EXERCISE 2.4 [1-24]

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(1) - (4) Compute derivates using:

(a) multiplying and derivating

(b) Product rule. $[f(x)g(x) = uv' + vu'$

Verify if (a) & (b) yield same result.

(1) $f(x) = \overset{u}{(x+1)}\overset{v}{(2x-1)}$

Conclusion:

Yielding same result.

(a) $f(x) = 2x^2 - x + 2x - 1$

$= 2x^2 + x - 1$

$f'(x) = 4x + 1.$

(b) $(x+1)(2) + (2x-1)(1)$

$2x + 2 + 2x - 1$

$f'(x) = 4x + 1.$

(2) $f(x) = \overset{u}{(3x^2-1)}\overset{v}{(x^2+2)}$

Conclusion:

Yielding same result.

(a) $f(x) = 3x^4 + 6x^2 - x^2 - 2$

$= 3x^4 + 5x^2 - 2$

$f'(x) = 12x^3 + 10x$

(b) $(3x^2-1)(2x) + (x^2+2)(6x)$

$6x^3 - 2x + 6x^3 + 12x$

$f'(x) = 12x^3 + 10x$

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(3) $f(x) = \overset{u}{(x^2+1)}\overset{v}{(x^2-1)}$

Conclusion:
Yielding same result.

(a) $f(x) = x^4 - \cancel{x^2} + \cancel{x^2} - 1$

$= x^4 - 1$

$f'(x) = 4x^3$.

(b) $(x^2+1)(2x) + (x^2-1)(2x)$

$2x^3 + \cancel{2x} + 2x^3 - \cancel{2x}$

$f'(x) = 4x^3$

(4) $f(x) = (x+1)(x^2-x+1)$

Conclusion:
Yielding same result.

(a) $f(x) = x^3 - \cancel{x^2} + \cancel{x} + \cancel{x^2} - \cancel{x} + 1$

$= x^3 + 1$

$f'(x) = 3x^2$

(b) $(x+1)(2x-1) + (x^2-x+1)(1)$

$2x^2 - \cancel{x} + \cancel{2x} - \cancel{1} + x^2 - \cancel{x} + \cancel{1}$

$f'(x) = 3x^2$

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(5) - (20) Find $f'(x)$.

(5) $f(x) = (\overset{u}{3x^2+6})(\overset{v}{2x - \frac{1}{4}})$

Using Product Rule:

$(3x^2+6).(2) + (2x - \frac{1}{4})(6x)$

$18x^2 + 12 + 12x^2 - \frac{3}{2}x$

$18x^2 + 12x - \frac{3}{2}x$

$\times \; [f'(x) = 6x^2 + \frac{45}{2}x] \; \times$

$f'(x) = 18x^2 + 12x - \frac{3}{2}x.$

(6) $f(x) = (2 - x - 3x^3)(7 + x^5)$

$= 14 + 2x^5 - 7x - x^6 - 21x^3 - 3x^8$

$f'(x) = 10x^4 - 7 - 6x^5 - 63x^2 - 24x^7$

(8) $f(x) = (x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})$

$= 2 + x^{-1} + 14x^{-1} + 7x^{-2} - 16x^{-3} - 8x^{-4}.$

$f'(x) = -x^{-2} - 14x^{-2} - 14x^{-3} + 48x^{-4} + 32x^{-5}$

$= \frac{-1}{x^2} - \frac{14}{x^2} - \frac{14}{x^3} + \frac{48}{x^4} + \frac{32}{x^5}$

$= \frac{-15}{x^2} - \frac{14}{x^3} + \frac{48}{x^4} + \frac{32}{x^5}$

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(8) $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$.

$= (x^{-1} + x^{-2})(3x^3 + 27)$.

$= 3x^2 + 27x^{-1} + 3x + 27x^{-2}$

$f'(x) = 6x - 27x^{-2} + 3 - 54x^{-3}$

$= 6x - \frac{27}{x^2} + 3 - \frac{54}{x^3}$

(9) $f(x) = \underset{u}{(x-2)}\,\underset{v}{(x^2+2x+4)}$

Using Product Rule:

$(x-2)(2x+2) + (x^2+2x+4)(1)$

$2x^2 + 2x - 4x - 4 + x^2 + 2x + 4$

$3x^2 + \cancel{4x} - \cancel{4x} + \cancel{4} - \cancel{4}$

$f'(x) = 3x^2$

(10) $f(x) = (x^2+x)(x^2-x)$

$= x^4 - x^2$

$f'(x) = 4x^3 - 2x$

(11) $f(x) = \dfrac{\overset{u}{3x+4}}{\underset{v}{x^2+1}}$. $\qquad QR = \dfrac{vu' - uv'}{v^2}$.

Using Quotient rule:

$$\frac{(x^2+1)(3) - (3x+4)(2x)}{(x^2+1)^2}.$$

$$\frac{3x^2 + 3 - 6x^2 + 8x}{(x^2+1)^2}.$$

$$f'(x) = \frac{-3x^2 + 8x + 3}{(x^2+1)^2}$$

(12) $f(x) = \dfrac{x-2}{x^4+x+1}$.

Using Q.R :

$$\frac{(x^4+x+1)(1) - (x-2)(4x^3+1)}{(x^4+x+1)^2}$$

$$\frac{x^4+x+1 - [4x^4 + x - 8x^3 - 2]}{(x^4+x+1)^2}$$

$$\frac{x^4 + \cancel{x} + 1 - 4x^4 - \cancel{x} + 8x^3 + 2}{(x^4+x+1)^2}$$

$$f'(x) = \frac{-3x^4 + 8x^3 + 3}{(x^4+x+1)^2}$$

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(13) $f(x) = \dfrac{x^2}{3x-4}$.

Using Q.R :

$$\frac{(3x-4)(2x) - (x^2)(3)}{(3x-4)^2}.$$

$$\frac{6x^2 - 8x - 3x^2}{(3x-4)^2}$$

$$f'(x) = \frac{3x^2 - 8x}{(3x-4)^2}$$

(14) $f(x) = \dfrac{2x^2+5}{3x-4}$.

Using QR :

$$\frac{(3x-4)(4x) - (2x^2+5)(3)}{(3x-4)^2}$$

$$\frac{12x^2 - 16x - [6x^2+15]}{(3x-4)^2}$$

$$\frac{12x^2 - 6x^2 - 16x - 15}{(3x-4)^2}$$

$$f'(x) = \frac{6x^2 - 16x - 15}{(3x-4)^2}$$

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(15) $f(x) = \dfrac{(2\sqrt{x}+1)(x-1)}{x+3} = \dfrac{(2x^{\frac{1}{2}}+1)(x-1)}{x+3}.$

$= \dfrac{2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x - 1}{x+3.} \begin{matrix}\to u \\ \to v.\end{matrix}$

$\dfrac{du}{dx} = 3x^{\frac{1}{2}} - x^{-\frac{1}{2}} + 1$

Using QR:

$\dfrac{(x+3)(3x^{\frac{1}{2}} - x^{-\frac{1}{2}} + 1) - (2x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x - 1)(1)}{(x+3)^2}$

$\dfrac{3x^{\frac{3}{2}} - x^{\frac{1}{2}} - \cancel{x} + 9x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} + 3 - 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \cancel{+x} + 1}{(x+3)^2}$

$\dfrac{3x^{\frac{3}{2}} - 2x^{\frac{3}{2}} + 9x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 2^{-\frac{1}{2}} + 4}{(x+3)^2}$

$f'(x) = \dfrac{x^{\frac{3}{2}} + 10x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} + 4}{(x+3)^2}.$

(16) $f(x) = (2\sqrt{x}+1)\left(\dfrac{2-x}{x^2+3x}\right) = \dfrac{4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} + 2 - x}{x^2+3x} \begin{matrix}\to u \\ \to v\end{matrix}$

$\dfrac{du}{dx} = 2x^{-\frac{1}{2}} - 3x^{\frac{1}{2}} - 1$

$\dfrac{(x^2+3x)(2x^{-\frac{1}{2}} - 3x^{\frac{1}{2}} - 1) - (4x^{\frac{1}{2}} - 2x^{\frac{3}{2}} + 2 - x)(2x+3)}{(x^2+3x)^2}$

$2x^{\frac{3}{2}} - 3x^{\frac{5}{2}} - x^2 + 6x^{\frac{1}{2}} - 9x^{\frac{3}{2}} - 3x - [8x^{\frac{3}{2}} - 4x^{\frac{5}{2}} + 4x - 2x^2 + 12x^{\frac{1}{2}} - 6x^{\frac{3}{2}} + 6 - 3x]$

$\dfrac{2x^{\frac{3}{2}} - 3x^{\frac{5}{2}} - x^2 + 6x^{\frac{1}{2}} - 9x^{\frac{3}{2}} - 3x - 8x^{\frac{3}{2}} + 4x^{\frac{5}{2}} - 4x + 2x^2 - 12x^{\frac{1}{2}} + 6x^{\frac{3}{2}} - 6 + 3x}{(x^2+3x)^2}$

$(x^{5/2} + x^2 - 9x^{3/2} - 4x - 6x^{\frac{1}{2}} - 6)/(x^2+3x)^2.$

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(17). $f(x) = (2x+1)\left(1+\frac{1}{x}\right)(x^{-3}+7)$

$f(x) = (2x+1)(1+x^{-1})(x^{-3}+7)$

$= (2x+2+1+x^{-1})(x^{-3}+7)$

$= (2x+3+x^{-1})(x^{-3}+7)$

$= 2x^{-2}+14x+3x^{-3}+21+x^{-4}+7x^{-1}.$

$= -4x^{-3}+14-9x^{-4}+0-4x^{-5}-7x^{-2}.$

$= 14-7x^{-2}-4x^{-3}-9x^{-4}-4x^{-5}.$

(18) $f(x) = x^{-5}(x^2+2x)(4-3x)(2x^9+1)$

$= x^{-5}(4x^2-3x^3+8x-6x^2)(2x^9+1)$

$= x^{-5}(-2x^2-3x^3+8x)(2x^9+1)$

$= x^{-5}(-4x^{11}-2x^2-6x^{12}-3x^3+16x^{10}+8x)$

$= -4x^6-2x^{-3}-6x^7-3x^{-2}+16x^5+8x^{-4}$

$f'(x) = -24x^5+6x^{-4}-42x^6+6x^{-3}+80x^4-32x^{-5}.$

(19) $f(x) = (x^7+2x-3)^3.$

$f'(x) = 3(x^7+2x-3)^2 \times (7x^6+2)$

$= 21x^6(x^7+2x-3)^2+6(x^7+2x-3)^2.$

(20) $f(x) = (x^2+1)^4$

$f'(x) = 4(x^2+1)^3 \times 2x$

$= 8x(x^2+1)^3.$

(21) - (24) Find $dy/dx \big|_{x=1}$.

(21) $y = \dfrac{2x-1}{x+3}$ $\begin{matrix} \to u \\ \to v \end{matrix}$

Using QR.

$$\frac{(x+3)(2) - (2x-1)(1)}{(x+3)^2}$$

$$\frac{2x+6 - [2x-1]}{(x+3)^2}$$

$$\frac{\cancel{2x}+6-\cancel{2x}+1}{(x+3)^2}$$

$$f'(x)\Big|_{x=1} \quad \frac{7}{(1+3)^2} = \frac{7}{16}$$

(22) $y = \dfrac{4x+1}{x^2-5}$. $\begin{matrix} \to u \\ \to v \end{matrix}$

Using QR.

$$\frac{(x^2-5)(4) - (4x+1)(2x)}{(x^2-5)^2}.$$

$$\frac{4x^2-20 - [8x^2+2x]}{(x^2-5)^2}$$

$$\frac{4x^2-20-8x^2-2x.}{(x^2-5)^2}$$

$$\frac{-4x^2-2x-20}{(x^2-5)^2}\Bigg|_{x=1} \Rightarrow -\frac{13}{8}.$$

(23) $f(x) = \left(\dfrac{3x+2}{x}\right)(x^{-5}+1)$.

$f = \dfrac{3x^{-4}+3x+2x^{-5}+2}{x}$ → u

$\dfrac{du}{dx} = -12x^{-5}+3-10x^{-6}$

Using QR:

$\dfrac{(x)(-12x^{-5}+3-10x^{-6}) - (3x^{-4}+3x+2x^{-5}+2)(1)}{x^2}$

$y' = \dfrac{-12x^{-4}+3x-10x^{-5}-3x^{-4}-3x-2x^{-5}-2}{x^2}$

$y'(1) = -12(1)^{-4}+3(1)-10(1)^{-5}-3(1)^{-4}-3(1)-2(1)^{-5}-2$.

$y'(1) = -29$.

(24) $y = (2x^7-x^2)\left(\dfrac{x-1}{x+1}\right) = \dfrac{2x^8-2x^7-x^3+x^2}{x+1}$ → u → v.

$\dfrac{du}{dx} = 16x^7-14x^6-3x^2+2x$

$\dfrac{(x+1)(16x^7-14x^6-3x^2+2x) - (2x^8-2x^7-x^3+x^2)(1)}{(x+1)^2}$.

$\dfrac{16x^8-14x^7-3x^3+2x^2-2x^8+2x^7+x^3-x^2}{(x+1)^2}$.

$y'\big|_{x=1} \cdot \dfrac{16-14-3+2-\cancel{2}+\cancel{2}+\cancel{1}-\cancel{1}}{4}$

$\Rightarrow \dfrac{1}{4}$