EXERCISE 1.2 [1-32]

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(1) Given that

$\lim_{x \to a} f(x) = 2$ $\qquad \lim_{x \to a} g(x) = -4$ $\qquad \lim_{x \to a} h(x) = 0.$

find the limits.

(a) $\lim_{x \to a} [f(x) + 2g(x)]$

$[2 + 2(-4)]$

$2 - 8 = -6$

(c) $\lim_{x \to a} [f(x)\, g(x)]$

$[(2)(-4)]$

$= -8$

(b) $\lim_{x \to a} [h(x) - 3g(x) + 1]$

$[0 - 3(-4) + 1]$

$12 + 1 = 13$

(d) $\lim_{x \to a} \dfrac{2}{g(x)}$

$\dfrac{\cancel{2}}{-4/2} = -\dfrac{1}{2}$

(e) $\lim_{x \to a} \sqrt[3]{6 + f(x)}$

$\sqrt[3]{6 + 2} =$

$= \sqrt[3]{8} = 2.$

(d) $\lim_{x \to a} [g(x)]^2$

$(-4)^2$

$= 16.$

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(2) Find Limits using graphs f and g. If non existing, explain why.

(a) $\lim_{x \to 2} [f(x) + g(x)]$

$\lim_{x \to 2} f(x) + \lim_{x \to 2} g(x)$

$0 + 0 = 0.$

(b) $\lim_{x \to 0} [f(x) + g(x)]$

$\lim_{x \to 0} f(x) + \lim_{x \to 0} g(x)$

Limit DNE as at f it doesn't exist while at g it does.

(c) $\lim_{x \to 0^+} [f(x) + g(x)]$

$\lim_{x \to 0^+} f(x) + \lim_{x \to 0^+} g(x)$

$-2 + 2 = 0.$

(d) $\lim_{x \to 0^-} [f(x) + g(x)]$

$\lim_{x \to 0^-} f(x) + \lim_{x \to 0^-} g(x)$

$1 + 2 = 3$

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(e) $\lim_{x \to 2} \dfrac{f(x)}{1+g(x)}$

$\dfrac{0}{1+0} = \dfrac{0}{1} = 0.$

(f) $\lim_{x \to 2} \dfrac{1+g(x)}{f(x)}$

$\dfrac{1+0}{0} = \dfrac{1}{0} =$ DNE as denominator is 0 while numerator is not.

(g) $\lim_{x \to 0^+} \sqrt{f(x)}$

$\sqrt{-2}$ = DNE as it is not defined for real numbers.

(3) - (30) Find the limits.

(3) $\lim_{x \to 2} x(x-1)(x+1)$

$2(2-1)(2+1)$

$2 \times 1 \times 3$

$= 6$

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(4) $\lim_{x \to 3} x^3 - 3x^2 + 9x$

$3^3 - 3(3)^2 + 9(3)$

$\cancel{27} - 27 + \cancel{27}$

$= 27$

(5) $\lim_{x \to 3} \frac{x^2 - 2x}{x+1}$

$\frac{3^2 - 2(3)}{3+1}$

$\frac{3}{4}$

(6) $\lim_{x \to 0} \frac{6x - 9}{x^3 - 12x + 3}$

$\frac{6(0) - 9}{0 - 0 + 3} = \frac{-\cancel{9}^{3}}{\cancel{3}_{1}} = -3$

(7) $\lim_{x \to 1^+} \frac{x^4 - 1}{x - 1} \to 0.$

Simplify:

$(x^4 - 1)(x-1)^{-1}$

$(x^4 - 1)(x^{-1} - 1)$

$x^3 - x^4 - x^{-1} + 1$

X

(7) $\lim_{x \to 1^+} \frac{x^4 - 1}{x - 1}$ $\frac{0}{0}$.

$\lim_{x \to 1^+} \frac{\cancel{(x-1)}(x^3 + x^2 + x + 1)}{\cancel{x-1}}$

$\lim_{x \to 1^+} 1^3 + 1^2 + 1 + 1 = 4.$

(8) $\lim_{t \to -2} \frac{t^3 + 8}{t + 2}$ $\frac{0}{\to 0}$.

$\frac{\cancel{(t+2)}(t^2 - 2x + 4)}{\cancel{t+2}}$

$\lim_{t \to -2} ((-2)^2 - 2(-2) + 4)$

$4 + 4 + 4 = 12$

(9) $\lim_{x \to -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4}$ $\frac{0}{\to 0}$.

$\frac{\cancel{(x+1)}(x+5)}{(x-4)\cancel{(x+1)}}$

$\lim_{x \to -1} \frac{(-1+5)}{(-1-4)} = -\frac{4}{5}$

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(10) $\lim\limits_{x \to 2} \dfrac{x^2 - 4x + 4}{x^2 + x - 6} \begin{matrix} \to 0 \\ \to 0. \end{matrix}$ $\qquad \dfrac{x^2 - 2x - 2x + 4}{x^2 - 4x + 4.}$

$\dfrac{(x-2)\cancel{(x-2)}}{\cancel{(x-2)}(x+3)}$

$\lim\limits_{x \to 2} \dfrac{2-2}{2+3} = \dfrac{0}{5} = 0$

(11) $\lim\limits_{x \to -1} \dfrac{2x^2 + x - 1}{x+1} \begin{matrix} 0 \\ \to 0. \end{matrix}$

$\dfrac{(x - \frac{1}{2})\cancel{(x+1)}}{\cancel{x+1}}$

$\lim\limits_{x \to -1} -1 - \frac{1}{3} = -\frac{3}{2}.$

(12) $\lim\limits_{x \to 1} \dfrac{3x^2 - x - 2}{2x^2 + x - 3} \begin{matrix} 0 \\ \to 0. \end{matrix}$

$\dfrac{\cancel{(x-1)}\cancel{(x+\frac{2}{3})}}{\cancel{(x-1)}\cancel{(x+\frac{3}{2})}.} = 1$

(13) $\lim_{t \to 2} \dfrac{t^3 + 3t^2 - 12t + 4}{t^3 - 4t} \to 0 \quad \dfrac{0}{0}$

$\dfrac{t^2 + 5t - 2}{t^2 + 2t}$

$\lim_{t \to 2} \dfrac{2^2 + 5(2) - 2}{2^2 + 2(2)} = \dfrac{3}{2}$

(14) $\lim_{x \to 1} \dfrac{t^3 + t^2 - 5t + 3}{t^3 - 3t + 2} = \dfrac{0}{0}$

$\dfrac{(x^2+3)\cancel{(x-1)}\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}$

$\lim_{x \to 1} \dfrac{(1+3)\cancel{(1-1)}}{1+2} = \dfrac{4}{3} = \dfrac{4}{3}$

(15) $\lim_{x \to 3^+} \dfrac{x}{x-3} = \dfrac{3}{0} = +\infty$

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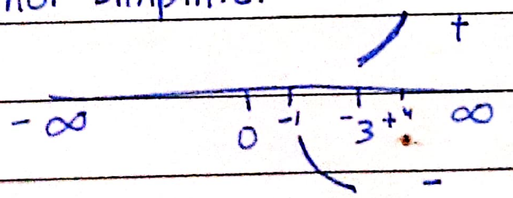
Cannot simplifie.



Hence

(16) $\lim_{x \to 3^-} \dfrac{x}{x-3} = \dfrac{3}{0} = -\infty$

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(17) $\lim_{x \to 3} \frac{x}{x-3}$

$\frac{3}{3-3} = \frac{3}{0}$

Does not exist as $\lim_{x \to 3^+} \neq \lim_{x \to 3^-}$

(18) $\lim_{x \to 2^+} \frac{x}{x^2-4}$

$\frac{2}{4-4} = \frac{2}{0} = +\infty$

(19) $\lim_{x \to 2^-} \frac{x}{x^2-4} =$

$\frac{2}{4-4} = \frac{2}{0} = -\infty$

(20) $\lim_{x \to 2} \frac{x}{x^2-4}$

Limit Donot exist at $x \to 2$ as $\text{limit}_{x \to 2^+} \neq \text{limit}_{x \to 2^-}$

(21) $\displaystyle\lim_{y \to 6^+} \frac{y+6}{y^2-36} = \frac{12}{0} = +\infty$

(22) $\displaystyle\lim_{y \to 6^-} \frac{y+6}{y^2-36} = \frac{12}{0} = -\infty$

(23) $\displaystyle\lim_{y \to 6} \frac{y+6}{y^2-36} =$ DNE as $\displaystyle\lim_{y \to 6^+} f(x) \neq \lim_{y \to 6^-} f(x)$.

(24) $\displaystyle\lim_{x \to 4^+} \frac{3-x}{x^2-2x-8} = \frac{3-4}{16-8-8} = \frac{-1}{0} = -\infty$

(25) $\displaystyle\lim_{x \to 4^-} \frac{3-x}{x^2-2x-8} = \cancel{3}\ \frac{-1}{0} = +\infty$

(26) $\displaystyle\lim_{x \to 4} \frac{3-x}{x^2-2x-8} =$ DNE as $\displaystyle\lim_{x \to 4^-} \neq \lim_{x \to 4^+}$

(27) $\displaystyle\lim_{x \to 2^+} \frac{1}{|2-x|} = \frac{1}{0} = +\infty$

(28) $\displaystyle\lim_{x \to 3^-} \frac{1}{|x-3|} = \frac{1}{0} = +\infty$

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(29) $\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} = \frac{0}{0} = \neq / \infty$

$\sqrt{x}+3 = \sqrt{9}+3 = 3+3.$

$= 6.$

(30) $\lim_{y \to 4} \frac{4-y}{2-\sqrt{y}}$

$2+\sqrt{y} = 2+\sqrt{4} + 2+2$

$= 4.$

(31) Let

$$f(x) = \begin{cases} x-1 & , x \leq 3 \\ 3x-7 & , x > 3. \end{cases}$$

Find:

(a) $\lim_{x \to 3^-} f(x) = 3-1 = 2.$

(b) $\lim_{x \to 3^+} f(x) = 3(3)-7 = 2.$

(c) $\lim_{x \to 3} f(x) = 3-1 = 2.$

(32) Let

$$g(t) = \begin{cases} t-2, & t<0 \\ t^2, & 0 \le t \le 2 \\ 2t, & t>2. \end{cases}$$

Find:

(a) $\lim_{t \to 0} g(t) = t^2 = 0^2 = 0.$

(b) $\lim_{t \to 1} g(t) = t^2 = 1^2 = 1$

(c) $\lim_{t \to 2} g(t) = t^2 = 2^2 = 4.$