Ex# 7.1

Date

(Q1) $\int (4-2x)^3 dx$

$\Rightarrow \int (4-2x)^3 \left(\frac{-2}{-2}\right) dx$

$\Rightarrow -\frac{1}{2}\int (4-2x)^3(-2)\,dx$

$\Rightarrow -\frac{1}{2}\left[\frac{(4-2x)^4}{4}\right] + c$

$\Rightarrow -\frac{1}{8}(4-2x)^4 + c$

2. $\int 3\sqrt{4+2x}\,dx$

$\Rightarrow 3\int (4+2x)^{\frac{1}{2}}\left(\frac{2}{2}\right) dx$

$\Rightarrow \frac{3}{2}\int (4+2x)^{\frac{1}{2}}(2)\,dx$

$\Rightarrow \frac{\cancel{3}}{\cancel{2}}\left[\frac{\cancel{2}}{\cancel{3}}(4+2x)^{\frac{3}{2}}\right] + c$

$\Rightarrow (4+2x)^{\frac{3}{2}} + c$

3. $\int x\sec^2(x^2)\,dx$

Let $u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$dx = du/2x$

$\Rightarrow \int \cancel{x}\sec^2 u \cdot \frac{1}{2\cancel{x}}\,du$

$\Rightarrow \frac{1}{2}\int \sec^2 u\,du$

$\Rightarrow \frac{1}{2}\tan u + c \Rightarrow \frac{1}{2}\tan x^2 + c$

4. $\int 4x\tan(x^2)\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = dx$

$\Rightarrow \int \cancel{4}^{2}x\tan u \cdot \frac{du}{2x}$

$\Rightarrow 2\int \tan u\,du$

$\Rightarrow -2\ln|\cos x^2| + c$

5. $\int \frac{\sin 3x}{2+\cos 3x}\,dx$

$u = 2+\cos 3x$

$\frac{du}{dx} = -3\sin 3x$

$\frac{du}{-3\sin 3x} = dx$

$\Rightarrow \int \frac{\cancel{\sin 3x}}{u}\cdot\frac{du}{-3\cancel{\sin 3x}}$

$\Rightarrow -\frac{1}{3}\int \frac{du}{u}$

$\Rightarrow -\frac{1}{3}\ln(2+\cos 3x) + c$

6. $\int \frac{dx}{9+4x^2}$

$\Rightarrow \int \frac{dx}{3^2+(2x)^2}$

$\Rightarrow \frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right)\cdot\frac{1}{2} + c$

$\Rightarrow \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right) + c$

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7. $\int e^x \sinh(e^x)\,dx$

$\Rightarrow \cosh(e^x)+c$

8. $\int \frac{\sec(\ln x)\tan(\ln x)}{x}\,dx$

$\Rightarrow \int \sec(\ln x)\tan(\ln x)\left(\frac{1}{x}\right)dx$

$\Rightarrow \sec(\ln x)+c$

9. $\int e^{\tan x}\sec^2 x\,dx$

$\Rightarrow \{\int e^{\square}\square' d\square = e^{\square}+c\}$

$\rightarrow e^{\tan x}+c$

10. $\int \frac{x}{\sqrt{1-x^4}}\,dx$

$\Rightarrow \int \frac{x}{\sqrt{1^2-(x^2)^2}}\,dx$

$\rightarrow \int \frac{2x}{2\sqrt{1-(x^2)^2}}\,dx$

$\rightarrow \frac{1}{2}\int \frac{2x\,dx}{\sqrt{1-(x^2)^2}}$

$\rightarrow \frac{1}{2}\cancel{\ln\sqrt{1-x^4}}+c$

$\rightarrow \frac{1}{2}\sin^{-1}(x^2)+c$

11. $\int \cos^5 5x \sin 5x\,dx$

$\Rightarrow \int (\cos 5x)^5\left(\frac{-5\sin 5x}{-5}\right)dx$

$\Rightarrow -\frac{1}{5}\int(\cos 5x)^5(-5\sin 5x)\,dx$

$\rightarrow -\frac{1}{5}\left[\frac{(\cos 5x)^6}{6}\right]+c$

$\Rightarrow -\frac{1}{30}\cos^6 5x+c$

12. $\int \frac{-\cos x}{\sin x\sqrt{\sin^2 x+1}}\,dx$

$\Rightarrow -\int \frac{\cos x}{\sin x\sqrt{1+\sin^2 x}}\,dx$

$\left\{\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right|+c\right\}$

$\rightarrow -\ln\left|\frac{1+\sqrt{1+\sin^2 x}}{\sin x}\right|+c$

13. $\int \frac{e^x}{\sqrt{4+e^{2x}}}\,dx$

$\Rightarrow \int \frac{e^x}{\sqrt{2^2+e^{2x}}}\,dx$

$\Rightarrow \ln\left(e^x+\sqrt{e^{2x}+4}\right)+c$

14. $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

$\Rightarrow \int e^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)dx$

$\rightarrow e^{\tan^{-1}x}+c$

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15. $\int \frac{e^{\sqrt{x-1}}}{\sqrt{x-1}}\,dx$

$\Rightarrow \int e^{\sqrt{x-1}}\left(\frac{1}{\sqrt{x-1}}\right)dx$

$\Rightarrow \int e^{\sqrt{x-1}}\left(\frac{2}{2\sqrt{x-1}}\right)dx$

$\Rightarrow 2\int e^{\sqrt{x-1}}\left(\frac{1}{2\sqrt{x-1}}\right)dx$

$\Rightarrow 2e^{\sqrt{x-1}} + c$

16. $\int (x+1)\cot(x^2+2x)\,dx$

$\int \cot(x^2+2x)\left(\frac{2x+2}{2}\right)dx$

$\Rightarrow \frac{1}{2}\int \cot(x^2+2x)(2x+2)\,dx$

$\Rightarrow \frac{1}{2}\ln|\sin(x^2+2x)| + c$

17. $\int \frac{\cosh\sqrt{x}}{\sqrt{x}}\,dx$

$\Rightarrow \int \cosh\sqrt{x}\left(\frac{2}{2\sqrt{x}}\right)dx$

$\Rightarrow 2\int \cosh\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)dx$

$\Rightarrow 2\sinh\sqrt{x} + c$

18. $\int \frac{dx}{x(\ln x)^2}$

$\Rightarrow \int (\ln x)^{-2}\left(\frac{1}{x}\right)dx$

$\Rightarrow -\frac{1}{\ln x} + c$

19. $\int \frac{dx}{\sqrt{x}\cdot 3^{\sqrt{x}}}$

$\Rightarrow \int \frac{2}{2\sqrt{x}\cdot 3^{\sqrt{x}}}\,dx$

$\Rightarrow 2\int \frac{dx}{2\sqrt{x}\cdot 3^{\sqrt{x}}}$

$\Rightarrow 2\int -3^{-\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)dx$

$\Rightarrow 2\left[\frac{-3^{-\sqrt{x}}}{\ln 3}\right] + c$

20. $\int \sec(\sin\theta)\tan(\sin\theta)\cos\theta\,d\theta$

$\Rightarrow \int \cancel{\frac{1}{\cos\theta}(\sin\theta)\cdot\frac{\sin\theta}{\cos\theta}(\sin\theta)}\cdot\cos\theta\,d\theta$

$\Rightarrow \int \sec(\sin\theta)\tan(\sin\theta)\cos\theta\,d\theta$

$\Rightarrow \sec(\sin\theta) + c$

21. $\int \frac{\operatorname{cosech}^2(2/x)}{x^2}\,dx$

$\Rightarrow \int \operatorname{cosech}^2\left(\frac{2}{x}\right)\left(\frac{-2}{-2x^2}\right)dx$

$\Rightarrow -\frac{1}{2}\int \operatorname{cosech}^2\left(\frac{2}{x}\right)\left(-\frac{2}{x^2}\right)dx$

$\Rightarrow -\frac{1}{2}\left(-\coth^2(2/x)\right) + c$

$\Rightarrow \frac{1}{2}\coth^2(2/x) + c$

22. $\int \frac{dx}{\sqrt{x^2-4}}$

$\Rightarrow \ln\left|x+\sqrt{x^2-4}\right| + c$

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23. $\int \frac{e^{-x}}{4-e^{-2x}}\,dx$

$\Rightarrow \int \frac{e^{-x}}{2^2-(e^{-x})^2}\,dx$

$\Rightarrow -\frac{1}{4}\ln\left|\frac{2+e^{-x}}{2-e^{-x}}\right|+c$

24. $\int \frac{\cos\ln x}{x}\,dx$

$\Rightarrow \int \cos\ln x\left(\frac{1}{x}\right)dx$

$\Rightarrow \sin\ln x + c$

25. $\int \frac{e^{x}}{\sqrt{1-e^{2x}}}\,dx$

$\Rightarrow \sin^{-1}e^{x}+c$

26. $\int \frac{\sinh(x^{-\frac{1}{2}})}{x^{\frac{3}{2}}}\,dx$

$\Rightarrow \int \sinh x(x^{-\frac{1}{2}})\left(\frac{1}{x^{\frac{3}{2}}}\right)dx$

$\Rightarrow \int \sinh x(x^{-\frac{1}{2}})\left(\frac{-2}{-2x^{\frac{3}{2}}}\right)dx$

$\Rightarrow -2\int \sinh x(x^{-\frac{1}{2}})\left(-\frac{1}{2x^{\frac{3}{2}}}\right)dx$

$\Rightarrow -2\cosh x(x^{-\frac{1}{2}})+c$

27. $\int \frac{x}{\operatorname{cosec}(x^2)}\,dx$

$\Rightarrow \int \sin x^2(x)\,dx$

$\frac{1}{2}\int \sin x^2(2x)\,dx$

$\Rightarrow -\frac{1}{2}\cos x^2+c$

28. $\int \frac{e^{x}}{\sqrt{4-e^{2x}}}\,dx$

$\Rightarrow \int \frac{e^{x}}{\sqrt{2^2-(e^{x})^2}}\,dx$

$\Rightarrow \sin^{-1}\frac{e^{x}}{2}+c$

29. $\int x4^{-x^2}\,dx$

$\Rightarrow \int 4^{-x^2}\left(\frac{-2x}{-2}\right)dx$

$\Rightarrow -\frac{1}{2}\int 4^{-x^2}(-2x)\,dx$

$\Rightarrow -\frac{1}{2}\left[\frac{4^{-x^2}}{\ln 4}\right]+c$

30. $\int 2^{\pi x}\,dx$

$\Rightarrow \int 2^{\pi x}\left(\frac{\pi}{\pi}\right)dx$

$\Rightarrow \frac{1}{\pi}\int 2^{\pi x}(\pi)\,dx$

$\Rightarrow \frac{1}{\pi}\left[\frac{2^{\pi x}}{\ln 2}\right]+c$

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Ex# 7.2

1. $\int xe^{-2x}\,dx$

$u = x \quad v = e^{-2x}$

$\frac{du}{dx} = 1 \quad \int v\,dx = \int e^{-2x}\,dx$

$\boxed{du = dx} \quad \int v\,dx = \frac{e^{-2x}}{-2} + c$

$\int xe^{-2x}\,dx = x\left[\frac{e^{-2x}}{-2}\right] - \int\left(\int\left[\frac{e^{-2x}}{-2}\right]\cancel{dx}\right)dx$

$\Rightarrow -\frac{1}{2}xe^{-2x} - \int \frac{e^{-2x}}{-2}\,dx$

$\Rightarrow -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$

$\Rightarrow -\frac{1}{2}xe^{-2x} + \frac{1}{2}\left[\frac{e^{-2x}}{-2}\right] + c$

$\Rightarrow -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + c$

2. $\int xe^{3x}\,dx$

$u = x \quad v = e^{3x}$

$\frac{du}{dx} = 1 \quad \int v\,dx = \frac{e^{3x}}{3} + c$

$du = dx$

$\int xe^{3x}\,dx = x\left[\frac{e^{3x}}{3}\right] - \int \frac{e^{3x}}{3}\,dx$

$\Rightarrow \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx$

$\Rightarrow \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$

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3. $\int x^2 e^x dx$

$u = x^2 \qquad v = e^x$

$\frac{du}{dx} = 2x \qquad \int v dx = e^x$

$\therefore \int x^2 e^x dx = x^2 e^x - \int 2x e^x dx$

$= x^2 e^x - I_1 \rightarrow (1)$

$I_1 = \int 2x e^x dx$

$u = 2x \qquad v = e^x$

$\frac{du}{dx} = 2 \qquad \int v dx = e^x$

$I_1 = 2x e^x - \int 2 e^x dx$

$I_1 = 2x e^x - 2 e^x$

$\therefore (1) \Rightarrow x^2 e^x - [2x e^x - 2e^x]$

$\Rightarrow x^2 e^x - 2x e^x + 2e^x$

$\Rightarrow e^x (x^2 - 2x + 2) + c$

5. $\int x \sin 3x \, dx$

$u = x \qquad v = \sin 3x$

$\frac{du}{dx} = 1 \qquad \int v dx = \int \sin 3x \left(\frac{3}{3}\right) dx$

$\int v dx = -\frac{1}{3} \cos 3x$

$\Rightarrow \int x \sin 3x \, dx = x\left[\frac{\cos 3x}{3}\right] + \int \frac{\cos 3x}{3} dx$

$= \frac{1}{3} x \cos 3x - \frac{1}{3} \int \cos 3x \left(\frac{3}{3}\right) dx$

$= \frac{1}{3} x \cos 3x - \frac{1}{9} \sin 3x + c$

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7. $\int x^2 \cos x \, dx$

$u = x^2 \qquad v = \cos x$

$du/dx = 2x \quad \int v\,dx = \sin x$

$\int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx$

$= x^2 \sin x - I_1 \rightarrow (i)$

$I_1 = \int 2x \sin x \, dx$

$u = 2x \qquad v = \sin x$

$du/dx = 2 \qquad \int v\,dx = -\cos x$

$\therefore I_1 = -2x\cos x - \int -2\cos x \, dx$

$I_1 = -2x \cos x + 2\sin x$

$(i) \Rightarrow \quad x^2 \sin x + 2x\cos x - 2\sin x + C$

9. $\int x \ln x \, dx$

$u = \ln x \qquad v = x$

$du/dx = \frac{1}{x} \quad \int v\,dx = \frac{x^2}{2}$

$\Rightarrow \ln x\left[\frac{x^2}{2}\right] - \int \frac{1}{\cancel{x}}\left[\frac{x^{\cancel{2}}}{2}\right] dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\left[\frac{x^2}{2}\right] + C$

$= \frac{1}{2}\left[x^2 \ln x - \frac{x^2}{2}\right] + C$

11. $\int (\ln x)^2 dx$

$u = (\ln x)^2 \qquad v = 1$

$\frac{du}{dx} = \frac{2\ln x}{x} \quad \int v\,dx = x$

$\Rightarrow x(\ln x)^2 - \int \frac{2\ln x}{\cancel{x}} \cdot \cancel{x} \, dx$

$x(\ln x)^2 - 2\int \ln x \, dx$

$x(\ln x)^2 - 2 I_1$

$I_1 = \int \ln x$

$u = \ln x \qquad v = 1$

$\frac{du}{dx} = \frac{1}{x} \qquad \int v\,dx = x$

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$\Rightarrow x\ln x - \int \frac{1}{x}\cdot x\, dx$

$\Rightarrow x\ln x - x$

$\therefore x(\ln x)^2 - 2[x\ln x - x] + c$

13. $\int \ln(3x-2)\, dx$

$u = \ln(3x-2)$ $\qquad v = 1$

$\frac{du}{dx} = \frac{1}{3x-2}\cdot 3$ $\qquad \int v\,dx = x$

$\frac{du}{dx} = \frac{3}{3x-2}$

$\Rightarrow x\ln(3x-2) - \int \frac{3}{3x-2}\cdot x\, dx$

$\Rightarrow x\ln(3x-2) - \int \frac{3x}{3x-2}\, dx$

$$3x-2 \,\overline{)\, 3x} \quad 1$$
$$-(3x-2)$$
$$2$$

$\Rightarrow x\ln(3x-2) - \int \left(1 + \frac{2}{3x-2}\right) dx$

$\Rightarrow x\ln(3x-2) - \int dx \mp 2\int \frac{dx}{3x-2}$

$\Rightarrow x\ln(3x-2) - x \mp \frac{2}{3}\int \frac{3}{3x-2}\, dx$

$\Rightarrow x\ln(3x-2) - x \mp \frac{2}{3}\ln(3x-2) + C$

14. $\int \ln(x^2+4)\, dx$

$u = \ln(x^2+4)$ $\qquad \int v\,dx = x$

$\frac{du}{dx} = \frac{1}{x^2+4}\cdot 2x$

$\frac{du}{dx} = \frac{2x}{x^2+4}$

$\Rightarrow x\ln(x^2+4) - \int \frac{2x}{x^2+4}\cdot x\, dx$

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$$\Rightarrow x\ln(x^2+4) - \int \frac{2x^2}{x^2+4}\,dx$$

$$\begin{array}{r|l} & 2 \\ x^2+4 & 2x^2 \\ & -2x^2 \pm 8 \\ \hline & -8 \end{array}$$

$$\Rightarrow x\ln(x^2+4) - \int \left(2 - \frac{8}{x^2+4}\right)dx$$

$$\Rightarrow x\ln(x^2+4) - 2\int dx + \int \frac{8}{x^2+4}\,dx$$

$$\Rightarrow x\ln(x^2+4) - 2x + \frac{8}{2}\int \frac{dx}{x^2+2^2}$$

$$\Rightarrow x\ln(x^2+4) - 2x + 8\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right] + c$$

$$\Rightarrow x\ln(x^2+4) - 2x + 4\tan^{-1}(x/2) + c$$

15. $\int \sin^{-1}x\,dx$

$u = \sin^{-1}x \qquad \int v\,dx = x$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow x\sin^{-1}x - \int\left(\frac{1}{\sqrt{1-x^2}}\cdot x\right)dx$$

$$\Rightarrow x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow x\sin^{-1}x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow x\sin^{-1}x + \frac{1}{2}\sin^{-1}\sqrt{1-x^2} + c$$

17. $\int \tan^{-1}(3x)\,dx$

$u = \tan^{-1}(3x) \qquad \int v = x$

$$\frac{du}{dx} = \frac{1}{1+(3x)^2}\cdot 3$$

$$\frac{du}{dx} = \frac{3}{1+9x^2}$$

$$\Rightarrow x\tan^{-1}(3x) - \int \frac{3}{1+9x^2}\cdot x\,dx$$

$$\Rightarrow x\tan^{-1}(3x) - 3\int \frac{x}{1+9x^2}\,dx$$

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$\rightarrow x\tan^{-1}(3x) - 3\int \frac{18x}{18(1+9x^2)} dx$

$\rightarrow x\tan^{-1}(3x) - \frac{1}{6}\ln(1+9x^2) + c$

19. $\int e^x \sin x \, dx$

$u = \sin x \qquad v = e^x$
$\frac{du}{dx} = \cos x \qquad \int v\,dx = e^x$

$\rightarrow e^x \sin x - \int \cos x \, e^x \, dx$

$\rightarrow e^x \sin x - \int e^x \cos x \, dx$

$\rightarrow e^x \sin x - I_1$

$I_1 = \int e^x \cos x \, dx$

$u = \cos x \qquad v = e^x$
$\frac{du}{dx} = -\sin x \qquad \int v\,dx = e^x$

$I_1 \rightarrow e^x \cos x + \int e^x \sin x$

$I_1,\ e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$
$I_1 = e^x \sin x - e^x \cos x - I_1$
$2I_1 = e^x \sin x - e^x \cos x$
$I_1 = \frac{1}{2} e^x (\sin x - \cos x) + c$

21. $\int \sin(\ln x) \, dx$
$u = \sin(\ln x) \qquad \int v = x$
$\frac{du}{dx} = \frac{\cos \ln x}{x}$

$I \rightarrow x\sin(\ln x) - \int \cos(\ln x) \, dx \rightarrow (A)$

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Consider

$$\int \cos(\ln x)\,dx$$

$u = \cos \ln x$ $\qquad \int v = x$

$\frac{du}{dx} = -\frac{\sin \ln x}{x}$

$\Rightarrow x \cos \ln x + \int \sin(\ln x)\,dx$

(A) $\Rightarrow$

$I = x \sin \ln x - x \cos \ln x - \int \sin(\ln x)\,dx$

$I = x \sin \ln x - x \cos \ln x - I$

$2I = x \sin \ln x - x \cos \ln x$

$I = \frac{1}{2} x(\sin(\ln x) - \cos(\ln x)) + c$

23. $\int x \sec^2 x\,dx$

$u = x$ $\qquad v = \sec^2 x$

$\frac{du}{dx} = 1$ $\qquad \int v\,dx = \int \sec^2 x\,dx$

$\int v\,dx = \tan x$

$I = x \tan x - \int \tan x\,dx$

$I = x \tan x - \ln|\sec x| + c$

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$\Rightarrow t^{-\frac{1}{2}} e^{t} - \int -\frac{t^{-\frac{3}{2}}}{2} \cdot e^{t}$

$\rightarrow t^{-\frac{1}{2}} e^{t} + \frac{1}{2} \int t^{-\frac{3}{2}} e^{t}$

$\rightarrow x^{-1} e^{x^2} + \frac{1}{2} \int x^{-3} e^{x^2}$

25. $\int x^3 e^{x^2} dx$

$\int x^2 \cdot x e^{x^2} dx$

$t = x^2$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2x} = dx$

$\Rightarrow \int x^3 e^{x^2} dx \Rightarrow \int \cancel{x} \cdot t \cdot e^{t} \cdot \frac{dt}{2\cancel{x}}$

$\rightarrow \frac{1}{2} \int t e^{t} dt$

$u = t \qquad v = e^{t}$

$\frac{du}{dt} = 1 \qquad \int v = e^{t}$

$\Rightarrow t e^{t} - \int e^{t} dt$

$\Rightarrow t e^{t} - e^{t}$

$\Rightarrow x^2 e^{x^2} - e^{x^2}$

$\Rightarrow \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C$

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27. $\int_0^2 xe^{2x}\,dx$

$u = x \qquad v = e^{2x}$

$du/dx = 1 \qquad \int v = \frac{e^{2x}}{2}$

$I \Rightarrow \frac{1}{2}xe^{2x} - \int \frac{e^{2x}}{2}\,dx$

$I = \frac{1}{2}xe^{2x} - \frac{1}{2}\left[\frac{e^{2x}}{2}\right]$

$I = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\Big|_0^2$

$I = \left(\frac{1}{\cancel{2}}(\cancel{2})e^4 - \frac{1}{4}e^4\right) + \frac{1}{4}$

$I = e^4 - \frac{e^4}{4} + \frac{1}{4}$

$I = \frac{3e^4}{4} + \frac{1}{4}$

26. $\int \frac{xe^x}{(x+1)^2}\,dx$

$\Rightarrow \int \frac{xe^x}{(1+x)^2}\,dx$

$\Rightarrow \int e^x \frac{1+x-1}{(1+x)^2}\,dx$

$\Rightarrow \int e^x \left(\frac{\cancel{1+x}}{(1+x)^{\cancel{2}}} - \frac{1}{(1+x)^2}\right)dx$

$\Rightarrow \int e^x \left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right)dx$

$\Rightarrow \frac{e^x}{1+x} + c$

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30. $\int_{\sqrt{e}}^{e} \frac{\ln x}{x^2}\, dx$

$u = \ln x \qquad v = \frac{1}{x^2}$

$\frac{du}{dx} = \frac{1}{x} \qquad \int v = -1/x$

$$I = -\frac{1}{x}\ln x - \int \frac{1}{x} \cdot -\frac{1}{x}\, dx$$

$$I = -\frac{1}{x}\ln x + \int \frac{1}{x^2}\, dx$$

$$I = -\frac{1}{x}\ln x + \int x^{-2}\, dx$$

$$I = -\frac{1}{x}\ln x + \left[\frac{x^{-1}}{-1}\right] \cancel{c}$$

$$I = -\frac{1}{x}\ln x - \frac{1}{x}\Bigg|_{\sqrt{e}}^{e}$$

$$I = \left[-\frac{\ln e}{e} - \frac{1}{e}\right] - \left[-\frac{\ln\sqrt{e}}{\sqrt{e}} - \frac{1}{\sqrt{e}}\right]$$

$$I = \left[-\frac{1}{e} - \frac{1}{e}\right] - \left[-\frac{1}{2\sqrt{e}} - \frac{1}{\sqrt{e}}\right]$$

$$I = \frac{-2}{e} - \left[\frac{-1-2}{2\sqrt{e}}\right]$$

$$I = \frac{-2}{e} + \frac{3}{2\sqrt{e}}$$

Ex#7.4

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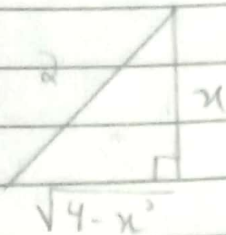
1. $\int \sqrt{4-x^2}\,dx$

$\Rightarrow \int \sqrt{2^2-(x)^2}\,dx$

$x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$



$dx = 2\cos\theta\,d\theta$

$\Rightarrow \int \sqrt{4-4\cos^2\theta}\,d\theta$

$\Rightarrow \int \sqrt{4(1-\cos^2\theta)}\,d\theta$

$\rightarrow 2\int \sin\theta\,d\theta$

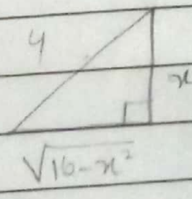
$\rightarrow -2\cos\theta + c$

$\Rightarrow -\cancel{2}\cdot\left[\frac{\sqrt{4-x^2}}{\cancel{2}}\right] + c$

$\rightarrow -\sqrt{4-x^2} + c$

3. $\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

$\Rightarrow \int \frac{x^2}{\sqrt{4^2-(x)^2}}\,dx$



$x = 4\cancel{2}\sin\theta$

$\frac{dx}{d\theta} = 4\cancel{2}\cos\theta$

$dx = 4\cos\theta\,d\theta$

$\Rightarrow \int \frac{16\sin^2\theta}{\sqrt{16-16\cos^2\theta}} \cdot 4\cos\theta\,d\theta$

$\Rightarrow \int \frac{16\sin^2\theta}{\cancel{4}\sin\theta} \cdot \cancel{4}\cos\theta\,d\theta$

$\Rightarrow 16\int \sin^2\theta\cos\theta\,d\theta$

$\Rightarrow 16\frac{\sin^3\theta}{3} + c$

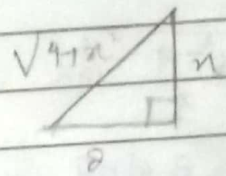
$\Rightarrow \left[\cancel{16}\cdot\frac{x^3}{\cancel{64}}\right] \div 3 + c$

$\rightarrow \frac{x^3}{4} * \frac{1}{3} + c$

$\rightarrow \frac{1}{12}x^3 + c$

5. $\int \frac{dx}{(4+x^2)^2}$

$\Rightarrow \int \frac{dx}{(2^2+(x)^2)^2}$



$x = 2\tan\theta$

$\frac{x}{2} = \tan\theta$

$\frac{dx}{d\theta} = 2\sec^2\theta$

$dx = 2\sec^2\theta\,d\theta$

$\Rightarrow \int \frac{2\sec^2\theta\,d\theta}{(4+4\tan^2\theta)^2}$

$\Rightarrow \int \frac{2\sec^2\theta\,d\theta}{(4(1+\tan^2\theta))^2}$

$\Rightarrow \frac{1}{8}\int \frac{\sec^2\theta\,d\theta}{\sec^4\theta}$

$\Rightarrow \frac{1}{8}\int \cos^2\theta\,d\theta$

$\Rightarrow \frac{1}{8}\int \frac{1+\cos 2\theta}{2}\,d\theta$

$\Rightarrow \frac{1}{8}\int \left(\frac{1}{2} + \frac{\cos 2\theta}{2}\right) d\theta$

$\Rightarrow \frac{1}{16}\int d\theta + \frac{1}{16}\int \cos 2\theta\left(\frac{2}{2}\right) d\theta$

$\Rightarrow \frac{1}{16}\theta + \frac{1}{32}\int \cos 2\theta (2)\, d\theta$

$\Rightarrow \frac{1}{16}\theta + \frac{1}{32}\sin 2\theta + c$

$\Rightarrow \frac{1}{16}\theta + \frac{1}{32_{16}} \cdot \cancel{2}\sin\theta\cos\theta + c$

$\Rightarrow \frac{1}{16}\left[\theta + \sin\theta\cos\theta\right] + c$

$\Rightarrow \frac{1}{16}\left[\tan^{-1}\left(\frac{x}{2}\right) + \left(\frac{x}{\sqrt{4+x^2}}\right)\left(\frac{2}{\sqrt{4+x^2}}\right)\right] + c$

$\Rightarrow \frac{1}{16}\left[\tan^{-1}\left(\frac{x}{2}\right) + \frac{2x}{4+x^2}\right] + c$

7. $\int \frac{\sqrt{x^2-9}}{x}\, dx$

~~$x = 3\sin\theta$~~

~~$\frac{x}{3} = \sin\theta$~~

~~$\frac{dx}{d\theta} = 3\cos\theta$~~

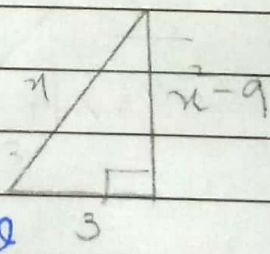
~~$dx = 3\cos\theta\, d\theta$~~

$x = 3\sec\theta$

$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$

$dx = 3\sec\theta\tan\theta\, d\theta$

$\Rightarrow \int \sqrt{x^2 - 9\sec^2\theta}$



$\Rightarrow \int \frac{\sqrt{9\sec^2\theta - 9}}{\cancel{3}\cancel{\sec\theta}} \cdot \cancel{3\sec\theta}\tan\theta\, d\theta$

$\Rightarrow \int \sqrt{9(\sec^2\theta - 1)} \cdot \tan\theta\, d\theta$

$\Rightarrow 3\int \tan^2\theta\, d\theta$

$\Rightarrow 3\int (\sec^2\theta - 1)\, d\theta$

$\Rightarrow 3\int \sec^2\theta\, d\theta - 3\int d\theta$

$\Rightarrow 3\tan\theta - 3\theta + c$

$\Rightarrow \cancel{3}\frac{\sqrt{x^2-9}}{\cancel{3}} - 3\sec^{-1}\left(\frac{x}{3}\right) + c \quad \Rightarrow \sqrt{x^2-9} - 3\sec^{-1}\left(\frac{x}{3}\right) + c$

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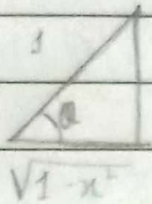
9. $\int \frac{3x^3}{\sqrt{1-x^2}}\,dx$

$\Rightarrow 3\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

$x = \sin\theta$

$\theta = \sin^{-1}x$



$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

$\Rightarrow 3\int \frac{\sin^3\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\, d\theta$

$\Rightarrow 3\int \frac{\sin^3\theta}{\cos\theta}\cdot\cos\theta\, d\theta$

$\Rightarrow 3\int \sin^3\theta\, d\theta$

$\Rightarrow 3\int \sin^2\theta \sin\theta\, d\theta$

$\Rightarrow 3\int (1-\cos^2\theta)\sin\theta\, d\theta$

$\Rightarrow 3\int d\theta - 3\int \cos^2\theta \sin\theta\, d\theta$

$\Rightarrow 3\theta + 3\int \cos^2\theta(-\sin\theta)\, d\theta$

$\Rightarrow 3\theta + \frac{\cancel{3}\cos^3\theta}{\cancel{3}} + c$

$\Rightarrow 3\theta + \cos^3\theta + c$

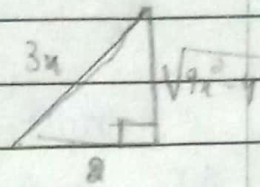
$\Rightarrow 3\sin^{-1}x + \left(\sqrt{1-x^2}\right)^3 + c$

$\Rightarrow -3\sqrt{1-x^2} + (1-x^2)^{3/2} + c$

11. $\int \frac{dx}{x^2\sqrt{9x^2-4}}$

$\Rightarrow \int \frac{dx}{x^2\sqrt{(3x)^2-(2)^2}}$

$\Rightarrow \int \frac{dx}{x^2\sqrt{3\left[x^2-\left(\frac{2}{3}\right)^2\right]}}$



$\Rightarrow \frac{1}{3}\int \frac{dx}{x^2\sqrt{x^2-(2/3)^2}}$

$x = \frac{2}{3}\sec\theta$

$\frac{dx}{d\theta} = \frac{2}{3}\sec\theta\tan\theta$

$dx = \frac{2}{3}\sec\theta\tan\theta\, d\theta$

$\Rightarrow \frac{1}{3}\int \frac{\sec\theta\tan\theta\, d\theta}{\left(\frac{2}{3}\sec\theta\right)^2\sqrt{\left(\frac{2}{3}\sec\theta\right)^2-\left(\frac{2}{3}\right)^2}}$

$\Rightarrow \frac{1}{3}\int \frac{\cancel{\sec\theta}\tan\theta\, d\theta}{\frac{4}{9}\sec^2\theta\sqrt{\frac{4}{9}\sec^2\theta - \frac{4}{9}}}$

$\Rightarrow \frac{3}{4}\int \frac{\cancel{\tan\theta}\, d\theta}{\frac{2}{3}\cancel{\tan\theta}\cdot\sec\theta}$

$\Rightarrow \frac{3}{4}\int \cos\theta\, d\theta$

$\Rightarrow \frac{3}{4}\sin\theta + c$

$\Rightarrow \frac{\cancel{3}}{4}\left[\frac{\sqrt{9x^2-4}}{\cancel{3}x}\right] + c$

$\Rightarrow \frac{1}{4}\left[\frac{\sqrt{9x^2-4}}{x}\right] + c$

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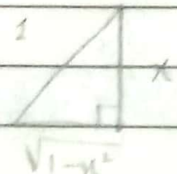
13. $\int \frac{dx}{(1-x^2)^{3/2}}$

$\Rightarrow \int \frac{dx}{(\sqrt{1-x^2})^3}$



$x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

$\Rightarrow \int \frac{\cos\theta\, d\theta}{(\sqrt{1-\sin^2\theta})^3}$

$\Rightarrow \int \frac{\cancel{\cos\theta}\, d\theta}{\cos^{3}\theta}$

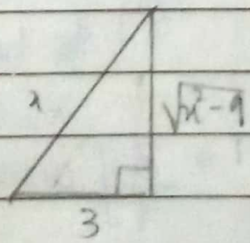
$\Rightarrow \int \frac{d\theta}{\cos^2\theta}$

$\Rightarrow \int \sec^2\theta\, d\theta$

$\Rightarrow \tan\theta + c$

$\Rightarrow \frac{x}{\sqrt{1-x^2}} + c$

15. $\int \frac{dx}{\sqrt{x^2-9}}$



$\Rightarrow \int \frac{dx}{\sqrt{x^2-3^2}}$

$x = 3\sec\theta$

$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$

$dx = 3\sec\theta\tan\theta\, d\theta$

$\Rightarrow \int \frac{3\sec\theta\tan\theta\, d\theta}{\sqrt{9\sec^2\theta - 9}}$

$\Rightarrow \int \frac{\cancel{3}\sec\theta\,\cancel{\tan\theta}\, d\theta}{\cancel{3}\,\cancel{\tan\theta}}$

$\Rightarrow \int \sec\theta\, d\theta$

$\Rightarrow \ln|\sec\theta + \tan\theta| + c$

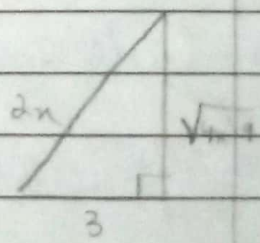
$\Rightarrow \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + c$

17. $\int \frac{dx}{(4x^2-9)^{3/2}}$

$\Rightarrow \int \frac{dx}{(\sqrt{4x^2-9})^{3/2}}$

$\Rightarrow \int \frac{dx}{\left(2\sqrt{x^2-(\frac{3}{2})^2}\right)^{3/2}}$

$\Rightarrow \int \frac{dx}{2\sqrt{2}\left(\sqrt{x^2-(\frac{3}{2})^2}\right)^{3/2}}$



$x = \frac{3}{2}\sec\theta$

$\frac{dx}{d\theta} = \frac{3}{2}\sec\theta\tan\theta\, d\theta$

$\Rightarrow \frac{1}{2\sqrt{2}} \int \frac{\frac{3}{2}\sec\theta\tan\theta\, d\theta}{\left(\sqrt{\frac{9}{4}\sec^2\theta - \frac{9}{4}}\right)^{3/2}}$

$\Rightarrow \frac{3}{4\sqrt{2}} \int \frac{\sec\theta\tan\theta\, d\theta}{(\frac{3}{2}\tan\theta)^{3/2}}$

$\Rightarrow \frac{3}{4\sqrt{2}} \int \frac{\sec\theta\tan\theta\, d\theta}{(\frac{3}{2})^{3/2} \cdot \tan^{3/2}\theta}$

$\Rightarrow \frac{\cancel{3}}{2} \int \frac{\sec\theta\,\cancel{\tan\theta}\, d\theta}{9\,\cancel{27}\tan^2\theta}$

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$$\Rightarrow \frac{1}{18}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$\Rightarrow \frac{1}{18}\int \sec\theta \cdot \frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$\Rightarrow \frac{1}{18}\int \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$$

$$\Rightarrow \frac{1}{18}\int (\sin\theta)^{-2}\cos\theta\,d\theta$$

$$\Rightarrow \frac{1}{18}\left[\frac{(\sin\theta)^{-1}}{-1}\right] + c$$

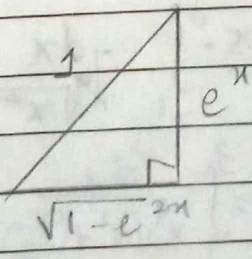
$$\Rightarrow -\frac{1}{18}\left[\frac{1}{\sin\theta}\right] + c$$

$$\Rightarrow -\frac{1}{18}\operatorname{cosec}\theta + c$$

$$\Rightarrow -\frac{1}{9\cancel{18}}\left[\frac{\cancel{2}x}{\sqrt{4x^2-9}}\right] + c$$

$$\Rightarrow -\frac{1}{19}\left[\frac{x}{\sqrt{4x^2-9}}\right] + c$$

19. $\int e^x\sqrt{1-e^{2x}}\,dx$



$e^x = \sin\theta$

$e^x \cdot \frac{dx}{d\theta} = \cos\theta$

$e^x dx = \cos\theta\,d\theta$

$$\Rightarrow \int e^x dx\sqrt{1-e^{2x}}$$

$$\Rightarrow \int \cos\theta\sqrt{1-\sin^2\theta}\,d\theta$$

$$\Rightarrow \int \cos^2\theta\,d\theta$$

$$\Rightarrow \int\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$\Rightarrow \frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta\left(\frac{2}{2}\right)d\theta$$

$$\Rightarrow \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\,d\theta$$

$$\Rightarrow \frac{1}{2}\sin^{-1}e^x + \frac{1}{\cancel{4}\,2}\cdot \cancel{2}\sin\theta\cos\theta\,d\theta$$

$$\Rightarrow \frac{1}{2}\left[\sin^{-1}e^x + e^x\sqrt{1-e^{2x}}\right] + c$$

21. $\int_0^1 5x^3\sqrt{1-x^2}\,dx$

$$\Rightarrow 5\int_0^1 x^3\sqrt{1-x^2}\,dx$$

$x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\,d\theta$

$$\Rightarrow 5\int_0^1 \sin^3\theta\sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$$

$$\Rightarrow 5\int_0^1 \sin^3\theta\cdot\cos^2\theta\,d\theta$$

$$\Rightarrow 5\int_0^1 \sin^2\theta\cdot\sin\theta\cdot\cos^2\theta\,d\theta$$

$$\Rightarrow 5\int_0^1 (1-\cos^2\theta)\cos^2\theta\sin\theta\,d\theta$$

$$\Rightarrow 5\int_0^1 \cos^2\theta\sin\theta\,d\theta - 5\int_0^1 \cos^4\theta\sin\theta\,d\theta$$

$$\Rightarrow 5\int_0^1 \cos^2\theta\left(\frac{-\sin\theta}{-1}\right)d\theta - 5\int_0^1 \cos^4\theta\left(\frac{-\sin\theta}{-1}\right)d\theta$$

$$\Rightarrow -5\int_0^1 \cos^2\theta(-\sin\theta)\,d\theta + 5\int_0^1 \cos^4\theta(-\sin\theta)\,d\theta$$

$$\Rightarrow -5\left[\frac{\cos^3\theta}{3}\right]_0^1 + 5\left[\frac{\cos^5\theta}{5}\right]_0^1$$

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Changing limits:

$1 = \sin\theta \qquad 0 = \sin\theta$

$\theta = \sin^{-1}(1) \qquad \theta = 0$

$\theta = \frac{\pi}{2}$

$\Rightarrow -5\left[\frac{\cos^3\theta}{3}\right]_0^{\pi/2} + 5\left[\frac{\cos^5\theta}{5}\right]_0^{\pi/2}$

$\Rightarrow -5\left[\frac{(\cos\frac{\pi}{2})^3}{3} - \frac{\cos 0}{3}\right] + 5\left[\frac{(\cos\frac{\pi}{2})^5}{5} - \frac{\cos 0}{5}\right]$

$\Rightarrow -5\left[0 - \frac{1}{3}\right] + 5\left[0 - \frac{1}{5}\right]$

$\Rightarrow \frac{5}{3} - 1$

$\Rightarrow \frac{2}{3}$

23. $\int_{\sqrt{2}}^{2} \frac{dx}{x^2\sqrt{x^2-1}}$

$\rightarrow x = \sec\theta$

$\frac{dx}{d\theta} = \sec\theta\tan\theta$

$dx = \sec\theta\tan\theta\, d\theta$

Changing limits:

$2 = \sec\theta \qquad \sqrt{2} = \frac{1}{\cos\theta}$

$2 = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{2} \qquad \theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{3}$

$\Rightarrow \int_{\pi/4}^{\pi/3} \frac{\sec\theta\tan\theta\, d\theta}{\sec^2\theta\sqrt{\sec^2\theta - 1}}$

$\Rightarrow \int_{\pi/4}^{\pi/3} \frac{\tan\theta\, d\theta}{\tan\theta\sec\theta}$

$\Rightarrow \int_{\pi/4}^{\pi/3} \cos\theta\, d\theta$

$\Rightarrow \sin\theta \Big|_{\pi/4}^{\pi/3}$

$\Rightarrow \sin\frac{\pi}{3} - \sin\frac{\pi}{4} \Rightarrow \frac{\sqrt{3} - \sqrt{2}}{2}$

25. $\int_1^3 \frac{dx}{x^4\sqrt{x^2+3}}$

$\Rightarrow \int_1^3 \frac{dx}{x^4\sqrt{x^2+(\sqrt{3})^2}}$

$\Rightarrow \int_1^3 \frac{dx}{x^4\sqrt{(\sqrt{3})^2+x^2}}$

$x = \sqrt{3}\tan\theta$

$\frac{dx}{d\theta} = \sqrt{3}\sec^2\theta$

$dx = \sqrt{3}\sec^2\theta\, d\theta$

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Changing limits

$3 = \sqrt{3}\tan\theta$ $\quad$ $1 = \sqrt{3}\tan\theta$

$\tan\theta = \frac{3}{\sqrt{3}}$ $\quad$ $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \pi/3$ $\quad$ $\theta = \pi/6$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\sqrt{3}\sec^2\theta\, d\theta}{(\sqrt{3}\tan\theta)^4\sqrt{(\sqrt{3}\tan\theta)^2+3}}$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\sqrt{3}\sec^2\theta\, d\theta}{9\tan^4\theta\sqrt{3\tan^2\theta+3}}$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} \frac{\sqrt{3}}{9}$$

$$\Rightarrow \frac{\sqrt{3}}{9}\int_{\pi/6}^{\pi/3} \frac{\sec^{\cancel{2}}\theta\, d\theta}{\sqrt{3}\tan^4\theta \cdot \cancel{\sec\theta}}$$

$$\Rightarrow \frac{1}{9}\int_{\pi/6}^{\pi/3} \frac{\sec\theta\, d\theta}{\tan^4\theta}$$

$$\Rightarrow \frac{1}{9}\int_{\pi/6}^{\pi/3} \frac{1}{\cos\theta}\cdot\frac{\cos^4\theta}{\sin^4\theta}\, d\theta$$

$$\Rightarrow \int_{\pi/6}^{\pi/3} (\sin\theta)^{-2}\cos^2\theta\, d\theta$$

$$\Rightarrow \left.\frac{(\sin\theta)^{-1}}{-1}\right]_{\pi/6}^{\pi/3}$$

$$\Rightarrow \left.\frac{-\csc}{\cancel{\operatorname{cosec}\theta}}\right|_{\pi/6}^{\pi/3}$$

$$\Rightarrow \frac{-1}{\operatorname{cosec}\pi/3} - \cancel{\left(\frac{-1}{\operatorname{cosec}\pi/6}\right)}$$

$$\Rightarrow \frac{-1}{\sin\pi/3} - \left(\frac{-1}{\sin\pi/6}\right)$$

$$\Rightarrow \frac{10\sqrt{3}+18}{243}$$

37. $\int \frac{dx}{x^2-4x+5}$

$$\Rightarrow L.T = \frac{(M.T)^2}{4\times F.T} = \frac{16}{4\times 1} \Rightarrow 4$$

$$\Rightarrow \int \frac{dx}{x^2-4x+5+4-4}$$

$$\Rightarrow \int \frac{dx}{x^2-4x+4+1}$$

$$\Rightarrow \int \frac{dx}{(x-2)^2+1}$$

$$\Rightarrow \int \frac{dx}{1^2+(x-2)^2}$$

$$\Rightarrow \tan^{-1}(x-2)+C$$

39. $\int \frac{dx}{\sqrt{3+2x-x^2}}$

$$\Rightarrow L.T. = \frac{(M.T)^2}{4\times F.T} = \frac{4}{-4} \Rightarrow -1$$

$$\Rightarrow \int \frac{dx}{\sqrt{3+1-1+2x-x^2}}$$

$$\Rightarrow \int \frac{dx}{4-1+2x-x^2}$$

$$\Rightarrow \int \frac{dx}{\sqrt{4-(x-1)^2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{2^2-(x-1)^2}}$$

$$\Rightarrow \sin^{-1}\left(\frac{x-1}{2}\right)+c$$

41. $$\int \frac{dx}{\sqrt{x^2-6x+10}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-6x+10+9-9}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-6x+9+1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2+1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} \Rightarrow \ln\left(x+\sqrt{x^2+a^2}\right)+c$$

$$\Rightarrow \ln\left(x-3+\sqrt{(x-3)^2+1}\right)+c$$

43. $$\int \sqrt{3-2x-x^2}\,dx$$

$$\Rightarrow \int \sqrt{3+1-1-2x-x^2}\,dx$$

$$\Rightarrow \int \sqrt{4-1-2x-x^2}\,dx$$

$$\Rightarrow \int \sqrt{4-(x+1)^2}\,dx$$

$$\Rightarrow \int \sqrt{2^2-(x+1)^2}\,dx$$

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$x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

$dx = 2\cos\theta \, d\theta$

$\Rightarrow \int \sqrt{4-4\cos^2\theta} \; d\theta$

$\Rightarrow \int 2\sin\theta \, d\theta$

$\Rightarrow 2\int \sin\theta \, d\theta$

$\Rightarrow -2\cos\theta + C$

$\Rightarrow -2\sin^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{2}(x+1)\left(\sqrt{3-2x-x^2}\right) + C$

47. $\int_1^2 \frac{dx}{\sqrt{4x-x^2}}$

$\Rightarrow \int_1^2 \frac{dx}{\sqrt{x(4-x)}}$

$\Rightarrow \int \frac{dx}{x\sqrt{}}$

$\Rightarrow \int_1^2 \frac{dx}{\sqrt{4-4+4x-x^2}}$

$\Rightarrow \int_1^2 \frac{dx}{\sqrt{4-(4-4x+x^2)}}$

$\Rightarrow \int_1^2 \frac{dx}{\sqrt{4-(x-2)^2}}$

$\Rightarrow \sin^{-1}\left(\frac{x-2}{2}\right)\Big|_1^2$

$\Rightarrow \sin^{-1}(0) - \sin^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow \frac{\pi}{6}$ Ans

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Ex# 7.5

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9. $\int \frac{dx}{x^2-3x-4}$

$\Rightarrow \int \frac{dx}{x^2-3x-4} \rightarrow (A)$

$\Rightarrow \int \frac{dx}{x^2-3x-4}$

$\Rightarrow \int \frac{dx}{x^2-4x+x-4}$

$\Rightarrow \int \frac{dx}{x(x-4)+1(x-4)}$

$\Rightarrow \int \frac{dx}{(x+1)(x-4)}$

$\Rightarrow \int \frac{dx}{(x-4)(x+1)} = \int \frac{A\,dx}{x+1} + \int \frac{B\,dx}{x-4} \rightarrow (B)$

$\Rightarrow \frac{1}{(x-4)(x+1)} = \frac{A}{x+1} + \frac{B}{x-4}$

$\Rightarrow \frac{1}{(x-4)(x+1)} = \frac{A(x-4)+B(x+1)}{(x-4)(x+1)}$

$\Rightarrow 1 = A(x-4) + B(x+1) \rightarrow (C)$

For 'A':

$x=4$

$\therefore 1 = A(5)$

$A = 1/5$

For 'B':

$x=-1$

$\therefore 1 = B(-5)$

$B = -1/5$

(B) $\Rightarrow \int \frac{dx}{(x+1)(x-4)} \Rightarrow \frac{1}{5}\int \frac{dx}{x+1} - \frac{1}{5}\int \frac{dx}{x-4}$

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$$\Rightarrow \frac{1}{5}\ln(x+1) - \frac{1}{5}\ln|x-4|$$

$$\Rightarrow \ln(x+1)^{\frac{1}{5}} - \ln(x-4)^{\frac{1}{5}}$$

$$\Rightarrow \ln\left(\frac{x+1}{x-4}\right)^{1/5} + C$$

11. $\int \frac{11x+17}{2x^2+7x-4}\,dx$

$$\Rightarrow \int \frac{11x+17}{2x^2+8x-x-4}\,dx$$

$$\Rightarrow \int \frac{11x+17}{2x(x+4)-1(x+4)}\,dx$$

$$\Rightarrow \int \frac{11x+17}{(2x-1)(x+4)}\,dx \Rightarrow \int \frac{A}{2x-1}\,dx + \int \frac{B}{x+4}\,dx \rightarrow (A)$$

$$\frac{11x+17}{(2x-1)(x+4)} = \frac{A}{2x-1} + \frac{B}{x+4}$$

$$\frac{11x+17}{\cancel{(2x-1)(x+4)}} = \frac{A(x+4)+B(2x-1)}{\cancel{(2x-1)(x+4)}}$$

$$11x+17 = A(x+4)+B(2x-1)$$

For 'A':

$2x-1=0$

$\boxed{x=\frac{1}{2}}$

$$\Rightarrow 11\left(\frac{1}{2}\right)+17 = A\left(\frac{1}{2}+4\right)$$

$$\Rightarrow \frac{11}{2}+17 = \frac{9}{2}A$$

$$\Rightarrow \frac{45}{\cancel{2}} = \frac{9}{\cancel{2}}A$$

$\boxed{A=5}$

For 'B':

$\boxed{x=-4}$

$$\Rightarrow 11(-4)+17 = B(-8-1)$$

$$\cancel{-}27 = \cancel{-}9B$$

$\boxed{B=3}$

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(A) ⇒

$$\int \frac{11x+17}{(2x-1)(x+4)}dx \Rightarrow 5\int \frac{dx}{2x-1} + 3\int \frac{dx}{x+4}$$

$$\Rightarrow 5\int \frac{2\,dx}{2(2x-1)} + 3\int \frac{dx}{x+4}$$

$$\Rightarrow \frac{5}{2}\ln(2x-1) + 3\ln(x+4) + c$$

$$\Rightarrow \ln(2x-1)^{5/2} + \ln(x+4)^{3} + c$$

$$\Rightarrow \ln\left((2x-1)^{5/2}(x+4)^3\right) + c$$

13. $\int \frac{2x^2-9x-9}{x^3-9x}$

$$\Rightarrow \int \frac{2x^2-9x-9}{x(x^2-9)}dx$$

$$\Rightarrow \int \frac{2x^2-9x-9}{x(x+3)(x-3)} \Rightarrow \int \frac{A}{x}dx + \int \frac{B}{x+3}dx + \int \frac{C}{x-3}dx \rightarrow (A)$$

$$\Rightarrow \frac{2x^2-9x-9}{x(x+3)(x-3)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-3}$$

$$\Rightarrow \frac{2x^2-9x-9}{\cancel{x(x+3)(x-3)}} = \frac{A(x+3)(x-3)+Bx(x-3)+Cx(x+3)}{\cancel{x(x+3)(x-3)}}$$

$$\Rightarrow 2x^2-9x-9 = A(x+3)(x-3)+Bx(x-3)+Cx(x+3)$$

For 'A':

$x=0$

$\therefore -9 = A(3)(-3)$

$-9 = -9A$

$\boxed{A=1}$

For 'B':

$x=-3$

$\therefore 2(9)+27-9 = -3B(-6)$

$36 = 18B$

$\boxed{B=2}$

For 'C':

$x=3$

$18-27-9 = C\times(6)$

$-18 = 6C$

$\boxed{C=-3}$

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(A) ⇒

$$\int \frac{2x^2-9x-9}{x(x+3)(x-3)}\,dx = \int \frac{dx}{x} + 2\int \frac{dx}{x+3} - 3\int \frac{dx}{x-3}$$

$$\Rightarrow \ln x + 2\ln(x+3)^2 - 3\ln(x-3)^3$$

$$\Rightarrow \ln\left[\frac{x(x+3)^2}{(x-3)^3}\right] + C$$

15. $\int \frac{x^2-8}{x+3}\,dx$

$$\Rightarrow \int \frac{x^2-8}{x+3}\,dx$$

$$\begin{array}{r} x-3 \\ x+3 \overline{)\, x^2-8} \\ -\,x^2 \mp 3x \\ \hline -3x-8 \\ -\,3x+9 \\ \hline 1 \end{array}$$

$$\Rightarrow \int \left(x - 3 + \frac{1}{x+3}\right) dx$$

$$\Rightarrow \int x-3\,dx + \int \frac{dx}{x+3}$$

$$\Rightarrow \int x\,dx - 3\int dx + \int \frac{dx}{x+3} \Rightarrow \frac{x^2}{2} - 3x + \ln|x+3| + C$$

17. $\int \frac{3x^2-10}{x^2-4x+4}\,dx$

$$\Rightarrow \int \frac{3x^2-10}{x^2-4x+4}\,dx$$

$$\begin{array}{r} 3 \\ x^2-4x-4 \overline{)\, 3x^2-10} \\ -\,3x^2 \mp 12 \pm 12x \\ \hline -4x+2 \end{array}$$

$$\Rightarrow \int \left(3 + \frac{12x-22}{x^2-4x+4}\right) dx$$

$\Rightarrow 3\int dx + \int \frac{12x-22}{x^2-4x+4}$

$\Rightarrow 3x + \int \frac{12x-22}{x^2-4x+4} \rightarrow (A)$

Consider

$\rightarrow \int \frac{12x-22}{x^2-4x+4}\, dx$

$\Rightarrow \int \frac{12x-22}{(x-2)^2}\, dx$

$\Rightarrow \int \frac{12x-22\, dx}{(x-2)(x-2)} \Rightarrow \int \frac{A\, dx}{x-2} + \int \frac{B\, dx}{(x-2)^2} \rightarrow (B)$

13.

$\Rightarrow \frac{12x-22}{(x-2)(x-2)} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$

$\rightarrow \cancel{\frac{12x-22}{(x-2)(x-2)}} = \frac{A(x-2)^2 + B(x-2)}{\cancel{(x-2)(x-2)}}$

$\Rightarrow 12x-22 = A(x-2)^2 + B(x-2)$

$\Rightarrow 12x-22 = A(x^2-4x+4) + B(x-2)$

$\Rightarrow 12x-22 = Ax^2 - 4Ax + 4A + Bx - 2B$

For 'A':

$12 = -4A + B \rightarrow (i)$

For 'B':

$-22 = 4A - 2B \rightarrow (ii)$

$12 = -\cancel{4A} + B$

$-22 = \cancel{4A} - 2B$

$-10 = -B$

$\therefore B = 2, \quad A = 12$

$\Rightarrow 12\ln(x-2) - \frac{2}{x-2} + C$

$\Rightarrow \int \frac{12\, dx}{x-2} + 2\int \frac{dx}{(x-2)^2}$

$\Rightarrow 12\ln(x-2) + 2\left[\frac{(x-2)^{-1}}{-1}\right] + C$

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19. $\int \frac{2x-3\,dx}{x^2-3x-10}$

$\Rightarrow \int \frac{2x-3\,dx}{x^2-5x+2x-10}$

$\Rightarrow \int \frac{2x-3\,dx}{x(x-5)+2(x-5)}$

$\rightarrow \int \frac{2x-3\,dx}{(x+2)(x-5)} \rightarrow \int \frac{A\,dx}{x+2} + \int \frac{B\,dx}{x-5} \rightarrow (A)$

$\rightarrow \frac{2x-3\,dx}{(x+2)(x-5)} = \frac{A}{x+2} + \frac{B}{x-5}$

$\rightarrow \frac{2x-3}{\cancel{(x+2)(x-5)}} = \frac{A(x-5)+B(x+2)}{\cancel{(x+2)(x-5)}}$

$\rightarrow 2x-3 = A(x-5)+B(x+2)$

For 'A':

$x=-2$

$\therefore 2(-2)-3 = A(-7)$

$-7=-7A$

$\boxed{A=1}$

For 'B':

$x=5$

$10-3 = B(7)$

$7=7B$

$\boxed{B=1}$

$(A) \Rightarrow$

$\rightarrow \int \frac{2x-3\,dx}{(x+2)(x-5)} \Rightarrow \int \frac{dx}{x+2} + \int \frac{dx}{x-5}$

$\Rightarrow \ln(x+2) + \ln(x-5) + c$

21. $\int \frac{x^5+x^2+2}{x^3-x}\,dx$

$\Rightarrow \int \frac{x^5+x^2+2}{x^3-x}\,dx$

$$\begin{array}{r|l} & x^2+1 \\ x^3-x & x^5+x^2+2 \\ & -\,x^5 \mp x^3 \mp 2x \\ & -2x^3+2x^2+2x \\ & x^3-x \\ & x^3+x+2 \end{array}$$

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$$\Rightarrow \int\left(x^2+1 + \frac{x^2+x+2}{x^3-x}\right)dx$$

$$\Rightarrow \int x^2+1\, dx + \int \frac{x^2+x+2}{x^3-x}\, dx$$

$$\Rightarrow \int x^2 dx + \int dx + \int \frac{x^2+x+2}{x^3-x}\, dx$$

$$\Rightarrow \frac{x^3}{3} + x + \int \frac{x^2+x+2}{x^3-x}\, dx \rightarrow (A)$$

Consider

$$\rightarrow \int \frac{x^2+x+2}{x^3-x}\, dx$$

$$\Rightarrow \int \frac{x^2+x+2}{x(x^2-1)}\, dx$$

$$\Rightarrow \int \frac{x^2+x+2}{x(x+1)(x-1)}\, dx \Rightarrow \int \frac{A}{x}\, dx + \int \frac{B}{x+1}\, dx + \int \frac{C}{x-1}\, dx \rightarrow (B)$$

$$\frac{x^2+x+2}{x(x+1)(x-1)} \Rightarrow \frac{A(x+1)(x-1)+Bx(x-1)+Cx(x+1)}{\cancel{x(x+1)(x-1)}}$$

$$x^2+x+2 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$$

For 'A':

$x=0$

$2 = A(3)\; 2=-A$

$\cancel{A = 2/3}$

$\boxed{A=-2}$

For 'B':

$x=-1$

$\cancel{+2} = -B(-2)$

$\cancel{+2} = 2B$

$\boxed{B=+\frac{1}{2}}$

For 'C':

$x=1$

$4=2C$

$\boxed{C=2}$

(B)

$$\Rightarrow \int \frac{x^2+x+2}{x(x+1)(x-1)}\, dx = -2\int \frac{dx}{x} + \int \frac{dx}{x+1} + 2\int \frac{dx}{x-1}$$

$$\Rightarrow -2\ln x + \ln(x+1) + 2\ln(x-1)$$

$\rightarrow \cancel{\ln}$

$$(A) \Rightarrow \frac{x^3}{3} + x + \left[-2\ln x + \ln(x+1) + 2\ln(x-1)\right] + c$$

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23. $\int \frac{2x^2+3}{x(x-1)^2}\,dx$

Ss.

$$\Rightarrow \int \frac{2x^2+3}{x(x-1)^2}\,dx \rightarrow \int \frac{A}{x}\,dx + \int \frac{B}{x-1}\,dx + \int \frac{C}{(x-1)^2}\,dx \rightarrow (A)$$

$$\frac{2x^2+3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$\frac{2x^2+3}{\cancel{x(x-1)^2}} = \frac{A(x-1)^2+Bx(x-1)+Cx}{\cancel{x\,(x-1)(x-1)^2}}$$

$$2x^2+3 = A(x-1)^2 + Bx(x-1) + Cx$$

For 'A':

$x=0$

$3 = A(0-1)^2$

$\boxed{3 = A}$

For 'B':

~~$2x^2+3$~~

$2x^2+3 = A(x^2-2x+1) + Bx^2 - Bx + Cx$

$2x^2+3 = Ax^2 - 2Ax + 2A + Bx^2 - Bx + Cx$

$2 = A + B$

$2 = 3 + B$

$\boxed{B = -1}$

For 'C':

$x = 1$

$2+3 = C$

$\boxed{C=5}$

(A) $\Rightarrow \int \frac{2x^2+3}{x(x-1)^2}\,dx \Rightarrow 3\int \frac{dx}{x} - \int \frac{dx}{x-1} + 5\int \frac{dx}{(x-1)^2}$

$\rightarrow 3\ln x - \ln(x-1) + \cancel{5\ln(x} \; 5\left(\frac{(x-1)^{-1}}{-1}\right) + c$

$\rightarrow 3\ln x - \ln(x-1) + 5\left(\frac{-1}{x-1}\right) + c$

$\rightarrow 3\ln x - \ln(x-1) - \frac{5}{x-1} + c$

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25. $\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}$

$\Rightarrow \int \frac{2x^2-10x+4}{(x+1)(x-3)^2} \Rightarrow \int \frac{A}{x+1}\,dx + \int \frac{B}{x-3}\,dx + \int \frac{C}{(x-3)^2}\,dx \rightarrow (A)$

$\rightarrow \frac{2x^2-10x+4}{(x+1)(x-3)^2} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$\Rightarrow \frac{2x^2-10x+4}{(x+1)(x-3)^2} = \frac{A(x-3)^2+B(x+1)(x-3)+C(x+1)}{(x+1)(x-3)^2}$

$\Rightarrow 2x^2-10x+4 = A(x-3)^2+B(x+1)(x-3)+C(x+1)$

For 'A':

$x=-1$

$\Rightarrow 2+10+4 = A(-4)^2$

$16 = 16A$

$A = 1$

For 'C':

$x = 3$

$18-30+4 = 4C$

$-8 = 4C$

$\boxed{C=-2}$

For 'B':

$2x^2-10x+4 = A(x^2-6x+9) + B[x^2-3x+x-3] + Cx + C$

$2x^2-10x+4 = Ax^2-6Ax+9A + Bx^2-3Bx+Bx-3B+Cx+C$

$2 = A+B$

$2 = B+1$

$\boxed{B=1}$

$(A) \Rightarrow \int \frac{2x^2-10x+4}{(x+1)(x-3)^2} \Rightarrow \int \frac{dx}{x+1} + \int \frac{dx}{x-3} - 2\int \frac{dx}{(x-3)^2}$

$\Rightarrow \ln(x+1) + \ln(x-3) - 2\left[\frac{-1}{x-3}\right] + c$

$\Rightarrow \ln(x+1)(x-3) + \frac{2}{x-3} + c$

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27. $\int \frac{x^2}{(x+1)^3} dx$

$\Rightarrow \int \frac{x^2}{(x+1)^3} dx = \int \frac{A}{x+1} dx + \int \frac{B}{(x+1)^2} dx + \int \frac{C\,dx}{(x+1)^3} \rightarrow (A)$

$\frac{x^2}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$

$\frac{x^2}{\cancel{(x+1)^3}} = \frac{A(x+1)^2 + B(x+1) + C}{\cancel{(x+1)^3}}$

$x^2 = A(x+1)^2 + B(x+1) + C$

For 'c':

$x = -1$

$\boxed{1 = C}$

For A & B:

$x^2 = A(x^2 + 2x + 1) + Bx + B + C$

$x^2 = Ax^2 + 2Ax + 2A + Bx + B + C$

$\boxed{A = 1}$

$0 = 2A + B$

$0 = 2 + B$

$\boxed{B = -2}$

$(A) \Rightarrow \int \frac{x^2}{(x+1)^3} dx \Rightarrow \int \frac{dx}{x+1} - 2\int \frac{dx}{(x+1)^2} + \int \frac{dx}{(x+1)^3}$

$\Rightarrow \ln(x+1) - 2\left[\frac{-1}{x+1}\right] + \left[\frac{-1}{2(x+1)^2}\right] + c$

$\Rightarrow \ln(x+1) + \frac{2}{x+1} - \frac{1}{2(x+1)^2} + c$

30. $\int \frac{dx}{x^3 + x}$

$\Rightarrow \int \frac{dx}{x(x^2+1)} \Rightarrow \int \frac{A}{x} dx + \int \frac{Bx + C}{x^2+1} \rightarrow (A)$

$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

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$$\Rightarrow \frac{1}{x(x^2+1)} = \frac{A(x^2+1)+x(Bx+C)}{x(x^2+1)}$$

$$\Rightarrow 1 = A(x^2+1) + x(Bx+C)$$

For 'A': For 'B',

$x = 0$

$\rightarrow 1 = A(x^2+1)$

$\boxed{A = 1}$

$\Rightarrow 1 = Ax^2 + A + Bx^2 + Cx$

$0 = A + B$

$0 = 1 + B$

$\boxed{B = -1}$

$\boxed{0 = C}$

(A) $\Rightarrow \int \frac{1}{x(x^2+1)} \Rightarrow \int \frac{dx}{x} + \int \frac{(-x)}{x^2+1}\,dx$

$\Rightarrow \int \frac{dx}{x} - \int \frac{x}{x^2+1}\,dx$

$\rightarrow \int \frac{dx}{x} - \frac{1}{2}\int \frac{2x}{x^2+1}\,dx$

$\rightarrow \ln|x| - \frac{1}{2}\ln|x^2+1|$

$\rightarrow \ln|x| - \ln|x^2+1|^{\frac{1}{2}} + C$

$\rightarrow \ln\left(\frac{x}{\sqrt{x^2+1}}\right) + C$

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15. $\int \frac{dx}{1+\sin x+\cos x} \rightarrow (i)$

$\Rightarrow \quad u = \tan\left(\frac{x}{2}\right) \rightarrow (A)$

$x = 2\tan^{-1}u \rightarrow (B)$

$\frac{dx}{du} = \frac{2}{1+u^2}$

$\boxed{dx = \frac{2}{1+u^2}du} \rightarrow (C)$

$\sin x = \frac{2u}{1+u^2} \rightarrow (D)$

$\cos x = \frac{1-u^2}{1+u^2} \rightarrow (E)$

Put (C), (D) and (E) in (i)

$(i) \Rightarrow \int \left[\frac{1}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}}\right] \cdot \left[\frac{2}{1+u^2}du\right]$

$\rightarrow \int 1 \div \left[\frac{1+\cancel{u^2}+2u+1-\cancel{u^2}}{1+u^2}\right] \cdot \left[\frac{2\,du}{1+u^2}\right]$

$\rightarrow \int \left(1 * \frac{1+u^2}{2+2u}\right) \cdot \frac{2\,du}{1+u^2}$

$\rightarrow \int \frac{2(1+\cancel{u^2})}{(2+2u)} \cdot \frac{du}{\cancel{1+u^2}}$

$\rightarrow \int \frac{2\,du}{2u+2}$

$\rightarrow \int \frac{\cancel{2}\,du}{\cancel{2}(u+1)}$

$\rightarrow \int \frac{du}{u+1}$

$\rightarrow \ln|u+1| + c$

$\rightarrow \ln|\tan(x/2)+1| + c$

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67. $\int \frac{d\theta}{1-\cos\theta}$

$\Rightarrow \int \frac{d\theta}{1-\cos\theta}$

$\left\{ 1-\cos x \Rightarrow 2\sin^2\frac{x}{2} \right\}$

$\Rightarrow \int \frac{d\theta}{2\sin^2(\theta/2)}$

$\Rightarrow \frac{1}{2}\int \csc^2\left(\frac{\theta}{2}\right)\left(\frac{\cancel{2}}{2}\right)d\theta$

$\Rightarrow \int \csc^2\left(\frac{\theta}{2}\right)\left(\frac{1}{2}\right)d\theta$

$\Rightarrow -\cot\left(\frac{\theta}{2}\right)+c$

68. $\int \frac{dx}{4\sin x-3\cos x}$

$u = \tan\left(\frac{x}{2}\right) \rightarrow (A)$

$x = 2\tan^{-1}u \rightarrow (B)$

$\frac{dx}{du} = \frac{2}{1+u^2}$

$dx = \frac{2}{1+u^2}\,du \rightarrow (C)$

$\sin x = \frac{2u}{1+u^2}$

$\cos x = \frac{1-u^2}{1+u^2}$

$\therefore \int \frac{1}{4\left(\frac{2u}{1+u^2}\right)-3\left(\frac{1-u^2}{1+u^2}\right)} \cdot \left[\frac{2}{1+u^2}du\right]$

$\Rightarrow \int \frac{1}{\frac{8u}{1+u^2}-\left[\frac{3-3u^2}{1+u^2}\right]} \cdot \frac{2\,du}{1+u^2}$

$\Rightarrow \int \frac{1}{\frac{8u-(3-3u^2)}{1+u^2}} \cdot \frac{2u}{1+u^2}du$

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$$\Rightarrow \int \frac{1+u^2}{8u-3+3u^2} \cdot \frac{2}{1+u^2}\, du$$

$$\Rightarrow \int \frac{2}{3u^2+8u-3}\, du$$

$$I.T. = \frac{(b)^2}{4 \times F.T} = \frac{64}{12}$$

$$\left[\frac{8}{3}\right]^2 \div [4]$$

$$\frac{64}{3} \times \frac{1}{4}$$

$$\Rightarrow \int \frac{2\, du}{3u^2+9u-u-3}$$

$$\Rightarrow 2\int \frac{du}{3u(u+3)-1(u+3)}$$

$$\Rightarrow 2\int \frac{du}{(3u-1)(u+3)} \Rightarrow \int \frac{A}{3u-1}\, du + \int \frac{B\, du}{u+3}$$

$$\Rightarrow \frac{1}{(3u-1)(u+3)} = \frac{A}{3u-1} + \frac{B}{u+3}$$

$$\Rightarrow \frac{1}{\cancel{(3u-1)(u+3)}} = \frac{A(u+3)+B(3u-1)}{\cancel{(3u-1)(u+3)}}$$

$$\Rightarrow 1 = A(u+3) + B(3u-1)$$

$u = \frac{1}{3}$

$\Rightarrow 1 = A\left(3+\frac{1}{3}\right)$

$\Rightarrow 1 = \frac{10A}{3}$

$\frac{3}{10} = A$

$u = \frac{1}{3} - 3$

$\Rightarrow 1 = B(-10)$

$B = -\frac{1}{10}$

$$\Rightarrow 2\left[\frac{3}{10}\int \frac{3}{3(3u-1)}\, du - \frac{1}{10}\int \frac{du}{u+3}\right]$$

$$\Rightarrow 2\left[\frac{1}{10}\ln|3u-1| - \frac{1}{10}\ln|u+3|\right]$$

$$\Rightarrow \frac{1}{5}\ln|3u-1| - \frac{1}{5}\ln|u+3|$$

$$\Rightarrow \ln\left|\frac{3\tan(x/2)-1}{\tan(x/2)+3}\right|^{1/5}$$

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$$\int \frac{dx}{\sin x + \tan x}$$

$u = \tan\left(\frac{x}{2}\right)$

$x = 2\tan^{-1} u$

$dx = \frac{2}{1+u^2}\, du$

$\sin x = \frac{2u}{1+u^2}$

$\tan x = \frac{\sin x}{\cos x} = \frac{2u}{1+u^2} \div \left[\frac{1-u^2}{1+u^2}\right] = \frac{2u}{1-u^2}$

$$\Rightarrow \int \frac{1}{\frac{2u}{1+u^2} + \frac{2u}{1-u^2}} \cdot \left[\frac{2\,du}{1+u^2}\right]$$

$$\Rightarrow \int 1 \div \left[\frac{2u(1-u^2) + 2u(1+u^2)}{(1+u^2)(1-u^2)}\right] \cdot \frac{2\,du}{1+u^2}$$

$$\Rightarrow \int \frac{\cancel{(1+u^2)}(1-u^2)}{2u - \cancel{2u^3} + \cancel{2u^3} + 2u} \cdot \frac{2\,du}{\cancel{1+u^2}}$$

$$\Rightarrow \int \frac{\overset{1}{\cancel{2}}(1-u^2)\,du}{{}_2\cancel{4}u}$$

$$\Rightarrow \frac{1}{2}\int \frac{1-u^2}{u}\, du$$

$$\Rightarrow \frac{1}{2}\int \left(\frac{1}{u} - \frac{u^{\cancel{2}}}{\cancel{u}}\right) du$$

$$\Rightarrow \frac{1}{2}\int u^{-1}\, du - \frac{1}{2}\int u\, du$$

$$\Rightarrow \frac{1}{2}\ln|u| - \frac{1}{2}\left[\frac{u^2}{2}\right] + c$$

$$\Rightarrow \ln|\tan(x/2)|^{2} - \frac{1}{4}\tan\left(\frac{x}{2}\right) + c$$

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Ex# 7.8

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3. $\int_0^{+\infty} e^{-2x}\,dx$

$\Rightarrow \int_0^{+\infty} e^{-2x}\,dx \Rightarrow \lim_{b\to+\infty} \int_0^b e^{-2x}\,dx$

$\Rightarrow -\frac{1}{2} e^{-2x} \Big|_0^b$

$\Rightarrow \lim_{b\to+\infty} \left[-\frac{1}{2} e^{-2x}\right]_0^b$

$\Rightarrow -\frac{1}{2e^{2\times\infty}} + \frac{1}{2} e^0$

$\Rightarrow \frac{1}{2}$ convergent

5. $\int_3^{+\infty} \frac{2}{x^2-1}\,dx$

$\Rightarrow \cancel{\int_3^{+\infty} \frac{2}{-(}}$

$\Rightarrow \int_3^{+\infty} \frac{2}{x^2-1}\,dx \Rightarrow \lim_{b\to+\infty} \int_3^b \frac{2}{x^2-1}\,dx \Rightarrow$

$\Rightarrow \lim_{b\to\infty} \int_3^b \frac{-2}{1-x^2}\,dx$

$\Rightarrow \lim_{b\to+\infty} \left[-2\coth^{-1}x \;dx\right]_3^b$

$\Rightarrow \lim_{b\to+\infty} \left[-2\coth^{-1}(b) + 2\coth^{-1}(3)\right] \Rightarrow 2\coth^{-1}3$ convergent

7. $\int_e^{+\infty} \frac{1}{x\ln^3 x}\,dx$

$\Rightarrow \int_e^{+\infty} (\ln^{-3} x)\frac{1}{x}\,dx \Rightarrow \lim_{b\to+\infty} \int_e^{+\infty} (\ln x)^{-3}\left(\frac{1}{x}\right)dx$

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$$\Rightarrow \lim_{b\to+\infty}\left[\frac{(\ln x)^{-2}}{-2}\right]_e^b$$

$$\Rightarrow \lim_{b\to+\infty}\left[\frac{-1}{2\ln^2 x}\right]_e^b$$

$$\Rightarrow \lim_{b\to+\infty}\left[\frac{-1}{2\ln^2 b}+\frac{1}{2(\ln e)^2}\right]$$

$$\Rightarrow \frac{-1}{2\ln\infty}+\frac{1}{2} \Rightarrow \frac{1}{2} \text{ convergent}$$

9. $\displaystyle\int_{-\infty}^{0}\frac{dx}{(2x-1)^3}$

$$\Rightarrow \int_{-\infty}^{0}\frac{dx}{(2x-1)^3} \Rightarrow \lim_{b\to-\infty}\int_b^0 (2x-1)^{-3}\left(\frac{2}{2}\right)dx$$

$$\Rightarrow \frac{1}{2}\lim_{b\to-\infty}\int_b^0 (2x-1)^{-3}(2)\,dx$$

$$\Rightarrow \frac{1}{2}\lim_{b\to-\infty}\left[\frac{(2x-1)^{-2}}{-2}\right]_b^0$$

$$\Rightarrow \frac{1}{2}\lim_{b\to-\infty}\left[-\frac{1}{2(2x-1)^2}\right]_b^0$$

$$\Rightarrow \frac{1}{2}\left[\frac{-1}{2(-1)^2}+\frac{1}{2(-\infty-1)^2}\right] \Rightarrow -\frac{1}{4} \text{ convergent}$$

11. $\displaystyle\int_{-\infty}^{0} e^{3x}\,dx$

$$\Rightarrow \int_{-\infty}^{0} e^{3x}\,dx \Rightarrow \lim_{b\to-\infty}\int_b^0 e^{3x}\left(\frac{3}{3}\right)dx$$

$$\Rightarrow \lim_{b\to-\infty}\frac{1}{3}\int_b^0 e^{3x}(3)\,dx$$

$$\Rightarrow \frac{1}{3}\lim_{b\to-\infty} e^{3x}\Big|_b^0$$

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$$\Rightarrow \frac{1}{3}\left[e^0 - \frac{1}{e^{-3(-\infty)}}\right]$$

$$\Rightarrow \frac{1}{3} \text{ convergent}$$

13. $\int_{-\infty}^{+\infty} x \, dx$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{+\infty} f(x)\,dx$$

$$\Rightarrow \int_{-\infty}^{0} x\,dx + \int_{0}^{+\infty} x\,dx$$

$$\Rightarrow \lim_{b\to-\infty} \int_{b}^{0} x\,dx + \lim_{b\to+\infty} \int_{0}^{b} x\,dx$$

$$\Rightarrow \lim_{b\to-\infty} \frac{x^2}{2}\Big|_{b}^{0} + \lim_{b\to+\infty} \frac{x^2}{2}\Big|_{0}^{b}$$

$$\Rightarrow \left[-0 + \frac{\infty}{2}\right] + \left[0 - \frac{\infty}{2}\right]$$

$$\Rightarrow +\infty \text{ divergent}$$

15. $\int_{-\infty}^{+\infty} \frac{x}{(x^2+3)^2}\,dx$

$$\int_{-\infty}^{+\infty} \frac{x}{(x^2+3)^2}\,dx \Rightarrow \int_{-\infty}^{0} (x^2+3)^{-2}\,dx + \int_{0}^{+\infty} (x^2+3)^{-2}(x)\,dx$$

$$\Rightarrow \lim_{b\to-\infty} \int_{b}^{0} (x^2+3)^{-2}\left(\frac{2x}{2}\right)dx + \int_{0}^{b} (x^2+3)^{-2}\left(\frac{2x}{2}\right)dx$$

$$\Rightarrow \frac{1}{2}\lim_{b\to-\infty} \int_{b}^{0} (x^2+3)^{-2}(2x)\,dx + \frac{1}{2}\int_{0}^{b} (x^2+3)^{-2}(2x)\,dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{(x^2+3)^{-1}}{-1}\right]_{b}^{0} + \frac{1}{2}\left[\frac{(x^2+3)^{-1}}{-1}\right]_{0}^{b}$$

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$\Rightarrow \frac{1}{2}\left[\frac{-1}{(x^2+3)^2}\right]_b^0 + \frac{1}{2}\left[\frac{-1}{(x^2+3)^2}\right]_0^b$

$\Rightarrow \frac{1}{2}\left[-\frac{1}{9}+0\right] + \frac{1}{2}\left[0+\frac{1}{9}\right]$

$\Rightarrow -\frac{1}{18}+\frac{1}{18} \Rightarrow 0$ convergent

17. $\int_0^4 \frac{dx}{(x-4)^2}$

$\Rightarrow \lim_{x\to 4^-} \int_0^4 \frac{dx}{(x-4)^2}$

$\Rightarrow \lim_{x\to 4^-} \left[\frac{-1}{x-4}\right]_0^4$

$\Rightarrow \frac{-1}{4-4} + \frac{1}{-4}$

$\Rightarrow -\infty$ divergent

19. $\int_0^{\pi/2} \tan x\, dx$

$\Rightarrow \lim_{x\to \frac{\pi}{2}^-} \int_0^{\pi/2} \tan x\, dx$

$\Rightarrow \ln|\sec x| \Big|_0^{\pi/2}$

$\Rightarrow \ln \sec\frac{\pi}{2} - \ln\sec 0$

$\Rightarrow -\ln\cos\frac{\pi}{2} \Rightarrow +\infty$ divergent

21. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\Rightarrow \lim_{x\to 1^-} \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\Rightarrow \lim_{x\to 1^-} \left[\sin^{-1}x\right]_0^b$

$\Rightarrow \sin^{-1} 1 \Rightarrow \frac{\pi}{2}$ convergent

23. $\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\, dx$

$\Rightarrow \lim_{x\to \frac{\pi}{3}^+} \int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\, dx$

$\Rightarrow \lim_{x\to \frac{\pi}{3}^+} \int_{\pi/3}^{\pi/2} (1-2\cos x)^{-\frac{1}{2}}\left(\frac{2\sin x}{2}\right) dx$

$\Rightarrow \frac{1}{2}\lim_{x\to \frac{\pi}{3}^+} \int_{\pi/3}^{\pi/2} (1-2\cos x)^{-\frac{1}{2}}(2\sin x)\, dx$

$\Rightarrow \frac{1}{\cancel{2}}\lim_{x\to \frac{\pi}{3}^+} \left[\cancel{2}\sqrt{1-2\cos x}\right]_{\pi/3}^{\pi/2}$

$\Rightarrow \frac{1}{2}\left[\sqrt{1-2\cos\frac{\pi}{2}}\right] - \left[\sqrt{1-2\cos\frac{2\pi}{3}}\right]$

$\Rightarrow 1$ Convergent

25. $\int_0^3 \frac{dx}{x-2}$

$\Rightarrow \lim_{x\to 2^-} \int_0^2 \frac{dx}{x-2}$

$\Rightarrow \ln|x-2| \Big|_0^2$

$\Rightarrow \ln 0 - \ln(-2) \Rightarrow -\infty$

divergent

27. ~~26.~~ $\int_{-1}^{8} x^{-\frac{1}{3}}\, dx$

$\Rightarrow \int_{-1}^{8} \frac{1}{x^{1/3}}\, dx$

$\Rightarrow \int_{-1}^{0} \frac{dx}{x^{1/3}} + \int_0^8 \frac{dx}{x^{1/3}}$

$\Rightarrow \lim_{b\to 0^-} \int_{-1}^{b} \frac{dx}{x^{1/3}} + \lim_{a\to 0^+} \int_a^8 \frac{dx}{x^{1/3}}$

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$$\Rightarrow \lim_{b\to 0^-}\left[\frac{3}{2}x^{\frac{2}{3}}\right]_{-1}^{b} + \lim_{a\to 0^+}\left[\frac{3}{2}x^{\frac{2}{3}}\right]_{a}^{8}$$

$$\Rightarrow \frac{3}{2}(0) - \frac{3}{2}(-1)^{\frac{2}{3}} + \left(\frac{3}{2}(8)^{\frac{2}{3}} - \frac{3}{2}(0)\right)$$

$$\Rightarrow 0 - \frac{3}{2} + 6 \Rightarrow \frac{9}{2} \text{ Convergent}$$

29. $$\int_0^{+\infty} \frac{dx}{x^2}$$

$$\Rightarrow \int_0^{b} \frac{dx}{x^2} + \int_b^{+\infty} \frac{dx}{x^2}$$

$$\Rightarrow \left[-\frac{1}{x}\right]_0^{b} + \left[-\frac{1}{x}\right]_b^{+\infty}$$

$$\Rightarrow \left[-\frac{1}{b} + \frac{1}{0}\right] + \left[-\frac{1}{\infty} + \frac{1}{b}\right] = +\infty \text{ Divergent}$$

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