Wave Problems.

QUESTION 01

$$y(x,t) = \frac{2}{(x-3t)^2+1}$$

x & y are in cm

plot the wave function at

i) $t=0$

ii) $t=1s$

iii) $t=2s$

01) At $t=0$

$$y(x,t) = \frac{2}{(x-3t)^2+1} \quad (\because t=0)$$

$$y(x,t) = y(x,0) = \frac{2}{x^2+1} \text{ cm}$$

02) At $t=1s$

$$y(x,t) = \frac{2}{(x-3)^2+1}$$

$$= \frac{2}{x^2-6t+9+1}$$

$$y(x,t) = y(x,1) = \frac{2}{x^2-6t+10} \text{ cm}$$

iii) At $t = 2s$

$$y(x, 2) = \frac{2}{(x - 3(2))^2 + 1}$$

$$= \frac{2}{(x-6)^2 + 1}$$

$$= \frac{2}{x^2 - 12x + 36 + 1}$$

$$\boxed{y(x, 2) = \frac{2}{x^2 - 12x + 37} \text{ cm}}$$

Ans.

QUESTION 02

Data:

$V = 50 m/s$

$l = 5m$

$m = 0.0600 kg$

$T = ?$

Solution :-

The speed of wave in a taut string is given by,

$$V = \sqrt{\frac{T}{\mu}} \quad (\because \mu = m/l)$$

$$V = \sqrt{\frac{T}{m/l}}$$

$$V = \sqrt{\frac{lT}{m}} \quad \text{(squaring B.S)}$$

$$V^2 = \frac{lT}{m}$$

$$T = \frac{mV^2}{l} = \frac{(0.06)(50)^2}{5m}$$

$$\boxed{T = 30\,N}$$ Ans

QUESTION 03

Data:-

$V_1 = 20 m/s$

$T_1 = 6 N$

$V_2 = 30 m/s$

$T_2 = ?$

$\mu = ?$

Solution:-

1) For μ:-

velocity is given by:

$V = \sqrt{T/\mu}$

$V_1^2 = T_1/\mu$

$\mu = T_1/V_1^2 = 6/400$

$\mu = 0.015 kg/m$

FOR T_2:-

The velocity of a string is given by,

$$V = \sqrt{\frac{T}{\mu l}}$$

The formula becomes,

$$V_2 = \sqrt{\frac{T_2}{\mu l}} \quad \text{(squaring B.S.)}$$

$$V_2^2 = \frac{T_2}{\mu l}$$

$$T_2 = V_2^2 \times \mu l$$

$$= 30^2 \times (0.015)$$

$$\boxed{T_2 = 13.5\ N}$$

QUESTION 04:-

Data:-

a) $A = y_m = 8cm = 0.08m$

$\lambda = 80\ cm = 0.8m$

$f = 3Hz$

$y(0,t) = 0$ at $t = 0$

$y(x,t) = ?$

b) $y(x,t) = ?$ at

$y(x,0) = 0$ & $x = 10cm = 0.1m$

Solution:-

Let $y(x,t) = y_m \sin(Kx + \omega t + \phi)$ be the equation of the wave since it is traveling in negative direction,

a)

$$y(x,t) = y_m \sin(Kx + \omega t + \phi) - (1)$$

→ For K:-

$$K = \frac{2\pi}{\lambda} = \frac{2 \times 3.142}{0.8}$$

$$K = 7.855 \text{ rad/m}$$

→ For ω:-

$$\omega = \frac{2\pi}{T} = 2\pi \times f = 2 \times 3.142 \times 3$$

$$\omega = 18.85 \text{ rad/s}$$

→ FOR ϕ:-

Since $y(0,t) = 0$ & $t = 0$

$$y(0,t) = y_m \sin(Kx + \omega t + \phi)$$

$\therefore y(0,t) = 0$; $t = 0$; $x = 0$

$$0 = y_m \sin(0 + 0 + \phi)$$

$$0 = \sin(\phi)$$

$$\phi = \sin^{-1}(0)$$

$\phi = 0$

now

(1) $\Rightarrow y(x,t) = y_m \sin(kx - \omega t + \phi)$

$= 0.08 \sin(7.855x - 18.85t + 0)$

$y(x,t) = 0.08 \sin(7.855x - 18.85t)\ m$

b) For, $y(x,t)$ when $y(x,0) = 0$ & $x = 0.1m$

$y(x,t) = y_m \sin(kx - \omega t + \phi)$ —(1)

$\because t = 0$; $y(x,0) = 0$ & $x = 0.1$

$0 = y_m \sin(k(0.1) - 0 + \phi)$

$0 = \sin(k(0.1) + \phi)$

$\because k = 7.855\ rad/m$

$0 = \sin(7.855 \times 0.1 + \phi)$

$0 = \sin(0.7855 + \phi)$

$0.7855 = \phi = \sin^{-1}(0)$

$\boxed{\phi = -0.7855}$

Now,

~~$y(x,t) = 0.08 \sin(7.855(0.1) + 18.85(0) - 0.7855)$~~

$\boxed{y(x,t) = 0.08 \sin(7.855(x) + 18.85t - 0.7855)}$

Ans

QUESTION NO.05

Data :-

$y = (0.25\,m)\ \sin(0.3x - 40t)$ —(1)

a) Amplitude = ?

b) Angular frequency = ?

c) Angular wave number = ?

d) Wavelength = ?

e) Wave speed = ?

f) direction of motion = ?

Solution :-

a) Amplitude :-

Consider eq (1)

$$y_m = 0.25\,m$$

b) Angular frequency :-

Consider eq (1)

$$\omega = -40\ rad/s$$

c) Angular wave number:

Consider eq (i)

$$K = 0.3 \text{ rad/m}$$

d) Wave length:

we know that $K = \frac{2\pi}{\lambda}$

$$\lambda = \frac{2\pi}{K} = \frac{2 \times 3.142}{0.3}$$

$$\lambda = 20.9 \text{ m}$$

e) Wave speed:

we know that

$$V = \omega / K = \frac{-40}{0.3}$$

$$V = -133.33 \text{ m/s}$$

f) direction of motion:

Since $\omega < 0$ & considering eq (i)

Direction = left ~~for~~ for right i.e. +ve

QUESTION 06

Data:-

~~$y = (0.51\,cm)\sin(Kx - \omega t) - (1)$~~

$y = (0.005\,m)\sin(Kx - \omega t)$

~~$K = 3.1\,rad/cm =$~~

$y = (0.51\,cm)\sin(Kx - \omega t)$

$K = 3.1\,rad/cm$

$\omega = 9.3\,rad/s$

x = displacement = ?

direction = ?

$t = 10s$

Solution:-

$y = 0.51\,cm \sin(3.1x - 9.3t)$

For x (displacement)

We know that

$x = V \times t \quad -(1)$

V_{13} given by,

$V = \omega / k = 9.3 / 3.1$

$V = 3\ cm/s$

Thus

~~$x = 3/10$~~

~~$x = 0.3cm$~~

$x = 3 \times 10$

$x = 30\ cm$

OR

$x = 0.3m$

For direction:-

Since $x > 0$ Therefore

direction = left to right i.e +ve

QUESTION 07

Data

$y = (2cm)\ Sin(Kx - \omega t)$

$y = (0.02m)\ Sin(Kx - \omega t)$ —(1)

$K = 2.11\ rad/m$

$\omega = 3.62\ rad/s$

i) amplitude = ?

ii) wavelength = ?

iii) frequency = ?

iv) speed = ?

Solution

i) For amplitude:

Consider eq (1)

$y_m = 0.02m$

2) Wavelength

we know that

$$K = \frac{2\pi}{\lambda} \; ; \quad \lambda = \frac{2\pi}{K} = \frac{2 \times 3.142}{2.11}$$

$$\lambda = 2.97 \text{ m}$$

03) Frequency :-

$$\omega = 2\pi f \; ; \quad f = \frac{\omega}{2\pi} = \frac{3.62}{2 \times 3.142}$$

$$\omega = 0.576 \text{ rad/s}$$

04) Speed :-

$$V = \frac{\omega}{K} = \frac{0.576}{2.11}$$

$$V = 0.272 \text{ m/s}$$

QUESTION 08:-

Data

$y = (0.15m) \sin(0.8x - 50t)$

$\mu = 12 g/m = 0.012 kg/m$

i) $V = ?$

ii) $\lambda_2 = ?$

iii) $f_2 = ?$

Solution:-

i) For V:-

We know that

$$V = \frac{\omega}{k} = \frac{50}{0.8}$$

$$\boxed{V = 62.5 m/s}$$

ii) For λ:-

We know that

$$K = \frac{2\pi}{\lambda} \quad ; \quad \lambda = \frac{2\pi}{K} = \frac{2 \times 3.142}{0.8}$$

$$\lambda = 7.585 \text{ m}$$

iii) For f:-

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{50}{2 \times 3.142} = 15.91 \text{ Hz}$$

$$f = 15.91 \text{ Hz}$$