EXERCISE 2.6 [7-40]

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(7)-(26) Find $f'(x)$.

(7) $f(x) = (x^3+2x)^{37}$

$f'(x) = 37(x^3+2x)^{36} \times 3x^2+2$

$= (3x^2+2)\left[37(x^3+2x)^{36}\right]$

(8) $f(x) = (3x^2+2x-1)^6$

$f'(x) = 6(3x^2+2x-1)^5 \times 6x+2.$

$= (6x+2)\left[6(3x^2+2x-1)^5\right]$

(9) $f(x) = \left(x^3 - \frac{7}{x}\right)^{-2} \Rightarrow (x^3-7x^{-1})^{-2}$

$f'(x) = -2(x^3-7x^{-1})^{-3} \times (3x^2+7x^{-2})$

$= (3x^2+7x^{-2})\left[-2(x^3-7x^{-1})^{-3}\right]$

(10) $f(x) = \frac{1}{(x^5-x+1)^9} = (x^5-x+1)^{-9}.$

$f'(x) = -9(x^5-x+1)^{-10} \times 5x^4-1.$

$= (5x^4-1)\left[-9(x^5-x+1)^{-10}\right]$

(11) $f(x) = \frac{4}{(3x^2-2x+1)^3} = 4(3x^2-2x+1)^{-3}$

$f'(x) = -12(3x^2-2x+1)^{-4} \times 6x-2$

$= (6x-2)\left[-12(3x^2-2x+1)^{-4}\right]$

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(12) $f(x) = \sqrt{x^3 - 2x + 5} = (x^3 - 2x + 5)^{\frac{1}{2}}$.

$f'(x) = \frac{1}{2}(x^3 - 2x + 5)^{-\frac{1}{2}} \times (3x^2 - 2)$

$= (3x^2 - 2)\left[\frac{1}{2}(x^3 - 2x + 5)^{-\frac{1}{2}}\right]$

(13) $f(x) = \sqrt{4 + \sqrt{3x}} = \left(4 + (3x)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}\left(4 + 3x^{\frac{1}{2}}\right)^{-\frac{1}{2}} \times \left(\left(\frac{3}{2}x\right)^{-\frac{1}{2}}\right)$

$= \frac{1}{2\sqrt{4 + \sqrt{3x}}}\left(\frac{\sqrt{3}}{2\sqrt{x}}\right)$

$= \frac{\sqrt{3}}{4\sqrt{x}\sqrt{4 + \sqrt{3x}}}$.

(14) $f(x) = \sqrt[3]{12 + \sqrt{x}} = \left(\left(12 + (x)^{\frac{1}{2}}\right)\right)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}\left(12 + (x)^{\frac{1}{2}}\right)^{-\frac{2}{3}} \times \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{3(12 + \sqrt{x})^{\frac{2}{3}}} \times \frac{1}{2\sqrt{x}}$

$= \frac{1}{6(12 + \sqrt{x})^{\frac{2}{3}}}$

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$\frac{d}{dx} \sin x \Rightarrow \cos x \times 1$

$x^{-2} \Rightarrow -2x^{-3}$

(15) $f(x) = \sin\left(\frac{1}{x^2}\right)$

$= \cos\left(\frac{1}{x^2}\right) \times -2x^{-3}$

$= \cos x \left(\frac{1}{x^2}\right) \times -2x^{-3}$

$= \frac{-2}{x^3} \cdot \cos\left(\frac{1}{x^2}\right)$

(16) $f(x) = \tan\sqrt{x}. \quad = \tan x^{\frac{1}{2}}$

$= \sec^2(x^{\frac{1}{2}}) \times \frac{d}{dx}(x^{\frac{1}{2}}).$

$= \sec^2(\sqrt{x}) \times \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2\sqrt{x}} \sec^2\sqrt{x}$

(17) $f(x) = 4\cos^5 x$

$f'(x) = 20\cos^4 x \times -\sin x \times 1$

$= -20\sin x\cos^4 x$

(18) $f(x) = 4x + 5\sin^4 x$

$f'(x) = 4 + 20\sin^3 x \times \cos x \times 1.$

$= 4 + 20\sin^3 x\cos x.$

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$$\frac{d}{dx}(3x^{\frac{1}{2}}) = \frac{3}{2}x^{-\frac{1}{2}}$$

(19) $f(x) = \cos^2(3\sqrt{x})$ $\quad = \quad 3x^{\frac{1}{2}}$

$f'(x) = 2\cos(3\sqrt{x}) \cdot -\sin(3\sqrt{x}) \times \frac{3}{x} x^{-\frac{1}{2}}$

$= -\frac{3}{\sqrt{x}} \cos(3\sqrt{x}) \sin(3\sqrt{x})$

(20) $f(x) = \tan^4(x^3).$

$f'(x) = 4\tan^3(x^3) \times \sec^2(x^3) \times 3x^2.$

$= 12x^2 \tan^3(x^3) \sec^2(x^3).$

(21) $f(x) = 2\sec^2(x^7)$

$f'(x) = 4\sec(x^7) \times \sec(x^7) \tan(x^7) \times 7x^6.$

$= 28x^6 \sec^2(x^7) \tan(x^7)$

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(22) $f(x) = \cos^3\left(\frac{x}{x+1}\right)$;

$$f'(x) = 3\cos^2\left(\frac{x}{x+1}\right) \times -\sin\left(\frac{x}{x+1}\right) \times \frac{d}{dx}\left(\frac{x}{x+1}\right)$$

Using QR on $\frac{x}{x+1}$:

$$\frac{(x+1)(1) - (x)(1)}{(x+1)^2}$$

$$\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{\cancel{x}+1 - \cancel{x}}{(x+1)^2} = \frac{1}{(x+1)^2}.$$

$$f'(x) = \frac{-3\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)}{(x+1)^2}$$

(23) $f(x) = \sqrt{\cos(5x)} = (\cos(5x))^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}\left[\cos(5x)\right]^{-\frac{1}{2}} \times -\sin 5x \times 5.$$

$$= \frac{-5\sin 5x}{2\sqrt{\cos(5x)}}$$

(24) $f(x) = \sqrt{3x - \sin^2(4x)}. = \left[3x - \sin^2(4x)\right]^{\frac{1}{2}}.$

$$f'(x) = \frac{1}{\cancel{2}_1}\left[3x - \sin^2 4x\right]^{-\frac{1}{2}} \times \left[3 - 2\overset{\sin 4x}{\cos}(4x)\right] \times \cancel{4}.2$$

$$= \frac{6 - 4\overset{\sin 4x}{\cos}(4x)}{\sqrt{3x - \sin^2 4x}}$$

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(25) $f(x) = [x + \operatorname{cosec}(x^3+3)]^{-3}$

$f'(x) = -3[x + \operatorname{cosec}(x^3+3)]^{-4} \times (1 - \operatorname{cosec}(x^3+3)\cot(x^3+3) \times 3x^2)$.

$= -3[x + \operatorname{cosec}(x^3+3)]^{-4} \times [1 - \operatorname{cosec}(x^3+3)\cot(x^3+3)] \times 3x^2]$

$= -3(x + \cos(x^3+3))^{-4} \times [1 - 3x^2 \operatorname{cosec}(x^3+3)\cot(x^3+3)]$

(26) $f(x) = [x^4 - \sec(4x^2-2)]^{-4}$.

$f'(x) = -4[x^4 - \sec(4x^2-2)]^{-5} \times [4x^3 - \sec(4x^2-2)\tan(4x^2-2) \times 8x]$

$= \dfrac{-4}{(x^4 - \sec(4x^2-2))^5} \times 4x^3 - 8x \sec x(4x^2-2)\tan(4x^2-2)$

$= \dfrac{-16x^3 \times -8x \sec x (4x^2-2)\tan(4x^2-2)}{[x^4 - \sec(4x^2-2)]^5}$

(27) - (40) Find dy/dx.

(27) $y = x^3 \sin^2(5x)$

Using PR:

$\frac{dy}{dx} \rightarrow (x^3)(2\sin(5x)\cos(5x)) + (\sin^2(5x))(3x^2)$

$2x^3\sin(5x)\cos(5x) + 3x^2\sin^2(5x)$.

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(28) $y = \sqrt{x}\tan^3(\sqrt{x}) = x^{\frac{1}{2}}\tan^3(x^{\frac{1}{2}})$.

Using PR:

$$\frac{dy}{dx} = (x^{\frac{1}{2}})(3\tan^2(x^{\frac{1}{2}})\sec^2(x^{\frac{1}{2}})\tfrac{1}{2}x^{-\frac{1}{2}}) + \tan^3(x^{\frac{1}{2}})(\tfrac{1}{2}x^{-\frac{1}{2}}).$$

$$= \frac{3\tan^2(\sqrt{x})\sec^2(\sqrt{x})}{2} + \frac{1}{2\sqrt{x}}\tan^3(\sqrt{x})$$

(29) $y = x^5\sec(\frac{1}{x}) \Rightarrow y = x^5\sec(x^{-1})$.

Using PR:

$$\frac{dy}{dx} = (x^5)\left[\sec(\tfrac{1}{x})\tan(\tfrac{1}{x}) \times -x^{-2}\right] + \sec(x^{-1}) \times 5x^4.$$

$$= -x^3\sec\left(\tfrac{1}{x}\right)\tan\left(\tfrac{1}{x}\right) + 5x^4\sec\left(\tfrac{1}{x}\right).$$

(30) $y = \dfrac{\sin x}{\sec(3x+1)}$.

Using QR.

$$\frac{dy}{dx} = \frac{\sec(3x+1)(\cos x) - (\sin x)(\sec(3x+1)\tan(3x+1) \times 3)}{[\sec(3x+1)]^2}$$

$$= \frac{\cos x\sec(3x+1) - 3\sin x\sec(3x+1)\tan(3x+1)}{[\sec(3x+1)]^2}$$

$$= \frac{\cancel{\sec(3x+1)}\,[\cos x - 3\sin x\tan(3x+1)]}{(\sec(3x+1))^{\cancel{2}}}$$

$$= \frac{\cos x - 3\sin x\tan(3x+1)}{\sec(3x+1)}$$

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(31) $y = \cos(\cos x)$.

$\frac{dy}{dx} = -\sin(\cos x) \times -\sin x \times 1$.

$= \sin(\cos x)\sin x$.

$\tan 3x$.

$3\sec^2(3x) \times 3$

(32) $y = \sin(\tan 3x)$

$\frac{dy}{dx} = \cos(\tan 3x) \times \sec^2(3x) \times 3$

$= \cos(\tan 3x) 3\sec^2(3x)$

$= 3\sec^2(3x)\cos(\tan 3x)$.

(33) $y = \cos^3(\sin 2x)$.

$\frac{dy}{dx} = 3\cos^2(\sin 2x) \times -\sin(\sin 2x) \times 2\cos 2x$.

$= -6\cos^2(\sin 2x)\sin(\sin 2x)\cos 2x$

(34) $y = \frac{1 + \csc(x^2) \rightarrow u}{1 - \cot(x^2) \rightarrow v}$

$\frac{du}{dx} = -\csc(x^2)\cot(x^2) \times 2x$

$= -2x\csc(x^2)\cot(x^2)$.

$\frac{dv}{dx} = -(-\csc^2(x^2) \times 2x)$

$= 2x\csc^2(x^2)$

Using QR:

$\frac{dy}{dx} = \frac{(1 - \cot x^2)(-2x\csc x^2 \cot x^2) - (1 + \csc(x^2))(2x\csc^2 x^2)}{1 - \cot x^2}$

$= \frac{-2x\csc x^2 \cot x^2 + 2x\csc x^2 \cot^2 x^2 - 2x\csc^2 x^2 - 2x\csc^3 x^2}{1 - \cot x^2}$

$= -2x\csc x^2 \left[\frac{1 - \cot^2 x^2 + \csc x^2}{1 - \cot x^2}\right]$.

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$$-x^{\frac{1}{2}}$$
$$-\frac{1}{2}x^{-\frac{1}{2}}$$

(35) $y = \overset{u}{(5x+8)^7}\,\overset{v}{(1-\sqrt{x})^6}$.

$\frac{du}{dx} = 7(5x+8)^6 \times 5 = 35(5x+8)^6$

$\frac{dv}{dx} = \overset{3}{\cancel{6}}(1-\sqrt{x})^5 \times -\frac{1}{\cancel{2}}x^{-\frac{1}{2}} = -3x^{-\frac{1}{2}}(1-\sqrt{x})^5$.

Using Product rule:

$\frac{dy}{dx} = (5x+8)^7\left(-3x^{-\frac{1}{2}}(1-\sqrt{x})^5\right) + (1-\sqrt{x})^6\left[35(5x+8)^6\right]$

$= -15\cancel{x^{\frac{1}{2}}(1-\sqrt{x})^5}$

(36) $y = (x^2+x)^5\sin^8 x$.

$u' = 5(x^2+x)^4 \times (2x+1)$ $\qquad$ $v' = 8\sin^7 x \times \cos x \times 1$

$u' = 10x(x^2+x)^4 + 5(x^2+x^4)^4$. $\qquad$ $v' = 8\cos x\sin^7 x$.

Using PR:

$\frac{dy}{dx} = (x^2+x)^5.(8\cos x\sin^7 x) + (\sin^8 x)(10x(x^2+x)^4 + 5(x^2+x^4)^4)$

$= \cancel{8x\cos x\sin^7 x} + \cancel{8x\cos x\sin x}$

(37) $y = \left(\frac{x-5}{2x+1}\right)^3$.

Using QR:

$\frac{(2x+1)(1)-(x-5)(2)}{(2x+1)^2} = \frac{\cancel{2x}+1-\cancel{2x}+10}{(2x+1)^2} = \frac{11}{(2x+1)^2}$.

$\frac{dy}{dx} = 3\left(\frac{x-5}{2x+1}\right)^2 \times \left(\frac{11}{(2x+1)^2}\right) = 3\left(\frac{(x-5)^2}{(2x+1)^2}\right) \times \frac{11}{(2x+1)^2}$

$= \frac{33(x-5)^2}{(2x+1)^4}$

(38) $y = \left(\dfrac{1+x^2}{1-x^2}\right)^{17}$, $y = 1+x^2$

Using QR:

$$\frac{(1-x^2)(2x) - (1+x^2)(-2x)}{(1-x^2)^2} \qquad -2x \cdot x \;\; 2x^3$$

$$\frac{2x - \cancel{2x^3} + 2x + \cancel{2x^3}}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2}.$$

$$\frac{dy}{dx} = 17\left(\frac{1+x^2}{1-x^2}\right)^{16} \times \frac{4x}{(1-x^2)^2}.$$

$$= 17\left(\frac{(1+x^2)^{16}}{(1-x^2)^{16}}\right) \times \frac{4x}{(1-x^2)^2}$$

$$= \cancel{17}\ \frac{68x(1+x^2)^{16}}{(1-x^2)^{18}}.$$

(39) $y = \dfrac{(2x+3)^3}{(4x^2-1)^8}$ $\begin{array}{l} \longrightarrow 3(2x+3)^2 \times 2 = 6(2x+3)^2 \\ \longrightarrow 8(4x^2-1)^7 \times 8x = 64x(4x^2-1)^7 \end{array}$

Using QR:

$$\frac{dy}{dx} = \frac{(4x^2-1)^8\left(6(2x+3)^2\right) - (2x+3)^3\left(64x(4x^2-1)^7\right)}{(4x^2-1)^{16}}.$$

$$= \frac{\cancel{(4x^2-1)^7}\left[(4x^2-1)\left(6(2x+3)^2\right) - (2x+3)64x\right]}{(4x^2-1)^{\cancel{16}\ 9}}$$

$$= \frac{6(4x^2-1)(2x+3)^2 - 64x(2x+3)}{(4x^2-1)^9}$$

(40) $y = \left[1 + \sin^3(x^5)\right]^{12}$.

$$\frac{dy}{du} = 12\left[1+\sin^3(x^5)\right]^{11} \times \frac{d}{du}\left[1+\sin^3(x^5)\right] \times \frac{d}{dx}(x^5).$$

$$\frac{d}{du}\left[1+\sin^3(x^5)\right] = 0 + 3\sin^2(x^5) \times \cos(x^5) \cancel{\times 5x^4} \times 5x^4$$

$$= 15x^4 \sin^2(x^5)\cos(x^5).$$

$$\left.\frac{d}{dx}\left[x^5\right] = 5x^4.\right]$$ IGNORE!

$$\frac{dy}{du} = 12\left[1+\sin^3(x^5)\right]^{11} \times 15x^4 \sin^2(x^5)\cos(x^5)$$

$$= 180\left[1+\sin^3(x^5)\right]^{11} x^4 \sin^2(x^5)\cos(x^5)$$