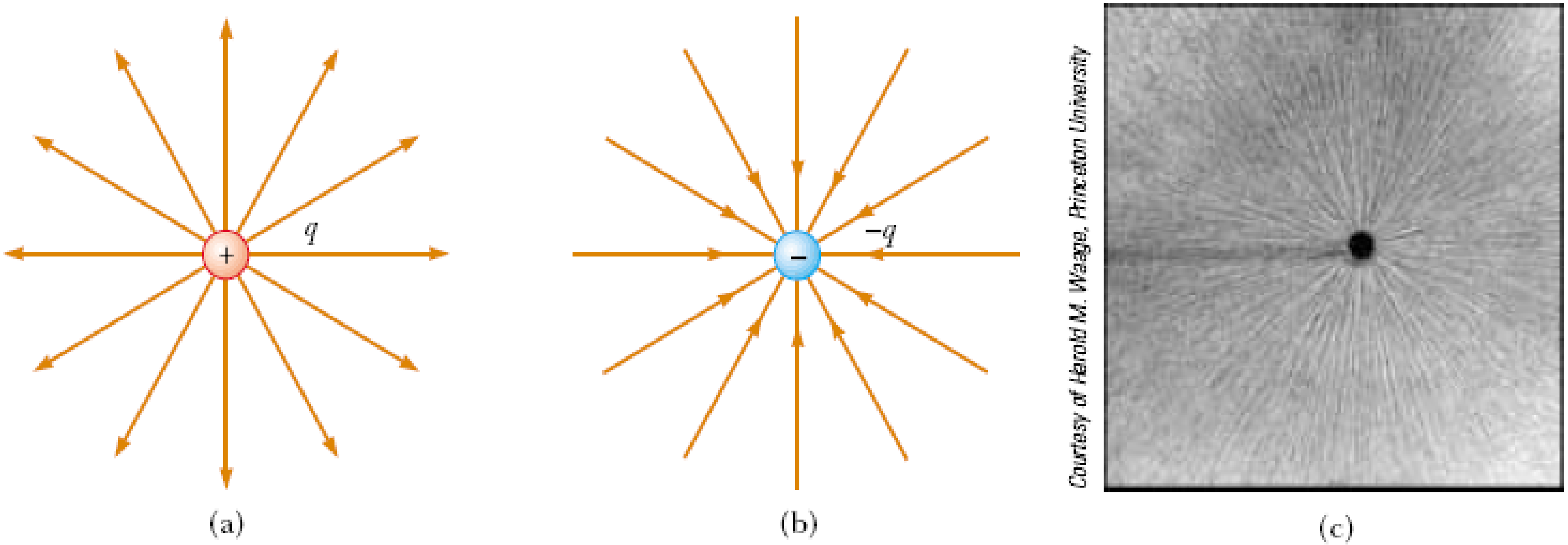


Figure 23.21 The electric field lines for a point charge. (a) For a positive point charge, the lines are directed radially outward. (b) For a negative point charge, the lines are directed radially inward. Note that the figures show only those field lines that lie in the plane of the page. (c) The dark areas are small pieces of thread suspended in oil, which align with the electric field produced by a small charged conductor at the center.

Electric field lines represent the field at various locations. Except in very special cases, they *do not* represent the path of a charged particle moving in an electric field.

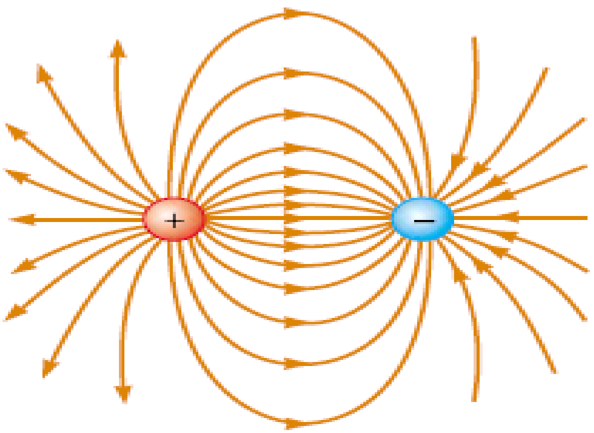
The rules for drawing electric field lines are as follows:

• The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.

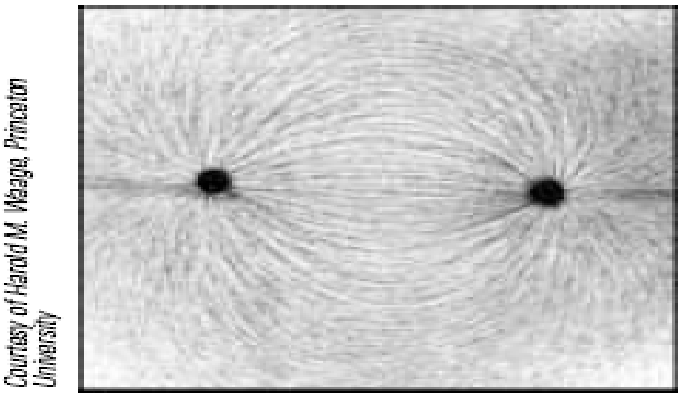
• The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

• No two field lines can cross.

Electric Field Lines of a dipole



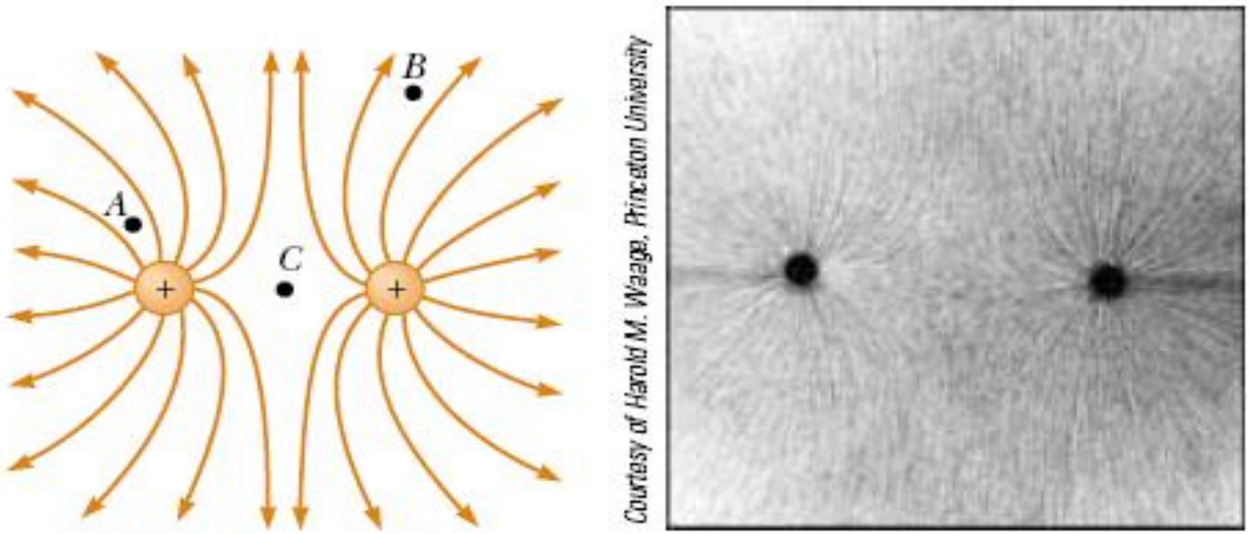
(a)



(b)

Figure 23.22 (a) The electric field lines for two point charges of equal magnitude and opposite sign (an electric dipole). The number of lines leaving the positive charge equals the number terminating at the negative charge. (b) The dark lines are small pieces of thread suspended in oil, which align with the electric field of a dipole.

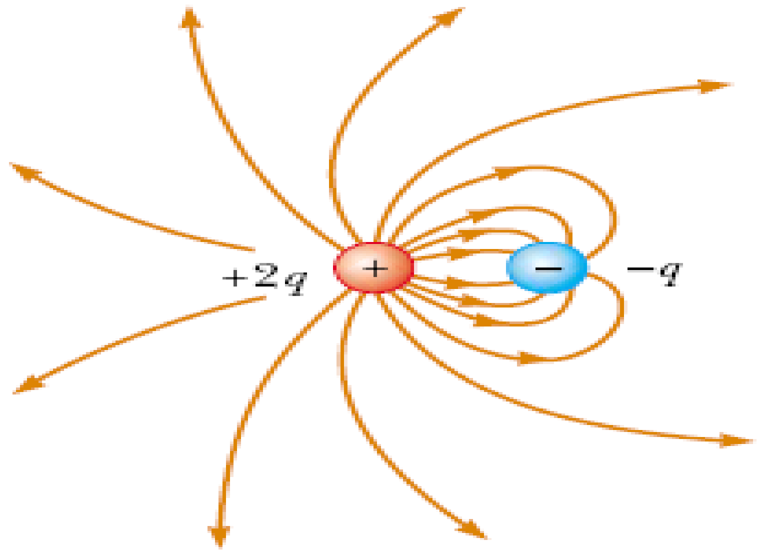
Electric Field Lines due to two same Charges



(a) (b)

Figure 23.23 (a) The electric field lines for two positive point charges. (The locations *A*, *B*, and *C* are discussed in Quick Quiz 23.7.) (b) Pieces of thread suspended in oil, which align with the electric field created by two equal-magnitude positive charges.

Electric Field Lines due to two Charges of unequal magnitude



Active Figure 23.24 The electric field lines for a point charge $+2q$ and a second point charge $-q$. Note that two lines leave $+2q$ for every one that terminates on $-q$.

Motion of Charged Particles in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field $\mathbf{E}$, the electric force exerted on the charge is $q\mathbf{E}$ according to Equation 23.8. If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law. Thus,

$$\mathbf{F}_e = q\mathbf{E} = m\mathbf{a}$$

The acceleration of the particle is therefore

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \tag{23.12}$$

If $\mathbf{E}$ is uniform (that is, constant in magnitude and direction), then the acceleration is constant. If the particle has a positive charge, its acceleration is in the direction of the electric field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

An accelerating Positive Charge

A positive point charge q of mass m is released from rest in a uniform electric field $\mathbf{E}$ directed along the x axis, as shown in Figure 23.25. Describe its motion.

Solution The acceleration is constant and is given by $q\mathbf{E}/m$. The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one dimension (see Chapter 2):

$$x_f = x_i + v_i t + \tfrac{1}{2}at^2$$

$$v_f = v_i + at$$

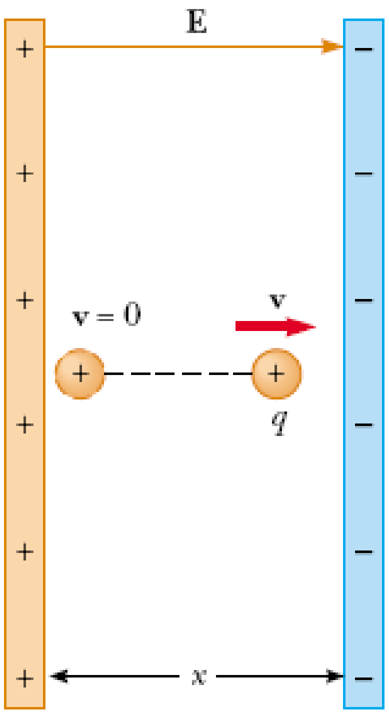
$$v_f{}^2 = v_i{}^2 + 2a(x_f - x_i)$$

Choosing the initial position of the charge as $x_i = 0$ and assigning $v_i = 0$ because the particle starts from rest, the position of the particle as a function of time is

$$x_f = \tfrac{1}{2}at^2 = \frac{qE}{2m}t^2$$

The speed of the particle is given by

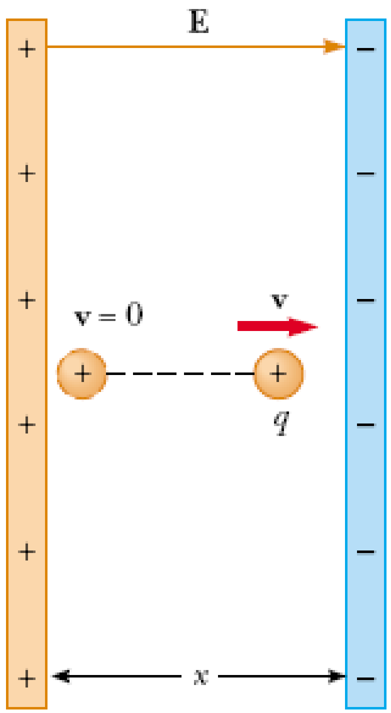
$$v_f = at = \frac{qE}{m}t$$



from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_f - x_i$:

$$K = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{2qE}{m}\right)\Delta x = qE\Delta x$$

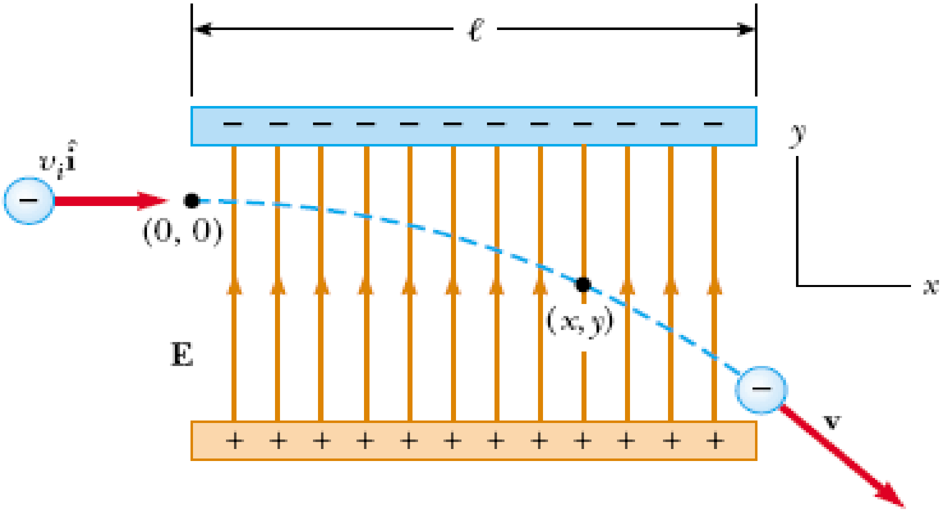
We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is $F_e\Delta x = qE\Delta x$ and $W = \Delta K$.



The electric field in the region between two oppositely charged flat metallic plates is approximately uniform (Fig. 23.26). Suppose an electron of charge $-e$ is projected horizontally into this field from the origin with an initial velocity $v_i\hat{\mathbf{i}}$ at time $t = 0$. Because the electric field $\mathbf{E}$ in Figure 23.26 is in the positive y direction, the acceleration of the electron is in the negative y direction. That is,

$$\mathbf{a} = -\frac{eE}{m_e}\hat{\mathbf{j}} \qquad (23.13)$$

Because the acceleration is constant, we can apply the equations of kinematics in two dimensions (see Chapter 4) with $v_{xi} = v_i$ and $v_{yi} = 0$. After the electron has been in the



electric field for a time interval, the components of its velocity at time t are

$$v_x = v_i = \text{constant}$$

$$v_y = a_y t = -\frac{eE}{m_e} t$$

Its position coordinates at time t are

$$x_f = v_i t$$

$$y_f = \tfrac{1}{2} a_y t^2 = -\tfrac{1}{2} \frac{eE}{m_e} t^2$$

Substituting the value $t = x_f / v_i$ from Equation 23.16 into Equation 23.17, we see that y_f is proportional to $x_f^{\,2}$. Hence, the trajectory is a parabola. This should not be a surprise—consider the analogous situation of throwing a ball horizontally in a uniform gravitational field (Chapter 4). After the electron leaves the field, the electric force vanishes and the electron continues to move in a straight line in the direction of **v** in Figure 23.26 with a speed $v > v_i$.

An accelerated Electron

An electron enters the region of a uniform electric field as shown in Figure 23.26, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

Solution The charge on the electron has an absolute value of 1.60×10^{-19} C, and $m_e = 9.11 \times 10^{-31}$ kg. Therefore, Equation 23.13 gives

$$\mathbf{a} = -\frac{eE}{m_e}\hat{\mathbf{j}} = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\hat{\mathbf{j}}$$

$$= \boxed{-3.51 \times 10^{13}\hat{\mathbf{j}}\ \text{m/s}^2}$$

(B) If the electron enters the field at time $t = 0$, find the time at which it leaves the field.

Solution The horizontal distance across the field is $\ell = 0.100$ m. Using Equation 23.16 with $x_f = \ell$, we find that the time at which the electron exits the electric field is

$$t = \frac{\ell}{v_i} = \frac{0.100 \text{ m}}{3.00 \times 10^6 \text{ m/s}} = \boxed{3.33 \times 10^{-8} \text{ s}}$$

(C) If the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

Solution Using Equation 23.17 and the results from parts (A) and (B), we find that

$$y_f = \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(3.51 \times 10^{13} \text{ m/s}^2)(3.33 \times 10^{-8} \text{ s})^2$$

$$= -0.019\,5 \text{ m} = \boxed{-1.95 \text{ cm}}$$

If the electron enters just below the negative plate in Figure 23.26 and the separation between the plates is less than the value we have just calculated, the electron will strike the positive plate.