



TRIGONOMETRY

Chapter 17

3rd
SECONDARY

REDUCCIÓN AL PRIMER
CUADRANTE II



 **SACO OLIVEROS**



¿ CÓMO REPRESENTAMOS EL SENO Y EL COSENO DE UN ÁNGULO NEGATIVO ?

Trigonometría					
α	0°	30°	45°	60°	90°
$\text{sen } \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\text{cos } \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\text{tg } \alpha$	0	$\frac{\sqrt{3}}{2}$	1	$\sqrt{3}$	∞

Producido por *Kharla Mérida*

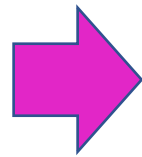


REDUCCIÓN AL PRIMER CUADRANTE II

3er CASO : PARA ÁNGULOS MAYORES A UNA VUELTA

Si a un ángulo positivo α mayor de una vuelta, se le elimina de su medida el número entero de vueltas que contiene, entonces los valores de sus razones trigonométricas no varían, es decir :

$$\begin{array}{c} \alpha \\ (\theta) \end{array} \left| \frac{360^\circ}{n} \right.$$



$$RT(\alpha) = RT(360^\circ n + \theta) = RT(\theta)$$

$$n \in \mathbb{Z}^+$$

$$0^\circ < \theta < 360^\circ$$

Nota : n indica el número entero positivo de vueltas contenidas en el ángulo, que podemos eliminar.



Ejemplo :

$$\tan 765^\circ = \tan(\cancel{360^\circ \cdot 2} + 45^\circ) = \tan 45^\circ = 1$$

$$\begin{array}{r|l} 765^\circ & 360^\circ \\ \hline 720^\circ & 2 \\ (45^\circ) & \end{array}$$

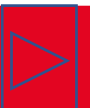
4to CASO : PARA ARCOS NUMÉRICOS CON FACTOR π

A) Para arcos fraccionarios de la forma $\frac{a\pi}{b}$; donde $a > 2b$

$$\begin{array}{r|l} a & 2b \\ (r) & q \end{array} \quad \rightarrow \quad \boxed{\text{RT} \left(\frac{a\pi}{b} \right) = \text{RT} \left(\frac{r\pi}{b} \right)}$$

$$\begin{array}{r|l} 33 & 8 \\ \hline 32 & 4 \\ (1) & \end{array}$$

Ejemplo : $\csc \left(\frac{33\pi}{4} \right) = \csc \left(\frac{1\pi}{4} \right) = \sqrt{2}$





4to CASO : PARA ARCOS NUMÉRICOS CON FACTOR π

B) Para arcos enteros de la forma $n\pi$; donde $n \in \mathbb{Z}$

$$\text{RT (par. } \pi \pm \theta) = \text{RT}(\pm \theta)$$

$$\text{RT (impar. } \pi \pm \theta) = \text{RT}(\pi \pm \theta)$$

Ejemplos :

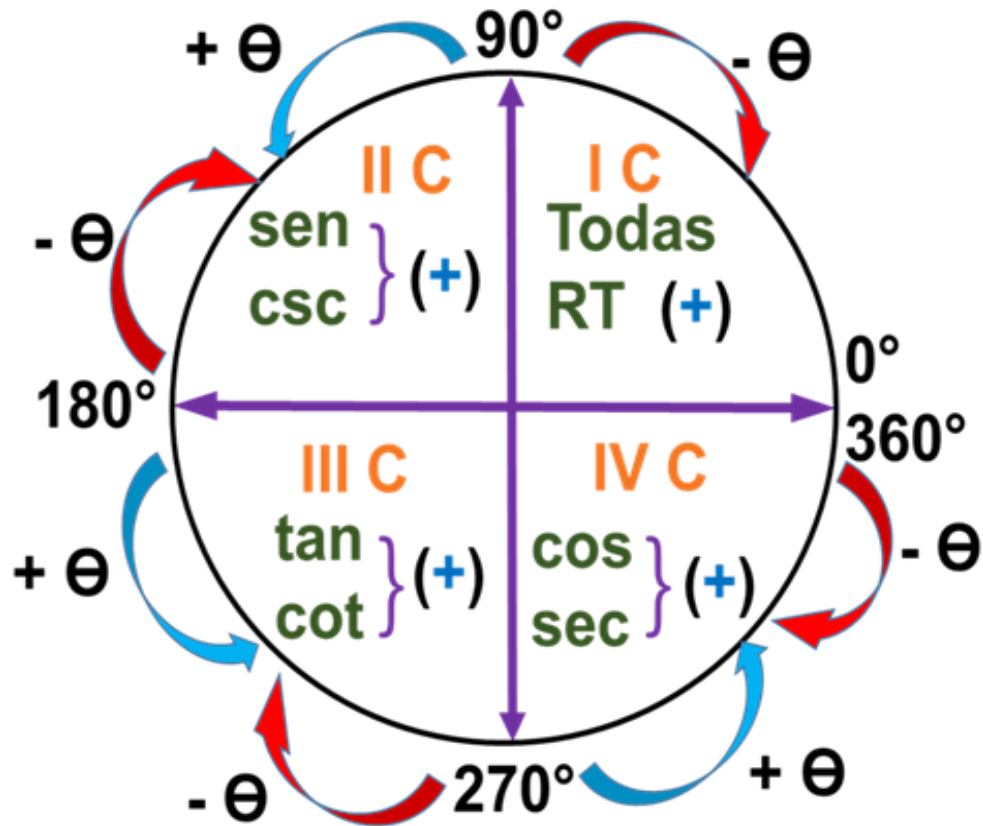


$$\cot(\underbrace{6\pi}_{\text{par}} - \frac{\pi}{3}) = \cot(-\frac{\pi}{3}) = -\cot\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$$

$$\text{sen}(\underbrace{9\pi}_{\text{impar}} - \frac{\pi}{6}) = \text{sen}(\underbrace{\pi - \frac{\pi}{6}}_{\text{II C}}) = \text{sen}\frac{\pi}{6} = \frac{1}{2}$$



DEBEMOS RECORDAR



$$\text{RT} \left\{ \begin{array}{l} 180^\circ \pm \theta \\ 360^\circ - \theta \end{array} \right\} = \pm \text{RT}(\theta)$$

$$\text{RT} \left\{ \begin{array}{l} 90^\circ \pm \theta \\ 270^\circ \pm \theta \end{array} \right\} = \pm \text{Co_RT}(\theta)$$

$$\begin{aligned} \cos(-\alpha) &= \cos\alpha \\ \sec(-\alpha) &= \sec\alpha \end{aligned}$$

$$\begin{aligned} \sin(-\alpha) &= -\sin\alpha \\ \tan(-\alpha) &= -\tan\alpha \\ \cot(-\alpha) &= -\cot\alpha \\ \csc(-\alpha) &= -\csc\alpha \end{aligned}$$



1) Calcule $\cos 1110^\circ$

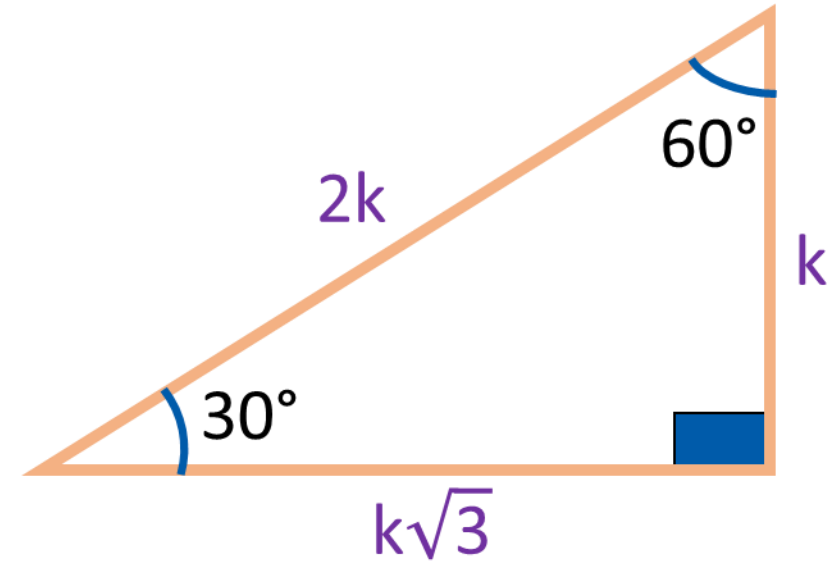
RESOLUCIÓN

$$\frac{1110^\circ}{1080^\circ} \Bigg| \frac{360^\circ}{3}$$

(30°)



$$\cos \theta = \frac{CA}{H}$$



Piden : $\cos 1110^\circ = \cos 30^\circ$

$$\therefore \cos 1110^\circ = \frac{\sqrt{3}}{2}$$

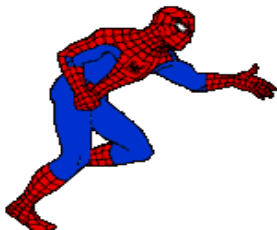




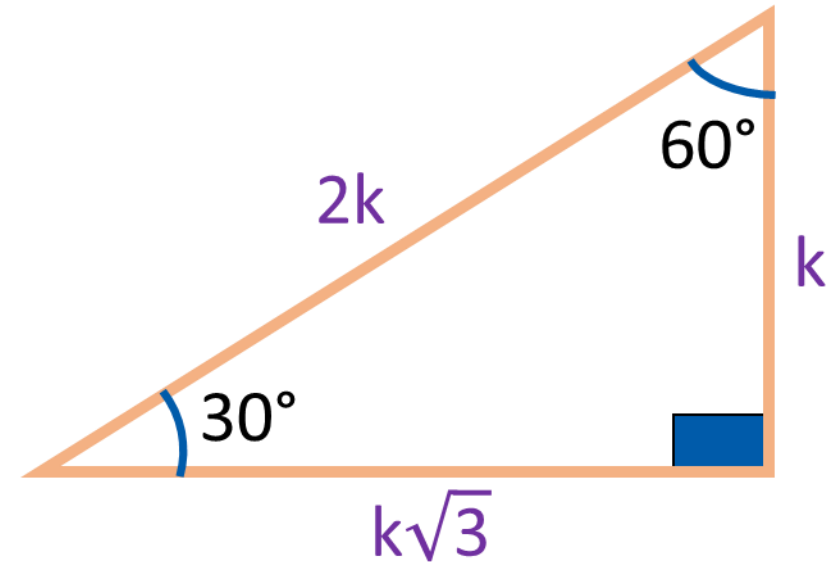
2) Calcule $\text{sen}4020^\circ$

RESOLUCIÓN

$$\begin{array}{r|l} 4020^\circ & 360^\circ \\ 3960^\circ & 11 \\ \hline & (60^\circ) \end{array}$$



$$\text{sen}\theta = \frac{\text{CO}}{\text{H}}$$



Piden : $\text{sen}4020^\circ = \text{sen}60^\circ$

$$\therefore \text{sen}4020^\circ = \frac{\sqrt{3}}{2}$$



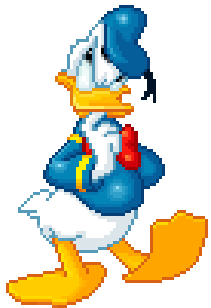
3) Reduzca

$$E = \cos 780^\circ \cdot \sec 1485^\circ$$

RESOLUCIÓN

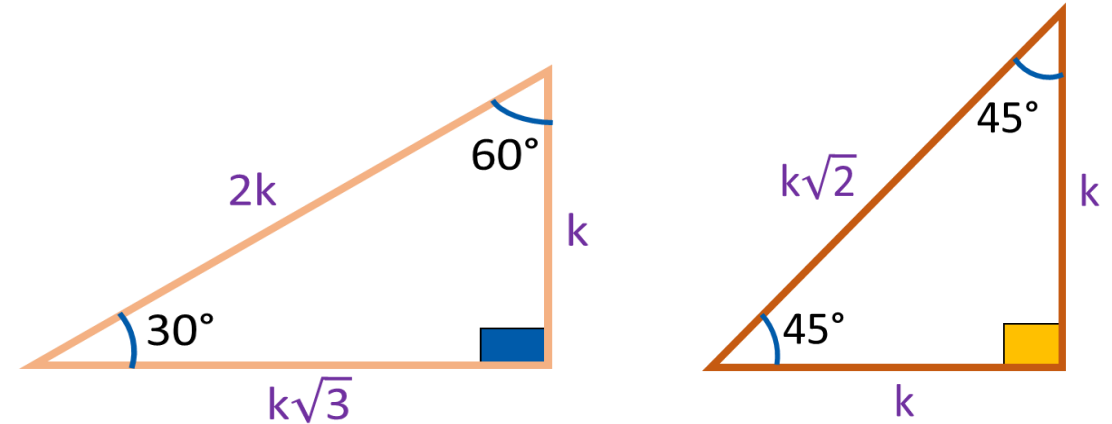
$$\frac{780^\circ}{720^\circ} \bigg| \frac{360^\circ}{2} \quad \frac{1485^\circ}{1440^\circ} \bigg| \frac{360^\circ}{4}$$

(60°) (45°)



$$\cos \theta = \frac{CA}{H}$$

$$\sec \theta = \frac{H}{CA}$$



Piden : $E = \cos 780^\circ \cdot \sec 1485^\circ$

$$E = \cos 60^\circ \cdot \sec 45^\circ$$

$$E = \left(\frac{1}{2} \right) (\sqrt{2})$$

$$\therefore E = \frac{\sqrt{2}}{2}$$



4) Reduzca

$$A = \text{sen}(24\pi + x)$$

$$B = \text{tan}(12\pi - x)$$

RESOLUCIÓN

Recordemos que :

$$\text{RT}(\text{par. } \pi \pm \theta) = \text{RT}(\pm \theta)$$

$$\text{RT}(\text{impar. } \pi \pm \theta) = \text{RT}(\pi \pm \theta)$$

$$\tan(-\alpha) = -\tan\alpha$$

Luego :

$$A = \text{sen}(24\pi + x)$$

par

$$\therefore A = \text{sen}x$$

$$B = \text{tan}(12\pi - x)$$

par

$$B = \text{tan}(-x)$$

$$\therefore B = -\text{tan}x$$



5) Reduzca

$$a) \operatorname{sen}\left(\frac{13\pi}{2} + x\right)$$

$$b) \tan\left(\frac{23\pi}{2} + x\right)$$

RESOLUCIÓN

Recordemos que :

$$\operatorname{RT}\left(\frac{a\pi}{b}\right) = \operatorname{RT}\left(\frac{r\pi}{b}\right)$$

$$\begin{array}{l} a \\ (r) \end{array} \left| \frac{2b}{q} \right.$$

$$\operatorname{RT}\left\{\begin{array}{l} 90^\circ \pm \theta \\ 270^\circ \pm \theta \end{array}\right\} = \pm \operatorname{Co_RT}(\theta)$$

Luego :

$$a) \operatorname{sen}\left(\frac{13\pi}{2} + x\right) = \operatorname{sen}\left(\frac{1\pi}{2} + x\right) \quad \text{II C}$$

$$\begin{array}{l} 13 \\ 12 \\ (1) \end{array} \left| \frac{4}{3} \right.$$

$$= \cos x$$

$$b) \tan\left(\frac{23\pi}{2} + x\right) = \tan\left(\frac{3\pi}{2} + x\right) \quad \text{IV C}$$

$$\begin{array}{l} 23 \\ 20 \\ (3) \end{array} \left| \frac{4}{5} \right.$$

$$= -\cot x$$



6) Reduzca

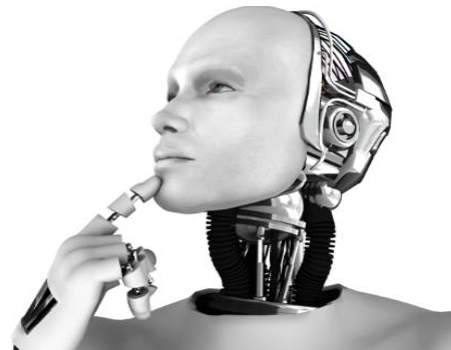
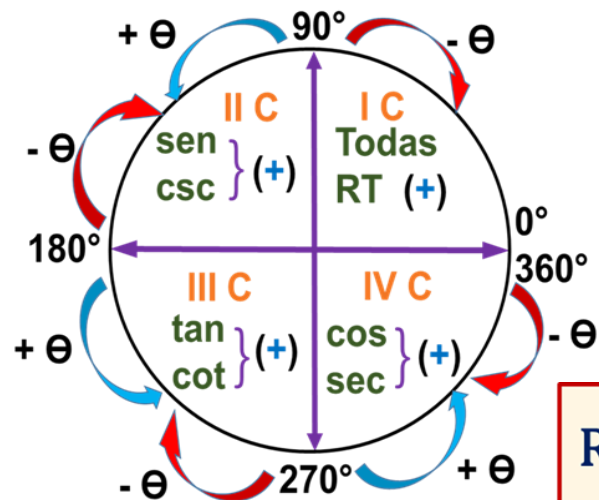
$$E = \cos(35\pi + x) \cdot \sec(23\pi + x)$$

RESOLUCIÓN

Recordemos que :

$$RT(\text{par. } \pi \pm \theta) = RT(\pm \theta)$$

$$RT(\text{impar. } \pi \pm \theta) = RT(\pi \pm \theta)$$



$$RT\left\{\begin{matrix} 180^\circ \pm \theta \\ 360^\circ - \theta \end{matrix}\right\} = \pm RT(\theta)$$

Luego :

$$E = \underbrace{\cos(35\pi + x)}_{\text{impar}} \cdot \underbrace{\sec(23\pi + x)}_{\text{impar}}$$

$$E = \underbrace{\cos(\pi + x)}_{\text{III C}} \cdot \underbrace{\sec(\pi + x)}_{\text{III C}}$$

$$E = (-\cos x) (-\sec x)$$

$$E = \cos x \cdot \sec x$$

$$\therefore E = 1$$



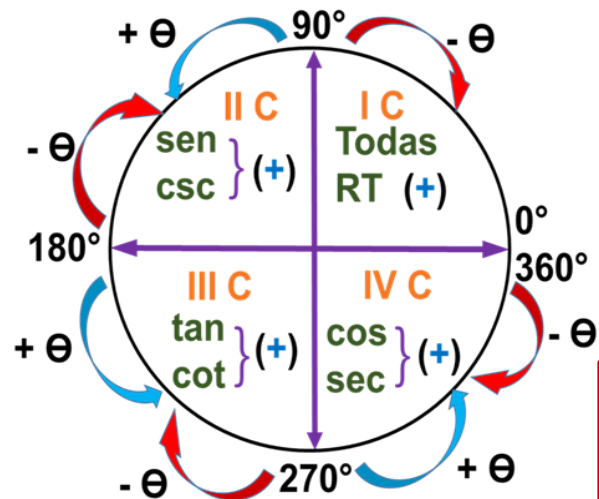
7) Reduzca

$$T = \tan\left(\frac{25\pi}{2} + x\right) - 3 \tan\left(\frac{17\pi}{2} + x\right)$$

RESOLUCIÓN

Recordemos que :

$$RT\left(\frac{a\pi}{b}\right) = RT\left(\frac{r\pi}{b}\right) \quad a \left| \frac{2b}{q} \right. \quad (r)$$



$$RT\left\{\begin{matrix} 90^\circ \pm \theta \\ 270^\circ \pm \theta \end{matrix}\right\} = \pm \text{Co_RT}(\theta)$$

Luego :

$$\frac{25}{24} \left| \frac{4}{6} \right. \quad (1)$$

$$\frac{17}{16} \left| \frac{4}{4} \right. \quad (1)$$

$$T = \tan\left(\frac{1\pi}{2} + x\right) - 3 \tan\left(\frac{1\pi}{2} + x\right)$$

$$T = -2 \tan\left(\frac{\pi}{2} + x\right)$$

II C

$$T = -2 (-\cot x)$$

$$\therefore T = 2 \cot x$$



8) El gasto diario de Jhon en pasajes es de S/. A ¿Cuál será el gasto total a la semana ?.- Para calcular dicho valor deberás reducir lo siguiente :

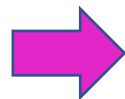
$$A = \frac{\text{sen}(42\pi + x)}{\text{sen}(31\pi + x)} + \frac{\tan(\frac{21\pi}{2} - x)}{\tan(\frac{39\pi}{2} + x)} + 3$$

RESOLUCIÓN

$$A = \frac{\overbrace{\text{sen}(42\pi + x)}^{\text{par}}}{\underbrace{\text{sen}(31\pi + x)}_{\text{impar}}} + \frac{\tan(\frac{21\pi}{2} - x)}{\tan(\frac{39\pi}{2} + x)} + 3 = \frac{\text{sen}x}{\underbrace{\text{sen}(\pi + x)}_{\text{III C}}} + \frac{\overbrace{\tan(\frac{1\pi}{2} - x)}^{\text{I C}}}{\underbrace{\tan(\frac{3\pi}{2} + x)}_{\text{IV C}}} + 3$$

$$A = \frac{\text{sen}x}{-\text{sen}x} + \frac{\text{cot}x}{-\text{cot}x} + 3 = -1 - 1 + 3 = 1$$

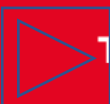
Piden : $7A = 7(1) = 7$



∴ Gasto semanal = S/. 7

$$\begin{array}{r} 21 \overline{) 4} \\ 20 \\ \hline (1) \end{array}$$

$$\begin{array}{r} 39 \overline{) 4} \\ 36 \\ \hline (3) \end{array}$$



COLEGIOS

 **SACO OLIVEROS**  **APEIRON**
SISTEMA HELICOIDAL

**MUCHAS GRACIAS POR
TU ATENCIÓN**

Tu curso amigo
TRIGONOMETRÍA