



TRIGONOMETRY

5th
SECONDARY

ADVISORY



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1. Simplifique la expresión: $E = \frac{\text{sen}(x + y)}{\cos x \cos y} - \tan y$

RESOLUCIÓN

$$\text{sen}(x + y) = \text{sen} x \cdot \cos y + \cos x \cdot \text{sen} y$$

$$E = \frac{\text{sen}(x + y)}{\cos x \cos y} - \tan y$$

$$E = \frac{\text{sen} x \cos y + \cos x \text{sen} y}{\cos x \cos y} - \tan y$$

$$E = \frac{\cancel{\text{sen} x \cos y}}{\cancel{\cos x} \cos y} + \frac{\cos x \cancel{\text{sen} y}}{\cancel{\cos x} \cos y} - \tan y$$

$$E = \frac{\text{sen} x}{\cos x} + \frac{\text{sen} y}{\cos y} - \tan y$$

$$E = \tan x + \cancel{\tan y} - \cancel{\tan y}$$

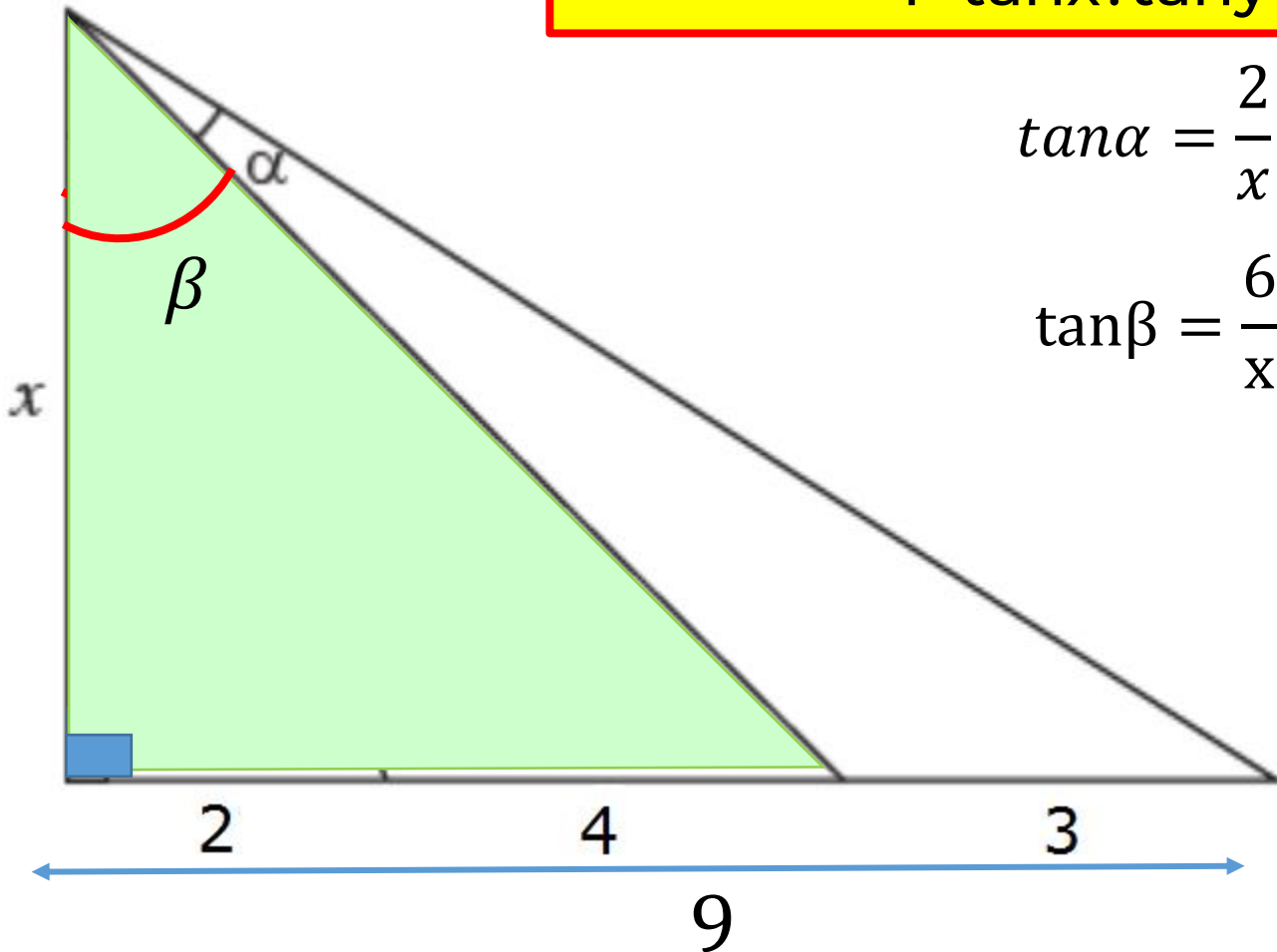
$$\therefore E = \tan x$$



2. Del gráfico, halle el valor de x .

RESOLUCIÓN

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$



$$\tan \alpha = \frac{2}{x}$$

$$\tan \beta = \frac{6}{x}$$

Se observa que: $\tan(\alpha + \beta) = \frac{9}{x}$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta} = \frac{9}{x}$$

$$\frac{\frac{2}{x} + \frac{6}{x}}{1 - \left(\frac{2}{x}\right) \cdot \left(\frac{6}{x}\right)} = \frac{9}{x}$$

$$\frac{\frac{8}{x}}{1 - \frac{12}{x^2}} = \frac{9}{x}$$

$$8 = 9 \left(1 - \frac{12}{x^2}\right)$$

$$8 = 9 - \frac{9(12)}{x^2}$$

$$\frac{9(12)}{x^2} = 1$$

$$x^2 = 36 * 3$$

$$\therefore x = 6\sqrt{3}$$



3. Efectúe: $M = (1 + \tan 19^\circ)(1 + \tan 26^\circ)$

RESOLUCIÓN

$$\tan x + \tan y + \tan(x + y) \cdot \tan x \cdot \tan y = \tan(x + y)$$

$$M = (1 + \tan 19^\circ)(1 + \tan 26^\circ)$$

$$M = 1 + \tan 26^\circ + \tan 19^\circ + 1 \cdot \tan 19^\circ \cdot \tan 26^\circ$$

$$M = 1 + (\tan 26^\circ + \tan 19^\circ + \tan 45^\circ \cdot \tan 19^\circ \cdot \tan 26^\circ)$$

$$M = 1 + (\tan(26^\circ + 19^\circ))$$

$$M = 1 + \tan 45^\circ = 1 + 1$$

$$\therefore M = 2$$



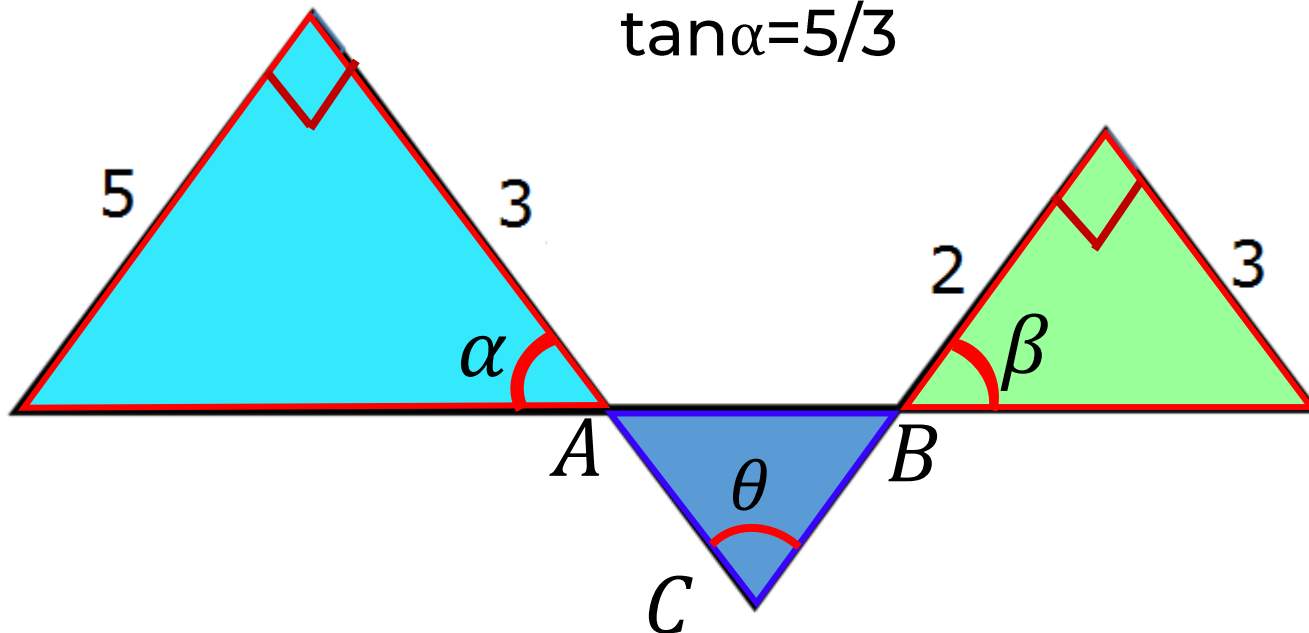


4. Del gráfico, halle el valor de $\tan\theta$.

RESOLUCIÓN

$$\tan\beta = 3/2$$

$$\tan\alpha = 5/3$$



Si $x + y + z = 180^\circ$, se cumple:

$$\tan x + \tan y + \tan z = \tan x \cdot \tan y \cdot \tan z$$

▲ ABC: $\alpha + \beta + \theta = 180^\circ$

$$\tan\alpha + \tan\beta + \tan\theta = \tan\alpha \tan\beta \tan\theta$$

$$\frac{5}{3} + \frac{3}{2} + \tan\theta = \frac{5}{3} \cdot \frac{3}{2} \cdot \tan\theta$$

$$\frac{19}{6} + \tan\theta = \frac{5}{2} \cdot \tan\theta$$

$$\frac{19}{6} = \frac{3}{2} \cdot \tan\theta$$

$$\therefore \tan\theta = 19/9$$





5. Sabiendo que $\pi/24$; determine el valor de :
 $E = \text{sen}x\cos^3x - \cos x\text{sen}^3x$

RESOLUCIÓN

$$E = \text{sen}x\cos^3x - \cos x\text{sen}^3x$$

$$E = \text{sen}x\cos x(\underbrace{\cos^2x - \text{sen}^2x}_{\cos 2x})$$

$$2E = \underbrace{2\text{sen}x\cos x}_{\text{sen}2x}\cos 2x$$

$$2E = \text{sen}2x\cos 2x$$

$$4E = \underbrace{2\text{sen}2x\cos 2x}_{\text{sen}4x}$$

$$E = \frac{1}{4}\text{sen}(30^\circ)$$

$$E = \frac{1}{4}\left(\frac{1}{2}\right)$$

$$4x = \frac{\pi}{6}$$

$$4x = 30^\circ$$

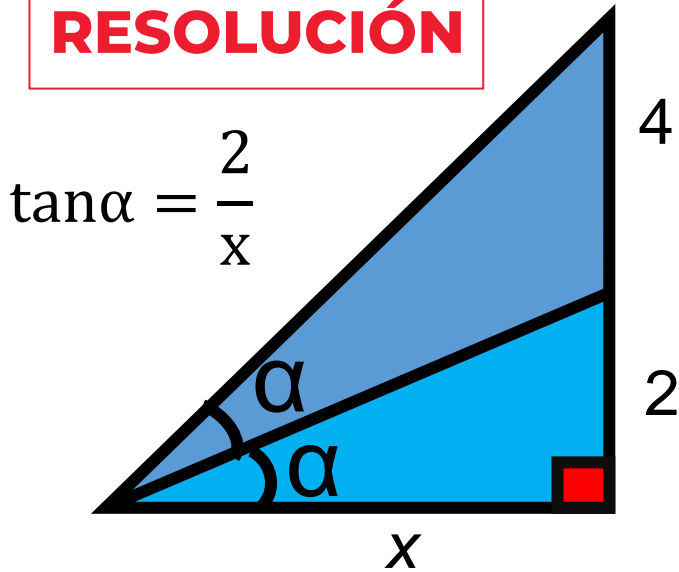
$$\therefore E = 1/8$$





6. Del gráfico, calcule el valor de x .

RESOLUCIÓN



$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\tan 2\alpha = \frac{6}{x}$$

$$\frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \frac{6}{x}$$

$$\frac{2 \left(\frac{2}{x} \right)}{1 - \left(\frac{2}{x} \right)^2} = \frac{6}{x}$$

$$\frac{\frac{4}{x}}{1 - \frac{4}{x^2}} = \frac{6}{x}$$

$$2 = 3 \left(1 - \frac{4}{x^2} \right)$$

$$2 = 3 - \frac{12}{x^2}$$

$$\frac{12}{x^2} = 1 \quad \Rightarrow \quad x^2 = 12$$

$$\therefore x = 2\sqrt{3}$$





7. Si : $\alpha + \beta = 45^\circ$; halle: $E = (1 - \cot\alpha)(1 - \cot\beta)$

RESOLUCIÓN

$$E = (1 - \cot\alpha)(1 - \cot\beta)$$

$$E = \left(1 - \frac{1}{\tan\alpha}\right)\left(1 - \frac{1}{\tan\beta}\right)$$

$$E = \left(\frac{\tan\alpha - 1}{\tan\alpha}\right)\left(\frac{\tan\beta - 1}{\tan\beta}\right)$$

$$E = \frac{\tan\alpha \cdot \tan\beta - \tan\alpha - \tan\beta + 1}{\tan\alpha \cdot \tan\beta}$$

$$E = \frac{\tan\alpha \cdot \tan\beta + 1 - \tan\alpha - \tan\beta}{\tan\alpha \cdot \tan\beta}$$

Del dato: $\tan(\alpha + \beta) = \tan 45^\circ$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = 1$$

$$\tan\alpha + \tan\beta = 1 - \tan\alpha \cdot \tan\beta$$

$$\tan\alpha \cdot \tan\beta = 1 - \tan\alpha - \tan\beta$$

Reemplazando en E:

$$E = \frac{\tan\alpha \cdot \tan\beta + \tan\alpha \cdot \tan\beta}{\tan\alpha \cdot \tan\beta}$$

$$E = \frac{2\tan\alpha \cdot \tan\beta}{\tan\alpha \cdot \tan\beta}$$

$$\therefore E = 2$$



8. Simplifica la expresión:

$$T = \frac{\text{sen}^3 x + \text{cos}^3 x}{\text{sen} x + \text{cos} x} + 3\text{sen} x \text{cos} x$$

RESOLUCIÓN

$$T = \frac{\text{sen}^3 x + \text{cos}^3 x}{\text{sen} x + \text{cos} x} + 3\text{sen} x \text{cos} x$$

$$T = \frac{(\cancel{\text{sen} x + \text{cos} x})(\text{sen}^2 x - \cancel{\text{sen} x \text{cos} x} + \text{cos}^2 x)}{\cancel{\text{sen} x + \text{cos} x}} + 3\text{sen} x \text{cos} x$$

$$T = \text{sen}^2 x + \text{cos}^2 x - \text{sen} x \text{cos} x + 3\text{sen} x \text{cos} x$$

$$T = 1 + \underbrace{2\text{sen} x \text{cos} x}_{\text{sen} 2x}$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

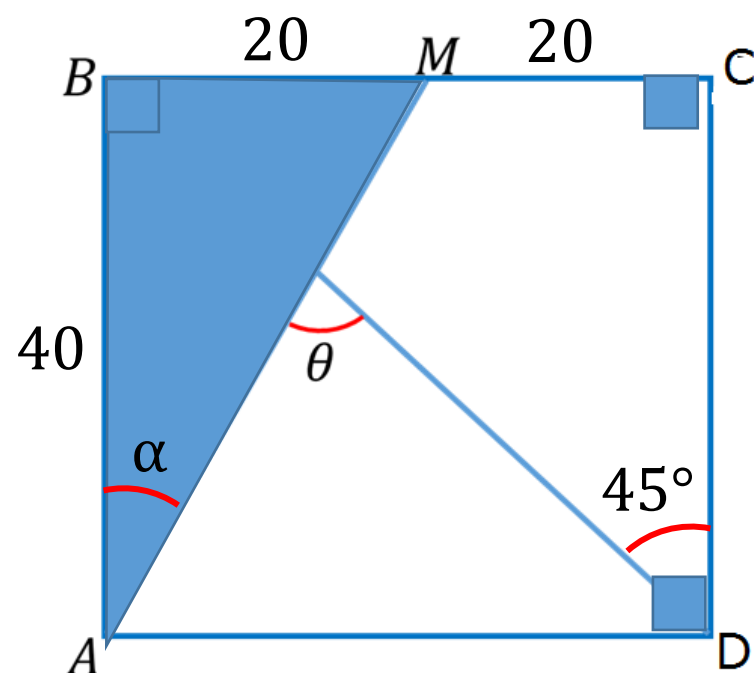
$$\therefore T = 1 + \text{sen} 2x$$



9.

Una baldosa de forma cuadrada ABCD es dividida para que sus partes sean pintadas de diferentes colores; de acuerdo con un cierto diseño. Para dividirla se consideran los trazos BD y AM, siendo M el punto medio de BC. Si AB=40 cm, hallas $\tan\theta$

RESOLUCIÓN



$$\theta = \alpha + 45^\circ$$

$$\tan\theta = \tan(\alpha + 45^\circ)$$

$$\tan\theta = \frac{\tan\alpha + \tan 45^\circ}{1 - \tan\alpha \cdot \tan 45^\circ}$$

$$\tan\alpha = \frac{20}{40} \quad \tan\alpha = \frac{1}{2}$$

$$\tan\theta = \frac{\frac{1}{2} + 1}{1 - \frac{1}{2} \cdot 1} \Rightarrow \tan\theta = \frac{3}{1}$$

$$\therefore \tan\theta = 3$$





10. Si A, B, y C son los ángulos internos de un triángulo y $\text{sen}(A+B)\cos(A+B)=1/2$, calcule $\tan C$

RESOLUCIÓN

Dato:

$$\text{sen}(A + B) \cos(A + B) = \frac{1}{2}$$

$$\underbrace{2\text{sen}(A + B) \cos(A + B)} = 1$$

$$\text{sen}(2A + 2B)$$

$$\Rightarrow \text{sen}(2A + 2B) = 1$$

$$2A + 2B = 90^\circ$$

$$A + B = 45^\circ$$

Además:

$$\underbrace{A + B}_{45^\circ} + C = 180^\circ \longrightarrow C = 135^\circ$$

Piden: $\tan C = \tan 135^\circ$

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$$\tan C = \tan(\overbrace{180^\circ - 45^\circ})$$

$$\tan C = -\tan 45^\circ$$

$$\therefore \tan C = -1$$

