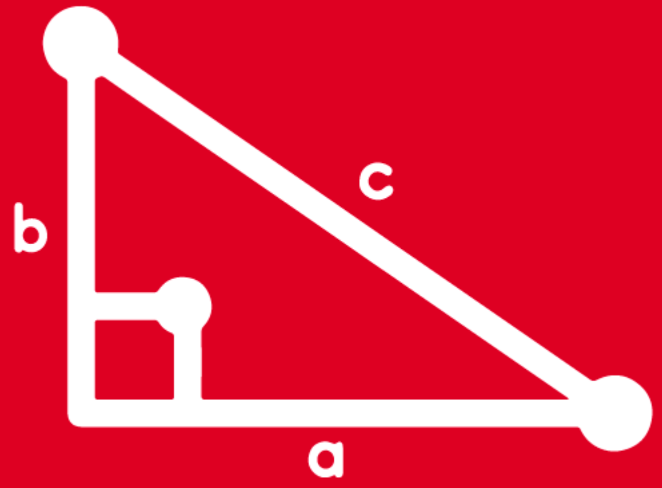
TRIGONOMETRY

Chapter 22



5th
SECONDARY

FUNCIONES TRIGONOMÉTRICAS INVERSAS

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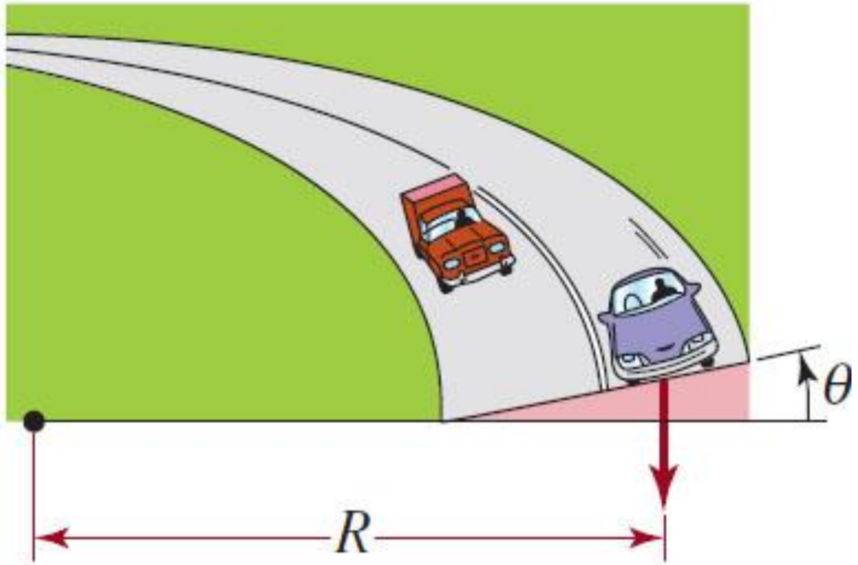
En el diseño de las carreteras y los ferrocarriles, las curvas tienen un peralte (inclinación) para producir una fuerza centrípeta que proporcione seguridad. El ángulo θ optimo, se calcula así:

$$\theta = \arctan\left(\frac{V^2}{R.g}\right)$$

Donde **V** es la velocidad promedio del vehículo, **R** es el radio de la curva y **g** la aceleración de la gravedad.



Pregunta:

Calcule el ángulo θ del peralte, para una velocidad promedio de 20 m/s , radio de la curva de 120 m y g = 10 m/s^2

Rpta: 18º30’

FUNCIONES TRIGONOMÉTRICA INVERSAS

Notación: **Se lee:**

arcsen(x) …… arcoseno de x

arccos(x) …… arcocoseno de x

arctan(x) …… arcotangente de x

arccot(x) …… arcocotangente de x

arcsec(x) …… arcosecante de x

arccsc(x) …… arcocosecante de x

PROPIEDAD FUNDAMENTAL

$$\boxed{FT(\theta) = N \iff \theta = arcFT(N)}$$

Ejemplos:

- $\text{Si: } sen\alpha = \frac{1}{3} \Rightarrow \alpha = arcsen\left(\frac{1}{3}\right)$
- $\text{Si: } \cos\beta = \frac{2}{5} \Rightarrow \beta = \arccos\left(\frac{2}{5}\right)$
- $\text{Si: } \tan\theta = 1 \Rightarrow \theta = \arctan(1) = \frac{\pi}{4}$

Ejemplos:

- $\alpha = \operatorname{arcsen}\left(\frac{3}{4}\right) \Rightarrow \operatorname{sen}\alpha = \frac{3}{4}$
- $\beta = \arctan\left(\frac{\sqrt{2}}{3}\right) \Rightarrow \tan\beta = \frac{\sqrt{2}}{3}$
- $\theta = \operatorname{arcsen}\left(\frac{1}{2}\right) \Rightarrow \operatorname{sen}\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$
- $\phi = \arctan\left(\sqrt{3}\right) \Rightarrow \tan\phi = \sqrt{3} \Rightarrow \phi = \frac{\pi}{3}$

Propiedad 1:

$$\operatorname{arcsen}(x) + \arccos(x) = \frac{\pi}{2} \ ; \ x \in [-1;1]$$

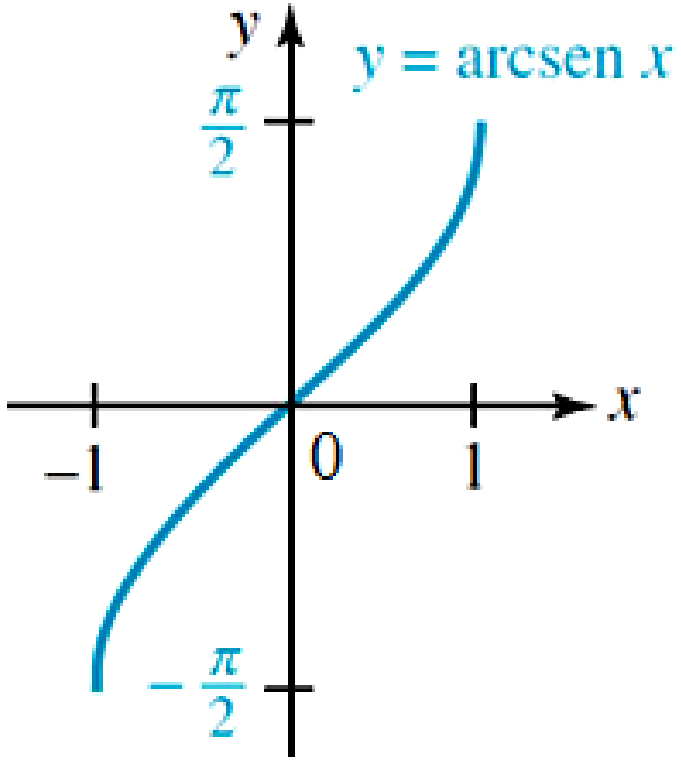
$$\arctan(x) + \operatorname{arccot}(x) = \frac{\pi}{2} \ ; \ x \in R$$

$$\operatorname{arcsec}(x) + \operatorname{arccsc}(x) = \frac{\pi}{2} \ ; \ x \in R - \langle -1;1 \rangle$$

Ejemplos:

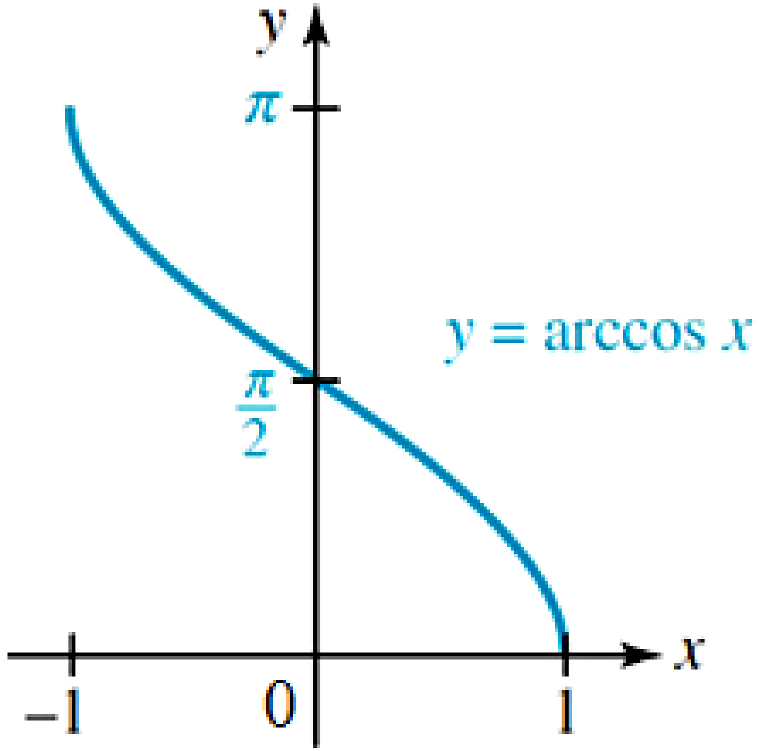
- $\operatorname{arcsen}\left(\frac{1}{3}\right) + \arccos\left(\frac{1}{3}\right) = \frac{\pi}{2}$
- $\arctan(4) + \operatorname{arccot}(4) = \frac{\pi}{2}$

1. Función Arcoseno:



- **Dominio:** $-1 \leq x \leq 1$
- **Rango:** $-\dfrac{\pi}{2} \leq \operatorname{arcsen}(x) \leq \dfrac{\pi}{2}$

2. Función Arcocoseno:



- **Dominio:** $-1 \leq x \leq 1$
- **Rango:** $0 \leq \arccos(x) \leq \pi$

1. Escriba verdadero (V) o falso (F) según corresponda

a. $\arcsin\left(\dfrac{\sqrt{2}}{2}\right)=\dfrac{\pi}{4}$ ....( ) b. $\arcsin\left(\dfrac{1}{2}\right)=\dfrac{\pi}{6}$ ....( )

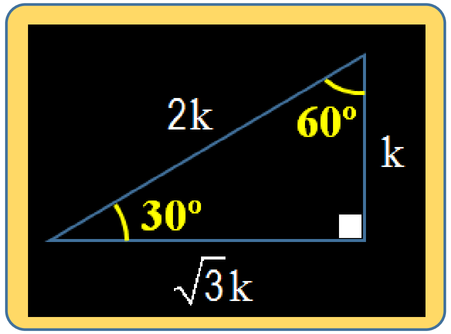
c. $\arctan\left(\sqrt{3}\right)=\dfrac{\pi}{3}$ ....( )

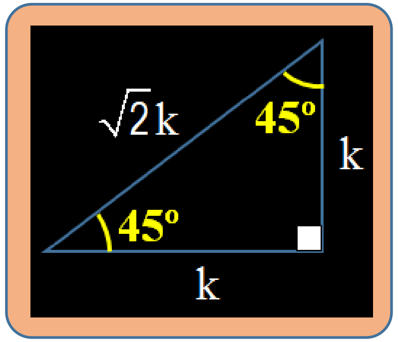
Resolución:

a. $\arcsin\left(\dfrac{\sqrt{2}}{2}\right)=\dfrac{\pi}{4} \Leftrightarrow \text{sen}\left(\dfrac{\pi}{4}\right)=\dfrac{\sqrt{2}}{2}$ ....(V)

b. $\arcsin\left(\dfrac{1}{2}\right)=\dfrac{\pi}{6} \Leftrightarrow \text{sen}\left(\dfrac{\pi}{6}\right)=\dfrac{1}{2}$ ....(V)

c. $\arctan\left(\sqrt{3}\right)=\dfrac{\pi}{3} \Leftrightarrow \tan\left(\dfrac{\pi}{3}\right)=\sqrt{3}$ ....(V)





2. Halle el valor de: $M = \arctan(1) + \operatorname{arcsen}\left(\frac{1}{2}\right)$

Resolución:

$$M = \underbrace{\arctan(1)}_{\alpha} + \underbrace{\operatorname{arcsen}\left(\frac{1}{2}\right)}_{\theta}$$

- $\alpha = \arctan(1) \Rightarrow \tan\alpha = 1 \rightarrow \alpha = \frac{\pi}{4}$
- $\theta = \operatorname{arcsen}\left(\frac{1}{2}\right) \Rightarrow \operatorname{sen}\theta = \frac{1}{2} \rightarrow \theta = \frac{\pi}{6}$

Luego:

$$M = \alpha + \theta = \frac{\pi}{4} + \frac{\pi}{6}$$

$$\therefore \boxed{M = \frac{5\pi}{12}}$$

3. Halle el valor de: $T = \operatorname{sen}\left[\arccos\left(\frac{\sqrt{2}}{2}\right)\right] + \cos\left[\arctan(1)\right]$

Resolución:

$$T = \operatorname{sen}\underbrace{\left[\arccos\left(\frac{\sqrt{2}}{2}\right)\right]}_{\alpha} + \cos\left[\underbrace{\arctan(1)}_{\theta}\right]$$

- $\alpha = \arccos\left(\frac{\sqrt{2}}{2}\right) \Rightarrow \cos\alpha = \frac{\sqrt{2}}{2} \rightarrow \alpha = \frac{\pi}{4}$

- $\theta = \arctan(1) \Rightarrow \tan\theta = 1$

$\rightarrow \theta = \frac{\pi}{4}$

Piden: $T = \operatorname{sen}\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)$

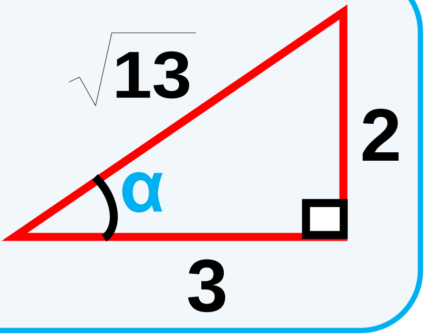
$T = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$\therefore \boxed{T = \sqrt{2}}$

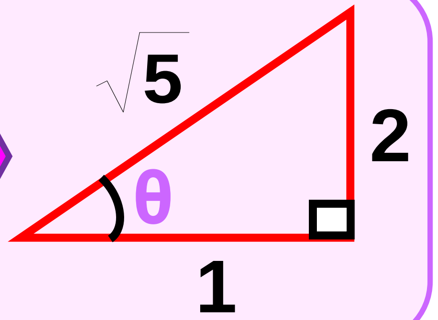
4. Halle el valor de: $E = \sqrt{13}\ \text{sen}[\arctan(\frac{2}{3})] + \sqrt{5}\ \cos[\arctan(2)]$

Resolución: $E = \sqrt{13}\ \text{sen}[\underbrace{\arctan(\tfrac{2}{3})}_{\alpha}] + \sqrt{5}\ \cos[\underbrace{\arctan(2)}_{\theta}]$

$\tan\alpha = \frac{2}{3} = \frac{Co}{Ca}$



$\tan\theta = \frac{2}{1} = \frac{Co}{Ca}$



Reemplazando:

$$E = \sqrt{13}\,\text{sen}(\alpha) + \sqrt{5}\cos(\theta)$$

$$E = \cancel{\sqrt{13}}\left(\frac{2}{\cancel{\sqrt{13}}}\right) + \cancel{\sqrt{5}}\left(\frac{1}{\cancel{\sqrt{5}}}\right)$$

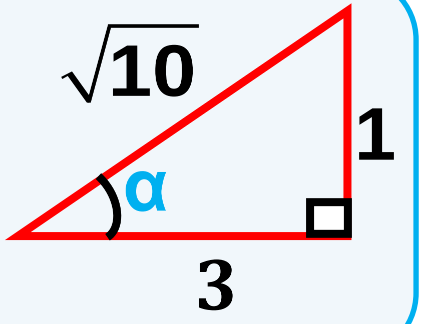
$\therefore\ \boxed{E = 3}$

5. Halle el valor de: $P = \tan\left[\frac{\pi}{4} + \text{arcsen}\left(\frac{1}{\sqrt{10}}\right)\right]$

Resolución:

$$P = \tan\left[\frac{\pi}{4} + \underbrace{\text{arcsen}\left(\frac{1}{\sqrt{10}}\right)}_{\alpha}\right]$$

$$\text{sen}\,\alpha = \frac{1}{\sqrt{10}} = \frac{Co}{H}$$



Piden:

$$P = \tan[45° + \alpha]$$

$$P = \frac{\tan 45° + \tan\alpha}{1 - \tan 45° . \tan\alpha} = \frac{1 + \frac{1}{3}}{1 - 1 . \frac{1}{3}}$$

$$P = \frac{\frac{4}{\cancel{3}}}{\frac{2}{\cancel{3}}} = \frac{4}{2}$$

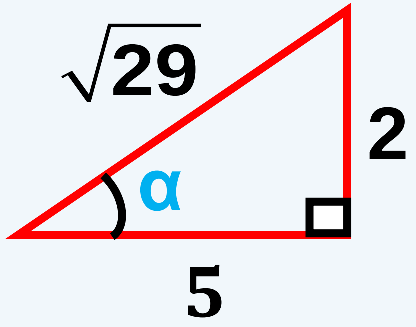
$\therefore$ $\boxed{P = 2}$

6. Halle el valor de: $T = \text{sen}\left[2\arctan\left(\frac{2}{5}\right)\right]$

Resolución:

$$T = \text{sen}[2\underbrace{\arctan\left(\tfrac{2}{5}\right)}_{\alpha}]$$

$$\tan\alpha = \frac{2}{5} = \frac{Co}{Ca}$$



Piden:

$$T = \text{sen}[2\alpha]$$

$$T = 2\,\text{sen}\alpha\cos\alpha$$

$$T = 2\left(\frac{2}{\sqrt{29}}\right)\left(\frac{5}{\sqrt{29}}\right)$$

$$T = \frac{20}{29}$$

$$\therefore \boxed{T = \frac{20}{29}}$$

7. Halle el valor de x de la siguiente igualdad :

$$3\text{arcsenx} + 2\text{arccosx} = \frac{7\pi}{6}$$

Resolución:

$$3\text{arcsenx} + 2\text{arccosx} = \frac{7\pi}{6}$$

$$\text{arcsenx} + 2\underbrace{(\text{arcsenx} + \text{arccosx})}_{\frac{\pi}{2}} = \frac{7\pi}{6}$$

$$\text{arcsenx} + \text{arccosx} = \frac{\pi}{2}$$

$$\text{arcsenx} + \pi = \frac{7\pi}{6}$$

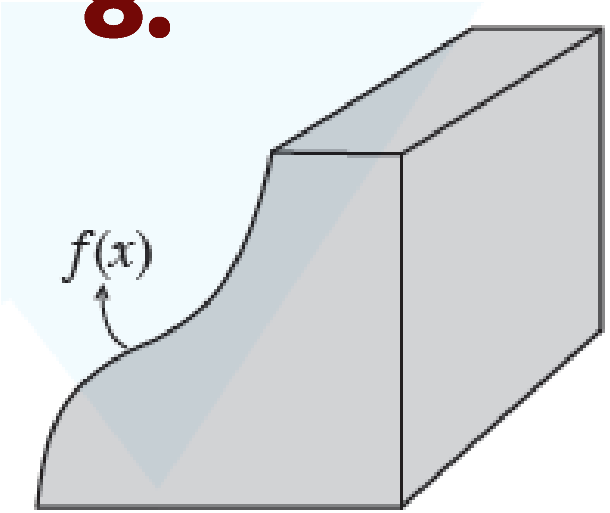
$$\text{arcsenx} = \frac{\pi}{6}$$

$$x = \text{sen}\left(\frac{\pi}{6}\right)$$

$$x = \frac{1}{2}$$

$$\therefore \boxed{x = \frac{1}{2}}$$

8.

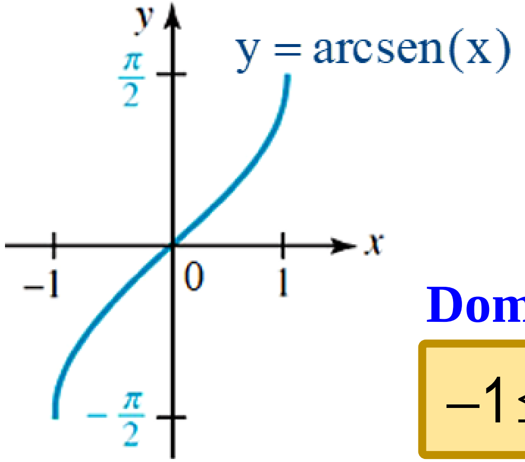


Un arqueólogo descubrió un santuario de una determinada cultura, tal como se muestra la figura.

Para lo cual le pide a un matemático hallar el dominio de la función que representa la pared lateral : *f(x) = arcsen(x+2)*

¿Cuál es el dominio de la función?

Resolución:



Dominio:

$-1 \leq x \leq 1$

$$El\ dominio\ de\ f(x) = arcsen(x+2);\ cumple:$$

$$-1 \leq x+2 \leq 1 \quad (-2)$$

$$-3 \leq x \leq -1$$

$$\therefore \boxed{\text{Dom } f = [-3;-1]}$$