Std. XI : Physics
1. Measurements

Shortcuts, Important Results & Formulae

1. If $x = a^m b^n c^p$, then fractional error in 'x' can be calculated as

$$\frac{\Delta x}{x} = m\frac{\Delta a}{a} + n\frac{\Delta b}{b} + p\frac{\Delta c}{c}.$$

2. For vernier callipers, least count = s – v.
(s = length of one division on main scale, v = length of one division on vernier scale.)

3. Length measured by vernier calliper = reading of main scale + reading of vernier scale × least count

4. For Screw Gauge, least count = $\dfrac{\text{pitch of the screw}}{\text{no. of divisions on the circular scale}}$.

where, Pitch = $\dfrac{\text{distance travelled on the pitch scale}}{\text{no. of rotations}}$

5. Length measured by screw gauge = Reading of main scale + Reading of circular scale × least count.

Significant figures and error analysis

- The limit of accuracy of a measuring instrument is equal to the least count of the instrument.
- When a quantity is squared, the number of significant digits is not squared.

Algebraic operations with significant figures

General rule : Final result shall have significant figure corresponding to the number of significant digits in the **least accurate variable.**

(i) Addition and subtraction

Suppose in the measured values to be added or subtracted the **least number of significant digits after the decimal** is n. Then in the sum or difference also, the number of significant digits after the decimal should be n.

Example : $1.9 + 2.77 + 3.456 = 8.126 \approx 8.1$ (to correct significant digits)

Here, the least number of significant digits after the decimal is one. Hence, the result will be 8.1 (when rounded off to smallest number of decimal places).

Example : $17.36 - 11.4 = 5.96 \approx 6.0$ (to correct significant digits)

(ii) Multiplication or division

Suppose in the measured values to be multiplied or divided the **least number of significant digits** be n. Then in the product or quotient, the number of significant digits should also be n.

Example : $2.5 \times 13.41 = 33.525 \approx 34$ (to correct significant digits)

The least number of significant digits in the measured values are two. Hence the result when rounded off to two significant digits become 34. Therefore the answer is 34.

Example : $\dfrac{3570}{11.4} = 313.157895 \approx 313$

- When two quantities are multiplied, the maximum relative error in the result is the sum of maximum relative errors in those two quantities.
- When we are considering result involving quotient of two quantities, the maximum relative error in the result is the sum of maximum relatives errors in those quantities.

Multiple Choice Questions

1. Let $x = \left[\frac{a^2b^2}{c}\right]$ be the physical quantity. If the percentage error in the measurement of physical quantities a, b and c is 2, 3 and 4 percent respectively then percentage error in the measurement of x is **[MHT-CET 2018]**
(A) 7% (B) 14% (C) 21% (D) 28%

2. The radius of the earth and the radius of orbit around the sun are 6371 km and 149×10^6 km respectively. The order of magnitude of the diameter of the orbit is greater than that of earth by **[MHT–CET 2019]**
(A) 10^2 (B) 10^5 (C) 10^4 (D) 10^3

3. The force 'F' acting on a body of density 'd' are related by the relation $F = \frac{y}{\sqrt{d}}$. The dimensions of 'y' are **[MHT–CET 2019]**
(A) $\left[L^{-\frac{1}{2}} M^{\frac{3}{2}} T^{-2}\right]$ (B) $\left[L^{-1} M^{\frac{1}{2}} T^{-2}\right]$ (C) $\left[L^{-1} M^{\frac{3}{2}} T^{-2}\right]$ (D) $\left[L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-2}\right]$

4. The dimensions of self or mutual inductance are given as **[MHT–CET 2019]**
(A) $[L^{-2} M^1 T^{-2} I^{-2}]$ (B) $[L^2M^1T^{-2}I^{-2}]$ (C) $[L^2M^2T^{-2}I^{-1}]$ (D) $[L^2M^2 T^{-2}I^{-2}]$

5. The ratio of the dimensions of Planck's constant to that of moment of inertia is the dimensions of **[MHT–CET 2019]**
(A) time (B) velocity (C) angular momentum (D) frequency

6. $[L^2M^1 T^{-2}]$ are the dimensions of **[MHT–CET 2019]**
(A) torque (B) angular momentum
(C) angular acceleration (D) force

7. In case of dimensions of electric field and electric dipole moment the power of mass is respectively **[MHT–CET 2019]**
(A) 1, 1 (B) 1, 0 (C) 0, 1 (D) 0, 0

8. In the equation, force F = A/Linear density, the dimensions of 'A' are **[MHT–CET 2019]**
(A) $[L^1 M^0 T^{-2}]$ (B) $[L^0 M^1 T^{-1}]$ (C) $[L^1 M^0 T^{-1}]$ (D) $[L^0 M^2 T^{-2}]$

9. If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities, then the dimensions of surface tension are **[MHT–CET 2019]**
(A) $[E^1 V^{-1}T^{-2}]$ (B) $[E^1 V^{-2}T^{-1}]$ (C) $[E^1 V^{-2}T^{-2}]$ (D) $[E^{-2} V^{-1}T^{-3}]$

10. A physical quantity 'p' is related to another physical quantity 'q' by the equation $p = kq^a$ where 'k' and 'a' are constants. If percentage error in the measurement of 'q' is 'x', then the percentage error in 'p' depends upon **[MHT–CET 2019]**
(A) x and a (B) q and a (C) x, k and a (D) k and a

SOLUTIONS

1. (B)

$$x = \frac{a^2b^2}{c}$$

$$\frac{\Delta x}{x}\% = \frac{2\Delta a}{a}\% + \frac{2\Delta b}{b}\% + \frac{\Delta c}{c}$$
$$= 2 \times 2 + 2 \times 3 + 4 = 4 + 6 + 4 = 14\%$$

2. (C)

$R = 6371\text{ km} = 0.6371 \times 10^4\text{ km}$

$h = 149 \times 10^6\text{ km} = 1.49 \times 10^8\text{ km}$

$\therefore \ \frac{h}{R} \approx 10^4$

3. (A)

$\lambda = F\sqrt{d} = MLT^{-2}M^{1/2}L^{-3/2} = M^{3/2}L^{-1/2}T^{-2}$

4. (B)

$F = Bqv$

$\therefore \ B = \frac{F}{qV} = \frac{MLT^{-2}}{ITLT^{-1}} = MI^{-1}T^{-2}$

$Q = A\,.\,B = MI^{-1}T^{-2}L^2$

$Q = LI$

$L = \frac{Q}{I} = \frac{MI^{-1}T^{-2}L^2}{I} = MI^{-2}T^{-2}L^2$

5. (D)

$h = \frac{E}{\nu} = ET = ML^2T^{-2}T$

$\frac{h}{I} = \frac{ML^2T^{-2}T}{ML^2} = T^{-1}$

6. (A)

Torque = Force × distance $= MLT^{-2}L = ML^2T^{-2}$

7. (B)

$\bar{E} = \frac{F}{Q} = \frac{MLT^{-2}}{IT} \Rightarrow M^1$

$\tau = q \times 2\ell\ \sin\theta = [I\,T][L]$

$\Rightarrow M^0$

8. (D)

$A = F \times \frac{M}{L} = [MLT^{-2}][ML^{-1}] = [M^2L^0T^{-2}]$

9. (C)

$S = \frac{F}{L} = \frac{MLT^{-2}}{L} = MT^{-2}$

Now $E = ML^2T^{-2}$

$V = LT^{-1}$

$T = T$

$\therefore \ S = MT^{-2} = EL^{-2} = EV^{-2}T^{-2}$

10. (A)

$p = kq^a$

$dp = k\,a\,q^{a-1}\,dq$

$\frac{dp}{p} = \frac{k\,a\,q^{a-1}\,dq}{k\,q^a}, \ a\frac{dq}{q} = ax$

2. Scalars and Vectors

Shortcuts, Important Results & Formulae

1. $R = |\vec{P}+\vec{Q}| = \sqrt{P^2+Q^2+2PQ\cos\theta}, \theta = \widehat{\vec{P},\vec{Q}}$

   $\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta,} \alpha = \widehat{\vec{P},\vec{R}}$

2. $\vec{P} = P_x\hat{i}+P_y\hat{j}+P_z\hat{k},$

   Direction cosines, $\cos\alpha = \dfrac{P_x}{P}, \cos\beta = \dfrac{P_y}{P}, \cos\gamma = \dfrac{P_z}{P}$

3. $\vec{P} = P_x\hat{i}+P_y\hat{j}+P_z\hat{k},\ \vec{Q} = Q_x\hat{i}+Q_y\hat{j}+Q_z\hat{k},$

   $R = |\vec{P}+\vec{Q}| = \sqrt{(P_x+Q_x)^2+(P_y+Q_y)^2+(P_z+Q_z)^2}$

4. $\vec{P}\cdot\vec{Q} = PQ\cos\theta = P_xQ_x+P_yQ_y+P_zQ_z,$

   $P\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{Q} = \dfrac{P_xQ_x+P_yQ_y+P_zQ_z}{\sqrt{Q_x^2+Q_y^k+Q_z^2}}$

   $Q\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{P} = \dfrac{P_xQ_x+P_yQ_y+P_zQ_z}{\sqrt{P_x^2+P_y^2+P_z^2}}$

5. $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1, \quad \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

6. $R = |\vec{P}\times\vec{Q}| = PQ\sin\theta$

7. $\vec{R} = \vec{P}\times\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix},$

   $R_x = P_yQ_z - P_zQ_y, \quad R_y = P_zQ_x - P_xQ_z, \quad R_z = P_xQ_y - P_yQ_x$

8. $\hat{i}\times j = \hat{k}, \quad \hat{j}\times\hat{k} = \hat{i}, \quad \hat{k}\times\hat{i} = \hat{j},$

   $\hat{j}\times\hat{i} = -\hat{k}, \quad \hat{k}\times\hat{j} = -\hat{i}, \quad \hat{i}\times\hat{k} = -\hat{j},$

   $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

➤ $\vec{A}\times\vec{B} = \vec{C}$ and then $\vec{C}\perp\vec{A}$ as well as $\vec{B}$.

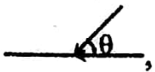
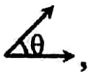
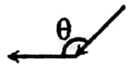
➤ $|\vec{A}\times\vec{B}| = \vec{A}\cdot\vec{B}$ when $\theta = 45°$.

➤ The rectangular components can't have magnitude greater than vector itself. [$\because \cos\theta \le 1$]

➤ If $\vec{A_1}+\vec{A_2}+\vec{A_3}+.....+\vec{A_n} = 0$.

$A_1 = A_2 = A_3 = ... = A_n$, then the adjacent vectors can be inclined to each other at angle $\dfrac{2\pi}{n}$.

- If any two vectors are parallel or equal, then the scalar triple product is zero.
- The magnitude of $\vec{P}+\vec{Q}$ can vary from maximum value $|\vec{P}|+|\vec{Q}|$ to minimum value $||\vec{P}|-|\vec{Q}||$.
- While finding the angle between two vectors one should check that the two vectors are directed towards the point or away from point.

  [(1) Angle = θ , (2) , (3) angle = π − θ ]

- For resultant of two vectors of equal magnitude.

θ	magnitude of resultant
60°	$\sqrt{3}a$
90°	$\sqrt{2}a$
120°	a

where 'a' stands for magnitude of each vector.

- $\vec{a}=\lambda\vec{b}$, (a) $\vec{a}$ is parallel to $\vec{b}$, if $\lambda>0$; (b) $\vec{a}$ is antiparallel to $\vec{b}$, if $\lambda<0$.

Multiple Choice Questions

1. If $\vec{A}=3\hat{i}-2\hat{j}+\hat{k}$, $\vec{B}=\hat{i}-3\hat{j}+5\hat{k}$ and $\vec{C}=2\hat{i}+\hat{j}-4\hat{k}$ form a right angled triangle then out of the following which one is satisfied? **[MHT-CET 2018]**

   (A) $\vec{A}=\vec{B}+\vec{C}$ and $A^2=B^2+C^2$ (B) $\vec{A}=\vec{B}+\vec{C}$ and $B^2=A^2+C^2$
   (C) $\vec{B}=\vec{A}+\vec{C}$ and $B^2=A^2+C^2$ (D) $\vec{B}=\vec{A}+\vec{C}$ and $A^2=B^2+C^2$

2. A unit vector is represented as $(0.8\hat{i}+b\hat{j}+0.4\hat{k})$. Hence the value of 'b' must be **[MHT-CET 2018]**

   (A) 0.4 (B) $\sqrt{0.6}$ (C) 0.2 (D) $\sqrt{0.2}$

3. The resultant $\vec{R}$ of $\vec{P}$ and $\vec{Q}$ is perpendicular to $\vec{P}$. Also $|\vec{P}|=|\vec{R}|$. The angle between $\vec{P}$ and $\vec{Q}$ is [tan 45° = 1] **[MHT-CET 2019]**

   (A) $\frac{7\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{5\pi}{4}$

4. If $\sqrt{A^2+B^2}$ represents the magnitude of resultant of two vectors $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$, then the angle between the two vectors is **[MHT-CET 2019]**

   (A) $\cos^{-1}\left[-\frac{(A^2-B^2)}{A^2+B^2}\right]$ (B) $\cos^{-1}\left[-\frac{2(A^2-B^2)}{(A^2+B^2)}\right]$

   (C) $\cos^{-1}\left[-\frac{(A^2-B^2)}{A^2B^2}\right]$ (D) $\cos^{-1}\left[-\frac{(A^2+B^2)}{2(A^2-B^2)}\right]$

5. $\vec{P}$ and $\vec{Q}$ are two non-zero vectors inclined to each other at an angle 'θ', '$\hat{p}$', and '$\hat{q}$' are unit vectors along $\vec{P}$ and $\vec{Q}$ respectively. The component of $\vec{Q}$ in the direction of $\vec{P}$ **[MHT-CET 2019]**

(A) $\frac{\vec{P}.\vec{Q}}{Q}$ (B) $\frac{\vec{P}\times\vec{Q}}{P}$ (C) $\hat{P}.\vec{Q}$ (D) $\vec{P}.\hat{q}$

6. The vectors $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$ are at right angles to each other. This is possible under the condition **[MHT-CET 2019]**

(A) $|\vec{A}|=|\vec{B}|$ (B) $\vec{A}\cdot\vec{B}=1$ (C) $\vec{A}\cdot\vec{B}=0$ (D) $\vec{A}\times\vec{B}=0$

7. A vector '$\vec{P}$' has 'X' and 'Y' components of magnitude 2 units and 4 units respectively. A vector '$\vec{Q}$', along negative X axis, has magnitude 6 units. The vector $(\vec{Q}-\vec{P})$ will be **[MHT-CET 2019]**

(A) $-4(2\hat{i}-\hat{j})$ (B) $4(2\hat{i}-\hat{j})$ (C) $-4(2\hat{i}+\hat{j})$ (D) $4(2\hat{i}+\hat{j})$

8. Two perpendicular force of magnitude 1 N and 4 N are inclined to positive X axis and positive Y axis at 45° respectively. Another force of magnitude 2 N acts in opposite direction of 4 N force. To have the resultant force only along Y axis, the minimum force which should be added to this system of forces is $(\sin 45° = \cos 45° = \frac{1}{\sqrt{2}})$ **[MHT-CET 2019]**

(A) $-\frac{\sqrt{3}}{2}\hat{i}$ (B) $+\frac{1}{\sqrt{2}}\hat{i}$ (C) $-\frac{1}{\sqrt{2}}\hat{i}$ (D) $+\frac{1}{2}\hat{i}$

9. If the angle between the vectors $\vec{P}$ and $\vec{Q}$ is θ, then the value of the product $(\vec{Q}\times\vec{P})\cdot\vec{P}$ is **[MHT-CET 2019]**

(A) $PQ\sin\theta$ (B) $P^2Q^2\cos\theta$ (C) P^2Q^2 (D) zero

10. If '$\vec{P}$' and '$\vec{Q}$' are two non–zero vectors, then angle between '$\vec{P}$' and the resultant vector of $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ is **[MHT-CET 2019]**

(A) zero (B) $\tan^{-1}\left(\frac{P-Q}{P+Q}\right)$ (C) $\tan^{-1}\left(\frac{Q}{P}\right)$ (D) $\tan^{-1}\left(\frac{P}{Q}\right)$

11. The resultant of two non–zero vectors '$\vec{A}$' and '$\vec{B}$' is '$\vec{R}_1$'. On reversing vector $\vec{B}$, the resultant becomes '$\vec{R}_2$'. The value of $(R_1^2+R_2^2)$ will be **[MHT-CET 2019]**

(A) $2(A^2-B^2)$ (B) A^2+B^2 (C) $2(A^2+B^2)$ (D) A^2-B^2

12. The angle made by a vector $4\hat{j}+5\hat{k}$ with z-axis is **[MHT-CET 2019]**

(A) $\cos^{-1}\left(\frac{4}{5}\right)$ (B) $\tan^{-1}\left(\frac{4}{5}\right)$ (C) $\sin^{-1}\left(\frac{4}{5}\right)$ (D) $\tan^{-1}\left(\frac{5}{4}\right)$

SOLUTIONS

1. (B)

$A = 3\hat{i} - 2\hat{j} + \hat{k}$ $\quad A = \sqrt{9+4+1} = \sqrt{14}$

$B = \hat{i} - 3\hat{j} + 5\hat{k}$ $\quad B = \sqrt{1+9+25} = \sqrt{35}$

$C = 2\hat{i} + \hat{j} - 4\hat{k}$ $\quad C = \sqrt{4+1+16} = \sqrt{21}$

$\vec{A} = \vec{B} + \vec{C}$ $\quad B^2 = A^2 + C^2$

2. (D)

$0.8\hat{i} + b\hat{j} + 0.4\hat{k}$ is a unit vector

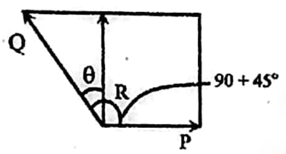
$\therefore\ 0.64 + b^2 + 0.16 = 1$

$0.80 + b^2 = 1$

$b^2 = 1 - 0.8 = 0.2$

$b = \sqrt{0.2}$

3. (B)



$R^2 = P^2 + Q^2 + 2PQ\cos(90° + \theta)$

$P^2 = P^2 + Q^2 - 2PQ\sin\theta$

$Q = 2P\sin\theta \qquad \rightarrow (1)$

$$\tan\alpha = \frac{Q\sin(\theta + 90°)}{P + Q\cos(\theta + 90°)}$$

$0 = P + Q\cos(\theta + 90°)$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{2}} \quad \therefore\ \theta = 45°$$

$\therefore$ Angle between P and Q is $90° + 45° = 135°$

4. (D)

$P = A + B \qquad Q = A - B$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$A^2 + B^2 = A^2 + B^2 + 2AB + A^2 + B^2 - 2AB + 2(A^2 - B^2)\cos\theta$

$O = A^2 + B^2 + 2(A^2 - B^2)\cos\theta$

$$\cos\theta = -\frac{1}{2}\left[\frac{(A^2 + B^2)}{(A^2 - B^2)}\right]$$

5. (C)

$Q\cos\theta$ along $\vec{P}$

$\therefore\ \vec{Q}.\hat{P}.$

6. (A)

$(A+B)\cdot(A-B)=0$

$A^2+AB-AB-B^2=0$

$A^2=B^2$

$|A|=|B|$

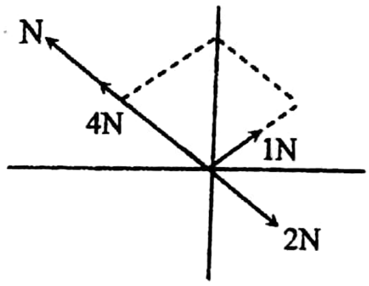
7. (C)

$P=2i+4j, Q=-6i$

$Q-P = -6i-2i-4j=-4(2i+j)$

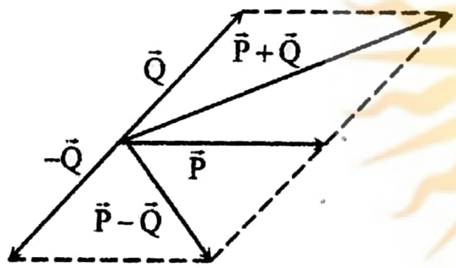
8. (B)

1 N force should be added with the existing 1 N force or in x–axis direction $\frac{1}{\sqrt{2}}\hat{i}$ N force should be added.



9. (D)

10. (A)



11. (C)

$R_1^2=A^2+B^2+2AB\cos\theta$

$R_2^2=A^2+B^2+2AB\cos(\pi-\theta)$

$\quad = A^2+B^2-2AB\cos\theta$

$R_1^2+R_2^2=2\left(A^2+B^2\right)$

12. (B)

$A=4j+5k,\quad B=k$

$\bar{A}\cdot\bar{B}=|A||B|\cos\theta$

$$\cos\theta=\frac{A\cdot B}{|A||B|}=\frac{5}{\sqrt{41}}$$

$$\therefore \tan\theta=\frac{4}{5}$$

$$\theta=\tan^{-1}\left(\frac{4}{5}\right)$$

3. Force

Shortcuts, Important Results & Formulae

1. $\Sigma\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$, $\quad \vec{J} = \vec{F}\,\Delta t = m\vec{v} - m\vec{u}$

2. $F_g = \frac{Gm_1m_2}{r^2}$ $\quad [G = 6.67\times10^{-11}\,N.m^2/kg^2]$

3. $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2$

4. $W = \Delta KE = \int_{x_1}^{x_1} F dx$

5. **Elastic one-dimensional head-on collision:** $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

   $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$, $\quad u_1 - u_2 = v_2 - v_1$,

   $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$, $\quad v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$

   If $u_2 = 0$, (stationary target), then $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$ and $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$

   If $u_2 = 0$, and $m_1 = m_2$, then $v_1 = 0$ and $v_2 = u_1$
   If $u_2 = 0$, and $m_1 >> m_2$, then $v_1 \cong u_1$ and $v_2 \cong 2u_1$
   If $u_2 = 0$, and $m_1 << m_2$, then $v_1 \cong -u_1$ and $v_2 \cong 0$
   If $u_2 = 0$, then the fractional decrease in the kinetic energy of the first particle

   $= \frac{4m_1m_2}{(m_1 + m_2)^2}$ and the loss in the kinetic energy of the first particle

   $= \frac{1}{2}m_1u_1^2 \times \frac{4m_1m_2}{(m_1 + m_2)^2}$

6. **Perfectly inelastic one-dimensional head-on collision :** $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

   Loss of kinetic energy $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \frac{1}{2}(m_1 + m_2)v^2 = \frac{m_1m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$

   If $u_2 = 0$, the loss of kinetic energy $= \frac{1}{2}m_1u_1^2 \times \left(\frac{m_2}{m_1 + m_2}\right)$

   So that, the fractional decrease in kinetic energy $= \left(\frac{m_2}{m_1 + m_2}\right)$

7. $e = \frac{v_2 - v_1}{u_1 - u_2} = \sqrt{\frac{h_2}{h_1}}$, $e = 1$ for elastic collision, $0 < e < 1$ for partially inelastic collision,

   $e = 0$ for perfectly inelastic collision

8. $\vec{\tau} = \vec{r}\times\vec{F}$, $\tau = rF\sin\theta$ = magnitude of force × moment arm, τ of a couple = magnitude of any one of the forces × arm of the couple

9. $\vec{R}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_N\vec{r}_N}{m_1 + m_2 + \ldots + m_N}$, $\quad X_{CM} = \frac{m_1x_1 + m_2x_2 + \ldots + m_Nx_N}{m_1 + m_2 + \ldots + m_N}$

   $Y_{CM} = \frac{m_1y_1 + m_2y_2 + \ldots + m_Ny_N}{m_1 + m_2 + \ldots + m_N}$, $\quad Z_{CM} = \frac{m_1z_1 + m_2z_2 + \ldots + m_Nz_N}{m_1 + m_2 + \ldots + m_N}$

10. $\Sigma\vec{F} = 0$ (translational equilibrium), $\quad \Sigma\vec{\tau} = 0$ (rotational equilibrium)

Newton's Law

- If a body is in equilibrium, then it does not mean that no force acts on the body but it simply means that the net force (resultant of any number of forces) acting on the body is zero.
- $\vec{F} = \frac{d}{dt}(m\vec{v})$ If m is constant, then $\vec{F} = m\frac{d\vec{v}}{dt}$
- Mass of a body is a measure of the resistance offered by the body to the change in velocity of the body. In other words, it is a measure of inertia of the body.
- Whenever a body loses a contact with the surface, the normal force becomes zero. In problems where a body loses contact, this concept should be used.
- Area under the force time graph gives magnitude of impulse of the given force at given time.

Tension

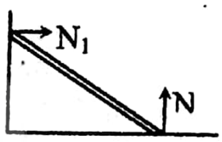
- Tension force always pulls a body.
- Tension across massless pulley or frictionless pulley remains constant.

Application of Newton's law to circular motion type of problems

- Draw free body diagram.
- Identify the direction of acceleration.
- Write equation of motion.

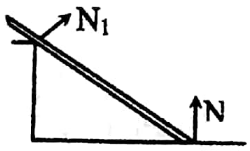
To find out the direction of normal force

- Normal force will be perpendicular to the surface of the body.



or

- If perpendicular to the surface of contact can't be drawn, the normal force will act perpendicular to the surface of the body.



or

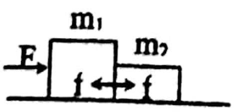
- If neither can be done, normal force has to be drawn as two components one in the x-direction and one in the y-direction.

For two bodies in contact

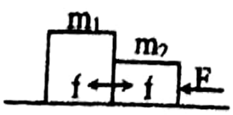
Force at contact (between two bodies)

$$= \frac{\text{Mass on which force is not directly applied} \times \text{Applied force}}{\text{Total mass of system}}$$

Note : Not applicable for more than two bodies.



$$f = \frac{m_2 F}{m_1 + m_2}$$



$$f = \frac{m_1 F}{m_1 + m_2}$$

- Bodies, which move together, can be considered as one system. If bodies have different motions, they should be considered as separate bodies. To find internal forces for bodies moving together, treat them as single system to find acceleration then to find internal forces consider one of the bodies as system.

Multiple Choice Questions

1. A sphere of mass 'm' moving with velocity 'v' collides head-on on another sphere of same mass which is at rest. The ratio of final velocity of second sphere to the initial velocity of the first sphere is (e is coefficient of restitution and collision is inelastic) **[MHT-CET 2018]**

(A) $\frac{e-1}{2}$ (B) $\frac{e}{2}$ (C) $\frac{e+1}{2}$ (D) e

2. A bomb at rest explodes into 3 parts of same mass. The momentum of two parts is $-3p\hat{i}$ and $2P\hat{j}$ respectively. The magnitude of momentum of the third part is **[MHT-CET 2018]**

(A) P (B) $\sqrt{5}\,P$ (C) $\sqrt{11}\,P$ (D) $\sqrt{13}\,P$

3. The dimensions of torque are same as that of **[MHT-CET 2019]**

(A) pressure (B) impulse (C) moment of force (D) acceleration

4. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 (as shown in figure) in the gravitational field of the point mass 'm'. Find the correct relation between 'W_1', 'W_2' and 'W_3'. **[MHT-CET 2019]**

(A) $W_1 < W_3 < W_2$ (B) $W_1 > W_3 > W_2$ (C) $W_1 = W_2 = W_3$ (D) $W_1 < W_2 < W_3$

5. An aircraft is moving with uniform velocity 150 m/s in the space. If all the forces acting on it are balanced, then it will **[MHT-CET 2019]**

(A) fall down on earth (B) keep moving with same velocity
(C) escape in space (D) remain floating at its place.

6. A force $(\vec{F}) = -5\hat{i} - 7\hat{j} + 3\hat{k}$ acting on a particle causes a displacement $(\vec{S}) = 3\hat{i} - 2\hat{j} + a\hat{k}$ in its own direction. If the work done is 14 J, then the value of 'a' is **[MHT-CET 2019]**

(A) 0 (B) 15 (C) 5 (D) 1

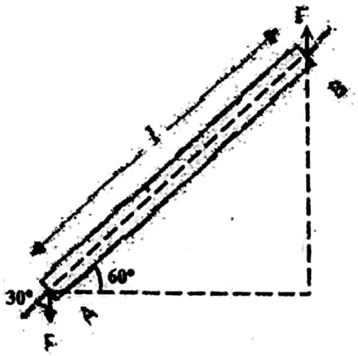
7. A block of mass 'm' moving on a frictionless surface at speed 'V' collides elastically with a block of same mass, initially at rest. Now the first block moves at an angle 'θ' with its initial direction and has speed 'V_1'. The speed of the second block after collision is **[MHT-CET 2019]**

(A) $\sqrt{V - V_1}$ (B) $\sqrt{V^2 + V_1^2}$ (C) $\sqrt{V^2 - V_1^2}$ (D) $\sqrt{V_1^2 - V^2}$

8. A block of mass 'M' is pulled along a smooth horizontal surface with a rope of mass 'm' by force 'F'. The acceleration of the block will be **[MHT-CET 2019]**

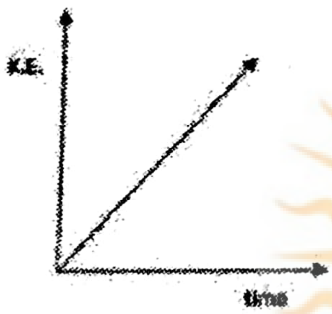
(A) $\frac{F}{(M-m)}$ (B) $\frac{F}{M}$ (C) $\frac{F}{(M+m)}$ (D) $\frac{F}{m}$

9. A rod 'ℓ' m long is acted upon by a couple as shown in figure. The moment of couple is 'τ' Nm. If the force at each end of the rod, the magnitude of each force is ($\sin 30^\circ = \cos 60^\circ = 0.5$) **[MHT-CET 2019]**



(A) $\frac{\ell}{2\tau}$ (B) $\frac{\tau}{\ell}$ (C) $\frac{2\ell}{\tau}$ (D) $\frac{2\tau}{\ell}$

10. A body moves along a straight line and the variation of its kinetic energy with time is linear as shown in the figure below. Then the force acting on the body is **[MHT-CET 2019]**



(A) zero (B) constant greater than zero
(C) inversely proportional to velocity (D) directly proportional to velocity

11. If bullet of mass 'm_1' is fired from a gun of mass 'm_2' with a speed of 'V_1', then the recoil velocity of gun is **[MHT-CET 2019]**

(A) $-\frac{m_1V_1}{m_2}$ (B) $-\frac{m_2}{m_1V_1}$ (C) $-\frac{m_2}{m_1V_1}$ (D) $\frac{m_1V_1}{m_2}$

12. A force $\vec{F} = 3\hat{i} + 6\hat{j} + 2\hat{k}$ acting on a particle causes displacement $\vec{S} = -4\hat{i} + x\hat{j} + 3\hat{k}$ in the direction of $\vec{F}$. If the work done is 12 J, then value of 'x' is **[MHT-CET 2019]**

(A) zero (B) 1 (C) 3 (D) 6

13. A bullet of mass 'm' hits a target of mass 'M' hung to a string and gets embedded in it. If the block with embedded bullet swings and rises to a height 'h' as a result of this inelastic collision, the velocity of the bullet before collision is **[MHT-CET 2019]**

(A) $\sqrt{(M+m)gh}$ (B) $\left(\frac{M+m}{m}\right)\sqrt{2gh}$ (C) $\left(\frac{M+m}{m}\right)2gh$ (D) $(m+M)gh$

14. The kinetic energy acquired by a body of mass 'M' in travelling a certain distance 'd', starting from rest, under the action of constant force is **[MHT-CET 2019]**

(A) directly proportional to $\sqrt{M}$ (B) inversely proportional to $\sqrt{M}$
(C) independent of M (D) directly proportional to $\sqrt{M}$

SOLUTIONS

1. (C)

$$e = \frac{\upsilon_2 - \upsilon_1}{u_1 - u_2} \qquad \frac{\upsilon_2}{v} = ?$$

$u_1 = v, u_2 = 0$

$$e = \frac{\upsilon_2 - \upsilon_1}{v} = \frac{\upsilon_2}{v} - \frac{\upsilon_1}{v} \qquad ...(1)$$

By law of conservation of momentum:

$$mv = m\upsilon_1 + m\upsilon_2$$

$$v = \upsilon_1 + \upsilon_2$$

$$1 = \frac{\upsilon_1}{v} + \frac{\upsilon_2}{v}$$

$$\therefore \frac{\upsilon_1}{v} = 1 - \frac{\upsilon_2}{v} \qquad ...(2)$$

Putting in equation (1) : $e = \frac{\upsilon_2}{v} - \left(1 - \frac{\upsilon_2}{v}\right)$

$$\therefore e = \frac{\upsilon_2}{v} - 1 + \frac{\upsilon_2}{v}$$

$$e = \frac{2\upsilon_2}{v} - 1$$

$$\therefore \frac{2\upsilon_2}{v} = e + 1 \qquad \therefore \frac{\upsilon_2}{v} = \frac{e+1}{2}$$

2. (D)

$P_A = -3Pi$

$P_B = 2Pj$

$P_A + P_B + P_C = 0$

$\therefore -2Pi + 2Pj + P_C = 0$

$\therefore P_C = 2Pi - 2Pj$

$\therefore P_C = 3Pi - 2Pj$

$\therefore |P_C| = \sqrt{9P^2 + 4P^2} = \sqrt{13}P$

3. (C)

Torque = force × distance

Moment of force = force × distance

4. (C)

Since gravitational field is conservative type, so $W_1 = W_2 = W_3$

5. (B)

6. (C)

$F = -5i - 7j + 3k$

$S = 3i - 2j + ak$

$14 = -15 + 14 + 3a$

$$a = \frac{15}{3} = 5.$$

7. (C)

$$\frac{1}{2}mV^2 + 0 = \frac{1}{2}m V_1^2 + \frac{1}{2} m V_2^2$$

$$V_2 = \sqrt{V^2 - V_1^2}$$

8. (C)

Total mass $(m + M)$

$$\therefore \text{ acceleration} = \frac{F}{M+m}$$

9. (D)

$$\tau = F \times \ell \cos 60^\circ = \frac{F\ell}{2}$$

$$\therefore F = \frac{2\tau}{\ell}$$

10. (B)

KE is constantly increasing

$\therefore$ $\frac{1}{2}mv^2$ = constantly increasing

v^2 = constantly increasing

v = constantly increasing

$\therefore$ acceleration = constant and positive

$\therefore$ force is constant and greater than zero.

11. (A)

$$m_1V_1 - m_2V_2 = 0$$

$$\therefore V_2 = \frac{-m_1V_1}{m_2}$$

12. (C)

$$F = 3i + 6j + 2k$$

$$s = -4i + xj + 3k$$

$$12 = -12 + 6x + 6$$

$$\therefore x = 3$$

13. (B)

$$mv = (M + m)V$$

$$v = \frac{(M+m)}{m}V$$

$$\text{But } (M + m)gh = \frac{1}{2}(M+m)V^2$$

$$\therefore V = \sqrt{2gh}$$

$$\therefore v = \left(\frac{M+m}{m}\right)\sqrt{2gh}$$

14. (C)

$$v^2 = u^2 + 2ad$$

$$v^2 = 2 \times \frac{F}{m}.d$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2\frac{F}{m}d = Fd$$

4. Friction in Solids and Liquids

Shortcuts, Important Results & Formulae

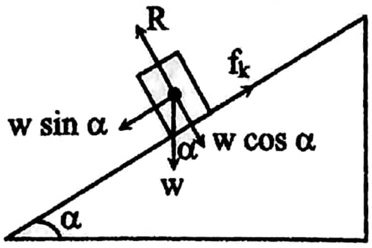
1. $\mu_s = \frac{f_s}{N}, \mu_k = \frac{f_k}{N}$
2. At angle of repose θ_r, $\mu_s = \tan\theta_r$
3. For a body sliding down an inclined plane, $a = g(\sin\theta - \mu_k \cos\theta)$
4. Gauge pressure due to a fluid column = $h\rho g$, Absolute pressure = $p_0 + h\rho g$
5. Equation of continuity = $A_1\rho_1 v_1 = A_2\rho_2 v_2$ For an incompressible fluid, $A_1v_1 = A_2v_2$
6. Pascal's law : $\frac{F_1}{A_1} = \frac{F_2}{A_2}$
7. $\frac{F}{A} = \eta\frac{dv}{dy}, \eta = \frac{\frac{F}{A}}{\frac{dv}{dy}}$
8. $|f| = 6\pi\eta r v_0$
9. $v_t = \frac{2}{9}\frac{r^2(\rho - \rho_L)g}{\eta}$
10. $N = \frac{v_c d\rho}{\eta}$
11. $p + \frac{1}{2}\rho v^2 + \rho g y =$ constant or $\frac{p}{\rho} + \frac{1}{2}v^2 + gy =$ constant
12. Venturi meter : $v_{inlet} = \sqrt{\frac{2\rho_m gh}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$

    OR $v_{inlet} = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right) - 1}}$

13. Velocity of efflux, $v = \sqrt{2\left(\frac{p - p_0}{\rho} + gh\right)}$ For a tank open to atmosphere, $v = \sqrt{2gh}$

- Frictional force varies depending on whether the body is in motion or not
  - If the body is at rest with respect to the surface then $f < \mu_s N$
  - If the body is just in motion with respect to the surface $f = \mu_s N$
  - If the body is in motion with respect to the surface $f = \mu_k N$
- If the maximum force of friction is greater than the applied force, then the force of friction will be equal to the applied force.

➢ Acceleration of a body sliding down an inclined plane –

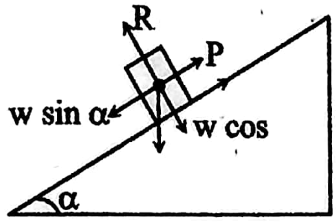


$a = g(\sin\alpha - \mu_k \cos\alpha)$ if $\sin\alpha > \mu_s \cos\alpha$

$\Rightarrow \mu_s < \tan\alpha$

$a = 0$ if $\mu_s \geq \tan\alpha$

➢ Work done in moving a body up an inclined plane through a distance x along the incline.



$W = mg(\sin\theta + \mu_k \cos\theta)x$

x = distance

➢ If the body is moved down an inclined plane with constant speed then work done is given by $W = mg(\mu_k \cos\theta - \sin\theta)x$

Multiple Choice Questions

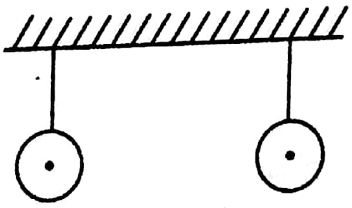
1. A vessel completely filled with water has holes 'A' and 'B' at depths 'h' and '3h' from the top respectively. Hole 'A' is a square of side 'L' and 'B' is circle of radius 'r'. The water flowing out per second from both the holes is same. Then 'L' is equal to **[MHT-CET 2018]**

   (A) $r^{\frac{1}{2}}(\pi)^{\frac{1}{2}}(3)^{\frac{1}{2}}$ (B) $r\cdot(\pi)^{\frac{1}{4}}(3)^{\frac{1}{4}}$ (C) $r\cdot(\pi)^{\frac{1}{2}}(3)^{\frac{1}{4}}$ (D) $r^{\frac{1}{2}}\cdot(\pi)^{\frac{1}{3}}(3)^{\frac{1}{2}}$

2. A metal sphere of radius 'R' and density 'ρ_1' is dropped in a liquid of density 'σ' moves with terminal velocity 'V'. Another metal sphere of same radius and density 'ρ_2' is dropped in the same liquid, its terminal velocity will be **[MHT-CET 2019]**

   (A) $V[(\rho_2 - \sigma)/(\rho_1 - \sigma)]$
   (B) $V[(\rho_2 + \sigma)/(\rho_1 + \sigma)]$
   (C) $V[(\rho_1 + \sigma)/(\rho_2 + \sigma)]$
   (D) $V[(\rho_1 - \sigma)/(\rho_2 - \sigma)]$

3. Two light balls are suspended as shown in figure. When a stream of air passes through the space between them, the distance between the balls will **[MHT-CET 2019]**



   (A) may increase or decrease, depending on speed of air
   (B) increase
   (C) decrease
   (D) remain same

4. By considering frictional force for a vehicle of mass 'm' moving along rough curved road, banked at an angle 'θ' the maximum safety speed of a vehicle is (R = radius of circular path, g = acceleration due to gravity) **[MHT-CET 2019]**

   (A) $V_m = \sqrt{Rg\left[\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right]}$
   (B) $V_m = \sqrt{Rg\left[\frac{\mu_s + \tan\theta}{1 + \mu_s \tan\theta}\right]}$
   (C) $V_m = \sqrt{Rg\left[\frac{\mu_s + \tan\theta}{1 + \tan\theta}\right]}$
   (D) $V_m = \sqrt{\frac{1}{Rg}\left[\frac{1 + \mu_s \tan\theta}{\mu_s + \tan\theta}\right]}$

5. A large vessel completely filled with water has two holes 'A' and 'B' at depths 'h' and '4h' from the top. Hole 'A' is a square of side 'L' and hole 'B' is circle of radius 'R'. If from both the holes same quantity of water is flowing per second then side of square hole is **[MHT-CET 2019]**

(A) $\sqrt{2\pi R}$ (B) R/2 (C) $\sqrt{2\pi}\cdot R$ (D) $2\pi R$

6. The average velocity of water flowing through a pipe of radius 0.5 cm is 10 cm/s. The nature of flow is **[MHT-CET 2019]**

(Coefficient of viscosity $\eta_{water} = 10^{-3}$ Ns/m^2, density $\rho_{water} = 10^3$ kg/m^3)

(A) turbulent (B) neither turbulent nor streamline

(C) streamline (D) either turbulent or streamline

7. A vehicle of mass 'M' is moving with momentum 'P' on a rough horizontal road. The coefficient of friction between the tyres and the road is 'μ'. The stopping distance is **[MHT-CET 2019]**

(A) $\dfrac{P^2M^2}{2\mu g}$ (B) $\dfrac{P^2}{\mu g M}$ (C) $\dfrac{2P^2}{\mu g M^2}$ (D) $\dfrac{P^2}{2\mu g M^2}$

8. In hydraulic press, the diameters of piston in smaller cylinder and larger cylinder are 'd_1' and 'd_2' respectively. If 'F_1' is the force applied on smaller piston, the force 'F_2' on the larger piston is **[MHT-CET 2019]**

(A) $F_2 = \left(\dfrac{d_2}{d_1}\right)^2 \dfrac{1}{F_1}$ (B) $F_2 = \left(\dfrac{d_1}{d_2}\right)^2 F_1$ (C) $F_2 = \left(\dfrac{d_1}{d_2}\right)^2 \dfrac{1}{F_1}$ (D) $F_2 = \left(\dfrac{d_2}{d_1}\right)^2 F_1$

9. Eight identical drops of water falling through air with uniform velocity of 10 cm/s combine to form a single drop of big size, then terminal velocity of the big drop will be **[MHT-CET 2019]**

(A) 30 cm/s (B) 80 cm/s (C) 10 cm/s (D) 40 cm/s

SOLUTIONS

1. (C)

Velocity of efflux for A : $\upsilon_A = \sqrt{2gh}$

Velocity of efflux for B : $\upsilon_B = \sqrt{2g \times 3h} = \sqrt{6gh}$

Water flowing out from A = Water flowing out from B

$\upsilon_A \times (\text{Area of A}) = \upsilon_B (\text{Area of B})$

$\sqrt{2gh} \times L^2 = \sqrt{6gh} \times \pi r^2$

$\therefore \quad L^2 = \dfrac{\sqrt{6gh}}{\sqrt{2gh}} \cdot \pi r^2 = \sqrt{3}\pi r^2$

$L = 3^{\frac{1}{4}} \cdot \pi^{\frac{1}{2}} \cdot r = r \cdot (\pi)^{\frac{1}{2}} \cdot (3)^{\frac{1}{4}}$

2. (A)

$$V_1 = \frac{2}{9}\frac{(\rho_1 - \sigma)R^2 g}{\eta}$$

$$\therefore \quad \frac{V_1}{\rho_1 - \sigma} = \frac{2}{9}\frac{R^2 g}{\eta}$$

$$V_2 = \frac{2}{9}\frac{(\rho_2 - \sigma)R^2 g}{\eta} = \frac{\rho_2 - \sigma}{\rho_1 - \sigma} \cdot V_1$$

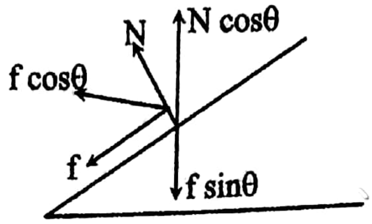
3. (C)

4. (A)

$N\cos\theta = mg + f\sin\theta$

$mg = N\cos\theta - f\sin\theta$

$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$



$\frac{mv^2}{R} = N\sin\theta + f\cos\theta$

$\frac{v^2}{Rg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$

$$V_m = \sqrt{Rg\left[\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}\right]} = \sqrt{Rg\left[\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right]}$$

5. (C)

$L^2V_1 = \pi R^2 V_2$

$\therefore\ V_1 = \sqrt{2gh} \quad \Rightarrow V_2 = \sqrt{8gh}$

$L^2\sqrt{2gh} = \pi R^2\sqrt{8gh} \Rightarrow L = \sqrt{2\pi}\,R$

6. (C)

Raynold's no. $= \frac{V_c \rho D}{\eta} = \frac{10\times10^{-2}\times10^3\times2\times0.5\times10^{-2}}{10^{-3}} = 1000$

$1000 << 2000$ $\quad\therefore$ stream liquid motion.

7. (D)

$P = MV \Rightarrow V = \frac{P}{M}$

$V_2^2 = V^2 - 2fs$

$0 = V^2 - 2fs$

$s = \frac{V^2}{2f} = \frac{P^2}{2M^2\mu g}$

$F = \mu Mg \Rightarrow Mf = \mu Mg \Rightarrow f = \mu g$

8. (D)

$\frac{F_1}{\pi d_1^2} = \frac{F_2}{\pi d_2^2} \Rightarrow F_2 = F_1\left(\frac{d_2}{d_1}\right)^2$

9. (D)

$v \propto r^2$

$8\times\frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi r_2^3$

$\frac{v_1}{v_2} = \frac{r_1^2}{r_2^2} \qquad \therefore 2r_1 = r_2$

$\therefore\ \frac{\frac{10}{100}}{V^2} = \frac{r_1^2}{r_2^2} = \frac{1}{4}$

$\therefore\ v_2 = 0.4$ m/s $= 40$ cm/s

5. Refraction of Light

Shortcuts, Important Results & Formulae

1. $c_0 = f\lambda_0, \; n = \dfrac{c_0}{c} = \dfrac{\lambda_0}{\lambda}$

2. ${}_1n_2 = \dfrac{n_2}{n_1} = \dfrac{c_1}{c_2} = \dfrac{\lambda_1}{\lambda_2}, \quad {}_2n_1 = \dfrac{n_1}{n_2} = \dfrac{1}{{}_1n_2}$

3. ${}_1n_2 = \dfrac{n_2}{n_1} = \dfrac{\sin i}{\sin r}$ or $n_1 \sin i = n_2 \sin r$

4. ${}_1n_2 = \dfrac{\text{real depth (D)}}{\text{apparent depth (d)}}$ (for $i \simeq 0°$)

5. $\sin i_C = {}_dn_r = \dfrac{1}{{}_rn_d}$

6. ${}_1n_2 = \dfrac{\sin i}{\sin r_1} = \dfrac{\sin e}{\sin r_2}, \; A = r_1 + r_2, \; i + e = A + \delta$

7. For same δ, $i_1 + e_1 = A + \delta = i_2 + e_2$, $i_1 = e_2$ and $i_2 = e_1$
   $\therefore \; i_1 + i_2 = A + \delta = e_1 + e_2$

8. For $\delta = \delta_m, \; i = e, \; i = \dfrac{A + \delta_m}{2}, \quad r_1 = r_2 = \dfrac{A}{2}, \; {}_1n_2 = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin(A/2)}$

9. $\Delta = \delta_V - \delta_R, \; \omega = \dfrac{\delta_V - \delta_R}{\delta_Y}$

10. For thin prism, $\delta = A(n - 1)$,
    $\delta = A(n_V - 1)$, $\delta_R = A(n_R - 1)$, $\delta_Y = A(n_Y - 1)$
    $\omega = \dfrac{\delta_V - \delta_R}{\delta_Y} = \dfrac{n_V - n_R}{n_Y - 1}$ or $\omega = \dfrac{\delta_V - \delta_R}{\dfrac{\delta_V + \delta_R}{2}} = \dfrac{n_V - n_R}{\left(\dfrac{n_V + n_R}{2}\right) - 1}$

Multiple Choice Questions

1. A ray of light is incident normally on a glass slab of thickness 5 cm and refractive index 1.6. The time taken to travel by a ray from source to surface of slab is same as to travel through glass slab. The distance of source from the surface is **[MHT-CET 2018]**
   (A) 4 cm (B) 8 cm (C) 12 cm (D) 16 cm

2. A thin hollow prism of refracting angle 3°, filled with water gives a deviation of 1°. The refractive index of water is **[MHT-CET 2019]**
   (A) 1.33 (B) 1.51 (C) 1.59 (D) 1.46

3. The critical angle for light going from medium 'x' to medium 'y' is 'θ'. The speed of light in medium 'x' 'V_x'. The speed of light in medium 'y' is **[MHT-CET 2019]**
   (A) $V_x/\sin\theta$ (B) $V_x \tan\theta$ (C) $V_x/\tan\theta$ (D) $V_x \sin\theta$

4. Glass has refractive index 'μ' with respect to air and the critical angle for a ray of light going from glass to air is 'θ'. If a ray of light is incident from air on the glass with angle of incidence 'θ', corresponding angle of refraction is **[MHT-CET 2019]**
   (A) $\sin^{-1}\left(\dfrac{1}{\sqrt{\mu}}\right)$ (B) $\sin^{-1}\left(\sqrt{\mu}\right)$ (C) $\sin^{-1}\left(\dfrac{1}{\mu^2}\right)$ (D) $\sin^{-1}\left(\dfrac{1}{\mu}\right)$

5. The refractive index of the material of crystal is 1.68 and that of castor oil is 1.2. When a ray of light passes from oil to glass, its velocity will change by a factor [MHT-CET 2019]

(A) 3/4 (B) 5/7 (C) 2/3 (D) 5/6

6. Absolute refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively. The ratio of velocity of light in glass and water will be [MHT-CET 2019]

(A) 8 : 9 (B) 4 : 3 (C) 8 : 7 (D) 3 : 4

7. A ray of light is incident on a medium of refractive index 'μ' at an angle of incidence 'i'. After refraction the angle of deviation is 'δ'. Then $\frac{1}{\mu}$ is [MHT-CET 2019]

(A) $\sin\delta - \cos\delta \tan i$
(B) $\cos\delta - \sin\delta \tan i$
(C) $\cos\delta - \sin\delta \cot i$
(D) $\sin\delta - \cos\delta \cot i$

8. Which one of the following is NOT the correct formula for refractive index of glass w.r.t. air (${}_a\mu_g$) (i = angle of incidence, r = angle of refraction) [MHT-CET 2019]

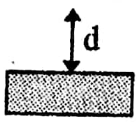
(A) ${}_a\mu_g = \frac{\lambda_{air}}{\lambda_{glass}}$ (B) ${}_a\mu_g = \frac{\sin i}{\sin r}$ (C) ${}_a\mu_g = \frac{V_{glass}}{V_{air}}$ (D) ${}_a\mu_g = \frac{\lambda_{glass}}{\lambda_{air}}$

9. Which one of the following statements is NOT the property of light? [MHT-CET 2019]
(A) Light has finite speed
(B) Light involves transportation of energy
(C) Light can travel through vacuum
(D) Light requires material medium for propagation

10. The brilliance of diamonds is due to [MHT-CET 2019]
(A) dispersion of light
(B) polarisation of light
(C) refraction of light
(D) total internal reflection of light

SOLUTIONS

1. (B)
$t = 5$ cm
$\mu = 1.6$
$d = \mu t = 5 \times 1.6 = 8.0$ cm



2. (A)
$A = 3°, d = 1°, \mu = ?$

$$d = A(\mu - 1)$$
$$1 = 3(\mu - 1)$$
$$\frac{1}{3} = \mu - 1$$
$$\mu = \frac{1}{3} + 1 = \frac{4}{3} = 1.33$$

3. (A)
$$\mu = \frac{V_x}{V_y} \quad \therefore V_y = \frac{V_x}{\mu} = \frac{V_x}{\sin\theta}$$

4. (C)

$$\sin\theta = \frac{1}{\mu}$$

$$\frac{\sin\theta}{\sin r} = \mu$$

$$\therefore \quad \sin r = \frac{\sin\theta}{\mu} = \frac{1}{\mu^2}$$

$$r = \sin^{-1}\frac{1}{\mu^2}$$

5. (B)

$$\mu = \frac{c}{v} \quad \therefore \mu \propto \frac{1}{v}$$

$$\frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{1.2}{1.68} = \frac{5}{7}$$

6. (A)

$$\mu_g = \frac{3}{2} \quad \mu_\omega = \frac{4}{3}$$

$$\frac{V_g}{V_\omega} = \frac{\mu_\omega}{\mu_g} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

7. (C)

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin(i-\delta)}$$

$$\frac{1}{\mu} = \frac{\sin(i-\delta)}{\sin i} = \frac{\sin i \cos\delta - \cos i \sin\delta}{\sin i} = \cos\delta - \sin\delta \cot i$$

8. (A)

$$_a\mu_g = \frac{c}{v} = \frac{\lambda_a \nu}{\lambda_g \nu} = \frac{\lambda_a}{\lambda_g}$$

9. (D)

10. (D)

6. Ray Optics

Shortcuts, Important Results & Formulae

1. Spherical mirror : $f = \frac{R}{2}$, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

2. Single spherical refracting surface : $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

3. Thin lens : $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = ({}_1n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, $m = \frac{h_i}{h_0} = \frac{v}{u}$

   $P = \frac{1}{f}$, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots$

4. Simple microscope : $M = \frac{D}{u} = \frac{D}{f}$ (image at ∞) $= 1 + \frac{D}{f}$ (image at the least distance of distinct vision)

5. Compound microscope :

$$M = m_0 \times M_e = \frac{v_0}{u_0} \times \frac{D}{u_e}$$

$$= \frac{v_0}{u_0} \times \frac{D}{f_e} = \left(\frac{f_0}{f_0 + u_0}\right)\left(\frac{D}{f_0}\right) \quad \text{(image at } \infty)$$

$$= \frac{v_0}{u_0} \times \left(1 + \frac{D}{f_0}\right) = \left(\frac{f_0}{f_0 + u_0}\right)\left(1 + \frac{D}{f_e}\right) \quad \text{(image at D)}$$

6. Astronomical telescope :

$$M = \frac{f_0}{f_e}, L = f_0 + f_e \text{ (normal adjustment)}$$

➢ When an object is placed with its length along the principal axis, then the magnification is known as longitudinal magnification and is denoted by m_L.

In this case,

$$m_L = \frac{I}{O} = -\left[\frac{(v_2 - v_1)}{(u_2 - u_1)}\right] = -\frac{dv}{du} \text{ (for small objects)}$$

We know, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$-\frac{dv}{v^2} - \frac{du}{u^2} = 0 \text{ or, } \frac{dv}{du} = -\left(\frac{v}{u}\right)^2$$

$$\therefore \quad m_L = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = m^2.$$

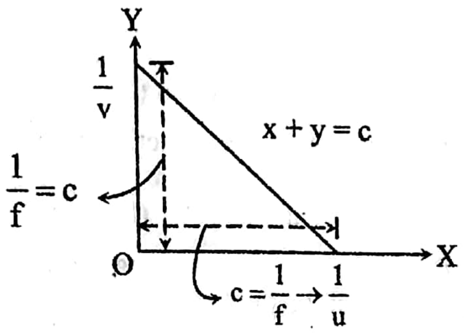
where 'm' is known as transverse magnification (m) which is defined as

$$m = \frac{I}{O} = -\left(\frac{v}{u}\right)$$

➢ When a two dimensional object is placed with its plane perpendicular to principal axis, then it's magnification is known as superficial magnification (m_s).

$$m_s = \frac{\text{area of image}}{\text{area of object}} = \frac{(ma)(mb)}{a \times b} = m^2$$

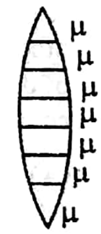
➤ $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$



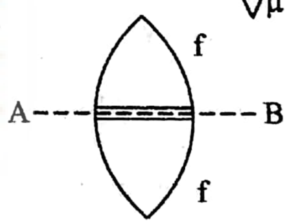
➤ If the object is placed between 'F' and 'P', then concave mirrors give enlarged, erect and virtual image. Due to their converging property, they used as reflectors in automobile head lights and search lights.

➤ The focal length of a lens depends upon 'μ. Actually $\frac{1}{f}\propto(\mu-1)$. The refractive index will be different for different colours. Hence the focal length of a lens is different for different wavelengths. For red colour it is maximum and for violet colour it is minimum irrespective of the nature of lens.

➤ If a lens is made of a number of layers of different refractive indices, then for a given wavelength of light, the lens will have as many focal lengths as the number of μ's.

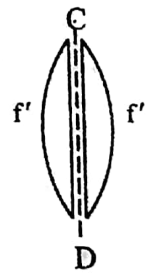


➤ When an equi–convex lens of focal length 'f' is cut into two equal parts by a horizontal plane as shown in figure below, then focal length of each part remains the same but intensity of image formed by each part is reduced to half.



➤ When a equi–convex lens is cut into the two equal parts by a vertical plane CD (see figure). Then the focal length of each part (f') becomes twice. i.e.

$f' = 2f$



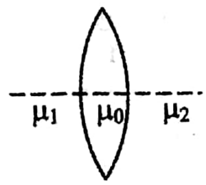
➤ **Limitations of the lens maker's formula:**

(A) The lens should be thin so that the separation between the two refracting surface should be small.

(B) The medium on either side of the lens should be same.

If any of the limitation is violated then we have to use the refraction at the curved surface formula for both the surfaces.

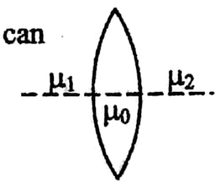
➤ It should be remembered that both the focal lengths f_1 and f_2 of a thin lens are not always equal. Actually it is $\frac{f_1}{f_2}=\frac{\mu_1}{\mu_2}$.



➤ When two sides of a given equi–convex lens have different medium then we can write

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{1}{f_0}\left[\frac{2\mu_0-\mu_1-\mu_2}{2(\mu_0-1)}\right]$$

(where μ_0 is the refractive index of material and f_0 is the focal length of lens in axis.)



Multiple Choice Questions

1. The angle made by incident ray of light with the reflecting surface is called [MHT-CET 2018]
(A) glancing angle (B) angle of incidence (C) angle of deviation (D) angle of refraction

2. In compound microscope, the focal length and aperture of the objective used is respectively [MHT-CET 2018]
(A) large and large (B) large and small (C) short and large (D) short and small

3. The equiconvex lens has a focal length 'f'. If the lens is cut along the line perpendicular to principal axis and passing through the pole, what will be the focal length of any half part? [MHT-CET 2019]
(A) $\frac{3f}{2}$ (B) $\frac{f}{2}$ (C) f (D) 2f

4. A convex lens of focal length 'f' is placed in contact with a concave lens of the same focal length. The equivalent focal length of the combination is [MHT-CET 2019]
(A) zero (B) infinity (C) f (D) $\frac{f}{2}$

5. A plano convex lens fits exactly into a plano–concave lens with plane surfaces parallel to each other. The radius of curvature of the curved surface of the lenses is 'R'. If the lenses are made of different materials of refractive indices 'μ_1' and 'μ_2' respectively, then the focal length of the combination is [MHT-CET 2019]
(A) $\frac{R}{(\mu_1-\mu_2)}$ (B) $\frac{R}{2(\mu_1-\mu_2)}$ (C) $\frac{2R}{(\mu_1-\mu_2)}$ (D) $\frac{R}{2(\mu_1+\mu_2)}$

6. For concave mirror, if the object is just beyond the focal length, the image formed is [MHT-CET 2019]
(A) real, inverted and magnified (B) real, inverted and diminished
(C) real, erect and magnified (D) real, erect and diminished

7. For convex mirrors, whatever may be the position of the object, the image formed is always on the [MHT-CET 2019]
(A) same side, real, erect and diminished (B) opposite side, virtual, inverted & diminished
(C) same side, virtual, inverted, & diminished (D) opposite side , virtual, erect & diminished

SOLUTIONS

1. (A) 2. (D)

3. (D)
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_1}=\frac{2}{f_1}$$
$\therefore$ $F_1=2f$

4. (B)
$$\frac{1}{F}=\frac{1}{f}-\frac{1}{f}=0$$
$\therefore$ $F=\infty$

5. (A)
$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{(\mu_1-1)}{R} \quad \text{AND} \quad \frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{\mu_2-1}{-R}$$
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{\mu_1-1}{R}-\frac{\mu_2-1}{R}=\frac{1}{R}[\mu_1-1-\mu_2+1]=\frac{\mu_1-\mu_2}{R}$$

6. (A) 7. (D)