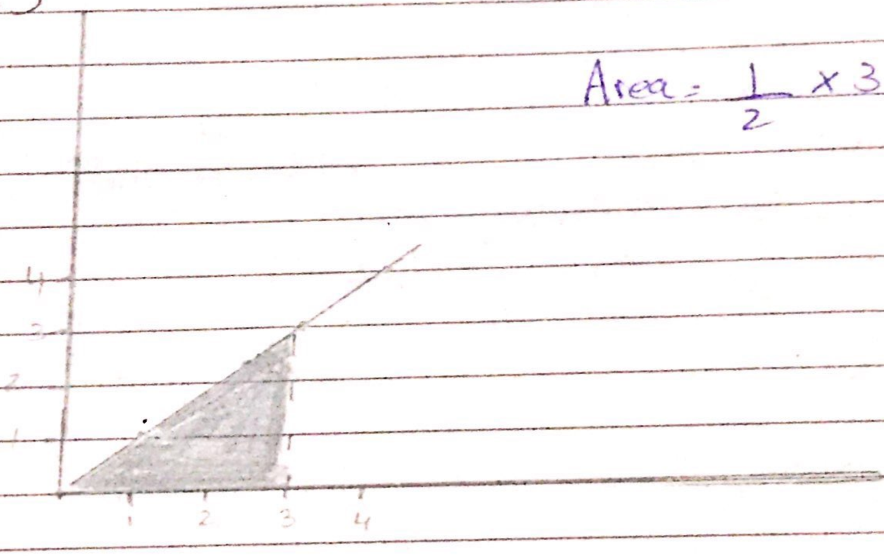
Date

Exercise 5.5 (13-24)

Q13-16 Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using an appropriate formula formula from geometry where needed

Q 13)

a) $\int_0^3 x\,dx$



Area $= \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

$\left. \frac{x^2}{2} \right|_0^3$

upper limit - Lower limit

$\frac{3^2}{2} - \frac{0^2}{2} = \frac{9}{2}$

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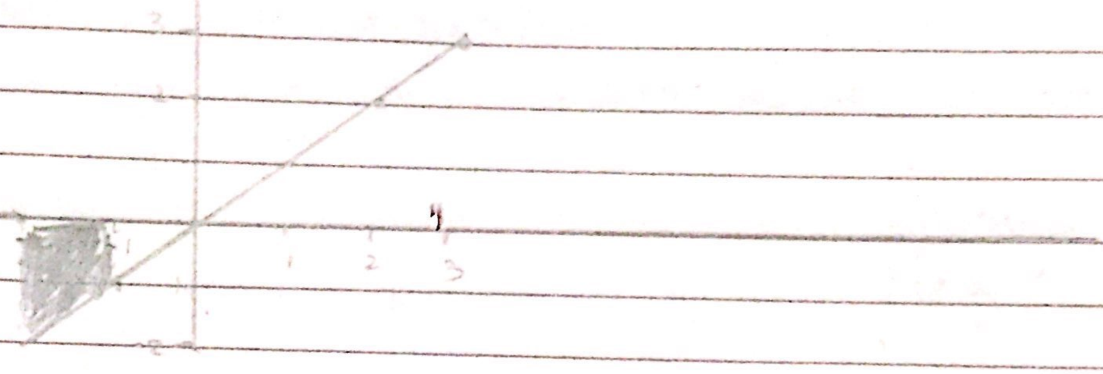
b)

$y = \frac{6}{(4.51)^2+3}$

$\int_{-2}^{-1} x \, dx$



-> By geometry

Area = area of triangle + area of square

~~$\frac{1}{2} \times 2 \times 2 = 2$ squnits~~

$\frac{1}{2} \times 1 \times 1 + 1 \times 1$

$\frac{1}{2} + 1 = \frac{3}{2}$

-> By Integral

$\int_{-2}^{-1} x \, dx$

$\left. \frac{x^2}{2} \right|_{-2}^{-1}$

$\frac{(-1)^2}{2} - \frac{(-2)^2}{2}$

$\frac{1}{2} - 2 = \frac{-3}{2}$

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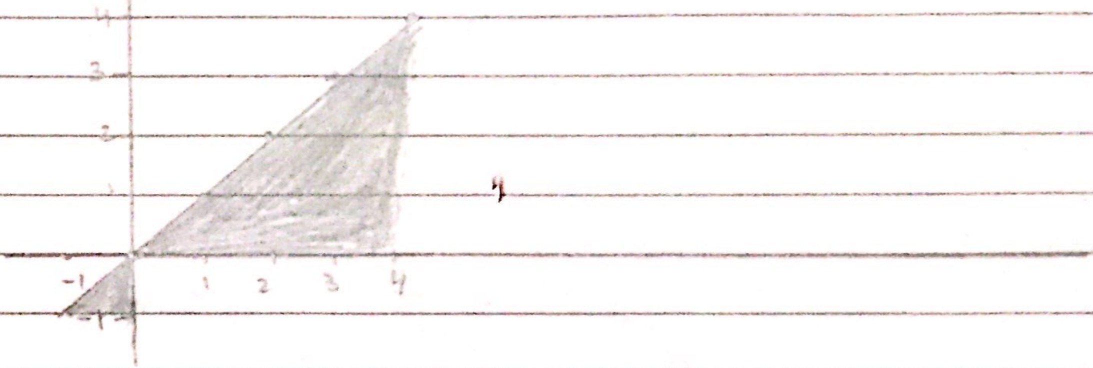
( ) $\int_{-1}^{4} x \, dx$



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-> By Geometry $\quad$ Area $= \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 4 \times 4$

$$\frac{1}{2} + \frac{16}{2} = \frac{17}{2}$$

-> By integration $\quad \int_{-1}^{4} x \, dx$

$$\left[\frac{x^2}{2}\right]_{-1}^{4} \rightarrow \left[\frac{x^2}{2}\right]_{0}^{4} + \left[\frac{x^2}{2}\right]_{-1}^{0}$$

$$\frac{\cancel{4}\,16}{2} + \frac{1}{2} = \frac{17}{2}$$

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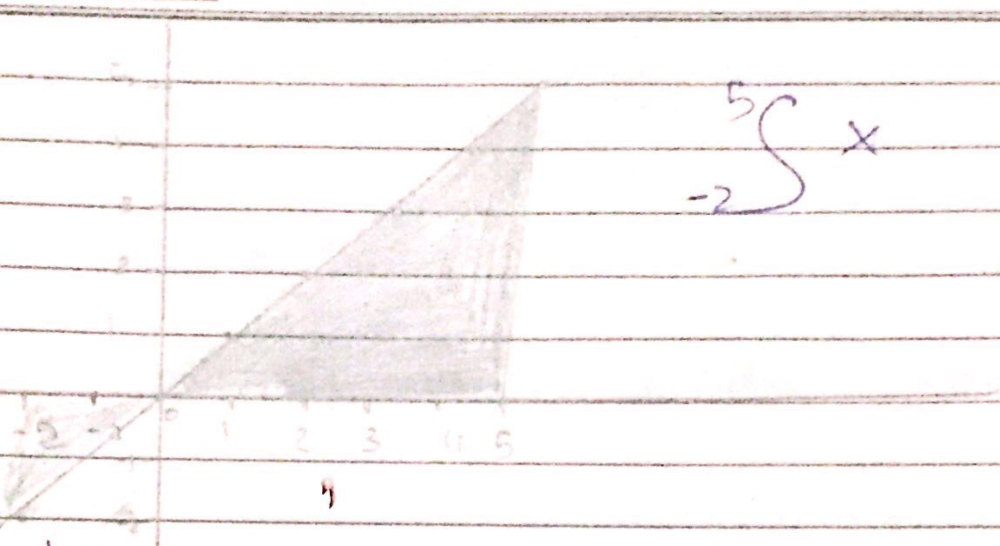
d)

$$\int_{-2}^{5} x$$



→ By geometry

$$\text{Area} = \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 5 \times 5$$

$$2 + \frac{25}{2} = \frac{29}{2}$$

→ By integration

$$\int_{-2}^{5} x$$

$$\left. \frac{x^2}{2} \right|_{-2}^{5} \rightarrow \left. \frac{x^2}{2} \right|_{-2}^{0} + \left. \frac{x^2}{2} \right|_{0}^{5}$$

$$2 + \frac{25}{2} = \frac{29}{2}$$

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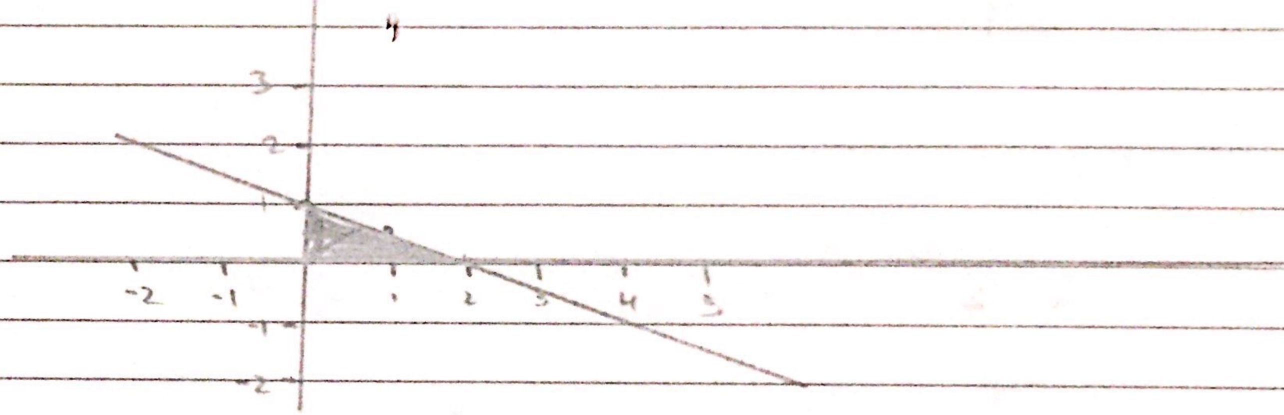
$y = 1 - \frac{1}{2}x$

Q14

$\int_0^2 1 - \frac{1}{2}x$



-> By geometry

Area $= \frac{1}{\cancel{2}} \times 1 \times \cancel{2} = 1$

-> By integration

~~Area = 1~~

$\int_0^2 1 - \frac{1}{2}x \; dx$

$x - \frac{1}{2}\frac{x^2}{2}$

$\left[x - \frac{x^2}{4} \right]_0^2$

$(2 - 1) - (0)$

1

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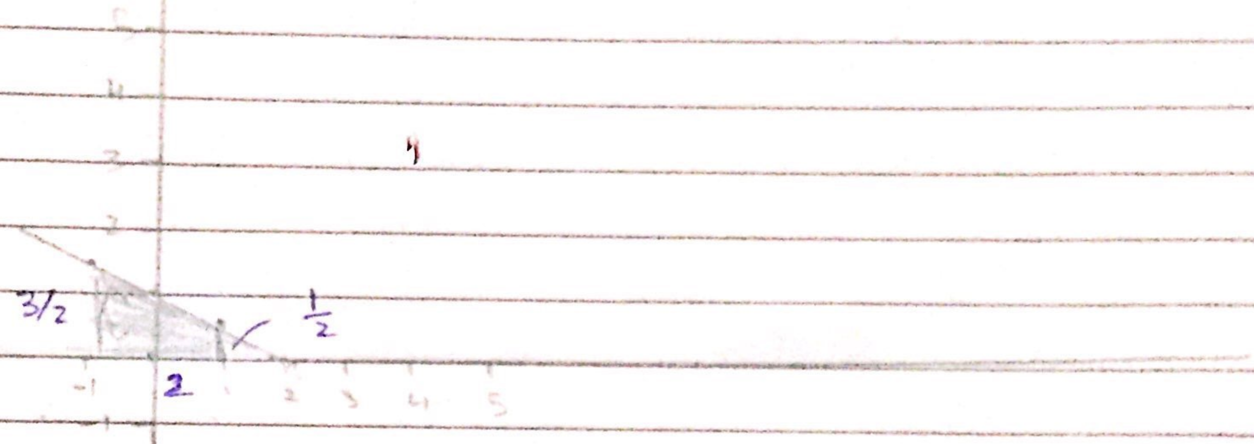
b)

$$\int_{-1}^{1} 1-\frac{1}{2}x$$



-> By geomory

$$A = \frac{1}{\cancel{2}} \times \left(\frac{3}{2}+\frac{1}{2}\right) \times \cancel{2} = 2$$

-> By integralion

$$\int_{-1}^{1} 1-\frac{1}{2}x$$

$$\left[x-\frac{x^2}{4}\right]_{-1}^{1}$$

$$\left(+1-\frac{(+1)^2}{4}\right) - \left(-1-\frac{(-1)^2}{4}\right)$$

$$\frac{3}{4} - \left(-\frac{5}{4}\right) \qquad -\frac{5}{4} \; \cancel{\frac{3}{4}}$$

$$\frac{8}{4} = 2$$

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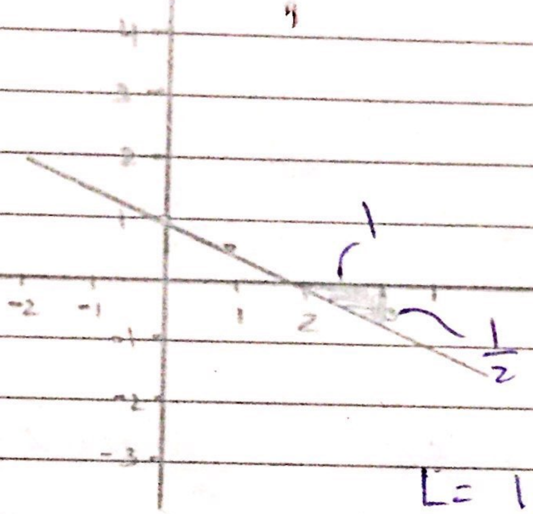
c)

$$\int_2^3 \left(1 - \frac{1}{2}x\right) dx$$



$L = 1 \quad h = \frac{1}{2}$

→ By geometry

$A = \frac{1}{2} \times L \times h = \frac{1}{2} \times \frac{1}{2} \times 1 = \frac{1}{4}$

→ By integration

$\int_2^3 1 - \frac{1}{2}x$

$\left[x - \frac{x^2}{4}\right]_2^3$

$\left(3 - \frac{9}{4}\right) - \left(2 - \frac{4}{4}\right)$

$\frac{3}{4} - 1 = -\frac{1}{4}$

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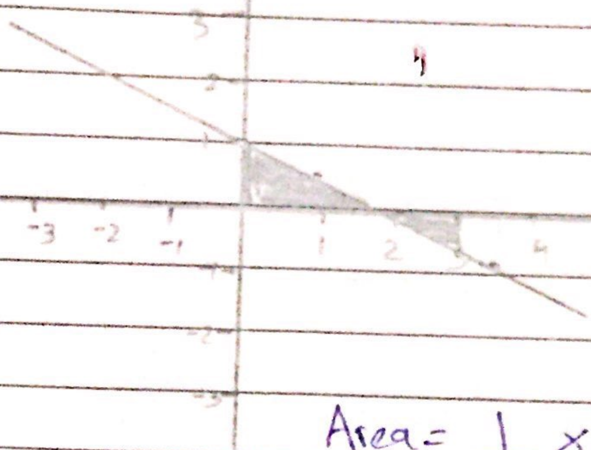
d)

$$\int_0^3 1-\frac{1}{2}x$$



Area $= \frac{1}{2}\times 1\times 2 + \frac{1}{2}\times 1\times 1$

$1+\frac{1}{2} = \frac{3}{2}$

By integration

$$\int_0^3 1-\frac{1}{2}x$$

$$\left[x-\frac{x^2}{4}\right]_0^3 = \left[x-\frac{x^2}{4}\right]_0^2 + \left[x-\frac{x^2}{4}\right]_2^3$$

$$4(2-1) + \left(\left(3-\frac{9}{4}\right)-(1)\right)$$

$$\frac{8}{4}-\frac{9}{4}$$

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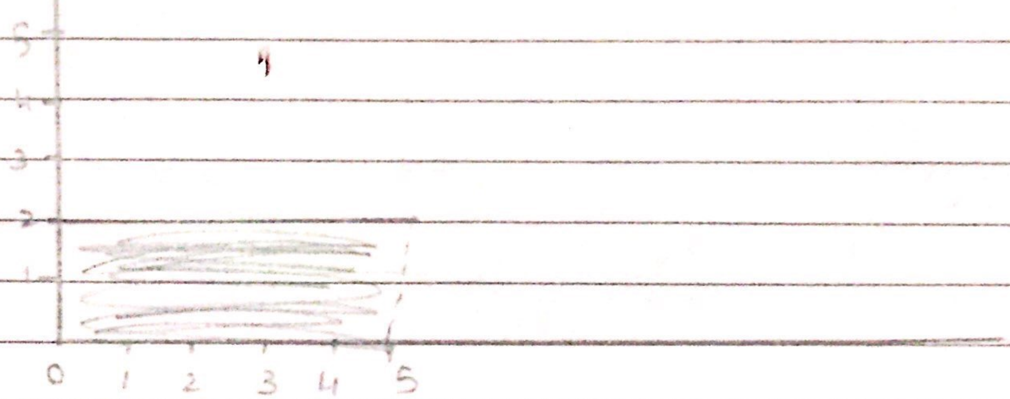
16) a)



$\int_0^5 2\,dx$

→ By geomary
A = l x B
5 x 2 = 10

→ By integration
$\left[2x\right]_0^5$
10 − 0 = 10

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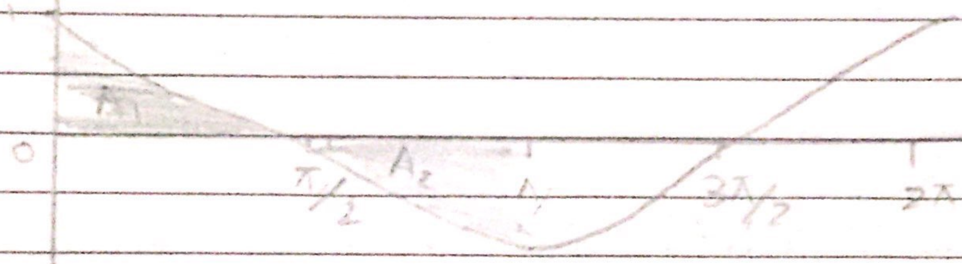
b)



$A_1 = A_2$

$\int_0^{\pi} \cos x$

$\left|\sin x\right|_0^{\pi} = \left|\sin x\right|_0^{\pi/2} + \left|\sin x\right|_{\pi/2}^{\pi}$

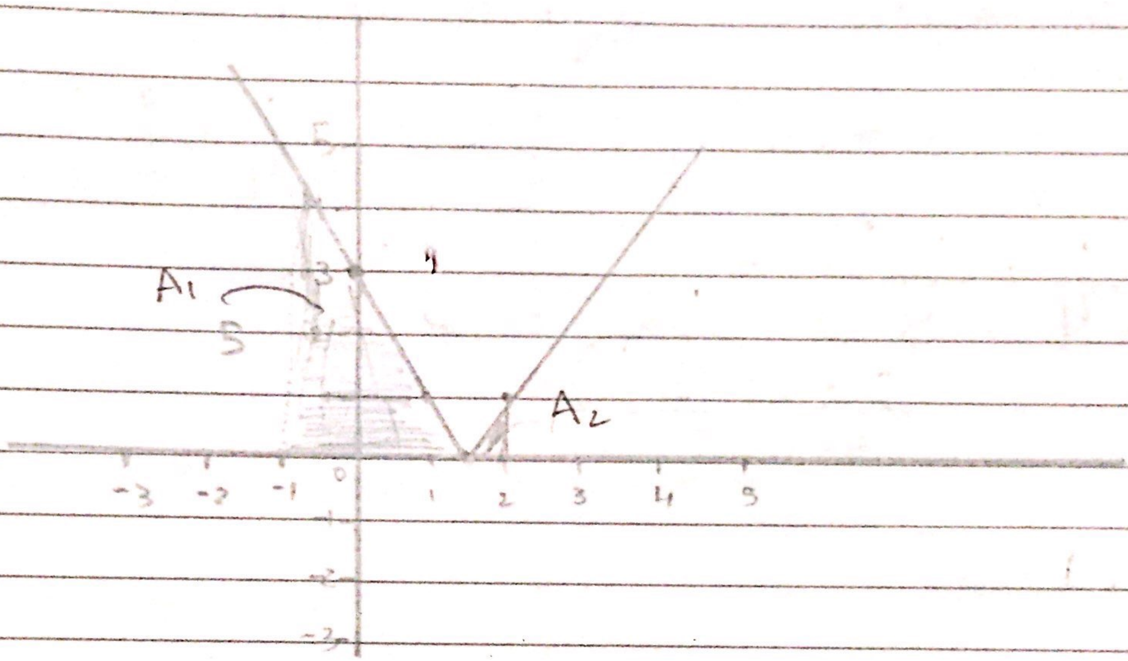
$(1) + (0 - 1)$

$1 + 1 = 2$

c)

$$\int_{-1}^{2} |2x-3|$$

$x = \frac{3}{2}$



→ By geometry

$$\text{Area} = \left(\frac{1}{2} \times 5 \times \frac{5}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times 1\right)$$

$$\frac{25}{4} + \frac{1}{4} = \frac{26}{4} = \frac{13}{2}$$

→ By integration

$$\int_{-1}^{2} \begin{cases} 2x-3 & x > \frac{3}{2} \\ 3-2x & x < \frac{3}{2} \end{cases}$$

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Area $\int_{-1}^{\frac{3}{2}} 3-2x \quad + \quad \int_{\frac{3}{2}}^{2} 2x-3$

$\left. 3x - x^2 \right|_{-1}^{\frac{3}{2}} \quad + \quad \left. x^2-3x \right|_{\frac{3}{2}}^{2}$

$\left[\left(\frac{9}{2}-\frac{9}{4}\right)-\left(-3-1\right)\right] + \left[-2-\left(\frac{9}{4}-\frac{9}{2}\right)\right]$

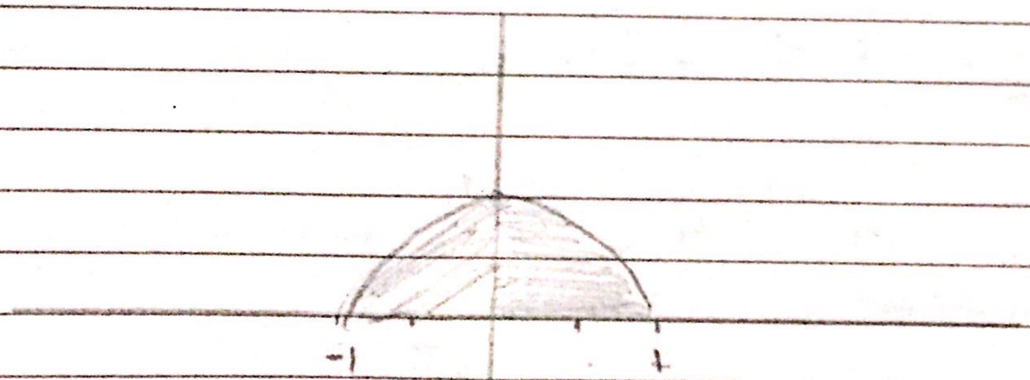
$\left[\frac{9}{4}+\frac{16}{4}\right] + \left[\frac{1}{4}\right]$

$\frac{26}{4} = \frac{13}{2}$

d) $\int_{-1}^{1} \sqrt{1-x^2}$

$y^2 + x^2 = (1)^2$

$r = 1$ equation of circle at origin



$A = \frac{\pi r^2}{2}$

$A = \frac{\pi}{2}$

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→ By integration

$$\int_{-1}^{1} \sqrt{1-x^2}\, dx$$

$a^2 - x^2$

$\sqrt{1-\sin^2\theta}$ $\qquad$ $x = a\sin\theta$

$\sqrt{\cos^2\theta}$ $\cos\theta$ $\qquad$ $x = \sin\theta$

$\int_{-1}^{1} \cos^2\theta$ $\qquad$ $dx = \cos\theta\, d\theta$

$\int \frac{\cos 2\theta + 1}{2}$ $\qquad$ N.LL $\qquad$ NUL

$\sin\theta = -1$ $\qquad$ $\sin\theta = 1$

$\frac{1}{2} \int \cos 2\theta + 1\, d\theta$ $\qquad$ $\theta = \frac{3\pi}{2}$ $\qquad$ $\theta = \sin^{-1}(1)$

$\theta = \pi/2$

$$\frac{1}{2} \left| \frac{1}{2}\left[2\sin 2\theta + \theta\, d\theta\right] \right|_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$\frac{1}{2}\left[\left(\frac{3\pi}{2}\right) - \frac{\pi}{2}\right]$$

$$\frac{1}{2}\left(\frac{2\pi}{2}\right)$$

$$\frac{\pi}{2}$$

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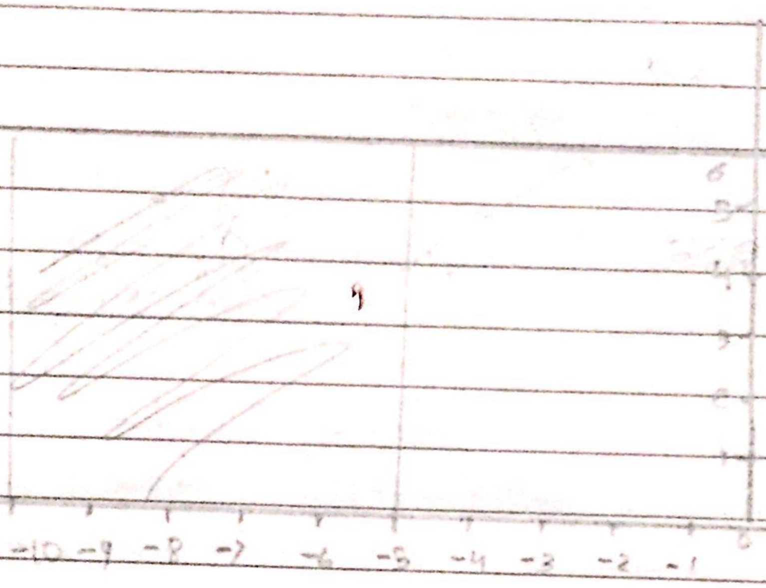
Q16) $\int_{-10}^{-5} 6\,dx$



→ By geometry

$A = L \times B$

$5 \times 6 = 30$

→ By integration

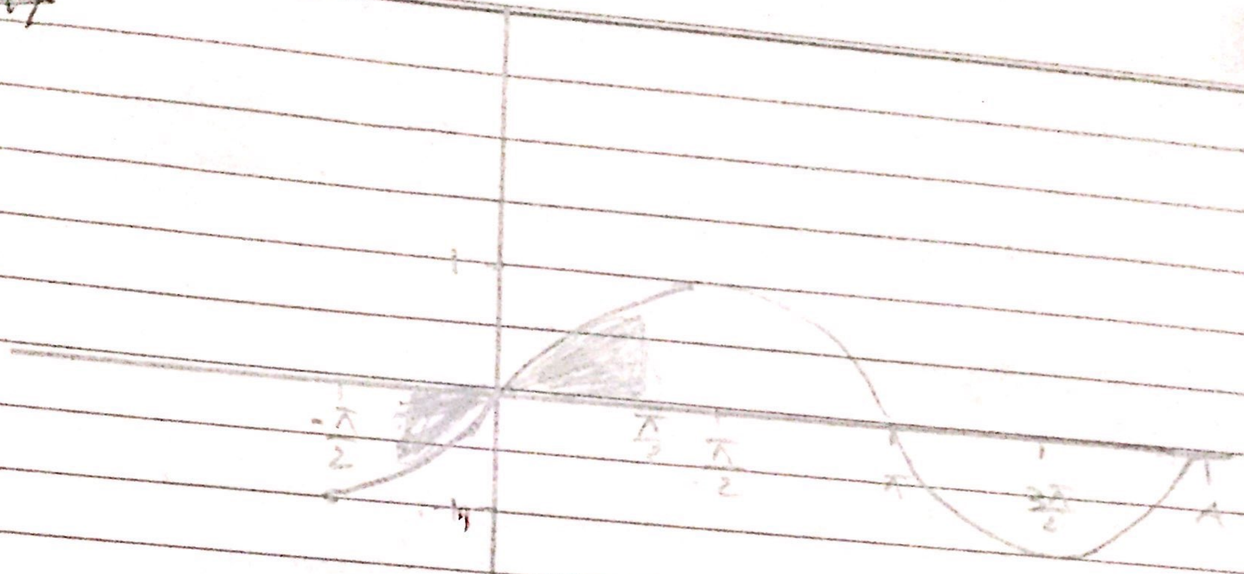
$\int_{-10}^{-5} 6\,dx$

$\left| 6x \right|_{-10}^{-5}$

$-30 - (-60) = 30$

~~Q17~~

b)



-) By Integralron

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sin x \, dx$$

$$\left. \cos x \right|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = \left. \cos x \right|_{-\frac{\pi}{3}}^{0} + \left. \cos x \right|_{0}^{\frac{\pi}{3}}$$

$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - 1\right)$$

$$\frac{1}{2} + \left(-\frac{1}{2}\right)$$ Area is Always postive

$$\frac{1}{2} + \frac{1}{2} = \frac{1}{\cancel{4}} \quad 1$$

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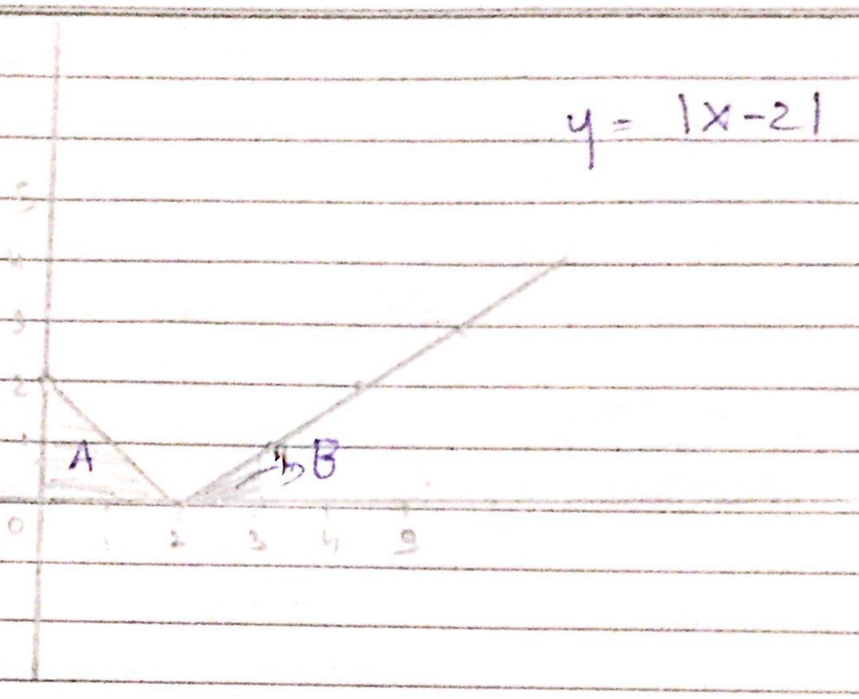
1b (c)

$y = |x-2|$



Area = A + B

$\left(\frac{1}{2} \times 2 \times 2\right) + \frac{1}{2} \times 1 \times 1$

$2 + \frac{1}{2}$

$\frac{\cancel{13}}{2} \cdot 3 \quad \frac{5}{2}$

→ By Integration

$\int_0^3 |x-2|$ $\qquad f(x) = \begin{cases} x-2 & x \geq 2 \\ 2-x & x < 2 \end{cases}$

$\int_0^3 |x-2| = \int_0^2 2-x + \int_2^3 x-2$

$\left[2x - \frac{x^2}{2}\right] + \left[\frac{x^2}{2} - 2x\right]$

$2 + \left(-\frac{3}{2} - (-2)\right)$

$2 + \frac{1}{2} = \frac{5}{2}$

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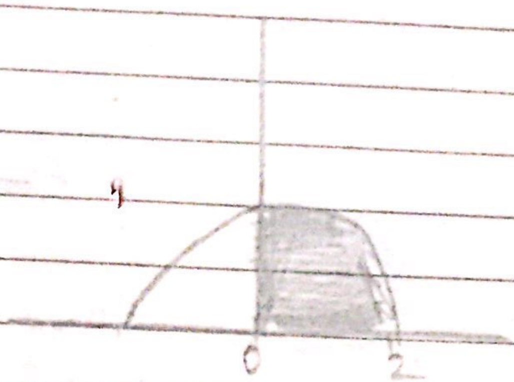
d) $\int_0^2 \sqrt{4-x^2}$

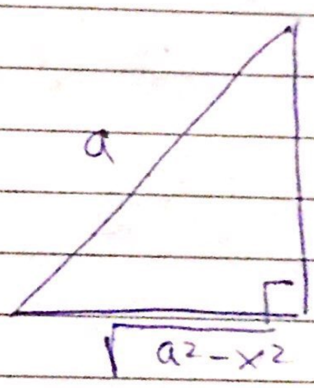
$y^2 + x^2 = 2^2$ radius = 2   Circle at origin



-> By geometry

$A = \frac{\pi r^2}{4} = \frac{\pi \times \cancel{4}}{\cancel{4}} = \pi$

-> By Integration



$\int \sqrt{(2)^2 - x^2}\, dx$

$\int \sqrt{2^2 - 4\sin^2\theta}\, dx$

$\int \sqrt{4 - 4\sin^2\theta}$

$\int \sqrt{4(1-\sin^2\theta)}$

$\int \sqrt{4}\, \cancel{\sin^2\theta} \sqrt{\cos^2\theta}$

$\int 2\cos\theta$

$2\sin\theta \Big|_0^{\frac{\pi}{2}}$

$\left(2\sin\frac{\pi}{2} - 2\sin 0\right)$

$\sin\theta = \frac{p}{h}$

$x = a\sin\theta$

$x = 2\sin\theta$

$dx = 2\cos\theta$

at $x = 2$

$\theta = \sin^{-1}(1)$

$\theta = \frac{1}{2}\pi$

at $x = 0$

$\sin\theta = 0$

$\theta = 0$

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$\int \sqrt{2^2 - x^2}\, dx$

$\cancel{x} = 2\sin\theta$

$\int \sqrt{4 - 4\sin^2\theta} \times 2\cos\theta\, d\theta$ $\quad dx = 2\cos\theta\, d\theta$

$\int 2\sqrt{1 - \sin^2\theta}\ \ 2\cos\theta\, d\theta$

$4\cos\theta \sqrt{\cos^2\theta}$,

$\int 4\cos^2\theta\, d\theta$

$24 \int 4\ \frac{\cos 2\theta + 1}{2}$

$2\int \cos 2\theta + 1\ d\theta$

$2\left[2\sin 2\theta + \theta\right]$

$\left| 4\sin 2\theta + 2\theta \right|_0^{\frac{\pi}{2}}$

$\left(4\sin\pi + \cancel{2}\frac{\pi}{2}\right) - 0$

π

$\cos 2\theta = C^2 - S^2$

$\cancel{C^2 - \cos}$

$\cos 2\theta = C^2 - (1 - C^2)$

$\cos 2\theta = 2C^2 - 1$

$C^2 = \frac{\cos 2\theta + 1}{2}$

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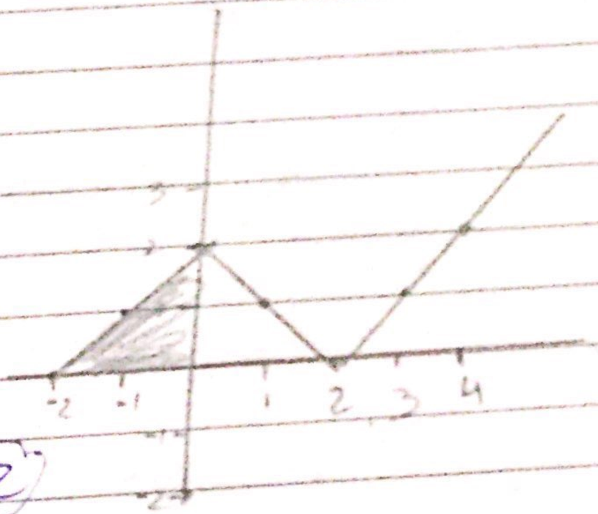
Q17) In each part, evaluate the integral, given that.

$$f(x) = \begin{cases} |x-2|, & x \geq 0 \\ x+2, & x < 0 \end{cases}$$

a) $\int_{-2}^{0} f(x)\,dx$



$\int_{0}^{-2} x+2$

$\left. \frac{x^2}{2} + 2x \right|_{0}^{-2}$

$\ominus - \left(\frac{4}{2} + -4 \right) \longrightarrow \ominus$

$\ominus - \frac{4}{2} - \frac{8}{2} = \frac{+4}{2} = +2$

Triangle of length 2 and width 2 so Area is 2

b) $\int_{-2}^{2} f(x)\,dx$

$\int_{-2}^{0} x+2 \quad + \quad \int_{0}^{2} |2-x|$

$\left. \frac{x^2}{2} + 2 \right| \quad + \quad \left. \frac{x^2}{2} - 2x \right|_{0}^{2}$

$2 \qquad \frac{4}{2} - 4$

(-2) Area is always positive

$2 + 2$

Area $= 4$

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b) $\int_{-2}^{2} f(x)\,dx$

$\int_{-2}^{0} x+2 \quad + \quad \int_{0}^{2} 2-x$

$\left. \frac{x^2}{2} + 2x \right|_{-2}^{0} \quad + \quad \left. 2x - \frac{x^2}{2} \right|_{0}^{2}$

$2 \quad + \quad 2 \quad = 4$

c) $\int_{0}^{6} f(x)\,dx$

$\int_{0}^{2} 2-x \quad + \quad \int_{2}^{6} x-2$

$\cancel{2 \quad +}$

$\left. 2x - \frac{x^2}{2} \right|_{0}^{2} \quad + \quad \left. \frac{x^2}{2} - 2x \right|_{2}^{6}$

$\cancel{(4-2) \quad + \quad (8-8) - (-2)}$

$2 \cancel{+ 2 = 4}$

$(2) + \left[(18 - 12) - (-2) \right]$

$2 + (6+2) = 10$

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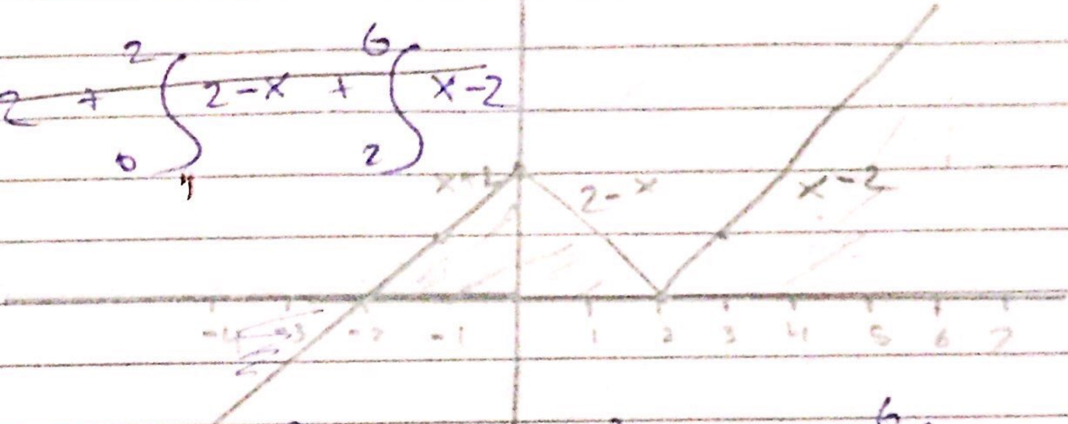
d) $\int_{-4}^{6} f(x)\, dx$

$\int_{-4}^{6} f(x) = \int_{-4}^{0} x+2 + \int_{0}^{2} 2-x + \int_{2}^{6} x-2$



$\int_{-4}^{6} f(x) = \int_{-4}^{-2} x+2 + \int_{-2}^{0} x+2 + \int_{0}^{2} 2-x + \int_{2}^{6} x-2$

$\left|\frac{x^2}{2} + 2x\right|_{-4}^{-2} + \left|\frac{x^2}{2} + 2x\right|_{-2}^{0} + \left|2x - \frac{x^2}{2}\right]_{0}^{2} + \left|\frac{x^2}{2} - 2x\right|_{2}^{6}$

$\left[(-2) - (0)\right] + \left[0 - (-2)\right] + (2) + \left[6 - (-2)\right]$

$\cancel{-2} \; \cancel{+2} + 2 + 8 = 10$

~~Area~~ triangle of L=2 & B=2 below x axis

" " above x axis

"

triangle of L=4 and B=4 above x axis

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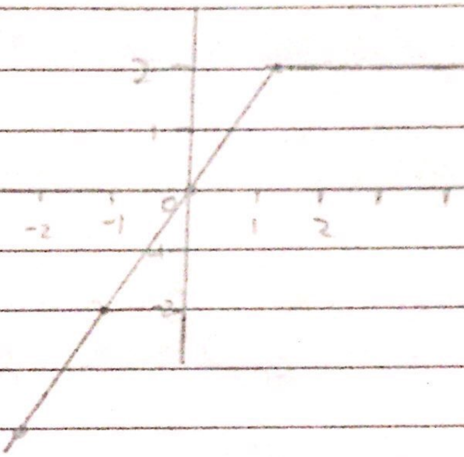
Q18) In each part evaluate the integral given that

$$f(x) = \begin{cases} 2x & x \le 1 \\ 2 & x > 1 \end{cases}$$



a) ~~f(x)~~ $\int_0^1 f(x)\,dx$

$\int_0^1 2x$

$\left.\frac{\cancel{2}x^2}{\cancel{2}}\right|_0^1$

$(1)^2 - (0)^2 = 1$

b) $\int_{-1}^1 f(x)\,dx$

$\int_{-1}^0 2x + \int_0^1 2x$

$\left. x^2 \right|_{-1}^0 + \left. x^2 \right|_0^1$

$1 + 1 = 2$

c) $\int_0^6 f(x)\,dx$

$\int_0^1 2x + \int_1^6 2$

$\left. x^2 \right|_0^1 + \left. 2x \right|_1^6$

$1 + (12 - 2)$

$1 + 10 = 11$

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c) $\int_{1}^{10} f(x)\, dx$

$\int_{1}^{10} 2\, dx$

$\left| 2x \right|_{1}^{10}$

$20 - 2 = 18$

d) $\int_{1/2}^{5} f(x)\, dx$

$\int_{1/2}^{1} 2x\, dx + \int_{1}^{5} 2\, dx$

$\left| x^2 \right|_{\frac{1}{2}}^{1} + \left| 2x \right|_{1}^{5}$

$\left(1 - \frac{1}{4}\right) + (8)$

$\frac{3}{4} + 8 = \frac{35}{4}$

Q21) find $\int_{-1}^{2} [f(x) + 2g(x)]\, dx$

if $\int_{-1}^{2} f(x)\, dx = 5$ and $\int_{-1}^{2} g(x)\, dx = -3$

$\int_{-1}^{2} [f(x) + 2g(x)]\, dx = \int_{-1}^{2} f(x)\, dx + 2\int_{-1}^{2} g(x)\, dx$

$= 5 + 2(-3) = -1$

Q22) Find $\int_{1}^{4} [3f(x) - g(x)]\, dx$ if

$\int_{1}^{4} f(x)\, dx = 2$ and $\int_{1}^{4} g(x)\, dx = 10$

Sol:=

$\int_{1}^{4} [3f(x) - g(x)] = 3\int_{1}^{4} f(x)\, dx - \int_{1}^{4} g(x)\, dx$

$3(2) - 10 = -4$

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23) Find $\int_1^5 f(x)\,dx$ if

$$\int_0^1 f(x)\,dx = -2 \quad \text{and} \quad \int_0^5 f(x)\,dx = 1$$

sol

$$\int_1^5 f(x)\,dx = \int_0^5 f(x)\,dx - \int_0^1 f(x)\,dx$$

$$1 - (-2) = 3$$

24) Find $\int_3^{-2} f(x)\,dx$ if

$$\int_{-2}^1 f(x)\,dx = 2 \quad \text{and} \quad \int_1^3 f(x)\,dx = -6$$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^1 f(x) = 2 + \int_1^3 f(x)\,dx = -6$$

$$\int_{-2}^3 \quad 2 - 6 = -4$$

$$\int_3^{-2} f(x)\,dx = -\int_{-2}^3 f(x)\,dx$$

$$\int f(x)\,dx = -(-4) = +4$$

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