

Lecture# 04

De Morgan's laws



De Morgan's laws state that:

The negation of an **and** proposition is logically equivalent to the **or** proposition in which each component is negated.

$$1. \neg(p \wedge q) \equiv \neg p \vee \neg q$$

The negation of an **or** proposition is logically equivalent to the **and** proposition in which each component is negated.

$$2. \neg(p \vee q) \equiv \neg p \wedge \neg q$$

Applying De-Morgan's Law

Question: Negate the following compound Propositions

1. John is six feet tall and he weights at least 200 pounds.
2. The bus was late or Tom's watch was slow.

Applying De-Morgan's Law

Question: Negate the following compound Propositions

1. John is six feet tall and he weights at least 200 pounds.
2. The bus was late or Tom's watch was slow.

Solution

- a) John is not six feet tall or he weighs less than 200 pounds.
- b) The bus was not late and Tom's watch was not slow.

Inequalities and De Morgan's Laws



Question Use De Morgan's laws to write the negation of

$$-1 < x \leq 4$$

Solution: The given proposition is equivalent to

$$-1 < x \text{ and } x \leq 4,$$

By De Morgan's laws, the negation is

$$-1 \geq x \text{ or } x > 4.$$

Laws of Logic

1. Commutative laws

$$p \wedge q \equiv q \wedge p ; \quad p \vee q \equiv q \vee p$$

2. Associative laws

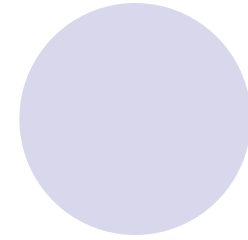
$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r ; \quad p \vee (q \vee r) \equiv (p \vee q) \vee r$$

3. Distributive laws

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Laws of Logic



4. Identity laws

$$p \wedge t \equiv p \quad ; \quad p \vee c \equiv p$$

5. Negation laws

$$p \vee \neg p \equiv t \quad ; \quad p \wedge \neg p \equiv c$$

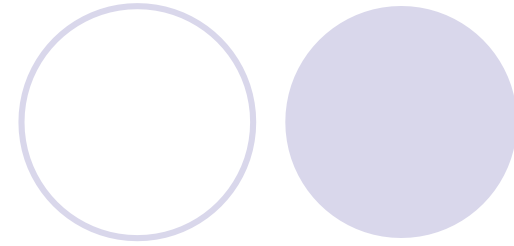
6. Double negation law

$$\neg(\neg p) \equiv p$$

7. Idempotent laws

$$p \wedge p \equiv p \quad ; \quad p \vee p \equiv p$$

Laws of Logic



8. Universal bound laws

$$p \vee t \equiv t ; p \wedge c \equiv c$$

9. Absorption laws

$$p \wedge (p \vee q) \equiv p ; p \vee (p \wedge q) \equiv p$$

10. Negation of t and c

$$\neg t \equiv c ; \neg c \equiv t$$

Example

Show that the proposition form $p \vee \neg p$ is a tautology and the proposition form $p \wedge \neg p$ is a contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

Exercise: If t is a tautology and c is contradiction, show that $p \vee t \equiv p$ and $p \wedge c \equiv c$?

Propositional Equivalences

Constructing New Logical Equivalences

- Example: Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent.

Solution:

$$\begin{aligned}\neg(p \rightarrow q) &\equiv \neg(\neg p \vee q) && \text{by example discussed in slide 66} \\ &\equiv \neg(\neg p) \wedge \neg q && \text{by the second De Morgan law} \\ &\equiv p \wedge \neg q && \text{by the double negation law}\end{aligned}$$

- Example: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Solution: To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to T.

$$\begin{aligned}(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) && \text{by example already discussed} \\ &\equiv (\neg p \vee \neg q) \vee (p \vee q) && \text{by the first De Morgan law} \\ &\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{by the associative and} \\ &&& \text{communicative law for disjunction} \\ &\equiv T \vee T \\ &\equiv T\end{aligned}$$

- Note: The above examples can also be done using truth tables.

Exercise

Using laws of logic, show that

$$r - (r - p \wedge q) \vee A(p \vee q) = p.$$

Solution

Take $r - (r - p \wedge q) \vee A(p \vee q)$

$$= (r - (r - p) \vee + q) \vee A(p \vee q), \quad (\text{by De Morgan's laws})$$

$$= (p \vee r - q) \vee A(p \vee q), \quad (\text{by double negative law})$$

$$= p \vee (r - q \vee q), \quad (\text{by distributive law})$$

$$= p \vee (q \vee \neg q), \quad (\text{by the commutative law})$$

$$= p \vee c, \quad (\text{by the negation law})$$

$$= p, \quad (\text{by the identity law})$$

Skill in simplifying proposition forms is useful in constructing logically efficient computer programs and in designing digital circuits.

Exercise

Prove that $\neg[r \vee (q \wedge (\neg r \rightarrow \neg p))] \equiv \neg r \wedge (p \vee \neg q)$

$$\begin{aligned} & \neg[r \vee (q \wedge (\neg r \rightarrow \neg p))] \\ & \equiv \neg r \wedge \neg(q \wedge (\neg r \rightarrow \neg p)), \\ & \equiv \neg r \wedge \neg(q \wedge (\neg r \vee \neg p)), \\ & \equiv \neg r \wedge \neg(q \wedge (r \vee \neg p)), \\ & \equiv \neg r \wedge (\neg q \vee \neg(r \vee \neg p)), \\ & \equiv \neg r \wedge (\neg q \vee (\neg r \wedge p)), \\ & \equiv (\neg r \wedge \neg q) \vee (\neg r \wedge (\neg r \wedge p)), \\ & \equiv (\neg r \wedge \neg q) \vee ((\neg r \wedge \neg r) \wedge p), \\ & \equiv (\neg r \wedge \neg q) \vee (\neg r \wedge p), \\ & \equiv \neg r \wedge (\neg q \vee p), \\ & \equiv \neg r \wedge (p \vee \neg q), \end{aligned}$$

De Morgan's law

Conditional rewritten as disjunction

Double negation law

De Morgan's law

De Morgan's law, double negation

Distributive law

Associative law

Idempotent law

Distributive law

Commutative law

Exercise

Prove that:

$$\neg(P \leftrightarrow Q) \equiv P \leftrightarrow \neg Q$$

$$\begin{aligned} & \neg(P \leftrightarrow Q) \\ & \equiv \neg((\neg P \vee Q) \wedge (\neg Q \vee P)) \quad \text{##} \\ & \equiv \neg(\neg P \vee Q) \vee \neg(\neg Q \vee P) \quad \text{De Morgan's Laws} \\ & \equiv (P \wedge \neg Q) \vee (Q \wedge \neg P) \quad \text{De Morgan's Laws} \\ & \equiv ((P \wedge \neg Q) \vee Q) \wedge ((P \wedge \neg Q) \vee \neg P) \quad \text{Distributive Laws} \\ & \equiv ((P \vee Q) \wedge (\neg Q \vee Q)) \wedge ((P \vee \neg P) \wedge (\neg Q \vee \neg P)) \quad \text{Distributive Laws} \\ & \equiv (P \vee Q) \wedge T \wedge T \wedge (\neg Q \vee \neg P) \quad \text{Negation Laws} \\ & \equiv (P \vee Q) \wedge (\neg Q \vee \neg P) \quad \text{Identify Laws} \\ & \equiv P \leftrightarrow \neg Q \quad \text{##} \end{aligned}$$

$$\begin{aligned} ** \quad P \rightarrow Q & \equiv \neg P \vee Q \\ \text{##} \quad P \leftrightarrow Q & \equiv (P \rightarrow Q) \wedge (Q \rightarrow P) \\ & \equiv (\neg P \vee Q) \wedge (\neg Q \vee P) \end{aligned}$$