Lecture# 04

De Morgan's laws

De Morgan's laws state that:

The negation of an **and** proposition is logically equivalent to the **or** proposition in which each component is negated.

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$

The negation of an **or** proposition is logically equivalent to the **and** proposition in which each component is negated.

2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Applying De-Morgan's Law

Question: Negate the following compound Propositions

1. John is six feet tall and he weights at least 200 pounds.

2. The bus was late or Tom's watch was slow.

Applying De-Morgan's Law

Question: Negate the following compound Propositions

1. John is six feet tall and he weights at least 200 pounds.
2. The bus was late or Tom's watch was slow.

Solution

a) John is not six feet tall or he weighs less than 200 pounds.

b) The bus was not late and Tom's watch was not slow.

Inequalities and De Morgan's Laws

Question Use De Morgan's laws to write the negation of

$$-1 < x \leq 4$$

Solution: The given proposition is equivalent to

$$-1 < x \quad \text{and} \quad x \leq 4,$$

By De Morgan's laws, the negation is

$$-1 \geq x \quad \text{or} \quad x > 4.$$

Laws of Logic

1. Commutative laws

   $p \wedge q \equiv q \wedge p$; $p \vee q \equiv q \vee p$

2. Associative laws

   $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$; $p \vee (q \vee r) \equiv (p \vee q) \vee r$

3. Distributive laws

   $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
   $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Laws of Logic

4. Identity laws

   $p \wedge t \equiv p$; $p \vee c \equiv p$

5. Negation laws

   $p \vee \neg p \equiv t$; $p \wedge \neg p \equiv c$

6. Double negation law

   $\neg(\neg p) \equiv p$

7. Idempotent laws

   $p \wedge p \equiv p$; $p \vee p \equiv p$

Laws of Logic

8. Universal bound laws

   $p \vee t \equiv t$; $p \wedge c \equiv c$

9. Absorption laws

   $p \wedge (p \vee q) \equiv p$; $p \vee (p \wedge q) \equiv p$

10. Negation of *t* and *c*

    $\neg t \equiv c$; $\neg c \equiv t$

Example

Show that the proposition form $p \vee \neg p$ is a tautology and the proposition form $p \wedge \neg p$ is a contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Exercise: If t is a tautology and c is contradiction, show that $p \vee t \equiv p$ and $p \wedge c \equiv c$?

Propositional Equivalences

Constructing New Logical Equivalences

- Example: Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

  Solution:

  $\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$ by example discussed in slide 66

  $\equiv \neg(\neg p) \wedge \neg q$ by the second De Morgan law

  $\equiv p \wedge \neg q$ by the double negation law

- Example: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

  Solution: To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to T.

  $(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$ by example already discussed

  $\equiv (\neg p \vee \neg q) \vee (p \vee q)$ by the first De Morgan law

  $\equiv (\neg p \vee p) \vee (\neg q \vee q)$ by the associative and communicative law for disjunction

  $\equiv T \vee T$

  $\equiv T$

- Note: The above examples can also be done using truth tables.

Exercise

Using laws of logic, show that

r-(r-p /\ q) A(p V q) = p.

Solution

Take r-(r-p x q) A(p v q)

=(r-(r-p) v +q) A(p v q), (by De Morgan's laws)

=(p v r-q) A(p v q), (by double negative law)

=p v(r-q "q), (by distributive law)

=p v(q" ·q), (by the commutative law)

=p v c, (by the negation law)

=p, (by the identity law)

Skill in simplifying proposition forms is useful in constructing logically efficient computer programs and in designing digital circuits.

Exercise

Prove that $\neg[r \vee (q \wedge (\neg r \rightarrow \neg p))] \equiv \neg r \wedge (p \vee \neg q)$

$\neg[r \vee (q \wedge (\neg r \rightarrow \neg p))]$	
$\equiv \neg r \wedge \neg(q \wedge (\neg r \rightarrow \neg p))$,	De Morgan's law
$\equiv \neg r \wedge \neg(q \wedge (\neg\neg r \vee \neg p))$,	Conditional rewritten as disjunction
$\equiv \neg r \wedge \neg(q \wedge (r \vee \neg p))$,	Double negation law
$\equiv \neg r \wedge (\neg q \vee \neg(r \vee \neg p))$,	De Morgan's law
$\equiv \neg r \wedge (\neg q \vee (\neg r \wedge p))$,	De Morgan's law, double negation
$\equiv (\neg r \wedge \neg q) \vee (\neg r \wedge (\neg r \wedge p))$,	Distributive law
$\equiv (\neg r \wedge \neg q) \vee ((\neg r \wedge \neg r) \wedge p)$,	Associative law
$\equiv (\neg r \wedge \neg q) \vee (\neg r \wedge p)$,	Idempotent law
$\equiv \neg r \wedge (\neg q \vee p)$,	Distributive law
$\equiv \neg r \wedge (p \vee \neg q)$,	Commutative law

Exercise

Prove that:

$\neg(P \leftrightarrow Q) \equiv P \leftrightarrow \neg Q$

$\neg(P \leftrightarrow Q)$

$\equiv \neg((\neg P \vee Q) \wedge (\neg Q \vee P))$ ##

$\equiv \neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)$ *De Morgan's Laws*

$\equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$ *De Morgan's Laws*

$\equiv ((P \wedge \neg Q) \vee Q) \wedge ((P \wedge \neg Q) \vee \neg P)$ *Distributive Laws*

$\equiv ((P \vee Q) \wedge (\neg Q \vee Q)) \wedge ((P \vee \neg P) \wedge (\neg Q \vee \neg P))$ *Distributive Laws*

$\equiv (P \vee Q) \wedge T \wedge T \wedge (\neg Q \vee \neg P)$ *Negation Laws*

$\equiv (P \vee Q) \wedge (\neg Q \vee \neg P)$ *Identify Laws*

$\equiv P \leftrightarrow \neg Q$ ##

** $P \rightarrow Q \equiv \neg P \vee Q$

$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

$\equiv (\neg P \vee Q) \wedge (\neg Q \vee P)$