Chapter#1
Practice Questions

Negation

- Find the negation of the proposition
    - "Vandana's smartphone has at least 32 GB of memory"
    - "Michael's PC runs Linux", and express this in simple English.

Conjunction

- **Note** that in logic the word "but" sometimes is used instead of "and" in a conjunction. For example, the statement "The sun is shining, but it is raining" is another way of saying "The sun shining and it is raining." (In natural language, there is a subtle difference in meaning between "and" and "but"; we will not be concerned with this nuance here.)

- Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

Disjunction (inclusive-OR)

- Note: The use of the connective or in a disjunction corresponds to one of the two ways the word or is used in English, namely, as an inclusive or. A disjunction is true when at least one of the two propositions is true. That is, p V q is true when both p and q are true or when exactly one of p and q is true.

- Translate the statement "Students who have taken calculus or introductory computer science can take this class" in a statement in propositional logic using the propositions p: "A student who has taken calculus can take this class" and q: "A student who has taken introductory computer science can take this class."

- Find the disjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."

Disjunction (exclusive-OR)

- Express the statement "I will use all my savings to travel to Europe or to buy an electric car" in propositional logic using the statement p: "I will use all my savings to travel to Europe" and the statement q: "I will use all my savings to buy an electric car."

- Let p and q be the propositions that state "A student can have a salad with dinner" and "A student can have soup with dinner," respectively. What is p ⊕ q, the exclusive or of p and q?

Conditional/Implications

- Note: The statement p in an implication p⇒q is called its **hypothesis**, **premise**, or **antecedent**, and q the **conclusion** or **consequence**.

- Let p, q, and r represent the following statements:

p:	Sam had pizza last night.
q:	Chris finished her homework.
r:	Pat watched the news this morning.

Write a symbolic statement for each of the following:

a. If Sam had pizza last night then Chris finished her homework.

b. Pat watched the news this morning only if Sam had pizza last night.

c. Chris finished her homework if Sam did not have pizza last night.

d. If it is not the case that Sam had pizza last night, then Pat watched the news this morning.

e. Sam did not have pizza last night and Chris finished her homework implies that Pat watched the news this morning.

- Define the propositional variables as in Problem 1. Express in words the following logic statements:

a. $q \Rightarrow r$

b. $p \Rightarrow (q \wedge r)$

c. $\bar{p} \Rightarrow (q \vee r)$

d. $r \Rightarrow (p \vee q)$

Conditional/Implications

- Original statement: If I do not eat diner, I will wake up early.

  (a) Find the converse, inverse, and contrapositive of the original statement.

  (b) Which of the statements you wrote in (a) have the same meaning as the original statement?

- Construct the truth tables for the following expressions:

  a. $(p \wedge q) \vee r$
  b. $(p \vee q) \Rightarrow (p \wedge r)$

Bi-Conditional/Bi-Implications

- Let p, q, and r represent the following statements:

p:	Sam had pizza last night.
q:	Chris finished her homework.
r:	Pat watched the news this morning.

Write a symbolic statement for each of these:

(a) Sam had pizza last night if and only if Chris finished her homework.

(b) Pat watched the news this morning iff Sam did not have pizza last night.

(c) Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night as well.

(d) In order for Pat to watch the news this morning, it is necessary and sufficient that both Sam had pizza last night and Chris finished her homework.

- Insert parentheses in the following formula

$$p \wedge q \Leftrightarrow \bar{p} \vee \bar{q}.$$

to identify the proper procedure for evaluating its truth value. Construct its truth table.

Bi-Conditional/Bi-Implications

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Logical Equivalencies

- Use truth tables to verify these logical equivalences.

    a. $(p \wedge q) \Leftrightarrow p \equiv p \Rightarrow q$

    b. $(p \wedge q) \Rightarrow r \equiv p \Rightarrow (\bar{q} \vee r)$

    c. $(p \Rightarrow \bar{q}) \wedge (p \Rightarrow \bar{r}) \equiv p \wedge \overline{(q \vee r)}$

- Use only the properties of logical equivalences to verify (b) and (c) in Problem

- Construct a truth table for each formula below. Which ones are tautologies?

    a. $(\bar{p} \vee q) \Rightarrow p$

    b. $(p \Rightarrow q) \vee (p \Rightarrow \bar{q})$

    c. $(p \Rightarrow q) \Rightarrow r$

Logical Equivalencies

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Answer

(a) $p \Leftrightarrow q$
(b) $r \Leftrightarrow \bar{p}$
(c) $r \Leftrightarrow (q \wedge \bar{p})$
(d) $r \Leftrightarrow (p \wedge q)$