LINEAR MOTION

Muhammad Adeel

Contents

Introduction to Linear Motion

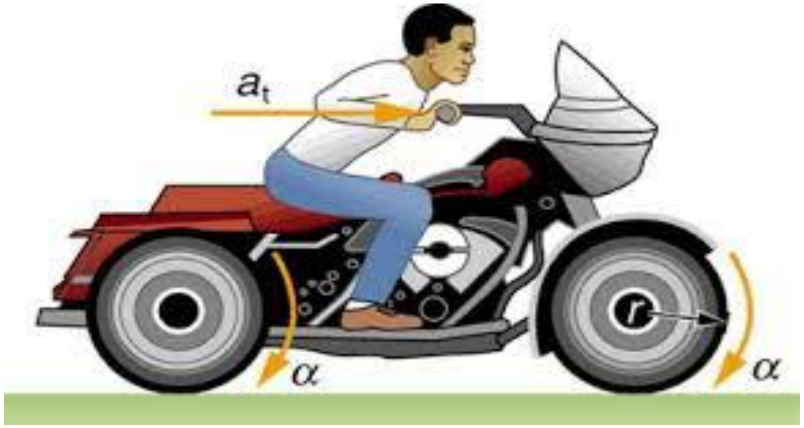
- The word "**Linear**" means "**Straight**" and the word "**Motion**" means **"change in position of a body with respect to frame of reference(surrounding)"**

To Remember:

- Linear Motion is the motion in **One Dimension**
- The motion should be in a **straight line** only.
- The line may be vertical, horizontal, or slanted, but it must be straight.
- All parts of the body move through the same distance in the same direction in same time.
- The motion of the body may not be uniform.

Muhammad Adeel

Examples of Linear Motion



Motion & Rest

- **MOTION :**

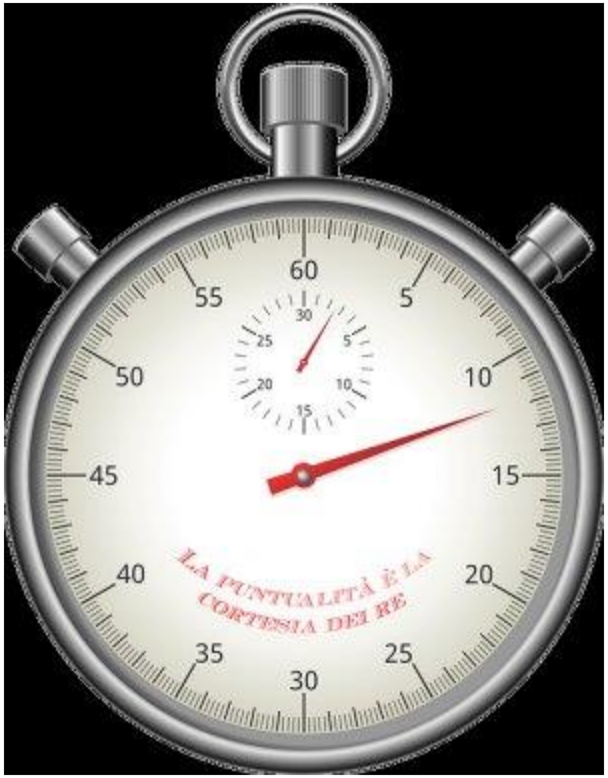
  A body is said to be in Motion if its position changes with respect to its surrounding .

- **REST:**

  A body is said to be in Rest if its position does not change with respect to its surrounding.

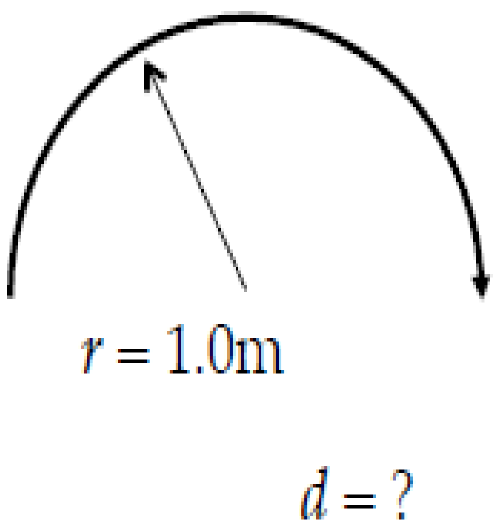
Time

- Time refers to how long an object is in motion for.
- Here, we’ll usually use seconds, but we might use minutes, hours, years, milliseconds, or any other unit of time.



Distance

- Distance is simply how far something travels along its path, whether measured in miles, kilometers, meters, centimeters, feet, or any other unit.



Speed

- Speed = how fast you're going
- Speed is simply a measure of how quickly an object is moving: how much distance it travels in a given time.
- It is a scalar quantity

Formula:

$$speed = \frac{distance}{time}$$

Unit: Standard unit in MKS system is meter/sec (m/s)

Average Speed

- The average speed (where average will be represented by a bar on top of the quantity involved) is the distance traveled in any direction, Δs divided by the time Δt

$$\overline{\text{speed}} = \frac{\Delta s}{\Delta t} \longrightarrow \textit{Eq. 1}$$

Where, Δ *(anything)= final value – initial value*

Example Problem

- A swimmer travels one complete lap in a pool that is 50.0-meters long. The first lap is covered in 20.0 seconds, the second lap is covered in 25.0 seconds. What was his average speed for the lap?

Solution:

$$speed = \frac{distance}{time}$$

$$speed = \frac{50+50}{20+25} = 2.22 m/s$$

Displacement

- Displacement is a measure of how far you have “displaced,” or changed your position.
- Displacement is a vector quantity, you need to specify a direction for your displacement.

For instance;

Q# What was your displacement coming to this class?

A# 152 meters, East

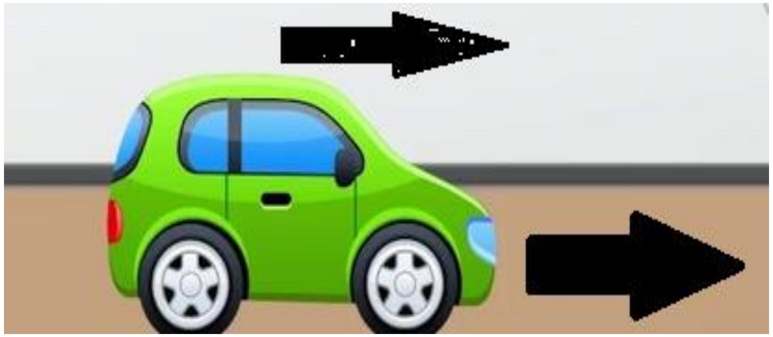
Q# How high can you jump?

A# 1 meters up.

Types of Displacement

Mainly two types that we'll go through;

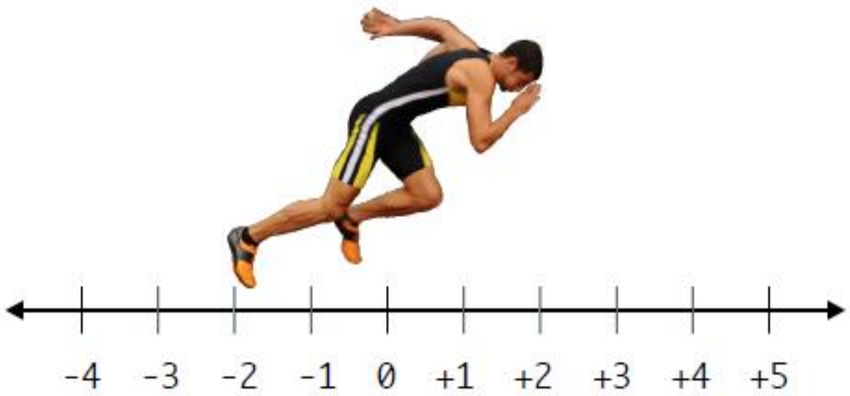
1. Horizontal Displacement
- Example: motion of a car

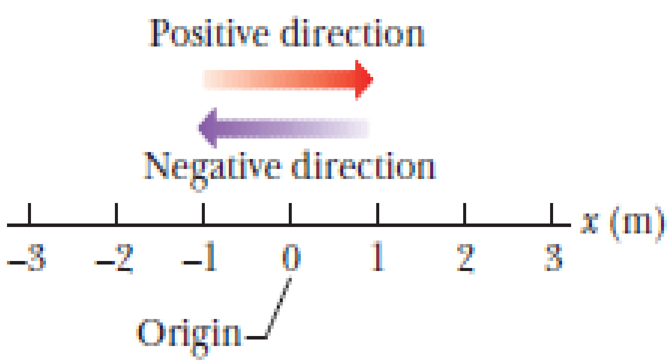


2. Vertical Displacement
- Example : launching of rocket

Horizontal Displacement

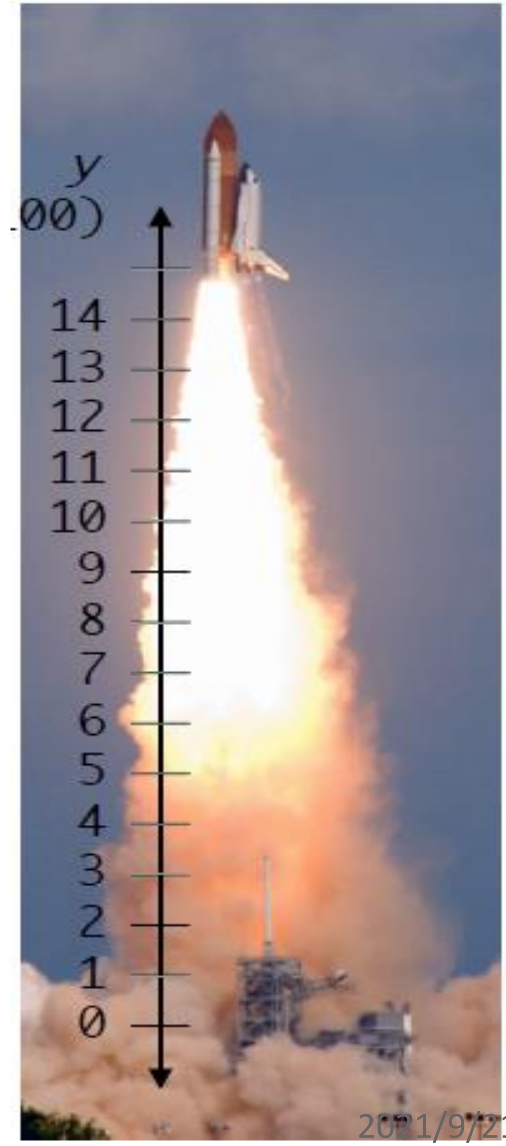
- For horizontal motion, we'll often describe the displacement in regards to an imaginary number line, with "to the right" being the positive-x direction, and "to the left" being the negative-x direction.





Vertical Displacement

- For vertical motion, we'll often describe the displacement in regards to an imaginary number line, with "up" being the positive *y*-direction, and "down" being the negative-*y* direction.



Velocity

- It is the rate of change of displacement.
- It is a vector quantity.
- Formula :

$$v = \vec{r}/t$$

Unit:

It is measured in MKS system of units that is meter per second(m/s)

Average Velocity

Consider a particle moving in space. Let the particle be at point P in Fig. **1** at some initial time t_0 and at point P' some later time t_f.

The initial position of the particle can be specified by a *position vector* $\mathbf{r}_0$ obtained by drawing an arrow from the origin of the coordinate system to point P. Similarly, the position at the later time is specified by a second position vector $\mathbf{r}_f$ that results when an arrow is drawn from the origin to point P'. The position at any other point in the motion is specified by a corresponding position vector $\mathbf{r}$. We can now define the *displacement vector* $\Delta\mathbf{r}$ as the vector difference between the final and the initial position vectors, namely, $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_0$ (see Fig. **1**). Correspondingly, we define the *average velocity* $\overline{\mathbf{v}}$ as the ratio of the displacement vector to the time taken for the displacement to occur, namely,

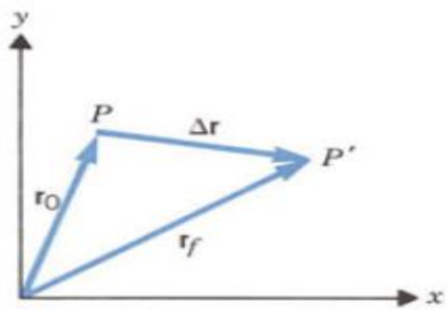


FIGURE 1 The displacement vector $\Delta\mathbf{r}$ is obtained by drawing an arrow from the initial position vector $\mathbf{r}_0$ to the final position vector $\mathbf{r}_f$.

$$\overline{\mathbf{v}} = \frac{\mathbf{r}_f - \mathbf{r}_0}{t_f - t_0} = \frac{\Delta\mathbf{r}}{\Delta t} \quad \text{——— Eq. 2}$$

Avg. Speed Vs Avg.Velocity

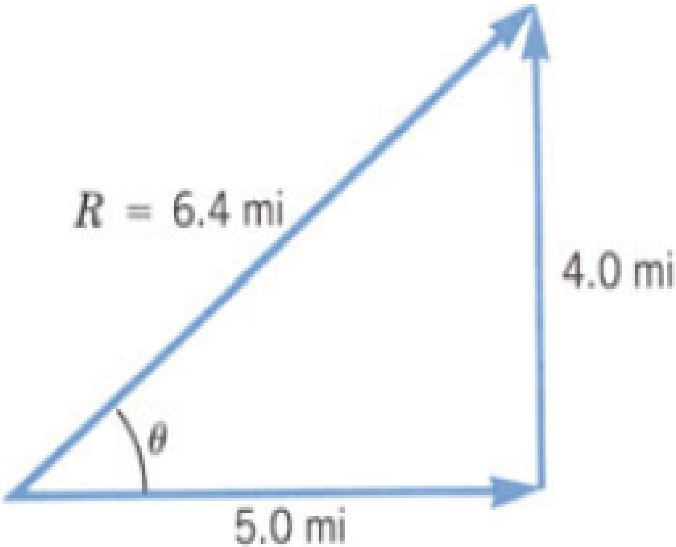
Consider the walk taken in Fig.

Suppose it took 1 h.

Then, by definition, $\overline{\text{speed}} = \dfrac{\Delta s}{\Delta t}$

the average speed of walking was $\Delta s/\Delta t$

$(4.0\,\text{mi} + 5.0\,\text{mi})/1\,\text{h} = 9\,\text{mi/h}$



whereas the average velocity was $\Delta \mathbf{r}/\Delta t = 6.4\,\text{mi}/1\,\text{h}$

6.4 mi/h in the direction 39° north of east.

Example of Avg.Speed & Avg.Velocity

Suppose a race car is traveling around a circular track of 1-mi diameter and its speedometer reads 100 mi/h. This is the speed.

The time taken to reach any point is, from Eq $\Delta t = \Delta s/\overline{\text{speed}}$.

Because the track length is $\pi \times \text{diameter} = 3.14\,\text{mi}$,

the time to complete one circuit is

$$\Delta t = \frac{3.14\,\text{mi}}{100\,\text{mi/h}} = 3.14 \times 10^{-2}\,\text{h}$$

and the time to go halfway around is $1.57 \times 10^{-2}\,\text{h}$.

However, the car's average velocity depends on its position.

When the car has gone halfway around, say from the western-most to eastern-most position on the track, then the magnitude of the vector displacement from the starting point is the diameter or 1 mi. Hence, its average velocity to that point is

$$\bar{\mathrm{v}} = \frac{1\,\text{mi}}{1.57 \times 10^{-2}\,\text{h}} = 63.7\,\text{mi/h in the east direction.}$$

In one complete circuit, as the car passes the starting point its vector displacement is zero and hence

$$\bar{\mathbf{v}} = \frac{0\,\text{mi}}{3.14 \times 10^{-2}\,\text{h}} = 0\,\text{mi/h}$$

This seeming contradiction has occurred because we have taken large displacements for $\Delta\mathbf{r}$. If we shrink the displacement to a minute amount by taking the limit

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \qquad \textbf{Eq. 3}$$

then the magnitude of the velocity, which is now called the *instantaneous velocity*, at any point on the track will equal the speed. This can be seen in Fig. 2 (a,b,c,d)

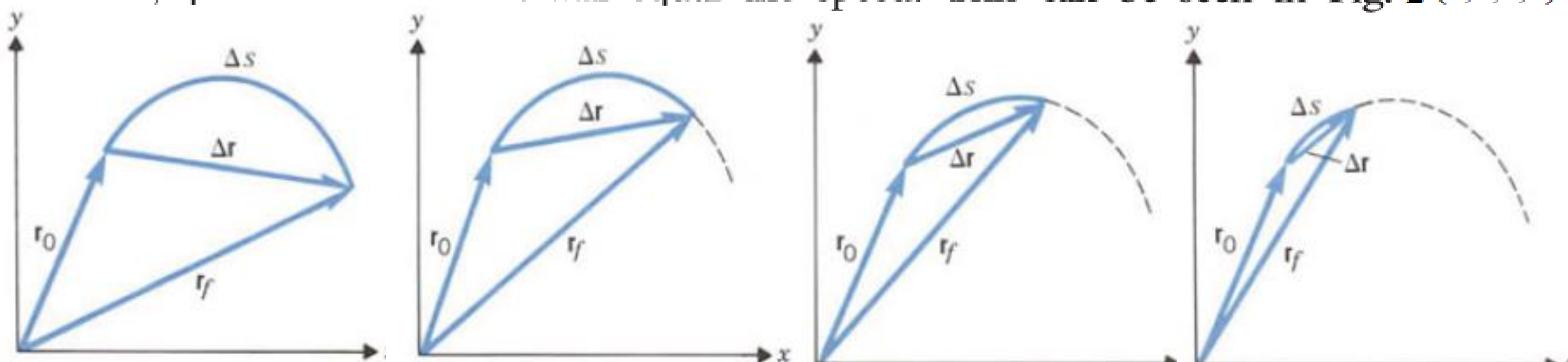


Fig. 2(a) Fig. 2(b) Fig. 2(c) Fig. 2(d)

FIG 2 (a,b,c,d) A curved path of a car traveling clockwise. Δs is the distance traveled by the car, and $\Delta\mathbf{r}$ is the displacement vector between the position of the car $\mathbf{r}_f$ at some instant and the position $\mathbf{r}_0$ at the initial time. As the distance Δs becomes smaller, its value approaches $\Delta\mathbf{r}$.

we notice that as Δs becomes smaller, the difference between Δs and the corresponding $\Delta \mathbf{r}$ decreases. We should also note that in the limit where $\Delta \mathbf{r}$ becomes infinitesimally small, it becomes tangential to the path, and therefore the direction of the instantaneous velocity is the tangent to the path.

while the magnitude of the instantaneous velocity remains equal to the speed, the direction part of the instantaneous velocity is changing.

Velocity is a vector because it is equal to a vector displacement divided by time, which is scalar, and the division of a vector by a scalar does not remove the vector property.

We should note Eq. 3

instantaneous velocity $\mathbf{v}$ is the first derivative of the position vector with respect to time. It should also be pointed out that because Eq. 3 is a vector equation, it holds for each of the cartesian components of the vectors $\mathbf{v}$ and $\mathbf{r}$, namely,

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}$$

where v_x, v_y, and v_z are the cartesian components of $\mathbf{v}$ and x, y, and z are those of $\mathbf{r}$.

Acceleration

If there is a velocity change $\Delta\mathbf{v}$ in a certain time Δt, we define the average acceleration as

$$\bar{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t} \qquad (4)$$

or

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_0}{t_f - t_0}$$

where the subscripts f and 0 represent final and initial values, respectively. Usually in a problem we start our stopwatch at $t_0 = 0$, so the elapsed time is simply t_f and we drop the subscript f. We may define an instantaneous acceleration as

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \qquad (5)$$

which is the first derivative of $\mathbf{v}$ with respect to time. Substituting Eq. 3 for $\mathbf{v}$, we write

$$\mathbf{a} = \frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\right) = \frac{d^2\mathbf{r}}{dt^2} \qquad (6)$$

which is the second derivative of $\mathbf{r}$ with respect to time.

Example Problem

The position of a body on the x axis varies as a function of time according to the following equation

$$x(\text{meters}) = (3t + 2t^2)\text{m}$$

Find its velocity and acceleration when $t = 3$ sec.

Solution

Because the body moves in a straight line, $\mathbf{r} = x$. From Eq. 3

$$v = \frac{dx}{dt} = \frac{d}{dt}(3t + 2t^2) = (3 + 4t)\ \text{m/sec}$$

The velocity of the body at $t = 3$ sec is therefore

$$v(t = 3\ \text{sec}) = 3 + 4 \times 3 = 15\ \text{m/sec}$$

From Eq. 5

$$a = \frac{dv}{dt} = \frac{d}{dt}(3 + 4t) = 4\ \text{m/sec}^2$$

Notice that a is a constant, and therefore $a(t = 3\ \text{sec}) = 4\ \text{m/sec}^2$.

Tasks for Assignment

Exercise Problems

Q 1 A student drives to college 15 km away from home in half an hour. After classes, he returns home in 20 min. Find (a) the average speed on his way to college, (b) the average speed for the round trip, (c) his average velocity for the entire trip.

Q.2 The position of a particle moving along the x axis is given by $x = 3 + 17t - 5t^2$, where x is in meters and t is in seconds. (a) What is the position of the particle at $t = 1$, 2, and 3 sec? (b) At what time does the particle return to the origin? (c) What is the instantaneous velocity at $t = 1$, 2, and 3 sec? (d) At what time is the instantaneous velocity of the particle zero? (e) What is the velocity of the particle as it passes through the origin? (f) What is the acceleration of the particle as it passes the origin?

Exercise Problems

Q.3 The position of a particle moving in a straight line is given by $x = 5 + 2t + 4t^2 - t^3$, where x is in meters. (a) Find an expression for the instantaneous velocity as a function of time. (b) Find the position of the particle at $t = 0, 1, 0.1$, and 0.01 sec. (c) What is the average velocity between $t = 0$ sec and $t = 1$ sec, between $t = 0$ sec and $t = 0.1$ sec, and between $t = 0$ sec and $t = 0.01$ sec? (d) What is the instantaneous velocity at $t = 0$ sec? (e) What conclusion do you draw from the answers in (c) and (d)?

Q 4 A car is driving east at 60 km/h, it then makes a turn and travels north at 50 km/h. If it takes 2 sec to make the turn, what is the average acceleration of the car over this 2 second interval?

Answer: 10.85 m/sec^2, directed 39.8° north of west.

Exercise Problems

Q.5 Consider the particle of problem 3. (a) Find an expression for the acceleration of the particle as a function of time. (b) What is the instantaneous velocity of the particle at $t = 0$, 1, 0.1, and 0.01 sec? (c) What is the average acceleration between $t = 0$ sec and $t = 1$ sec, between $t = 0$ sec and $t = 0.1$ sec, between $t = 0$ sec and $t = 0.01$ sec? (d) What is the instantaneous acceleration at $t = 0$ sec? (e) What conclusion can you draw from the answers in (c) and (d)?

Muhammad Adeel

Summary

- Speech: Rate of change of distance $speed = \dfrac{distance}{time}$
- Velocity: Rate of change of displacement $\vec{v} = \dfrac{\vec{r}}{t}$
- Avg. Speed: $\overline{\text{speed}} = \dfrac{\Delta s}{\Delta t}$
- Avg. Velocity: $\bar{\mathbf{v}} = \dfrac{\mathbf{r}_f - \mathbf{r}_0}{t_f - t_0} = \dfrac{\Delta \mathbf{r}}{\Delta t}$
- Instantaneous Velocity: $\mathbf{v} = \lim_{\Delta t \to 0} \dfrac{\Delta \mathbf{r}}{\Delta t} = \dfrac{d\mathbf{r}}{dt}$
- Acceleration: Rate of change of velocity $\vec{a} = \dfrac{\vec{v}}{t}$
- Avg. Acceleration: $\bar{\mathbf{a}} = \dfrac{\Delta \mathbf{v}}{\Delta t}$
- Instantaneous Acceleration: $\mathbf{a} = \lim_{\Delta t \to 0} \dfrac{\Delta \mathbf{v}}{\Delta t} = \dfrac{d\mathbf{v}}{dt}$