EXERCISE 1.5 [1-6, 11-22, 29, 30, 35, 36]

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(1) - (4) On which intervals is f continous?

(1) (a) [1,3] = Not continous.

(b) (1,3) = Not continous

(c) [1,2] = Not continous

(d) (1,2) = Continous

(e) [2,3] = Continous

(f) (2,3) = Continous.

(2) (a) [1,3] = Not continous

(b) (1,3) = Not Continous

(c) [1,2] = Not continous

(d) (1,2) = Continous

(e) [2,3] = Not continous

(f) (2,3) = Continous

(3) (a) [1,3] = Not continous

(b) (1,3) = Continous

(c) [1,2] = Not continous

(d) (1,2) = Continous

(e) [2,3] = Not continous

(f) (2,3) = Continous.

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(4) (a) $[1,3]$ = Not continous

(b) $(1,3)$ = Continous

(c) $[1,2]$ = ~~Not~~ continous.

(d) $(1,2]$ = Continous

(e) $[2,3]$ = Not continous

(f) $(2,3)$ = Continous.

(5) $f(x) = \begin{cases} 1, & x \neq 4 \\ -1, & x = 4 \end{cases}$ $g(x) \begin{cases} 4x-10, & x \neq 4 \\ -6, & x = 4. \end{cases}$

Is given function continous at $x=4$.

(a) $f(x)$: $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = \lim_{x \to 4}$

$1 = 1 \neq -1$ (Not continous).

(b) $g(x)$: $\lim_{x \to 4^-} g(x) = \lim_{x \to 4^+} g(x) = \lim_{x \to 4}$.

$4(4) - 10 = 6 \neq -6 = -6$. (Not cont)

(c) $-g(x)$: $\lim_{x \to 4^-} -g(x) = \lim_{x \to 4^+} -g(x) = \lim_{x \to 4} -g(x)$

$-6 \neq 6 = 6$ (Not continous).

(d) $|f(x)|$: $\lim_{x \to 4^-} |f(x)| = \lim_{x \to 4^+} |f(x)| = \lim_{x \to 4} |f(x)|$

$1 = 1 = 1$ (Continous).

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(e) $f(x)\,g(x)$: $\lim_{x \to 4^-} f(x)g(x) = \lim_{x \to 4^+} f(x)g(x) = \lim_{x \to 4} f(x)g(x)$

$4(4) - 10 = 6 \quad = \quad 6 \quad = 6$ (Continous)

(f) $g(f(x))$: $\lim_{x \to 4^-} g(f(x)) = \lim_{x \to 4^+} g(f(x)) = \lim_{x \to 4} g(f(x))$

$4-10 = -6 \quad = \quad -6 \quad =$ DNE (Not cont)

(g) $g(x) - 6f(x) \Rightarrow (4x-10) - 6(1)$

$\Rightarrow 4x - 10 - 6$

$\Rightarrow 4x - 16.$ for $x \to 4^{\pm}$

$g(x) - 6f(x) \Rightarrow -6 - 6(-1)$

$\Rightarrow -6 + 6$

$\Rightarrow 0$ for $x \to 4.$

$4(4) - 16 \quad = \quad 4(4) - 16 \quad = \quad 0$

$0 \quad = \quad 0 \quad = 0$ (Continous).

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(6) $f(x) = \begin{cases} 1, & 0 \le x \\ 0, & x < 0 \end{cases}$ $\quad g(x) = \begin{cases} 0, & 0 \le x \\ 1, & x < 0. \end{cases}$

Is function continous at $x = 0$.

(a) $f(x) = \lim_{x \to 0^+} = \lim_{x \to 0^-} = \lim_{x \to 0}$

$1 \neq 0 = 1$. (Not continous).

(b) $g(x) = 0 \neq 1 = 0$ (Not continous).

(c) $-g(x) = -0 \neq -1 = -0$ (Not continous).

(d) $|f(x)| = 1 \neq 0 = 1$ (Not continous).

(e) $f(x)g(x) = 0 = 0 = 0$ (Continous).

(f) $g(f(x)) = 0 = 0 = 0$ (Continous).

(g) $g(x) - 6f(x) = (0^{\pm})$ (+) $\cancel{0} - \cancel{6}(1) = -6$

(−) $1 - 6(0) = 1$

(g) $f(x) + g(x) =$ (+) $1 + 0 = 1$

(−) $0 + 1 = 1$

$1 + 0 = 1$ (Continous).

(11) - (22) Find x if any at which f is not catinous.

(11) $f(x) = 5x^4 - 3x + 7$

Cantinous for all x values.

(12) $f(x) = \sqrt[3]{x-8}$

~~Not~~ catinous for all $x < 8$ values.

(13) $f(x) = \dfrac{x+2}{x^2+4}$

Cantinous for all x values.

(14) $f(x) = \dfrac{x+2}{x^2-4}$.

Not cantinous for $x = 2$ and $x = -2$.

(15) $f(x) = \dfrac{x}{2x^2+x}$

$2x^2 + x \neq 0$. ~~$x \neq$~~ $x = -\frac{1}{2}$ $x = 0$.

~~x~~ Not cantinous for $x = -\frac{1}{2}$ and $x = 0$.

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(16) $f(x) = \frac{2x+1}{4x^2+4x+5}$.

$4x^2+4x+5 \neq 0$.

Cantinous for all real x values.

(17) $f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$

Not cantinous for $x=0$, $x=1$, $x=-1$.

(18) $f(x) = \frac{5}{x} + \frac{2x}{x+4}$

Not cantinous for $x=0$, $x=-4$.

(19) $f(x) = \frac{x^2+6x+9}{|x|+3}$.

Cantinous for all real x values.

(20) $f(x) = \left| 4 - \frac{8}{x^4+x} \right|$

Not cantinous for $x=0$, $x=-1$.

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(21) $f(x) = \begin{cases} 2x+3, & x \leq 4 \\ 7 + \frac{16}{x}, & x > 4. \end{cases}$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$$

$$11 = 7 + \frac{16}{4} = 11 = 11$$

This function is cantinous everywhere

(22) $f(x) = \begin{cases} \frac{3}{x-1}, & x \neq 1 \\ 3, & x = 1. \end{cases}$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \cancel{\lim} f(1)$$

$$\frac{3}{1-1} = -\infty = \frac{3}{1-1} = +\infty = 3 \quad \text{DNE}$$

This function is not cantinous at $x = 1$.

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(29) -(30) Find k, which makes function continous.

(29) (a) $f(x) = \begin{cases} 7x-2 & x \leq 1 \\ kx^2 & x > 1 \end{cases}$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1).$$

$$7(1) - 2 = k(1)^2 = k = 7(1) - 2 = 5.$$

$$5 = k = 5.$$

If $k = 5$, then $f(x)$ is catinous everywhere.

(b) $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + k, & x > 2. \end{cases}$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

$$k(2)^2 = 2(2) + k = 2 + k. = k(2)^2 = 4k.$$

$$4k = 4 + k. = 4k.$$

$$4k = 4 + k$$

$$3k = 4$$

$$k = \frac{3}{4}.$$

If $k = \frac{3}{4}$, then $f(x)$ is continous everywhere.

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(30) (a) $f(x) = \begin{cases} 9 - x^2 & , x \geq -3 \\ \frac{k}{x^2} & , x < -3. \end{cases}$

$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x) = f(-3)$

$\frac{k}{(-3)^2} = 0. = 9 - (-3)^2 = 0.$

$\frac{K}{9} = 0 = 0.$

If $K = 0$, then $f(x)$ is continous everywhere.

(b) $f(x) = \begin{cases} 9 - x^2 & x \geq 0 \\ k/x^2 & x < 0. \end{cases}$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = \lim_{x} f(0)$

$\frac{K}{0^2} = 9 = 9 - 0^2 = 9.$

If $K \neq 0$, then $f(x)$ is contin{o}us
K is not continous anywhere.

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(35)-(36) Find x at which f is not continous, and determine if it is a removable discontinuity.

(35) (a) $f(x) = \frac{|x|}{x}$

→ f is not continous at $x = 0$.

→ Non removable hole discontinuity

(b) $f(x) = \frac{x^2+3x}{x+3}$

$(-3)^2 + 3(-3)$
$\cancel{9} - 9 = 0$
$\frac{0}{0}$.

→ f is not continous at $x = -3$

→ $\frac{x^2+3x}{x+3} = \frac{\cancel{(x+3)}\,x}{\cancel{x+3}} = -3$.

→ Removable discontinuity.

(c) $f(x) = \frac{x-2}{|x|-2}$

→ f is not continous at $x = 2$ and $x = -2$

$\frac{x-2}{x-2} \neq \frac{x-2}{-x-2}$

→ Non removable discontinuity.

(36) (a) $f(x) = \frac{x^2-4}{x^3-8}$.

→ f is not continous at $x = 2$

$\frac{x^2-4}{x^3-8} = \frac{0}{0}$.

$\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)} = \frac{2}{6} = \frac{1}{3}$.

→ Removable discontinuity.

(b) $f(x) = \begin{cases} 2x-3, & x \leq 2 \\ x^2, & x > 2. \end{cases}$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$1 = 2^2 = 4 = 2(2) - 3 = 1$

→ f is not continous at $x=2$.

→ Non removable discontinuity (hole).

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(c) $f(x) = \begin{cases} 3x^2+5 , & x \neq 1 \\ 6 , & x = 1 . \end{cases}$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$8 = 3(1)+5 = 8 \neq 6 .$

→ f is not continous at $x=1$

→ Removable discontinuity .