Lecture Notes for

Machine Learning in Python

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Professor Eric Larson

MLP History

Class Logistics and Agenda

- Logistics:
  - Grading Update
  - **Next time: Flipped Module on back propagation**
- Multi Week Agenda:
  - Today: Neural Networks History, up to 1980
  - Today: Multi-layer Architectures
  - Town Hall, Lab 4 (probably next time)
  - **Flipped**: Programming Multi-layer training

Class Overview, by topic

Table Data Visualization

Numpy, Pandas, Seaborn
Overviews with some in-depth discussion

Dimension Reduction and Image Processing

Scikit-learn, Scikit Image,
Intuition only, Some mathematics

Linear and Logistic Regression

Numpy, Recreate API for Scikit-learn
Detailed mathematics for simple optimization
intuition for advanced optimization

Neural Networks and Back Prop.

Numpy
Detailed mathematics for NN operations

Wide and Deep Networks | **Convolutional Networks** | **Recurrent Networks**

Keras, Tensorflow
Intuition, Detailed implement.

Ethics in Language Models

ConceptNet
Case studies

Lab 3, Town Hall (if needed)

Tyler Rablin @Mr_Rablin · 2d

You're not grading assignments.

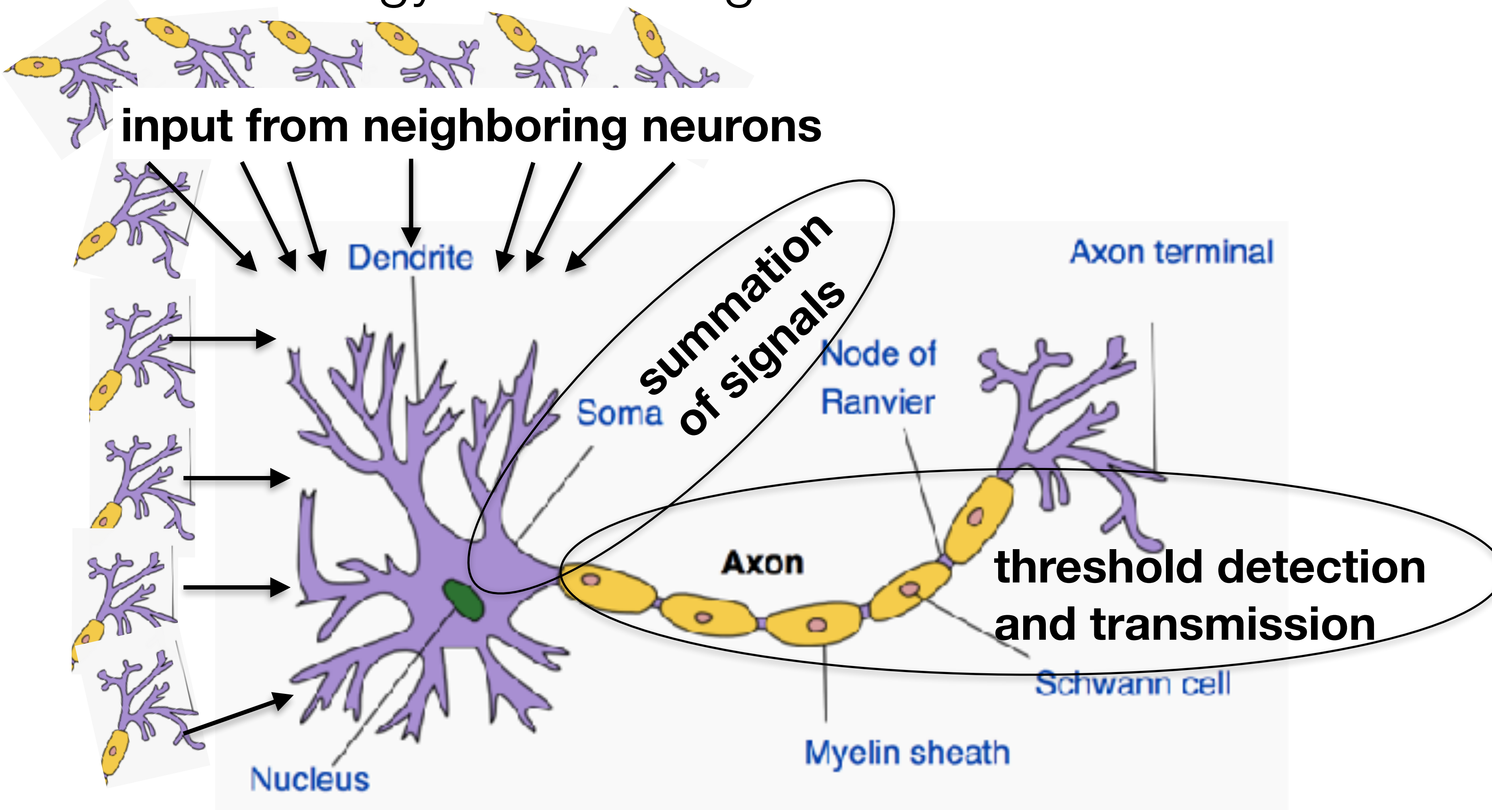
You're collecting evidence to determine student progress and pointing them towards their next steps.

Make the mental switch. It matters.

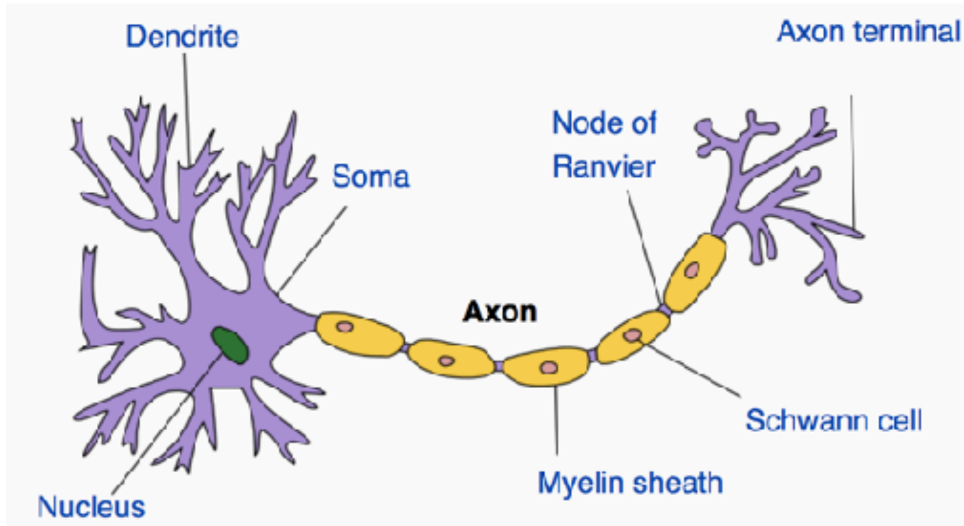
A History of Neural Networks

Neurons

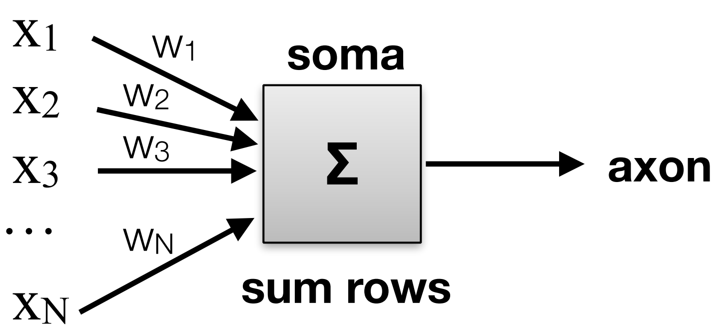
- From biology to modeling:



McCulloch and Pitts, 1943



dendrite



input

$$\mathbf{x} \cdot \mathbf{w} = a$$

for each neuron

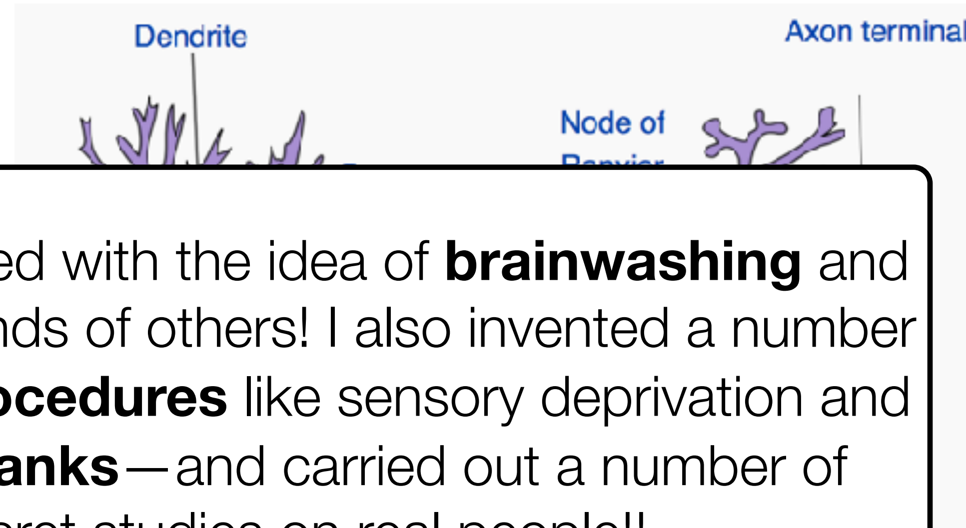
logic gates of the mind

Warren McCulloch

Walter Pitts

Neurons

- McCulloch and Pitts, 1943
- Donald Hebb, 1949
  - Hebb's Law: close neurons fire together
  - neurons "learn
  - easier synaptic
  - basis of neura



I was infatuated with the idea of **brainwashing** and controlling minds of others! I also invented a number of **torture procedures** like sensory deprivation and **isolation tanks**—and carried out a number of secret studies on real people!!

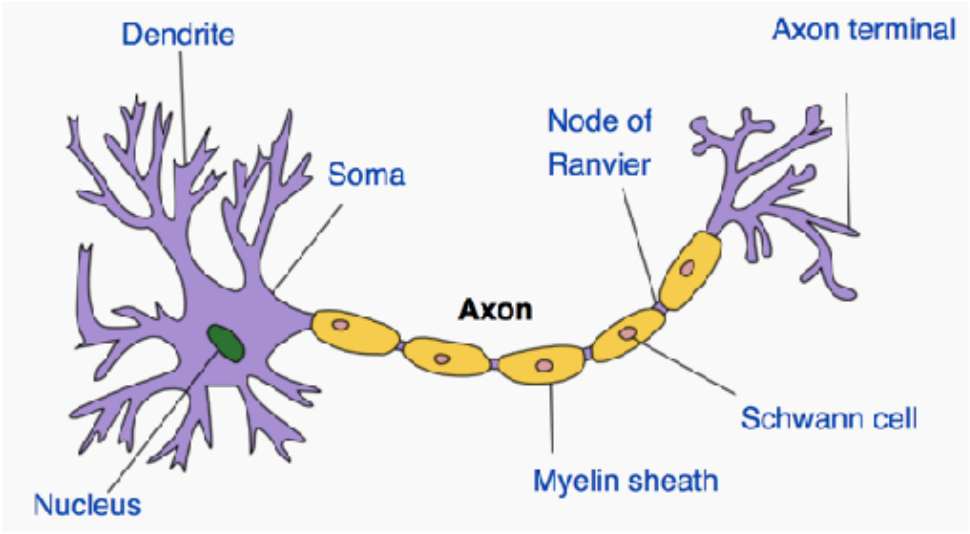
Donald O. Hebb

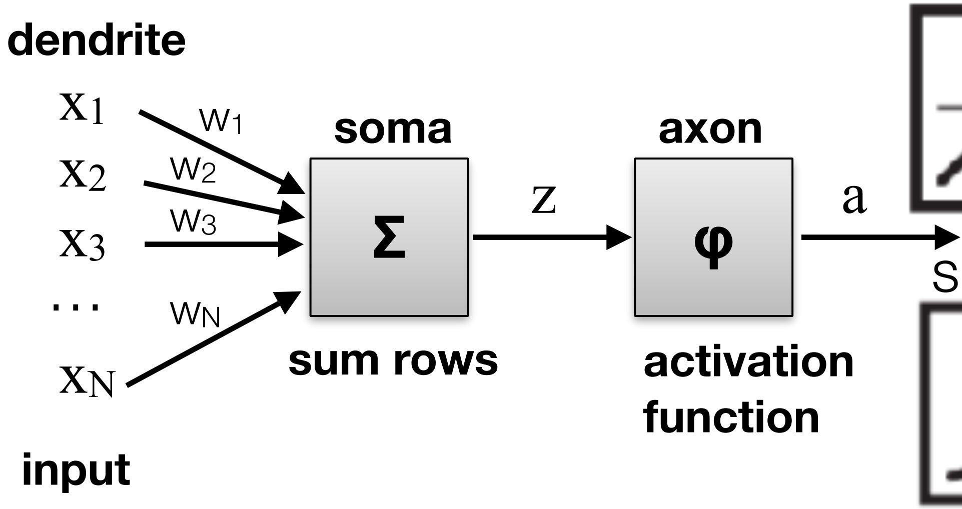
Warren McCulloch

Walter Pitts

Rosenblatt’s perceptron, 1957

Frank Rosenblatt





hard limit

```
a=-1    z< 0
a= 1    z>=0
```

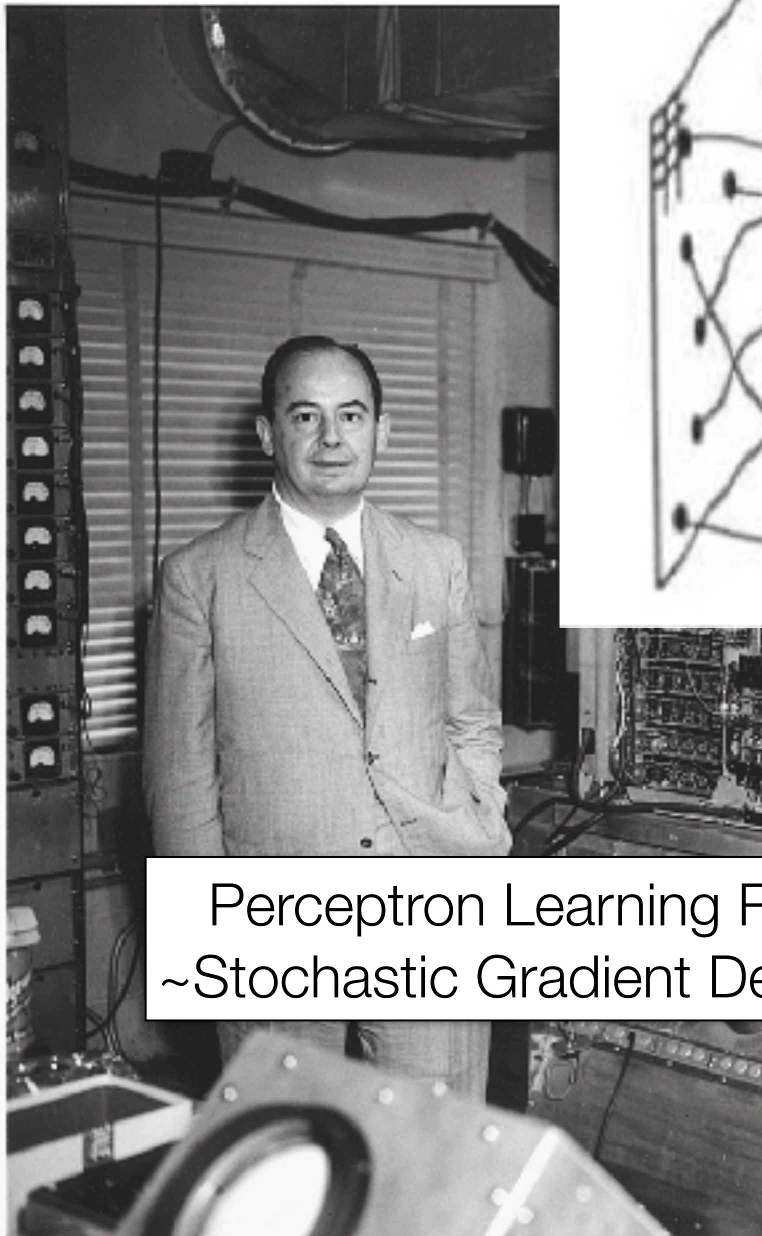
linear

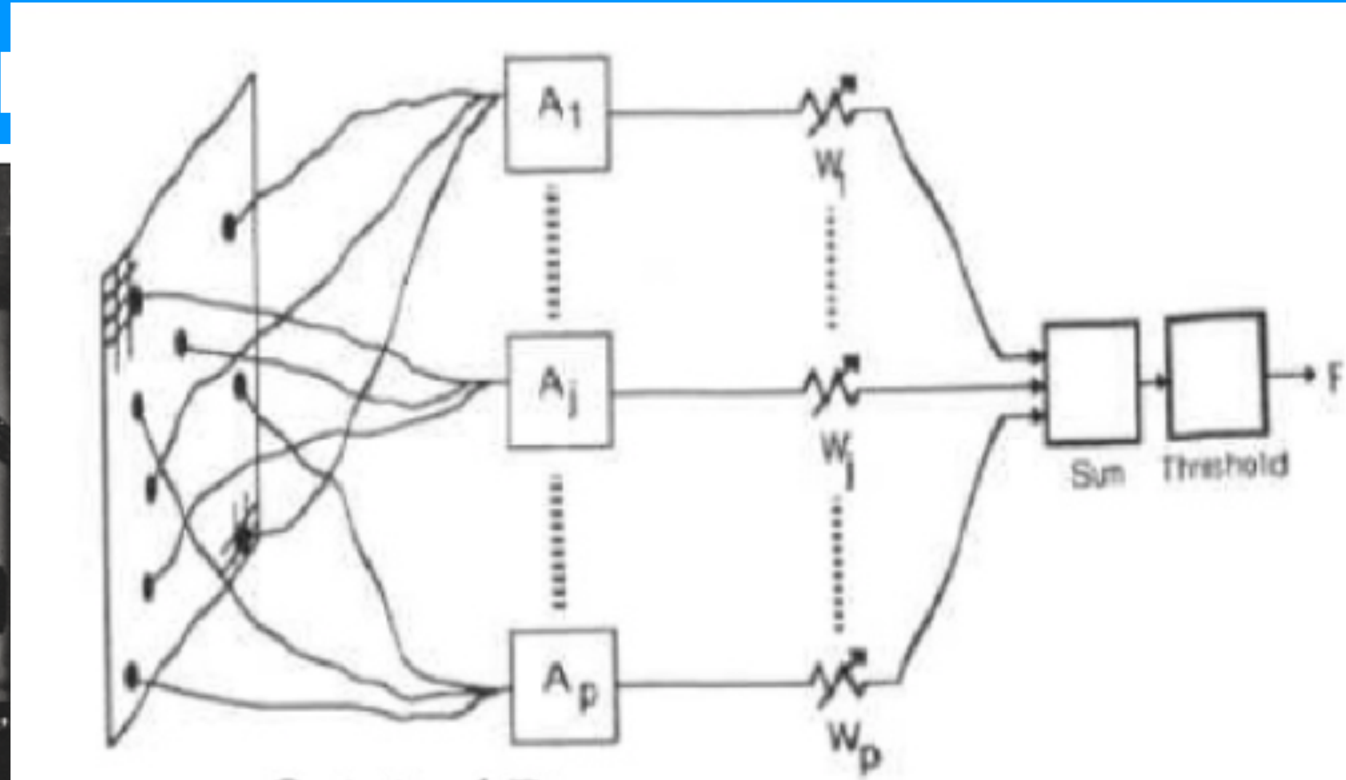
$a=z$

sigmoid

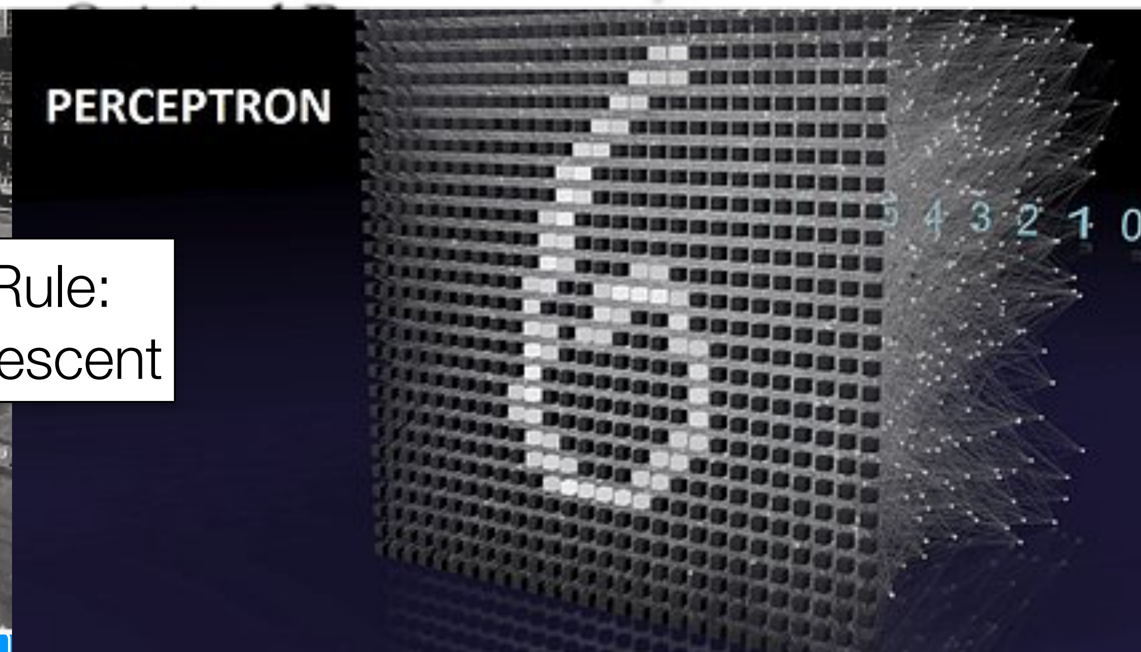
$$a=\frac{1}{1+\exp(-z)}$$

The Mark 1

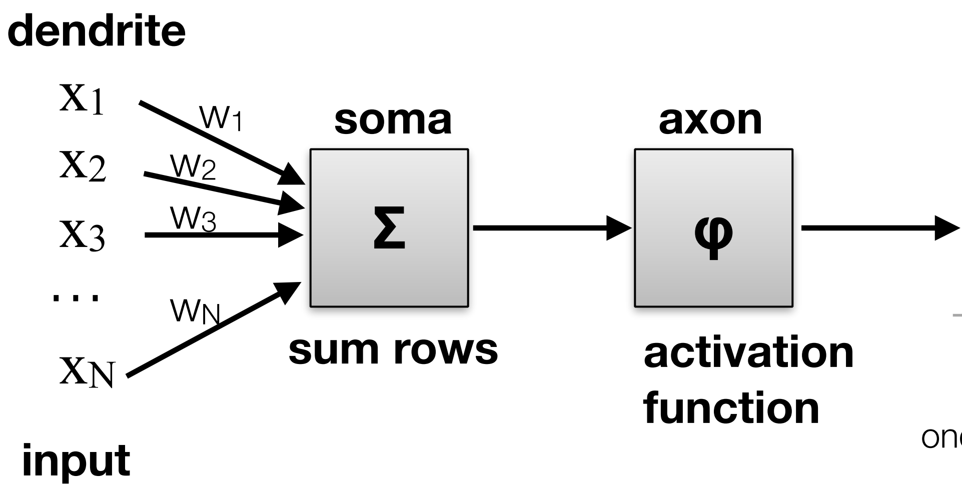




Perceptron Learning Rule:
~Stochastic Gradient Descent



One Neuron



a=x

$$\mathbf{x}^{(i)} = \begin{bmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_N \end{bmatrix}^{(i)} = [\mathbf{a}^{(1)}]^{(i)}$$

$$\mathbf{z} = \mathbf{W} \cdot \mathbf{x}^{(i)} + \mathbf{b}$$

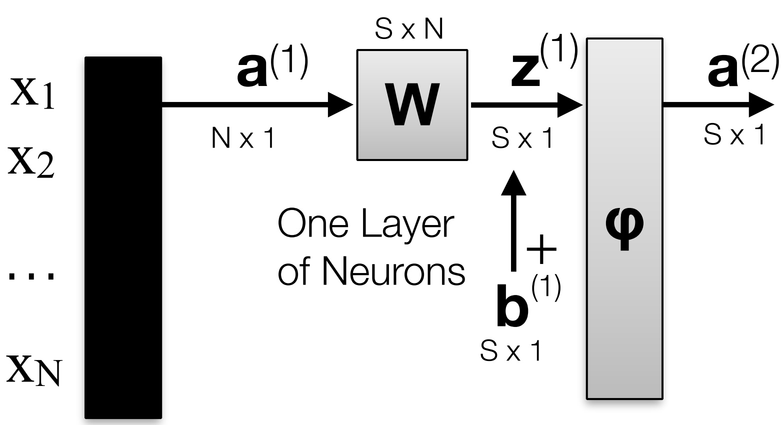
one neuron →

$$\mathbf{W} = \begin{bmatrix} \boxed{w_{1,1} \;\; \cdots \;\; w_{1,N}} \\ w_{2,1} \;\; \cdots \;\; w_{2,N} \\ \vdots \\ w_{S,1} \;\; \cdots \;\; w_{S,N} \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_S \end{bmatrix}$$

$$[\mathbf{z}^{(1)}]^{(i)} = \mathbf{W}^{(1)} \cdot [\mathbf{a}^{(1)}]^{(i)} + \mathbf{b}^{(1)}$$

$$\mathbf{a}^{next} = \phi(\mathbf{z}^{\mathbf{current}})$$

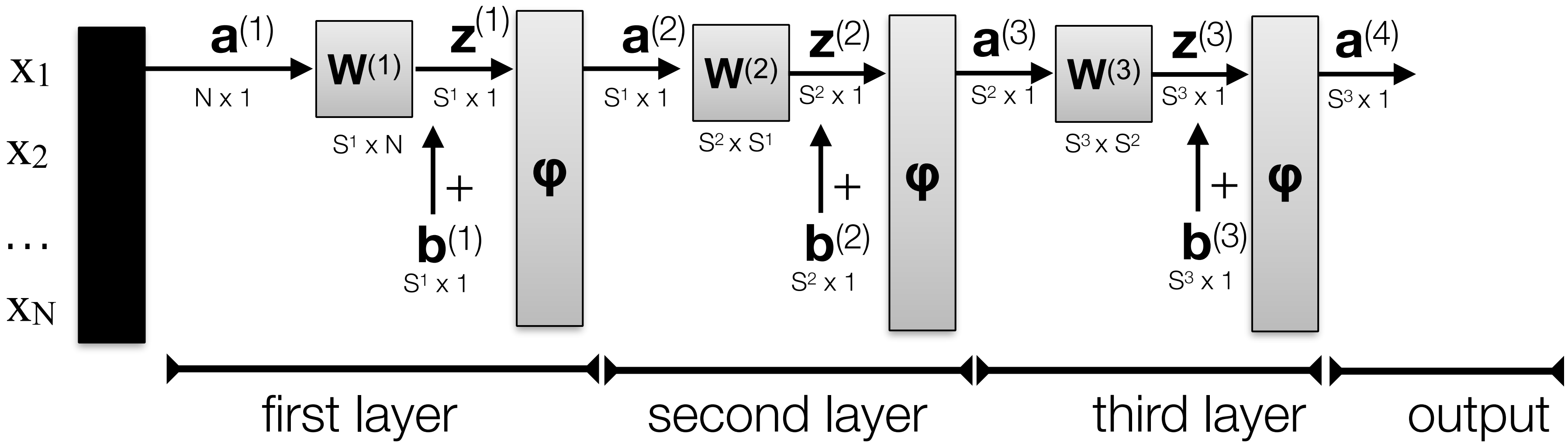
$$\mathbf{a}^{(next)} = \begin{bmatrix} \phi(z_1^{curr}) \\ \vdots \\ \phi(z_N^{curr}) \end{bmatrix} \rightarrow \mathbf{a}^{(L)} = \begin{bmatrix} \phi(z_1^{L-1}) \\ \vdots \\ \phi(z_N^{L-1}) \end{bmatrix}$$



$\mathbf{x}^{(i)}$ One row from Table data becomes input column to model

notation adapted from *Neural Network Design*, Hagan, Demuth, Beale, and De Jesus

Generic Multiple Layers Notation



$$\mathbf{a}^{(L+1)} = \phi(\mathbf{z}^{(L)})$$

$\mathbf{a}^{(final)}$ size=unique classes, C

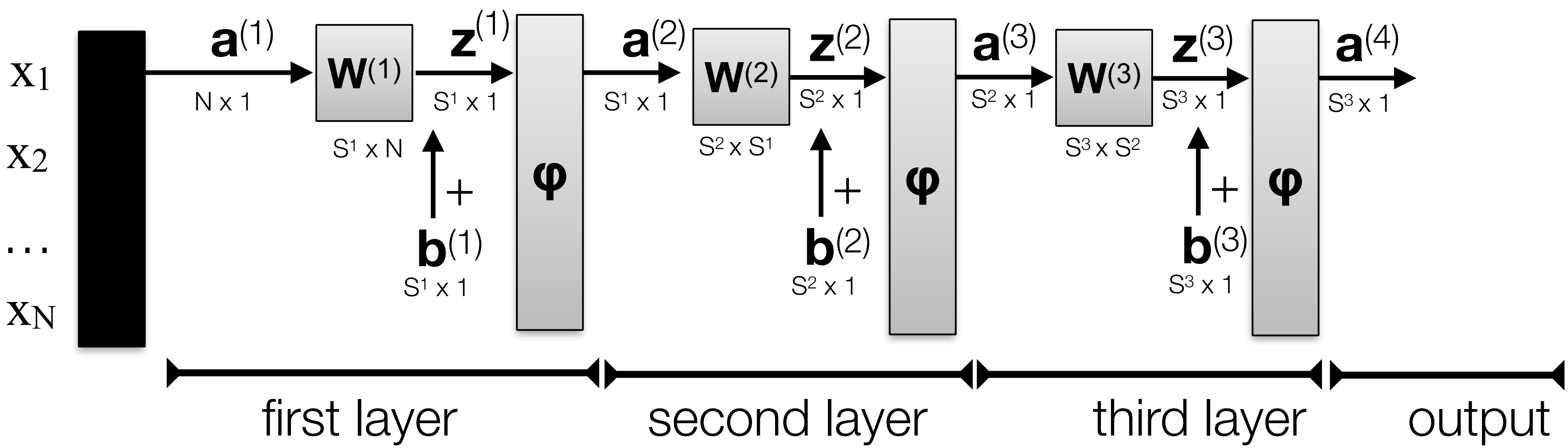
$$\mathbf{z}^{(L)} = \mathbf{W}^{(L)} \cdot \mathbf{a}^{(L)} + \mathbf{b}^{(L)}$$

$$\mathbf{z}^{(L)} = \mathbf{W}^{(L)} \cdot \phi(\mathbf{z}^{(L-1)}) + \mathbf{b}^{(L)}$$

$$\mathbf{z}^{(L)} = \mathbf{W}^{(L)} \cdot \phi\left(\mathbf{W}^{(L-1)} \cdot \phi(\mathbf{z}^{(L-2)}) + \mathbf{b}^{(L-1)}\right) + \mathbf{b}^{(L)}$$

notation adapted from *Neural Network Design*, Hagan, Demuth, Beale, and De Jesus

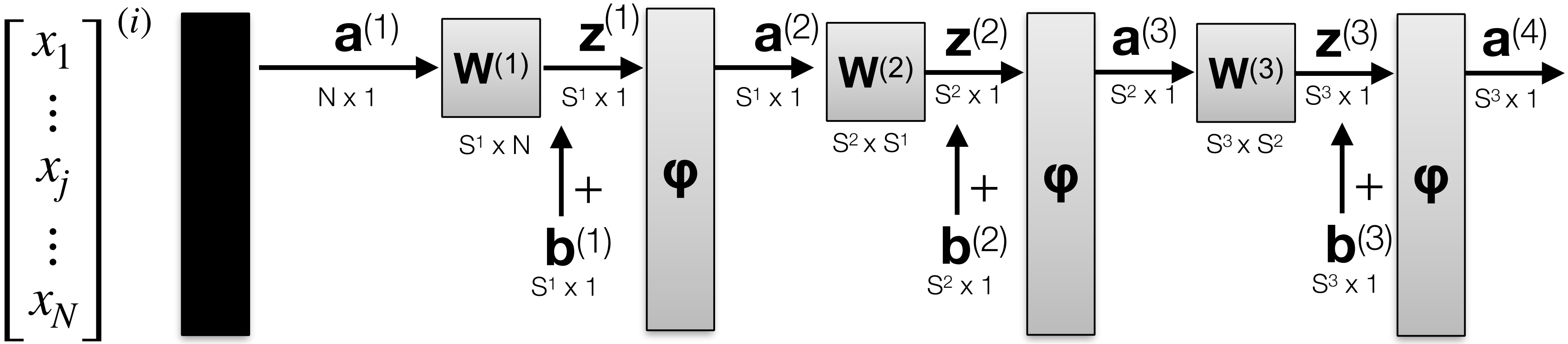
Multiple layers notation



- **Self test**: How many parameters need to be trained in the above network?
  - A. $[(N+1) \times S^1] + [(S^1+1) \times S^2] + [(S^2+1) \times S^3]$
  - B. $|\mathbf{W}^{(1)}| + |\mathbf{W}^{(2)}| + |\mathbf{W}^{(3)}| + |\mathbf{b}^{(1)}| + |\mathbf{b}^{(2)}| + |\mathbf{b}^{(3)}|$
  - C. can't determine from diagram
  - D. it depends on the sizes of intermediate variables, $\mathbf{z}^{(i)}$

notation adapted from *Neural Network Design*, Hagan, Demuth, Beale, and De Jesus

Compact feedforward notation

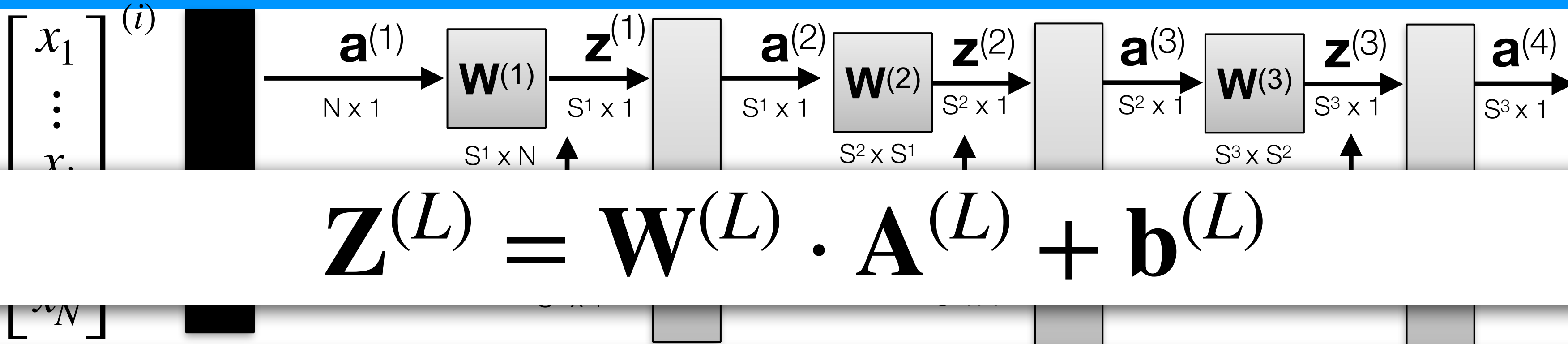


$$\mathbf{X}^T = \left[\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}^{(1)}, \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}^{(2)} \cdots \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}^{(M)} \right] = \left[\begin{bmatrix} a_0^{(1)} \\ a_1^{(1)} \\ \vdots \\ a_N^{(1)} \end{bmatrix}^{(1)}, \begin{bmatrix} a_0^{(1)} \\ a_1^{(1)} \\ \vdots \\ a_N^{(1)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} a_0^{(1)} \\ a_1^{(1)} \\ \vdots \\ a_N^{(1)} \end{bmatrix}^{(M)} \right] = \mathbf{A}^{(1)}$$

Table Data

Table Data, in Neural Net Notation

Compact feedforward notation



$$\mathbf{Z}^{(L)} = \mathbf{W}^{(L)} \cdot \mathbf{A}^{(L)} + \mathbf{b}^{(L)}$$

$$\mathbf{Z}^{(L)} = \mathbf{W}^{(L)} \cdot \phi(\mathbf{Z}^{(L-1)}) + \mathbf{b}^{(L)}$$

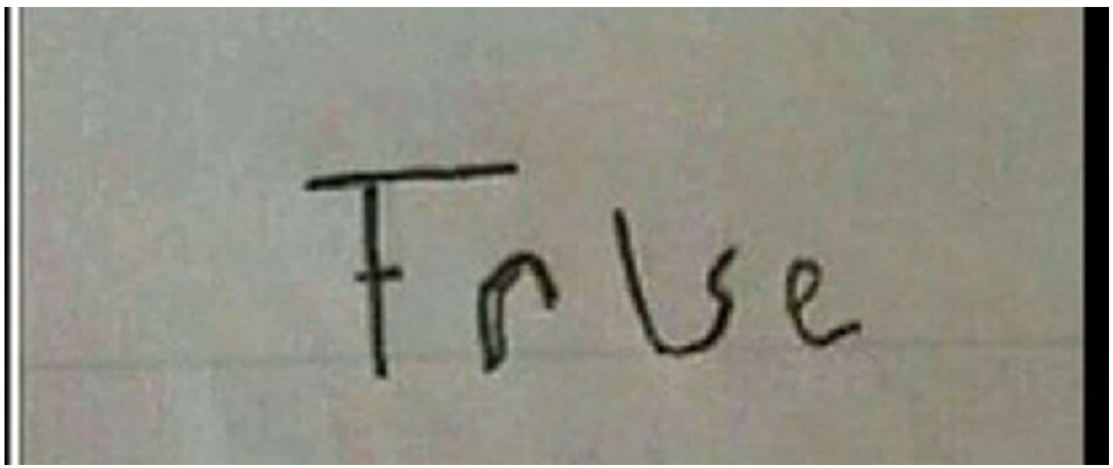
$$[\mathbf{z}^{(L)}]^{(i)} = \mathbf{W}^{(L)} \cdot [\mathbf{a}^{(L)}]^{(i)} + \mathbf{b}^{(L)}$$

$$\begin{bmatrix} z_2 \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix} = \mathbf{W}^{(L)} \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix} + \mathbf{b}^{(L)}$$

$$\left[\begin{bmatrix} z_1^{(L)} \\ z_2^{(L)} \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix}^{(1)}, \begin{bmatrix} z_1^{(L)} \\ z_2^{(L)} \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} z_1^{(L)} \\ z_2^{(L)} \\ \vdots \\ z_{S^L}^{(L)} \end{bmatrix}^{(M)} \right] = \mathbf{W}^{(L)} \cdot \left[\begin{bmatrix} a_0^{(L)} \\ a_1^{(L)} \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix}^{(1)}, \begin{bmatrix} a_0^{(L)} \\ a_1^{(L)} \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} a_0^{(L)} \\ a_1^{(L)} \\ \vdots \\ a_{S^{L-1}}^{(L)} \end{bmatrix}^{(M)} \right] + \mathbf{b}^{(L)}$$

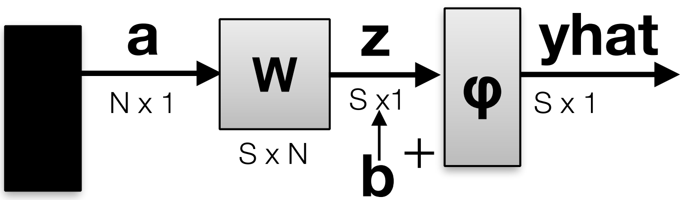
b is broadcast added

Training Neural Network Architectures



When a binary classification model outputs 0.5

Start Simple: Simplifying to One Layer



where ground truth $\mathbf{Y}$ is one-hot encoded!

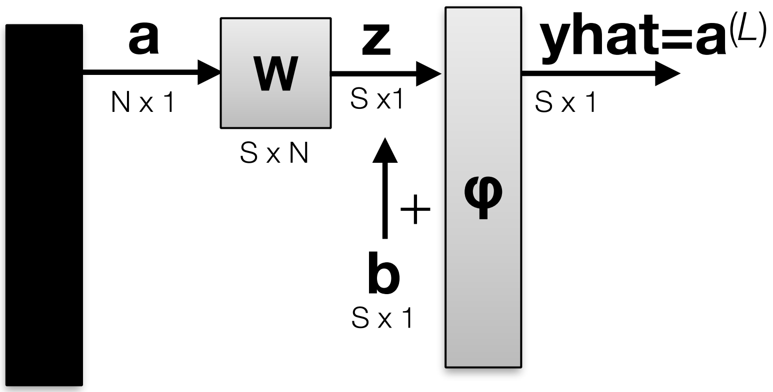
Need objective Function, minimize MSE $J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2$

$$\mathbf{Y} = \left[\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(1)} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(2)} \cdots \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(M)} \right] \rightarrow \left[\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^{(1)} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}^{(2)} \cdots \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}^{(M)} \right]$$

$$J(\mathbf{W}) = \left| \underbrace{\left[\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(1)} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(2)} \cdots \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(M)} \right]}_{\mathbf{Y}} - \underbrace{\left[\begin{bmatrix} a_1^{(L)} \\ a_2^{(L)} \\ \vdots \\ a_C^{(L)} \end{bmatrix}^{(1)} \begin{bmatrix} a_1^{(L)} \\ a_2^{(L)} \\ \vdots \\ a_C^{(L)} \end{bmatrix}^{(2)} \cdots \begin{bmatrix} a_1^{(L)} \\ a_2^{(L)} \\ \vdots \\ a_C^{(L)} \end{bmatrix}^{(M)} \right]}_{\hat{\mathbf{Y}}} \right|^2$$

One Layer Architectures

- Rosenblatt's perceptron, 1957

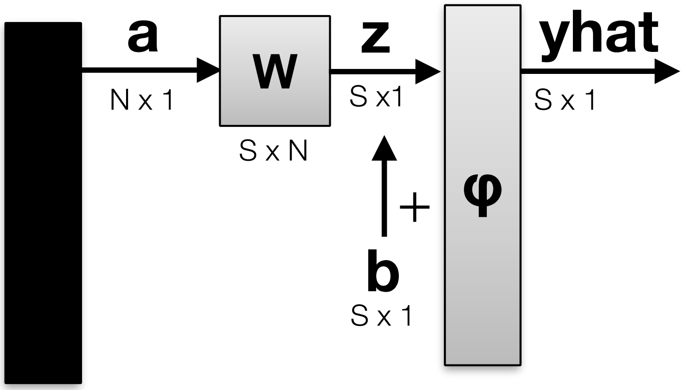


Self Test - If this is a binary classification problem, how large is *S*, the length of **yhat** and number of rows in **W**?

A. Can't determine
B. 2
C. 1
D. N

One Layer Architectures

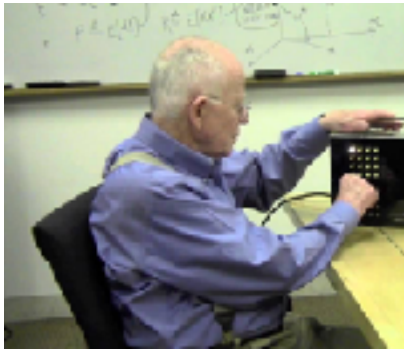
- Adaline network, Widrow and Hoff, 1960



linear

phi

Marcian “Ted” Hoff



Bernard Widrow

Simplify Objective Function:

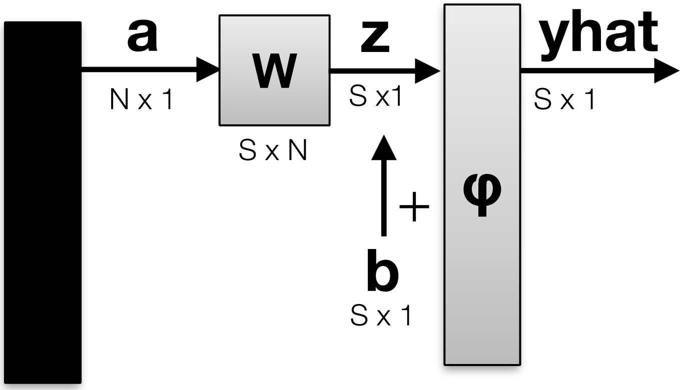
$$J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2 \longrightarrow J(\mathbf{w}) = \left\| \mathbf{Y} - \mathbf{A} \cdot \mathbf{w} \right\|^2$$

Need gradient $\nabla J(\mathbf{w})$ for update equation $\mathbf{w} \leftarrow \mathbf{w} + \eta \nabla J(\mathbf{w})$

We have been using the **Widrow-Hoff Learning Rule**

One Layer Architectures

- Adaline network, Widrow and Hoff, 1960



linear

phi

Marcian “Ted” Hoff

Bernard Widrow

Need gradient $\nabla J(\mathbf{w})$ for update equation $\mathbf{w} \leftarrow \mathbf{w} + \eta \nabla J(\mathbf{w})$

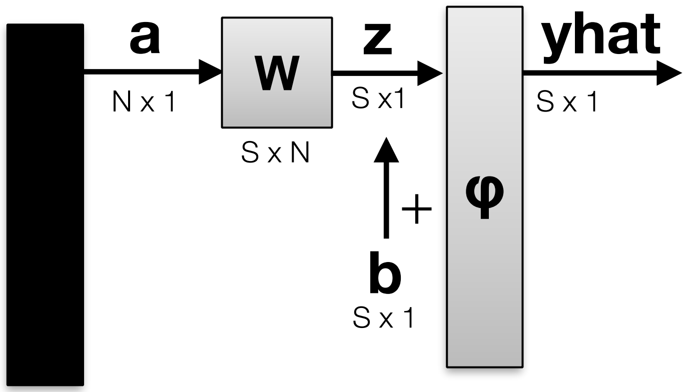
For case S=1, $\mathbf{W}$ has only one row, $\mathbf{w}$
this is just **linear regression…**

$$J(\mathbf{w}) = \sum_{i=1}^{M} (y^{(i)} - \mathbf{x}^{(i)} \cdot \mathbf{w})^2$$

$$\mathbf{w} = (\mathbf{X}^T \cdot \mathbf{X})^{-1} \cdot \mathbf{X}^T \cdot y$$

One Layer Architectures

- Modern Perceptron network



sigmoid

$$g(z) = \phi(z) = \frac{1}{1 + \exp(-z)}$$

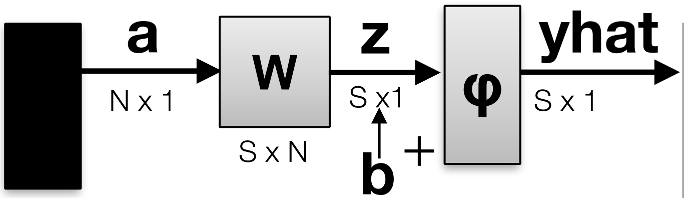
Need gradient $\nabla J(\mathbf{w})$ for update equation $\mathbf{w} \leftarrow \mathbf{w} + \eta \nabla J(\mathbf{w})$

For case S=1, this is just **logistic regression…**
and **we have already solved this**!

$$\mathbf{w} \leftarrow \mathbf{w} + \eta \cdot \text{mean}\left(\underbrace{(\mathbf{y} - g(\mathbf{X} \cdot \mathbf{w}))}_{\mathbf{y}_{diff}} \odot \mathbf{X} \right)_{cols}$$

What happens when S > 1 ?

One Layer Architectures of Many Classes



$$J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2$$

$$J(\mathbf{W}) = \left\| \mathbf{Y} - \phi\,(\mathbf{W} \cdot \mathbf{X}) \right\|^2$$

$$J(\mathbf{w}_{row=1}) = \sum_i (y_1^{(i)} - \phi(\mathbf{x}^{(i)} \cdot \mathbf{w}_{row=1}))^2$$

$$\cdots$$

$$J(\mathbf{w}_{row=C}) = \sum_i (y_C^{(i)} - \phi(\mathbf{x}^{(i)} \cdot \mathbf{w}_{row=C}))^2$$

$$\mathbf{Y} = \left[\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(1)} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(2)} \cdots \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \end{bmatrix}^{(M)} \right] \rightarrow \left[\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^{(1)} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}^{(2)} \cdots \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}^{(M)} \right]$$

Each target class in $\mathbf{Y}$ can be independently optimized

$$\hat{\mathbf{Y}} = \left[\begin{bmatrix} \phi(\mathbf{x}^{(1)} \cdot \mathbf{w}_{row=1}) \\ \phi(\mathbf{x}^{(1)} \cdot \mathbf{w}_{row=2}) \\ \vdots \\ \phi(\mathbf{x}^{(1)} \cdot \mathbf{w}_{row=C}) \end{bmatrix} \begin{bmatrix} \phi(\mathbf{x}^{(2)} \cdot \mathbf{w}_{row=1}) \\ \phi(\mathbf{x}^{(2)} \cdot \mathbf{w}_{row=2}) \\ \vdots \\ \phi(\mathbf{x}^{(2)} \cdot \mathbf{w}_{row=C}) \end{bmatrix} \cdots \begin{bmatrix} \phi(\mathbf{x}^{(M)} \cdot \mathbf{w}_{row=1}) \\ \phi(\mathbf{x}^{(M)} \cdot \mathbf{w}_{row=2}) \\ \vdots \\ \phi(\mathbf{x}^{(M)} \cdot \mathbf{w}_{row=C}) \end{bmatrix} \right]$$

which is one versus-all!

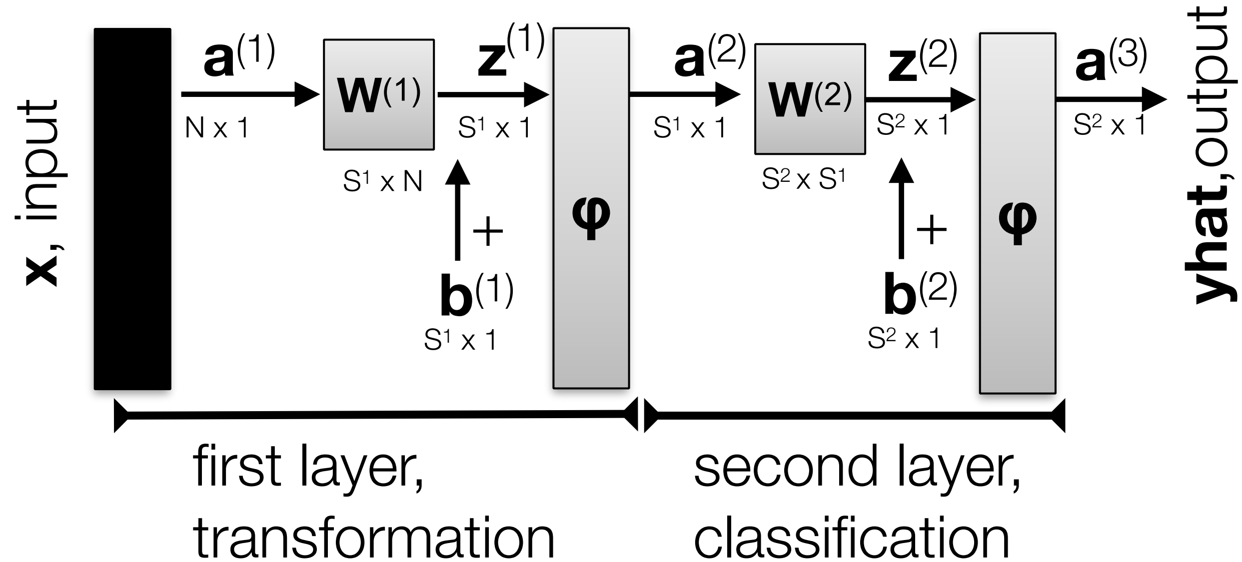
Early Architectures: Summary

- Adaline network, Widrow and Hoff, 1960
  - linear regression, iterative updates
- Perceptron
  - *with sigmoid*: logistic regression
- One-versus-all implementation is the same as having $\mathbf{w}_{class}$ be rows of weight matrix, $\mathbf{W}$

- **But what about when we have more than one layer?**

Moving to multiple layers…

- The multi-layer perceptron (MLP):
  - two layers shown, but could be arbitrarily many layers



each element of **yhat** is no longer independent of the rows in $\mathbf{W}^{(1)}$ so we cannot optimize using one versus all 😢

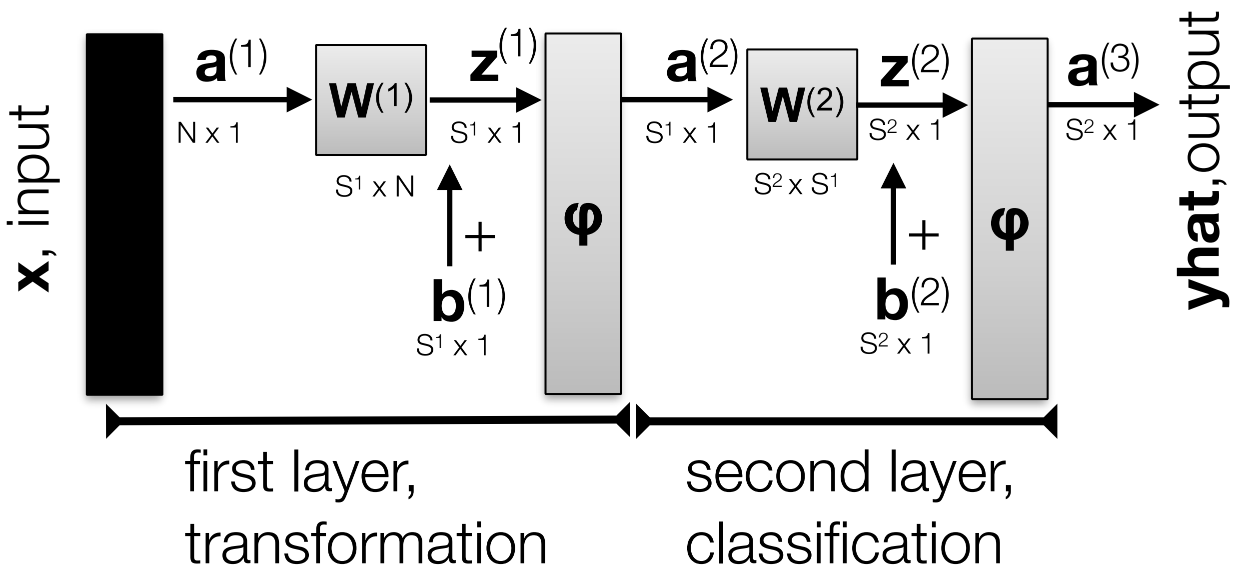
$$\begin{bmatrix}\hat{y}_1\\ \hat{y}_2\\ \vdots\\ \hat{y}_C\end{bmatrix} = \begin{bmatrix}\phi(\phi(\mathbf{z}^{(1)})\cdot\mathbf{w}^{(2)}_{row=1})\\ \phi(\phi(\mathbf{z}^{(1)})\cdot\mathbf{w}^{(2)}_{row=2})\\ \vdots\\ \phi(\phi(\mathbf{z}^{(1)})\cdot\mathbf{w}^{(2)}_{row=C})\end{bmatrix}$$

$$\boxed{\mathbf{z}^{(1)} = \mathbf{W}^{(1)}\cdot\mathbf{a}^{(1)}}$$

Back propagation

- Optimize all weights of network at once
- Steps:
  1. Forward propagate to get all $\mathbf{Z}^{(l)}$, $\mathbf{A}^{(l)}$
  2. Get final layer gradient
  3. Back propagate sensitivities
  4. Update each $\mathbf{W}^{(l)}$

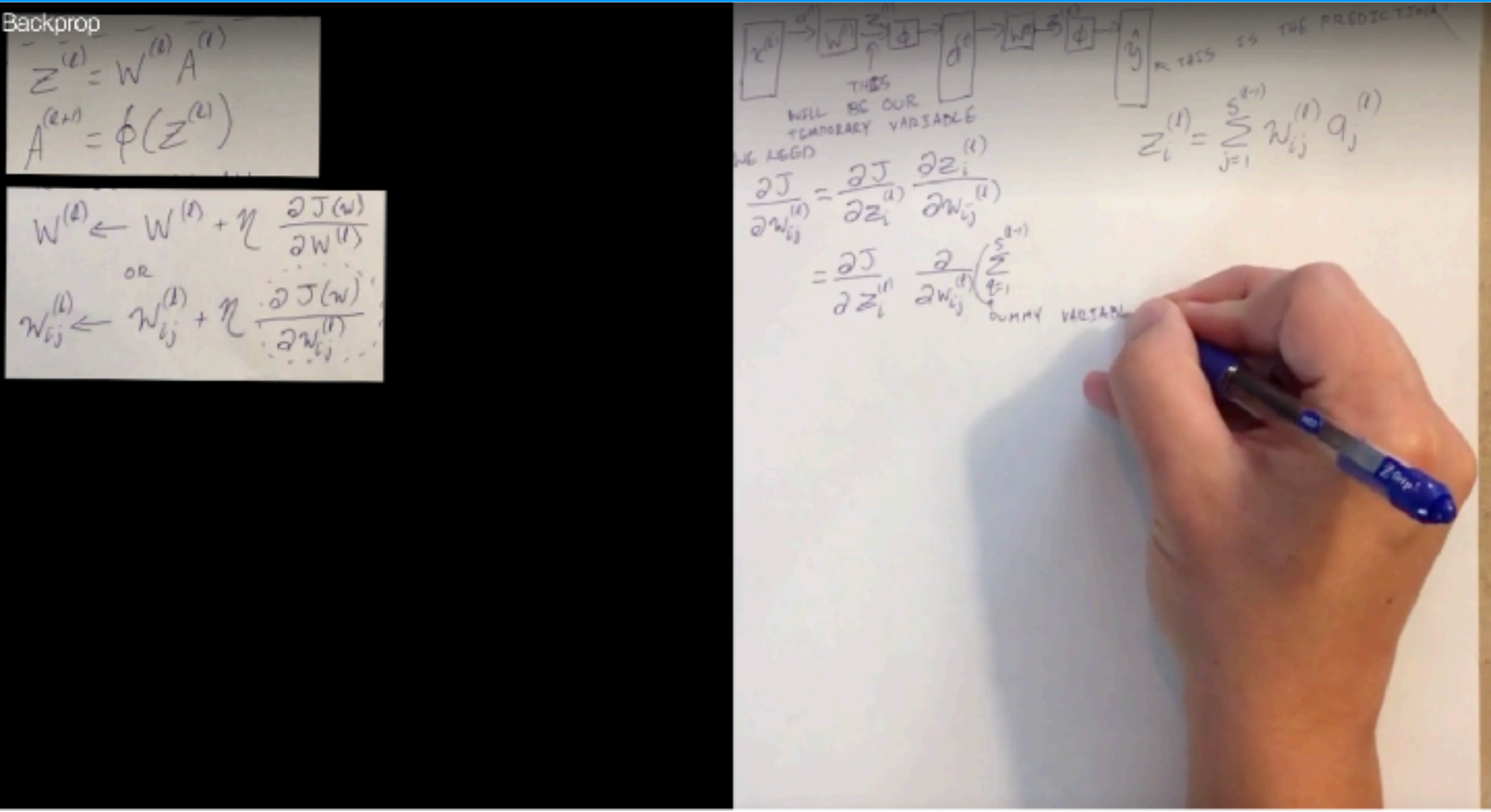
Back-propagation is solved in flipped assignment!!



$$J(\mathbf{W}) = \left\| \mathbf{Y} - \hat{\mathbf{Y}} \right\|^2$$

$$w_{i,j}^{(l)} \leftarrow w_{i,j}^{(l)} - \eta \frac{\partial J(\mathbf{W})}{\partial w_{i,j}^{(l)}}$$

Back propagation Preview

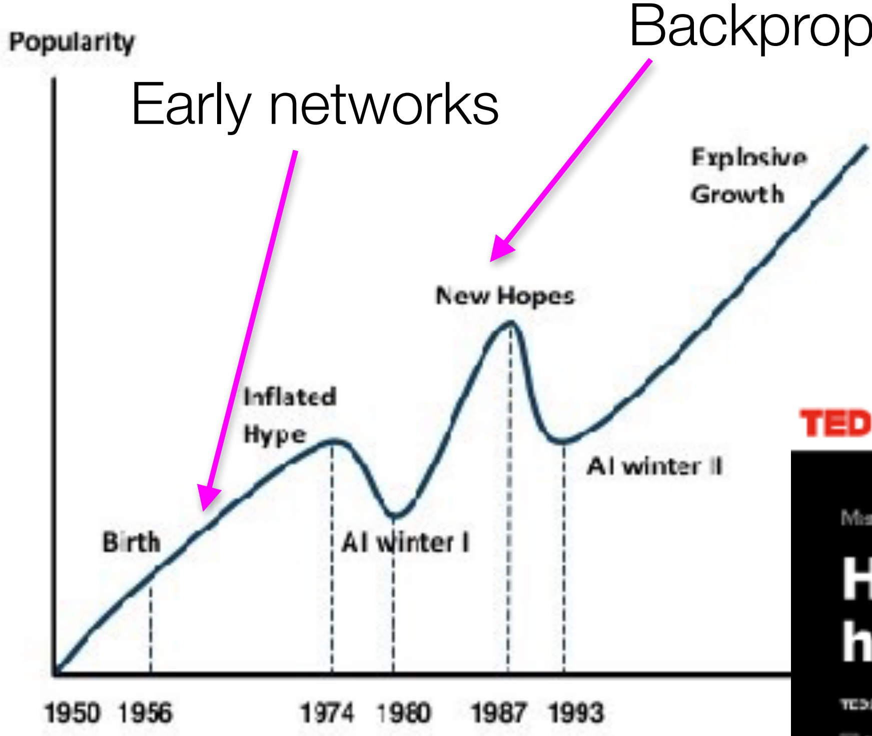


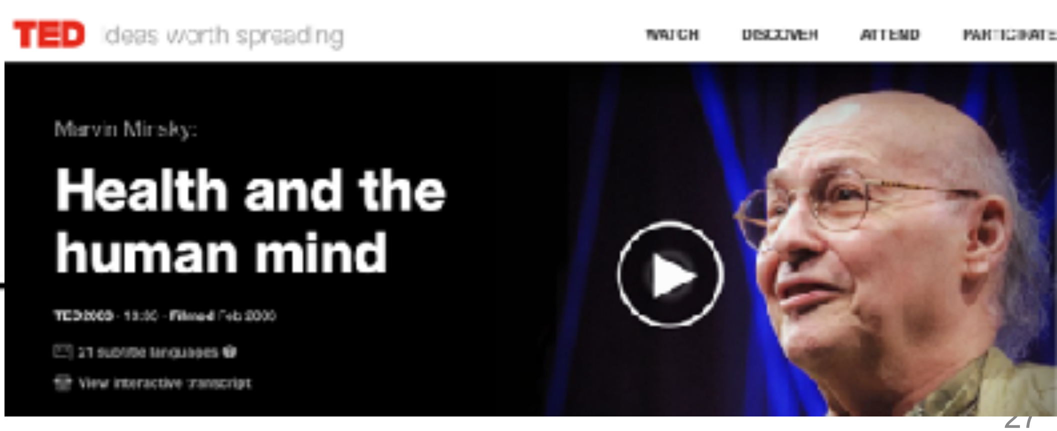
This is solved in explainer video for next flipped assignment!

The First AI Winter (if time)

Backpropagation

Early networks





The Rosenblatt-Widrow-Hoff Dilemma

- 1960's: Rosenblatt got into a public academic argument with Marvin Minsky and Seymour Papert

  "Given an elementary α-perceptron, a stimulus world W, and any classification C(W) for which a solution exists; let all stimuli in W occur in any sequence, provided that each stimulus must reoccur in finite time; then beginning from an arbitrary initial state, an error correction procedure will always yield a solution to C(W) in finite time…"

- Minsky and Papert publish limitations paper, 1969:

  "the style of research being done on the perceptron is doomed to failure because of these limitations."

- Widrow and Rosenblatt try to build bigger networks without limitations and fail
  - Neural Networks research **basically stops** for **17 years**
- **Until**: researchers revisit training bigger networks
  - Neural Networks with multiple layers

Stable Training of Multi-layer Architectures: history

- 1986: *Rumelhart*, *Hinton*, and *Williams* popularize gradient calculation for multi-layer network
  - *technically* introduced by Werbos in 1982
- **difference**: Rumelhart *et al.* validated ideas with a computer
- until this point no one could train a multiple layer network consistently
- algorithm is popularly called **Back-Propagation**
- wins pattern recognition prize in 1993, becomes de-facto machine learning algorithm until: SVMs and Random Forests in ~2004
- would eventually see a resurgence for its ability to train algorithms for Deep Learning applications: **Hinton is widely considered the founder of deep learning**

David Rumelhart

Geoffrey Hinton

End of Session

- thanks! **Next time is Flipped Assignment!!!**

More help on neural networks to prepare for next time:

Dr. Sebastian Raschka

https://github.com/rasbt/python-machine-learning-book/blob/master/code/ch12/ch12.ipynb

Dr. Martin Hagan

https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwioprvn27fPAhWMx4MKHYbwDlwQFggeMAA&url=http%3A%2F%2Fhagan.okstate.edu%2FNNDesign.pdf&usg=AFQjCNG5YbM4xSMm6K5HNsG-4Q8TvOu_Lw&sig2=bgT3k-5ZDDTPZ07Qu8Oreg

Dr. Michael Nielsen

http://neuralnetworksanddeeplearning.com