EXERCISE 0.2 [27-34, 53-63, 66-67]

(27)-(28) Find $f+g$, $f-g$, fg and f/g, state domains.

(27) $f(x) = 2\sqrt{x-1}$, $g(x) = \sqrt{x-1}$.

$$f+g = 2\sqrt{x-1} + \sqrt{x-1}$$
$$= 3\sqrt{x-1} \qquad D \Rightarrow x \geq 1.$$

$$f-g = 2\sqrt{x-1} - \sqrt{x+1}$$
$$= \sqrt{x-1} \qquad D \Rightarrow x \geq 1$$

$$fg = 2\sqrt{x-1} \times \sqrt{x-1}$$
$$= 2(x-1) \qquad D \Rightarrow x \geq 1$$

$$f/g = \frac{2\sqrt{\cancel{x-1}}}{\sqrt{\cancel{x-1}}} , \qquad D \Rightarrow x > 1.$$

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(28) $f(x) = \dfrac{x}{1+x^2}$, $g(x) = \dfrac{1}{x}$.

$f+g \Rightarrow \dfrac{x}{1+x^2} + \dfrac{1}{x} = \dfrac{x^2 + 1+x^2}{x(1+x^2)} = \dfrac{2x^2+1}{x(1+x^2)}$

$D \Rightarrow (-\infty, 0) \cup (0, +\infty)$.

$f-g \Rightarrow \dfrac{x}{1+x^2} + \dfrac{1}{x} = \dfrac{\cancel{x^2}+1-\cancel{x^2}}{x(1+x^2)} = \dfrac{-1}{x(1+x^2)}$

$D \Rightarrow (-\infty, 0) \cup (0, +\infty)$

$fg \Rightarrow \dfrac{\cancel{x}}{1+x^2} \times \dfrac{1}{\cancel{x}} = \dfrac{1}{1+x^2}$ $D = (-\infty, +\infty)$.

$f/g = \dfrac{\dfrac{x}{1+x^2}}{\dfrac{1}{x}} = \dfrac{x}{1+x^2} \times x = \dfrac{x^2}{1+x^2} = D(-\infty, +\infty)$.

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(29) Let $f(x) = \sqrt{x}$ and $g(x) = x^3 + 1$. Find

(a) $f(g(2)) =$

$g(2) = 8 + 1 = 9.$

$f(9) = \sqrt{9} = 3.$

(b) $g(f(4)) =$

$f(4) = 2.$

$g(2) = 2^3 + 1 = 9.$

(c) $f(f(16)) =$

$f(16) = \sqrt{16} = 4.$

$f(4) = \sqrt{4} = 2.$

(d) $g(g(0)) =$

$g(0) = 1$

$g(1) = 2.$

(e) $f(2+h) =$

$f(2+h) = \sqrt{2+h}.$

(f) $g(3+h) = (3+h)^3 + 1$

$= 27 + 9h + h^3 + 1$

$= h^3 + 9h + 28.$

(30) Let $g(x) = \sqrt{x}$. find.

(a) $g(5s+2) = \sqrt{5s+2}$

(b) $g(\sqrt{x}+2) = \sqrt{\sqrt{x}+2}$.

(c) $3g(5x) =$

$g(5x) = \sqrt{5x}$.

$3g(5x) = 3\sqrt{5x}$.

(d) $\frac{1}{g(x)} = \frac{1}{\sqrt{x}}$

(e) $g(g(x)) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$ $\quad ((x)^{\frac{1}{2}})^{\frac{1}{2}}$.

(f) $(g(x))^2 - g(x^2)$

$(\sqrt{x})^2 - \sqrt{x^{\cancel{2}}}$.

$x - x = 0$.

(g) $g(1/\sqrt{x}) = \sqrt{\frac{1}{\sqrt{x}}}$

(h) $g((x-1)^2) = \sqrt{(x-1)^{\cancel{2}}} = x-1$.

(i) $g(x+h) = \sqrt{x+h}$.

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(31) - (34) Find fog and gof, state domains.

(31) $f(x) = x^2$, $g(x) = \sqrt{1-x}$

$\left(\sqrt{1-x}\right)^{\cancel{2}} = 1-x.$

$D(-\infty, +\infty).$

(32) $f(x) = \sqrt{x-3}$ $\quad g(x) = \sqrt{x^2+3}.$

$fog = \sqrt{\sqrt{x^2+3} - 3}.$ $\quad gof = \sqrt{(\sqrt{x-3})^2 + 3}$

$D(-\infty, +\infty).$ $\quad = \sqrt{x \cancel{-3} \cancel{+3}} = x$

(33) $f(x) = \dfrac{1+x}{1-x}$, $g(x) = \dfrac{x}{1-x}.$

$$fog: = \frac{1 + \frac{x}{1-x}}{1 - \frac{x}{1-x}} = \frac{\frac{1-\cancel{x}+\cancel{x}}{\cancel{1-x}}}{\frac{1-x-x}{\cancel{1-x}}} = \frac{1}{1-2x}$$

$= D = (-\infty, \tfrac{1}{2}) \cup (\tfrac{1}{2}, +\infty).$

$$gof: = \frac{\frac{1+x}{1-x}}{1 - \frac{1+x}{1-x}} = \frac{\frac{1+x}{\cancel{1-x}}}{\frac{\cancel{1}-x-\cancel{1}-x}{\cancel{1-x}}} = \frac{1+x}{-2x}.$$

$D = (-\infty, 0) \cup (0, +\infty).$

(34) $f(x) = \dfrac{x}{1+x^2}$, $g(x) = \dfrac{1}{x}$.

fog: $\dfrac{\frac{1}{x}}{1+\left(\frac{1}{x}\right)^2} = \dfrac{\frac{1}{x}}{1+\frac{1}{x^2}} = \dfrac{\frac{1}{\cancel{x}}}{\frac{x^2+1}{x^{\cancel{2}}}} = \dfrac{1}{x^2+1}$

$D(-\infty, +\infty)$.

gof: $\dfrac{1}{\frac{x}{1+x^2}} = \dfrac{1+x^2}{x}$. $D(-\infty, 0) \cup (0, +\infty)$

(53) - (56) Find (a) $\dfrac{f(x+h) - f(x)}{h}$ and (b) $\dfrac{f(w) - f(x)}{w-x}$.

Simplify as much as possible:

(53) $f(x) = 3x^2 - 5$

$f(x+h) = 3(x+h)^2 - 5$.

$\dfrac{3(x+h)^2 - 5 - 3x^2 - 5}{h}$

$\dfrac{3(x^2 + 2hx + h^2) - 5 - (3x^2 - 5.)}{h}$

$\dfrac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{5} - \cancel{3x^2} + \cancel{5}}{h}$.

$\dfrac{3h^2 + 6hx}{h} = 3h + 6x$

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(b) $f(\omega) = 3\omega^2 - 5.$

$$\frac{3\omega^2 - 5 - (3x^2 - 5)}{\omega - x}$$

$$\frac{3\omega^2 - \cancel{5} - 3x^2 + \cancel{5}}{\omega - x.}$$

$\cancel{3\omega(3\omega - 3x}$ $\quad \frac{3(\omega^2 - x^2)}{\omega - x}$

$$\frac{3(\omega - x)\cancel{(\omega - x)}}{\cancel{\omega - x}} = 3(\omega - x).$$

(54) $f(x) = x^2 + 6x$

(a) $f(x+h) = (x+h)^2 + 6x$

$$\frac{(x+h)^2 + 6x - (x^2 + 6x)}{h.}$$

$$\cancel{x^2} + 2hx + h^2 + \cancel{6x} - \cancel{x^2} - \cancel{6x}$$

$$\frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + h + 6.$$

(b) $f(\omega) = \omega^2 + 6\omega$

$$\frac{\omega^2 + 6\omega - (x^2 + 6x)}{\omega - x.}$$

$$\frac{\omega^2 + 6\omega - x^2 + 6x.}{\omega - x} = \frac{\omega(\omega+6) - x(x+6)}{\omega - x}$$

$$\frac{\omega^2 - x^2 + 6(\omega + x)}{\omega - x} = \frac{(\omega^2 - x^2) + 6(\omega + x)}{\omega - x.}$$

$$\frac{(\omega - x)(\omega + x) + 6(\omega + x)}{\omega - x} = \frac{\cancel{\omega - x}[\omega + x + 6]}{\cancel{\omega - x}}$$

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(55) $f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} \Rightarrow \frac{\frac{x - (x+h)}{(x)(x+h)}}{h} = \frac{\frac{-\cancel{h}}{x(x+h)}}{\cancel{h}}$$

$$\Rightarrow \frac{-1}{x^2+hx}$$

$f(w) = \frac{1}{w}$

$$\frac{\frac{1}{w} - \frac{1}{x}}{w-x} = \frac{-\frac{(x-w)}{wx}}{w-x} = -\frac{\frac{\cancel{w-x}}{wx}}{\cancel{w-x}} = -\frac{1}{wx}.$$

(56) $f(x) = \frac{1}{x^2}$

$f(x+h) = \frac{1}{(x+h)^2}$

$$\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{\frac{x^2 - (x+h)^2}{(x^2)(x+h)^2}}{h}$$

$$\frac{x^2 - (x^2 + 2hx + h^2)}{x^2 (x+h)^2 h} = \frac{\cancel{x^2} - \cancel{x^2} - 2\cancel{h}x - h^2}{x^2(x+h)^2 \cancel{h}}$$

$$\frac{-(2x+h)}{x^2(x+h)^2}$$

(b) $f(w) = \frac{1}{w^2}$ $f(x) = \frac{1}{x^2}$.

$$\frac{\frac{1}{w^2} - \frac{1}{x^2}}{w-x} = \frac{\frac{x^2 - w^2}{w^2x^2}}{w-x} = -\frac{(x+w)\cancel{(x-w)}}{w^2x^2\cancel{(w-x)}}$$

$$\Rightarrow \frac{-w-x}{w^2x^2}$$

(57) Classify functions whose values are in table, as even, odd or neither.

x	-3	-2	-1	0	1	2	3
$f(x)$	5	3	2	3	1	-3	5
$g(x)$	4	1	-2	0	2	-1	-4.
$h(x)$	2	-5	8	-2	8	-5	2

→ $f(x)$ is neither

→ $g(x)$ is odd

→ $h(x)$ is even

(58) Complete table for $y = f(x)$ symmetric about (a) y axis (b) origin

(a)

x	-3	-2	-1	0	1	2	3
$f(x)$	1	-5	-1	0	-1	-5	1

(b)

x	-3	-2	-1	0	1	2	3
$f(x)$	1	5	-1	0	1	-5	-1

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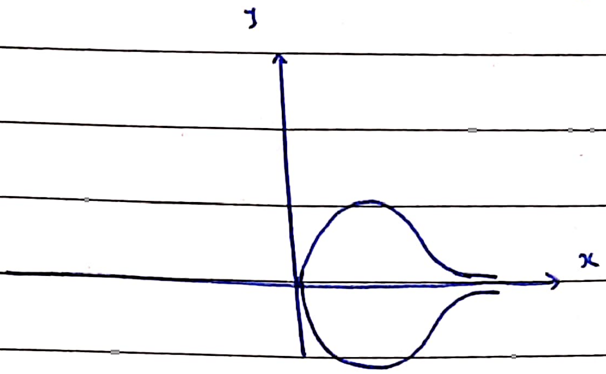
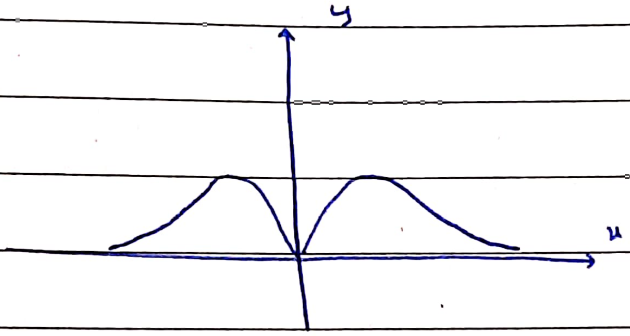
(59) Complete graph so entire graph is symmetric about (a) x axis (b) y axis (c) origin.

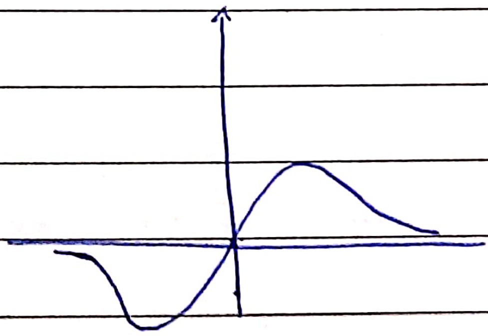
(a)



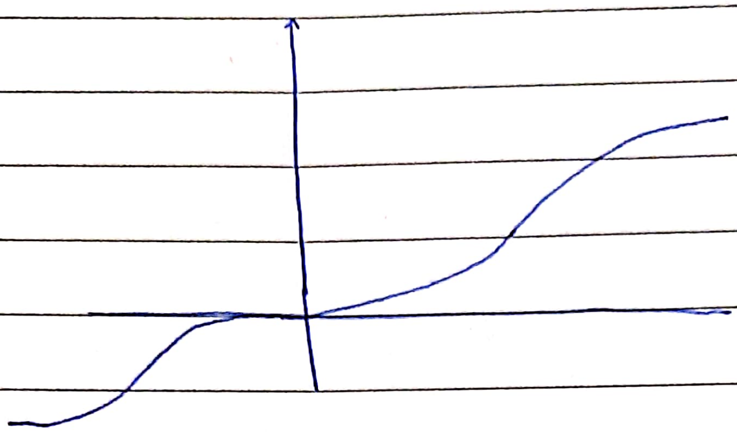
(b)

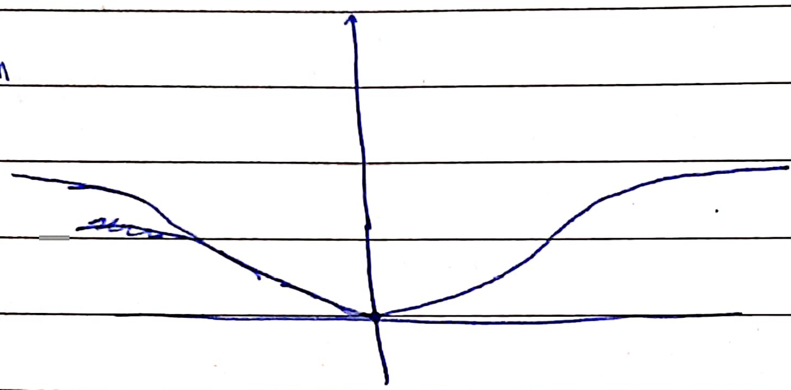


(c)



(60) Complete graph assuming that
(b) f is odd function (a) f is even function

(b)


(a)


(61) - (62) Classify the function graphed as even, odd, neither.

(61) (a) Even
(b) Odd

(62) (a) Odd
(b) Neither even nor odd.

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(63) In each part classify function is neither, even or odd.

(a) $f(x) = x^2$

$f(-x) = (-x)^2$

$= x^2$ (Even function).

(b) $f(x) = x^3$

$f(-x) = (-x)^3$

$= -x^3$ (Odd)

(c) $f(x) = |x|$

$f(-x) = |-x| = x$ (Even)

(d) $f(x) = x+1$

$f(-x) = -x+1$ (Neither)

(e) $f(x) = \dfrac{x^5 - x}{1+x^2}$.

$f(-x) = \dfrac{(-x)^5 - x}{1+(-x)^2} = \dfrac{-x^5 - (-x)}{1+x^2} = \dfrac{-x^5 + x}{1+x^2}$ (Odd).

(f) $f(x) = 2$ (Even).

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(66) - (67) Use theorem 0.2.3 to determine where graph has symmetries about x, y or origin axes.

66 (a) $x = 5y^2+9$

$-x = 5y^2+9$ (Y axis)

$x = 5(-y)^2+9 = 5y^2+9$ (x axis).

(b) $x^2-2y^2=3.$

$(-x)^2-2y^2=3$

$x^2-2y=3.$ (Y axis)

$x^2-2(-y)^2=3$

$x^2-2y^2=3$ (X axis).

$(-x)^2-2(-y)^2=3$

$x^2-2y^2=3$ (Origin).

(c) $xy=5.$

$(-x)(-y)=5$

$xy=5$ (Origin)

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67 (a) $x^4 = 2y^3 + y$.

$(-x)^4 = 2y^3 + y$

$x^4 = 2y^3 + y$ (Y axis).

$x^4 = 2(-y)^3 + (-y)$

$x^4 = -2y^3 - y$ (x axis).

(b) $y = \frac{x}{3+x^2}$.

$y = \frac{-x}{3+(-x)^2} = \frac{-x}{3+x^2}$

$-y = \frac{x}{3+x^2}$

$\cancel{-}y = \cancel{-}\frac{x}{3+(-x)^2}$ (Origin).

(c) $y^2 = |x| - 5$

$y^2 = |-x| - 5$

$y^2 = x - 5$ (Y axis).

$(-y)^2 = |x| - 5$

$y^2 = x - 5$ (x axis)

$(-y)^2 = |-x| - 5$

$y^2 = x - 5$ (Origin).