

## Artificial Intelligence - Probability;

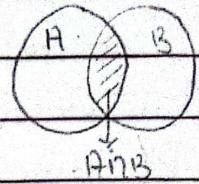
⇒ Probability =  $\frac{\text{favourable}}{\text{Total}}$ ; Prob (I win in race with 3 others) =  $\frac{1}{4}$   $\rightarrow$  Total People

Favourable (Me)

1

4 → Total People

⇒ Conditional probability:-  $P(A|B) = \frac{P(A \cap B)}{P(B)}$  → Joint  
P(B) → Marginal



↓ Find Probability of A given that B already occurred!!

So; let see:-

~~B~~ Dependent variable

$$P(A \cap B) = P(A) \cdot P(B|A)$$

Independent variable;

$$P(A \cap B) = P(A) \cdot P(B)$$

Question:- Bag contains 5 red balls and 11 green balls.

Event (A)  $\Rightarrow \{ \text{First ball is red} \}$

Event (B)  $\Rightarrow \{ \text{Second ball is red} \}$ .

$$P(A) = 5/16$$

$$P(A) = 5/9$$

$$P(B) = 5/9 \text{ Now; if dependency then } P(B) = 5/9$$

~~P(A ∩ B)~~, if first ball already taken out.

and it is red; Now; find second ball probability.

$$P(B|A) = 4/8 = 1/2$$

$$P(A \cap B) = P(A) \cdot P(B).$$

$$= 5/9 \cdot 5/9$$

↓ Because 4 red ball left; and total 8 left.

Now;

$$P(A \cap B) = 5/9 \times 1/2 = 0.2777$$

⇒ Means we don't replace our red ball hence  
Dependency occurred.

⇒ Means we replace red ball; means

put back red ball again in the bag  
hence, No dependency occurred!

Likelihood.

⇒ Baye's Theorem:-  $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$  → Prior  
↓ Posterior.  $\rightarrow$  Marginal

Single evidence case  
No comparison.

↓ Find probability of event "P(A|B)" when the given hypothesis is true!!

Multiple Evidence case  $\Rightarrow P(A|B_1, B_2, \dots, B_n) = P(B_1|A)P(B_2|A) \dots P(B_n|A)P(A)$   
 { Comparison, { Yes, No? }}

$$P(B_1) \cdot P(B_2) \dots P(B_n)$$



- Joint Probability Distribution:-
- Move bottom to top.
  - What is known keep on left side.

Date \_\_\_\_\_

## Bayesian Belief Network - Example 01:

Q01  $\Rightarrow$  Probability that alarm has sounded but neither burglary nor earthquake occurred but both John and Mary call?

Using concept of Joint Probability;

$$P(\text{Alarm} \cap \text{Mary calls} \cap \text{John calls} \cap \text{No Burglary} \cap \text{No Earthquake})$$

$\hookrightarrow$  Now; we need to find relation in these 5 variables.

i) P(Mary Calls given that Alarm rang)

ii) John Calls " " " "

iii) Alarm rings given that No Burglary And No Earthquake.

Likelihood with its prior.

$$\Rightarrow P(\text{M}/A) P(J/A) P(E/B/E) P(A/\neg B, \neg E) P(\neg B) P(\neg E).$$

$$= 0.70 \times 0.90 \times 0.001 \times 0.999 \times 0.998$$

$$P(\neg B, \neg E/A) = 0.00062$$

Ans.

Q02  $\Rightarrow$  Probability that John calls?

Now; here we don't have info about mary, Earthquake, Burglary;

$\Rightarrow$  We skip Mary;

Note;

$$P(\text{John}) = P(J/A) P(A) + P(J/\neg A) P(\neg A).$$

Now; Alarm depends upon burglary and earthquake; so  
we need Total probability; All 4,

$\begin{matrix} \text{TT} \\ \text{TF} \\ \text{FT} \\ \text{FF} \end{matrix} \left\{ \right. \text{and add all those.}$

So; Independent event Hence  $P(B) \cdot P(E)$ .

$$P(J) = P(J/A) \left[ P(A/B, E) * P(B, E) + P(A/\neg B, E) * P(\neg B, E) + P(A/B, \neg E) * P(B, \neg E) + P(A/\neg B, \neg E) * P(\neg B, \neg E) \right]$$

+

$$P(J/\neg A) \left\{ P(\neg A/B, E) * P(B, E) \dots \right\}.$$

$$P(J) = 0.90 * 0.00252 + 0.05 * 0.9974$$

$$P(J_{\text{John}}) = 0.0521.$$

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Q-03  $\rightarrow$  Probability that there is burglary given that John and Mary call?

Here, No information about the Earthquake and Alarm.

So, we need to cater all conditions!!

$P_{\downarrow}$  John and Mary both can  
Misinterpret alarm.

Here, we will find whether there is burglary or not burglary

$P$  written here as it is true that  
 $\uparrow$  burglary occurred

$$P(B|J,M) = \alpha P(B) \sum_A P(J|A) P(M|A) \sum_E [P(A|B,E) P(E)]$$

$\downarrow$  cater both A and  $\neg A$        $\downarrow$  cater both E and  $\neg E$ .

$$= \alpha P(B) \sum_A \{ P(J|A) P(M|A) [P(A|B,E) P(E) + P(A|B, \neg E) P(\neg E)] \}$$

$\downarrow$  Now open for A and  $\neg A$ .

$$= \alpha P(B) \left\{ P(J/A) P(M/\neg A) \{ P(A|B,E) P(E) + P(A|B, \neg E) P(\neg E) \} + P(J/\neg A) P(M/\neg A) \{ P(\neg A|B,E) P(E) + P(\neg A|B, \neg E) P(\neg E) \} \right\}$$

= Then put values and we do same with  $P(\neg B|J,M)$ .

$\hookrightarrow$  jahan B hai wahan  $\neg B$  kardo.

$$\text{So, } P(B|J,M) = \alpha * 0.00059$$

$$P(\neg B|J,M) = \alpha * 0.0015$$

Now, finding  $\alpha$  ;

Formula:-  $\alpha = \frac{1}{(P(B|J,M) + P(\neg B|J,M))}$

$$(P(B|J,M) + P(\neg B|J,M)) \rightarrow \text{Both Numerator part.}$$

$$\Rightarrow \alpha = \frac{1}{0.00059 + 0.0015} \Rightarrow 478.5$$

Now, put  $\alpha$  value;

$$P(B|J,M) = 478.5 * 0.00059 = 0.287$$

$$P(\neg B|J,M) = 478.5 * 0.0015 = \frac{0.72}{1.00} + 1.00$$

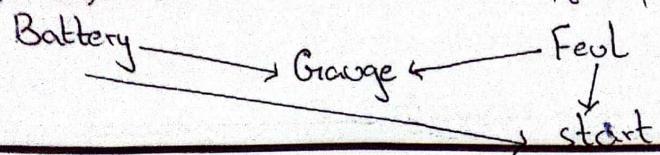
$\hookrightarrow$  Thus means our answer is write

Thus means there is no burglar in the home!!



$$P(B = \text{bad}) = 0.1$$

$$P(F: \text{empty}) = 0.2$$



*Date*

$P(\text{Gauge})$	$\text{Gauge}$	$\text{Battery}$	$\text{Fuel}$	$\text{Value}$	$P(\text{Start})$	$\text{Start}$	$\text{Battery}$	$\text{Fuel}$	$\text{Value}$
$\text{F}$	$\text{F}$	$\text{Empty}$	$\text{Good}$	$\text{Not Empty}$	0.1	$\text{No}$	$\text{Good}$	$\text{Not Empty}$	0.1
$\text{F}$	$\text{F}$	"	$\text{Good}$	$\text{Empty}$	0.8	"	$\text{Good}$	$\text{Empty}$	0.8
$\text{F}$	$\text{F}$	"	$\text{Bad}$	$\text{Not Empty}$	0.2	"	$\text{Bad}$	$\text{Not Empty}$	0.9
$\text{F}$	$\text{F}$	"	$\text{Bad}$	$\text{Empty}$	0.9	"	$\text{Bad}$	$\text{Empty}$	1.0

1)  $P(B=Good, F=Empty, G=Empty, S=Yes)$ .

$$\underline{P(0.8) P(0.9) \times P(0.2) \times P(0.2)} = 0.0288$$

a) Given that Battery is bad, Compute the probability that car will start.

$P(\text{Battery} = \text{Bad} \vee \text{Car Start} = \text{Yes}) \rightarrow$  No info about Fuel.

$$\begin{aligned}
 &= P(S=\text{Yes} | B=\text{Bad}, F_{\text{col}}=\text{Empty}) + P(S=\text{Yes} | B=\text{Bad}, F=\text{Not Empty}) \\
 &P(\text{Yes} | \text{Bad}, F_{\text{col}}) P(F_{\text{col}}) + P(\text{Yes} | \text{Bad}, F) P(F) \\
 &= (0.5 \times 0.2) + (0.1 \times 0.8)
 \end{aligned}$$

{ Why we don't have  $P(\text{Bad})$ , As  
because it's already given }.

Mileage

X - X - X - X

3)  $P(B = \text{Good}, S = \text{Empty})$

~~Engines~~ ~~for Conventional~~  
~~X X Cyl~~  
~~X Valves~~

- Used in - Binary Classification
- Spam Detection/Filtering
- Sentiment Analysis
- Classifying Articles.

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## NATURAL BAYES CLASSIFIER:-

$W \rightarrow \text{white}$      $S \rightarrow \text{short}$      $Y \rightarrow \text{yes}$   
 $G \rightarrow \text{green}$ ;     $T \rightarrow \text{tall}$ ;     $N \rightarrow \text{no}$

No	Color	Legs	Height	Smelly	Species	
1	W	3	S	Y	M	
2	G	2	T	N	M	
3	G	3	S	Y	M	
4	W	3	S	Y	M	$P(M) = 4/8 = 0.5$
5	G	2	S	N	H	$P(H) = 4/8 = 0.5$
7	W	2	T	N	H	
8	W	2	T	N	H	
9	W	2	S	Y	H	

Find probability:- (Color = G; Legs = 2; Height = T; Smelly = No).

Color	M	H	Legs	T	-T
White	2/4	3/4	3	3/4	0/4
Green	2/4	1/4	2	1/4	4/4

Height	M	H	Smelly	M	H
Tall	3/4	2/4	Yes	3/4	1/4
Short	1/4	2/4	No	1/4	3/4

$$P(M | \text{New Instance}) = P(M) * P(\text{Color: Green} | M) + P(\text{Legs=2} | M) + P(\text{Height=tall} | M) * P(\text{Smelly=No} | M)$$

$$= 0.5 * 2/4 * 1/4 * 3/4 * 1/4$$

$$P(M | \text{New Instance}) = 0.0117 \quad \checkmark$$

$$P(H | \text{New Instance}) = 0.047 \quad 0.047 > 0.0117$$

Hence New Instance = H

We can normalize the value by dividing by Total probability;

$$P(M | \text{Instance}) = 0.0117, \quad P(H | \text{Instance}) = 0.047$$

$$0.0117 + 0.047$$

$$0.047 + 0.0117$$

$$= 0.1993$$

$$= 0.8006 \quad \checkmark$$

## ⇒ Naïve Bayes - Text Classification:-

→ Includes = {Sentiment analysis, Spam detection, language identification} word occurrence in either class (+/-)

Formula:- +1 smoothing:-  $P(w_i | c) = \frac{\text{count}(w_i, c) + 1}{(\sum_{w \in c} \text{count}(w, c)) + 1}$  → smoothing

In Given sentence

$$(\sum_{w \in c} \text{count}(w, c)) + 1$$

Note:- If word not present in

the document then use sum of length of sentence. Unique word present in docs

simply drop it.

Question:-	Document	$P(\text{spam}) = 4/6 = 0.6666$
Category	Document	$P(\text{Non-Spam}) = 2/6 = 0.3333$
Spam	Send us your password	$\text{count}(w_i, c) \Rightarrow$
Spam	Review us	$\text{count}(\text{Review}, \text{spam}) = 1$
Spam	Send us your account	$\text{count}(\text{Review}, \text{spam}) = 1$
Spam	Send your password	$\text{count}(\text{us}, \text{spam}) = 3$
Non-Spam	password review	$\text{count}(\text{now}, \text{spam}) = 0$
Non-Spam	Send us your review	$\text{Vocabulary} = 6$
?	review us now	$\sum_{w \in c} \text{count}(w, c; \text{spam}) = 13$
?	Review account	$0.6666 \times 0.1052 \times 0.2105 = 0.0145$

$$P(\text{Review} | \text{spam}) = (1+1)/13+6 = 0.1052$$

$$P(\text{us} | \text{spam}) = (3+1)/13+6 = 0.2105$$

$$P(\text{spam}) P(w_i | \text{spam}) = 0.6666 \times 0.1052 = 0.0704$$

$$0.6666 \times 0.1052 \times 0.2105 = 0.0145$$

Same Do with non-spam:

{We will drop now as it is not

present neither in spam nor in the non-spam.

If we are not using +1 smoothing technique then:-

$$\text{formula:- } P(\text{spam}) \times P(w_i | \text{spam})$$

{here also we will ignore

word in target which not

exist in any document

remember: We stop for further calculation when our  $\{$  Example: Cloudy - Yes  $\}$  hence we stop there and don't calculate further.  $\{$  Day: {3, 7, 12, 13}  $\}$  for Cloudy attribute!! Date

## DECISION TREE:

Used for both classification and Regression tasks.

Steps to make decision tree - i) Calculate Entropy

ii) Calculate Information Gain;

Gradesmaster:-

Entropy of complete dataset ..

$$S[+9, 5-] = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.940.$$

Now find Entropy for all Attributes in Dataset;

{Temperature, Weather, Humidity, Wind}.

i) Weather:-

Yes No

$$\text{Entropy of Sunny} = [+2, 3-] = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$\text{Entropy of Cloudy} = [+4, 0-] = -\frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4} = 0$$

$$\text{Entropy of Wind} = [+3, 2-] = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97.$$

Step 2:- Information Gain for Weather:-

= Entropy of whole dataset  $\rightarrow \{$  All Entropy of that column (Weather)  $\}$  \*  $\frac{\text{Data}}{\text{Total}}$

$$= 0.940 - \frac{2}{5} \text{Ent}(S) - \frac{4}{14} \text{Ent}(C) - \frac{5}{14} \text{Ent}(R)$$

$$= 0.940 - 0.346 - 0 - 0.3484$$

$$IG(\text{Weather}) = 0.248.$$

Now; we find IG for other attributes- {Temp, Humidity, Wind}

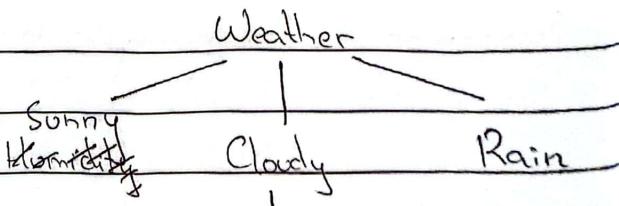
After calculation select highest IG as root and repeat process;

$$IG(\text{Weather}) = 0.248$$

$$IG(\text{Temp}) = 0.029$$

$$IG(\text{Humidity}) = 0.15$$

$$IG(\text{Wind}) = 0.0478$$



Now; which attribute to choose next?

$\Rightarrow$  Only pick {Sunny} data and check

Entropy for all remaining attributes

which are {Temp, Humidity, Wind}.

and same with Rain

{Cloud  $\rightarrow$  Yes; As because if there are all Yes. Hence we reach our first destination!}.



1	Sunny	Hot	High	Weak	No	Total value = 5
2	"	Hot	Normal	Strong	Yes	
3	"	Mild			No	
4	"	Cool				

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Entropy of whole dataset which is only Weather = Sunny;  $[+2, 3-] = 0.97$

Entropy of other attributes:

$$\text{for Temperature: - Hot } [+0, -2] = -0/2 \log_2 0/2 - 2/2 \log_2 2/2 = 0$$

$$\text{Mild } [+1, 1-] = 1.0$$

$$\text{Cool } [+1, 0-] = 0.$$

$$\begin{aligned} \text{Information Gain for Temperature} &= \text{Entropy (Sunny)} - \frac{2}{5} \text{Ent(Hot)} - \frac{2}{5} \text{Ent(Mild)} - \frac{1}{5} \text{Ent(Cool)} \\ &= 0.97 - \frac{2}{5}(0) - \frac{2}{5}(1) - \frac{1}{5}(0). \end{aligned}$$

$$IG(Temperature) = 0.57,$$

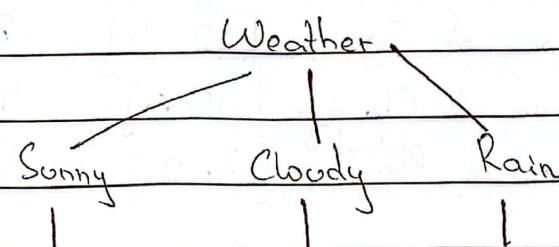
Find other attributes IG w.r.t sunny = {Humidity, Windy, Rain}.

$$IG(\text{Humidity}) = 0.97$$

$$IG(\text{Wind}) = 0.01$$

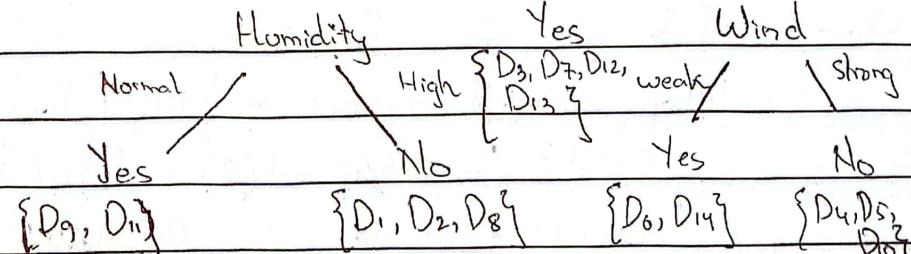
Same with Rain; As we done

with Sunny!!



Question:- Where does the

temperature attribute?



This is final Decision Tree;

Ans  $\rightarrow$  Multiple reason can be there:-

i) Due to low IG; it is not significantly impacting the decision.

ii) We have seen in example that our all days are set  $\{D_1 - D_{14}\}$  based on attributes So. we stop by further calculations as we achieved on target table  $\{\text{No}, \text{Yes}\}$

iii) Pre-pruning or Post-pruning due to attribute not contributing in Information Gain!

for each Day

10 find best splitting attribute:-

- i) Highest the gain best attribute
- ii) Lowest the gini Index best attribute.

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## GINI INDEX Splitting Attribute Decision Tree.

First of all formal :-  $Gini = 1 - \sum_{i=1}^n (p_i)^2$

$$\text{Gini Index} = \sum_{\text{VECTORS } S} \frac{|S_y|}{|S|} Gini(S_y).$$

Weekend	Weather	Parents	Money	Decision-
W <sub>1</sub>	Sunny	Yes	Rich	Cinema
W <sub>2</sub>	Sunny	No	R	Tennis
⋮	Windy	Y	R	C
⋮	Rainy	Y	Poor	C
⋮	R	N	R	Stay In
⋮	R	Y	P	C
⋮	W	N	P	C
⋮	W	N	R	Shopping
⋮	W	Y	R	C
W <sub>10</sub>	S	N	R	T

i) Gini for overall collection / Dataset;

$$\text{Decision} \Rightarrow \{C=6; T=2; \text{Stay}=1; \text{Shopping}=1\}.$$

$$Gini(S) = 1 - \left[ \left(\frac{6}{10}\right)^2 + \left(\frac{2}{10}\right)^2 + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2 \right] = 0.58$$

ii) ⇒ Gini Index for Money :-

Poor = 3 value → all correspond to Cinema;

$$Gini(S) = 1 - \left[ \left(\frac{3}{3}\right)^2 \right] = 0.$$

⇒ Rich = 7 value → In which  $\{Tennis=2; Cinema=3; Stay-In=1; Shopping=1\}$

$$Gini(S) = 1 - \left[ \left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(\frac{1}{7}\right)^2 \right] = 0.694.$$

$$\rightarrow \text{Gini Index for Money} := 0 * \left(\frac{3}{10}\right) + 0.694 * \left(\frac{7}{10}\right) = 0.486.$$

Now, Same with Gini Index for Weather!!

Date \_\_\_\_\_

Weather:-

$\Rightarrow$  Sunny = 3 values  $\rightarrow \{ \text{Cinema} = 2; \text{Tennis} = 1 \}$ ,

$$Gini(S) = 1 - \left\{ \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right\} = 0.444.$$

$\Rightarrow$  Now come for Windy and Rainy;

$$Gini(\text{Windy}) = 0.374; \quad Gini(\text{Rainy}) = 0.444.$$

$$\text{Gini Index for Weather: } 0.444 * \left(\frac{3}{10}\right) + 0.374 \left(\frac{4}{10}\right) + 0.444 * \left(\frac{3}{10}\right) = 0.416.$$

$\Rightarrow$  Directly calculated Gini Index for Parents = 0.36.

So, Parents Gini Index = 0.36 is lowest hence best attribute!!

Now; Split Dataset into Parent = No and Yes.

Parent

Yes /      No \

w<sub>1</sub>, w<sub>3</sub>, w<sub>4</sub>, w<sub>6</sub>, w<sub>9</sub>      w<sub>2</sub>, w<sub>5</sub>, w<sub>7</sub>, w<sub>8</sub>, w<sub>10</sub>

Repeat all process Again!!.

We can see from Dataset that When Parents are Yes than Decision = Cinema.

So; we achieved our first Destination hence Not to calculate on Parent = Yes.

So; let do: The No Data will seem as:-

Parent

Yes /      No \

Weekend	Weather	Parents	Money	Decision.	Cinema	?
w <sub>2</sub>	S	No	R	Tennis		
w <sub>5</sub>	R	!	R	Stay-In		
w <sub>7</sub>	W	!	P	Cinema		
w <sub>8</sub>	W	!	R	Shopping		
w <sub>10</sub>	S	No	R	Tennis		

Parent

Yes /      No \

Cinema      Weather

Gini Index for weather = 0.2 ✓

Gini Index for Money = 0.5

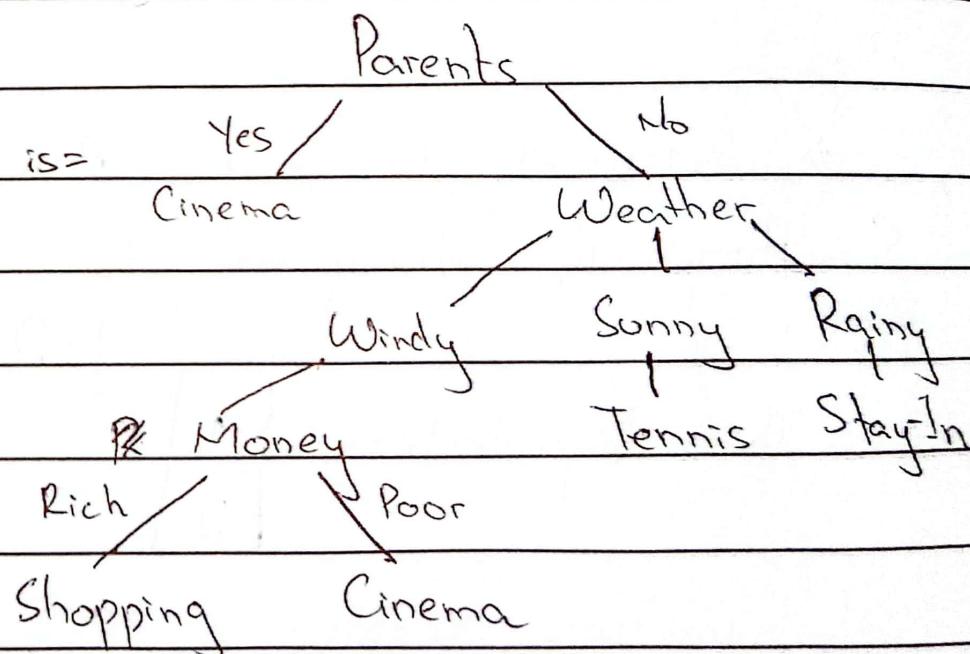
Now; there is catch; we didn't get to our destination.

Date \_\_\_\_\_

We see that when  $\{ \text{Sunny} = \text{Tennis}; \text{Rainy} = \text{Stay-In} \}$

but when  $\{ \text{Windy} = \{ \text{Cinema or Shopping} \} \} \Rightarrow$  Hence need to reduced it.

So; here the final Decision Tree is:



X — X — X — X  
DIMENSIONALITY      REFINEMENT.

These value of  $B_0$  and  $B_1$  is found by:-

$$B_0, B_1 = \frac{\sum((n-\bar{n})(y-\bar{y}))}{\sum(n-\bar{n})^2}$$

;  $y/n \rightarrow$  value from table

;  $\bar{y}/\bar{y} \rightarrow$  mean of the whole table column value.

$B_0 \rightarrow$  put value  $\{\hat{y}, \bar{y}, B_1\}$  and find value!!

formula:  $y = \hat{a} + \hat{b}x$  ;

n	y	$n^2$	$y^2$	$ny$
-1	-1	1	1	1
1	2	1	4	2
2	3	4	9	6
4	3	16	9	12
$\Sigma$	6	7	22	23

Formula:-

$$\hat{a} = (\sum n)(\sum y) - n \sum ny$$

$$(\sum n)^2 - n \sum n^2$$

$$\hat{b} = (\sum n)(\sum ny) - (\sum y)(\sum n^2)$$

$$(\sum n)^2 - n \sum n^2$$

standard deviation  $S_n = \sqrt{\frac{\sum n^2 - \frac{1}{n}(\sum n)^2}{n-1}}$

$$S_y = \sqrt{\frac{\sum y^2 - \frac{1}{n}(\sum y)^2}{n-1}}$$

Regression Correlation:-  $r = \frac{(S_n)}{(S_y)} \hat{a}$  or  $r = \frac{\sum ny - \frac{1}{n}(\sum n)(\sum y)}{\sqrt{(n-1) S_n S_y}}$

$x \rightarrow x \rightarrow x$ .

Parameter Estimation:-

$$B_1 = \frac{\sum_{i=1}^n X_i Y_i - \left( \frac{\sum_{i=1}^n X_i}{n} \right) \left( \frac{\sum_{i=1}^n Y_i}{n} \right)}{\sum_{i=1}^n X_i^2 - \left( \frac{\sum_{i=1}^n X_i}{n} \right)^2}$$

In the Slides

$$\sum_{i=1}^n X_i^2 - \left( \frac{\sum_{i=1}^n X_i}{n} \right)^2$$

$$B_0 = \bar{Y} - B_1 \bar{X}$$



## QUESTION - 01 (a)

$$a = \bar{Y} - b_1 \bar{x}_1 - b_2 \bar{x}_2$$

Given:-

$$b_1 = (\Sigma n_i^2)(\Sigma n_i y) - (\Sigma n_i n_2)(\Sigma n_i y)$$

Age =  $n_1$ 

$$(\Sigma n_i^2)(\Sigma n_i^2) - (\Sigma n_i n_2)^2$$

Blood Pressure =  $n_2$ Cholesterol Level =  $y$ .

$$b_2 = (\Sigma n_i)(\Sigma n_i y) - (\Sigma n_i n_2)(\Sigma n_i y)$$

$$(\Sigma n_i^2)(\Sigma n_i^2) - (\Sigma n_i n_2)^2$$

Patient	$\bar{Y}$	$n_1$	$n_2$	$\Sigma n_i$	$\Sigma n_i n_2$	$\Sigma n_i^2$	$\Sigma n_i y$	$\Sigma n_i^2 y$	$\Sigma n_i y^2$
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1	190	45	120	2025	14400	5400	8550	22800	36100
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2	200	50	130	2500	16900	6500	10000	26000	40000
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3	210	55	140	3025	19600	7700	13550	29400	44100
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4	220	60	150	3600	22500	9000	13200	33000	48400
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5	230	65	160	4225	25600	10400	14950	36800	52900
---	-----	----	-----	------	-------	-------	-------	-------	-------

6	240	70	170	4900	28900	11900	16800	40580	57600
---	-----	----	-----	------	-------	-------	-------	-------	-------

7	250	75	180	5625	32400	13500	18750	48000	62500
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8	260	80	190	6400	36100	15200	20800	49400	67600
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9	270	85	200	7225	40000	17000	22950	54000	72900
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10	280	90	210	8100	44100	18900	25200	58800	78400
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$$\Sigma = 2350 \quad 675 \quad 1650 \quad 47625 \quad 280500 \quad 118500 \quad 162750 \quad 396000 \quad 560500$$

Necessary Terms to solve :-

$$\Sigma n_i^2 = \Sigma n_i n_1 - (\Sigma n_i)(\Sigma n_1) / N \Rightarrow 47625 - (675)(675) / 10 = 2062.5$$

$$\Sigma n_i^2 n_2 = (\Sigma n_i)(\Sigma n_2) / N \Rightarrow 280500 - (1650)(1650) / 10 = 8250$$

$$\Sigma n_i y = \Sigma n_i y - (\Sigma n_i)(\Sigma y) / N \Rightarrow 162750 - (675)(2350) / 10 = 4125$$

$$\Sigma n_i y^2 = \Sigma n_i y^2 - (\Sigma n_i)(\Sigma y^2) / N \Rightarrow 396000 - (1650)(2350) / 10 = 8250$$

$$\Sigma n_i n_2 = \Sigma n_i n_2 - (\Sigma n_i)(\Sigma n_2) / N \Rightarrow 118500 - (675)(1650) / 10 = 4125$$

$$b_1 = (8250)(4125) - (4125)(8250) \quad , \quad b_2 = (2062.5)(8250) - (4125)(4125)$$

$$(2062.5)(8250) - (4125)$$

$$(2062.5)(8250) - (4125)$$

$$b_1 = 0$$

$$b_2 = 0;$$



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•) Regression.  $a = \bar{Y} + b_1 x_1 + b_2 x_2$ .

$$a = \frac{2350}{10} + 0 + 0 \Rightarrow a = 235$$

Anc

$$\therefore S_{n_1} = \sqrt{\frac{\sum (u_1 - \bar{x}_1)^2}{n-1}} ; S_{n_2} = \sqrt{\frac{\sum (u_2 - \bar{x}_2)^2}{n-1}}$$

$$\Rightarrow \frac{\sum u_1^2 - \frac{1}{n}(\sum u_1)^2}{n-1} = \frac{417625 - \frac{1}{10}(675)^2}{10-1} = S_{n_1} = 15.13.$$

$$S_{n_2} = \sqrt{\frac{\sum u_2^2 - \frac{1}{n}(\sum u_2)^2}{n-1}} = \sqrt{\frac{880500 - \frac{1}{10}(2350)^2}{10-1}} = S_{n_2} = 30.27$$

$$S_y = \sqrt{\frac{\sum u_1^2 - \frac{1}{n}(\sum u_1)^2}{n-1}} = \sqrt{\frac{560500 - \frac{1}{10}(2350)^2}{10-1}} \rightarrow S_y$$

Anc

•) formula:-  $r = \left( \frac{S_n}{S_y} \right) \hat{a}$

⇒ Between  $X_1$  and  $X_2$ ;  $r = \left( \frac{S_{n_1}}{S_{n_2}} \right) \hat{a} \Rightarrow r = 117.46$ .

⇒ Between  $X_1$  and  $Y$ ;  $r = 117.46$ .

⇒ Between  $X_2$  and  $Y$ ;  $r = 235.0$ .

Anc

•) For  $r^2 \Rightarrow$  Between  $X_1$  and  $X_2$ ;  $r^2 = 13796.85$

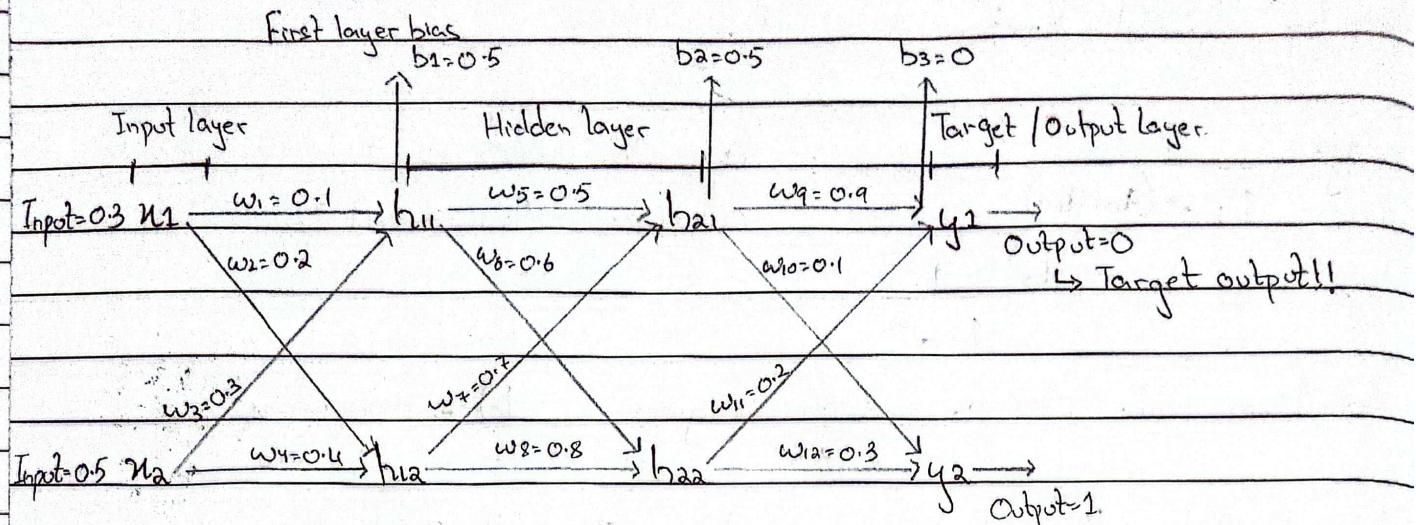
Between  $X_1$  and  $Y$ ;  $r^2 = 13796.85$

Between  $X_2$  and  $Y$ ;  $r^2 = 55225$

Anc

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## FEED FORWARD NEURAL NETWORK:-



$\Rightarrow$  If sigmoid function is calculated =  $\delta(Q) = \frac{1}{1+e^{-Q}}$   $\rightarrow$  Output  $\Rightarrow \frac{w_1 \quad w_2}{w_3 \quad w_4} \delta(Q) \rightarrow \frac{w_5 \quad w_6}{w_7 \quad w_8} \delta(Q)$

$\Rightarrow Q = w_1 n_1 + w_2 n_2 + \dots + w_n n_n + b_1$        $\because w_n = \text{weights}$   
 $n_n = \text{Input}$

lets start calculation:-

$$h_{11} = w_1 n_1 + w_3 n_2 + b_1$$

$$\Rightarrow h_{12} = w_2 n_1 + w_4 n_2 + b_1$$

$$h_{11} = 0.1 \times 0.3 + 0.3 \times 0.5 + 0.5$$

$$h_{12} = 0.2 \times 0.3 + 0.4 \times 0.5 + 0.5$$

$$h_{11} = 0.68$$

$$h_{12} = 0.76$$

$$\delta(h_{11}) = \frac{1}{1+e^{-0.68}} = 0.66$$

$$\delta(h_{12}) = \frac{1}{1+e^{-0.76}} = 0.68$$

$$\Rightarrow h_{21} = w_5 \times \delta(h_{11}) + w_7 \times \delta(h_{12}) + b_2$$

$$\Rightarrow h_{22} = w_6 \times \delta(h_{11}) + w_8 \times \delta(h_{12}) + b_2$$

$$h_{21} = 0.5 \times 0.66 + 0.7 \times 0.68 + 0.5$$

$$h_{22} = 0.6 \times 0.66 + 0.8 \times 0.76 + 0.5$$

$$h_{21} = 1.306$$

$$h_{22} = 1.504$$

$$\delta(h_{21}) = 0.786$$

$$\delta(h_{22}) = 0.818$$

$$\Rightarrow y_1 = w_9 \times \delta(h_{21}) + w_{11} \delta(h_{22}) + b_3$$

$$\Rightarrow y_2 = w_{10} \times \delta(h_{21}) + w_{12} \times \delta(h_{22}) + b_3$$

$$y_1 = 0.9 \times 0.786 + 0.2 \times 0.818 + 0$$

$$y_2 = 0.1 \times 0.786 + 0.3 \times 0.818 + 0$$

$$y_1 = 0.1636$$

$$y_2 = 0.324$$

$$\delta(y_1) = 0.54$$

$$\delta(y_2) = 0.58$$

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Now calculate loss/Error:-

E/L = Mean Squared Error.

$$= \frac{1}{2} \left[ (y_{A_1}' - y_{T_1})^2 + (y_{A_2} - y_{T_2})^2 \right]$$

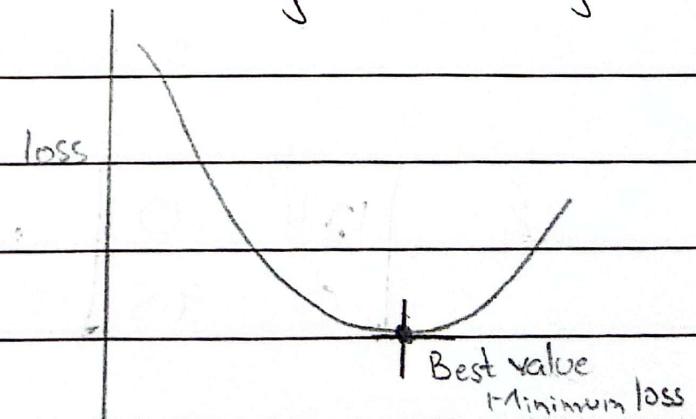
$$= \frac{1}{2} \left[ (0.54 - 0)^2 + (0.58 - 1)^2 \right].$$

$$= \frac{1}{2} (0.2916 + 0.1764) = 0.468/2 = 0.234$$

So, this is our whole one iteration;

We will do same iteration until loss is minimized;

But we can't change our nodes or input so we change our weight



K-Mean Clustering algorithm Disadvantage:-

- i) Requires to specify the number of cluster ( $k$ ) in advanced.
- ii) We need to select the initial centroids for each of the cluster.

So, to cater these two problems we introduce k-mean++ Algorithm.

Step-01:- select randomly one point as centroid.

Step-02:- for each data point compute its distance to all the centroids.

Step-03:- Select next centroid as the maximum distance from all centroids.

Now, Question arises how to find best  $k$  value:-

Elbow Method:- Every centroid means  $k = \{1, 2, 3, \dots\}$  we calculate WCSS;

Step-01:- Formula WCSS =  $\sum_{c_k} \sum_{d_i \in C_i} \text{distance}(d_i, c_k)^2$

$$\sum_{c_k} \left( \sum_{d_i \in C_i} \text{distance}(d_i, c_k)^2 \right)$$

$\hookrightarrow$  distance<sup>2</sup> of every point in a cluster.

Step-02:- Map on the graph.

X-axis =  $k$ -value ; Y-axis = WCSS

Step-03:- The point we ( $k$ -value) we find that no difference is occurring when increasing  $k$ -value that's the best  $k$ -value (No of cluster).

X — X — X ...

Example:- K-mean:-

Distance to

Cluster

New Cluster.

Initial Centroids:- Data points

2 10 5 8

3

A1: (2, 10) } A1 2 10 0.00 3.61 1

B1: (5, 8) } A2 8 4 8.49 5.00 2

A3 7 5 7.07 3.61 2

A4 5 8 0.00 3.61 2

After 1<sup>st</sup> iteration, we will find mean of each cluster;

B1 = (5, 8) :- points = {A2, A3, A4}; mean =  $x = \frac{8+7+5}{3}$ ;  $y = \frac{4+5+8}{3}$ .

A1 = (2, 10) :- points = {A1}; mean = (2, 10).

$\hookrightarrow (6.6, 5.6)$ .

So; new points = B1(6.6, 5.6); A1 = (2, 10).

we will do second iterate;

if previous and new clustering ~~value~~ be that is point those previously lie in cluster

after new points lies within those, only

Teacher's Signature