Lecture Notes for

Machine Learning in Python

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Professor Eric Larson

Convolutional Neural Networks

Logistics and Agenda

- Logistics
  - Wide/Deep due soon!
  - late turn in… reminder
- Agenda
  - Basic CNN architectures and Gradient

Class Overview, by topic

Table Data Visualization

Numpy, Pandas, Seaborn

Overviews with some in-depth discussion

Dimension Reduction and Image Processing

Scikit-learn, Scikit Image,

Intuition only, Some mathematics

Linear and Logistic Regression

Numpy, Recreate API for Scikit-learn

Detailed mathematics for simple optimization

intuition for advanced optimization

Neural Networks and Back Prop.

Numpy

Detailed mathematics for NN operations

Wide and Deep Networks

Convolutional Networks

Recurrent Networks

Keras, Tensorflow

Intuition, Detailed implement.

Ethics in Language Models

ConceptNet

Case studies

Convolutional Neural Networks

Reminder: Convolution

$$\sum \left(\mathbf{I}\left[i \pm \frac{r}{2}, j \pm \frac{c}{2}\right] \odot \mathbf{f} \right) = \mathbf{O}[i,j]$$

output image at pixel $\mathbf{i,j}$

input image at r x c range of pixels centered in i,j

kernel of size, r x c usually $r=c$

0	0	0	0	0	0	0	0	0
0	1	2	3	4	12	9	8	0
0	5	2	3	4	12	9	8	0
0	5	2	1	4	10	9	8	0
0	7	2	1	4	12	7	8	0
0	7	2	1	4	14	9	8	0
0	5	2	3	4	12	7	8	0
0	5	2	1	4	12	9	8	0
0	0	0	0	0	0	0	0	0

input image, $\mathbf{I}$

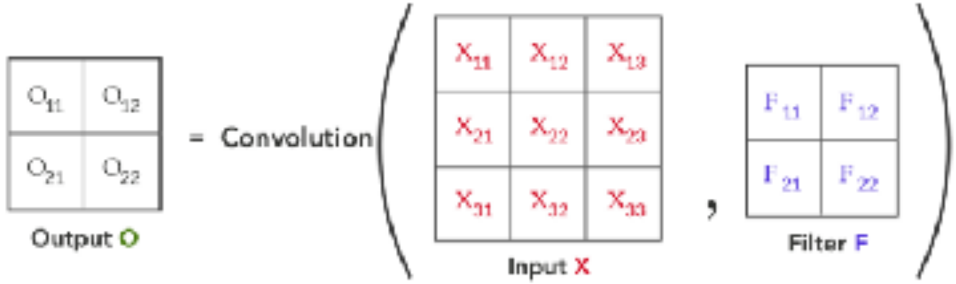
1	2	1
2	4	2
1	2	1

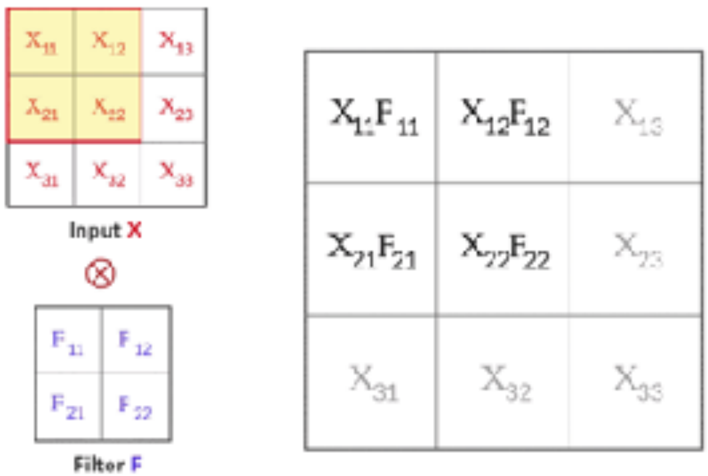
kernel filter, $\mathbf{f}$ 3x3

20	21	36	…	…	…	…
…	…	…	…	…	…	…
…	…	…	…	…	…	…
…	…	…	…	…	…	…
…	…	…	…	…	…	…
…	…	…	…	…	…	…
…	…	…	…	…	…	…

output image, $\mathbf{O}$

Breaking Apart Convolution Operations

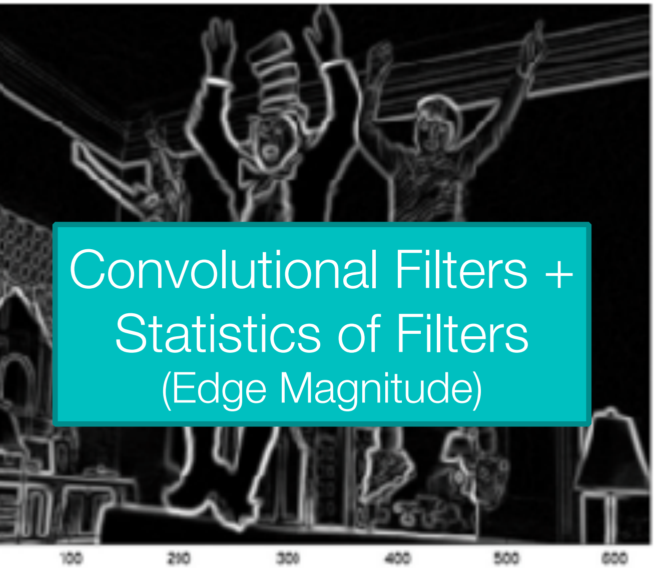




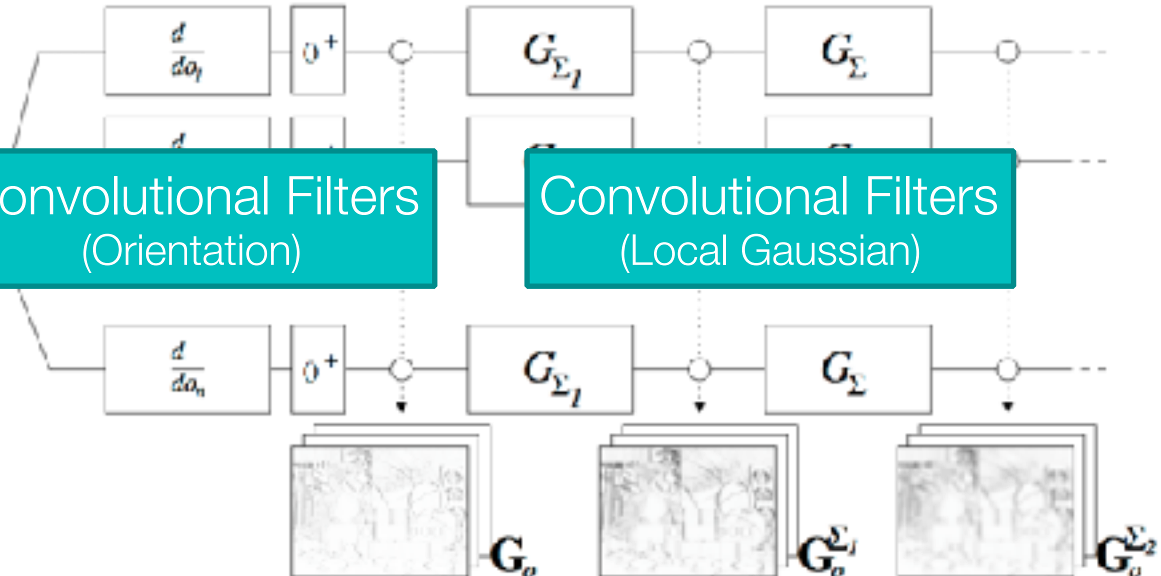
$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

What we did before (Daisy)



Convolutional Filters +
Statistics of Filters
(Edge Magnitude)



Convolutional Filters
(Orientation)

Convolutional Filters
(Local Gaussian)

Statistics of Filter Outputs
(Histograms)

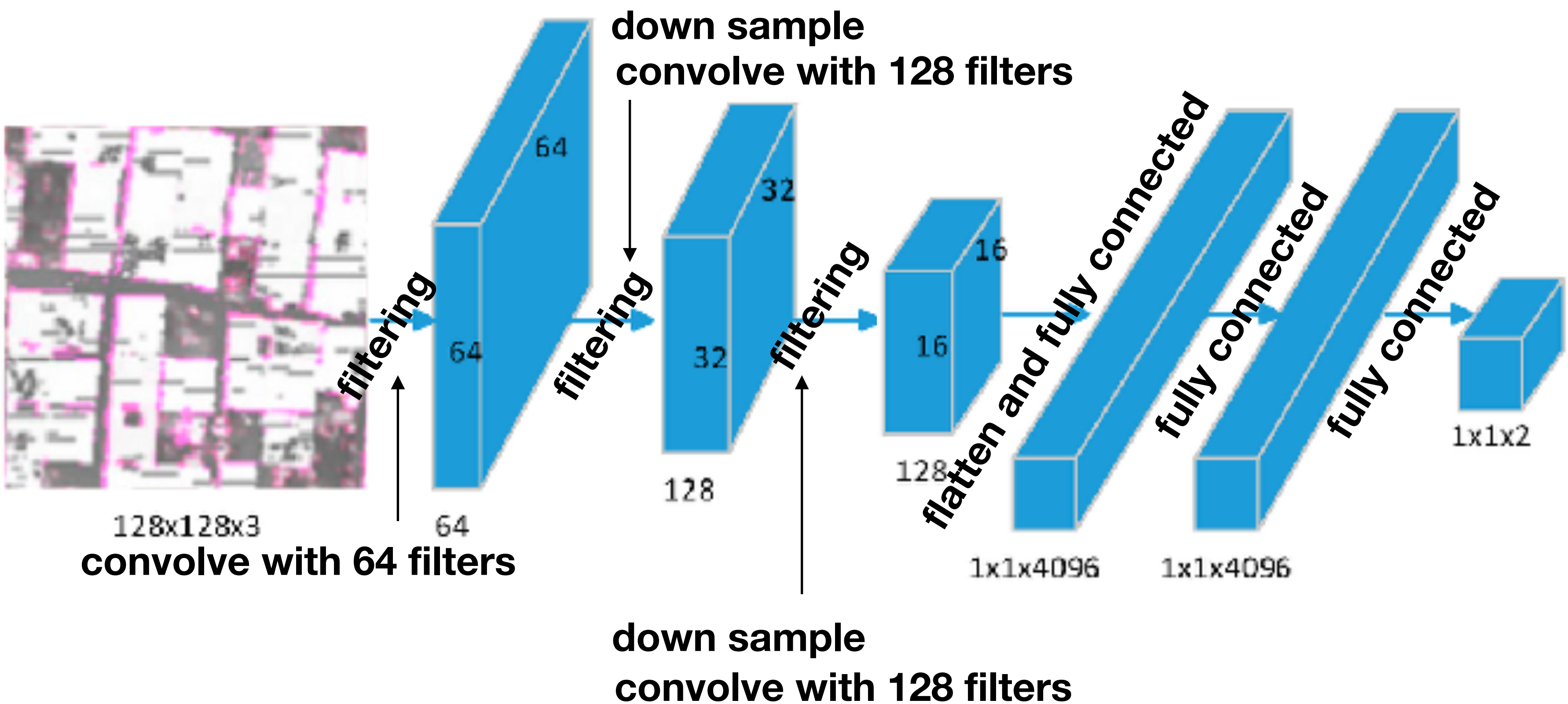
take norma ... nt u,v

$$\tilde{\mathbf{h}}_\Sigma(u,v) = \left\|\left[\mathbf{G}_1^\Sigma(u,v),\ldots,\mathbf{G}_H^\Sigma(u,v)\right]^\top\right\|$$

$$\begin{aligned}
&\mathcal{D}(u_0,v_0) = \\
&[\quad \tilde{\mathbf{h}}_{\Sigma_1}^\top(u_0,v_0), \\
&\tilde{\mathbf{h}}_{\Sigma_1}^\top(\mathbf{l}_1(u_0,v_0,R_1)),\cdots,\tilde{\mathbf{h}}_{\Sigma_1}^\top(\mathbf{l}_T(u_0,v_0,R_1)), \\
&\tilde{\mathbf{h}}_{\Sigma_2}^\top(\mathbf{l}_1(u_0,v_0,R_2)),\cdots,\tilde{\mathbf{h}}_{\Sigma_2}^\top(\mathbf{l}_T(u_0,v_0,R_2)),
\end{aligned}$$

Tola et al. “Daisy: An efficient dense descriptor applied to wide- baseline stereo.” Pattern Analysis and Machine Intelligence, IEEE Transactions

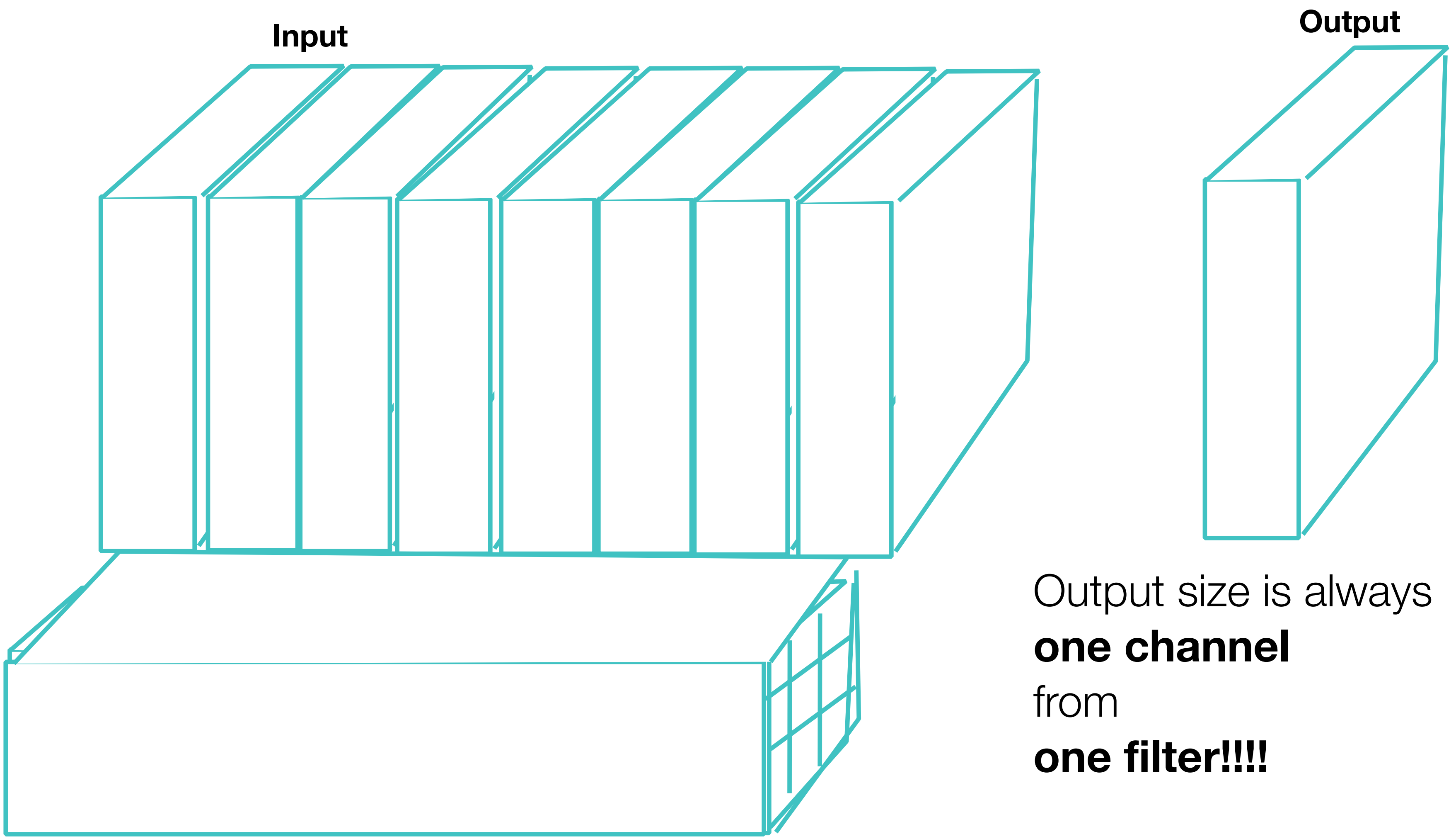
Anatomy of a convolutional network



Blue Tensors: Outputs tensors of Each Layer

Learned Params: Weights in Each Arrow

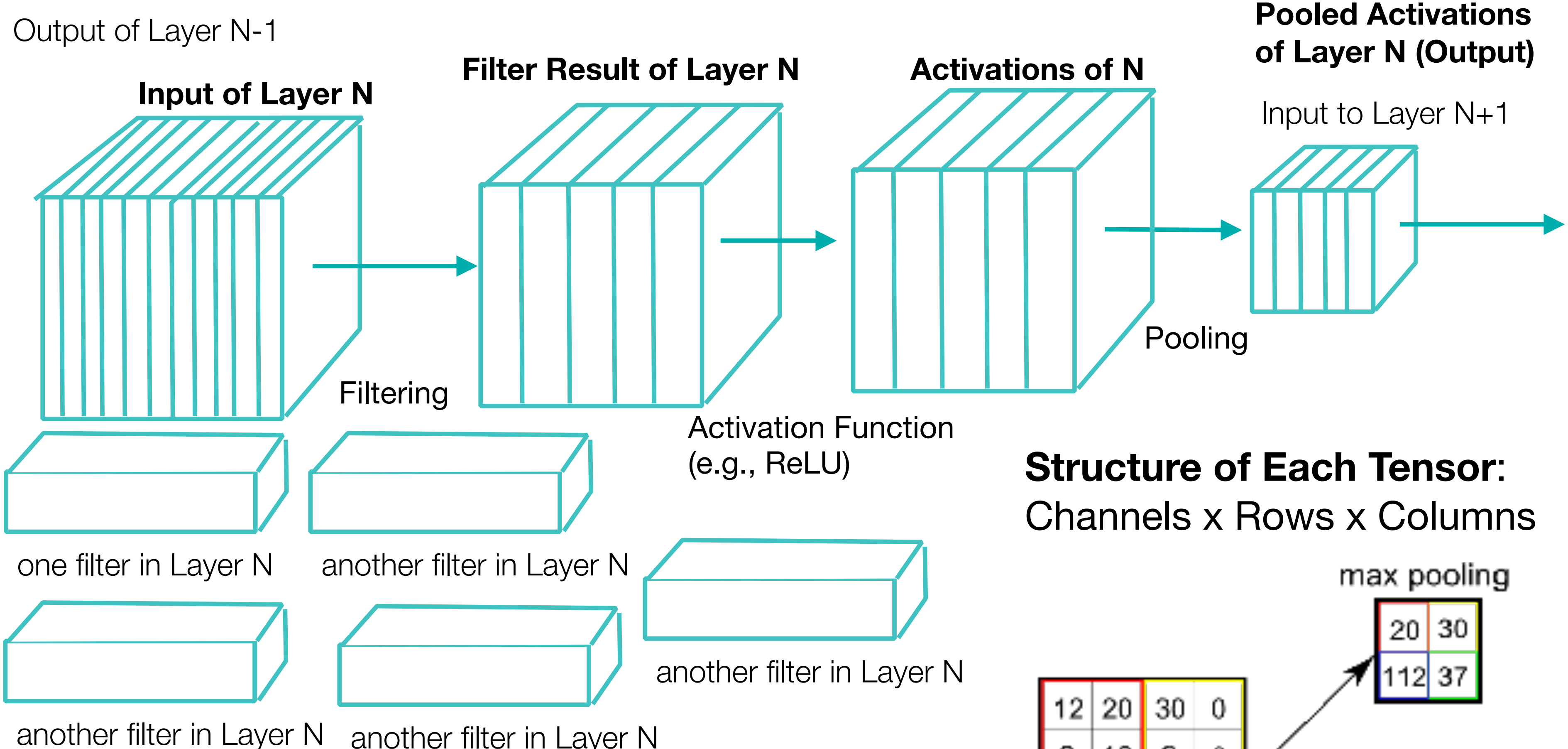
Convolution in a CNN



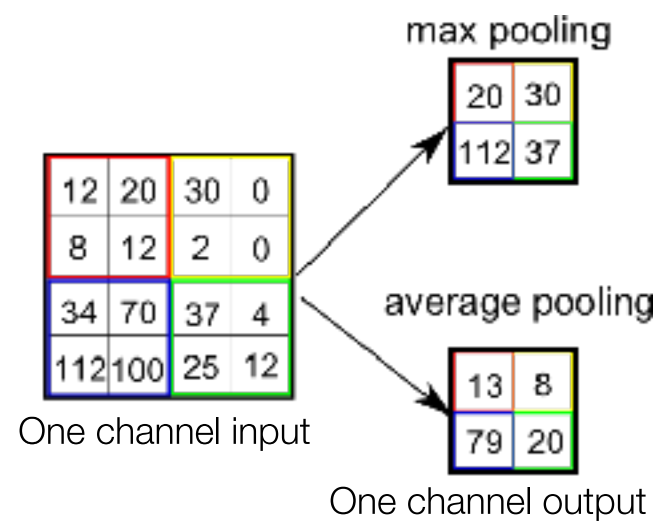
Output size is always
one channel
from
one filter!!!!

3x3 Filter (Kernel) one filter in Layer N

CNNs: Putting it together



Structure of Each Tensor:
Channels x Rows x Columns

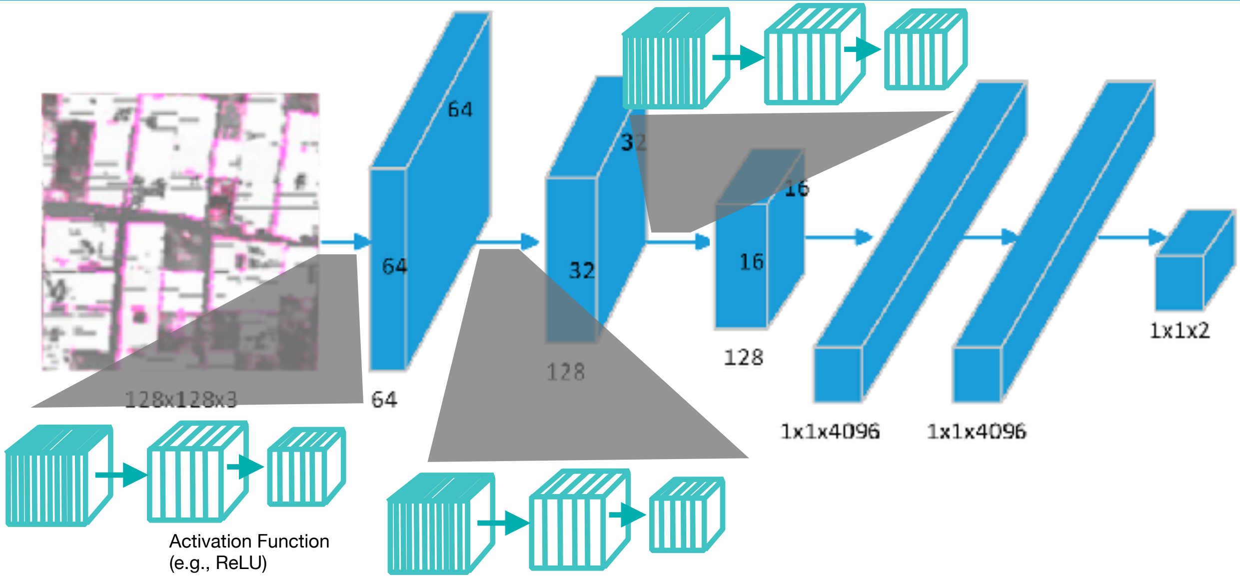


Self Test: What are the learned parameters?

A. Activations
B. Pooling Weights
C. Filter Weights

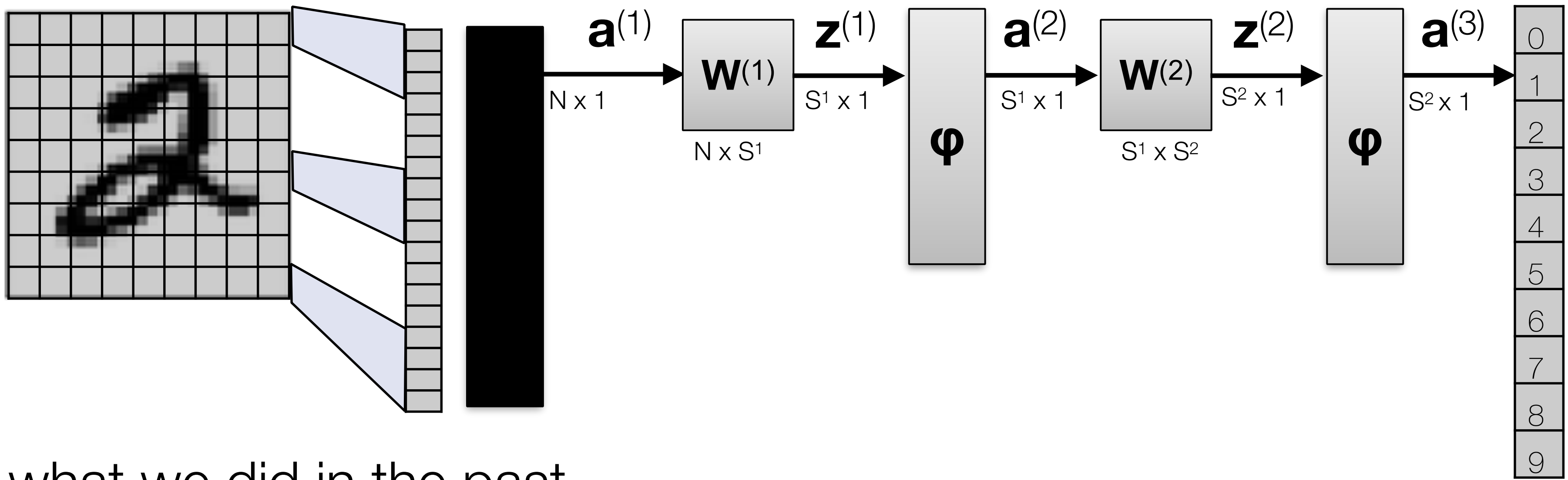
D. All of these

CNN Overview



- Conv. layer(s):
  - filtering
  - activation
  - pooling
  - Each pooling layer *can* make the input image “smaller”
    - allows for “Information Distillation”
    - less dependence on exact pixels
- Final layers are densely connected
  - typically multi-layer perceptrons

Simple Example: From Fully Connected to CNN



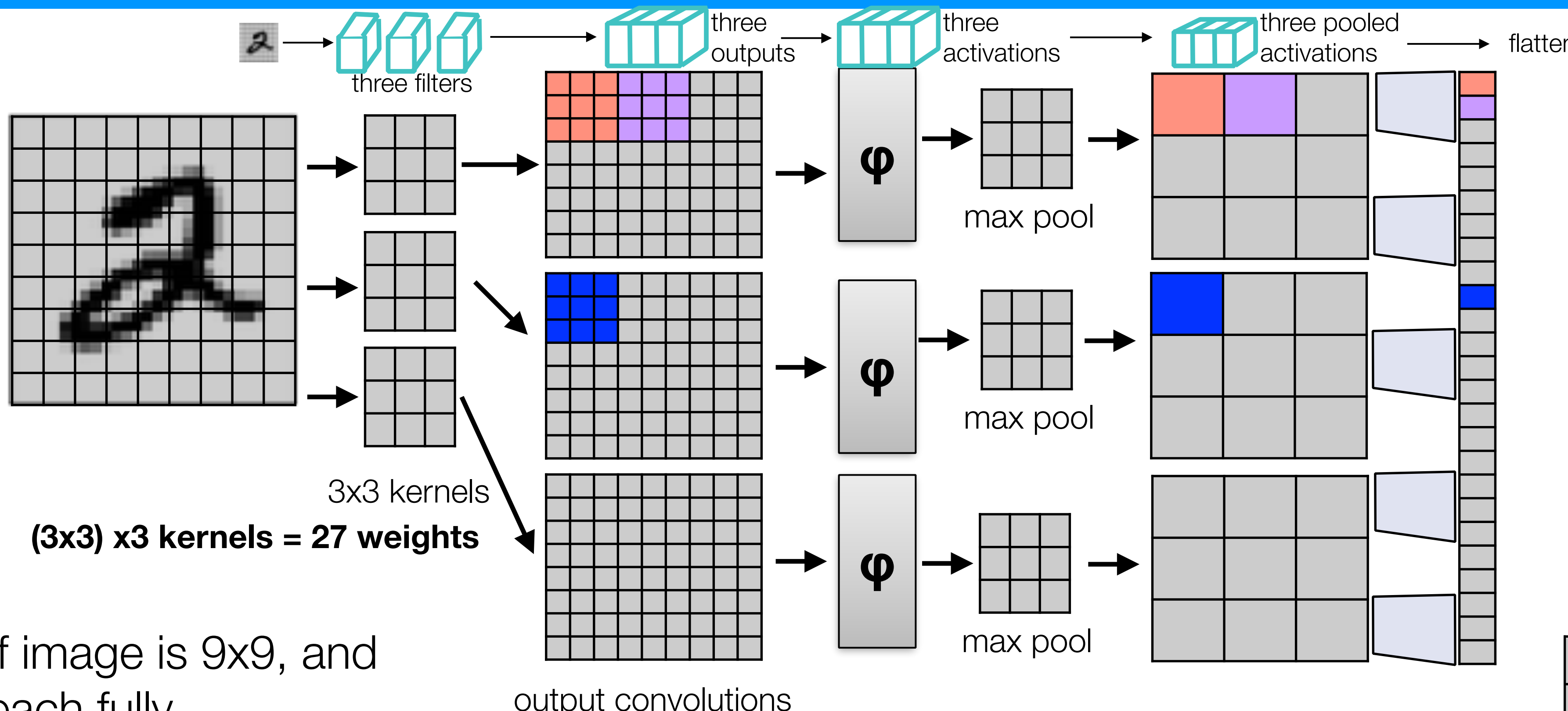
what we did in the past

If image is 9x9, and each fully connected layer is 20 hidden neurons wide, how many parameters are in this NN (ignore bias)?

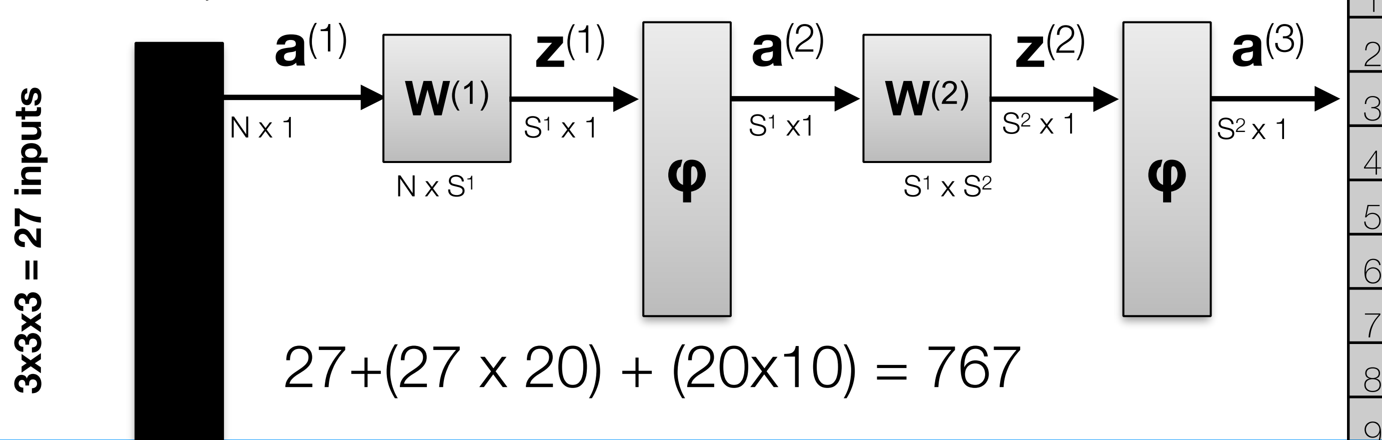
for 9x9, 9^2x20 + (20x10) = 1,820 parameters

$(K^2 \times 20) + (20\text{x}10) = 200 + 20\,K^2$

Simple Example: From Fully Connected to CNN

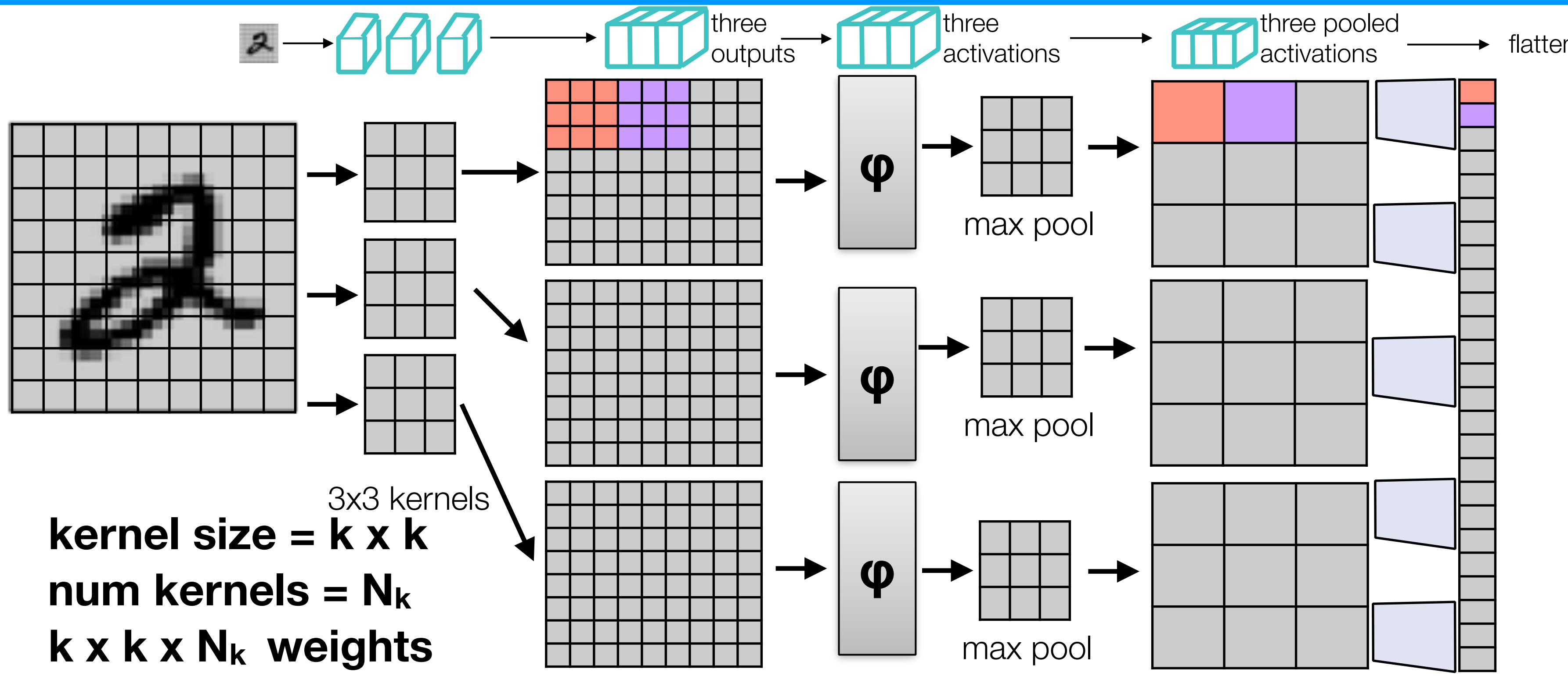


If image is 9x9, and each fully connected layer is 20 hidden neurons wide, how many parameters are in this NN (ignore bias)?



$$27+(27 \times 20) + (20 \times 10) = 767$$

Simple Example: From Fully Connected to CNN



kernel size = k x k

num kernels = N_k

k x k x N_k weights

output convolutions

convolutional params

N_k x k^2

filter dimension

num filters

pooling outputs

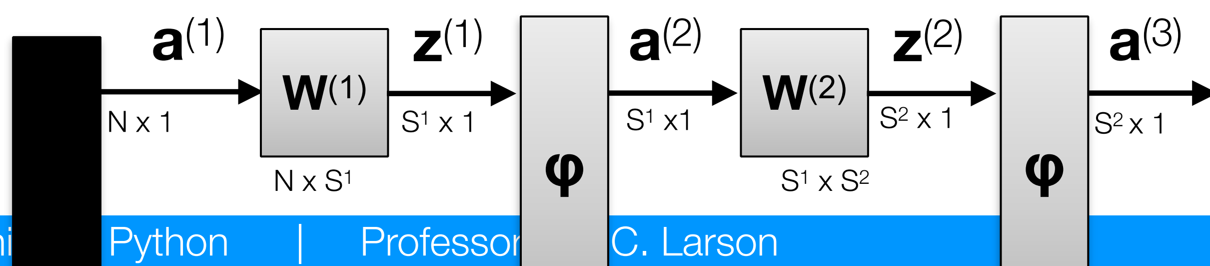
if stride = k

(K/k) x (K/k) x N_k

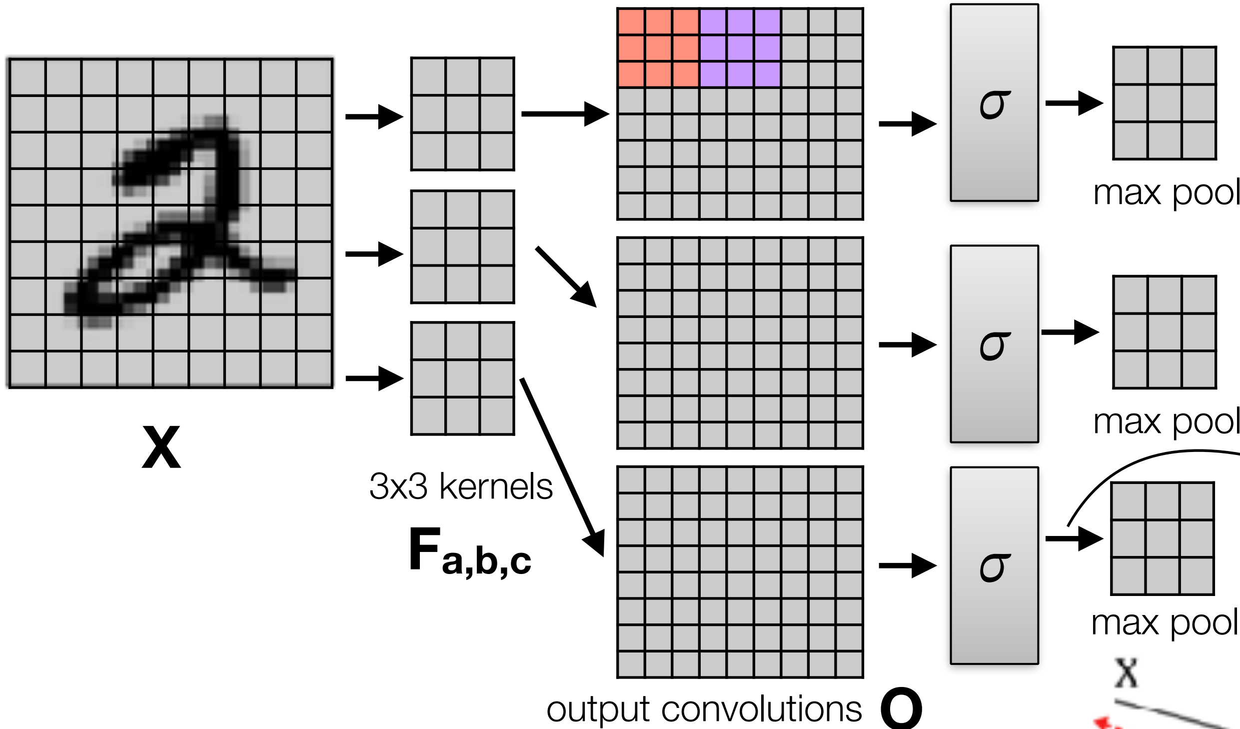
Input to MLP

N_k x (K^2/k^2)

image dimension



CNN gradient setup



Derivative of max pool is easy to compute:

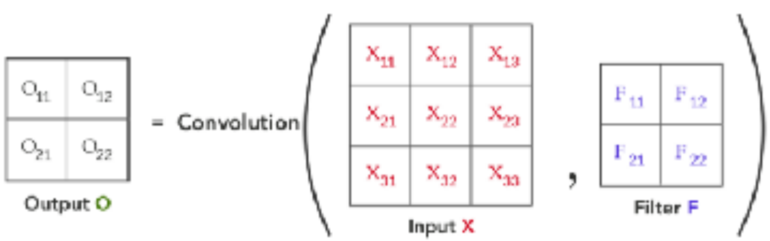
$$\frac{\partial}{L_i}\text{pool}(L_i) = 1, \text{ if } L_i \text{ is max}$$
$$0, \text{ else}$$

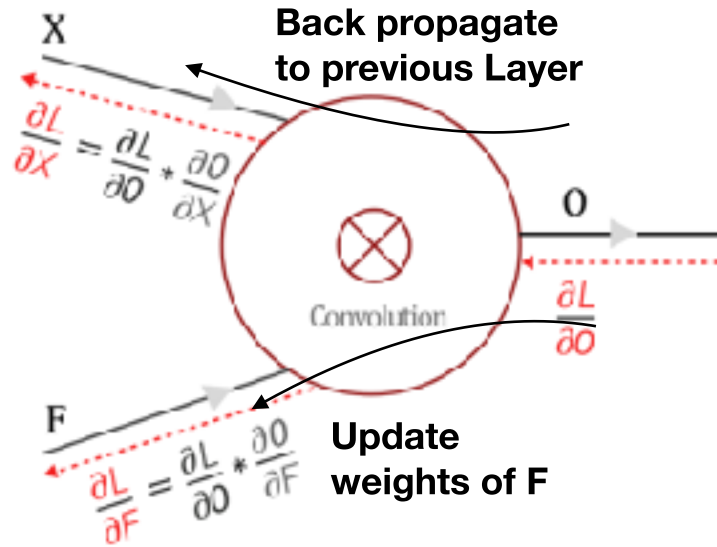
output activations

L = σ(**O**)

X$^{\textbf{(Next)}}$ = pool(σ(**O**))

Derivative of convolution is more involved:

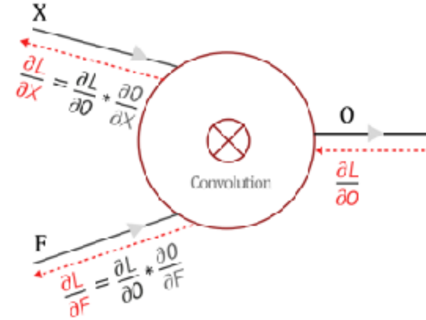


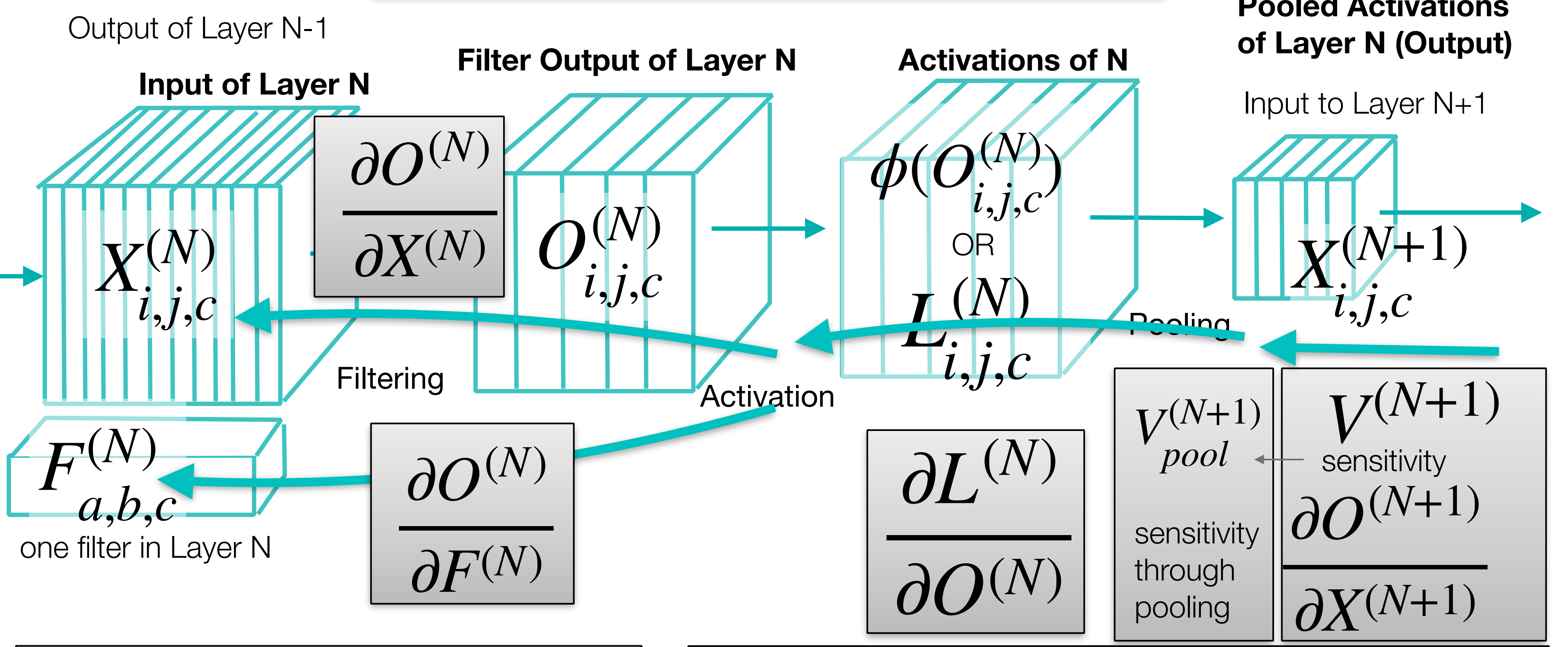


CNNs Back Propagation

Sensitivity to layer in back propagation

$$V^{(N)} = \frac{\partial O^{(N)}}{\partial X^{(N)}} \cdot \frac{\partial L^{(N)}}{\partial O^{(N)}} \cdot V_{pool}^{(N+1)}$$





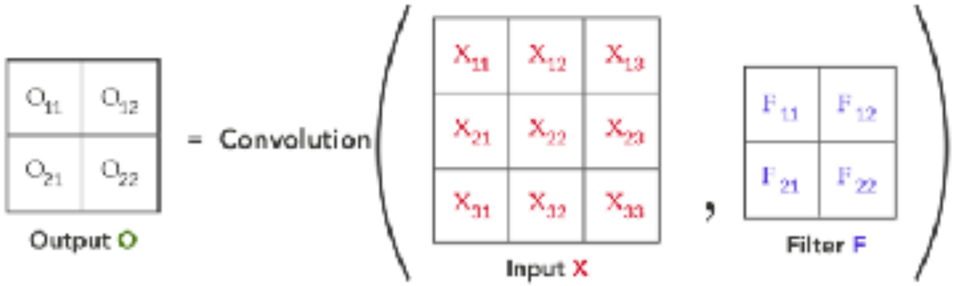
Now we can calc partial derivative

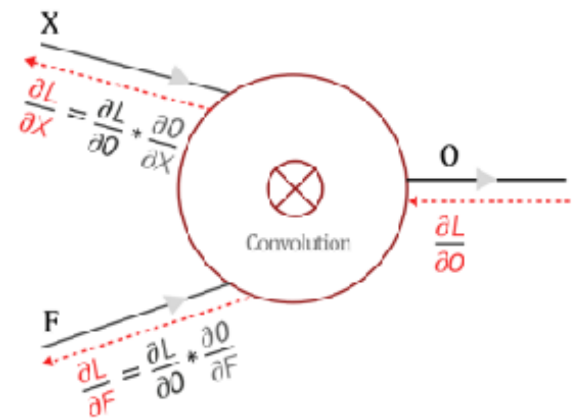
$$\frac{\partial L^{(N)}}{\partial F^{(N)}} = \frac{\partial O^{(N)}}{\partial F^{(N)}} \cdot \frac{\partial L^{(N)}}{\partial O^{(N)}}$$

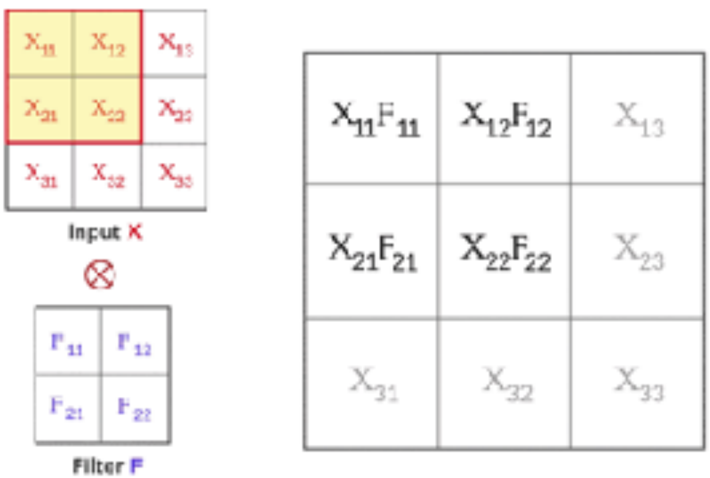
Just incorporate sensitivity, to get weight update

$$\frac{\partial J_{obj}}{\partial F^{(N)}} = \frac{\partial O^{(N)}}{\partial F^{(N)}} \cdot \frac{\partial L^{(N)}}{\partial O^{(N)}} \cdot V_{pool}^{(N+1)}$$

Reminder: Convolution







$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

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$$O_{12} = X_{12}F_{11} + X_{13}F_{12} + X_{22}F_{21} + X_{23}F_{22}$$
$$O_{21} = X_{21}F_{11} + X_{22}F_{12} + X_{31}F_{21} + X_{32}F_{22}$$
$$O_{22} = X_{22}F_{11} + X_{23}F_{12} + X_{32}F_{21} + X_{33}F_{22}$$

https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

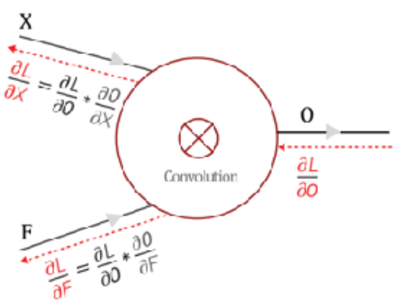
Gradient of Convolution

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial O}\frac{\partial O}{\partial X}$$

for back propagation

$$\frac{\partial L}{\partial F} = \frac{\partial L}{\partial O}\frac{\partial O}{\partial F}$$

for weight updates



$$\underset{\text{Output } O}{\begin{bmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{bmatrix}} = \text{Convolution}\left(\underset{\text{Input } X}{\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}}, \underset{\text{Filter } F}{\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}} \right)$$

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$
$$O_{12} = X_{12}F_{11} + X_{13}F_{12} + X_{22}F_{21} + X_{23}F_{22}$$
$$O_{21} = X_{21}F_{11} + X_{22}F_{12} + X_{31}F_{21} + X_{32}F_{22}$$
$$O_{22} = X_{22}F_{11} + X_{23}F_{12} + X_{32}F_{21} + X_{33}F_{22}$$

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Finding derivatives with respect to F_{11} , F_{12} , F_{21} and F_{22}

$$\frac{\partial O_{11}}{\partial F_{11}} = X_{11} \quad \frac{\partial O_{11}}{\partial F_{12}} = X_{12} \quad \frac{\partial O_{11}}{\partial F_{21}} = X_{21} \quad \frac{\partial O_{11}}{\partial F_{22}} = X_{22}$$

derivative of every O_{ij} w.r.t. F_{11}

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{11}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{11}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{11}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{11}}$$
$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{12}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{12}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{12}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{12}}$$
$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{21}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{21}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{21}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{21}}$$
$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * \frac{\partial O_{11}}{\partial F_{22}} + \frac{\partial L}{\partial O_{12}} * \frac{\partial O_{12}}{\partial F_{22}} + \frac{\partial L}{\partial O_{21}} * \frac{\partial O_{21}}{\partial F_{22}} + \frac{\partial L}{\partial O_{22}} * \frac{\partial O_{22}}{\partial F_{22}}$$

$$\frac{\partial L}{\partial F_{11}} = \frac{\partial L}{\partial O_{11}} * X_{11} + \frac{\partial L}{\partial O_{12}} * X_{12} + \frac{\partial L}{\partial O_{21}} * X_{21} + \frac{\partial L}{\partial O_{22}} * X_{22}$$
$$\frac{\partial L}{\partial F_{12}} = \frac{\partial L}{\partial O_{11}} * X_{12} + \frac{\partial L}{\partial O_{12}} * X_{13} + \frac{\partial L}{\partial O_{21}} * X_{22} + \frac{\partial L}{\partial O_{22}} * X_{23}$$
$$\frac{\partial L}{\partial F_{21}} = \frac{\partial L}{\partial O_{11}} * X_{21} + \frac{\partial L}{\partial O_{12}} * X_{22} + \frac{\partial L}{\partial O_{21}} * X_{31} + \frac{\partial L}{\partial O_{22}} * X_{32}$$
$$\frac{\partial L}{\partial F_{22}} = \frac{\partial L}{\partial O_{11}} * X_{22} + \frac{\partial L}{\partial O_{12}} * X_{23} + \frac{\partial L}{\partial O_{21}} * X_{32} + \frac{\partial L}{\partial O_{22}} * X_{33}$$

$$\underset{\text{Filter updates}}{\begin{bmatrix} \frac{\partial L}{\partial F_{11}} & \frac{\partial L}{\partial F_{12}} \\ \frac{\partial L}{\partial F_{21}} & \frac{\partial L}{\partial F_{22}} \end{bmatrix}} = \text{Convolution}\left(\underset{\text{Input}}{\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}}, \underset{\text{Derivative From activation!}}{\begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix}} \right)$$

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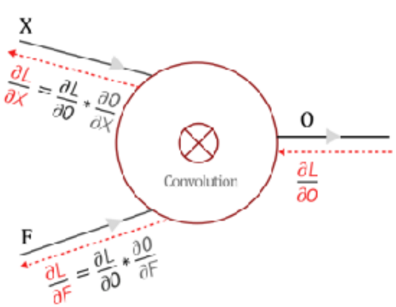

$$\underbrace{\begin{bmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{bmatrix}}_{\text{Output } O} = \text{Convolution}\left(\underbrace{\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}}_{\text{Input } X}, \underbrace{\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}}_{\text{Filter } F} \right)$$

$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$
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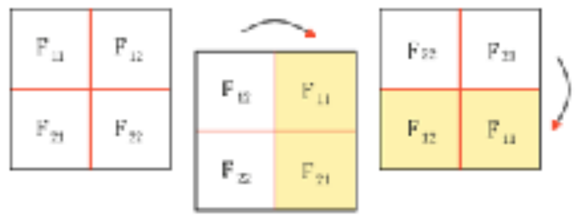
$$O_{11} = X_{11}F_{11} + X_{12}F_{12} + X_{21}F_{21} + X_{22}F_{22}$$

Differentiating with respect to X_{11}, X_{12}, X_{21} *and* X_{22}

$$\frac{\partial O_{11}}{\partial X_{11}} = F_{11} \quad \frac{\partial O_{11}}{\partial X_{12}} = F_{12} \quad \frac{\partial O_{11}}{\partial X_{21}} = F_{21} \quad \frac{\partial O_{11}}{\partial X_{22}} = F_{22}$$

Similarly, we can find local gradients for O_{12}, O_{21} *and* O_{22}

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial O_{11}} \cdot F_{11}$$
$$\frac{\partial L}{\partial X_{12}} = \frac{\partial L}{\partial O_{11}} \cdot F_{12} + \frac{\partial L}{\partial O_{12}} \cdot F_{11}$$
$$\frac{\partial L}{\partial X_{13}} = \frac{\partial L}{\partial O_{12}} \cdot F_{12}$$
$$\frac{\partial L}{\partial X_{21}} = \frac{\partial L}{\partial O_{11}} \cdot F_{21} + \frac{\partial L}{\partial O_{21}} \cdot F_{11}$$
$$\frac{\partial L}{\partial X_{22}} = \frac{\partial L}{\partial O_{11}} \cdot F_{22} + \frac{\partial L}{\partial O_{12}} \cdot F_{21} + \frac{\partial L}{\partial O_{21}} \cdot F_{12} + \frac{\partial L}{\partial O_{22}} \cdot F_{11}$$
$$\frac{\partial L}{\partial X_{23}} = \frac{\partial L}{\partial O_{12}} \cdot F_{22} + \frac{\partial L}{\partial O_{22}} \cdot F_{12}$$
$$\frac{\partial L}{\partial X_{31}} = \frac{\partial L}{\partial O_{21}} \cdot F_{21}$$
$$\frac{\partial L}{\partial X_{32}} = \frac{\partial L}{\partial O_{21}} \cdot F_{22} + \frac{\partial L}{\partial O_{22}} \cdot F_{21}$$
$$\frac{\partial L}{\partial X_{33}} = \frac{\partial L}{\partial O_{22}} \cdot F_{22}$$



$$\begin{bmatrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \frac{\partial L}{\partial X_{11}} & \frac{\partial L}{\partial X_{12}} & \frac{\partial L}{\partial X_{13}} \\ \frac{\partial L}{\partial X_{21}} & \frac{\partial L}{\partial X_{22}} & \frac{\partial L}{\partial X_{23}} \\ \frac{\partial L}{\partial X_{31}} & \frac{\partial L}{\partial X_{32}} & \frac{\partial L}{\partial X_{33}} \end{bmatrix}}_{\text{New sensitivity}} = \text{Full Convolution}\left(\underbrace{\begin{bmatrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{bmatrix}}_{\text{Rotated Filter}}, \underbrace{\begin{bmatrix} \frac{\partial L}{\partial O_{11}} & \frac{\partial L}{\partial O_{12}} \\ \frac{\partial L}{\partial O_{21}} & \frac{\partial L}{\partial O_{22}} \end{bmatrix}}_{\text{Derivative From activation! (zero padded)}} \right)$$

New sensitivity

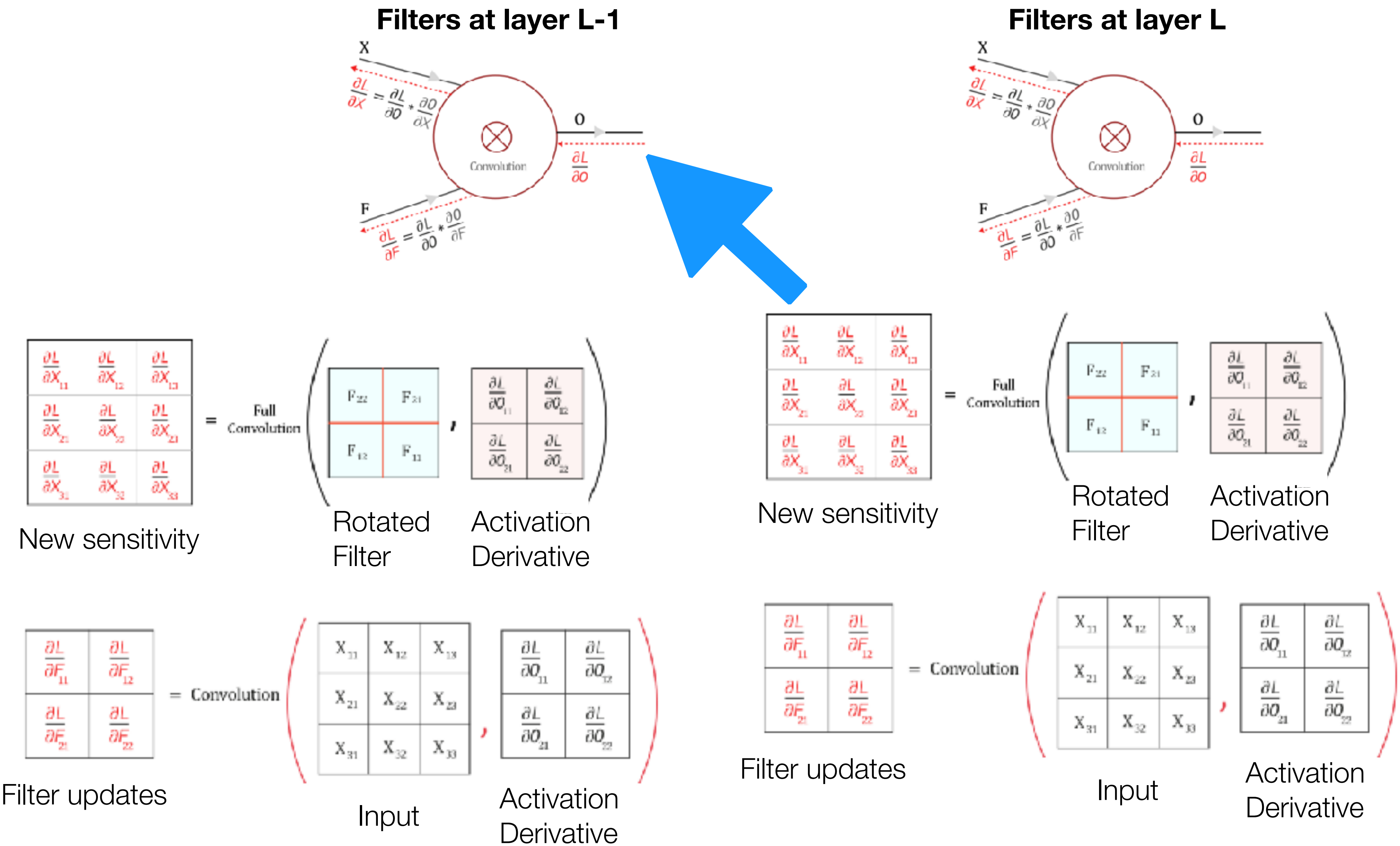
Rotated Filter

Derivative
From activation!
(zero padded)

0	0	0	0
0	$\frac{\partial L}{\partial O_{11}}$	$\frac{\partial L}{\partial O_{12}}$	0
0	$\frac{\partial L}{\partial O_{21}}$	$\frac{\partial L}{\partial O_{22}}$	0
0	0	0	0

https://medium.com/@pavisj/convolutions-and-backpropagations-46026a8f5d2c

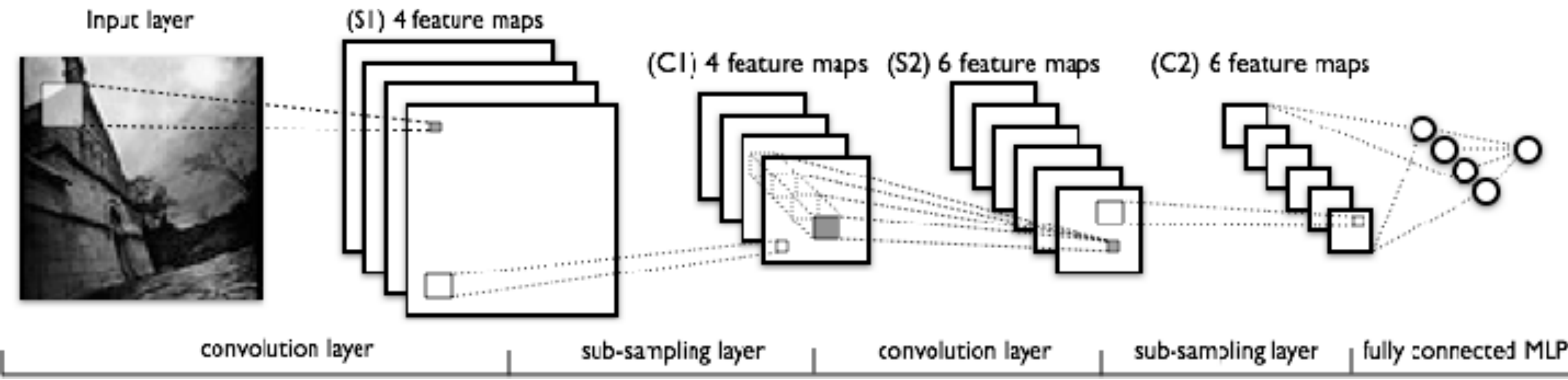
Summary

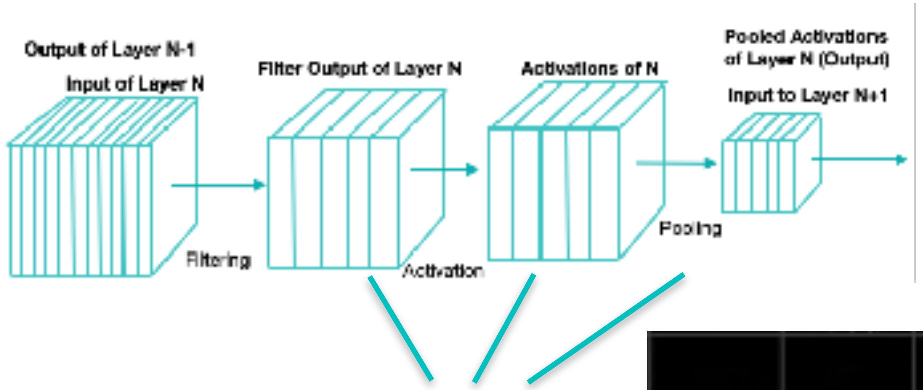

Filters at layer L-1
Filters at layer L
X
F
O
Convolution
New sensitivity
= Full Convolution
Rotated Filter
Activation Derivative
Filter updates
= Convolution
Input
Activation Derivative

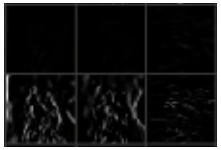
CNN Gradient

- Takeaways:
    - Derivative of a convolutional layer is calculated through two additional convolutions
        - One for filter updates
        - One for calculating a new sensitivity
    - We need to run convolution fast in order to speed up both:
        - feedforward operations (inference and training)
        - back propagation (training)
- Another great resource:
    - https://becominghuman.ai/back-propagation-in-convolutional-neural-networks-intuition-and-code-714ef1c38199

Some Example CNN Architectures

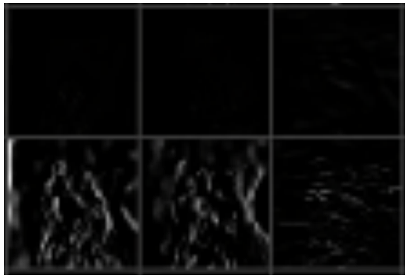






Visualize
each channel

(batch, channels, i, j)



Naming:
conv1 (output of conv)
p1 (output of pooling)
n1 (output of normalization)

CNN: Visuals

Deep Visualization Toolbox

yosinski.com/deepvis

#deepvis

Jason Yosinski | Jeff Clune | Anh Nguyen | Thomas Fuchs | Hod Lipson

Cornell University

University of Wyoming

Jet Propulsion Laboratory
California Institute of Technology

https://github.com/yosinski/deep-visualization-toolbox

Demo

Convolutional Neural Networks
in TensorFlow
with Keras

`11. Convolutional Neural Networks.ipynb`

Next Lecture

- More CNN architectures and CNN history