CHAP 02 Ex # 2.3

Question No. 1) $y = 4x^7$

$$\boxed{\frac{dy}{dx} = 28x^6}$$ Ans.

Question No. 2) $y = -3x^{12}$ $\Rightarrow$ $$\boxed{\frac{dy}{dx} = -36x^{11}}$$ Ans.

Question No. 3) $y = 3x^8 + 2x + 1$ $\Rightarrow$ $$\boxed{\frac{dy}{dx} = 24x^7 + 2}$$ Ans.

Question No. 4) $y = \frac{1}{2}(x^4 + 7)$ $\Rightarrow$ $$\frac{dy}{dx} = \frac{1}{\cancel{2}}(\cancel{4}^{2}x^3) = \boxed{2x^3}$$ Ans.

Question No. 5) $y = \pi^3$ $\Rightarrow$ $$\boxed{\frac{dy}{dx} = 0}$$ Ans.

Question No. 6) $y = \sqrt{2}x + (1/\sqrt{2})$ $\Rightarrow$ $$\boxed{\frac{dy}{dx} = \sqrt{2}}$$ Ans.

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Q7) $y = \frac{-1}{3}(x^7+2x-9) \Rightarrow \boxed{\frac{dy}{dx} = \frac{-1}{3}(7x^6+2)}$

Q8) $y = \frac{x^2+1}{5} \Rightarrow \frac{dy}{dx} = \frac{1}{5}(2x) \Rightarrow \boxed{\frac{2x}{5}}$

Q9) $f(x) = x^{-3} + \frac{1}{x^7} \Rightarrow f'(x) = -3x^{-4} - 7x^{-8} = \boxed{-3x^{-4} - 7x^{-8}}$

Q10) $f(x) = \sqrt{x} + \frac{1}{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}} - x^{-1} = \frac{1}{2\sqrt{x}} + (-1x^{-2})$

$\boxed{f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}}$

Q11) $f(x) = -3x^{-8} + 2\sqrt{x}$

$f'(x) = +24x^{-9} + \cancel{2}\frac{1}{\cancel{2}\sqrt{x}} \Rightarrow \boxed{24x^{-9} + \frac{1}{\sqrt{x}}}$

Q12) $f(x) = 7x^{-6} - 5\sqrt{x}$

$\boxed{f'(x) = -42x^{-7} - 5 \cdot \frac{1}{2\sqrt{x}}}$

Q13) $f(x) = x^e + \frac{1}{x^{\sqrt{10}}}$

$f'(x) = ex^{e-1} + x^{-\sqrt{10}} = \boxed{ex^{e-1} - \sqrt{10}\,x^{-\sqrt{10}-1}}$

Q14) $f(x) = \sqrt[3]{\frac{8}{x}} = (8x^{-1})^{1/3}$

$\cancel{f'(x) = \frac{1}{3}(8x^{-1})^{\frac{1}{3}-1} \cdot -10x^{-2}}$

$f(x) = \sqrt[3]{8x^{-1}} = 2x^{-1/3}$

$f'(x) = \boxed{\frac{-2}{3}x^{-4/3}}$

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Q15) $f(x) = (3x^2+1)^2$

$f'(x) = 2(3x^2+1).(6x) = 12x(3x^2+1) = \boxed{36x^3+12x}$ Ans.

Q16) $f(x) = ax^3 + bx^2 + cx + d$

$\boxed{f'(x) = 3ax^2 + 2bx + c + 0}$ Ans.

Q17) $y = 5x^2 - 3x + 1$

$y' = 10x - 3 \Rightarrow y'(1) = 10(1) - 3 = \boxed{7}$ Ans.

Q18) $y = \dfrac{x^{3/2}+2}{x} = \dfrac{x^{3/2}}{x} + \dfrac{2}{x}$

$\cancel{y' = \frac{3}{2}x^{3/2-1}} \quad y' = x^{3/2}.x^{-1} + 2x^{-1}$

$y' = x^{1/2} + 2x^{-1} = \frac{1}{2\sqrt{x}} + (-2x^{-2})$

$y' = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^2} \Rightarrow y'(1) = \dfrac{1}{2\sqrt{1}} - \dfrac{2}{(1)^2} = \boxed{\dfrac{-3}{2}}$ Ans.

Q19) $x = t^2 - t$

$\boxed{\dfrac{dx}{dt} = 2t - 1}$

Q20) $x = \dfrac{t^2+1}{3t} = \dfrac{t^2}{3t} + \dfrac{1}{3t}$

$x = \dfrac{t^{2-1}}{3} + \dfrac{t^{-1}}{3} \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{3} + \dfrac{(-1t^{-2})}{3} = \boxed{\dfrac{1}{3} - \dfrac{1}{3t^2}}$ Ans.

Q21) $y = 1 + x + x^2 + x^3 + x^4 + x^5$

$\dfrac{dy}{dx} = 0 + 1 + 2x + 3x^2 + 4x^3 + 5x^4$

$\left.\dfrac{dy}{dx}\right|_{x=1} = 1 + 2 + 3 + 4 + 5 = \boxed{15}$ Ans.

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Q22) $y = \dfrac{1+x+x^2+x^3+x^4+x^5+x^6}{x^3}$

$y = \dfrac{1}{x^3} + \dfrac{x}{x^3} + \dfrac{x^2}{x^3} + \dfrac{x^3}{x^3} + \dfrac{x^4}{x^3} + \dfrac{x^5}{x^3} + \dfrac{x^6}{x^3}$

$y = x^{-3} + x^{-2} + x^{-1} + 1 + x + x^2 + x^3.$

$\dfrac{dy}{dx} = -3x^{-4} + -2x^{-3} - 1x^{-2} + 0 + 1 + 2x + 3x^2.$

$\left.\dfrac{dy}{dx}\right|_{x=1} = \cancel{-3} - \cancel{2} - \cancel{1} + \cancel{1} + \cancel{2} + \cancel{3} = \boxed{0}$

Q23) $y = (1-x)(1+x)(1+x^2)(1+x^4)$

$y = (1-x^2)(1+x^2)(1+x^4)$

$y = (1-x^4)(1+x^4) \Rightarrow y = (1-x^8)$

$\dfrac{dy}{dx} = 0 - 8x^7 \Rightarrow \boxed{\left.\dfrac{dy}{dx}\right|_{x=1} = -8}$

Q24) $y = x^{24} + 2x^{12} + 3x^8 + 4x^6$

$\dfrac{dy}{dx} = 24x^{23} + 24x^{11} + 24x^7 + 24x^5$

$\boxed{\left.\dfrac{dy}{dx}\right|_{x=1} = 96}$

Q41) a) $y = 7x^3 - 5x^2 + x$

$\dfrac{dy}{dx} = 21x^2 - 10x + 1 \Rightarrow \boxed{\dfrac{d^2y}{dx^2} = 42x - 10}$

~~Q41~~ b) $y = 1/x = x^{-1}$

$\dfrac{dy}{dx} = -1x^{-2} = \dfrac{-1}{x^2} \Rightarrow \boxed{\dfrac{d^2y}{dx^2} = 2x^{-3}}$

b) $y = 12x^2 - 2x + 3$

$\dfrac{dy}{dx} = 24x - 2 \Rightarrow \boxed{\dfrac{d^2y}{dx^2} = 24}$

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c) $y = \frac{x+1}{x} \Rightarrow y = \frac{x}{x} + \frac{1}{x} = 1 + x^{-1}$

$\frac{dy}{dx} = 0 + -1x^{-2} \Rightarrow \boxed{\frac{d^2y}{dx^2} = 2x^{-3}}$

d) $y = (5x^2-3)(7x^3+x) \Rightarrow y = 35x^5 + 5x^3 - 21x^3 - 3x$

$\frac{dy}{dx} = 175x^4 + 15x^2 - 63x^2 - 3 \Rightarrow \boxed{\frac{d^2y}{dx^2} = 700x^3 - 96x}$ Ans.

$= 175x^4 - 48x^2 - 3$

Q42) a) $y = 4x^7 - 5x^3 + 2x.$

$\frac{dy}{dx} = 28x^6 - 15x^2 + 2 \Rightarrow \boxed{\frac{d^2y}{dx^2} = 168x^5 - 30x}$ Ans.

b) $y = 3x + 2$

$\frac{dy}{dx} = 3 \Rightarrow \boxed{\frac{d^2y}{dx^2} = 0}$ Ans.

c) $y = \frac{3x-2}{5x} = \frac{3x}{5x} - \frac{2x^{-1}}{5}$

$\frac{dy}{dx} = 0 - \frac{2}{5}(-1x^{-2}) = \frac{2}{5}x^{-2}$

$\frac{d^2y}{dx^2} = \frac{2}{5}(-2x^{-3}) = \boxed{\frac{-4x^{-3}}{5}}$ Ans.

Q45) (a) $f'''(2)$ where $f(x) = 3x^2 + 2.$

$f'(x) = 6x \Rightarrow f''(x) = 6 \Rightarrow f'''(x) = 0 \Rightarrow \boxed{f'''(2) = 0}$

Q43) a) $y = x^{-5} + x^5$

$y' = -5x^{-6} + 5x^4 \Rightarrow y'' = 30x^{-7} + 20x^3 \Rightarrow \boxed{y''' = -210x^{-8} + 60x^2}$ Ans.

b) $y = 1/x. = x^{-1}$

$y' = -1x^{-2} \Rightarrow y'' = 2x^{-3} \Rightarrow \boxed{y''' = -6x^{-4}}$

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c) $y = ax^3 + bx + c.$

$y' = 3ax^2 + b + 0 \Rightarrow y'' = 6ax \Rightarrow \boxed{y''' = 6a}$ Ans.

Q44) a) $y = 5x^2 - 4x + 7.$

$y' = 10x - 4 \Rightarrow y'' = 10 \Rightarrow \boxed{y''' = 0}$ Ans.

b) $y = 3x^{-2} + 4x^{-1} + x.$

$y' = -6x^{-3} - 4x^{-2} + 1 \Rightarrow y'' = 18x^{-4} + 8x^{-3} \Rightarrow \boxed{y''' = -72x^{-5} - 24x^{-4}}$ Ans.

c) $y = ax^4 + bx^2 + c$

$y' = 4ax^3 + 2bx \Rightarrow y'' = 12ax^2 + 2b \Rightarrow \boxed{y''' = 24ax}$ Ans.

Q45) a) find $f'''(2)$, $f(x) = 3x^2 - 2.$

$f'(x) = 6x - 0 \Rightarrow f''(x) = 6 \Rightarrow f'''(x) = 0 \Rightarrow \boxed{f'''(2) = 0}$ Ans.

b) $\left.\frac{d^2y}{dx^2}\right|_{x=1}$, $y = 6x^5 - 4x^2.$

$\frac{dy}{dx} = 30x^4 - 8x \Rightarrow \frac{d^2y}{dx^2} = 120x^3 - 8 \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=1} = 120 - 8$

$\boxed{= 112}$ Ans.

c) $\left.\frac{d^4(x^{-3})}{dx^4}\right|_{x=1}$

$\frac{d(x^{-3})}{dx} = -3x^{-4} \Rightarrow \frac{d^2(x^{-3})}{dx^2} = 12x^{-5} \Rightarrow \frac{d^3(x^{-3})}{dx^3} = -60x^{-6}$

$\Rightarrow \frac{d^4(x^{-3})}{dx^4} = 360x^{-7} \Rightarrow \boxed{\left.\frac{d^4(x^{-3})}{dx^4}\right|_{x=1} = 360}$ Ans.

Q46) a) $y'''(0)$, $y = 4x^4 + 2x^3 + 3.$

$y' = 16x^3 + 6x^2 + 0 \Rightarrow y'' = 48x^2 + 12x \Rightarrow y''' = 96x + 12$

$y'''(0) = 0 + 12 = \boxed{12}$ Ans.

b) d^4y

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b) $\left.\dfrac{d^4y}{dx^4}\right|_{x=1}$, $y = \dfrac{6}{x^4} = 6x^{-4}$

$\dfrac{dy}{dx} = -24x^{-5} \Rightarrow \dfrac{d^2y}{dx^2} = 120x^{-6} \Rightarrow \dfrac{d^3y}{dx^3} = -720x^{-7}$

$\dfrac{d^4y}{dx^4} = 5040x^{-8} \Rightarrow \boxed{\left.\dfrac{d^4y}{dx^4}\right|_{x=1} = 5040}$ Ans.

Q47) Show $y = x^3 + 3x + 1$ satisfy $y''' + xy'' - 2y' = 0$ — (A)

$y' = 3x^2 + 3 \Rightarrow y'' = 6x \Rightarrow y''' = 6$

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(A) $\Rightarrow 6 + x(6x) - 2(3x^2+3) = 0 \Rightarrow \cancel{6} + \cancel{6x^2} - \cancel{6x^2} - \cancel{6} = 0 \Rightarrow \boxed{0 = 0}$

Ex # 2.4

Q1) $f(x) = (x+1)(2x-1)$

for (a):- $f(x) = 2x^2 - x + 2x - 1$

$\boxed{f'(x) = 4x - 1 + 2} \Rightarrow \boxed{f'(x) = 4x + 1}$ Ans.

for (b):- $f(x) = (x+1)(2x-1)$

$f'(x) = [(x+1)(2-0) + (2x-1)(1+0)]$

$f'(x) = 2x + 2 + 2x - 1$ $\boxed{a = b}$

$\boxed{f'(x) = 4x + 1}$ Ans.

Q2) $f(x) = (3x^2-1)(x^2+2)$.

for a:- $f(x) = 3x^4 + 6x^2 - x^2 - 2. = 3x^4 + 5x^2 - 2$

$\boxed{f'(x) = 12x^3 + 10x}$ Ans.

for (b):- $f(x) = (3x^2-1)(x^2+2)$

$f'(x) = [(3x^2-1)(2x+0) + (x^2+2)(6x-0)]$

$f'(x) = 6x^3 - 2x + 6x^3 + 12x.$ $\boxed{a = b}$

$\boxed{f'(x) = 12x^3 + 10x}$ Ans.

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Q3) $f(x) = (x^2+1)(x^2-1)$

for (a):- $f(x) = x^4 - \cancel{x^2} + \cancel{x^2} - 1 = x^4 - 1$

$\boxed{f'(x) = 4x^3}$

for (b):- $f(x) = (x^2+1)(x^2-1)$

$f'(x) = [(x^2+1)(2x-0) + (x^2-1)(2x+0)]$

$f'(x) = 2x^3 + \cancel{2x} + 2x^3 - \cancel{2x} \Rightarrow \boxed{f'(x) = 4x^3}$

Q4) $f(x) = (x+1)(x^2-x+1)$

for (a):- $f(x) = x^3 - x^2 + x + x^2 - x + 1 = x^3+1$

$\boxed{f'(x) = 3x^2}$ Ans.

for (b):- $f'(x) = [(x+1)(2x-1+0) + (x^2-x+1)(1+0)]$

$f'(x) = 2x^2 - x + 2x - 1 + x^2 - x + 1$

$f'(x) = 3x^2 + x - x \Rightarrow \boxed{f'(x) = 3x^2}$ Ans.

Q5) $f(x) = (3x^2+6)(2x - 1/4)$

$f'(x) = [(3x^2+6)(2-0) + (2x - 1/4)(6x+0)]$

$f'(x) = [6x^2 + 12 + 12x^2 - 3/2\, x]$

$\boxed{f'(x) = 18x^2 - 3/2\, x + 12}$ Ans.

Q6) $f(x) = (2 - x - 3x^2)(7 + x^5)$

$f(x) = 14 + 2x^5 - 7x - x^6 - 21x^3 - 3x^8$

$f'(x) = 10x^4 - 7 - 6x^5 - 63x^2 - 24x^7$ Ans.

Q7) $f(x) = (x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})$

$f'(x) = [(x^3 + 7x^2 - 8)(-6x^{-4} - 4x^{-5}) + (2x^{-3} + x^{-4})(3x^2 + 14x - 0)]$

$f'(x) = -6x^{-1} - 4x^{-2} - 42x^{-2} - 28x^{-3} + 48x^{-4} + 32x^{-5} + 6x^{-1} + 28x^{-2} + 3x^{-2} + 14x^{-3}$

$f'(x) = -15x^{-2} - 14x^{-3} + 32x^{-5} + 48x^{-4}$ Ans.

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Q8) $(x^{-1} + x^{-2})(3x^3 + 27)$

$f(x) = 3x^2 + 27x^{-1} + 3x + 27x^{-2}$

$f'(x) = 6x - 27x^{-2} + 3 - 54x^{-3}$

$f'(x) = 6x - \frac{27}{x^2} + 3 - \frac{54}{x^3}$ Ans

Q9) $f(x) = (x-2)(x^2+2x+4)$

$f'(x) = [(x-2)(2x+2+0) + (x^2+2x+4)(1-0)]$

$f'(x) = 2x^2 + \cancel{2x} - \cancel{4x} - \cancel{4} + x^2 + \cancel{2x} + \cancel{4}$

$\boxed{f'(x) = 3x^2}$ Ans.

Q10) $f(x) = (x^2+x)(x^2-x)$

$f(x) = x^4 - x^2 \Rightarrow \boxed{f'(x) = 4x^3 - 2x}$ Ans.

Q11) $f(x) = \frac{3x+4}{x^2+1}$

$f'(x) = \left[\frac{(x^2+1)(3+0) - (3x+4)(2x+0)}{(x^2+1)^2}\right]$

$f'(x) = \left[\frac{3x^2+3-3x^2-8x}{(x^2+1)^2}\right] \Rightarrow f'(x) = \frac{-3x^2-8x+3}{(x^2+1)^2}$ Ans.

Q12) $f(x) = \frac{x-2}{x^4+x+1}$

$f'(x) = \left[\frac{(x^4+x+1)(1-0) - (x-2)(4x^3+1)}{(x^4+x+1)^2}\right]$

$f'(x) = \frac{x^4+\cancel{x}+1 - 4x^4 - \cancel{x} + 8x^3 + 2}{(x^4+x+1)^2}$

$f'(x) = \frac{-3x^4 + 8x^3 + 3}{(x^4+x+1)^2}$ Ans.

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Q13) $f(x) = \dfrac{x^2}{3x-4}$

$$f'(x) = \left[\frac{(3x-4)(2x) - (x^2)(3-0)}{(3x-4)^2}\right]$$

$$= \left[\frac{6x^2 - 8x - 3x^2}{(3x-4)^2}\right] \Rightarrow f'(x) = \left[\frac{3x^2 - 8x}{(3x-4)^2}\right] \text{ Ans.}$$

Q14) $f(x) = \dfrac{2x^2+5}{3x-4}$

$$f'(x) = \left[\frac{(3x-4)(4x-0) - (2x^2+5)(3-0)}{(3x-4)^2}\right]$$

$$f'(x) = \frac{12x^2 - 16x - (6x^2+15)}{(3x-4)^2} = \frac{12x^2 - 16x - 6x^2 - 15}{(3x-4)^2}$$

$$f'(x) = \frac{6x^2 - 16x - 15}{(3x-4)^2} \text{ Ans.}$$

Q15) $f(x) = \dfrac{(2\sqrt{x}+1)(x-1)}{x+3}$

$$f(x) = \frac{2x^{3/2} - 2x^{1/2} + x - 1}{x+3}$$

$$f'(x) = \left[\frac{(x+3)\left(\frac{3}{2} \times 2x^{1/2} - \frac{1}{2} \times 2x^{-1/2} + 1 - 0\right) - (2x^{3/2} - 2x^{1/2} + x - 1)(1+0)}{(x+3)^2}\right]$$

$$f'(x) = \left[\frac{3x^{3/2} - x^{1/2} + x + 9x^{1/2} - 3x^{-1/2} + 3 - 2x^{3/2} + 2x^{1/2} - x + 1}{(x+3)^2}\right]$$

$$f'(x) = \left[\frac{x^{3/2} + 10x^{1/2} - 3x^{-1/2} + 4}{(x+3)^2}\right] \text{ Ans.}$$

Q16) $f(x) = (2\sqrt{x}+1)\left(\dfrac{2-x}{x^2+3x}\right) = \dfrac{4x^{1/2} - 2x^{3/2} + 2 - x}{x^2+3x}$

$$f'(x) = \left[\frac{(x^2+3x)(2x^{-1/2} - 3x^{1/2} - 1) - (4x^{1/2} - 2x^{3/2} + 2 - x)(2x+3)}{(x^2+3x)^2}\right]$$

$$f'(x) = \frac{2x^{3/2} - 3x^{3/2} - x^2 + 6x^{1/2} - 9x^{3/2} - 3x - 8x^{3/2} + 4x^{5/2} - 4x + 2x^2 - 12x^{1/2} + 6x^{3/2} - 6 + 3x}{(x^2+3x)^2}$$

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$f'(x) = \frac{x^{5/2} + x^2 - 9x^{3/2} - 4x - 6x^{1/2} - 6}{(x^2+3x)^2}$ Ans.

Q17) $f(x) = (2x+1)\left(1+\frac{1}{x}\right)(x^{-3}+7)$

$f(x) = (2x+1)(1+x^{-1})(x^{-3}+7) = [2x+2+1+x^{-1}](x^{-3}+7)$

$f(x) = 2x^{-2} + 14x + 2x^{-3} + 14 + x^{-3} + 7 + x^{-4} + 7x^{-1}$

$f(x) = 14x + 7x^{-1} + 2x^{-2} + 3x^{-3} + x^{-4} + 21.$

$f'(x) = 14 - 7x^{-2} - 4x^{-3} - 9x^{-4} - 4x^{-5}$ Ans.

Q18) $f(x) = x^{-5}(x^2+2x)(4-3x)(2x^9+1)$

$f(x) = x^{-5}(4x^2 - 3x^3 + 8x - 6x^2)(2x^9+1)$

$f(x) = x^{-5}(-2x^2 - 3x^3 + 8x)(2x^9+1)$

$= x^{-5}(-4x^{11} - 2x^2 - 6x^{12} - 3x^3 + 16x^{10} + 8x)$

$= -4x^6 - 2x^{-3} - 6x^7 - 3x^{-2} + 16x^5 + 8x^{-4}$

$f'(x) = -24x^5 + 6x^{-4} - 42x^6 + 6x^{-3} + 80x^4 - 32x^{-5}$ Ans.

Q19) $f(x) = (x^7 + 2x - 3)^3$

$f'(x) = 3(x^7+2x-3)^2 \cdot (7x^6+2)$ Ans.

Q20) $f(x) = (x^2+1)^4$

$f'(x) = 4(x^2+1)^3 \cdot 2x \Rightarrow 8x(x^2+1)^3$ Ans.

Q21) ~~f(x) =~~ $y = \frac{2x-1}{x+3}$

$\frac{dy}{dx} = \left[\frac{(x+3)(2-0) - (2x-1)(1+0)}{(x+3)^2}\right]$

$\frac{dy}{dx} = \left[\frac{2x+6-2x+1}{(x+3)^2}\right] = \left(\frac{7}{(x+3)^2}\right)$

$\left.\frac{dy}{dx}\right|_{x=1} = \left[\frac{7}{(1+3)^2}\right] = \boxed{\frac{7}{16}}$ Ans.

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Q22) $y = \dfrac{4x+1}{x^2-5}$

$$\frac{dy}{dx} = \left[\frac{(x^2-5)(4+0) - (4x+1)(2x-0)}{(x^2-5)^2}\right] = \frac{4x^2-20-8x^2-2x}{(x^2-5)^2}$$

$$\frac{dy}{dx} = \frac{-4^2-2x-20}{(x^2-5)^2} \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = \frac{-13}{8} \text{ Ans.}$$

Q23) $y = \left(\dfrac{3x+2}{x}\right)\left(x^{-5}+1\right)$

$$\frac{dy}{dx} = \left[\left(\frac{3x+2}{x}\right).(-5x^{-6}) + (x^{-5}+1)\left[\frac{x(3+0)-(3x+2)(1)}{(x)^2}\right]\right]$$

$$\frac{dy}{dx} = \left(\frac{3x+2}{x}\right)(-5x^{-6}) + (x^{-5}+1)\left(\frac{\cancel{3x}-\cancel{3x}-2}{x^2}\right)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \left(\frac{3+2}{1}\right)(-5) + (1+1)\left(\frac{-2}{1^2}\right)$$

$$= -25 + 2(-2) \Rightarrow -25-4 \Rightarrow \boxed{-29} \text{ Ans}$$

Q24) $y = (2x^7 - x^2)\left(\dfrac{x-1}{x+1}\right)$

$$\frac{dy}{dx} = \left[(2x^7-x^2)\left[\frac{(x+1)(1-0)-(x-1)(1+0)}{(x+1)^2}\right] + \left(\frac{x-1}{x+1}\right)(14x^6-2x)\right]$$

$$\frac{dy}{dx} = (2x^7-x^2)\left[\frac{\cancel{x}+1-\cancel{x}+1}{(x+1)^2}\right] + \left(\frac{x-1}{x+1}\right)(14x^6-2x)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = (2-1)\left(\frac{2}{(1+1)^2}\right) + \left(\frac{1-1}{1+1}\right)(14-2)$$

$$= 1\left(\frac{2}{4}\right) + 0 \Rightarrow \boxed{f'(x) = \frac{1}{2}} \text{ Ans}$$

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Ex # 2.5

Q1) $f(x) = 4\cos x + 2\sin x$

$f'(x) = -4\sin x + 2\cos x$ Ans.

Q2) $f(x) = \frac{5}{x^2} + \sin x = 5x^{-2} + \sin x.$

$f'(x) = -10x^{-3} + \cos x$ Ans.

Q3) $f(x) = -4x^2\cos x$

$f'(x) = -4\left[(x^2)(-\sin x) + (\cos x)(2x)\right]$

$f'(x) = 4x^2\sin x - 8x\cos x$ Ans.

Q4) $f(x) = 2\sin^2 x$

$f'(x) = 2 \cdot 2\sin x \cdot \cos x = 4\sin x\cos x$ Ans.

Q5) $f(x) = \frac{5-\cos x}{5+\sin x}$

$f'(x) = \left[\frac{(5+\sin x)(0+\sin x) - (5-\cos x)(0+\cos x)}{(5+\sin x)^2}\right]$

$f'(x) = \frac{0 + 5\sin x + \sin^2 x - 5\cos x + \cos^2 x}{(5+\sin x)^2}$

$f'(x) = \frac{(\sin^2 x + \cos^2 x) + 5\sin x - 5\cos x}{(5+\sin x)^2}$

$f'(x) = \frac{1 + 5(\sin x - \cos x)}{(5+\sin x)^2}$ Ans

Q6) $f(x) = \frac{\sin x}{x^2 + \sin x}$

$f'(x) = \left[\frac{-(\sin x)(2x + \cos x) + (x^2 + \sin x)(\cos x)}{(x^2+\sin x)^2}\right]$

$f'(x) = \left[\frac{x^2\cos x + \sin x\cos x - [2x\sin x + \sin x\cos x]}{(x^2+\sin x)^2}\right]$

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$$f'(x) = \frac{x^2\cos x + \sin x\cos x - 2x\sin x - \sin x\cos x}{(x^2 + \sin x)^2}$$

$$f'(x) = \frac{x^2\cos x - 2x\sin x}{(x^2 + \sin x)^2} \quad \text{Ans.}$$

Q7) $f(x) = \sec x - \sqrt{2}\tan x.$

$f'(x) = \sec x\tan x - \sqrt{2}\sec^2 x.$ Ans.

Q8) $f(x) = (x^2+1)(\sec x)$

$f'(x) = [(x^2+1)(\sec x\tan x) + (\sec x)(2x+0)]$

$f'(x) = \sec x\,[(x^2+1)\tan x + 2x]$ Ans.

Q9) $f(x) = 4\operatorname{cosec} x - \cot x.$

$f'(x) = -4\operatorname{cosec} x\cot x + \operatorname{cosec}^2 x.$ Ans.

~~Q10) $f(x) = \cos x - x\cos x$~~

~~$f'(x) = -\sin x - [x(-\sin x) + (\cos x)(1)]$~~

~~$f'(x) = -\sin x + x\sin x - \cos x$ Ans.~~

Q10) $f(x) = \cos x - x\operatorname{cosec} x.$

$f'(x) = -\sin x - [x(-\operatorname{cosec} x\cot x) + (\operatorname{cosec} x)(1)]$

$f'(x) = -\sin x + x\operatorname{cosec} x\cot x + \operatorname{cosec} x$ Ans.

Q11) $f(x) = \sec x\tan x.$

$f'(x) = [(\sec x)(\cancel{\tan x}\ \sec^2 x) + (\tan x)(\sec x\tan x)]$

$f'(x) = \sec^3 x - \sec x\tan^2 x$ Ans.

Q12) $f(x) = \operatorname{cosec} x\cot x.$

$f'(x) = [(\operatorname{cosec} x)(-\operatorname{cosec}^2 x) + (\cot x)(-\operatorname{cosec} x\cot x)]$

$f'(x) = -\operatorname{cosec}^3 x - \operatorname{cosec} x\cot^2 x.$ Ans.

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$Q_{13})$ $f(x) = \dfrac{\text{Cot}x}{1+\text{Cosec}x}$

$f'(x) = \left[\dfrac{(1+\text{Cosec}x)(-\text{Cosec}^2x) - (\text{Cot}x)(0-\text{Cosec}x\,\text{Cot}x)}{(1+\text{Cosec}x)^2}\right]$

$f'(x) = \left[\dfrac{-\text{Cosec}^2x - \text{Cosec}^3x + \text{Cosec}x\,\text{Cot}^2x}{(1+\text{Cosec}x)^2}\right]$ $\therefore 1+\text{Cot}^2x = \text{Cosec}^2x$

$f'(x) = \dfrac{\text{Cosec}x(-\text{Cosec}x - \text{Cosec}^2x + \text{Cot}^2x)}{(1+\text{Cosec}x)^2}$ $-\text{Cosec}^2x + \text{Cot}^2x = -1$

$f'(x) = \dfrac{\text{Cosec}x(-1-\text{Cosec}x)}{(1+\text{Cosec}x)^2} = \dfrac{-\text{Cosec}x(1+\text{Cosec}x)}{(1+\text{Cosec}x)^2}$

$f'(x) = \dfrac{-\text{Cosec}x}{(1+\text{Cosec}x)}$ Ans.

$Q_{14})$ $f(x) = \dfrac{\text{Sec}x}{1+\tan x}$

$f'(x) = \left[\dfrac{(1+\tan x)(\text{Sec}x\tan x) - (\text{Sec}x)(0+\text{Sec}^2x)}{(1+\tan x)^2}\right]$

$f'(x) = \left[\dfrac{\text{Sec}x\tan x + \text{Sec}x\tan^2x - \text{Sec}^3x}{(1+\tan x)^2}\right]$

$f'(x) = \dfrac{\text{Sec}x(\tan x + \tan^2x - \text{Sec}^2x)}{(1+\tan x)^2}$

$f'(x) = \dfrac{-\text{Sec}x(-\tan x + \text{Sec}^2x - \tan^2x)}{(1+\tan x)^2} = \dfrac{-\text{Sec}x(-\tan x + 1)}{(1+\tan x)^2}$ Ans.

$Q_{15})$ $f(x) = \text{Sin}^2x + \text{Cos}^2x.$ $f(x) = 1 \Rightarrow \boxed{f'(x) = 0}$ Ans.

$Q_{16})$ $f(x) = \text{Sec}^2x - \tan^2x.$ $f(x) = 1 \Rightarrow \boxed{f'(x) = 0}$ Ans.

$Q_{17})$ $f(x) = \dfrac{\text{Sec}x\,\text{Sin}x}{1+x\tan x} = \dfrac{\tan x}{1+x\tan x}$

$f'(x) = \left[\dfrac{(1+x\tan x)(\text{Sec}^2x) - (\tan x)[0+(x)(\text{Sec}^2x)+(\tan x)(1)]}{(1+x\tan x)^2}\right]$

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$$f'(x) = \frac{(\sec^2 x + x\sec^2 x\tan x) - (\tan x)(x\sec^2 x + \tan x)}{(1+x\tan x)^2}$$

$$f'(x) = \frac{\sec^2 x + x\sec^2 x\tan x - x\sec^2 x\tan x - \tan^2 x}{(1+x\tan x)^2}$$

$$f'(x) = \frac{1}{(1+x\tan x)^2} \quad \text{Ans.}$$

Q18) $f(x) = \dfrac{(x^2+1)\cot x}{3-\cos x\csc x} = \dfrac{(x^2+1)\cot x}{3-\cot x}$

$$f'(x) = \left[\frac{(3-\cot x)[(x^2+1)(-\csc^2 x) + (\cot x)(2x)] - [(x^2+1)\cot x][0+\csc^2 x]}{(3-\cot x)^2}\right]$$

$$f'(x) = \left[\frac{(3-\cot x)(-x^2\csc^2 x - \csc^2 x + 2x\cot x) - (x^2\cot x + \cot x)(\csc^2 x)}{(3-\cot x)^2}\right]$$

$$f'(x) = \left[\frac{-3x^2\csc^2 x - 3\csc^2 x + 6x\cot x + x^2\cot x\csc^2 x + \cot x\csc^2 x - 2x\cot^2 x - x^2\cot x\csc^2 x - \cot x\csc^2 x}{(3-\cot x)^2}\right]$$

$$f'(x) = \left[\frac{6x\cot x - 2x\cot^2 x - 3(x^2-1)\csc^2 x}{(3-\cot x)^2}\right]$$

Q19) $y = x\cos x.$

$f'(x) = [(x)(-\sin x) + (\cos x)(1)] = -x\sin x + \cos x.$

$f''(x) = -[(x)(\cos x) + (\sin x)(1)] - \sin x.$

$= -x\cos x - \sin x - \sin x \Rightarrow -x\cos x - 2\sin x$ Ans.

Q20) $y = \csc x$

$\dfrac{dy}{dx} = -\csc x\cot x$

$\dfrac{d^2y}{dx^2} = -[(\csc x)(-\csc^2 x) + (\cot x)(-\csc x\cot x)]$

$= -[-\csc^3 x - \csc x\cot^2 x]$

$= \csc^3 x + \csc x\cot^2 x.$ Ans.

Q21) $y = x\sin x - 3\cos x.$

$f'(x) = [[(x)(\cos x) + (\sin x)(1)] + 3\sin x] \Rightarrow x\cos x + \sin x + 3\sin x$

$f'(x) = x\cos x + 4\sin x$

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$$\frac{d^2y}{dx^2} = \{[(x)(-\sin x) + (\cos x)(1)] + 4\cos x\}$$
$$= -x\sin x + \cos x + 4\cos x \Rightarrow -x\sin x + 5\cos x \text{ Ans.}$$

Q22) $y = x^2\cos x + 4\sin x.$

$$\frac{dy}{dx} = \{[(x^2)(-\sin x) + (\cos x)(2x)] + 4\cos x\}$$
$$= -x^2\sin x + 2x\cos x + 4\cos x.$$
$$\frac{d^2y}{dx^2} = \{[(x^2)(\cos x) + (\sin x)(2x)] + [(2x)(-\sin x) + (\cos x)(2)] - 4\sin x\}$$
$$= -x^2\cos x - 2x\sin x + -2x\sin x + 2\cos x - 4\sin x.$$
$$= (2-x^2)\cos x - 4(x+1)\sin x \text{ Ans.}$$

Q23) $y = \sin x\cos x.$

$$\frac{dy}{dx} = [(\sin x)(-\sin x) + (\cos x)(\cos x)]$$
$$= -\sin^2 x + \cos^2 x.$$
$$\frac{d^2y}{dx^2} = -2\sin x(\cos x) + 2\cos x(-\sin x)$$
$$= -2\sin x\cos x + -2\sin x\cos x \Rightarrow -4\sin x\cos x. \text{ Ans.}$$

Q24) $y = \tan x.$

$$\frac{dy}{dx} = \sec^2 x \Rightarrow \frac{d^2y}{dx^2} = 2\sec x.(\sec x\tan x)$$
$$= 2\sec^2 x\tan x. \text{ Ans.}$$

Ex # 2.6

Q7) $f(x) = (x^3+2x)^{37}$

$f'(x) = 37(x^3+2x).(3x+2)$ Ans.

Q8) $f(x) = (3x^2+2x-1)^6$

$f'(x) = 6(3x^2+2x-1).(6x+2).$

Q9) $f(x) = (x^3 - 7x^{-1})^{-2}$

$f'(x) = -2(x^3-7x^{-1}).(3x^2+7x^{-2})$ Ans.

Q10) $f(x) = (x^5-x+1)^{-9}$

$f'(x) = -9(x^5-x+1).(5x^4-1)$ Ans.

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Q11) $f(x) = \dfrac{4}{(3x^2-2x+1)^3} = 4(3x^2-2x+1)^{-3}$

$f'(x) = -12(3x^2-2x+1) \cdot 6x-2 \Rightarrow f'(x) = \dfrac{-24(3x-1)}{(3x^2-2x+1)^4}$ Ans.

Q12) $f(x) = \sqrt{x^2-2x+5} = (x^2-2x+5)^{1/2}$

$f'(x) = \frac{1}{2}(x^2-2x+5)^{-1/2} \cdot (2x^2-2)$.

$= \cancel{\dfrac{2(x-1)}{2\sqrt{x^2-2x+5}}} = \dfrac{(3x^2-2)}{2\sqrt{x^2-2x+5}}$ Ans.

Q13) $f(x) = \sqrt{4+\sqrt{3x}}$

$f'(x) = \dfrac{0 + \sqrt{3}\,\frac{1}{2}x^{-1/2}}{2\sqrt{4+\sqrt{3x}}} \Rightarrow \dfrac{\sqrt{3}}{4\sqrt{x}\sqrt{4+\sqrt{3x}}}$ Ans.

Q14) $f(x) = \sqrt[3]{12+\sqrt{x}} = (12+\sqrt{x})^{1/3}$

$f'(x) = \frac{1}{3}(12+\sqrt{x})^{-2/3} \cdot \left(0 + \frac{1}{2\sqrt{x}}\right) = \dfrac{1}{6\sqrt{x}\,(12+\sqrt{x})^{2/3}}$ Ans.

Q15) $f(x) = \sin(x^{-2})$

$f'(x) = \cos x^{-2} \cdot -2x^{-3} \cdot -2x^{-3}\cos x^{-2}$ Ans.

Q16) $f(x) = \tan\sqrt{x} = \tan x^{1/2}$

$f'(x) = \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$ Ans

Q17) $f(x) = 4\cos^5 x$.

$f'(x) = 20\cos^4 x \cdot -\sin x. = -20\cos^4 x \sin x.$ Ans.

Q18) $f(x) = 4x + 5\sin^4 x$.

$f'(x) = 4 + 20\sin^3 x \cdot \cos x$

Q19) $f(x) = \cos^2(3\sqrt{x})$

$f'(x) = 2\cos(3\sqrt{x}) \cdot -\sin(3\sqrt{x}) \cdot 3\,\frac{1}{2\sqrt{x}}$

$f'(x) = -3\sin(3\sqrt{x})\cos(3\sqrt{x})/\sqrt{x}$ Ans.

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Q20) $f(x) = \tan^4(x^3)$

$f'(x) = 4\tan^3(x^3) \cdot \sec^2(x^3) \cdot 3x^2$

$= 12x^2 \tan^3(x^3) \cdot \sec^2(x^3)$ Ans.

Q21) $f(x) = 2\sec^2(x^7)$

$f'(x) = 2(2\sec(x^7)) \cdot \sec(x^7)\tan(x^7) \cdot 7x^6$

$f'(x) = 28x^6 \sec^2(x^7)\tan(x^7)$ Ans.

Q22) $f(x) = \cos^3\left(\frac{x}{x+1}\right)$

$f'(x) = 3\cos^2\left(\frac{x}{x+1}\right) \cdot -\sin\left(\frac{x}{x+1}\right) \cdot \left[\frac{(x+1)(1) - (x)(1+0)}{(x+1)^2}\right]$

$f'(x) = -3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right) \cdot \left[\frac{\cancel{x}+1-\cancel{x}}{(x+1)^2}\right]$

$f'(x) = \frac{-3}{(x+1)^2}\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)$ Ans.

Q23) $f(x) = \sqrt{\cos(5x)} = [\cos(5x)]^{1/2}$

$f'(x) = \frac{-5\sin 5x}{2\sqrt{\cos(5x)}}$ Ans.

Q24) $f(x) = \sqrt{3x - \sin^2(4x)}$

$f'(x) = \frac{3 - 2\sin(4x) \cdot \cos(4x) \cdot 4}{2\sqrt{3x - \sin^2(4x)}} = \frac{3 - 8\sin 4x\cos 4x}{2\sqrt{3x - \sin^2(4x)}}$ Ans.

Q25) $f(x) = [x + \operatorname{cosec}(x^3+3)]^{-3}$

$f'(x) = -3(x + \operatorname{cosec}(x^3+3))^{-4} \cdot (1 \mp \operatorname{cosec}(x^3+3) \cdot \cot(x^3+3)) \cdot 3x^2$

$f'(x) = -3(x + \operatorname{cosec}(x^3+3))^{-4} \cdot [1 - 3x^2\operatorname{cosec}(x^3+3) \cdot \cot(x^3+3)]$ Ans.

Q26) $f(x) = (x^4 - \sec(4x^2-2))^{-4}$

$f'(x) = -4[x^4 - \sec(4x^2-2)]^{-5} \cdot (4x^3 - \sec(4x^2-2)\tan(4x^2-2)) \cdot 8x$

$= -4[x^4 - \sec(4x^2-2)]^{-5} \cdot [4x^3 - 8x\sec(4x^2-2)\tan(4x^2-2)]$ Ans.

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Q27) $y = x^3 \sin^2(5x)$.

$$\frac{dy}{dx} = \left[(x^3)\left(2\sin(5x).\cos(5x).5\right] + (\sin^2 5x)(3x^2)\right]$$

$$= 10x^3 \sin(5x)\cos(5x) + 3x^2 \sin^2(5x) \text{ Ans.}$$

Q28) $y = \sqrt{x}\tan^3(\sqrt{x})$

$$\frac{dy}{dx} = \left[(\cancel{\sqrt{x}})(3\tan^2\sqrt{x}).\sec^2\sqrt{x}.\frac{1}{2\cancel{\sqrt{x}}} + (\tan^3\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)\right]$$

$$\frac{dy}{dx} = \frac{3\tan^2\sqrt{x}\sec^2\sqrt{x}}{2} + \frac{\tan^3\sqrt{x}}{2\sqrt{x}} \text{ Ans.}$$

Q29) $y = x^5 \sec x^{-1}$

$$\frac{dy}{dx} = \left[(x^5)(\cancel{-1\sec x^{-2}}\ \sec x^{-1}.\tan x^{-1}.-1x^{-2}) + (\sec x^{-1})(5x^4)\right]$$

$$= -x^3 \sec x^{-1}.\tan x^{-1} + 5x^4\sec x^{-1} \text{ Ans.}$$

Q30) $y = \dfrac{\sin x}{\sec(3x+1)}$

$$\frac{dy}{dx} = \left[\frac{\sec(3x+1)(\cos x) - (\sin x)[\sec(3x+1)\tan(3x+1).3]}{[\sec(3x+1)]^2}\right]$$

$$\frac{dy}{dx} = \left[\frac{\cancel{\sec(3x+1)}[\cos x - 3\sin x\tan(3x+1)]}{[\sec(3x+1)]^{\cancel{2}}}\right]$$

$$\frac{dy}{dx} = \frac{\cos x - 3\sin x\frac{\sin(3x+1)}{\cos(3x+1)}}{\sec(3x+1)} = \frac{\cos x\cos(3x+1) - 3\sin x\sin(3x+1)}{\cos(3x+1).\frac{1}{\cos(3x+1)}}$$

$$\frac{dy}{dx} = \cos x\cos(3x+1) - 3\sin x\sin(3x+1) \text{ Ans.}$$

Q31) $y = \cos(\cos x)$.

$$\frac{dy}{dx} = -\sin(\cos x).-\sin x \Rightarrow \sin x.\sin(\cos x) \text{ Ans.}$$

Q32) $y = \sin(\tan 3x)$

$$\frac{dy}{dx} = \cos(\tan 3x).\sec^2(3x).3.$$

$$= 3\cos(\tan 3x)\sec^2(3x) \text{ Ans.}$$

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Q33) $y = \cos^3(\sin 2x)$.

$\frac{dy}{dx} = 3\cos^2(\sin 2x) \cdot -\sin(\sin 2x) \cdot \cos(2x) \cdot 2$.

$= -6\cos^2(\sin 2x) \cdot \sin(\sin 2x) \cdot \cos(2x)$ Ans.

Q34) $y = \dfrac{1+\operatorname{Cosec}(x^2)}{1-\operatorname{Cot}(x^2)}$

$\frac{dy}{dx} = \left[\dfrac{(1-\operatorname{Cot}x^2)[0-\operatorname{Cosec}(x^2)\operatorname{Cot}(x^2).2x] - (1+\operatorname{Cosec}(x^2))[0+\operatorname{Cosec}^2(x^2).2x]}{[1-\operatorname{Cot}(x^2)]^2}\right]$

$= \cancel{-2x\operatorname{Cosec}(x^2)}\left[\cancel{1-\operatorname{Cot}x^2}\right.$

$= \dfrac{-2x\operatorname{Cosec}(x^2)\operatorname{Cot}(x^2) + 2x\operatorname{Cosec}(x^2)\operatorname{Cot}^2(x^2) - 2x\operatorname{Cosec}^2(x^2) - 2x\operatorname{Cosec}^3(x^2)}{[1-\operatorname{Cot}(x^2)]^2}$

$= \dfrac{-2x\operatorname{Cosec}(x^2)\left[1 - \operatorname{Cot}^2x^2 + \operatorname{Cosec}x^2\right]}{[1-\operatorname{Cot}(x^2)]^2}$ Ans.

Q35) $y = (5x+8)^7(1-\sqrt{x})^6$

$\frac{dy}{dx} = \left[(5x+8)^7 \cdot \left[6(1-\sqrt{x})^5 \cdot \frac{-1}{2\sqrt{x}}\right] + (1-\sqrt{x})^6 \cdot 7(5x+6)^6 \cdot (5)\right]$

$= \dfrac{-3(5x+8)^7(1-\sqrt{x})^5}{\sqrt{x}} + 35(1-\sqrt{x})^6(5x+8)^6$.

Q36) $y = (x^2+x)^5 \sin^8 x$.

$\frac{dy}{dx} = \left[(x^2+x)^5 \cdot 8\sin^7 x \cdot \cos x + \sin^8 x\left(5(x^2+x)^4(2x+1)\right)\right]$

$= 8(x^2+x)^5\sin^7 x\cos x + 5(2x+1)(x^2+x^4)\sin^8 x$ Ans.

Q37) $y = \left(\dfrac{x-5}{2x+1}\right)^3$

$\frac{dy}{dx} = 3\left(\dfrac{x-5}{2x+1}\right)^2 \cdot \left[\dfrac{(2x+1)(1-0) - (x-5)(2+0)}{(2x+1)^2}\right]$

$= \dfrac{3(x-5)^2 \cdot (\cancel{2x}+1) - \cancel{2x}+10}{(2x+1)^4} = \dfrac{3(x-5)^2 \cdot 11}{(2x+1)^4} = \dfrac{33(x-5)^2}{(2x+1)^4}$ Ans.

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Q38) $y = \left(\frac{1+x^2}{1-x^2}\right)^{17}$

$$\frac{dy}{dx} = 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \cdot \left[\frac{(1-x^2)(0+2x) - (1+x^2)(0-2x)}{[1-x^2]^2}\right]$$

$$\frac{dy}{dx} = 17\frac{(1+x^2)^{16}}{(1-x^2)^{18}} \cdot [2x - \cancel{2x^3} + 2x + \cancel{2x^3}]$$

$$= 17\frac{(1+x^2)^{16}}{(1-x^2)^{18}} \cdot 4x = \frac{68x(1+x^2)^{16}}{(1-x^2)^{18}} \text{ Ans.}$$

Q39) $y = \frac{(2x+3)^3}{(4x^2-1)^8}$

$$\frac{dy}{dx} = \left[\frac{(4x^2-1)^8[3.(2x+3)^2.(2+0)] - (2x+3)^3[8(4x^2-1)^7.(8x)]}{(4x^2-1)^{16}}\right]$$

$$\frac{dy}{dx} = \left[\frac{6(4x^2-1)^8.(2x+3)^2 - 64x(4x^2-1)^7.(2x+3)^3}{(4x-1)^{16}}\right]$$

$$\frac{dy}{dx} = \frac{2\cancel{(4x^2-1)^7}.(2x+3)^2[3(4x^2-1) - 32x(2x+3)]}{(4x-1)^{16-9}}$$

$$\frac{dy}{dx} = \frac{2(2x+3)^2(12x^2 - 3 - 64x^2 - 96x)}{(4x-1)^9}$$

$$\frac{dy}{dx} = \frac{-2(2x+3)^2(52x^2+96x+3)}{(4x-1)^9} \text{ Ans.}$$

Q40) $y = [1 + \sin^3(x^5)]^{12}$

$$\frac{dy}{dx} = 12[1 + \sin^3(x^5)]^{11} . 0 + 3\sin^2(x^5).\cos(x^5).5x^4$$

$$= 180x^4(1+\sin^3(x^5))^{11}.\sin^2(x^5).\cos(x^5) \text{ Ans.}$$

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Ex # 3.4

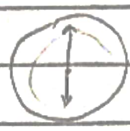
Q10) Data: $\left.\dfrac{dx}{dt}\right|_{x=1} = -2\ \text{unit/s}$ (decreasing), $\left.\dfrac{dy}{dt}\right|_{y=2} = 3\ \text{unit/s}$, $\dfrac{dz}{dt} = ?$

Solution: $z = x^3y^2 \Rightarrow \dfrac{dz}{dt} = \left[3x^2\dfrac{dx}{dt}(y^2) + (x^3).2y\dfrac{dy}{dt}\right]$

$$\frac{dz}{dt} = \left[3(1)^2(-2)(2)^2 + (1^3)(2)(2)(3)\right] = \boxed{-12\ \text{unit/s}}\ \text{Ans.}$$

z is decreasing.

Q11) Data: $r = 4\ \text{inch}$, $\dfrac{dA}{dt} = ?$, $\dfrac{d\theta}{dt} = \dfrac{\pi}{80}\ \text{rad/min}$.



full = 60 min

sector = 30 min

Solution: Area of Sector $= A = \frac{1}{2}\pi r^2\theta$.

Derivate on B.S $\qquad \theta = 180$.

$$\frac{dA}{dt} = \frac{1}{2}\pi \cancel{2r\frac{dr}{dt}}\, r^2 \frac{d\theta}{dt} \Rightarrow \frac{dA}{dt} = \boxed{\frac{4\pi}{15}\ \text{in}^2/\text{min}}\ \text{Ans.}$$

Q12) Data: $\left.\dfrac{dr}{dt}\right|_{t=10\text{sec}} = 3\ \text{ft/sec}$ $\quad$ r increasing at 3ft/s ×10, $r = 30\ \text{ft}$, $\left.\dfrac{dA}{dt}\right|_{t=10} = ?$

Solution: $A = \pi r^2 \Rightarrow \dfrac{dA}{dt} = \pi\, 2r\dfrac{dr}{dt} = \pi\, 2(30)(3) = \boxed{180\pi\ \text{ft}^2/\text{s}}$ Ans.

Q13) Data: $\dfrac{dA}{dt} = 6\ \text{mi}^2/\text{h}$, $A = 9\ \text{mi}^2$, $\left.\dfrac{dr}{dt}\right|_{A=9} = ?$, $A = \pi r^2$

$\dfrac{9}{\pi} = r^2 \Rightarrow r = 1.69\ \text{mi}$

Solution: $A = \pi r^2 \Rightarrow \left.\dfrac{dA}{dt}\right|_{A=9} = \pi\, 2r \left.\dfrac{dr}{dt}\right|_{A=9}$

$$\left.\frac{dr}{dt}\right|_{A=9} = \frac{6}{\pi \times 2(1.69)} = \boxed{0.565\ \text{mi/hr}}\ \text{Ans.}$$

Q14) Data: $\dfrac{dV}{dt} = 3\ \text{ft}^3/\text{min}$, $r = 1\ \text{ft}$, $\left.\dfrac{dD}{dt}\right|_{r=1} = ?$

Solution: $r = \dfrac{D}{2} \Rightarrow D = 2r \Rightarrow \boxed{D = 2}$

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Volume of Sphere:- $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$

$V = \frac{4}{3} \times \frac{D^3}{\cancel{8}_2} = \frac{1}{6}\pi D^3$

Derivate on B.S.

$\frac{dV}{dt}\Big|_{r=1} = \frac{1}{\cancel{6}_2}\pi\,\cancel{3}D^2\frac{dD}{dt}\Big|_{r=1} \Rightarrow 3 = \frac{\pi}{2}(2)^2\frac{dD}{dt}\Big|_{r=1}$

$\frac{dD}{dt}\Big|_{r=1} = \boxed{0.477\ ft/min.}$

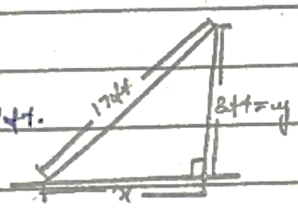
Q15) Data:- $\frac{dr}{dt}\Big|_{r=9cm} = -15\ cm/min$, $\frac{dV}{dt}\Big|_{r=9} = ?$, $r = 9cm$ Volume decreasing.

Solution:- $V = \frac{4}{3}\pi r^3 \Rightarrow$ Derivate on B.S.

$\frac{dV}{dt}\Big|_{r=9} = \frac{4}{3}\pi\, 3r^2\frac{dr}{dt}\Big|_{r=9} \Rightarrow = \frac{4}{\cancel{3}}\pi\,\cancel{3}(9)^2(-15)$

$\frac{dV}{dt}\Big|_{r=9} = \boxed{-4860\pi\ cm^3/min.}$

Q16) Data:- $\frac{dx}{dt} = 5ft/s$, $\frac{dy}{dt}\Big|_{y=8ft} = ?$ $y = 8ft$ - Hyp = 17ft.



Solution:- Using P.G.T

$x^2 + y^2 = 17^2 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x^2 + y^2 = 17^2$

$x^2 = 17^2 - 8^2$

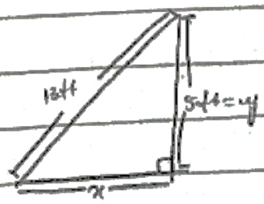
$\boxed{x = 15}$

$2(15)(5) + 2(8)\frac{dy}{dt} = 0 \Rightarrow 16\frac{dy}{dt}\Big|_{y=8} = 150$

$\frac{dy}{dt}\Big|_{y=8} = \boxed{9.375\ ft/sec}$

slipping down.

Q17) Data:- $\frac{dy}{dt}$ 2ft/sec, $y = 5ft$, $\frac{dx}{dt}\Big|_{y=5} = ?$



Solution:- Using P.G.T $\Rightarrow 13^2 = x^2 + 5^2 \Rightarrow \boxed{x = 12}$

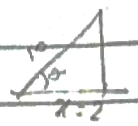
$x^2 + y^2 = 13^2$ Derivate on B.S.

$x^2 + y^2 = 13^2$ Derivate on B.S:

$2x \cdot \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow 2(12)\left(\frac{dx}{dt}\right) + 2(5)(-2) = 0$

$\frac{dx}{dt} = \frac{20}{24} = \boxed{0.833\,ft/s}$ Ans.

Q18) Data:- $\frac{dx}{dt} = 6\,inch/s$ (x-axis decreasing) $= -\frac{6}{12} = -\frac{1}{2}\,ft/s$, $x = 2ft$, $\left.\frac{d\theta}{dt}\right|_{x=2}$



Solution:- $\cos\theta = \frac{x}{10}$ Derivate on B.S.

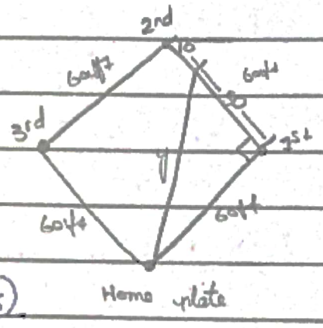
For θ:- $\theta = \cos^{-1}\left(\frac{x}{10}\right)$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{10}(1)\frac{dx}{dt} \Rightarrow$

$\theta = \cos^{-1}\left(\frac{2}{10}\right) = \boxed{78.46}$

$\frac{d\theta}{dt} = \frac{-1}{(10)(2) - \sin(78.46)} = \boxed{0.051\ rad/s}$

Q19) Data:- $\frac{dx}{dt} = 25\,ft/s$, $x = 50\,ft$, $\frac{dy}{dt} = ?$



Solution:- Using P.G.T

$y^2 = x^2 + 60^2$

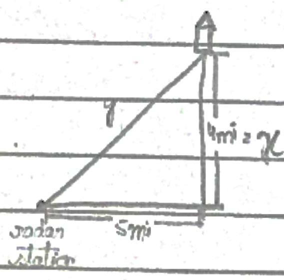
$y^2 = 50^2 + 60^2$

$\boxed{y = 10\sqrt{61}}$

$2y \cdot \frac{dy}{dt} = 2x\frac{dx}{dt} + 0 \Rightarrow \frac{dy}{dt} = \frac{2(50)(25)}{2(10\sqrt{61})}$

$\frac{dy}{dt} = \boxed{16\,ft/sec}$ Ans.

Q20) Data:- $\left.\frac{dy}{dt}\right|_{x=4} = 2000\,mi/hr$, $\frac{dx}{dt} = ?$



Solution:- $y^2 = x^2 + 5^2 \Rightarrow 2y\frac{dy}{dt} = 2x\frac{dx}{dt} + 0$

$\left.\frac{dx}{dt}\right|_{x=4} = \frac{2(\sqrt{41})(2000)}{2(4)} = \boxed{3201.56\ mi/hr}$ Ans.

$y^2 = x^2 + 5^2$

$y = \sqrt{41}$