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# LA Assignment 01

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$$Q_1 \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 2 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

$$Q_2 \begin{bmatrix} 2 & 1 & 3 \\ 0 & -2 & -29 \\ 3 & 4 & 5 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 2 & -1 & 4 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

$$R_3 \leftrightarrow R_1 \begin{bmatrix} 3 & 4 & 5 \\ 0 & -2 & -29 \\ 2 & 1 & 3 \end{bmatrix} \quad -R_1 + R_1 \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & -29 \\ 2 & 1 & 3 \end{bmatrix}$$

$$2R_1 - R_3 \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 2 & -1 & 4 \\ 0 & 1 & 3 & 9 \end{bmatrix}$$

$$-2R_1 + R_3 \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & -29 \\ 0 & -5 & -1 \end{bmatrix}$$

$$\frac{1}{2}R_2 \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -\frac{1}{2} & 2 \\ 0 & 1 & 3 & 9 \end{bmatrix}$$

$$-R_2 + R_3 \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & -29 \\ 0 & -3 & 28 \end{bmatrix}$$

$$R_2 - R_1 \begin{bmatrix} -1 & 0 & \frac{3}{2} & -4 \\ 0 & 1 & -\frac{1}{2} & 2 \\ 0 & 1 & 3 & 9 \end{bmatrix}$$

$$R_3 + R_1 \begin{bmatrix} 1 & 0 & 30 \\ 0 & -2 & -29 \\ 0 & -3 & 28 \end{bmatrix}$$

$$R_2 - R_3 \begin{bmatrix} -1 & 0 & -\frac{3}{2} & -4 \\ 0 & 1 & -\frac{1}{2} & 2 \\ 0 & 0 & \frac{7}{2} & -7 \end{bmatrix}$$

$$R_3 \leftrightarrow R_2 \begin{bmatrix} 1 & 0 & 30 \\ 0 & -3 & 28 \\ 0 & -2 & -29 \end{bmatrix}$$

$$-\frac{1}{2}R_3 \begin{bmatrix} -1 & 0 & -\frac{3}{2} & -4 \\ 0 & 1 & -\frac{1}{2} & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$-R_2 + R_3 \begin{bmatrix} 1 & 0 & 30 \\ 0 & -1 & 28 \\ 0 & -2 & -29 \end{bmatrix}$$

$$R_3 + R_2 \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 57 \\ 0 & 0 & 85 \end{bmatrix}$$

$$\frac{1}{2}R_3 + R_2 \begin{bmatrix} +1 & 0 & 0 & +1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$R_3 \times \frac{1}{85} \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 57 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x=1, \quad y=3, \quad z=2$$

$$-57R_3 + R_2 \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x=\pm 1, \quad y=\pm \sqrt{3}, \quad z=\pm \sqrt{2}$$

$$-30R_3 + R_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

 Reduced Row Echelon Form



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$$Q_3(a) \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 0 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 1 & 4 & 2 & 0 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 2 & -4 & 1 & 1 & 0 \end{bmatrix}$$

$$2R_1 - R_2 \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & -3 & 6 & 0 & 0 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$$R_1 - R_2 \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & -1 & -2 & 1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 2 & -4 & 1 & 1 & 0 \end{bmatrix}$$

$$R_1 + R_3 \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & -3 & 6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 \end{bmatrix}$$

$$R_1 - R_5 \begin{bmatrix} 0 & 5 & 1 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{3}R_4 \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & -3 & 6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \end{bmatrix}$$

$$2R_1 - R_4 \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & -1 & -2 & 1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & 10 & -1 & 1 & 0 \\ 0 & 5 & 1 & 0 & 0 \end{bmatrix}$$

$$-R_1 + R_4 \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & -3 & 6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$10R_2 + R_4 \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & -1 & -2 & 1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & 0 & 20 & -11 & 0 \\ 0 & 5 & 1 & 0 & 0 \end{bmatrix}$$

$$-3R_3 + R_2 \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$5R_2 + R_5 \begin{bmatrix} 1 & 3 & 0 & 1 & 0 \\ 0 & +1 & +2 & -1 & 0 \\ 0 & -2 & -2 & -1 & 0 \\ 0 & 0 & -21 & -11 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{bmatrix}$$

$$R_3 + R_1 \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 - R_2 \begin{bmatrix} 0 & +1 & +2 & -1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & 21 & -11 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{bmatrix}$$

$$x - w = -1 \Rightarrow -(x + w) = 1$$

$$w - x = 1 \quad [x = w + 1]$$

$$T_{xy} - 2z = 0 \quad (2)$$

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$$\frac{1}{2}R_3 \left[ \begin{array}{ccccc} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 21 & -11 & 0 \\ 0 & 0 & 9 & -5 & 0 \end{array} \right] \xrightarrow{\frac{1}{2}R_3 + R_1} \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$-2R_3 + R_4 \left[ \begin{array}{ccccc} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 9 & -5 & 0 \end{array} \right] \xrightarrow{\frac{1}{2}R_4 + R_3} \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$-9R_3 + R_5 \left[ \begin{array}{ccccc} 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \end{array} \right] \xrightarrow{-2R_3 + R_2} \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$3R_2 - R_1 \left[ \begin{array}{ccccc} 1 & 0 & 6 & -4 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \end{array} \right] \xrightarrow{\text{Reduced Row Echlon form}}$$

$$-\frac{1}{2}R_2 + R_4 \left[ \begin{array}{ccccc} 1 & 0 & 6 & -4 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{Elementary Method}} \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$6R_3 - R_1 \left[ \begin{array}{ccccc} 1 & 0 & 0 & -5 & 0 \\ 0 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{A^{-1} = C_{11}^T \times C_{21}^T} \left[ \begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(4)

$$b) \begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix} I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Elementary Method:-

$$R_2 \rightarrow R_2 - 4R_1 \quad \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3 + R_1 \rightarrow R_3 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$-\frac{3}{4}R_3 + R_2 \rightarrow R_2 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = C_3^T \cdot C_2^T \cdot C_1^T$$

$$C_2^T \cdot C_1^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+0+0 & 1(0)+0+0 & 1(2)+0+0 \\ 0+1(0) & 0+1(1)+0 & 0+0+\frac{-3}{4} \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3^T \cdot C_2^T \cdot C_1^T = \begin{bmatrix} 1(1)+0+0 & 0+0+0 & 0+0+0 \\ 0+1(0) & 0+\frac{1}{4}+0 & 0+0+\frac{-3}{4} \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & \frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix}$$

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Qs) (a)

$$\begin{array}{|c c c c|} \hline & -1 & -4 & 2 & 1 \\ \hline & 2 & -1 & 7 & 9 \\ \hline & -1 & 1 & 3 & 1 \\ \hline & 1 & -2 & 1 & -4 \\ \hline & -32 & 14 & 11 & -4 \\ \hline \end{array}$$

$$= -1 \begin{bmatrix} -1 & 7 & 9 \\ 1 & 3 & 1 \\ -5 & 1 & -4 \end{bmatrix} + 4 \begin{bmatrix} 2 & 7 & 9 \\ -1 & 3 & 1 \\ 1 & 1 & -4 \end{bmatrix} + 2 \begin{bmatrix} 2 & -1 & 9 \\ -1 & 1 & 1 \\ 1 & -2 & -4 \end{bmatrix} - 1 \begin{bmatrix} 2 & -1 & 7 \\ -1 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= (-1) \{-1(-12 - 1) - 7(-4 + 2) + 9(1 + 6)\} + 4 \{2(-12 - 1) - 7(4 - 1) + 9(-1 - 3)\} + 2 \{2(-4 + 2) + 1(4 - 1) + 9(2 - 1)\} - 1 \{2(1 + 6) + 1(-1 - 3) + 7(2 - 1)\}$$

$$= (-1)(90) + 4(-83) + 2(8) - 1(17) = -423.$$

$$|\Delta_1| = \begin{array}{|c c c c|} \hline & -32 & -4 & 2 & 1 \\ \hline & 14 & -1 & 7 & 9 \\ \hline & 11 & 1 & 3 & 1 \\ \hline & -4 & -2 & 1 & -4 \\ \hline & -32 & 14 & 11 & -4 \\ \hline \end{array}$$

$$= -32 \begin{bmatrix} -1 & 7 & 9 \\ 1 & 3 & 1 \\ -2 & 1 & -4 \end{bmatrix} + 4 \begin{bmatrix} 14 & 7 & 9 \\ 11 & 3 & 1 \\ -4 & 1 & -4 \end{bmatrix} + 2 \begin{bmatrix} 14 & -1 & 9 \\ 11 & 1 & 1 \\ -4 & -2 & -4 \end{bmatrix} - 1 \begin{bmatrix} 14 & -1 & 7 \\ 11 & 1 & 3 \\ -4 & -2 & 1 \end{bmatrix}$$

$$= -32(90) + 4(305) + 2(-230) - 1(-5) = -2115.$$

$$|\Delta_2| = \begin{array}{|c c c c|} \hline & -1 & -32 & 2 & 1 \\ \hline & 2 & 14 & 7 & 9 \\ \hline & -1 & 11 & 3 & 1 \\ \hline & 1 & -4 & 1 & -4 \\ \hline & -1 & -32 & 2 & 1 \\ \hline \end{array} = -305 - 2656 - 370 - 53$$

$$= -3384$$

$$|\Delta_3| = \begin{array}{|c c c c|} \hline & -1 & -4 & -32 & 1 \\ \hline & 2 & -1 & 14 & 9 \\ \hline & -1 & 1 & 11 & 1 \\ \hline & 1 & -2 & -4 & -4 \\ \hline & -1 & -4 & -32 & 1 \\ \hline \end{array} = -230 - 740 - 556 - 43$$

$$= -1269$$

$$|\Delta_4| = \begin{array}{|c c c c|} \hline & -1 & -4 & 2 & -32 \\ \hline & 2 & -1 & 7 & 14 \\ \hline & -1 & 1 & 3 & 11 \\ \hline & 1 & -2 & 1 & -4 \\ \hline & -5 & -212 & 86 & 544 \\ \hline \end{array} = 5 - 212 + 86 + 544$$

$$= 423$$

$$x_1 = \frac{|A_1|}{|A|} = \frac{-2115}{-423} = [5]$$

$$|A_1| = -423$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{-8884}{-423} = [8]$$

$$|A_2| = -423$$

$$x_3 = \frac{|A_3|}{|A|} = \frac{-1269}{-423} = [3]$$

$$|A_3| = -423$$

$$x_4 = \frac{|A_4|}{|A|} = \frac{423}{-423} = [-1]$$

$$|A_4| = -423$$

Q5) b)

$$\text{a)} \begin{array}{c|ccc|c} & 1 & 2 & -3 & 4 \\ \hline R_1 & 3 & -1 & 5 & 2 \\ \hline R_2 & 4 & 1 & (a^2 - 14) & a+2 \end{array}$$

$$-3R_1 + R_2 \quad \begin{array}{c|ccc|c} & 1 & 2 & -3 & 4 \\ \hline R_1 & 0 & -7 & 14 & -10 \\ \hline R_2 & 0 & -7 & a^2 - 2 & a - 14 \end{array}$$

$$R_2 - 4R_1 \quad \begin{array}{c|ccc|c} & 1 & 2 & -3 & 4 \\ \hline R_1 & 0 & -7 & 14 & -10 \\ \hline R_2 & 0 & 0 & a^2 - 16 & a - 4 \end{array}$$

$$-R_2 + R_3 \quad \begin{array}{c|ccc|c} & 1 & 2 & -3 & 4 \\ \hline R_1 & 0 & -7 & 14 & -10 \\ \hline R_2 & 0 & 0 & a^2 - 16 & a - 4 \end{array}$$

$$-\frac{1}{7}R_2 \quad \begin{array}{c|ccc|c} & 1 & 2 & -3 & 4 \\ \hline R_1 & 0 & 1 & -2 & \frac{10}{7} \\ \hline R_2 & 0 & 0 & a^2 - 16 & a - 4 \end{array}$$

$$a = \pm 4$$

$$a = 4$$

$$0 = -8$$

No Solution

$$a = +4$$

$$0 = 0$$

Infinitely many Solutions.



$$\textcircled{6} \quad \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{array} \right]$$

$$\begin{array}{l} 2R_1 + R_2 \\ 8a - R_1 + R_3 \end{array} \quad \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & -a^2+2 & a-2 \end{array} \right]$$

$$-\frac{1}{6} R_2 \quad \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} \\ 0 & 0 & -a^2+2 & a-2 \end{array} \right]$$

$$a = \pm \sqrt{2}$$

$$a = \sqrt{2} \quad \text{or} \quad a = -\sqrt{2}$$

The System has no Solution.

$$Q6(a) \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\frac{1}{2}R_2 \begin{bmatrix} 1 & 3 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inversion Algorithm.

$$\begin{bmatrix} 1 & 0 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 0 & 1 & 0 \\ 1 & 1 & 0 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & | & \frac{1}{12} & 0 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$R-R_3 \begin{bmatrix} 1 & 0 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 0 & 1 & 0 \\ 0 & -1 & 0 & | & 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & | & \frac{1}{12} & 0 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$R_2+R_3 \begin{bmatrix} 1 & 0 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 0 & 1 & 0 \\ 0 & 0 & 0 & | & 1 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & \frac{1}{2}/2 & \frac{2}{13}\sqrt{2} & 0 \\ 0 & 1 & 0 & | & \frac{1}{13}\sqrt{2} & \frac{2}{13}\sqrt{2} & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{2}R_3 \begin{bmatrix} 1 & 0 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$C \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$$

$$-R_3+R_2 \begin{bmatrix} 1 & 0 & 0 & | & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & | & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & | & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$R_4-R_3+R_1 \begin{bmatrix} 1 & 0 & 0 & | & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & | & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & | & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

~~-R<sub>1</sub>+R<sub>2</sub> R<sub>4</sub>-R<sub>3</sub> R<sub>2</sub>-R<sub>1</sub>~~

$$.b) \begin{bmatrix} \sqrt{2} & 3\sqrt{2} & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 3 & 5 & 0 \\ 0 & 3 & 5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & | & -1 & 1 & 0 & 0 \\ 0 & 3 & 5 & 0 & | & -1 & 0 & 1 & 0 \\ 0 & 3 & 5 & 7 & | & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Inversion Algorithm.} \begin{bmatrix} \sqrt{2} & 3\sqrt{2} & 0 & | & 0 & 1 & 0 & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & | & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 3 & 5 & 0 & | & -1 & 0 & 1 & 0 \\ 0 & 3 & 5 & 7 & | & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{2}R_1 \begin{bmatrix} 1 & 3 & 0 & | & \frac{1}{2} & 0 & 0 & 0 \\ -4\sqrt{2} & \sqrt{2} & 0 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & | & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 5 & 0 & | & 0 & -1 & 1 & 0 \\ 0 & 0 & 5 & 7 & | & 0 & -1 & 0 & 1 \end{bmatrix}$$



$$X_R_3 \begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ 0 \\ 5 \end{array} \begin{array}{r} 0 \\ 0 \\ 1 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ -1 \\ 0 \end{array} \begin{array}{r} 0 \\ 0 \\ 0 \\ 7 \end{array} \begin{array}{r} 1 \\ 0 \\ 0 \\ 0 \end{array}$$

$$-S_R_3 + R_4 \begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ -1 \\ 0 \end{array} \begin{array}{r} 0 \\ 0 \\ 1 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ 0 \\ 7 \end{array} \begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \end{array}$$

$$X_R_4 \begin{bmatrix} 1 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ -1 \\ 0 \end{array} \begin{array}{r} 0 \\ 0 \\ 1 \\ 0 \end{array} \begin{array}{r} 1 \\ 0 \\ 0 \\ 0 \end{array} \begin{array}{r} 0 \\ 0 \\ 0 \\ 1 \end{array}$$

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$$Q_3 \begin{bmatrix} -1 & -2 & -3 & -4 & | & \\ 4 & 3 & 2 & 1 & | & \\ 1 & 2 & 3 & 4 & | & \\ -4 & -3 & -2 & -1 & | & \end{bmatrix}$$

1 Determinant.

$$= (-1) \begin{bmatrix} 3 & 2 & 1 & | & +2 & 4 & 2 & 1 & | & -3 & 4 & 3 & 1 & | & +4 & 4 & 3 & 2 \\ 2 & 3 & 4 & | & 1 & 3 & 4 & | & 1 & 2 & 4 & | & 1 & 2 & 3 \\ -3 & -2 & -1 & | & -4 & -2 & -1 & | & -4 & 3 & -1 & | & -4 & -3 & -2 \end{bmatrix}$$

$$= (-1) \left\{ 3(-3+8) - 2(-2+12) + 1(-4+9) \right\} (+2) \left\{ 4(-3+8) - 2(-1+16) + 1(-2+12) \right\} \\ (-3) \left\{ 4(-2+12) - 2(-1+16) + 1(-3+8) \right\} (+4) \left\{ 4(-4+9) - 3(-2+12) + 2(-3+8) \right\}$$

$$= (-1)(0) + (2)(0) - (3)(0) + 4(0)$$

$$= \boxed{0}$$

Conclusion:

$$C_{11} = (-1) \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 4 \\ -3 & -2 & -1 \end{bmatrix} = 0$$



$$C_{12} = (+2) \begin{bmatrix} 4 & 2 & 1 \\ 1 & 3 & 4 \\ -4 & -2 & -1 \end{bmatrix} = 0, -C_{13} = (-3) \begin{bmatrix} 4 & 3 & 1 \\ 1 & 2 & 4 \\ -4 & -3 & -1 \end{bmatrix} = 0$$

$$C_{14} = (+4) \begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ -4 & -3 & -2 \end{bmatrix} = 0, C_{21} = (4) \begin{bmatrix} -2 & -3 & -4 \\ 2 & 3 & 4 \\ -3 & -2 & -1 \end{bmatrix} = 0, C_{22} = (-3) \begin{bmatrix} -1 & -3 & -4 \\ 1 & 3 & 4 \\ -4 & -2 & -1 \end{bmatrix} = 0$$

$$C_{23} = (2) \begin{bmatrix} -1 & -2 & -4 \\ 1 & 2 & 4 \\ -4 & -3 & -1 \end{bmatrix} = 0, C_{24} = (-1) \begin{bmatrix} -1 & -2 & -3 \\ 1 & 2 & 3 \\ -4 & -3 & -2 \end{bmatrix} = 0$$

$$C_{31} = (1) \begin{bmatrix} -2 & -3 & -4 \\ 3 & 2 & 1 \\ -3 & -2 & -1 \end{bmatrix} = 0, C_{32} = (-2) \begin{bmatrix} -1 & -3 & -4 \\ 4 & 2 & 1 \\ -4 & -2 & -1 \end{bmatrix} = 0$$

$$C_{33} = (+3) \begin{bmatrix} -1 & -2 & -4 \\ 4 & 3 & 1 \\ -4 & -3 & -1 \end{bmatrix} = 0, C_{34} = (-4) \begin{bmatrix} -1 & -2 & -3 \\ 4 & 3 & 2 \\ -4 & -3 & -2 \end{bmatrix} = 0$$

$$C_{41} = (-1) \begin{bmatrix} -2 & -3 & -4 \\ 3 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix} = 0, C_{42} = (+3) \begin{bmatrix} -1 & -3 & -4 \\ 4 & 2 & 1 \\ -4 & -2 & -1 \end{bmatrix} = 0$$

$$C_{42} = (-2) \begin{bmatrix} -1 & -2 & -4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} = 0, C_{43} = (+1) \begin{bmatrix} -1 & -2 & -3 \\ 4 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Q8)  $\begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = T(-1, 2, 4)$

$$w_1 = 3(-1) + 5(2) - 1(4) = 3$$

$$w_2 = -4(1) - 1(2) + 1(4) = -2$$

$$w_3 = 3(-1) + 2(2) + (-1)(4) = -3$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$