EXERCISE 2.3 [1-24]

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(1) - (8) Find dy/dx.

(1) $y = 4x^7$

$\frac{dy}{dx} = 28x^6$

(2) $y = -3x^{12}$

$\frac{dy}{dx} = -36x^{11}$

(3) $y = 3x^8 + 2x + 1$

$\frac{dy}{dx} = 24x^7 + 2$

(4) $y = \frac{1}{2}(x^4 + 7)$

$\frac{dy}{dx} = \frac{1}{2} \cdot \left[\frac{d}{dx}(x^4 + 7)\right]$

$= \frac{1}{2}(4x^3)$

$= 2x^3$

(5) $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

(6) $y = \sqrt{2}\,x + (1/\sqrt{2})$

$$\frac{dy}{dx} = \sqrt{2} + 0$$

$$= \sqrt{2}$$

(7) $y = -\frac{1}{3}(x^7 + 2x - 9)$

$$\frac{dy}{dx} = -\frac{1}{3}\frac{d}{dx}(x^7 + 2x - 9)$$

$$= -\frac{1}{3}\left\{7x^6 + 2 - 0\right\}$$

$$= -\frac{7}{3}x^6 - \frac{2}{3}$$

(8) $y = \frac{x^2 + 1}{5}$

$$\frac{dy}{dx} = \frac{x^2}{5} + \frac{1}{5}$$

$$= \frac{2x}{5} + 0$$

$$= \frac{2}{5}x$$

(11) $f(x) = -3x^{-8} + 2\sqrt{x}$

$f'(x) = 24x^{-9} + 2x^{\frac{1}{2}}$

$= 24x^{-9} + x^{-\frac{1}{2}}$

$= \frac{24}{x^9} + \frac{1}{\sqrt{x}}$

(12) $f(x) = 7x^{-6} - 5\sqrt{x}$

$f'(x) = -42x^{-7} - 5x^{\frac{1}{2}}$

$= -42x^{-7} - \frac{5}{2}x^{-\frac{1}{2}}$

$= \frac{-42}{x^7} - \frac{5}{2\sqrt{x}}$

(13) $f(x) = x^e + \frac{1}{x^{\sqrt{10}}}$

$f'(x) = ex^{e-1} - (x^{\sqrt{10}})^{-1}$

$= ex^{e-1} - \sqrt{10}\,x^{-\sqrt{10}-1}$ → $ex^{e-1} - x^{-\sqrt{10}}$ ↵

(14) $f(x) = \sqrt[3]{\frac{8}{x}}$

$f'(x) = \frac{8^{\frac{1}{3}}}{x^{\frac{1}{3}}} = 2x^{-\frac{1}{3}}$

$= \frac{-2}{3}x^{-\frac{4}{3}} = \frac{-2}{3.x^{\frac{4}{3}}}$

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(15) $f(x) = (3x^2+1)^2$

$f'(x) = 2(3x^2+1)^1 \times (3x^2+1)'$

$= 2(3x^2+1) \cdot 6x$

$= 12x(3x^2+1)$

$= 36x^3 + 12x$

(16) $f(x) = ax^3 + bx^2 + cx + d$ (a, b, c, d constant).

$f'(x) = 3ax^2 + 2bx + c + 0$

$= 3ax^2 + 2bx + c.$

(17) - (18) Find $y'(1)$.

(17) $y = 5x^2 - 3x + 1$

$y' = 10x - 3$

$y'(1) = 10(1) - 3$

$y = 7$

(18) $y = \frac{x^{\frac{3}{2}} + 2}{x} = x^{-1}(x^{\frac{3}{2}} + 2) = x^{\frac{3}{2}-1} + 2x^{-1}$

$= x^{\frac{1}{2}} + 2x^{-1}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2}$

$= \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$

$y'(1) = \frac{1}{2\sqrt{1}} - \frac{2}{(1)^2} = \frac{1}{2} - 2 = -\frac{3}{2}$

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(19) - (20) Find dx/dt.

(19) $x = t^2 - t$

$$\frac{dx}{dt} = 2t - 1$$

(20) $x = \frac{t^2+1}{3t} = (t^2+1)3t^{-1} = 3t^{2+(-1)} + 3t^{-1}$

$= (3)t^{2} + (3)t^{-1}$

$x = 3t + 3t^{-1}$

$$\frac{dx}{dt} = (3)^{-1} - (3)t^{-2}$$

$$= \frac{1}{3} - \frac{1}{3t^2}$$

(21) - (24) Find $dy/dx\big|_{x=1}$.

(21) $y = 1 + x + x^2 + x^3 + x^4 + x^5$

$$\frac{dy}{dx} = 0 + 1 + 2x + 3x^2 + 4x^3 + 5x^4$$

$$\frac{dy}{dx}\Big|_{x=1} = 1 + 2(1) + 3(1)^2 + 4(1)^3 + 5(1)^4$$

$$= 15.$$

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(22) $y = \dfrac{1 + x + x^2 + x^3 + x^4 + x^5 + x^6}{x^3}$

$= \dfrac{1}{x^3} + \dfrac{x}{x^3} + \dfrac{x^2}{x^3} + \dfrac{x^3}{x^3} + \dfrac{x^4}{x^3} + \dfrac{x^5}{x^3} + \dfrac{x^6}{x^3}$

$= x^{-3} + x^{-2} + x^{-1} + 1 + x + x^2 + x^3$

$\dfrac{dy}{dx} = -3x^{-4} - 2x^{-3} - x^{-2} + 0 + 1 + 2x + 3x^2$

$= -3(1)^{-4} - 2(1)^{-3} - (1)^{-2} + 0 + 1 + 2(1) + 3(1)^2$

$= -3 - 2 - 1 + 0 + 1 + 2 + 3$

$= 0$

(23) $y = (1-x)(1+x)(1+x^2)(1+x^4)$

$= (1-x^2)(1+x^2)(1+x^4)$

$= (1-x^4)(1+x^4)$

$= (1-x^8)$

$\dfrac{dy}{dx} = 0 - 8x^7$

$\left.\dfrac{dy}{dx}\right|_{x=1} = -8(1)^7 = -8.$

(24) $y = x^{24} + 2x^{12} + 3x^8 + 6x^6$

$\dfrac{dy}{dx} = 24x^{23} + 24x^{11} + 24x^7 + 24x^5$

$\left.\dfrac{dy}{dx}\right|_{x=1} = 24(1)^{23} + 24(1)^{11} + 24(1)^7 + 24(1)^5$

$= 24 + 24 + 24 + 24$

$= 96$

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(41 - 42) Find d^2y/dx^2.

(41) (a) $y = 7x^3 - 5x^2 + 1$

$$\frac{dy}{dx} = 21x^2 - 10x$$

$$\frac{d^2y}{dx^2} = 42x - 10.$$

(b) $y = 12x^2 - 2x + 3$

$$\frac{dy}{dx} = 24x - 2$$

$$\frac{d^2y}{dx^2} = 24$$

(c) $y = \frac{x+1}{x} = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x} = 1 + x^{-1}$

$$\frac{dy}{dx} = -x^{-2}$$

$$\frac{d^2y}{dx^2} = 2x^{-3}$$

$$= \frac{2}{x^3}$$

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(d) $y = (5x^2 - 3)(7x^3 + x)$

$= 35x^5 + 5x^3 - 21x^3 - 3x.$

$\frac{dy}{dx} = 175x^4 + 15x^2 - 63x^2 - 3$

$= 700x^3 + 30x - 126x$

$= 700x^3 - 96x$

(42) (a) $y = 4x^7 - 5x^3 + 2x$

$\frac{dy}{dx} = 28x^6 - 15x^2 + 2$

$\frac{d^2y}{dx^2} = 168x^5 - 30x$

(b) $y = 3x + 2$

$\frac{dy}{dx} = 3$

$\frac{d^2y}{dx^2} = 0.$

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(c) $y = \frac{3x-2}{5x} = \frac{3x}{5x} - \frac{2}{5x} = \frac{3}{5} - \frac{2}{5}x^{-1}$

$\frac{dy}{dx} = 0 + \frac{2}{5}x^{-2}$

$\frac{d^2y}{dx^2} = -\frac{4}{5}x^{-3}$

$= -\frac{4}{5x^3}$

(d) $y = (x^3-5)(2x+3)$

$= 2x^4 + 3x^3 - 10x - 15$

$\frac{dy}{dx} = 8x^3 + 9x^2 - 10$

$\frac{d^2y}{dx} = 24x^2 + 18x$

(43) - (44) Find y'''.

(43) (a) $y = x^{-5} + x^5$

$y' = -5x^{-6} + 5x^4$

$y'' = 30x^{-7} + 20x^3$

$y''' = -210x^{-8} + 60x^2$

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(b) $y = \frac{1}{x} = x^{-1}$

$y' = -x^{-2}$

$y'' = 2x^{-3}$

$y''' = -6x^{-4}$

(c) $y = ax^3 + bx + c$ (a, b, c constant).

$y' = 3ax^2 + b$

$y'' = 6ax$

$y''' = 6a$

(44) (a) $y = 5x^2 - 4x + 7$

$y' = 10x - 4$

$y'' = 10$

$y''' = 0.$

(b) $y = 3x^{-2} + 4x^{-1} + x$

$y' = -6x^{-3} + 4x^{-2} + 1$

$y'' = 18x^{-4} + 8x^{-3}$

$y''' = -72x^{-5} - 24x^{-4}$

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(c) $y = ax^4 + bx^2 + c$ (a, b, c constant).

$y' = 4ax^3 + 2bx$

$y'' = 12ax^2 + 2b$

$y''' = 24ax$

(45) Find

(a) $f'''(2)$, where $f(x) = 3x^2 - 2$

$f' = 6x$

$f'' = 6$

$f''' = 0.$

(b) $\left.\frac{d^2y}{dx^2}\right|_{x=1}$, where $y = 6x^5 - 4x^2$

$\frac{dy}{dx} = 30x^4 - 8x$

$\frac{d^2y}{dx^2} = 120x^3 - 8$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 120(1)^3 - 8 = 112.$

(c) $\left.\frac{d^4}{dx^4}[x^{-3}]\right|_{x=1}$. $\frac{dy}{dx} = -3x^{-4}$ $\frac{d^2y}{dx^2} = 12x^{-5}$

$\frac{d^3y}{dy^3} = -60x^{-6}$ $\frac{d^4y}{dy^4} = 360x^{-7} = 360(1)^{-7}$

$= 360$

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46. Find

(a) $y'''(0)$, where $y = 4x^4 + 2x^3 + 3$

$y' = 16x^3 + 6x^2$

$y'' = 48x^2 + 12x$

$y''' = 96x + 12$

$y'''(0) = 96(0) + 12$

$= 12.$

(b) $\left.\frac{d^4x}{dx^4}\right|_{x=1}$, where $y = \frac{6}{x^4}$.

$y = 6x^{-4}$

$\frac{dy}{dx} = -24x^{-5}$

$\frac{d^2y}{dx^2} = 120x^{-6}$

$\frac{d^3y}{dx^3} = -720x^{-7}$

$\frac{d^4y}{dx^4} = 5040x^{-8}$

$\frac{d^4y}{dx^4} = 5040(1)^{-8}$

$= 5040$

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(47). Show that $y = x^3 + 3x + 1$ satisfies $y''' + xy'' - 2y' = 0$.

$y' = 3x^2 + 3$

$y'' = 6x$

$y''' = 6.$

$y''' + xy'' - 2y' = 0$

$6 + x(6x) - 2(3x^2 + 3)$

$6 + 6x^2 - 6x^2 - 6 = 0$

$6 - 6 + 6x^2 - 6x^2 = 0$

$0 = 0$

Hence $y = x^3 + 3x + 1$ satisfies $y''' + xy'' - 2y' = 0$.