21K-3202 Syed Babar Ashraf Sukzwari
Sec - F

99/100

Day: Date:

ASSIGNMENT 03

Q1.

a) $19/7 = 7|19 \therefore q=2, r=5$
b) $-111/11 = 11/-111 : q=-11, r=10$
c) $789/23 = 23|789 : q=34, r=7$
d) $1001/13 = 13|1001 : q=77, r=0$
e) $10/19 = 19|10 : q=0, r=10$
f) $3/5 = 5|3 : q=0, r=5$
g) $-1/3 = 3|-1 : q=-1, r=2$
h) $4/1 = 1|4 : q=4, r=0$

Q2.

a) i) $q = a \text{ div } m$, $r = a \bmod m$
$a=-111, m=99$
$\therefore q=-2, r=87$

ii) $a=-9999, m=101$
$\therefore q=-99, r=0$

iii) $a=10299, m=999$
$\therefore q=10, r=309$

iv) $a=123456, m=1001$
$\therefore q=123, r=333$

b) For $a \equiv b \pmod m$ a number is congruent if
$\frac{a-b}{m} = k$, where k is an integer.

i) $80 = 5 \pmod{17} = \frac{80-5}{17} = \frac{98}{17} = 5.76 \neq 80$
NOT CONGRUENT

ii) $103 = 5 \pmod{17} = \frac{103-5}{17} = \frac{98}{17} = 4.4 \neq 103$
NOT CONGRUENT

iii) $-29 = 5 \pmod{17} = \frac{-29-5}{17} = \frac{-34}{17} = -2$
CONGRUENT

iv) $-122 = 5 \pmod{17} = \frac{-122-5}{17} = \frac{-127}{17} = 7.47$
NOT CONGRUENT

Day: Date:

Q3.

a) i) (11, 15, 19)

gcd(11, 15) = 1 gcd(11, 19) = 1, gcd(15, 19) = 1

∴ (11, 15, 19) are pairwise prime

ii) (14, 15, 21)

gcd(14, 15) = 1, gcd(15, 21) = 3, gcd(14, 21) = 7

∴ (14, 15, 21) are not pairwise primes

iii) (12, 17, 31, 37)

gcd(12, 17) = 1, gcd(12, 31) = 1, gcd(12, 37) = 1, gcd(17, 31) = 1,
gcd(17, 37) = 1, gcd(31, 37) = 1

∴ (12, 17, 31, 37) are pairwise primes

iv) (7, 8, 9, 11)

gcd(7, 8) = 1, gcd(7, 9) = 1, gcd(7, 11) = 1, gcd(8, 9) = 1, gcd(8, 11) = 1
gcd(9, 11) = 1 ∴ (7, 8, 9, 11) are pairwise prime

b) i) 88

2	88
2	44
2	22
11	11
	1

$2^3 \times 11$

ii) 126

2	126
2	63
3	21
7	7
	1

$2^1 \times 3^2 \times 7$

iii) 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

$\Rightarrow 3^6$

iv) 909

3	909
3	303
101	101
	1

$\Rightarrow 3^2 \times 101$

Day:

gcd(144, 89)

144 = (1)(89) + 55
89 = 1(55) + 34
55 = (1)(34) + 21
34 = (1)(21) + 13
21 = (1)(13) + 8
13 = (1)(8) + 5
8 = (1)(5) + 3
5 = (1)(3) + 2
3 = (1)(2) + 1

Backward substitution:

1 = 1·3 − 1·2 → (i)
2 = 1·5 − 1·3
put in (i)
1 = 1·3 − 1·5 + 1·3
1 = 2·3 − 1·5 → (ii)
3 = 1·8 − 1·5
put in (ii)
1 = 2·(1·8 − 1·5)
1 = 2·8 − 2·5 − 1·
1 = 2·8 − 3·5 −
5 = 1·13 − 1·8
put in (iii)
1 = 2·8 − 3·(1·1
1 = 2·8 − 3·13 +
1 = 5·8 − 3·13
8 = 1·21 − 1·13
1·13 = 1·21 −
put in (iv)
1 = 5·8 − 3(
1 = 5·8 − 3·2

Day: Date:

Q4.

) gcd (144, 89)

144 = (1)(89) + 55

89 = 1(55) + 34

55 = (1)(34) + 21

34 = (1)(21) + 13

21 = (1)(13) + 8

13 = (1)(8) + 5

8 = (1)(5) + 3

5 = (1)(3) + 2

3 = (1)(2) + 1

Backward substitution:

1 = 1·3 - 1·2 → (i)

2 = 1·5 - 1·3

put in (i)

1 = 1·3 - 1·5 + 1·3

8, 11) = 1

1 = 2·3 - 1·5 → (ii)

3 = 1·8 - 1·5

put in (ii)

1 = 2·(1·8 - 1·5) - 1·5

1 = 2·8 - 2·5 - 1·5

1 = 2·8 - 3·5 → (iii)

5 = 1·13 - 1·8

put in (iii)

1 = 2·8 - 3·(1·13 - 1·8)

1 = 2·8 - 3·13 + 3·8

1 = 5·8 - 3·13

8 = 1·21 - 1·13

1·13 = 1·21 - 1·8

put in (iv)

1 = 5·8 - 3(1·21 - 1·8)

1 = 5·8 - 3·21 + 3·8

1 = 8·8 - 3·21 → (v)

13 = 1·34 - 1·21

1·21 = 1·34 - 1·13

put in (v)

1 = 8·8 - 3(1·34 - 1·13)

1 = 8·8 - 3·34 + 3·13 → (vi)

21 = 1·55 - 1·34

1·34 = 1·55 - 1·21

put in (vi)

1 = 8·8 + 3·13 - 3·(1·55 - 1·21)

1 = 8·8 + 3·13 - 3·55 + 3·21 → (vii)

34 = 1·89 - 1·55

1·5 = 1·89 - 1·34

put in (vii)

1 = 8·8 + 3·13 - 3(1·89 - 1·34) + 3·21

1 = 8·8 + 3·13 + 3·21 + 3·34 - 3·89 → (viii)

1 = 8·8 + 3·13 + 3·21 + 3·34 - 3·144 + 3·55

1 = 8·21 - 8·13 + 3·13 + 3·21 + 3·34 - 3·144 + 355

1 = 11·21 - 5·13 + 3·34 + 3·144 + 3·55

1 = 11·21 - 5(1·34 - 1·21) + 3·34 - 3·144 + 3·55

1 = 16·21 - 2·34 - 3·144 + 3·55

1 = 16(1·55 - 1·34) - 2·34 - 3·144 + 3·55

1 = 16·55 - 16·34 - 2·34 - 3·144 + 3·55

1 = 19·55 - 18(1·89 - 1·55) - 3·144

1 = 19·55 - 18·89 + 18·55 - 3·144

1 = 37·55 - 18·89 - 3·144

1 = 37·144 - 37·89 - 3·144 - 18·89

1 = 34·144 - 55·89

1 = (34)(144) + (-55)(89)

∴ Linear combination of gcd (144, 89)

Day: Date:

2) gcd (1001, 100001)

$100001 = (0)(1001) + 100001$

$1001 = (1)(902) + 99$

$902 = (9)(99) + 11$

$99 = (9)(11) + 0$

Backward substitution,

$11 = 1 \cdot 902 - 9 \cdot 99 \rightarrow (i)$

$99 = 1 \cdot 1001 - 1 \cdot 902$

put in (i)

$11 = 1 \cdot 902 - 9(1 \cdot 1001 - 1 \cdot 902)$

$11 = -9 \cdot 1001 + 10 \cdot 902 \rightarrow (ii)$

$902 = 1 \cdot 100091 - 99 \cdot 1001$

put in (ii)

$11 = -9 \cdot 1001 + 10(1 \cdot 100001 - 99 \cdot 1001)$

$11 = -9 \cdot 1001 + 10 \cdot 100001 - 990 \cdot 1001$

$11 = -999 \cdot 1001 + 10 \cdot 10001$

$11 = (10)(100001) + (-999)(1001)$

∴ Linear combination of gcd (1001, 100001)

Q5.

a) $55x \equiv 34 \pmod{89}$

x both sides by m^{-1}

$34 \times 55x = 34 \cdot 34 \pmod{89}$

$x = 1156 \pmod{89}$

$x = 88$

b) $89x \equiv 2 \pmod{232}$

$89 = (0)(232) + 89$

$232 = (2)(89) + 54$

$89 = (1)(54) + 35$

$54 = (1)(35) + 19$

$35 = (1)(19) + 16$

$19 = (1)(16) + 3$

$16 = (5)(3) + 1$

Backward substitution

$1 = 1 \cdot 16 - 5 \cdot 3 \rightarrow (i)$

$3 = 1 \cdot 19 - 1 \cdot 16$

Put in (i)

$1 = 1 \cdot 16 - 5(1 \cdot 19 - 1 \cdot 16)$

$1 = 1 \cdot 16 - 5 \cdot 19 + 5 \cdot 16$

Day:

$1 = 6 \cdot 16 - 5 \cdot 19 \rightarrow$

$16 = 1 \cdot 35 - 1 \cdot 19$

put in (ii)

$1 = 6(1 \cdot 35 - 1 \cdot 1$

$1 = 6 \cdot 35 - 6 \cdot 19$

$1 = 6 \cdot 35 - 11 \cdot 1$

$19 = 1 \cdot 54 - 1 \cdot 3$

put in (iii)

$1 = 6 \cdot 35 - 11($

$1 = 6 \cdot 35 - 11 \cdot 5$

$1 = 17 \cdot 35 - 11$

$35 = 1 \cdot 89 - 1 \cdot 5$

put in (iv)

$1 = 17(1 \cdot 89 -$

$1 = 17 \cdot 89 - 17$

$1 = 17 \cdot 89 - 2$

$54 = 1 \cdot 232$

put in (v)

$1 = 17 \cdot 89 -$

$1 = 17 \cdot 89 -$

$1 = 73 \cdot 89$

$89 = 1 \cdot 89$

$1 = (73)(8$

inverse

$89x = 2($

x 73 on

73×89

$x = 14$

$x = 14$

Day: _____ Date: _____

$1 = 6 \cdot 16 - 5 \cdot 19 \rightarrow (ii)$

$16 = 1 \cdot 35 - 1 \cdot 19$

put in (ii)

$1 = 6(1 \cdot 35 - 1 \cdot 19) - 5 \cdot 19$

$1 = 6 \cdot 35 - 6 \cdot 19 - 5 \cdot 19$

$1 = 6 \cdot 35 - 11 \cdot 19$

$19 = 1 \cdot 54 - 1 \cdot 35$

put in (iii)

$1 = 6 \cdot 35 - 11(1 \cdot 54 - 135)$

$1 = 6 \cdot 35 - 11 \cdot 54 + 11 \cdot 35$

$1 = 17 \cdot 35 - 11 \cdot 54 \rightarrow (iv)$

$35 = 1 \cdot 89 - 1 \cdot 54$

put in (iv)

$1 = 17(1 \cdot 89 - 1 \cdot 54) + - 11 \cdot 54$

$1 = 17 \cdot 89 - 17 \cdot 54 - 11 \cdot 154$

$1 = 17 \cdot 89 - 28 \cdot 54 \rightarrow (v)$

$54 = 1 \cdot 232 - 2 \cdot 89$

put in (v)

$1 = 17 \cdot 89 - 28(1 \cdot 232 - 2 \cdot 89)$

$1 = 17 \cdot 89 - 28 \cdot 232 + 56 \cdot 89$

$1 = 73 \cdot 89 - 28 \cdot 232 \rightarrow (vi)$

$89 = 1 \cdot 89 - 0 \cdot 232$

$1 = (73)(89) + (-28)(232)$

inverse = 73

$89x \equiv 2 \pmod{32}$

x 73 on both sides

$73 \times 89x = 2 \times 73 \pmod{232}$

$x = 146 \pmod{232}$

$x = 146$

Ans

Day:

Q6.

i) $x \equiv 1 \pmod 5$, $x \equiv 2 \pmod 6$, $x \equiv 3 \pmod 7$

$\gcd(5,6)=1$ $\gcd(5,7)=1$ $\gcd(6,7)=1$

modulos are pairwise primes

$m = 5 \times 6 \times 7 = m_1 m_2 m_3 = 210$

$M_k = \frac{m}{m_k}$; $M_1 = \frac{210}{5} = 42$, $M_2 = \frac{210}{6} = 35$, $M_3 = \frac{210}{7} = 3$

$x = (a_1 y_1 M_1 + a_2 y_2 M_2 + a_3 y_3 M_3) \bmod m \rightarrow (1)$

$y_k = \overline{M_k} \bmod m_k$

$y_1 = 42^{-1} \bmod 5$

$42 = (8)(5) + 2$

$5 = (2)(2) + 1$

Backward substitution,

$1 = 1 \cdot 5 - 2 \cdot 2 \rightarrow (i)$

$2 = 1 \cdot 42 - 8 \cdot 5$

put in (i)

$1 = 1 \cdot 5 - 2(1 \cdot 42 - 8 \cdot 5)$

$1 = 1 \cdot 5 - 2 \cdot 42 + 32 \cdot 5$

$1 = 33 \cdot 5 + (-2)(42)$

$y_1 = -2 + 5$

$y_1 = 3$

$y_2 = 35^{-1} \bmod 6$

$35 = (5)(6) + 5$

$6 = (1)(5) + 1$

Backward substitution

$1 = 1 \cdot 6 - 1 \cdot 5 \rightarrow (ii)$

$5 = 1 \cdot 35 - 5 \cdot 6$

put in (ii)

$1 = 1 \cdot 6 - (1 \cdot 35 - 5 \cdot 6)$

$1 = 1 \cdot 6 - 1 \cdot 35 + 5 \cdot 6$

$1 = 6 \cdot 6 - 1 \cdot 35$

$1 = (6)(6) + (-1)(35)$

$y_2 = -1 + 6$

$y_2 = 5$

$y_3 = 30^{-1} \pmod 7$

$30 = (4)(7) + 2$

$7 = (3)(2) + 1$

Backward substitution:

$1 = 1 \cdot 7 - 3 \cdot 2 \rightarrow (i)$

$2 = 1 \cdot 30 - 4 \cdot 7$

put in (i)

$1 = 1 \cdot 7 - 3(1 \cdot 30 - 4 \cdot 7)$

$1 = 1 \cdot 7 - 3 \cdot 30 + 12 \cdot 7$

$1 = 13 \cdot 7 - 3 \cdot 30$

$1 = (13)(7) + (-3)(30)$

$y_3 = -3 + 7$

$y_3 = 4$

Day:

$x = [(1)(3)($

$x = (126 + 3$

$x = 836(m$

$x = 206$

$x \equiv 1 \pmod 2$

$\gcd(2,3)=1$

$\gcd(3,11)=1$

Modulos a

$m = m_1 m_2 m_3$

$M_1 = \frac{330}{2}$

$M_4 = \frac{330}{11}$

$x = (a_1 y_1 M$

$y_1 = 165^{-1} m$

$165 = (82)$

Substitut

$1 = 1 \cdot 165$

$y_1 = 1(16$

$y_1 = 1$

$y_2 = 110^{-1} m$

$110 = (36)$

$3 = (1)(2)$

Substituting

$1 = 1 \cdot 3 - 1 \cdot$

$2 = 1 \cdot 110$

put in

Day: Date:

$x = [(1)(3)(42) + (2)(5)(35) + (3)(4)(30)] \bmod 210$

$x = (126 + 350 + 360) \bmod 210$

$x = 836 \pmod{210}$

$x = 206$ ✱

1) $x \equiv 1 \pmod 2, x \equiv 2 \pmod 3, x \equiv (3 \bmod 5), x \equiv 4 \pmod{11}$

$\gcd(2,3)=1$ $\gcd(2,5)=1$ $\gcd(2,11)=1$ $\gcd(3,5)=1$

$\gcd(3,11)=1$ $\gcd(5,11)=1$

Modulos are pairwise primes

$m = m_1 m_2 m_3 m_4 = 2 \times 3 \times 5 \times 11 = 330$

$M_1 = \frac{330}{2} = 165$ $M_2 = \frac{330}{3} = 110$, $M_3 = \frac{330}{5} = 66$

$M_4 = \frac{330}{11} = 30$

$x = (a_1 y_1 M_1 + a_2 y_2 M_2 + a_3 y_3 M_3 + a_4 y_4 M_4) \bmod m \rightarrow (i)$

$y_1 = 165^{-1} \bmod 2$

$165 = (82)(2) + 1$

Substituting

$1 = 1 \cdot 165 - 82 \cdot 2$

$y_1 1 = 1(165) + (-82)(2)$

$y_1 = 1$

$y_2 = 110^{-1} \bmod 33$

$110 = (36)(3) + 2$

$3 = (1)(2) + 1$

Substituting

$1 = 1 \cdot 3 - 1 \cdot 2 \rightarrow (i)$

$2 = 1 \cdot 110 - 36 \cdot 3$

put in (i)

$1 = 1 \cdot 3 - 1 \cdot 110 + 36 \cdot 3$

$1 = 37 \cdot 3 - 1 \cdot 110$

$1 = (37)(3) + (-1)(110)$

$\therefore y_2 = -1 + 3 = 2$

$y_3 = 66^{-1} \bmod 5$

$66 = (13)(5) + 1 \rightarrow \bmod k$

$1 = 1 \cdot 66 - 13 \cdot 5$

$1 = (1)(66) + (-13)(5)$

$y_3 = 1$

$y_4 = 30^{-1} \bmod 11$

$y_4 = (2)(11) + 8$

$30 = (2)(11) + 8$

$11 = (1)(8) + 3$

$8 = (2)(3) + 2$

Day: Date:

$3 = (1)(2)+1$

Substituting

$1 = 1\cdot 3 - 1\cdot 2 \rightarrow (i)$

$2 = 1\cdot 8 - 2\cdot 3$

put in (i)

$1 = 1\cdot 3 - 1(1\cdot 8 - 2\cdot 3)$

$1 = 1\cdot 3 - 1\cdot 8 + 2\cdot 3$

$1 = 3\cdot 3 - 1\cdot 8 \rightarrow (ii)$

$3 = 1\cdot 11 - 1\cdot 8$

put in (ii)

$1 = 3(1\cdot 11 - 1\cdot 8) - 1\cdot 8$

$1 = 3\cdot 11 - 3\cdot 8 - 1\cdot 8$

$1 = 3\cdot 11 - 4\cdot 8 \rightarrow (iii)$

$8 = 1\cdot 30 - 2\cdot 11$

put in (iii)

$1 = 3\cdot 11 - 4(1\cdot 30 - 2\cdot 11)$

$1 = 3\cdot 11 - 4\cdot 30 + 8\cdot 11$

$1 = 11\cdot 11 - 4\cdot 30$

$1 = (11)(11) + (-4)(30)$

$y_4 = -4 + 11$

$y_4 = 7$

put in (i)

$x = [(1)(1)(165) + (2)(2)(110) + 3(1)(66) + (4)(7)(30)] \bmod 330$

$x = [165 + 440 + 198 + 840] \bmod 330$

$x = 1643 \bmod 330$

$x = 323$ Ans

Page No. 8

Lareb

Day:

) Case 01:

$x \equiv 3 (\text{mo}$

$\gcd(5,6) = 1$

$\gcd(7,11)$

Modulos

$m = 5 \times 6 \times$

$M_1 = \frac{2310}{5}$

$y_1 = 462^{-1} \text{ m}$

$462 = 92(5)$

$462 = (2)(2)$

Substitutin

$1 = 1 \cdot 5 - 5$

$2 = 1 \cdot 46$

put in (i)

$1 = 1 \cdot 5 -$

$1 = 1 \cdot 5 -$

$1 = 185 \cdot$

$1 = (185)($

$y_1 = -2 +$

$y_2 = 38$

$385 = (6$

$64 = Su$

$1 = 1 \cdot 3$

$\therefore y_2 = 1$

Page No.

Day: Date:

2) Case 01: $x \equiv 3 \pmod 5$ Case 02: $x \equiv 3 \pmod 6$ Case 03: $x \equiv 1 \pmod 7$ Case 04: $x \equiv 0 \pmod{11}$

$\gcd(5,6)=1$ $\gcd(5,7)=1$ $\gcd(5,11)=1$ $\gcd(6,7)=1$ $\gcd(6,11)=1$
$\gcd(7,11)=1$

Modulos are pairwise prime

$m = 5 \times 6 \times 7 \times 11 = 2310$

$M_1 = \frac{2310}{5} = 462$, $M_2 = \frac{2310}{6} = 385$, $M_3 = \frac{2310}{7} = 330$, $M_4 = \frac{2310}{11} = 210$

$y_1 = 462^{-1} \bmod 5$

$462 = 92(5) + 2$

$462 = (2)(2)^{5} + 1$

Substituting

$1 = 5 \cdot 5 - 2 \cdot 2 \rightarrow (i)$

$2 = 1 \cdot 462 - 92 \cdot 5$

put in (i)

$1 = 1 \cdot 5 - 2(1 \cdot 462 - 92 \cdot 5)$

$1 = 1 \cdot 5 - 2 \cdot 462 + 184 \cdot 5$

$1 = 185 \cdot 5 - 2 \cdot 462$

$1 = (185)(5) + (-2)(462)$

$y_1 = -2 + 5 \therefore y_1 = 3$

$y_2 = 385^{-1} \bmod 6$

$385 = (64)(6) + 1$

$64 =$ Substituting

$1 = 1 \cdot 385 - 64 \cdot 6$

$\therefore y_2 = 1$

$y_3 = 330^{-1} \bmod 7$

$330 = (47)(7) + 1$

$1 = 1 \cdot 330 - 47 \cdot 7$

$(1) = (1)(330) + (-47)(7)$

$y_3 = 1$

$y_4 = 210^{-1} \bmod 11$

$210 = (19)(11) + 1$

$(1) = 1 \cdot 210 - 19 \cdot 11$

$y_4 = 1$

$\therefore x = [(3)(3)(462) + (1)(3)(385) + (1)(1)(330) + (0)(1)(210)] \bmod 2310$

$x = [4158 + 1155 + 330] \bmod 2310$

$x = 5643 \bmod 2310$

$x = 1023$

He may have 1023 oranges

Day: Date:

Q7.

a) $a=2, m=17$

$2 = (0)(17)+2$

$17 = (8)(2)+1$

$1 = 1\cdot 17 - 8\cdot 2 \rightarrow (i)$

$2 = 1\cdot 2 - 0\cdot 17$

$1 = (1)(17)+(-8)(2)$

inverse $= \bar{a} = -8+17$

$\bar{a} = 9$

b) $a=34, m=89$

$34 = (0)(89)+34$

$89 = (2)(34)+21$ c

$34 = (1)(21)+13$ +

$21 = (1)(13)+8$ *

$13 = (1)(8)+5$ *

$8 = (1)(5)+3$

$5 = (1)(3)+2$

$3 = (1)(2)+1$

Substituting

$1 = 1\cdot 3 - 1\cdot 2 \rightarrow (i)$

$2 = 1\cdot 5 - 1\cdot 3$

in

put (i)

$1 = 1\cdot 3 - 1(1\cdot 5 - 1\cdot 3)$

$1 = 1\cdot 3 - 1\cdot 5 + 1\cdot 3$

$1 = 2\cdot 3 - 1\cdot 5 \rightarrow (ii)$

$3 = 1\cdot 8 - 1\cdot 5$

put in (ii)

$1 = 2(1\cdot 8 - 1\cdot 5) - 1\cdot 5$

substitute

$1 = 2\cdot 8 - 2\cdot 5 - 1\cdot 5$

$1 = 2\cdot 8 - 3\cdot 5$ * $\rightarrow (iii)$

$5 = 1\cdot 13 - 1\cdot 8 \Rightarrow$

Put in (iii)

$1 = 2\cdot 8 - 3(1\cdot 13 - 1\cdot 8)$

$1 = 2\cdot 8 - 3\cdot 13 + 3\cdot 8$

$1 = 5\cdot 8$ * $- 3\cdot 13 \rightarrow (iv)$

$8 = 1\cdot 21 - 1\cdot 13$

Put in (iv)

$1 = 5(1\cdot 21 - 1\cdot 13) - 3\cdot 13$

$1 = 5\cdot 21 - 5\cdot 13 - 3\cdot 13$

$1 = 5\cdot 21 - 8$ + $13 \rightarrow (v)$

$13 = 1\cdot 34 - 1\cdot 21$

Put in (v)

$1 = 5\cdot 21 - 8(1\cdot 34 - 1\cdot 21)$

$1 = 5\cdot 21 - 8\cdot 34 + 8\cdot 21$

$1 = 13\cdot 21$ c $- 8\cdot 34 \rightarrow (vi)$

$21 = 1\cdot 89 - 2\cdot 34$

Put in (vi)

$1 = 13(1\cdot 89 - 2\cdot 34) - 8\cdot 34$

$1 = 13\cdot 89 - 26\cdot 34 - 8\cdot 34$

$1 = 13\cdot 89 - 34\cdot 34 \rightarrow (vii)$

$\because 34 = 1\cdot 34 - 0\cdot 89$

$1 = 13\cdot 89 - 34(1\cdot 34 - 0\cdot 89)$

$1 = 13\cdot 89 - 34\cdot 34 \rightarrow (viii)$

$1 = (13)(89)+(-34)(34)$

$\bar{a} = -34+89$

$\bar{a} = 55$

Day:

$a=144, m=$

$144 = 10)($

$233 = (1$

$144 = (1)$

$89 = (1)$

$55 = (1)$

$34 = (1$

$21 = ($

$13 = (1$

$8 = ($

$5 = ($

$3 = (1$

Substitu

$1 = 1$

$2 = 1$

put i

$1 = 1 \cdot$

$1 = 1 \cdot$

$1 = 2 \cdot$

$3 = 1$

put

$1 = 2$

$1 = 2 \cdot$

$1 = 2 \cdot$

$5 = 1$

Put ir

$1 = 2$

$1 = 2$

$1 = 5$

$8 =$

Put i

$1 =$

Day: Date:

a) $a = 144, m = 233$

$144 = (0)(233) + 144$

$233 = (1)(144) + 89$

$144 = (1)(89) + 55$

$89 = (1)(55) + 34$

$55 = (1)(34) + 21$

$34 = (1)(21) + 13$

$21 = (1)(13) + 8$

$13 = (1)(8) + 5$

$8 = (1)(5) + 3$

$5 = (1)(3) + 2$

$3 = (1)(2) + 1$

Substituting

$1 = 1 \cdot 3 - 1 \cdot 2 \rightarrow (i)$

$2 = 1 \cdot 5 - 1 \cdot 3$

put in (i)

$1 = 1 \cdot 3 - 1(1 \cdot 5 - 1 \cdot 3)$

$1 = 1 \cdot 3 - 1 \cdot 5 + 1 \cdot 3$

$1 = 2 \cdot 3 - 1 \cdot 5 \rightarrow (ii)$

$3 = 1 \cdot 8 - 1 \cdot 5$

put in (ii)

$1 = 2(1 \cdot 8 - 1 \cdot 5) - 1 \cdot 5$

$1 = 2 \cdot 8 - 2 \cdot 5 - 1 \cdot 5$

$1 = 2 \cdot 8 - 3 \cdot 5 \rightarrow (iii)$

$5 = 1 \cdot 13 - 1 \cdot 8$

Put in (iii)

$1 = 2 \cdot 8 - 3(1 \cdot 13 - 1 \cdot 8)$

$1 = 2 \cdot 8 - 3 \cdot 13 + 3 \cdot 8$

$1 = 5 \cdot 8 - 3 \cdot 13 \rightarrow (iv)$

$8 = 1 \cdot 21 - 1 \cdot 13$

Put in (iv)

$1 = 5(1 \cdot 21 - 1 \cdot 13) - 3 \cdot 13$

$1 = 5 \cdot 21 - 5 \cdot 13 - 3 \cdot 13$

$1 = 5 \cdot 21 - 8 \cdot 13 \rightarrow (v)$

$13 = 1 \cdot 34 - 1 \cdot 21$

Put in (v)

$1 = 5 \cdot 21 - 8(1 \cdot 34 - 1 \cdot 21)$

$1 = 5 \cdot 21 - 8 \cdot 34 + 8 \cdot 21$

$1 = 13 \cdot 21 - 8 \cdot 34 \rightarrow (vi)$

$21 = 1 \cdot 55 - 1 \cdot 34$

Put in (vi)

$1 = 13(1 \cdot 55 - 1 \cdot 34) - 8 \cdot 34$

$1 = 13 \cdot 55 - 13 \cdot 34 - 8 \cdot 34$

$1 = 13 \cdot 55 - 21 \cdot 34 \rightarrow (vii)$

$34 = 1 \cdot 89 - 1 \cdot 55$

Put in (vii)

$1 = 13 \cdot 55 - 21(1 \cdot 89 - 1 \cdot 55)$

$1 = 13 \cdot 55 - 21 \cdot 89 + 21 \cdot 55$

$1 = 34 \cdot 55 - 1 \cdot 89 \rightarrow (viii)$

$55 = 1 \cdot 144 - 1 \cdot 89$

Put in (viii)

$1 = 34 \cdot (1 \cdot 144 - 1 \cdot 89) - 1 \cdot 89$

$1 = 34 \cdot 144 - 34 \cdot 89 - 1 \cdot 89$

$1 = 34 \cdot 144 - 55 \cdot 89 \rightarrow (ix)$

$89 = 1 \cdot 233 - 1 \cdot 144$

Put in (ix)

$1 = 34 \cdot 144 - 55(1 \cdot 233 - 1 \cdot 144)$

$1 = 34 \cdot 144 - 55 \cdot 233 + 55 \cdot 144$

$1 = 89 \cdot 144 - 55 \cdot 233 \rightarrow (x)$

$144 = 1 \cdot 144 - 0 \cdot 233$

$1 = (89)(144) + (-55)(233)$

$\boxed{a = 89}$

Day: Date:

d) $a = 200, m = 1001$

$200 = (0)(1001) + 200$

$1001 = (5)(200) + 1$

$1 = 1 \cdot 1001 - 5 \cdot 200$

$1 = (1)(1001) + (-5)(200)$

$\bar{a} = -5 + 1001$

$\bar{a} = 996$ Ans

Q8.

a) STOP POLLUTION

i) $f(p) = (p+4) \bmod 26$

18 19 14 15

STOP POLLUTION

15 14 11 11 20 19 8 14 13

$\because f(S) = 18 + 4 \pmod{26}$

$= 22 \pmod{26}$

$= 22 [W]$

$f(T) = 19 + 4 \pmod{26}$

$= 23 \pmod{26}$

$= 23 [X]$

$f(O) = 14 + 4 \pmod{26}$

$= 18 \pmod{26}$

$= 18 [S]$

$f(P) = 15 + 4 \pmod{26}$

$= 19 \bmod 26$

$= 19 [T]$

$f[L] = 11 + 4 \pmod{26}$

$= 15 \bmod 26$

$= 15 [P]$

$f(U) = 20 + 4 \pmod{26}$

$= 24 \pmod{26}$

$= 24 [Y]$

$f(I) = 8 + 4 \pmod{26}$

$= 12 \bmod 26$

$= 12 [M]$

$f(N) = 13 + 4 \pmod{26}$

$= 17 \pmod{26}$

$= 17 [R]$

$\therefore$ WXST TSPPYXMSR

Day:

$f(p) = (p+21) \text{ mo}$

$f(S) = 18 + 21 \text{ m}$

$f(S) = 39 \text{ mod } 2$

$= 13 [N]$

$f(T) = 19 + 21 \text{ mo}$

$= 40 \text{ mod}$

$= 14 [O]$

$f(O) = 14 + 21 \text{ m}$

$= 35 \text{ mo}$

$= 9 [J]$

$f(I) = 8 + 21$

$= 29 \text{ m}$

$= 3 [D]$

$f(P) = 15 + 2$

$= 36 +$

$= 10 [$

$f(L) = 11 + 2$

$= 32$

$= 6$

$f(U) = 20$

$= 4$

$= 1$

$f(N) = 13$

$=$

$=$

Day: Date:

$f(p) = (p+21) \bmod 26$

$f(S) = 18+21 \bmod 26$

$f(S) = 39 \bmod 26$

$= 13$ [N]

$f(T) = 19+21 \bmod 26$

$= 40 \bmod 26$

$= 14$ [O]

$f(O) = 14+21 \bmod 26$

$= 35 \bmod 26$

$= 9$ [J]

$f(I) = 8+21 \bmod 26$

$= 29 \bmod 26$

$= 3$ [D]

$f(P) = 15+21 \bmod 26$

$= 36 \bmod 26$

$= 10$ [K]

$f(L) = 11+21 \bmod 26$

$= 32 \bmod 26$

$= 6$ [G]

$f(U) = 20+21 \bmod 26$

$= 41 \bmod 26$

$= 15$ [P]

$f(N) = 13+21 \bmod 26$

$= 34 \bmod 26$

$= 8$ [I]

NOJK KJGGPODJI

b) $f(p) = (p+10) \bmod 26$

i) CEBBOX NOBXYG

$f(C) = 2-10 \bmod 26$

$= -8 \bmod 26$

$= (8+26) \bmod 26 = 18 \bmod 26 = 18$ [S]

$f(E) = (4-10) \bmod 26 = -6+26 \bmod 26$

$= 20 \bmod 26 = 20$ [U]

$f(B) = 1-10 \bmod 26 = -9+26 \bmod 26$

$= 20 \bmod 26 = 17$ [R]

$f(O) = 14-10 \bmod 26 = 4 \bmod 26 = 4$ [E]

$f(X) = 23-10 \bmod 26 = 13 \bmod 26 = 13$ [N]

$f(N) = 13-10 \bmod 26 = 3 \bmod 26 = 3$ [D]

$f(Y) = 24-10 \bmod 26 = 14 \bmod 26 = 14$ [O]

$f(G) = 6-10 \bmod 26 = -4+26 \bmod 26$

$= 22 \bmod 26 = 22$ [W]

SURRENDER NOW

ii) LO WI PBSOXN

$f(L) = 11-10 \bmod 26 = 1 \bmod 26 = 1$ [B]

$f(O) = 4 \bmod 26 = 4$ [E]

$f(W) = 22-10 \bmod 26 = 12 \bmod 26 = 12$ [M]

$f(I) = 15-10 \bmod 26 = 5 \bmod 26 = 5$ [F]

$f(B) = 17$ [R]

$f(S) = 18-10 \bmod 26 = 8 \bmod 26 = 8$ [I]

$f(O) = 4$ [E]

$f(X) = 13$ [N] BE MY FRIEND

$f(N) = 3$ [D]

Day: Date: Day:

Q9.

i) $5^{2003} \bmod 7$

$5^{7-1} = 1 \pmod 7$

$5^6 = 1 \pmod 7 \rightarrow (i)$

$a = qd + r$

$2003 = (333)(6) + 5$

$5^{333 \times 6 + 5} \pmod 7$

$(5^6)^{333} \cdot 5^5 \pmod 7 \rightarrow (ii)$

Put (i) in (ii)

$1^{333} \cdot 5^5 \pmod 7$

$3125 \pmod 7 = 3$ Ans

ii) $5^{2003} \bmod 11$

$5^{10} = 1 \pmod{11} \rightarrow (i)$

$a = qd + r$

$2003 = (200)(10) + 3$

$5^{200(10)+3} \bmod 11$

$(5^{10})^{200} \cdot 5^3 \bmod 11 \rightarrow (ii)$

Put (i) in (ii)

$= 1^{200} \cdot 5^3 \bmod 11$

$= 1 \cdot 125 \bmod 11$

$= 4$ Ans

iii) $5^{2003} \bmod 13$

$5^{12} = 1 \pmod{13} \rightarrow (i)$

$a = qd + r$

$2003 = (166)(12) + 11$

$5^{166(12)+11} \pmod{13}$

$(5^{12})^{166} \cdot 5^{11} \bmod 13 \rightarrow (ii)$

Put (i) in (ii)

$1^{166} \cdot 5^{11} \bmod 13 = 8$ Ans

Q10.

a) I LOVE DISCRETE MATHEMATICS

8 11 14 21 4 3 8 18 2 17 4 19 4

12 0 19 7 4 12 0 19 8 2 18

$f(p) = (p+k) \bmod 26$

for caesar cipher ; $k = 3$

$f(I) = 8+3 \bmod 26 = 11 \bmod 26 = 11$ [L]

$f(L) = 11+3 \bmod 26 = 14 \bmod 26 = 14$ [O]

$f(O) = 14+3 \bmod 26 = 17 \bmod 26 = 17$ [R]

$f(V) = 21+3 \bmod 26 = 24 \bmod 26 = 24$ [Y]

$f(E) = 4+3 \bmod 26 = 7 \bmod 26 = 7$ [H]

$f(D) = 3+3 \bmod 26 = 6 \bmod 26 = 6$ [G]

$f(S) = 18+3 \bmod 26 = 21 \bmod 26 = 21$ [V]

$f(C) = 2+3 \bmod 26 = 5 \bmod 26 = 5$ [F]

$f(R) = 17+3 \bmod 26 = 20 \bmod 26 = 20$ [U]

$f(T) = 19+3 \bmod 26 = 22 \bmod 26 = 22$ [W]

$f(M) = 12+3 \bmod 26 = 15 \bmod 26 = 15$ [P]

$f(A) = 0+3 \bmod 26 = 3 \bmod 26 = 3$ [D]

$f(H) = 7+3 \bmod 26 = 10 \bmod 26 = 10$ [K]

L ORYH GLVFUHWH PDWKHPDW

b) i) PLG WZR DVVLJ QPH QW

15 11 6 22 25 17 3 21 21 11 9 16 15 11

$f(P) = 15-3 \bmod 26 = 12 \bmod 26 = 12$ [M]

$f(L) = 11-3 \bmod 26 = 8 \bmod 26 = 8$ [I]

$f(G) = 6-3 \bmod 26 = 3 \bmod 26 = 3$ [D]

$f(W) = 22-3 \bmod 26 = 19 \bmod 26 = 19$ [T]

$f(Z) = 25-3 \bmo$

$(K) = 17-3 \bmod$

$(D) = 3-3 \bmod$

$(V) = 21-3 \bmod$

$(J) = 9-3 \bmod$

$(Q) = 16-3 \bmo$

$(H) = 7-3 \bmod$

MID TWO A

IDVW QXF

; 3 21 22 16 23

$F(I) = 8-3 m$

$F(D) = 3-3 r$

$f(V) = 21-3$

$f(W) = 22-3$

$f(Q) = 16-3$

$f(X) = 23-3$

$f(F) = 5-3$

$f(H) = 7-3$

$f(L) = 11-3$

$f(Y) = 24-$

$f(U) = 20$

$f(B) = 1-3 m$

FAST NU

Day: Date:

f(Z) = 25-3 mod 26 = 22 mod 26 = 22 [W]
f(K) = 17-3 mod 26 = 14 mod 26 = 14 [O]
f(D) = 3-3 mod 26 = 0 mod 26 = 0 [A]
f(V) = 21-3 mod 26 = 18 mod 26 = 18 [S]
f(J) = 9-3 mod 26 = 6 mod 26 = 6 [G]
f(Q) = 16-3 mod 26 = 13 mod 26 = 13 [N]
f(H) = 7-3 mod 26 = 4 mod 26 = 4 [E]

MID TWO ASSIGNMENT

1DVW QXFHV XQLVHUVLWB
(5 7, 24, 22)

8 3 21 22 16 23 21 23 16 11 7 20 21 11 1

F(1) = 8-3 mod 26 = 5 mod 26 = 5 [F]
F(D) = 3-3 mod 26 = 0 mod 26 ≡ A [A]
f(V) = 21-3 mod 26 = 18 mod 26 = 18 [S]
f(W) = 22-3 mod 26 = 19 mod 26 = 19 [T]
f(Q) = 16-3 mod 26 = 13 mod 26 = 13 [N]
f(X) = 23-3 mod 26 = 20 mod 26 = 20 [U]
f(F) = 5-3 mod 26 = 2 mod 26 = 2 [C]
f(H) = 7-3 mod 26 = 4 mod 26 = 4 [E]
f(I) = 11-3 mod 26 = 8 mod 26 = 8 [I]
f(V) = 24-3 mod 26 = 21 mod 26 = 21 [V]
f(U) = 20-3 (mod 26) = 17 mod 26 = 17 [R]
f(B) = 1-3 mod 26 = 24 mod 26 = 24 [Y]

FAST NUCES UNIVERSITY

Q11.

a) i) h(034567981) = 034567981 mod 97
= 91

ii) h(183211232) = 183211232 mod 97 = 57
iii) h(220195744) = 220195744 mod 97 = 21
iv) h(987255335) = 987255335 mod 97 = 5

b) i) h(104578690) = 104578690 mod 101 = 58
ii) h(432222187) = 432222187 mod 101 = 60
iii) h(372201919) = 372201919 mod 101 = 52
iv) h(5013387 53) = 5013387 53 mod 101 = 3

Q12.

$x_{n+1} = (4x_n + 1) \bmod 7 ; x_0 = 3$

$x_{0+1} = x_1 = (4x_0 + 1) \bmod 7 = 13 \bmod 7 = 6$
$x_{1+1} = x_2 = (24+1) \bmod 7 = 25 \bmod 7 = 4$
$x_{2+1} = x_3 = (16+1) \bmod 7 = 17 \bmod 7 = 3$
$x_{3+1} = x_4 = (12+1) \bmod 7 = 13 \bmod 7 = 6$
$x_{4+1} = x_5 = (24+1) \bmod 7 = 25 \bmod 7 = 4$

Constant repitition ⋮

$x_6 = 3$
$x_7 = 6$
$x_8 = 4$
$x_9 = 3$
$x_{10} = 6$
$x_{11} = 4$
$x_{12} = 3$
$x_{13} = 6$
$x_{14} = 4$
$x_{15} = 3$
$x_{16} = 6$
$x_{17} = 4$
$x_{18} = 3$
$x_{19} = 6$
$x_{20} = 4$

Day: Date:

Q13.

a)

i) 7323218443 4

$x_{12} = ?$

$\Rightarrow (3\cdot7)+3+(2\cdot3)+3+(2\cdot3)+1+(3\cdot8)+4+(3\cdot4)+4+(3\cdot4)+x_{12}$

$\Rightarrow 21+3+6+3+6+1+24+4+12+3+12+x_{12}$

$\Rightarrow 95+x_{12}$

$95+x_{12} \equiv 0 \pmod{10}$

$x_{12} = -95 \pmod{10}$

$\boxed{x_{12} = 5}$

ii) 63623991346

$x_{12} = ?$

$\Rightarrow (3\cdot6)+3+(3\cdot6)+2+(3\cdot3)+9+(3\cdot9)+1+(3\cdot3)+4+(3\cdot6)+x_{12}$

$\Rightarrow 18+3+18+2+9+9+27+1+9+9+18+x_{12}$

$\Rightarrow 118+x_{12}$

$118+x_{12} \equiv 0 \pmod{10}$

$x_{12} = -118 \bmod 10$

$\boxed{x_{12} = 2}$

b) i) 0360002914522 → validity check

$\Rightarrow 0+3+(3\cdot6)+0+0+0+(2\cdot3)+9+(1\cdot3)+4+(3\cdot5)+2$

$\Rightarrow 3+18+6+9+3+4+15+2$

$\Rightarrow 60$

$60 \equiv 0 \pmod{10}$

$0 = 0$

valid

ii) 0123456789033 → validity check

$\Rightarrow 0+1+(2\cdot3)+3+(3\cdot4)+5+(3\cdot6)+7+(3\cdot8)+9+0+3$

$\Rightarrow 1+6+3+12+5+18+7+24+9+3$

$\Rightarrow 88 \equiv 0 \bmod (10)$

$8 \neq 0$ invalid

0-07-1198

$x_{10} = ?$

$x_{10} = 0+0+(3\cdot$

$x_{10} = (21+4+$

$x_{10} = 213$ mo

$x_{10} = 4$

0-321-5

$x_{10} = 0+(2\cdot$

$x_{10} = (6+6+$

$x_{10} = (50+8$

$(50+8(1)$

$8 \equiv 6$m

$8-6 =$

finding in

$8 = 6(1$

$11 = (1)(1$

$8 = (2)(4$

$3 = (1)(3$

Backwa

$1 = 1\cdot2$

$2 = 1\cdot2$

Put in

$1 = 1\cdot3$

$1 = 1\cdot3$

$1 = 3\cdot$

$3 = 1$

Put i

$1 = 3$

Page

Day: Date:

Q14.

) 0-07-119881

$x_{10} = ?$

$x_{10} = 0+0+(3\cdot7)+(1\cdot4)+(1\cdot5)+(9\cdot6)+(8\cdot7)+(8\cdot8)+(9\cdot1) \bmod 11$

$x_{10} = (21+4+5+54+56+64+9) \bmod 11$

$x_{10} = 213 \bmod 11$

$x_{10} = 4$ Ans

) 0-321-500Q1-8

$x_{10} = 0+(2\cdot3)+(3\cdot2)+(4\cdot1)+(5\cdot5)+0+0+(8\cdot Q)+(9\cdot1) \bmod 11$

$x_{10} = (6+6+4+25+8Q_1+9) \bmod 11$

$x_{10} = (50+8Q_1) \bmod 11$

$(50+8Q_1) \bmod 11 \equiv 8 \quad \{x_{10}=8\}$

$8 \equiv 6 \bmod 11 + 8Q_1 \bmod 11$

$8-6 = 0 \bmod 11$

Finding inverse to eliminate 2

$8 = 0(11)+8$

$11 = (1)(8)+3$

$8 = (2)(3)+2$

$3 = (1)(2)+1$

Backward substituting

$1 = 1\cdot3 - 1\cdot2 \rightarrow (i)$

$2 = 1\cdot8 - 2\cdot3$

Put in (i)

$1 = 1\cdot3 - 1(1\cdot8 - 2\cdot3)$

$1 = 1\cdot3 - 1\cdot8 + 2\cdot3$

$1 = 3\cdot3 - 1\cdot8 \rightarrow (ii)$

$3 = 1\cdot11 - 1\cdot8$

Put in (ii)

$1 = 3\cdot(1\cdot11 - 1\cdot8) - 1\cdot8$

$\Rightarrow 3\cdot11 - 3\cdot8 - 1\cdot8$

$1 = 3\cdot11 - 4\cdot8 \rightarrow (iii)$

$1 = (3)(11)+(-4)(8)$

$\bar{a} = -4+11$

$\bar{a} = 7$

x eq $\rightarrow$ (A) with $\bar{a}$

$7 \times 8Q \bmod 11 = 2 \times 7 \bmod 11$

$8Q \bmod 11 = 14 \bmod 11$

$8Q = 3$

Day: Date:

Q15.

A	T	T	A	C	K
00	19	19	00	02	10

$n = 43 \cdot 59 = 2537$

$e = 13$

$k = (43-1)(59-1) = 42 * 58 = 2436$

$C = M^e \bmod n$

pair 1 = AT = $(0019)^{13} \bmod 2537$

pair 2 = TA = $(1900)^{13} \bmod 2537$

pair 3 = CK = $(0210)^{13} \bmod 2537$

Q16.

a) No. of floors = A = 27

No. of offices = O = 37

No. of offices in the building P = A * O = 27 * 37

= 999 offices

b) Colors of shirt = 12

2 genders

no. of sizes = 3

Different shirts required = 12 * 2 * 3 = 72 shirts

Q17.

a) AAA letter initials

$\therefore 26 * 26 * 26 = 26^3 = 17576$

b) $26 * 25 * 24 = 15600$ initials since a single cannot be repeated; they should be distinct

Day:

There are 16 l

$\therefore 16^{10} + 16^{28} + 1$

26 English alph

$26^4 - 25^4 =$

) {1, 2, ...}

Since each v

one or two

$\therefore 2 * 2 * 2$

Each successi

one option

function. So,

{3, 7, 9, 11, 2

For 3:

(3, 7), (3, 9)

For 7:

{(7, 9), (7, 1

For 11:

{∅}

For 24:

{∅}

Day: Date:

Q18.

i) There are 16 hexadecimal digits
$\therefore 16^{10} + 16^{28} + 16^{58}$ WEP keys are possible

ii) 26 English alphabets
$26^4 - 25^4 = 66351$ strings

Q19.

i) $\{1, 2, \ldots\ldots, m\}$
Since each value of the domain can be mapped to one or two values,
$\therefore 2 * 2 * 2 * \ldots\ldots m = 2^m$

ii) Each successive element from the domain will have one option that is predecessor as it is one-to-one function. So, number of functions are $5 * 4 * 3 * 2 * 1 = 120$

Q20.

$\{3, 7, 9, 11, 24\} \Rightarrow$

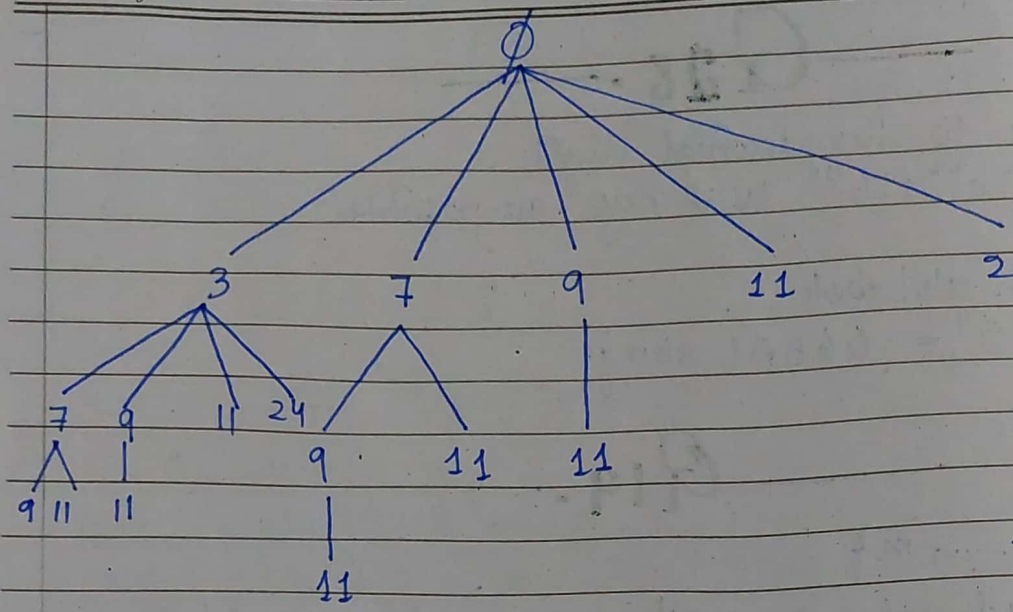
For 3:
$\{(3,7), (3,9), (3,11), (3,24)\}$

For 7:
$\{(7,9), (7,11)\}$

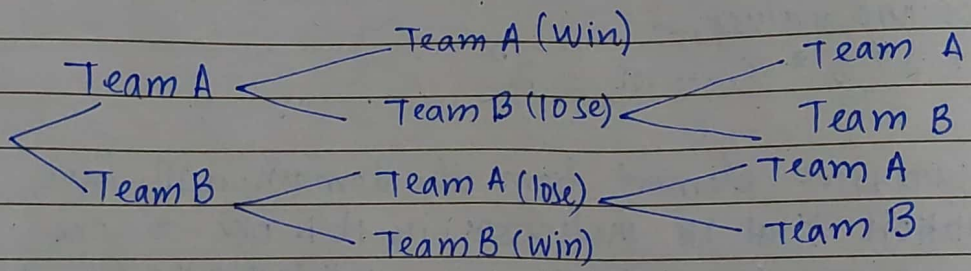
For 11: $\{\emptyset\}$

* For 9: $\{(9,11)\}$

For 24: $\{\emptyset\}$



b)



Q21.

a) No. of members = 8
No. of drum positions = 3
possible ways student can be chosen = ${}^8C_3 = 56$ ways

b) No. of CS elective courses = 6
No. of courses you are interested in = 12
Selection of courses = ${}^{12}C_6 = 924$ ways

c) No. of people in class = 9
Team with n-members = 5
No. of ways different teams can be chosen = ${}^9C_5 = 126$ teams

Day: Date:

) Since ordered
∴ P(20, 5) =
) P(16, 4) =
) P(15, 2) =

Types of meat
Types of br

) Combination
∴ 5C1 *

5C1 = 5

3C2 =

4C1 =

6C3 =

→ 5*

Day: Date:

Q22.

a) Since ordered arrangement

$\therefore P(20,5) = {}^{20}P_5 = 1,860,480$ ways

b) $P(16,4) = {}^{16}P_4 = 43,680$ ways

c) $P(15,2) = {}^{15}P_2 = 210$ ways

Q23.

Types of meat = 5
Types of bread = 3
Types of cheeses = 4
Types of condiments = 6

1) Combination of meat, bread, cheeses and condiments

$\therefore {}^5C_1 * {}^3C_2 * {}^4C_1 * {}^6C_3 =$

$${}^5C_1 = \frac{5!}{1! \times 4!} = \frac{5 \times 4!}{1 \times 4!} = 5$$

$${}^3C_2 = \frac{3!}{2! \times 1!} = \frac{3 \times 2!}{2! \times 1} = 3$$

$${}^4C_1 = \frac{4!}{1! \times 3!} = \frac{4 \times 3!}{3!} = 4$$

$${}^6C_3 = \frac{6!}{3! \times 3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

$\Rightarrow 5 * 3 * 4 * 20 = 1200$ students

Day: Date:

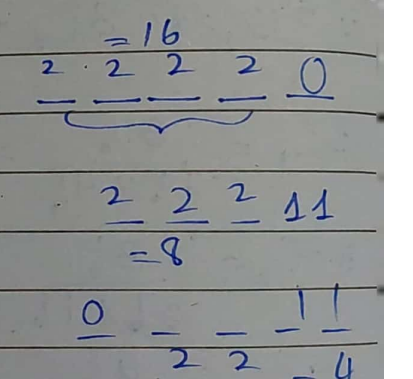
b) Unique faces
∴ 15 * 48 * 24 * 34 * 28 * 28 = 460,615,680 faces

c)

Q24.

a) A = String begins with three 0s = $2^7 = 128$
B = String ends with two 0s = $2^8 = 256$
$A \cap B = 2^5 = 32$ $A \cup B = ?$
Combining addition & sub rule:
$A \cup B = A + B - A \cap B$
$= 16 + 128 + 256 - 32 = 352$

b) A = String begins with 0s = $2^4 = 16$ →
B = String ends with two 1s = $2^3 = 8$
$A \cap B = 2^2 = 4$
∴ String with ~~neither~~ both = $A \cap B = 4$
string with 0s or 1s but not both
$A \cup B = A + B - A \cap B$
$= 16 + 8 - 4 = 20$



Q25.

a) Using Pigeonhole Principle:
$N = 30 \quad k = 26$
$\left\lceil \frac{N}{k} \right\rceil = \left\lceil \frac{30}{26} \right\rceil = 2$
∴ Two students have last names that begin with same letters

Day: Date:

Using Pigeonhole Principle:

$\left\lceil \frac{N}{k} \right\rceil$; $N = 8,008,278$, $k = 1,000,000$

$\therefore \left\lceil \frac{8,008,278}{1,000,000} \right\rceil = 9$

$N = 677$, $k = 38$

$\left\lceil \frac{N}{k} \right\rceil = \left\lceil \frac{677}{38} \right\rceil = 18$ classes

Q26.

coefficient of x^5 in $(1+x)^{11} = ?$

$\therefore {}^{n}C_{r} = {}^{11}C_{5} = 462$

$a^7 b^{17}$ coefficient in $(2a-b)^{24}$?

Using

$${}^{n}C_{r} = {}^{24}C_{17} (2)^7 (-1)^{17} = -44,301,312$$

Q27.

No. of men = 16 Total students = 36

No. of women = 20

$\therefore 36!$ ways students can be arranged in a row

$P(36,7) =$

Using product Rule:

$20! * 16!$ ways men & women can be arranged separately

Day: Date:

Q28.

a) Proof by exhaustion:

$\exists x [(x > 5) \rightarrow 2^x - 1$ is prime$]$

Let $n = 5$, by trivial proof

$2^n - 1 = 2^5 - 1 = 32 - 1 = 31$

31 is a prime number ∴ statement is true.

b) Proof by contradiction:

Contradictive statement ⇒ If $p \mid a$ then $p \nmid (a+1)$

Let k be an integer such that

$$\frac{a}{p} = k$$

$$a = kp \longrightarrow (1)$$

Let s be an integer such that

$$\frac{a+1}{p} = s$$

$$a + 1 = ps \longrightarrow (2)$$

(2) − (1)

$$a + 1 - a = ps - kp$$

$$1 = p(s-k) \qquad \{\because 1 = s-k\}$$

$$1 = pr$$

$$\frac{1}{p} = r$$

This statement says that p divides 1, which is impossible since primes are always > 1.

Hence our supposition is false

So, the given statement is true

Day: Date:

Q29.

a) Proof by exhaustion:

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

Let $a = 16$ and $b = 0$

$$\sqrt{16+0} = \sqrt{16} + \sqrt{0}$$

$$4 = 4$$

Proved!

b) 1* x is +ve
2* x is -ve

1* Let $x = 5$
$|5| = 5 > 1$

2* Let $x = -5$
$|-5| = 5 > 1$

Hence the statement is true

Q30.

For every prime, n+2 is prime
By counter example,

Let $n = 2$

$n + 2 = 2 + 2$

$n + 2 = 4$

4 is not a prime, hence disproved!

b) Proof by contradiction:

The set of prime numbers is infinite

$S = \{P_1, P_2, \ldots, P_n\}$; set of all prime no.s

Adding 1 to x:

$x = x + 1$

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Now, if this number x is divided by any of the prime in the set, then remainder is 1. So, this new number is either a prime proving our supposition wrong, or it is a composite such that a prime factor of this number exists.
Hence original statement is true.

Q31.

a) Proof by contradiction:
If n and m are odd, then n+m is odd integer

∴ Let $m = 2k+1$ and $n = 2k+1$ are two odd integers

$$m+n = 2k+1+2k+1$$
$$m+n = 4k+2$$
$$m+n = 2(2k+1)$$
$$m+n = 2r \quad \{r = 2k+1\}$$

$2r$ would always given an even output $\forall \mathbb{Z}$
∴ Original statement is true.

b) Proof by contraposition:
If m is odd and n is even, m+n is odd
Let $m = 2k+1$ is an odd integer
Let $n = 2k$ is an even integer

$$m+n = 2k+1+2k$$
$$m+n = 4k+1$$
$$m+n = 2(2k)+1$$
$$m+n = 2r+1 \quad \{r = 2k\}$$

$2r+1$ would give an odd output, hence original statement is true.

Day: Date:

Q32.

Proof by contradiction:

$6-7\sqrt{2}$ is irrational

Rational numbers can be represented as:

$x = p/q$

$\therefore 6 - 7\sqrt{2} = p/q$

$6 - p/q = 7\sqrt{2}$

$6q - p/q = 7\sqrt{2}$

$6q - p/7q = \sqrt{2}$

According to this statement, $6q-p/7q$ is rational but $\sqrt{2}$ is irrational, hence original statement is true.

Proof by contradiction:

$\sqrt{2}+\sqrt{3}$ is ~~irrational~~ rational

For rational numbers, the squares of their sum is also rational, so

$(\sqrt{2}+\sqrt{3})^2 \Rightarrow 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$

$\sqrt{6}$ is irrational, hence the original statement is true.

Day: Date:

Q33.

a) $1^2+2^2+3^2+\cdots+n^2=(n(n+1)(2n+1))/6$

① Basis case:

$P(1): 1^2+2^2+3^2+\cdots+1 = 1(1+1)(2+1)/6$

$= 6/6 = 1$

② Inductive case:

P(k) Ps true:

$P(k): 1^2+2^2+3^2+\cdots+k^2 = \frac{k(k+1)(2k+1)}{6} \rightarrow (i)$

P(k+1) is true:

$P(k+1): 1^2+2^2+3^2+\cdots+(k+1)^2 = 1^2+2^2+3^2+\cdots+k^2+(k+1)^2 \rightarrow (ii)$

Substitute (i) in (ii)

$(ii) \Rightarrow 1^2+2^2+3^2+\cdots+(k+1)^2 = \left[\frac{k(k+1)(2k+1)}{6}\right]+(k+1)^2$

$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$= (k+1)\left[\frac{k(2k+1)+6(k+1)}{6}\right]$

$= (k+1)\left[\frac{(k+6)(2k+1)}{6}\right]$

$= (k+1)\left[\frac{2k^2+k+12k+6}{6}\right]$

$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$

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$$= \left[\frac{2k^2+4k+3k+6}{6}\right](k+1)$$

$$= \frac{(k+1)\left[2k(k+2)+3(k+2)\right]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2k+2+1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$ Proved!

$1+2+2^2+\cdots+2^n = 2^{n+1}-1$ for all integers $n \geq 0$

① Base case:

$n=0$

$P(0): 1+2+2^2+\cdots+2^0 = 2^1-1 = 1$

② Inductive case:

$P(k)$ is true:

$P(k): 1+2+2^2+\cdots+2^k = 2^{k+1}-1 \rightarrow (i)$

$P(k+1)$ is true:

$P(k+1): 1+2+2^2+\cdots+2^{(k+1)} = 2^{k+1+1}-1$

$= 1+2+2^2+\cdots+2^k+2^{k+1} \rightarrow (ii)$

Substitute (i) in (ii)

(ii) $\Rightarrow$ $= 1+2^{k+1}-1+2^{k+1}$

$= 2^{k+2}-1$

$= 2^{k+1+1}-1$

$= 2^{n+1}-1$ Proved!

Day: Date:

c) $1^3+2^3+3^3+\cdots+n^3=\frac{1}{4}n^2(n+1)^2$

① Base case:

$P(1): 1^3+2^3+3^3+\cdots+1=\frac{1}{4}(1)(1+1)^2$

$1=\frac{4}{4}=1$

$1=1$

② Inductive case:

P(K) is true:

$P(k): 1^3+2^3+3^3+\cdots+k^3=\frac{1}{4}k^2(k+1)^2 \longrightarrow (1)$

P(K+1) is true:

$P(k+1): 1^3+2^3+3^3+\cdots+(k+1)^3 = 1^3+2^3+3^3+\cdots+k^3+(k+1)^3$

↳ (2)

Substitute (1) in (2)

(2) ⇒ $= 1^3+2^3+3^3+\cdots+k^3+(k+1)^3$

$= \frac{1}{4}k^2(k+1)^2+(k+1)^3$

$= \frac{k^2(k+1)^2+4(k+1)^3}{4}$

$= (k+1)^2\left[\frac{k^2+4(k+1)}{4}\right]$

$= (k+1)^2\left[\frac{k^2+4k+4}{4}\right]$

$= (k+1)^2\left[\frac{(k+2)^2}{4}\right]$

$\Rightarrow \frac{1}{4}(k+1)^2(k+2)^2$

$\Rightarrow \frac{1}{4}n^2(n+1)^2$ Proved

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Lareb PAPER PRODUCTS

Made in Pakistan

Day: Date:

Q34.

15+1

i) Constructing computer programs
ii) Designing hardware

i) Selection of menu, food and clothe items
ii) Lottery numbers

i) Used to study large data sets; concept of big data
ii) Selection of nominees for student council.

i) Handling input-output situations
ii) Calculating tax money on basis of salary.

i) Domino effect → ensuing each domino hits the domino next to it
ii) Used in solving puzzles.