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Section: 3-F

Date:

QUESTION#01:-

(i)

(a) $M \leftrightarrow N$

(b) $N \rightarrow K$

(c) $\neg K \vee I$

(d) $\neg M \rightarrow I$

(ii) $((p \vee q) \wedge (p \rightarrow r)) \rightarrow (q \vee r)$

$((p \vee q) \wedge (\neg p \vee q)) \rightarrow (q \vee r)$

$(p \wedge (\neg p \vee q) \vee q \wedge (\neg p \vee q)) \rightarrow (q \vee r)$

$((p \wedge \neg p) \vee (p \wedge q) \vee (q \wedge \neg p) \vee (q \wedge q)) \rightarrow (q \vee r)$

$(F \vee (p \wedge q) \vee (q \wedge \neg p) \vee q) \rightarrow (q \vee r)$

$((p \wedge q) \vee ((q \vee q) \wedge (\neg p \vee q))) \rightarrow (q \vee r)$

$((p \wedge q) \vee (q \wedge (\neg p \vee q)) \rightarrow (q \vee r)$

$((p \wedge q) \vee q) \rightarrow (q \vee r)$ $\therefore$ Absorption law

$q \rightarrow (q \vee r)$ $\therefore$ Absorption law

$\neg q \vee (q \vee r)$

$\neg q \vee q \vee r$

$T \vee r$

$= T$

$\therefore$ Hence, it is a tautology.

(iii) Premises:-

$(\neg r \rightarrow (s \rightarrow \neg t))$

$(\neg r \vee w)$

$(\neg p \rightarrow s)$

$(\neg w)$

Conclusion:-

$(t \rightarrow p)$

Date:

Sol

~~$\equiv \neg r \to (s \to \neg t)$~~ ~~∴ Premise 1~~

~~∴ Modus tollens~~

~~$= \neg r \lor w$~~ ~~Premise 3~~

~~$\equiv \neg r \lor$~~ ~~Premise 4~~

$= \neg r \lor w$ ∴ Premise 3

∴ $\neg w$ ∴ Premise 4

$= \neg r$

$= \neg r \to (s \to \neg t)$ ∴ Premise 1

$= s \to \neg t$ ∴ Modus Ponens

$= \neg p \to s$ ∴ Premise 2

$= \neg p \to \neg t$ ∴ Hypothetical Syllogism

$= t \to p$ ∴ Contrapositive

Conclusion

∴ Hence proved

(iv)

(a) There is a student in your class who understands all lecture notes.

(b) ~~Every~~ For every lecture notes, there is a student in your class who understands it.

(v) (a) $\forall p \, \exists q \, F(p, q)$

(b) $\exists q \, \forall p \, F(p, q)$

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(vi)

(a) True

(b) False

QUESTION#02:-

(i) $X - (X \cap Y) = (X - Y)$

Taking L.H.S,

$= X - (X \cap Y)$

$\cancel{= X \cap \overline{X \cap Y}} = X \cap \overline{X \cap Y}$

$\cancel{= X \cap \overline{X} \cup \overline{Y}}$

$= \{x \mid x \in X\} \wedge \{x \mid x \notin (X \cap Y)\}$

$= \{x \mid x \in X\} \wedge \{\neg x \mid x \in (X \cap Y)\}$

$= \{x \mid x \in X\} \wedge [\{x \mid x \notin X\} \vee \{x \mid x \notin Y\}]$

$\therefore$ Distributive law,

$= [\{x \mid x \in X\} \wedge \{x \mid x \notin X\}] \vee [\{x \mid x \in X\} \wedge \{x \mid x \notin Y\}]$

$= \{x \mid x \in \emptyset\} \vee [\{x \mid x \in X\} \wedge \{x \mid x \notin Y\}]$

$= \{x \mid x \in (X \wedge \overline{Y})\}$

$= X \cap \overline{Y}$

$= X - Y$

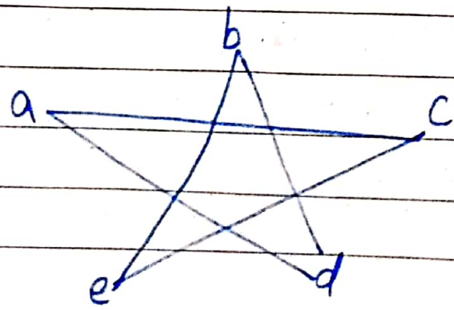
Ans

$R \circ S =$ $R = (a, b)$ $S = (b, d)$ (a, d)

(ii) ~~$R = \{a, b, c, d, e\}$~~

$R = \{(a, e), (e, d), (d, c), (c, b), (b, a)\}$

$R \circ R = \{(a, d), (e, c), (d, b), (c, a), (b, e)\}$



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(iii)

Reflexive: Not true because a player never plays herself.

Symmetric: Not true because if a beats b, then b does not beat a.

Transitive: Not true because if a beats b and b beats c, then a does not necessarily beat c.

∴ It is neither equivalence, nor partial ordering.

(iv) $g(n) = n^2 + 1$

sol

$g(1) = (1)^2 + 1$
$= 2$

$g(\{1\}) = (\{1\})^2 + 1$
$= \{2\}$

Hence, $g(1) \neq g(\{1\})$

(v) $f(0) = 0$

$f(n+1) = f(n) + 2n + 1$

$f(1) = 0 + 2(0) + 1 = 1$

∴ $f(2) = 1 + 2(1) + 1 = 4$

$f(3) = 3 + 2(2) + 1 = 9$

$f(4) = 8 + 2(3) + 1 = 16$

$f(5) = 15 + 2(4) + 1 = 25$

$f(6) = 24 + 2(5) + 1 = 36$

~~f(6) = 35 + 2(6) + 1 = 48~~

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(vi)

$m = 4$

$n = 3$

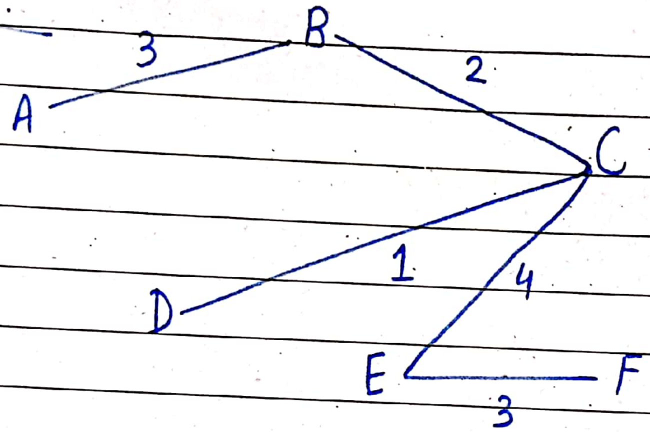
No. of functions $= n^m = 3 \times 3 \times 3 \times 3$

$\cancel{= 4 \times 3}$ $= 81$

$\cancel{= 12}$

QUESTION#03:-

Prim's:



MST cost $= 1 + 2 + 3 + 3 + 4$

$= 13$

Kruskal:

(C, D) = 1

(B, C) = 2

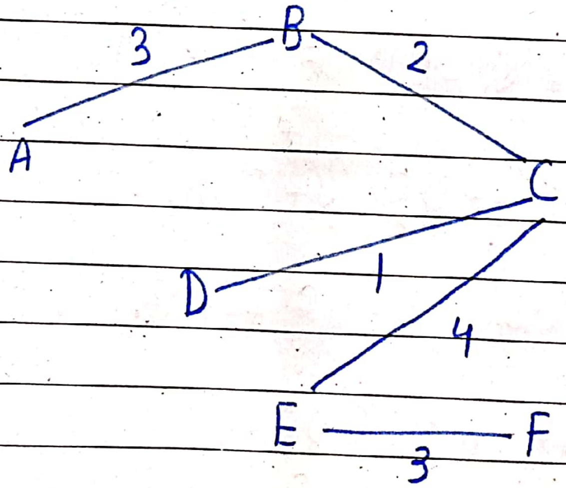
(A, B) = 3

~~(B, D) = 3~~

(E, F) = 3

(C, E) = 4

~~(A, F) = 10~~



MST Cost $= 1 + 2 + 3 + 3 + 4$

$= 13$

Date:

(ii) Nodes	n(A)	n(B)	n(C)	n(D)	n(E)
~~A~~C	1, c	7, c	~~0~~	2, c	
CA		~~3~~4, a		2, c	
CAD		4, a			9, d
CADB					5, b

Ques

(iii) This graph contains a Hamilton Cycle but it is not a Hamilton graph as edges are repeating.

iv) P(1) = S
P(2) = t
P(3) = u
P(4) = v
P(5) = w
P(6) = x
P(7) = y
P(8) = z

v) Pre Order = 60, 53, 33, 25, 42, 57, 95, 78, 71
Post Order = 25, 42, 33, 57, 53, 71, 78, 95
In Order = 25, 33, 42, 53, 57, 60, 71, 78, 95

vi) (A+B) x C - (D-E) x (F+G)
+AB x C - - DE + FG
+ABC - - DE x + FG

Pre = x + ABC x - DE + FG

(AB+) x C - (DE-) x (FG+)
AB+C x - DE - FG + x
Post AB+C x DE - FG + ~~x~~ x -

Pg No.

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Question 4

i) $P(n) = \frac{6!}{5^5} = \frac{5!}{6^4}$

ii) $P(n) = \frac{\binom{5}{2} \cdot 6 \times 5 \times 4 \times 3}{6^5} = \frac{100}{6^3}$

iii) $^5C_3$

$^6C_4$

$= {^5C_3} \times {^6C_4}$

iv) $C(n-2, K-2)$

$C(n,K) - C(n-2, K-2)$

v) $C(4,2) = 6$ possible ways

vi) a) $1 \times 26 \times 26 \times 10 \times 10 = 67600$

b) $26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11,232,000$

Question 5

i) $n \equiv 2 \bmod 3$ $n = 4 \bmod 5$ $n \equiv 5 \bmod 7$

$n \equiv 1 \bmod 11$

$M = 3 \times 5 \times 7 \times 11 = 1155$

$m_1 = 1155/3 = 385$

$m_2 = 1155/5 = 231$

$m_3 = 1155/7 = 165$

$m_4 = 1155/11 = 105$

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$n_1 = 385 \mod 3$

$n_2 = 231 \mod 5$

$n_3 = 165 \mod 7$

$n_4 = 105 \mod 11$

Using Euclidean

$y_1 = 1 \quad y_2 = 1 \quad y_3 = 2 \quad y_4 = 2$

Chinese remainder

$X = (2)(385)(1) + (1)(4)(231) + (2)(5)(165)$
$+ (2)(1)(105)$

$X = 770 + 924 + 1650 + 210$

$X = 3554 \text{ Mod } 1155$

$X = 89$ Ans

ii) Fermats theoram

$9^{78} = 1 \pmod{79}$

$9^{78 \times 33 + 5} \mod 79$

$(9^{78})^{33} \cdot 9^5 \mod 79$

$(1)^{33} \cdot 9^5 \mod 79$

$59049 \mod 79$

$= 36$ Ans

iii) $(x+3y)^9$

$$\begin{bmatrix} 9 \\ 3-1 \end{bmatrix} x^{9-(3-1)} (3y)^{3-1}$$

$$\begin{bmatrix} 9 \\ 2 \end{bmatrix} x^7 (3y)^2$$

$= 36x^7 9y^2$

$= 324x^7y^2$ Ans

iv) 25 students.

Assuming there are less then ~~8~~ or equal to 8 freshmen, sopho, junior

$8+8+8 = 24$

So assumption is wrong as there needs to be atleast 9 of either one to become class of 25.

v) Freshmen = 3

Sopho = 19

Juniors = 5

$F.L = 2$ $S.L = 18$ $J.L = 4$

$= 18+2+4 = 24$

which is less the total of 27

Hence Assumption is wrong

vi) UPC = 692771 98116

$= 3x_1 + x_2 + 3x_3 + x_4 + 3x_5 + x_6 + 3x_7 + x_8 + 3x_9 + x_{10} + 3x_{11} + x_{12}$

$= 0 \pmod{10}$

$$3(6) + 9 + 3(2) + 7 + 3(7) + 1 + 3(9)$$
$$+ 8 + 3(1) + 1 + 3(6) + u_{12} = 0 \mod 10$$
$$119 + u_{12} = 0 \mod 10$$
$$u_{12} = 0 \mod 10$$

Check digit = 1.

Question #6:

(i) if $a-2$ is divisible by 3 then there exists k as

$$\frac{a-2}{3} = k$$

$$a-2 = 3k$$

adding 3 on both sides

$$a-2+3 = 3k+3$$

$$a+1 = 3k+3$$

multiply by $a-1$ on both sides

$$a^2-1 = 3(k+3)$$

where $k+3$ is an integer let l

$$a^2-1 = 3l$$

$$\frac{a^2-1}{3} = l$$

so l results when a^2-1 is divided by 3 so it is divisible.

(ii) $x^2 = 4y+2$

let there be x and y.

$$x^2 = 2(2y+1)$$

so x is even

so let $x = 2k$

$(2k)^2 = 2(2y+2)$

so a is even and y is odd so they can't be equal so the statement give us a contradiction so assumption is false statement is true.

iii) Contraposition is that if a and b are even then a+b is even

let $a = 2k$ and $b = 2L$

$a + b = 2k + 2L = 2(k+L)$

where $(k+L)$ is an integer

So a+b is even. Since contraposition is true so the statement is also true

iv) $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

for P(1):

$$1^3 = \left(\frac{(1)(1+1)}{2}\right)^2$$

$$1 = \left(\frac{(1)(2)}{2}\right)^2 = (1)^2$$

$1 = 1$ prooved

for P(K):

Assuming p(K) is true

$1^3+2^3+3^3+\dots+K^3 = (K(K+1)/2)^2$ — ①

Add $(K+1)^3$ on b/s

$1^2+2^3+3^3+\dots+K^3+(K+1)^3 = [(K+1)(K+1+1/2)]^2$ — ②

put eq ① in ②

$(K(K+1)/2)^2+(K+1)^3 = [(K+1)(K+2)/2]^2$

$K^2(K+1)^2/4 + (K+1)^3$

$(K+1)^2[K^2+4K+4]/4$

$(K+1)^2(K+2)^2/4$

$[(K+1)^2(K+2)^2/2]^2 = [(K+1)(K+2)/2]^2$

LHS = RHS proved

v) $1^2+2^2 = 1+4 \rightarrow 5$ True

$3^2+4^2 = 25$ True

$1^2+3^2 = 10$ False

Hence disproof