EXERCISE 2.5 [1-24]

(1) - (18) Find $f'(x)$.

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

(1) $f(x) = 4\cos x + 2\sin x$

$f'(x) = 4(-\sin x) + 2(\cos x)$

$= -4\sin x + 2\cos x$

(2) $f(x) = \frac{5}{x^2} + \sin x$

$= 5x^{-2} + \sin x$

$f'(x) = -10x^{-3} + \cos x$

$= \frac{-10}{x^3} + \cos x$

(3) $f(x) = -4x^2 \cos x$

Using product rule:

$(-4x^2)(-\sin x) + (\cos x)(-8x)$

$f'(x) = 4x^2\sin x - 8x\cos x$

(4) $f(x) = 2\sin^2 x$

$f'(x) = 2.2\sin x \times \cos x \times 1.$

$= 4\sin x\cos x$

(5) $$f(x) = \frac{\overset{u}{5 - \cos x}}{\underset{v}{5 + \sin x}}$$

Using quotient rule:

$$\frac{(5+\sin x)(-(-\sin x)) - (5-\cos x)(\cos x)}{(5+\sin x)^2}$$

$$\frac{\sin x\,(5+\sin x) - [5\cos x - \cos^2 x]}{(5+\sin x)^2}$$

$$\frac{5\sin x + \sin^2 x - 5\cos x + \cos^2 x.}{(5+\sin x)^2} \qquad 1 = \sin^2 x + \cos^2 x$$

$$f'(x) = \frac{5(\sin x - \cos x) + 1}{(5+\sin x)^2}.$$

(6) $$f(x) = \frac{\overset{u}{\sin x}}{\underset{v}{x^2 + \sin x}}$$

Using QR:

$$\frac{(x^2+\sin x)(\cos x) - [(\sin x)(2x+\cos x)]}{(x^2+\sin x)^2}$$

$$\frac{x^2\cos x + \sin x\cos x - [2x\sin x + \sin x\cos x]}{(x^2+\sin x)^2}$$

$$\frac{x^2\cos x + \cancel{\sin x\cos x} - 2x\sin x - \cancel{\sin x\cos x}}{(x^2+\sin x)^2}$$

$$f'(x) = \frac{x^2\cos x - 2x\sin x}{(x^2+\sin x)^2}$$

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(7) $f(x) = \sec x - \sqrt{2}\tan x$

$f'(x) = \sec x \tan x - \sqrt{2}\sec^2 x.$

$= \sec x(\tan x - \sqrt{2}\sec x)$

(8) $f(x) = \overset{u}{(x^2+1)}\overset{v}{\sec x}$

Using product rule:

$(x^2+1)(\sec x \tan x) + (\sec x)(2x)$

$f'(x) = \sec x\left[(x^2+1)(\tan x) + 2x\right]$

(9) $f(x) = 4\operatorname{cosec} x - \cot x$ $\qquad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$f'(x) = 4(-\operatorname{cosec} x \cot x) - (-\operatorname{cosec}^2 x)$

$= -4\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x.$ $\qquad \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x.$

$= \operatorname{cosec} x(-4\cot x + \operatorname{cosec} x).$

(10) $f(x) = \cos x - x\cos x.$

Using product rule for $(\overset{u}{x}\overset{v}{\cos x})$.

$(x.-\sin x) + (\cos x)1.$

$-x\sin x + \cos x.$

$\cos x - [-x\sin x + \cos x]$

$\cancel{\cos x} + x\sin x - \cancel{\cos x}$

$f'(x) = x\sin x$

(11) $f(x) = \overset{u}{\sec x}\,\overset{v}{\tan x}$

Using product rule:

$(\sec x)(\sec^2 x) + (\tan x)(\sec x \tan x)$

$\sec^3 x + \sec x \tan^2 x.$

$f'(x) = \sec x\,(\sec^2 x + \tan^2 x).$

(12) $f(x) = \overset{u}{\operatorname{cosec} x}\,\overset{v}{\cot x}.$

Using PR:

$(\operatorname{cosec} x)(-\operatorname{cosec}^2 x) + (\cot x)(-\operatorname{cosec} x \cot x)$

$f'(x) = -\operatorname{cosec}^3 x - \cot^2 x \operatorname{cosec} x$

(13) $f(x) = \dfrac{\overset{u}{\cot x}}{\underset{v}{1+\operatorname{cosec} x}}.$

Using quotient rule:

$$\frac{(1+\operatorname{cosec} x)(-\operatorname{cosec} x \cot x) - (\cot x)(-\operatorname{cosec} x \cot x)}{(1+\operatorname{cosec} x)^2}.$$

$$\frac{-\operatorname{cosec} x \cot x - \operatorname{cosec}^2 x \cot x + \operatorname{cosec} x \cot^2 x}{(1+\operatorname{cosec} x)^2}.$$

$$f'(x) = \frac{-\operatorname{cosec} x \cot x\,[1+\operatorname{cosec} x - \cot x]}{(1+\operatorname{cosec} x)^2}$$

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(14) $f(x) = \dfrac{\sec x}{1 + \tan x}$

Using QR:

$$\frac{(1+\tan x)(\sec x \tan x) - (\sec x)(\sec^2 x)}{(1+\tan x)^2}.$$

$$\frac{\sec x \tan x + \sec x \tan^2 x - \sec^3 x}{(1+\tan x)^2}$$

$$f'(x) = \frac{\sec x\,[\tan x + \tan^2 x - \sec^2 x]}{(1+\tan x)^2}.$$

(15) $f(x) = \sin^2 x + \cos^2 x.$

$f'(x) = (2\sin x . \cos x . 1) + (2\cos x . -\sin x . 1)$

$= 2\sin x \cos x - 2\cos x \sin x$

$= 0$

(16) $f(x) = \sec^2 x - \tan^2 x$

$= (2\sec x . \sec x \tan x . 1) - (2\tan x . \sec^2 x . 1)$

$= 2\sec^2 x \tan x - 2\sec^2 \tan x$

$= 0.$

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(17) $f(x) = \dfrac{\sin x \sec x}{1 + x\tan x} \Rightarrow \dfrac{\sin x \times \frac{1}{\cos x}}{1 + x\tan x} \Rightarrow \dfrac{\overset{u}{\tan x}}{1 + x\tan x \cdot v}$

Using PR for: $x\tan x$.

$x\sec^2 x + \tan x\,(1)$.

$v' = x\sec^2 x + \tan x$.

Using QR:

$$\frac{(1 + x\tan x)(\sec^2 x) - (\tan x)(x\sec^2 x + \tan x)}{(1 + x\tan x)^2}.$$

$$\frac{\sec^2 x + \cancel{x\tan x\sec^2 x} - \cancel{x\tan x\sec^2 x} - \tan x}{(1 + x\tan x)^2}$$

$$\frac{\sec^2 x - \tan x}{(1 + x\tan x)^2} \qquad f'(x) = \frac{1}{(1 + x\tan x)^2}$$

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(18) $f(x) = \dfrac{(x^2+1)\cot x}{3 - \cos x \operatorname{cosec} x}$

$\cos x \cdot \dfrac{1}{\sin x} = \dfrac{\cos x}{\sin x} = \cot x.$

$f(x) = \dfrac{(x^2+1)\cot x}{3 - \cot x.}$

PR for $(x^2+1)\cot x.$

$(x^2+1)(-\operatorname{cosec}^2 x) + (\cot x)(2x)$

$-x^2\operatorname{cosec}^2 x - \operatorname{cosec}^2 x + 2x\cot x.$

Using QR:

$\dfrac{(3-\cot x)(-x^2\operatorname{cosec}^2 x - \operatorname{cosec}^2 x + 2x\cot x) - (x^2+1)\cot x \operatorname{cosec}^2 x}{(3-\cot x)^2.}$

$\dfrac{-3x^2\operatorname{cosec}^2 x - 3\operatorname{cosec}^2 x + 6x\cot x - x^2\cot x\operatorname{cosec}^2 x - \cot x\operatorname{cosec}^2 x.}{(3-\cot x)^2.}$

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(19-24) Find d^2y/dx^2.

(19) $y = x\cos x.$

Using PR:

$(x)(-\sin x) + (\cos x)1.$

$\frac{dy}{dx} \quad -x\sin x + \cos x$

(2) Using PR:

$-\left[(x)(\cos x) + (\sin x)(1)\right] + \cos x$

$-x\cos x - \sin x + \cos x.$

$-x\cos x - \sin x - \sin x$

$\frac{d^2y}{dx^2} = -x\cos x - 2\sin x.$

(20) $y = \underset{u}{\operatorname{cosec} x} \quad \underset{v}{}$

$\frac{dy}{dx} = -\operatorname{cosec} x \cot x.$

Using PR on $\frac{dy}{dx}$.

$-\left[(\operatorname{cosec} x)(-\operatorname{cosec}^2 x) + (\cot x)(-\operatorname{cosec} x \cot x)\right]$

$-\left[-\operatorname{cosec}^3 x - \operatorname{cosec} x \cot^2 x\right]$

$\operatorname{cosec}^3 x + \operatorname{cosec} x \cot^2 x.$

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(21) $y = x\sin x - 3\cos x$

$\frac{dy}{dx} = x\sin x - 3(-\sin x).1$

Using PR for $x\sin x$.

$x\cos x + \sin x\,(1)$

$x\cos x + \sin x$

$\frac{dy}{dx} = x\cos x + \sin x + 3\sin x.$

$= x\cos x + 4\sin x.$

Using PR for $x\cos x$:

$(x)(-\sin x) + (\cos x)(1).$

$-\sin^2 x + \cos x.$

$\frac{d^2y}{dx^2} = -\sin^2 x + \cos x + 4\sin x. \rightarrow 4\cos x.$

$= -\sin^2 x + 5\cos x$

(22) $y = x^2\cos x + 4\sin x.$

PR on $x^2\cos x$

$-x^2\sin x + \cos x . 2x.$

Let $f = -x^2\sin x + 2x\cos x. \leftarrow \frac{dy}{dx}$

$4\frac{d}{dx}(\sin x) = 4\cos x$

$4\frac{d^2}{dx^2}(\cos x) = -4\sin x$

PR on f :

$-[x^2\cos x + \sin x\, 2x] + [-2x\sin x + \cos x . 2]$

$-x^2\cos x - 2x\sin x - 2x\sin x + 2\cos x - 4\sin x$

$\frac{d^2y}{dx^2} = -x^2\cos x - 4x\sin x + 2\cos x - 4\sin x.$

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(23) $y = \sin x \cos x.$

Using PR :

$\sin x . -\sin x + \cos x . \cos x$

$-\sin^2 x + \cos^2 x$

$\frac{dy}{dx} \Rightarrow \cos^2 x - \sin^2 x$

$(2\cos x . -\sin x . 1) - (2\sin x . \cos x . 1).$

$-2\sin x \cos x - 2\sin x \cos x$

$\frac{d^2y}{dx^2} = -4\sin x \cos x.$

(24) $y = \tan x.$

$\frac{dy}{dx} = \sec^2 x$

$\frac{d^2y}{dx} = 2\sec x . \sec x \tan x . 1.$

$= 2\sec^2 x \tan x.$