Date:

$x_2 + 300 = 400 + x_3$
$x_1 + 100 = x_2 + 400$
$x_4 + 200 = x_1 + 300$
$x_3 + 750 = 250 + x_4$

$x_2 - x_3 = 100$
$x_1 - x_2 = 300$
$x_4 - x_1 = 100$
$x_3 - x_4 = -500$

$u = kv$

$-500 + 0$
500

$$\begin{bmatrix} 0 & 1 & -1 & 0 & 100 \\ 0 & 1 & 0 & -1 & 400 \end{bmatrix}$$

Vector space:- axioms → set of assumptions

u, v, w in V (non empty set of objects) with vector addition + scalar multiplication

1. $u + v$ must be in V
2. $u + v = v + u$
3. $u + (v + w) = (u + v) + w$
4. $0 + u = u + 0 \Rightarrow u$
5. $u + (-u) = 0$
6. ku must be in V
7. $k(u+v) = ku + kv$

Date:

(8) $(k+m)u = ku + mu$

(9) $(km)u = k(mu)$

(10) $1 \cdot u = u$

e.g 1 Let V consist of single object by 0

$0 + 0 = 0$

$k(0) = 0$

all axioms satisfy so zero vector space

eg 02 R^n = set of all ordered n-tuples is a vector space.

n-space (n=2, so 2 space)

(1) $u + v = (u_1, u_2, u_3, \ldots) + (v_1, v_2, v_3, \ldots)$

$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3, \ldots)$

$= (u_1, u_2, u_3, u_4, \ldots) + (v_1, v_2, v_3, v_4, \ldots)$

$= u + v$

n x 1 remains same $= (v_1, v_2, v_3, v_4, \ldots) + (u_1, u_2, u_3, \ldots)$

$\begin{bmatrix} w_1 \\ w_2 \\ w_n \end{bmatrix}$ $= v + u$

eg 04 The vector of 2x2 matrix.

V = set of all 2x2 matrices of real nos

let $u = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$ $v = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$ $w = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$

(1) $u + v = v$ order same.

(2) $u + v = v + u$ (3) $u + (v + w) = (u + v)$

Date:

(4) $0 + U = U$
(5) $U + (-U) = 0$
(6) kU = must in V
(7) $k(U+V) = kU + kV$
(8) $(k+m)V = kV + mU$
(9) $(k.m)U = (U.k).m$
(10) $1.U = U$

eg 03 $R^{\infty} = V$ is a vector space.

$V = R^2$ set of all ordered pair.
(1) $U+V = (U_1 + V_1, U_2 + V_2)$
(2) $kU = (kU_1, 0) \Rightarrow 3(5,6) = (15,0)$
(10) violated. $1(5,6) = (5,0) \neq U$

e.g. 0 V = set of +ve real numbers ↗ unusual vector space.
Let U + V be any vectors defined in real numbers
(1) $U + V = UV$
(2) $kU = U^k$

(2) $U + V = V + U$ ✓ (3) $U + (V+W) = (U+V) + W$ ✓
(4) $0 + U = U$ ✗ → zero not included
so take 1
(5) $U + (-U) = 0$
(7) point of origin is 1
if commutative on both sides so it's valid.

(5) $U + (-U) = \cancel{0}1$ → for this case.

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$$\begin{array}{r}25\\ \times 9\\ \hline 225\end{array}$$

Date:

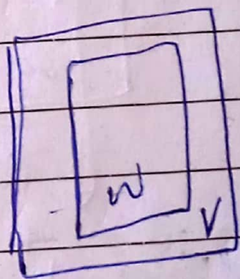
$= U(-U)$
$= U(-1)U$
$= UU^{-1}$
$= 1$

$k(U+V) = kU + kV$
$2(3+5) = 3^2 + 5^2$
$16 \neq 34$
$= 15^2 = 34$

$\boxed{225 = 225}$

$(km)U = k(mU)$ $m=2$ $k=3$ $U=4$
$(m^k)U = k(U^m)$
m^{k^U}
$m^{kU} = U^{mk}$
$2^{3\times 4} = 4^{2\times 3}$
$2^{12} = 2^{12}$

A subset W of a vector space V if W itself a vector space under addition and scalar multiplication defined on V.



subspaces — all 6 were inherited

① Closure of W under addition
④ Existence of a zero vector W.

} should be checked to prove it as a subspace

⑤ Existence of a -ve in W

⑥ Closure of W under scalar multiplication

it can go out

These Can't be in W.

Date:

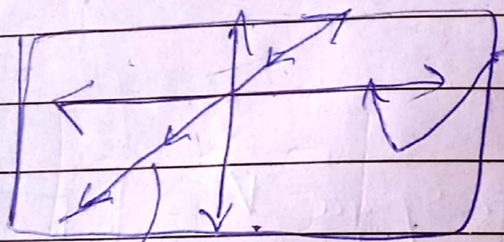
eg 1 The zero subspace.

If V is any vector space, if $W = \{0\}$ is the subset of V that consist zero vector only.

- $0 + 0 = 0$ in W • $k(0) = 0$ in W.

eg 02 Lines through the origin are subspaces of R^2 & R^3

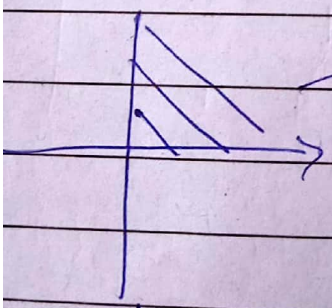
Also planes.



R^2 or R^3 → planes.

if we add or multiply will remain in zero origin.

eg 04 A subset of R^2 that is not a subspace.



→ it's not a vector subspace, no -ve value in w,

R^2

$U = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ $V = \begin{bmatrix} -1 & 2 \\ -2 & 5 \end{bmatrix}$ W = invertible 2×2 matrices

1. U must is invertible.
2. Addition $U + V$ = not invertible

not a subspace.

Date:

$V = R^3$ defined $(a, 0, 0)$.

① closure under addition

$(3,0,0) + (2,0,0) = (5,0,0)$ $(10,0,0)$ defined.

② closed under multiplication

③ zero vector exist $(0,0,0)$

④ -ve value exist.

$(a, 1, 1)$ ① not closed under addition

$(2, 2, 2)$

(a, b, c) $b = a + c + 1$ not a subspace.

Scalar addition not possible.

SPANNING SETS:-

Set of all possible linear combination of vectors in a vector space V

Span $(V_y\ V_x - V_x)$ = Set of all possible LC of V in V

det (A) = 0 so vectors don't span.

if consistent solution so spans

eg 01

The standard unit vectors. R^n

$e_1 = (1, 0, 0, 0, \ldots)$

$e_2 = (0, 1, 0, 0, \ldots)$

$\vdots$

$e_n = (0, 0, 0, \ldots, 1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since det is not zero. det = 1

$\therefore e^3$ span R^n

Date:

Q.2 $V_1 = (1,1,2)$, $V_2 = (1,0,1)$
$V_3 = (2,1,3)$ span R^3?

$$V = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix} = 1\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= (-1) - (1) + 2 = 0$$

$$|A| = 0$$

Hence, it doesn't span R^3

Q.3 $V_1 = (2,2,2)$ $V_2 = (0,0,3)$ $V_3 = (0,1,1)$

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow R_1 \times 1/2 \Rightarrow R_1 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow R_2 \times 1/3 \Rightarrow R_2 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow -R_2 + R_3 \Rightarrow R_3 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$x_1 + x_2 = 1$
$x_2 = 0$
$\boxed{x_1 = 1}$

$(-6) \neq 0$ so span.

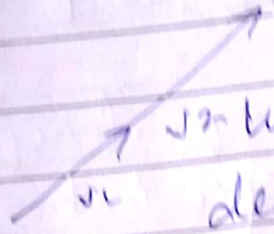
$\boxed{x_2 = 0}$

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

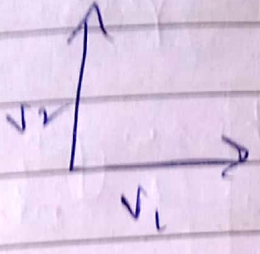
$$= \begin{bmatrix} x_1 = 0 \\ 3x_2 = 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & \end{bmatrix}$$

Date:

Unit vector $= \dfrac{V}{\|V\|}$ $\quad \|V\| = \sqrt{V_x^2 + V_y^2 + V_z^2}$



linearly dependent



linearly independent

$V = k_1 V_1 + k_2 V_2 + \cdots + k_r V_r$

$k_1 = k_2 = k_3 = \cdots k_r = 0$

independent if solution zero.

$v_1 = (1, 2, 2, -1)$
$v_2 = (4, 9, 9, -4)$
$v_3 = (5, 8, 9, -5)$

take transpose then apply

$$\begin{bmatrix} 1 & 4 & 5 & 0 \\ 2 & 9 & 8 & 0 \\ 2 & 9 & 9 & 0 \\ -1 & -4 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & 0 \\ 0 & 0 & -1 & 0 \\ 2 & 9 & 9 & 0 \\ -1 & -4 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & 0 \\ 0 & 0 & -1 & 0 \\ 2 & 9 & 9-1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = x_1 = 0$$

$x_2 = 0$

$x_3 = 0$

hence these vectors are independent

Date:

$v_1(-1,2,4)$ & $v_2 = (5,-10,-20)$

multiply by -5

$v_1\ v_2\ v_3$ scalar multiple. so dependent

b) $\begin{bmatrix} 3 & 4 & -4 \\ 4 & 5 & 7 \end{bmatrix}$ R^2

homogeneous and unequal rows and columns so dependent.

if rows > columns so independent.

Basis for a vector space:

$$\left[\begin{array}{cc|c} 1 & 5 & 0 \\ 2 & -10 & 0 \\ 4 & -20 & 0 \end{array}\right]$$

If vectors span a vector space, and they are linearly independent then They are basis vectors.

These are the foundations which form others vectors.

$v_1 = (1,2,1)$
$v_2 = (2,9,0)$
$v_3 = (3,3,4)$

$\det = (-1)$

$k_1 = 0 \quad k_2 = 0 \quad k_3 = 0$

They are the basis vectors.

If $S = \{v_1, v_2, v_3, \ldots\ldots, v_n\}$ is called basis.

for V if:

(i) S spans V

(ii) S is linearly independent.

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for span

$v_1 = (1, 2, 1)$ $\quad R^3 = V$

$v_2 = (2, 9, 0)$

$v_3 = (3, 3, 4)$.

$$V = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{bmatrix} \qquad \det(A) = 36 + 2 + -81$$

$$= 36 + 10 - 27 \quad = -1$$

Since det (A) is nonzero so

Therefore v_1, v_2, v_3 span R^3

for linear independence:-

$$k_1 v_1 + k_2 v_2 + k_3 v_3 = 0$$

$$k_1 = k_2 = k_3 = 0$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 9 & 3 & 0 \\ 1 & 0 & 4 & 0 \end{array}\right] = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 9 & -5 & 0 \\ 1 & 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 9 & -5 & 0 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad x_1 = 0 \quad x_2 = 0, \ x_3 = 0, \text{ or}$$

$$k_1 = k_2 = k_3 = 0$$

linearly independent

Hence v_1, v_2, v_3 are basis vectors

Date:

Q.2:- $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $V = M_{2\times2}$

$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\det(A) = 1$

So spanning.

$k_1 = k_2 = k_3 = k_4 = 0$

hence independent

Matrix M_1, M_2, M_3, M_4 are basis of vector V

Q.3 $v_1 = (2, -3, 1)$
$v_2 = (4, \cancel{1}, 1)$
$v_3 = (0, -7, 1)$

$A = \begin{bmatrix} 2 & 4 & 0 \\ -3 & \cancel{1} & -7 \\ 1 & 1 & 1 \end{bmatrix}$

$= 2(12) - 4(4) = 8$ $\det(A) \neq 0$ so spanning.

$= 2(8) - 4(4)$

$= 0$

$\det(A) = 0$ so not spanning

Q.4.- $v_1 = (3, 1, -4)$
$v_2 = (2, 5, 6)$
$v_3 = (1, 4, 8)$

$VA = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 5 & 4 \\ -4 & 6 & 8 \end{bmatrix}$

$= 3(16) - 2(24) + 1(26)$

$= 48 - 48 + 26 = 26$

so it is independent and it is basis vector

Date:

Coordinate Vector Relative to Basis:-

the coefficient of basis vector.

$v(s) = c_1 v_1 + c_2 v_2 + c_3 v_3$

$v_1 = (1,2,1) \quad v_2 = (2,9,0) \quad v_3 = (3,3,4)$

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Q. find the coordinate vectors $v = (5,-1,9)$ relative to $S = \{v_1, v_2, v_3\}$

$$A = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 2 & 9 & 3 & -1 \\ 1 & 0 & 4 & 9 \end{array}\right] = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 9 & 3 & -1 \\ 0 & -2 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 5 & -3 & -11 \\ 0 & -2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & -2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad k_1 = 1 \quad k_2 = -1, \quad k_3 = 2$$

$1(1,2,1) + (-2,-9,0) + (6,6,8)$

$v(5,-1,9) = c_1(v_1) + c_2 v_2 + c_3 v_3$

Date:

Show that matrices are basis:

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\det A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad |A| = -1 \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix}$$

$$|A| = -1 \begin{vmatrix} -1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = -1\left\{(-1)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}\right\}$$

$$= -1\{(-1)0 + 0\} = -1$$

$\boxed{\det A = -1} \neq 0$ so it spans.

for independence

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & -1 & -1 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

Date:

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = x_1 = x_2 = x_3 = x_4 = 0$$

$k_1 = k_2 = k_3 = k_4 = 0$

$\therefore$ they are linearly independent:
these matrices from the basis for a vector M_{22}

③ $(1+x), (1-x), (1-x^2), (1-x^3)$ form basis for $P(x^3)$

$$P^3(x^3) = C_1(1+x) + C_2(1-x) + C_3(1-x^2) + C_4(1-x^3) = 0$$

$$C_1 + C_1 x + C_2 - C_2 x + C_3 - C_3 x^2 + C_4 - C_4 x^3 = 0$$

$$C_1 x - C_2 x - C_3 x^2 - C_4 x^3 + C_1 + C_2 + C_3 + C_4 = 0$$

$$x(c_1 - c_2) - c_3 x^2 - c_4 x^3 + c_1 + c_2 + c_3 + c_4 = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = -1 \begin{vmatrix} 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix} = (-1)\left\{(1)\begin{vmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{vmatrix}\right\}$$

$$= (-1)\left\{1\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}\right\}$$

$= (-1)\{(1) + 1(1).\} = -2 \neq 0$ so spanning.

$$1\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{vmatrix}$$

$-1 - 1 = -2$

$R_1 - R_2$ $\quad (-R_2) + R_1 \rightarrow R_2$

$R_1 + (-R_2) \rightarrow R_2$

Date:

Find the coordinate vector relative to basis $S = \{v_1, v_2, v_3\}$ for R^3 basis vectors

$v = (2, -1, 3)$ $\quad v_1 = (1, 0, 0)$ $\quad v_2 = (2, 2, 0)$

$v_3 = (3, 3, 3)$

$c_1 v_1 + c_2 v_2 + c_3 v_3 \longrightarrow$ coordinates.

$$A = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 2 & 3 & -1 \\ 0 & 0 & 3 & 3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 3/2 & -1/2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 3/2 & -1/2 \\ 0 & 0 & 3/3 & 3/3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & 3/2 & -1/2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad = v_3 : (3, -2, 1)$$

Dimension:- The no. of vectors in a basis for V

$\dim(R^n) = n \quad \dim(P_n) = n+1$

$\dim(M_{mn}) = mn$

e.g #02

$$A = \left[\begin{array}{cccccc|c} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 0 & 6 & 0 & 8 & 4 & 18 & 0 \end{array}\right]$$

Find basis and dimension

Apply Gaus Jordan method.

Date:

$$\left[\begin{array}{ccccc|c|c} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = -3x_2 - 4x_4 - 2x_5 \\ x_3 = -2x_4 \\ x_6 = 0 \end{array}$$

$x_1 = -3s - 4t - 2y$

$x_3 = -2t$

$x_6 = 0$

$x_2 = s$ $x_4 = t$ $x_5 = y$

$s = (-3, 1, 0, 0, 0, 0)$

$t = (-4, 0, -2, 1, 0, 0)$

$y = (-2, 0, 0, 0, 1, 0)$

dimension (3).

Q.3 The plane in R^3

$3x - 2y + 5z = 0$ one equation so many solutions

$x = \frac{2y - 5z}{3}$ $x = \frac{2}{3}s - \frac{5}{3}t$

$\therefore y = s$ $z = t$

$s\left(\frac{2}{3}, 1, 0\right)$

$t\left(-\frac{5}{3}, 0, 1\right)$ dimension = 2.

Q. $x - y = 0$

R^3 $x = y$ $x = s$ $y = s$, $z = t$

$s: (1, 1, 0)$ dimension = 2

$t: (0, 0, 1)$

combinations :

Date:

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x - z = 0$

$y = 0$

$x = z$

$y = 0$

$z = s$ $\quad x = s$

$y = 0$ $\quad s(1, 0, 1)$ Dimension: 1

$z = s$

homogeneous system $\to$ solution is nullspace.

Row space , Column Space:-

Matrix's rows	Matrix's cols.
linear	linear
combinations	combination

If A is an $m \times n$ matrix, then the subspace.

Th 4.7.1

$Ax = b$ is consistent iff.

b is the column space of A.

$$\begin{array}{ccc} A & x & = b \end{array}$$

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}$$

$-\frac{3}{5}$ $\quad -\frac{13}{5}$ $\quad \frac{17}{5}$ $\quad \frac{5}{5}$ $\quad \frac{51}{5}$

Date:

$$\left[\begin{array}{ccc|c} -1 & 3 & 2 & 1 \\ 1 & 2 & -3 & -9 \\ 2 & 1 & -2 & -3 \end{array}\right] = \begin{bmatrix} 1 & -3 & -2 & -1 \\ 1 & 2 & -3 & -9 \\ 2 & 1 & -2 & -3 \end{bmatrix}$$

$\frac{39}{5} - \frac{29}{5}$

$$= \begin{bmatrix} 1 & -3 & -2 & -1 \\ 0 & -5 & 1 & 8 \\ 2 & 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 & -1 \\ 0 & -5 & 1 & 8 \\ 0 & 7 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -2 & -1 \\ 0 & 1 & -1/5 & -8/5 \\ 0 & 7 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -2 & -1 \\ 0 & 1 & -1/5 & -8/5 \\ 0 & 0 & 17/5 & \frac{51}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -2 & -1 \\ 0 & 1 & -1/5 & -8/5 \\ 0 & 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{13}{5} & -\frac{29}{5} \\ 0 & 1 & -1/5 & -8/5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -13/5 & -29/5 \\ 0 & 1 & 0 & -5/5 \\ 0 & 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$x_1 = 2$ $\quad \cancel{x_1 =}$

$x_2 = -1$

$x_3 = 3$

linear combination $= 2\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} - 3\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}$$

Hence b is in the column space of A.

Date:

$$= \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ -2 & 0 & 2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow x + z = 0$

$x = -z \qquad y - z = 0 \qquad \boxed{y = z} \qquad z = t$

$x = -t \qquad y = t \qquad z = t$

$t(-1, 1, 1)$

Change of Bases:-

$B = \{u_1, u_2, u_3\}, \; B' = \{u_1', u_2', u_3'\}$

$u_1 = (2, 1, 1) \quad u_2 = (2, -1, 1)$

$u_3 = (1, 2, 1)$

$u_1' = (3, 1, -5), \; u_2' = (1, 1, 3), \; u_3'(-1, 0, 2)$

a) Find the transition matrix from B to B'

b) Compute coordinate vector $[w]_{B'}$ where $W = (-5, 8, -5)$

c) Compute $[w]_{B'}$ directly.

$$[W.]_B = \underbrace{P}_{\text{transition matrix.}}\overset{\text{matrix.}}{[w]_{B'}} \longrightarrow \text{new bases}$$

transition matrix.

a) $\left[\begin{array}{ccc|ccc} 2 & 2 & 1 & 3 & 1 & -1 \\ 1 & -1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & -5 & 3 & 2 \end{array}\right]$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & -1 & 2 & 0 & -1 \\ 1 & -1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & -5 & 3 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & 5/2 \\ 0 & 1 & 0 & -2 & -3 & -1/2 \\ 0 & 0 & 1 & 5 & 1 & 6 \end{array}\right]$$

Date:

b) $\begin{bmatrix} 3 & 2 & 5/2 \\ -2 & -3 & -1/2 \\ 5 & 1 & 6 \end{bmatrix} \begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix} =$

b)

$$[w]_{B} = \left[\begin{array}{ccc|c} 2 & 2 & 1 & -5 \\ 1 & -1 & 2 & 8 \\ 1 & 1 & 1 & 5 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 1 & 1/2 & -5/2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

$$[W]_{B'} = \begin{bmatrix} 9 \\ -9 \\ -5 \end{bmatrix}$$

$$[w]_{B'} = P[w]_{B}$$

c) for directly

$$\begin{bmatrix} 3 & 2 & 5/2 \\ -2 & -3 & -1/2 \\ 5 & 1 & 6 \end{bmatrix} \begin{bmatrix} 9 \\ -9 \\ -5 \end{bmatrix} = \begin{bmatrix} 27-18-25/2 \\ -18+27+5/2 \\ 45-9-30 \end{bmatrix} = \begin{bmatrix} -7/2 \\ 23/2 \\ 6 \end{bmatrix}$$