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DATE

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— QUESTION #1 —

(i) (a) $M \leftrightarrow N$
(b) $N \rightarrow K$
(c) $\neg K \vee I$
(d) $\cancel{\neg I} \rightarrow \neg M \rightarrow I$

(ii) $((p \vee q) \wedge (p \rightarrow r)) \rightarrow (q \vee r)$
$((p \vee q) \wedge (\neg p \vee q)) \rightarrow (q \vee r)$
$\neg(((p \vee q) \wedge (\neg p \vee r))) \vee (q \vee r)$
$p \wedge q)$

$\Rightarrow \neg(((p \wedge (\neg p \vee q) \vee q \wedge (\neg p \vee q))) \rightarrow (q \vee r)$
$\Rightarrow ((p \wedge \neg p) \vee (p \wedge q) \vee (q \wedge \neg p) \vee (q \wedge q)) \rightarrow (q \vee r)$
$\Rightarrow ((F \vee (p \wedge q) \vee (q \wedge \neg p) \vee q)) \rightarrow (q \vee r)$
$\Rightarrow ((p \wedge q) \vee ((q \vee q) \wedge (\neg p \vee q))) \rightarrow (q \vee r)$
$\Rightarrow ((p \wedge q) \vee (q \wedge (\neg p \vee q))) \rightarrow (q \vee r)$
$\Rightarrow ((p \wedge q) \vee (q \wedge \neg p) \vee (\neg p \vee q)) \rightarrow (q \vee r)$ ∵ Absorption Law
$\Rightarrow ((p \wedge q) \vee q) \rightarrow (q \vee r)$ ∵ Absorption Law
$\Rightarrow \cancel{\neg} q \rightarrow q \vee r$
$\Rightarrow \neg q \vee q \vee r$
$\Rightarrow T \vee r$
$\Rightarrow T$

Hence T is tautology

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iii) $((\neg r \rightarrow (s \rightarrow \neg t)) \wedge (\neg r \vee w) \wedge (\neg p \rightarrow s) \wedge (\neg w)) \rightarrow (t \rightarrow p)$

$\neg r \rightarrow (s \rightarrow \neg t)$

$r \vee \neg(s \rightarrow \neg t)$

$r \vee \neg(\neg s \vee \neg t)$

$r \vee (s \wedge t)$

$\Rightarrow \neg r \vee w$ Premise 3

$\Rightarrow \neg r$

$\Rightarrow$ 1. $\neg w$ 1. Hypothesis

$\Rightarrow$ 2. $\neg r \vee w$ 2. Hypothesis

$\Rightarrow$ 3. $\neg r$ 3. Disjunctive syllogism

$\Rightarrow$ 4. $\neg r \rightarrow (s \rightarrow \neg t)$ 4. Hypothesis

$\Rightarrow$ 5. $s \rightarrow \neg t$ 5. Modes ponen 3. and 4.

$\rightarrow$ 6. $\neg p \rightarrow s$ 6. Hypothesis

$\Rightarrow$ 7. $\neg p \rightarrow \neg t$ 7. Hypothetical syllogism 6. and 5.

$\Rightarrow$ 8. $t \rightarrow p$ 8. contrapositive of 7.

iv) a) There exists a student in class who understands every example in lecture notes

b) For every lecture notes, there is a student in your class who understands it

(v) a) $\forall p \exists q\ F(p,q)$

b) $\exists q \forall p\ F(p,q)$

vi) Solution : True (a)

Solution : False (b)

— QUESTION #2 —

(i) $X-(X\cap Y)=(X-Y)$

L.H.S $X-Y = X\cap\bar{Y}$ Complement Law $\because \{A-B=A\cap\bar{B}\}$

$=\{x \mid (x\in X)\cap x\notin (X\cap Y)\}$

$=\{x \mid (x\in X)\wedge x\notin (X\wedge Y)\}$

$=\{x \mid (x\in X)\wedge \neg x\in (X\wedge Y)\}$

$=\{x \mid ((x\in X)\wedge(x\notin X))\vee((x\in X)\wedge(x\notin Y))\}$

Distributive Law

$=\{x \mid (\emptyset)\vee((x\in X)\wedge(x\notin Y))\}$

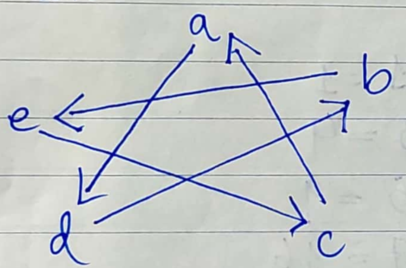
$=\{x \mid (x\in X)\wedge(x\notin Y)\}$

$=\{x \mid (x\in(X-Y)\}$ L.H.S = R.H.S

$R=\{a,b,c,d,e\},\{a,b,c,d,e\}$

(ii) $R=\{(e,c),(c,a),(a,d),(d,b),(b,e)\}$

$R\circ R=$



iii) 1. Transitive Property:

There exists a cycle of length 3 where x beats y, y beats z and z beats x. At a tournament, x cannot beat y in such situation. Not transitive

2. Reflexive: Not reflexive since a tournament graph has no self-loops

3. Symmetric: Not symmetric if x beats y then y doesnot beat x

a b c d

a. b c

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4. Anti-symmetric: It is antisymmetric because if x beat y then it need not be necessary that y beats x.

v) $g: Z \to Z$

$g(n) = n^2+1$

$g(1) = (1)^2+1 = 2$

$g\{\{1\}\} = \{2\}$

$\therefore 2 \neq \{2\}$

v) $f(n+1) = f(n) + 2n+1$

$f(0+1) = f(0) + 2(0)+1 = 0+1 = 1$

$f(1+1) = f(1)+2(1)+1 = 1+3 = 4$

$f(2+1) = f(2) + 2(2)+1 = 4+5 = 9$

$f(3+1) = f(3) + 2(3)+1 = 9+7 = 16$

$f(4+1) = f(4) + 2(4)+1 = 16+9 = 25$

$f(5+1) = f(5) + 2(5)+1 = 25+11 = 36$

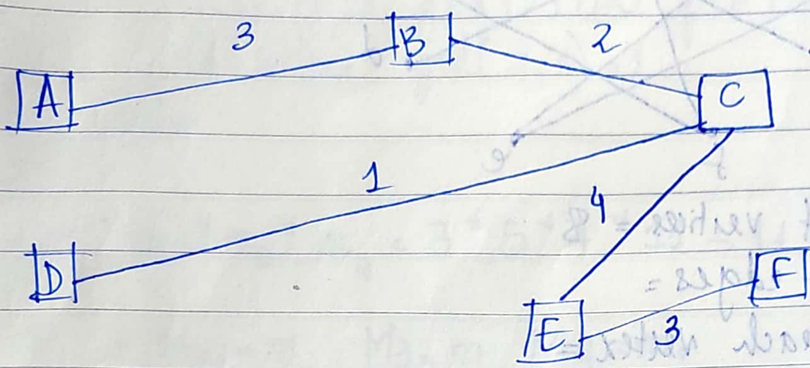
$\therefore f(6) = 36$

vi) m^n ; $m = 3$, $n = 4$

$\therefore 3^4 = 81$ different functions

QUESTION #3

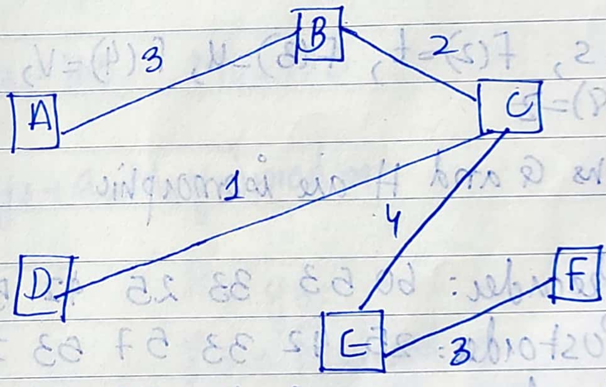
i) Prim's Algorithm:



M.S.T = 13

ii) Kruskal's Algorithm:

(C,D) = 1
(B,C) = 2
(A,B) = 3
(E,F) = 3
(C,E) = 4



M.S.T = 13

(iii)

Node	D(A)	D(B)	D(D)	D(E)
c	$\boxed{(1,c)}$	(7,c)	(2,c)	∞
CA	–	(4,A)	$\boxed{(2,c)}$	∞
CAD	–	$\boxed{(4,A)}$	–	(9,D)
CADB	–	–	–	$\boxed{(5,B)}$
CADBE				

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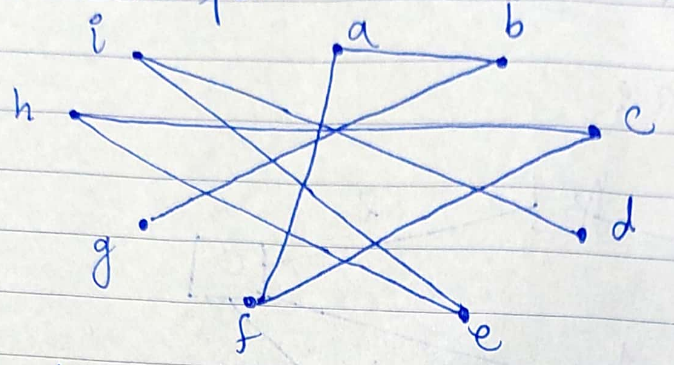
(iii) Hamiltonian path



iv) 1) Number of vertices = 8
2) Number of edges =
3) Degree of each vertex =
4)

f(1) = s, f(2) = t, f(3) = u, f(4) = v, f(5) = w, f(6) = x, f(7) = y
f(8) = z

Graphs G and H are isomorphic

(v) Preorder: 60 53 33 25 42 57 95 78 71
Postorder: 25 42 33 57 53 71 78 95 60.
Inorder: 25 33 42 53 57 60 71 78 95

(vi) (A+B)*C - (D-E) * (F+G)
Prefix: _ * + A B C * _ D E + F G
Postfix: AB + C * DE - FG + * _

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$((p \vee q) \wedge (p \to r)) \to (q \vee r)$

Q4 -i) Dice = 5
Values - 6

Solution- ~~Distinct values = 6^5~~
Total possible values = 6^5
~~Distinct values = 6^4~~
Possible outcomes = $6!$

Distinct values = $\frac{6!}{6^5}$ = 20 Ans

ii) Total possible values = 6^5
possible pairs = $\binom{5}{2}$

distinct values with pairs = $\frac{\binom{5}{2} \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6^5}$ =

= 0.462963 Ans

iii) Women choose chairs = ${}^5C_3$
Men choose chairs = ${}^6C_4$

Combination = ${}^5C_3 \times {}^6C_4$

$= \frac{5!}{(5-3)!\,3!} \times \frac{6!}{(6-4)!\,4!}$

$$= \frac{5!}{2!\cdot 3!} \times \frac{6!}{2!\cdot 4!}$$

$$= \frac{5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1}{2\cdot 1\cdot \cancel{3}\cdot \cancel{2}\cdot 1} \times \frac{6\cdot 5\cdot 4\cdot \cancel{3}\cdot \cancel{2}\cdot 1}{2\cdot 1\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1}$$

$$\frac{6\cdot 5^2\cdot 4}{2^2} = 150 \text{ ways}$$

) ~~$k = \{3 \ldots n\}$~~ $\times$

Choose $k = {}^{n}C_{k}\, C(n,k)$ $\therefore$ from $\{1 \ldots n)$
Choose $k = {}^{n-2}C_{(k-2)}\, C(n-2, k-2)$ from $(3 \ldots n)$

$$C(n,k) - C(n-2, k-2)$$

) balls $n = 4$
box $= 2$

$${}^{4}C_{2} = \frac{4!}{2!\cdot 2!} = 6 \text{ ans}$$

) Begins ~~with~~
Multiple for each digit $= 1 * 26 * 26 * 10 * 10 * 1$
$= 67600$ ans

) $26 * 25 * 24 * 10 * 9 * 8 = 11,232,000$
Ans

QUESTION #5

(i) According to given condition:

$x \equiv 2 \pmod 3$ $\quad x \equiv 4 \pmod 5$ $\quad x \equiv 5 \pmod 7$ $\quad x \equiv 1 \pmod{11}$

$\gcd(2,3)=1$ $\quad \gcd(2,5)=1$ $\quad \gcd(2,7)=1$ $\quad \gcd(2,11)$ $\quad \gcd(4,5)=1$

$\gcd(5,7)=1$ $\quad \gcd(5,11)=1$ $\quad \gcd(7,11)=1$

Pairwise prime

$\because m = m_1 * m_2 * m_3 * m_4 = 3*5*7*11 = 1155$

$M_1 = \frac{m}{m_1}$ $\quad M_2 = \frac{m}{m_2}$ $\quad M_3 = \frac{m}{m_3}$ $\quad M_4 = \frac{m}{m_4}$

$M_1 = \frac{1155}{3}$ $\quad M_2 = \frac{1155}{5}$ $\quad M_3 = \frac{1155}{7}$ $\quad M_4 = \frac{1155}{11}$

$M_1 = 385$ $\quad M_2 = 231$ $\quad M_3 = 165$ $\quad M_4 = 105$

$x = (a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3 + a_4 M_4 y_4) \bmod m$

For y_1:

$y_1 = \bar{M}_1 \bmod m$

$y_1 = 385^{-1} \bmod 3$

$385 = \overset{128}{\cancel{3}}(3) + 1$

$\cancel{128} = \cancel{(1)}$

$3 = (2)(1) + 1$

$\boxed{y_1 = 1}$

$\cancel{1 = 1 \cdot 3 - 1 \cdot 2} \rightarrow (1)$

For y_2:

$y_2 = \bar{M}_2 \bmod m$

$y_2 = 231^{-1} \bmod 5$

$231 = 46(5) + 1$

$5 = (5)(1) + 0$

$\boxed{y_2 = 0}$

For y_3:

$y_3 = \bar{M}_3 \bmod m$

$y_3 \; \cancel{165} = 165^{-1} \bmod 7$

$165 = 23(7) + 4$

$7 = (1)(4) + 3$

$4 = (1)(3) + 1$

$1 = 1 \cdot 4 - 1 \cdot 3 \rightarrow (i)$

$3 = 1 \cdot 7 - 1 \cdot 4 \rightarrow (ii)$

Put (ii) in (i)

$1 = 1 \cdot 4 - 1(1 \cdot 7 - 1 \cdot 4)$

$1 = 1 \cdot 4 - 1 \cdot 7 + 1 \cdot 4$

$1 = 2 \cdot 4 - 1 \cdot 7 \rightarrow (iii)$

$4 = 1 \cdot 165 - 23 \cdot 7 \rightarrow (iv)$

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$\because 1 = 2(1\cdot165 - 23\cdot7) - 1\cdot7$

$1 = 2\cdot165 - 46\cdot7 - 1\cdot7$

$1 = 2\cdot165 - 47\cdot7$

$1 = (165)(2) + (-7)(47)$

$\therefore y_3 = 2$

$y_4 = M_4 \bmod m$

$y_4 = 105^{-1} \bmod 11$

$105 = (9)(11) + 6$

$11 = (1)(6) + 5$

$6 = (1)(5) + 1$

$1 = 1\cdot6 - 1\cdot5 \rightarrow (i)$

$5 = 1\cdot11 - 1\cdot6 \rightarrow (ii)$

Put (ii) in (i)

$1 = 1\cdot6 - 1(1\cdot11 - 1\cdot6)$

$1 = 1\cdot6 - 1\cdot11 + 1\cdot6$

$1 = 2\cdot6 - 1\cdot11 \rightarrow (iii)$

$6 = 1\cdot105 - 9\cdot11 \rightarrow (iv)$

Put (iv) in (iii)

$1 = 2(1\cdot105 - 9\cdot11) - 1\cdot11$

$1 = 2\cdot105 - 18\cdot11 - 1\cdot11$

$1 = 2\cdot105 - 19\cdot11$

$1 = (105)(2) + (-19)(11)$

$\therefore y_4 = 2$

$\therefore x = ((2*385*1) + (4*231*1) + (5*165*2) + (1*105*2)) \bmod 2310$

$x = 3554 \pmod{1155} = 89$

89 coconuts

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ii) $9^{2579} \mod 79$

$a^{p-1} \equiv 1 \pmod p$

$9^{79-1} \equiv 1 \pmod{2579}$

$9^{78} \equiv 1 \pmod{2579} \rightarrow (i)$

$a = qd + r$

$2579 = (33)(78) + 5$

$\Rightarrow 9^{33 \times 78 + 5} \pmod{79}$

$\Rightarrow (9^{78})^{33} \cdot 9^5 \pmod{79}$

$\rightarrow (1)^{33} \cdot 9^5 \pmod{79}$

$\Rightarrow 59049 \mod 79 = 36$

iii) $(x+3y)^9$; $x^7y^2 = ?$

$(x+3y)^9 \Rightarrow \binom{9}{3-1} x^{9-(3-1)} (3y)^{3-1}$

$= \binom{9}{2} x^7 (3y)^2$

$= 36x^7 \cdot 9y^2$

$= 324x^7y^2$

$\therefore$ Coefficient of $x^7y^2 = 324$

iv) no. of freshmen $= n(f) \geq 9$

no. of sophomores $= n(s) \geq 9$

no. of juniors ≥ 9

- If there are less than or equal to 8 freshmen, sophomores or juniors, then there are no more than 24 students in class

$\therefore$ there are atleast 9 freshmen, sophomores or juniors in the class.

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(V) If there exist less than or equal to 2 freshmen, 18 sophomores or 4 juniors in class there are no more than 24 students in class

∴ there are atleast 3 freshmen, 19 sophomores or 5 juniors in class.

(vi) 69277198116

$$3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 3x_7 + x_8 + 3x_9 + x_{10} + 3x_{11} + x_{12} \equiv 0 \pmod{10}$$

$$3\cdot6 + 9 + 3\cdot2 + 7 + 3\cdot7 + 1 + 3\cdot9 + 8 + 3\cdot1 + 1 + 3\cdot6 + x_{12} = 0 \pmod{10}$$

$$119 + x_{12} \equiv 0 \pmod{10}$$

$$x_{12} = 0 \pmod{10}$$

check digit = 1

— QUESTION #6 —

(i) Let $a-2$ is divisible by 3 for integer

∵ Let $K = \frac{a-2}{3}$

$a-2 = 3k$ for some integer K

$a-2+3 = 3k+3$

$a+1 = 3(k+1)$

$(a+1)(a-1) = 3(k+1)(\cancel{k}^{a}-1)$

$a^2-1 = 3r$ $\{ r = (k+1)(a-1)$

∴ a^2-1 is divisibly by 3

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ii) Suppose x and y integers exist such that $x^2 = 4y+2$

$$x^2 = 2(2y+1)$$

x^2 is even

Let $x = 2k$ for some integer k

$$x^2 = 4k^2$$

$$\therefore \cancel{4}k^2 = \cancel{2}(2y+1)$$

$$2k^2 = 2y+1$$

$2k$ is even

$2y+1$ is odd

$\therefore$ Our assumption is false and the statement is true

Q)

i) let a and b be even integers then
$a = 2k$ and $b = 2l$.

Now let
$$a+b = 2k+2l = 2(k+l)$$

as long as $(k+l)$ is an integer $a+b$ is even.

ii) Solution:-
$$1^3 + 2^3 + 3^3 + \dots n^3 = (n(n+1)/2)^2$$

at $p(1)$:-
$$1^3 = 1 = 1^2(1+1)^2/4$$
$$1 = 1 = 1$$

at $p(k)$:-
$$1^3 + 2^3 + \dots k^3 = (k(k+1)/2)^2$$

add $(k+1)^3$

$$1^3 + 2^3 \dots + k^3 + (k+1)^3 = (k(k+1)/2)^2 + (k+1)^3$$

$$1^3 + 2^3 \dots + k^3 + (k+1)^3 = [(k+1)(k+2)/2]^2$$

check digit $= 1$

— QUESTION #6 —

(i) Let $a-2$ is divisible by 3 for integer

$\because$ Let $K = \dfrac{a-2}{3}$

$a-2 = 3k$ for some integer k

$a-2+3 = 3k+3$

$a+1 = 3(k+1)$

$(a+1)(a-1) = 3(k+1)(\overset{a}{\cancel{k}}-1)$

$a^2-1 = 3r$ $\quad \{ r = (k+1)(a-1)$

$\because a^2-1$ is divisibly by 3

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ii) Suppose x and y integers exist such that $x^2 = 4y+2$

$\Rightarrow$

$x^2 = 2(2y+1)$

x^2 is even

Let $x = 2k$ for some integer k

$x^2 = 4k^2$

$\because \cancel{4}k^2 = \cancel{2}(2y+1)$

$2k^2 = 2y+1$

$2k$ is even

$2y+1$ is odd

$\therefore$ Our assumption is false and the statement is true

Q2)

i) let a and b be even integers then
$a = 2k$ and $b = 2l$.

Now let
$$a + b = 2k + 2l = 2(k + l)$$

as long as $(k + l)$ is an integer $a + b$ is even.

ii) Solution:-
$$1^3 + 2^3 + 3^3 + \ldots\ n^3 = (n(n+1)/2)^2$$

at $p(1)$:-
$$1^3 = 1 = 1^3(1+1)^{3}/4$$
$$1 = 1 = 1$$

at $p(k)$:-
$$1^3 + 2^3 + \ldots\ k^3 = (k(k+1)/2)^3$$

add $(k+1)^3$

$$1^3 + 2^3 \ldots + k^3 + (k+1)^3 = (k(k+1)/2)^3 + (k+1)$$

$$1^3 + 2^3 \ldots + k^3 + (k+1)^3 = [(k+1)(k+2)/2]^2$$

$$[k(k+1)/2]^2 + (k+1)^3 = [(k+1)(k+1+1)/2]^2$$

$$[k^2(k+1)^2/4 + (k+1)^3$$

$$(k+1)^2[k^2+4k+4]/4$$

$$(k+1)^2[(k+2)^2/4]$$

$$[(k+1)(k+2)/2]^2 = [(k+1)(k+2)/2]^2$$

R.H.S = L.H.S

v) Solution:-

$1^2 + 2^2 = 1 + 4 = 5$ ✓

$3^2 + 4^2 = 9 + 16 = 25$ ✓

$1^2 + 3^2 = 1 + 9 = 10$ ✗

disproven

vi) $N = 55,\ p = 5,\ q = 11$

$d = e^{-1} \bmod (p-1)(q-1)$

$1 = 40 - 3(13)$

$1 = (40)(1) + (-3)(13)$

$\boxed{d = -13\ ,\ \bmod 40 \text{ is } 27}$

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* Minimum difference Problem (Undecidable / NP-complete)

* <u>NP-complete:</u>
  ↳ any of a class of computational problems for which no efficient solution algorithm has been found.
  Travelling salesman problem

* <u>POST-CORRESPONDENCE PROBLEM:</u> (PCP)

  ↳ compatibility theory
  ↳ game of dominoes

abc	aca	b
ac	ca	acab

abc	aca	aca	b
ac	ca	ca	acab

• Minimum number of ~~proble~~ dominoes required to reach a solution.

ac	ba	ac
abb	bba	acc

• No solution because upper tile.length() < lowertile.length()

* Oracles are instances of a problem with similar characteristics
* Problems defined with similar characteristics
  - ↳ Oracle of sophomore students
  - ↳ Oracle of PCP dominos

* Certificate is a particular instance from an oracle that shows some verifiable and optimal solutions.

* PCP set-3 problems ⟶ Oracle
  - ↳ optimal solution ⟶ Certificate

$\left[\frac{a}{b}\right] \quad \left[\frac{ab}{a}\right] \quad \left[\frac{b}{bab}\right] \Rightarrow$ Minimum solution is 44!

$\left[\frac{abb}{a}\right] \quad \left[\frac{b}{abb}\right] \quad \left[\frac{a}{b}\right] \Rightarrow$ Minimum solution is 75!

* <u>FORMAL DEFINITION OF TURING MACHINE:</u>

↳ computational model ~~used to determine~~ ~~to~~ used to perform computations by reading and writing to an infinite tape.

↳ TM is a 7-tuple.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

↳ can be used as recognizer and acceptor

* Busy Beaver ⟶ overly simplified Turing machine
  - ↳ largest possible integer with maximum number of 1s.

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* Halting problem: Problem of determining whether the program will finish running, or continue to run forever.

1) Stops and accepts
2) Stops and rejects
3) Loops forever.

* MAPPING PCP ON TURING MACHINE:

Certificate → Max certificate
Certificate → Min certificate

- Dominos mapped on tapes of TM
-

* CONCLUSION:

↳ There are very simple problems that computers can't solve
↳ PCP has more than 20 variants which are proved to be undecidable
↳ The limitation of a computer is evident in such cases.

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Summary:-

In our universe some ~~things~~ problems are considered solvable or are already solved mathematically. Solvability means to know ~~by~~ the correct steps needed to solve a problem at any point. Yet there are some problems that are considered universally unsolvable due to the undecidability that remains, computationally. E.g: chess, tic tac toe, etc

When attempting to solve a problem a method called PCP (Post-correspondence Problem) exists inorder to ~~solve~~ check a problem ~~to~~ is solvable in the minimum required steps. ~~E~~

Turing machine is method that can be used as a solver, recongnizer, checker, etc for all problems by changing ~~states~~ or keeping its current state. Busy Beaver is a simplified version of the turing machine. A PCP can be mapped to a turing machine to solve it, or see if it can be solved by computers in a minimum certificate set time.

There are more than 20 preven unsolvable PCPs, that ~~p~~show the limitations of computers.