M. Anas

21K-4556

Assignment # 02

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Exercise 4.1

Q1) $u = (u_1, u_2)$ $v = (v_1, v_2)$

$u+v = (u_1 + v_1, u_2 + v_2)$, $ku = (0, ku_2)$

a) $u = (-1, 2)$, $v = (3, 4)$ and $k = 3$

$u+v = (-1+3, 2+4) = (2, 6)$ $ku = (0, 3.2) = (0, 6)$

b) why! V is closed under addition and scalar multiplication.

- $u+v$ is ~~and~~ an ordered pair of real numbers
  - $\therefore$ $u+v$ is in V.
- ku is an ordered pair of real numbers
  - $\therefore$ ku is in V.

c) Certain vector space axioms hold for V because they are known to hold for R^2, in which axioms are they?

Axioms 1-5 holds for V.

d) Show that Axioms 7, 8 and 9 hold

7: $k((u_1, u_2) + (v_1, v_2)) = k(u_1 + v_1, u_2 + v_2) = (0, k(u_2 + v_2))$
$= (0, ku_2) + (0, kv_2)$

8: $(k+m)(u_1, u_2) = (0, (k+m)u_2) = (0, ku_2 + mu_2)$
$= (0, ku_2) + (0, mu_2)$

9: $(km)(u_1, u_2) = k(0, mu_2) = (0, kmu_2)$

e) Show Axioms 10 fails and hence that V is not vector space under the operation.

$1(u_1, u_2) \neq (0, u_2)$ V is not a vector space.

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Q2) $u+v = (u_1+v_1+1, u_2+v_2+1)$, $ku = (ku_1, ku_2)$

$k=2$

a) $u = (0,4)$, $v = (1,-3)$

$u = (0+1+1, 4-3+1) = (2,2)$

$ku = (2\cdot 0, 2\cdot 4) = (0,8)$

b) $(0,0) + (u_1, u_2) = (0+u_1+1, 0+u_2+1)$

$= (u_1+1, u_2+1) \neq (u_1, u_2)$

c) $(-1,-1) + (u_1, u_2) = (-1+u_1+1, -1+u_2+1)$

$= (u_1, u_2) = (u_1, u_2)$

d) $u = (u_1, u_2)$ $-u = (-2-u_1, -2-u_2)$

$u + (-u) = (u_1 + (-2-u_1)+1, u_2 + (-2-u_2)+1)$

$= (-1,-1) = 0$.

e) $k(u+v) = k(u_1+v_1+1, u_2+v_2+1) = (ku_1+kv_1+k, ku_2+kv_2+k)$

$ku+kv = (ku_1, ku_2) + (kv_1, kv_2) = (ku_1+kv_1+1, ku_2+kv_2+1)$

$k(u+v) \neq ku+kv$ Axioms 7 fails.

$(k+m)u = ((k+m)u_1, (k+m)u_2) = (ku_1+mu_1, ku_2+mu_2)$

$ku+mu = (ku_1, ku_2) + (mu_1, mu_2) = (ku_1+mu_1+1, ku_2+mu_2+1)$

Axioms 8 fails

Q3) 1) $x+y$ is in V

2) $x+y = y+x$ 3) $x+(y+z) = (x+y)+z$

4) $0+x = x+0 = x$

5) $x+(-x) = 0$ 6) kx is in V

7) $k(x+y) = kx+ky$ 8) $(k+m)x = kx+mx$

9) $(km)x = k(mx)$ 10) $1x = x$

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Q4) set of all pair of real numbers of the form $(x,0)$ R^2

1) $(x,0)+(y,0) = (x+y,0)$ is in V.

2) $(x,0)+(y,0) = (x+y,0) = (y+x,0) = (y,0)+(x,0)$

3) $(x,0)+((y,0)+(z,0)) = (x,0)(y+z,0) = (x+y+z,0)$

$= (x+y,0)+(z,0) = ((x,0)+(y,0))+(z,0)$

4) $0=(0,0)$ $\quad (0,0)+(x,0) = (x,0)$

and $\quad (x,0)+(0,0) = (x,0)$

5) $(x,0)+(-x,0) = (0,0)$

$(-x,0)+(x,0) = (0,0)$

6) $k(x,0) = (kx,0)$ is in V

7) $k((x,0)+(y,0)) = k(x+y,0) = (kx+ky,0)$

$= k(x,0)+k(y,0)$

8) $(k+m)(x,0) = ((k+m)x,0) = (kx+mx,0) = k(x,0)+m(x,0)$

9) $km(x,0) = k(mx,0) = (kmx,0) = k(m(x,0))$

10) $1(x,0) = (x,0)$

Q5) set of pair of real numbers form (x,y) where $x\geq 0$ R^2

5) fails whenever $x\neq 0$ $x'\geq 0$ for which

$(x,y)+(x',y') = (0,0)$

6) fails whenever $k<0$ and $x\neq 0$

Q6) set of n-tuples of real numbers $(x,x\ldots,x)$ on R^n.

1) $(x,x\ldots,x)+(y,y\ldots y) = (x+y,x+y,\ldots x+y)$ is in V.

2) $(x,x,\ldots x)+(y,y\ldots y) = (y,y,\ldots y)+(x,x\ldots x)$

3) $(x,x\ldots x)+((y,y\ldots y),(z,z\ldots z)) = ((x,x\ldots,x),(y,y\ldots y))+(z,z\ldots z)$

4) $(0,0,\ldots 0)+(x,x\ldots x) = (x,x,\ldots x)$.

5) $(x,x,\ldots x)+(-x,-x,\ldots -x) = (0,0,\ldots 0)$.

6) $k(x,x,\ldots x) = (kx,kx,\ldots kx)$ is in V

7) $k((x,x\ldots x)+(y,y\ldots y)) = k(x,x,\ldots x)+k(y,y\ldots y)$

8) $(k+m)(x,x,\ldots x) = ((k+m)x,(k+m)x,\ldots (k+m)x)$

$= k(x,x,\ldots x)+m(x,x,\ldots x)$

9) $k(m(x,x\ldots,x)) = (kmx,kmx,\ldots,kmx) = (km)(x,x,\ldots x)$

10) $1(x,x\ldots x) = (x,x\ldots x)$

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Q7) Set of triples real numbers defined by $k(x,y,z) = (k^2x, k^2y, k^2z)$

8) $(k+m)u = ((k+m)^2x, (k+m)^2y, (k+m)^2z)$

$ku + mu = (k^2x, k^2y, k^2z) + (m^2x, m^2y, m^2z)$

$= ((k^2+m^2)x, (k^2+m^2)y, (k^2+m^2)z)$

Axioms 8 Fails.

Q8) set of all 2x2 invertable matrices.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ are invertable.

but Axioms 1

1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not invertable.

Q9) set of all 2x2 matrix of form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

1) sum of two diagonal is also a diagonal.

2) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

3) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \left\{\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} e & 0 \\ 0 & d \end{bmatrix}\right\} = \left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}\right\} + \begin{bmatrix} e & 0 \\ 0 & d \end{bmatrix}$

4) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

5) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

6) Scaler multipikation of a diagonal is also a diagonal.

7) $k\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}\right\} = k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + k\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$

8) $(k+m)\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = k\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} + m\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

9) $(km)\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = k\begin{bmatrix} am & 0 \\ 0 & bm \end{bmatrix}$

10) $1\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

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Q10) set of real valued function f defined everywhere on the real line and such that $f(1) = 0$

1) If f and g are in v then f+g is a function defined for all real numbers

$(f+g)(1) = f(1) + g(1) = 0$

6) If k is scaler and f is in v then

$(kf)(1) = k(f(1)) = 0$

→ verifies all axioms

Q11) set of pair of all number in form $(1, x)$

$(1, y) + (1, y') = (1, y+y')$ and $k(1, y) = (1, ky)$

1) $(1, y) + (1, y') = (1, y+y')$

2) $(1, y) + (1, y') = (1, y+y') = (1, y') + (1, y)$

3) $(1, y) + ((1, y') + (1, y'')) = (1, y) + (1, y'+y'') = (1, y+y'+y'')$
$= ((1, y) + (1, y')) + (1, y'')$

4) $0 = (1, 0)$ $\quad (1,0) + (1, y) = (1, y)$

5) $(1, y) + (1, -y) = (1, 0)$

6) $k(1, y) = (1, ky)$

7) $k((1, y) + (1, y')) = k(1, y) + k(1, y')$

8) $(k+m)(1, y) = (1, (k+m)y) = k(1, y) + m(1, y)$

9) $k(m(1, y)) = k(1, my) = (1, kmy) = (km)(1, y)$

10) $1(1, y) = (1, y)$

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Q12) set of polynomials ~~to~~ of form ... $a_0 + a_1x$

$(a_0+a_1x) + (b_0+b_1x) = (a_0+b_0) + (a_1+b_1)x$

$k(a_0+a_1x) = (ka_0) + (ka_1)x$

1) $(a_0 + b_0 x) + (a_1+b_1x) = (a_0+a_1) + (b_0+b_1)x$

3) $(a_0+b_0x) + ((a_1+b_1x) + (a_2+b_2x)) =$

$((a_0+b_0x) + (a_1+b_1x)) + (a_2+b_2x)$.

4) $(0+0x) + (a+bx) = a+bx$.

5) $(a+bx) + (-a-bx) = 0+0x$.

6) $k(a+bx) = ka + (kb)x$

7) $k((a_0+b_0x) + (a_1+b_1x)) = k(a_0+b_0x) + k(a_1+b_1x)$

8) $(k+m)(a+bx) = (k+m)a + (k+m)bx$

$= k(a+bx) + m(a+bx)$

9) $k(m(a+bx)) = kma + kmbx = (km)(a+bx)$

10) $1(a+bx) = a+bx$.

Exercise 4.2

Q) use the subspaces test to determine which of the sets are subspaces of R^3.

Q1) a) All vectors of the form $(a, 0, 0)$.

1) Closure under Addition $(a,0,0) + (b,0,0) = (a+b,0,0)$

~~2)~~ Scalar Multiplication of vector $k(a,0,0) = (ka, 0, 0)$

$\therefore (a,0,0)$ is a vector Subspace.

b) All vectors of form $(a,1,1)$

Vector Addition $(a,1,1) + (b,1,1) = (a+b, 2, 2)$

$\therefore (a,1,1)$ is not a vector Subspace.

c) Form (a,b,c) where $b = a+c$

~~$u + (-u) =$~~ $(0,0,0) \Rightarrow b = a+c = b=0, a=0, c=0$.

Addition $(a, a+c, c) + (d, d+e, e) = (a+d, a+c+d+e, c+e)$

Multiplication $k(a,b,c) = (ka, k(a+c), kc)$

$= (ka, ka+kc, kc)$

$\Rightarrow (a,b,c)$ is a vector Subspace.

Q2) a) All vectors of form (a, b, c), where $b = a+c+1$

Addition: $(a, a+c+1, c) + (a', a'+c'+1, c') = (a+a', a+c+a'+c'+2, c+c')$

$\therefore$ ~~this is~~ (a, b, c) is not a vector subspace.

b) form $(a, b, 0)$

Addition $(a, b, 0) + (a', b', 0) = (a+a', b+b', 0)$

Multiplication $k(a, b, 0) = (ka, kb, 0)$

$\therefore$ $(a, b, 0)$ is a vector Subspace.

c) form (a, b, c) $a+b=7$

Addition $(a, b, c) + (a', b', c') = (a+a', b+b', c+c')$

Multiplication $k(a, b, c) = (ka, kb, kc)$

Q3) a) The set of all diagonal $n\times n$ matrices. are subspaces of M_{nn}

Addition
$$\begin{bmatrix} a_{11} & 0 - & 0 \\ 0 & a_{22} - & 0 \\ | & | & | \\ 0 & 0 - & a_{nn} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 - & 0 \\ 0 & b_{22} - & 0 \\ | & | & | \\ 0 & 0 - & b_{nn} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & 0 & - 0 \\ 0 & a_{22}+b_{22} & - 0 \\ | & | & | \\ 0 & 0 & - a_{nn}+b_{nn} \end{bmatrix}$$

Multiplication:
$$k\begin{bmatrix} a_{11} & 0 - & 0 \\ 0 & a_{22} - & 0 \\ 0 & 0 - & a_{nn} \end{bmatrix} = \begin{bmatrix} ka_{11} & 0 - & 0 \\ 0 & ka_{22} - & 0 \\ 0 & 0 - & ka_{nn} \end{bmatrix}$$

$\therefore$ Subspace ~~of~~ of M_{nn}

b) set of all $n\times n$ matrix A such that $\det(A) = 0$

Addition: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\cdot$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\det(\neq 0$

$\therefore$ Not a Subspace.

c) set of all $n\times n$ matrices A such that $tr(A) = 0$

let $A = [a_{ij}]$ and $B = [b_{ij}]$

Add. $tr(A+B) = (a_{11}+b_{11}) + (a_{22}+b_{22}) + \cdots + (a_{nn}+b_{nn})$

Mul. $k\,tr(A) = ka_{11} + ka_{22} + \cdots + ka_{nn}$

$\therefore$ ~~is~~ Subspace of M_{nn}

~~Q4) set of all n×n matrices A such that $A^T = -A$~~

d) set of all symmetric n×n matrices.
symmetric such that $A^T = A$
Addition :- $A^T = A$ and $B^T = B$
$(A+B)^T = A^T + B^T = A+B$
Multiplication :- $k(A^T) = kA^T = kA$
$\therefore$ Subspace of M_{nn}

Q4) a) Set of all n×n matrix A such $A^T = -A$.
$A^T = -A$ and $B^T = -B$
$\therefore$ Addition $(A+B)^T = A^T + B^T = -A-B = -(A+B)$
Multiplication $k(A)^T = kA^T = -kA$
$\therefore$ Subspace of M_{nn}.

b) n×n matrix A for which $Ax=0$ has only trivial solution.
Vector not closed under vector multiplication.
$\therefore$ W is not subspace.

c) n×n matrices A such that $AB = BA$ for some fixed n×n matrix B.
Let A and C are both in vector W
Add $AB = BA$ and $CB = BC$.
$(A+C)B = AB + CB = BA + BC$
Multiplication : $(kA)B = k(AB) = k(BA) = B(kA)$
$\therefore$ W is a subspace of M_{nn}

Q5) • use the Subspace test to determine which of the sets are subspaces P_3.

a) All polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ for which $a_0 = 0$.

set contains atleast one polynomial, $0 + 0x + 0x^2 + 0x^3 = 0$

Addition: $(0 + a_1x + a_2x^2 + a_3x^3) + (0 + b_1x + b_2x^2 + b_3x^3)$

$= 0 + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3.$

Multiplication: $k(0 + a_1x + a_2x^2 + a_3x^3) = 0 + k(a_1x) + k(a_2x^2) + k(a_3x^3)$

$\therefore$ W is a Subspace of P_3

b) All polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ which $a_0 + a_1 + a_2 + a_3 = 0$.

$a_0 = -a_1 - a_2 - a_3.$

vector can be expressed as $-a_1 - a_2 - a_3 + a_1x + a_2x^2 + a_3x^3$.

Addition: $(-a_1 - a_2 - a_3 + a_1x + a_2x^2 + a_3x^3) + (-b_1 - b_2 - b_3 + b_1x + b_2x^2 + b_3x^3)$

$= (-a_1 - a_2 - a_3 - b_1 - b_2 - b_3) + (a_1+b_1)x + (a_2+b_2)x^2 + (a_3+b_3)x^3$.

$= 0$

Multiplication: $k(-a_1 - a_2 - a_3 + a_1x + a_2x^2 + a_3x^3)$

$= (-ka_1 - ka_2 - ka_3) + (ka_1)x + (ka_2)x^2 + (ka_3)x^3. = 0.$

$\therefore$ W is subspace of P^3.

Q12) a) $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Addition: $A_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $A_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$A_1\begin{bmatrix} 1 \\ -1 \end{bmatrix} + A_2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(A_1 + A_2)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Multiplication: $k(A)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = k\left(A\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = k\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore$ W is subspace of M_{22}

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(b) $A \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} A$

Addition: $A \cdot \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} A$

$B \cdot \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} B$

$(A+B) \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = A \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} + B \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}$

$(A+B) \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} A + \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} B$

$(A+B) \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} (A+B)$

Multiplication: $(kA) \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} = k\left(A \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix}\right)$

$= k \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} A = \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} (kA)$

Hence A is a Subset of M_{22}

(c) A for which $\det(A) = 0$.

Addition: $A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$

$A+B = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$\det(A+B) \neq 0$

Hence A is not a Subspace

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Q13) a) All vectors of the form (a, a^2, a^3, a^4)

Addition $v = (1, 1, 1, 1)$ $w = (2, 4, 8, 16)$

$v + w = (3, 5, 9, 17)$

∴ w is not a subspace of R^4.

b) $(a, 0, b, 0)$

Addition: $v = (a, 0, b, 0)$. $w = (c, 0, d, 0)$.

$v + w = (a+c, 0, b+d, 0)$

Multiplication: $k(v) = k(a, 0, b, 0) = (ka, 0, kb, 0)$

Hence w is a subspace of R^4.

Q19) Determine whether $Ax=0$ is line through origin, plane through origin or origin. If plane or line, find its equation.

a) $A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & -1 & 0 \\ 2 & -4 & -5 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 3 & -1 & 0 & 0 \\ 2 & -4 & -5 & 0 \end{array}\right]$$

$2R_1 + R_3$ $$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & -2 & -3 & 0 \end{array}\right]$$

$3R_1 + R_2$ $$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & -2 & -3 & 0 \end{array}\right]$$

$R_2 + R_3$ & $\frac{1}{2}R_2$ $$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-R_2 + R_1$ & $-R_1$ $$\left[\begin{array}{ccc|c} +1 & 0 & +1/2 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + \frac{1}{2}z = 0$

$y + \frac{3}{2}z = 0$

let $z = t$

$x = -\frac{1}{2}t$

$y = -\frac{3}{2}t$

$z = t$

Line passing through Origin.

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

$-2R_1+R_2 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -3 \\ 1 & 0 & 8 \end{bmatrix}$

$-2R_2+R_1 \left[\begin{array}{ccc|c} 1 & 0 & 9 & 0 \\ 0 & 1 & -3 & 0 \\ 1 & 0 & 8 & 0 \end{array}\right]$

$-R_1+R_3$ & $-R_3$ $\left[\begin{array}{ccc|c} 1 & 0 & 9 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & +1 & 0 \end{array}\right] \Rightarrow 3R_2+R_1 \left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & +1 & 0 \end{array}\right]$

$3R_3+R_2$ & $-3R_2+R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$x_1 = 0$

$x_2 = 0$ At Origin

$x_3 = 0$

(c) $A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix}$

$-2R_1+R_2$ & $-3R_1+R_3$ $\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 - 3x_2 + x_3 = 0$

passes through Origin.

(d) $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 4 \\ 3 & 1 & 11 \end{bmatrix}$

$-2R_1+R_2$ & $-3R_1+R_3$ $\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 4 & 8 & 0 \end{array}\right] = R_2+R_1$ & $\frac{1}{4}R_3$ $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$-R_2+R_3 \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow x_1 + 3x_3 = 0 \Rightarrow x_1 = -3t$

$x_2 + 2x_3 = 0 \qquad x_2 = -2t$

let $x_3 = t$ $\qquad x_3 = t$

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Q20) Show sets of functions are subspaces of $F(-\infty,\infty)$

(a) All Continuous function on $(-\infty,\infty)$

Addition: Since sum of two continuous function is always continuous $\therefore f+g$ is also continuous on $(-\infty,\infty)$

Multiplication: (kf) Scalar multiple of continuous function is continuous $\therefore k(f)$ is also continuous on $(-\infty,\infty)$

$\therefore T$ is Subspace of $f(-\infty,\infty)$

(b) differentiable functions on $(-\infty,\infty)$.

$T = \{f : f \text{ is differentiable on } (-\infty,\infty)\}$

i) Addition: Sum of two differentiable function is always differentiable $\therefore (f+g)$ is diff on $(-\infty,\infty)$

ii) Multiplication: Scalar Multiple of diff function is diff $\therefore kf$ is diff on $(-\infty,\infty)$

$\therefore T$ is Subspace of $f(-\infty,\infty)$

(c) Diff function on $(-\infty,\infty)$ satisfy $f' + 2f = 0$.

$T = \{f : f \text{ is diff on } (-\infty,\infty), f' + 2f = 0\}$

Finding form of function f which is used in set T.

$$\frac{df}{dx} + 2f = 0 \Rightarrow \frac{df}{dx} = -2f \Rightarrow \frac{df}{f} = -2dx$$

Integrating:
$$\int \frac{df}{f} = \int -2dx$$

$$\ln f = -2x + \ln c \Rightarrow f = ce^{-2x}$$

$T = \{ce^{-2x}\}$

1) Addition: $f+g = c_1e^{-2x} + c_2e^{-2x} = (c_1+c_2)e^{-2x}$

2) Multiplication: $kf = k(c_1e^{-2x}) = (kc_1)e^{-2x}$

$\therefore T$ is a Subspace of $f(-\infty,\infty)$.

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Q21) continous function $f: f(x)$ on $[a,b]$ such that $\int_a^b f(x)dx = 0$ is a subspace of $C[a,b]$.

Addition: $f(x) + g(x) = \int_a^b f(x)dx + \int_a^b g(x)dx$
$= 0 + 0 = \boxed{0}$

Multiplication: $kf(x) = k \int_a^b f(x)dx = k(0) = \boxed{0}$

$\therefore$ Given Set is a subspace.

Q22) Show sol vectors of a consistent nonhomo system of m linear Eqn in n unknowns do not form subspace of R^n.

Non homogenous System $Ax = b$.

Addition: $Ax_1 + Ax_2 = b + b = 2b$

where $A = \begin{bmatrix} a_{11} & a_{12} - & a_{1n} \\ a_{21} & a_{22} - & a_{2n} \\ | & | & | \\ a_{m1} & a_{m2} - & a_{mn} \end{bmatrix}$ $, x = \begin{bmatrix} x_1 \\ x_2 \\ | \\ x_n \end{bmatrix}$ $, b = \begin{bmatrix} b_1 \\ b_2 \\ | \\ b_n \end{bmatrix}$

$\Rightarrow$ Not a Subspace of R^n.

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Exercise 4.3

Q3) Linear Combination of following?

$$A = \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix} ?$$

a) $\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$

$$\begin{array}{c} \\ \\ -R_3+R_4 \\ \\ \end{array}\left[\begin{array}{ccc|c} 4 & 1 & 0 & 6 \\ 0 & -1 & 2 & -8 \\ -2 & 2 & 1 & -1 \\ 0 & 1 & 3 & -7 \end{array}\right] = \begin{array}{c} \cancel{2R_3+R_1} \\ \frac{1}{4}R_1 \\ 2R_1+R_3 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & \frac{6}{4} \\ 0 & -1 & 2 & -8 \\ 0 & \frac{5}{2} & 1 & 2 \\ 0 & 1 & 3 & -7 \end{array}\right]$$

$$\begin{array}{c} -2R_3+R_2 \\ \cancel{-\frac{1}{6}R_2} \\ \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & \frac{6}{4} \\ 0 & -6 & 0 & -12 \\ 0 & \frac{5}{2} & 1 & 2 \\ 0 & 1 & 3 & -7 \end{array}\right] = -\frac{1}{6}R_2 \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & \frac{6}{4} \\ 0 & 1 & 0 & 2 \\ 0 & \frac{5}{2} & 1 & 2 \\ 0 & 1 & 3 & -7 \end{array}\right]$$

$$= \begin{array}{c} -\frac{1}{4}R_2+R_1 \\ \& \\ -\frac{5}{2}R_2+R_3 \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 1 & 3 & -7 \end{array}\right] = \begin{array}{c} -R_2+R_4 \\ \& -3R_3+R_4 \\ \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

System is Consistent $\therefore$ $\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$ is linear Combination.

b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

It is a linear Combination Since

$$0\begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + 0\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + 0\begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a[A] + b[B] + c[C] = [\text{given}]$

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(c) $\begin{bmatrix} -1 & 5 \\ 7 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 4 & 1 & 0 & -1 \\ 0 & -1 & 2 & 5 \\ -2 & 2 & 1 & 7 \\ -2 & 3 & 4 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$0 = 1$

∴ System not Consistant $\therefore \begin{bmatrix} -1 & 5 \\ 7 & 1 \end{bmatrix}$ not linear Combination.

Q4) which of the poly is a linear of

$p_1 = 2 + x + x^2$, $p_2 = 1 - x^2$, $p_3 = 1 + 2x$

(a) $1 + x$.

$$A = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 0 & 0 \end{array}\right]$$

$\Rightarrow R_1 \leftrightarrow R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \end{array}\right]$ $-2R_1 + R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & -1 \\ 1 & -1 & 0 & 0 \end{array}\right]$

$= -R_1 + R_3 \; R_2 \leftrightarrow R_3$ $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & -3 & -1 \end{array}\right]$ $= -R_1 + R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & -3 & 0 \end{array}\right]$

$R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & -5 & -1 \end{array}\right]$ $= \begin{matrix} -R_2 \\ -\frac{1}{5}R_3 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right]$

$\Rightarrow -2R_3 + R_2$
$\quad \& \; -2R_3 + R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{5} \\ 0 & 1 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & \frac{2}{5} \end{array}\right]$

$1+x$ is a linear Combination

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(b) $1+x^2$.

$$A = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right]$$

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4/5 \\ 0 & 1 & 0 & -1/5 \\ 0 & 0 & 1 & -2/5 \end{array}\right]$$

$\therefore$ $1+x^2$ is a linear Combination

(c) $1+x+x^2$.

$$A = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 1 & 1 \end{array}\right]$$

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/5 \\ 0 & 1 & 0 & -2/5 \\ 0 & 0 & 1 & 1/5 \end{array}\right]$$

$\therefore$ $1+x+x^2$ is a linear Combination.

Q9) Determine whether the poly span P_2.

$p_1 = 1-x+2x^2$, $p_2 = 3+x$, $p_3 = 5-x+4x^2$, $p_4 = -2-2x+2x^2$.

for ~~p2~~ P_2 p is $p = a_0+a_1x+a_2x^2$.

$$A = \left[\begin{array}{cccc|c} 1 & 3 & 5 & -2 & a_0 \\ -1 & 1 & -1 & -2 & a_1 \\ 2 & 0 & 4 & 2 & a_2 \end{array}\right]$$

$$R_1+R_2 \left[\begin{array}{cccc|c} 1 & 3 & 5 & -2 & a_0 \\ 0 & 4 & 4 & -4 & a_0+a_1 \\ 2 & 0 & 4 & 2 & a_2 \end{array}\right]$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & \frac{1}{4}a_0 - \frac{3}{4}a_1 \\ 0 & 1 & 1 & -1 & \frac{1}{4}a_0 + \frac{1}{4}a_1 \\ 0 & 0 & 0 & 0 & \frac{-1}{2}a_0 + \frac{3}{2}a_1 + a_2 \end{array}\right]$$

∴ System has no Solution
polynomials do not span P_2.

Q11) whether matrices span M_{22}

a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\det(A) = 1\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} - 1\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} + 0 - 0$$

$$1(-1) - 1(-1) = -1 + 1 = 0.$$

∴ Does not Span M_{22}.

b) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = 1\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 0 + 1\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} - 1\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= 1(1) + 1(0) - 1(1) = 1 - 1 = 0.$$

∴ Does not Span M_{22}

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(c) $\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}1&1\\0&0\end{bmatrix}, \begin{bmatrix}1&1\\1&0\end{bmatrix}, \begin{bmatrix}1&1\\1&1\end{bmatrix}$

$$A = \begin{bmatrix}1&1&1&1\\0&1&1&1\\0&0&1&1\\0&0&0&1\end{bmatrix} = 1\begin{vmatrix}1&1&1\\0&1&1\\0&0&1\end{vmatrix} - 1\begin{vmatrix}0&1&1\\0&1&1\\0&0&1\end{vmatrix} + 1\begin{vmatrix}0&1&1\\0&0&1\\0&0&1\end{vmatrix} - 1\begin{vmatrix}0&1&1\\0&0&1\\0&0&0\end{vmatrix}$$

$$= 1(1) - 1(0) + 1(0) - 1(0) = \boxed{1}$$

Spanning M_{22}.

Exercise 4.4

Q2) Show vector set forms basis for R^3.

$\{(3,1,-4),\ (2,5,6),\ (1,4,8)\}$

$$A = \begin{bmatrix}3&2&1\\1&5&4\\-4&6&8\end{bmatrix}$$

$\det(A) = 3\big(5(8)-4(6)\big) - 2\big(8+4(4)\big) + 1\big(6+20\big)$

$= 26 \neq 0$

Spans R^3

For Linear Independence

$$A = \left[\begin{array}{ccc|c}3&2&1&0\\1&5&4&0\\-4&6&8&0\end{array}\right] \quad R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|c}1&5&4&0\\3&2&1&0\\-4&6&8&0\end{array}\right]$$

$$\begin{array}{l}-3R_1+R_2\\ \& \ 4R_1+R_2\end{array}\left[\begin{array}{ccc|c}1&5&4&0\\0&-13&-11&0\\0&26&24&0\end{array}\right] \quad \tfrac{1}{2}R_3 \left[\begin{array}{ccc|c}1&5&4&0\\0&-13&-11&0\\0&13&12&0\end{array}\right]$$

$$R_2+R_3\left[\begin{array}{ccc|c}1&5&4&0\\0&-13&-11&0\\0&0&1&0\end{array}\right] \quad \begin{array}{l}11R_3+R_2\\ \& -4R_3+R_1\\ \& -\tfrac{1}{13}R_2\end{array}\left[\begin{array}{ccc|c}1&5&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$$

$$-5R_2+R_1\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$$

$K_1 = K_2 = K_3 = 0$ Linearly Independent

Q4) poly form basis for P_3.

$1+x, \quad 1-x, \quad 1-x^2, \quad 1-x^3$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

$$1\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} + 1\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} - 1\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 1(-1) - 1(-1) + 1(0) - 1(0) = \boxed{2}$$

Spans P_3.

For linearly Independent:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right] \Rightarrow \begin{array}{c} \\ -R_1+R_2 \\ -R_3 \\ -R_4 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 1 & 0 \\ 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 \end{array}\right]$$

$$\begin{array}{c} -R_3+R_2 \\ \& -R_4+R_2 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{array}{c} -R_3+R_1 \\ \& -R_4+R_1 \\ \& \frac{1}{2}R_2 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$-R_2+R_1 \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$k_1 = k_2 = k_3 = k_4 = 0$;

Linearly Independent.

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Q5) $\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}$, $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 0 & 0 & 1 \\ 6 & -1 & -8 & 0 \\ 3 & -1 & -12 & -1 \\ -6 & 0 & -4 & 2 \end{vmatrix} = 3\begin{vmatrix} -1 & -8 & 0 \\ -1 & -12 & -1 \\ 0 & -4 & 2 \end{vmatrix} - 0 + 0 - 1\begin{vmatrix} 6 & -1 & -8 \\ 3 & -1 & -12 \\ -6 & 0 & -4 \end{vmatrix}$$

$$= 3\{-1(-12(2) - (-1)(-4)) + 8((-1)(2) - 0) + 0\} - 1\{6(4-0) + 1(-12 - 6(12)) - 8(-6)\}$$

$$= 3(12) - 1(-12) = 48$$ Spans M_{22}.

For Linearly Independent:

$$= \left[\begin{array}{cccc|c} 3 & 0 & 0 & 1 & 0 \\ 6 & -1 & -8 & 0 & 0 \\ 3 & -1 & -12 & -1 & 0 \\ -6 & 0 & -4 & 2 & 0 \end{array}\right] \quad = \begin{array}{l} -2R_1+R_2 \\ -R_1+R_3 \\ 2R_1+R_4 \end{array} \begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & -1 & -8 & -2 \\ 0 & -1 & -12 & -2 \\ 0 & 0 & -4 & 4 \end{bmatrix}$$

$$= -2R_4+R_2 \begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & -1 & 0 & -10 \\ 0 & -1 & -12 & -2 \\ 0 & 0 & -4 & 4 \end{bmatrix} \quad = -R_2+R_3 \begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & -1 & 0 & -10 \\ 0 & 0 & -12 & 8 \\ 0 & 0 & -4 & 4 \end{bmatrix}$$

$$-2R_1+R_3 \begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & -1 & 0 & -10 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & -4 & 4 \end{bmatrix} \quad = \begin{array}{l} R_3+R_4 \\ \& -\tfrac{1}{4}R_3 \\ \& \tfrac{1}{4}R_4 \end{array} \begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & -1 & 0 & -10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{l} -1R_4+R_1 \\ \& \ 10R_4+R_2 \\ \& \ -R_2 \\ \& \ \tfrac{1}{3}R_1 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

∴ Linearly Independent

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Exercise 4.5

Q) basis for solution space of homo system and find dimension

Q1)
$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + x_3 = 0$$

$$\begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow \begin{matrix} R_1 + R_3 \\ 2R_1 + R_2 \end{matrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & \cancel{0}4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$-2R_3 + R_2 \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow \tfrac{1}{2}R_2 \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = x_3$, $x_2 = 0$, let $x_3 = t$

$x_1 = t$, $x_2 = 0$, $x_3 = t$

$t = (\cancel{t}, 1, 0, 1)$ dim $= 1$

3)
$$2x_1 + x_2 + x_3 = 0$$
$$x_1 + 5x_3 = 0$$
$$x_2 + x_3 = 0$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$-2R_1 + R_2 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} = -R_2 + R_3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \tfrac{1}{2}R_3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{matrix} R_3 + R_2 \\ \& -R_3 + R_1 \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x_1 = x_2 = x_3 = 0$ Dimension $= 0$

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4) $x_1 - 4x_2 + 3x_3 - x_4 = 0$

$2x_1 - 8x_2 + 6x_3 - 2x_4 = 0.$

$$\begin{bmatrix} 1 & -4 & 3 & -1 \\ 2 & -8 & 6 & -2 \end{bmatrix} \xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & -4 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 4x_2 + 3x_3 - x_4 = 0 \Rightarrow x_1 = 4x_2 - 3x_3 + x_4$

let $x_2 = r$, $x_3 = s$, $x_4 = t$.

$x_1 = 4r - 3s + t$

$x_2 = r$ $\qquad r = (4, 1, 0, 0)$

$x_3 = s$ $\qquad s = (-3, 0, 1, 0)$

$x_4 = t$ $\qquad t = (1, 0, 0, 1)$ $\qquad$ dim $= 3$.

Q7). All vectors of the form (a, b, c) where $b = a + c$.

$(a, a+c, c) = a(1, 1, 0) + c(0, 1, 1)$ we can express it as Span where $S = \{(1,1,0), (0,1,1)\}$ and S is linearly Independent

S form basis for the subspace dim $= 2$

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Exercise 4.7

Q3) ⓐ $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 2 \end{array}\right] \Rightarrow R_1 \leftrightarrow R_2 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & -1 \\ 2 & 1 & 3 & 2 \end{array}\right]$

$= -R_1 + R_2 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 3 & 2 \end{array}\right] \Rightarrow -2R_1 + R_3 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 2 \end{array}\right]$

$= -R_2 + R_3 \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ $Ax = b$ is inconsistent.

b is not a column space of A.

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Q4) (a) $A = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & 1 & 0 \end{array}\right]$

$\begin{array}{c} R_1+R_2 \\ R_1+R_3 \\ \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & -2 & 2 & 2 \end{array}\right]$

$= \frac{1}{2}R_3 \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & -1 & 1 & 1 \end{array}\right] \Rightarrow -R_3+R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & -1 & 1 & 1 \end{array}\right]$

$= R_2 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right]$ $Ax = b$ is inconsistent.

$\therefore$ b is not A domain space of A.

Q8) (a) $x_1 - 2x_2 + x_3 + 2x_4 = -1$ Find vector form of
$2x_1 - 4x_2 + 2x_3 + 4x_4 = -2$ $Ax = b$ then use it
$-x_1 + 2x_2 - x_3 - 2x_4 = 1$ to find $Ax = 0$
$3x_1 - 6x_2 + 3x_3 + 6x_4 = -3$

$= \left[\begin{array}{cccc|c} 1 & -2 & 1 & 2 & -1 \\ 2 & -4 & 2 & 4 & -2 \\ -1 & 2 & -1 & -2 & 1 \\ 3 & -6 & 3 & 6 & -3 \end{array}\right]$ $\begin{array}{c} = R_1+R_3 \\ -2R_1+R_2 \\ -3R_1+R_4 \\ \end{array} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 - 2x_2 + x_3 + 2x_4 = -1$ $\Rightarrow x_1 = 1 + 2x_2 - x_3 - 2x_4$

let $x_2 = r$; $x_3 = s$ $x_4 = t$.

$x_1 = -1 + 2r - s - 2t$. $(-1, 0, 0, 0)$

$x_2 = r$ $r(2, 1, 0, 0)$

$x_3 = s$ $s(-1, 0, 1, 0)$

$x_4 = t$ $t(-2, 0, 0, 1)$

~~for $Ax = 0$~~

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Q11) a) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ Col: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

Rows: $[1\ 0\ 2]$, $[0\ 0\ 1]$

(b) $\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Column: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

Rows: $[1\ -3\ 0\ 0]$, $[0\ 1\ 0\ 0]$

Q12) a) $\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Column: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$

Rows: $[1\ 2\ 4\ 5]$, $[0\ 1\ -3\ 0]$, $[0\ 0\ 1\ -3]$, $[0\ 0\ 0\ 1]$

(b) $\begin{bmatrix} 1 & 2 & -1 & 5 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Column: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 4 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 3 \\ -7 \\ 1 \end{bmatrix}$

Rows: $[1\ 2\ -1\ 5]$, $[0\ 1\ 4\ 3]$, $[0\ 0\ 1\ -7]$, $[0\ 0\ 0\ 1]$

Q14) $(1, 1, -4, -3)$, $(2, 0, 2, -2)$, $(2, -1, 3, 2)$.

$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & -1 \\ -4 & 2 & 3 \\ -3 & -2 & 2 \end{bmatrix} \Rightarrow R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 2 \\ -4 & 2 & 3 \\ -3 & -2 & 2 \end{bmatrix} = -R_1+R_2 \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ -4 & 2 & 3 \\ -3 & 2 & 2 \end{bmatrix}$

$-R_2+R_3$ & $3R_1+R_4$ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ -4 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix} = -R_1+R_2 \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 4 \\ -4 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix} = \frac{1}{4}R_2$ & $-\frac{1}{4}R_3$ $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

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$$= \xrightarrow{R_2+R_4} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \xrightarrow[\text{\& } -R_1+R_2]{R_2+R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

R

$$\xrightarrow[\text{\& } R_4 \leftrightarrow R_2,\ \frac{1}{2}R_2]{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Form the basis for col space.

Exercise 4.8

Q3) $A = \begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & -3 \\ 1 & 1 & 4 \end{bmatrix}$; $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

a) rank $(A) = 3$; nullity $(A) = 0$

b) rank (A) + nullity $(A) = 3+0 = 3$ (no. of col)

c) 3 leading variables ; 0 parameter in general solution $n-r$

Q6) $A = \begin{bmatrix} 0 & 2 & 2 & 4 \\ 1 & 0 & -1 & -3 \\ 2 & 3 & 1 & 1 \\ -2 & 1 & 3 & -2 \end{bmatrix}$; $R = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

a) rank $(A) = 3$; nullity $= 1$

b) rank (A) + nullity $(A) = 3+1 = 4$ (col)

c) 3 leading variables, 1 parameter in general solution

Q8) $m \times n$ matrix largest value possible for rank and smallest for nullity.

- largest possible value of rank is when every row and column contains 1 leading 1
- Smallest value possible for nullity is 0.

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Q13) $T: R^5 \to R^3$

$T(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_2,\ x_2 + x_3 + x_4,\ x_4 + x_5)$

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$-R_3 + R_2 \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

rank$(A) = 3$

nullity $(A) = 0$

$$-R_2 + R_1 \begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Exercise 5.1

Signature

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