Exercise 0.1

1(a) $x = -2$, $x = -2.9$, $x = 2.4$, $x = 2.9$

(b) $y \neq 3$ at any x point

(c) $y = 0$

(d) $-1.75 \leq x \leq 2.15$, $x = 3$, $x = -3$

(e) $Y_{max} = 2.8$ at $x = 2.55$

$Y_{min} = -2.15$ at $x = 1.2$

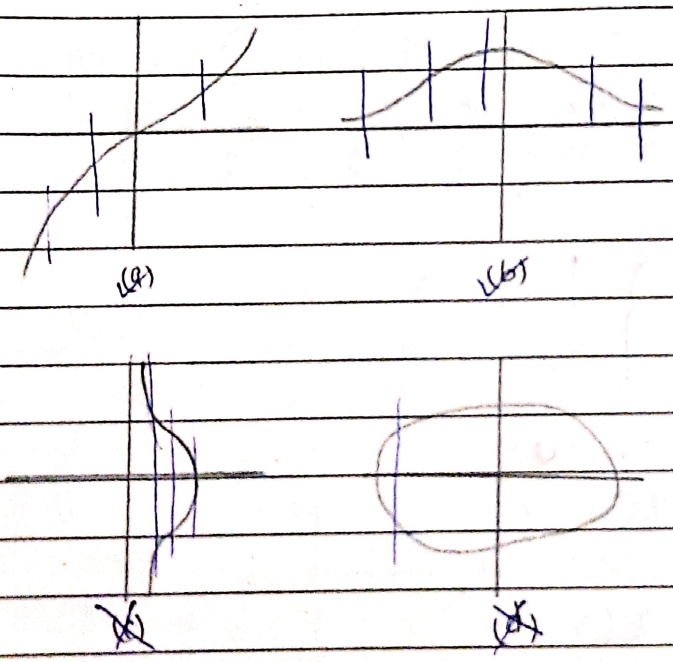
2(a) $x = -1$, $x = 4$

(b) None

(c) $y = -1$

(d) $x = 5$, $x = 3$, $x = 0$

(3)



4 (a) $f(x) = \dfrac{x^2 + x}{x+1}$, $g(x) = x$.

$x + 1 \neq 0$

$x \neq -1$.

$D = (-\infty, -1) \cup (-1, \infty)$ $D = (-\infty, \infty)$

$f(x) = g(x)$ on intersecting of their domains.

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4 (b) $f(x) = \frac{x\sqrt{x} + \sqrt{x}}{x+1}$; $g(x) = \sqrt{x}$.

$\sqrt{x} \geq 0$ $\quad x \neq -1$

$x \geq 0$

$(0, +\infty)$

$\sqrt{x} \geq 0$

$x \geq 0$

$(0, +\infty)$

Domains are same for both $f(x)$ and $g(x)$.

7. (a) $f(x) = 3x^2 - 2$

$f(0) = 3(0)^2 - 2 = -2$

$f(2) = 3(2)^2 - 2 = 10$

$f(-2) = 3(-2)^2 - 2 = 10$

$f(3) = 3(3)^2 - 2 = 25$

$f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4$

$f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$

(b) $f(x) = \begin{cases} \frac{1}{x}, & x > 3 \\ 2x, & x \leq 3 \end{cases}$

$f(0) = 2(0) = 0$

$f(2) = 2(2) = 4$

$f(-2) = 2(-2) = -4$

$f(3) = 2(3) = 6$

$f(\sqrt{2}) = 2(\sqrt{2}) = 2\sqrt{2}$

$f(3t) = \begin{cases} \frac{1}{t} ; t > 1 \\ 2t, t \leq 1 \end{cases}$

$x > 3 \quad 3t > 3$

$t > \frac{3}{3}$

$t > 1$

$= \begin{cases} \frac{1}{3t} & t > 1 \\ 2(3t) & t \leq 1 \end{cases}$

$x \leq 3 \quad 3t \leq 3$

$t \leq 1$

$= \begin{cases} \frac{1}{3t} & t > 1 \\ 6t & t \leq 1 \end{cases}$

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(8) (a) $g(x) = \frac{x+1}{x-1}$

$g(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$

$g(-1) = \frac{-1+1}{-1-1} = \frac{0}{-2} = 0.$

$g(\pi) = \frac{\pi+1}{\pi-1} = 1.934$

$g(-1.1) = \frac{-1.1+1}{-1.1-1} = 0.0476$

$g(t^2-1) = \frac{t^2-1+1}{t^2-1-1} = \frac{t^2}{t^2-2}$

(b) $g(x) = \begin{cases} \sqrt{x+1} & x \geq 1 \\ 3 & x < 1 \end{cases}$

$g(3) = \sqrt{3+1} = \sqrt{4} = 2$

$g(-1) = 3$

$g(\pi) = \sqrt{\pi+1} = 2.035$

$g(-1.1) = 3$

$g(t^2-1) = \begin{cases} \sqrt{x+1} & x \geq 1 \\ 3 & x < 1 \end{cases}$

$\begin{cases} \sqrt{x+1} & t \geq \sqrt{2} \\ 3. & t < \sqrt{2} \end{cases}$ $\quad t^2 - 1 \geq 1$, $t^2 \geq 2$, $t \geq \sqrt{2}$

$\sqrt{t^2-1+1} = \sqrt{t^2} = t.$

$\begin{cases} t, & t \geq \sqrt{2} \\ 3, & t < \sqrt{2}. \end{cases}$ $\quad t^2 - 1 < 1$, $t^2 < 2$, $t < \sqrt{2}$

9(a) $f(x) = \frac{1}{x-3}$

Domain:

$x - 3 \neq 0$.

$x \neq 3$.

$(-\infty, 3) \cup (3, +\infty)$

Range:

$x \neq 0$.

$y = \frac{1}{x-3}$ $\quad (-\infty, 0) \cup (0, +\infty)$

$x = \frac{1}{y-3}$

$y - 3 = \frac{1}{x}$

$y = \frac{1}{x} + 3$.

(b) $f(x) = \frac{x}{|x|}$

Domain:

$|x| \neq 0$

$(-\infty, 0) \cup (0, +\infty)$

Range:

$y = \frac{x}{|x|}$ $\quad y = \frac{x}{|x|}$

$x = \frac{y}{|y|}$

$\{-1, 1\}$ $\quad \frac{|y|}{y} = \frac{1}{x}$

$\frac{-y}{y} = \frac{1}{x}$ $\quad \frac{y}{y} = \frac{1}{x}$

$x = -1$ $\quad x = 1$.

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(c) $g(x) = \sqrt{x^2-3}$

Domain:

$\sqrt{x^2-3} \geq 0$

$x^2 \geq 3$

$x \geq \sqrt{3}$

$[\sqrt{3}, +\infty)$

Range:

$y = \sqrt{x^2-3}$

$y^2 = x^2-3$

$x^2 = y^2+3$

$y^2 = x^2+3$

$y = \sqrt{x^2+3}$.

$\sqrt{x^2+3} \geq 0$

$x^2+3 \geq 0$

$x^2 \geq -3$

$x \geq \sqrt{-3}$

(d) $g_1(x) = \sqrt{x^2-2x+5}$

Domain:

$\sqrt{x^2-2x+5} \geq 0$

$x^2-2x+5 \geq 0$

$(x+1+2i)(x-(1+2i)) \geq 0$

$(-\infty, +\infty)$.

Range:

$y^2 = x^2-2x+5$

$x^2 = y^2-2y+5$

$x^2-5 = y^2-2y$

$x^2-5 = y(y-2)$

$y = x^2-5$ $\quad y-2 = x^2-5$

$y = x^2-5$ $\quad y = x^2-3$

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(e) $h(x) = \dfrac{1}{1-\sin x}$

Domain :

$1 - \sin x \neq 0$

$\sin x \neq 1$

$x \neq \sin^{-1}(1)$

$x \neq \frac{\pi}{2}$

Range :

$y = \dfrac{1}{1-\sin x}$

$x = \dfrac{1}{1-\sin y}$

$1 - \sin y = \dfrac{1}{x}$

$\sin y = 1 - \dfrac{1}{x}$

$y = \sin^{-1}\left(1 - \dfrac{1}{x}\right)$

(f) $H(x) = \sqrt{\dfrac{x^2-4}{x-2}}$

Domain:

$\sqrt{\dfrac{x^2-4}{x-2}} \geq 0$

$x^2 - 4 \geq x - 2$

$x^2 - x - 2 \geq 0$

$x \geq 2 \quad x \geq 1$

$[-2, 2) \cup (2, +\infty)$

$y = \sqrt{\dfrac{x^2-4}{x-2}}$

Range :

$x^2 = \dfrac{y^2-4}{y-2}$

$y - 2 = \dfrac{y^2-4}{x^2}$

$\dfrac{y-2}{y^2-4} = \dfrac{1}{x^2}$

$\dfrac{y(1-\frac{2}{y})}{y(y-\frac{4}{y})} = \dfrac{1}{x^2}$

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10.(a) $f(x) = \sqrt{3-x}$

Domain:

$\sqrt{3-x} \geq 0$

$3-x \geq 0$

$x \leq 3$

$(-\infty, 3]$

Range:

$y = \sqrt{3-x}$

$x = \sqrt{3-y}$

$x^2 - 3 = -y$

$y = 3 - x^2$

$(-\infty, \infty)$

(b) $F(x) = \sqrt{4-x^2}$

Domain:

$\sqrt{4-x^2} \geq 0$

$4 - x^2 \geq 0$

$x^2 \leq 4$

$x \leq 2$

$[-2, 2]$

Range:

$y = \sqrt{4-x^2}$

$x^2 = 4 - y^2$

$y^2 = 4 - x^2$

$y = \sqrt{4-x^2}$

$[0, 2]$

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(c) $g(x) = 3 + \sqrt{x}$

Domain:

$3 + \sqrt{x} \geq 0$

$x \geq 0$.

$[0, +\infty)$

Range:

$y = 3 + \sqrt{x}$

$x = 3 + \sqrt{y}$

$x - 3 = \sqrt{y}$

$y = (x-3)^2$

$[3, +\infty)$

(d) $G(x) = x^3 + 2$.

Domain:

$(-\infty, +\infty)$

Range:

$y = x^3 + 2$

$x = y^3 + 2$

$y^3 = x - 2$

$y = \sqrt[3]{x-2}$

$(-\infty, +\infty)$

(e) $h(x) = 3\sin x$

D $(-\infty, \infty)$

Range:

$y = 3\sin x$

$x = 3\sin y$

$\sin y = \frac{x}{3}$

$y = \sin^{-1}\left(\frac{x}{3}\right)$

Range $[-3, 3]$

(f) $H(x) = (\sin\sqrt{x})^{-2}$

Domain:

$\sin\sqrt{x} \neq 0$

$\sqrt{x} \neq \sin^{-1}(0)$

$x = (\sin(0))^{-2}$

$(0, +$

Range:

$y = (\sin\sqrt{x})^{-2}$.

$y = \frac{1}{(\sin\sqrt{x})^2}$

$x = \frac{1}{(\sin\sqrt{y})^2}$

$(\sin\sqrt{y})^2 = \frac{1}{x}$

$\sin\sqrt{y} = \frac{1}{x^2}$

$\sqrt{y} = \left(\frac{1}{x^2}\right)\sin^{-1}$

$y = \sin^{-2}\left(\frac{1}{x^2}\right)$

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(27)-(28) Express functions in piecewise.

(27) (a) $f(x) = |x| + 3x + 1.$

$x + 3x + 1. = 4x + 1$

$-x + 3x + 1 = 2x + 1$

$$f(x) = \begin{cases} 4x+1, & x \geq 0 \\ 2x+1, & x < 0. \end{cases}$$

(b) $g(x) = |x| + |x-1|$

$x + x - 1 = 2x - 1.$

$-x - x + 1 = -2x + 1.$

$x - x + 1 = 1.$

$-x + x - 1 = -1.$

$$g(x) = \begin{cases} 2x-1, & x \geq 1 \\ 1, & 0 \leq x \leq 1 \\ 1-2x, & x < 0. \end{cases}$$

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(28) (a) $f(x) = 3 + |2x-5|$.

$3 + 2x - 5 = 2x - 2$ $\quad x = 2x-5 = \frac{5}{2}$

$3 - 2x + 5 = 8 - 2x$ $\quad -2x+5 = \frac{-5}{-}$

$$f(x) = \begin{cases} 2x-2, & x \geq \frac{5}{2} \\ 8-2x, & x \leq \frac{5}{2}. \end{cases}$$

(b) $g(x) = 3|x-2| - |x+1|$.

$3x - 6 - x - 1 = 2x - 7$ $\quad x-2=0$, $x=2$

$-3x + 6 - x - 1 = -4x + 5$ $\quad -1 \leq x < 2$

$-3x + 6 \pm x + 1 = -2x + 7$ $\quad x \geq 2$.

$$g(x) = \begin{cases} 2x-7, & x \geq 2. \\ 5-4x, & 1 \leq x < 2 \\ 7-2x., & x < -1 \end{cases}$$