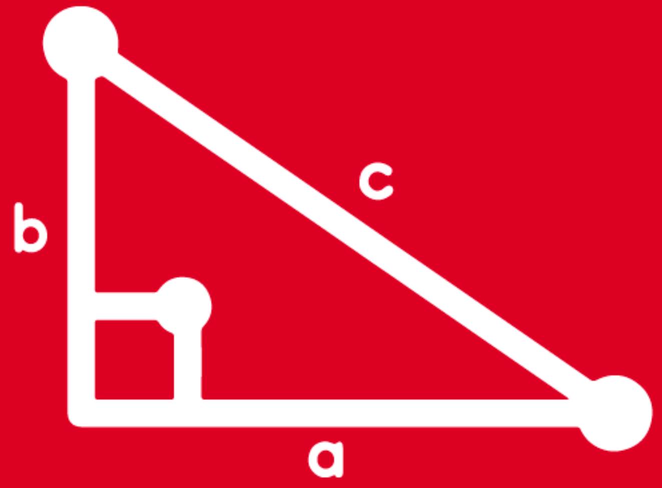
TRIGONOMETRY

Chapter 05

Sesión 2

4th
SECONDARY



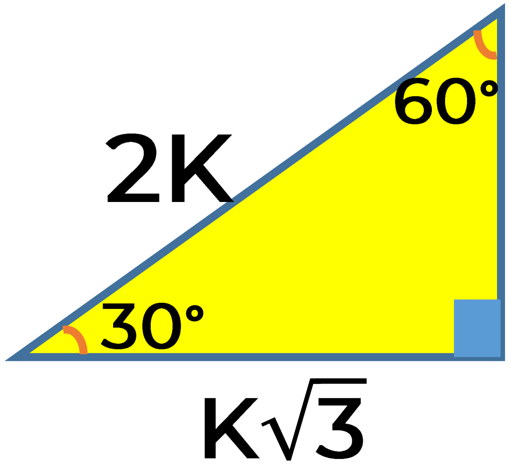
Razones trigonométricas de ángulos notables

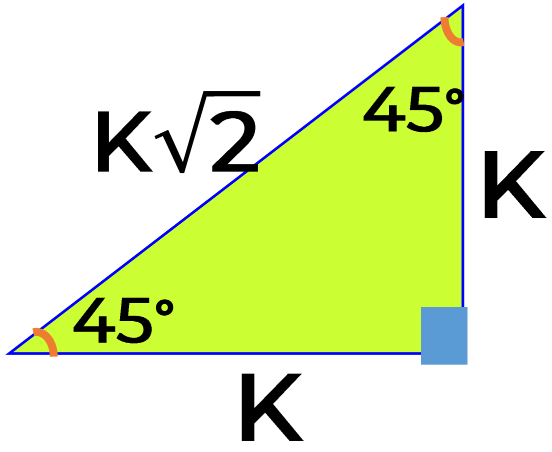
SACO OLIVEROS

MOTIVATING STRATEGY

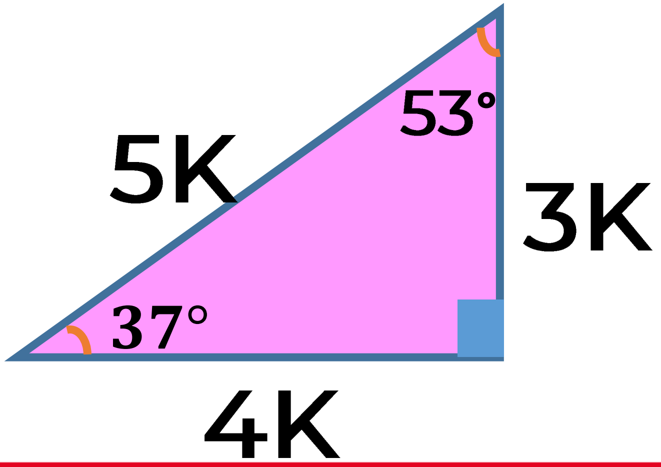
¿ SE PUEDE DETERMINAR LAS RAZONES TRIGONOMÉTRICAS CON LA MANO ?

Razones Trigonométricas de Ángulos Notables





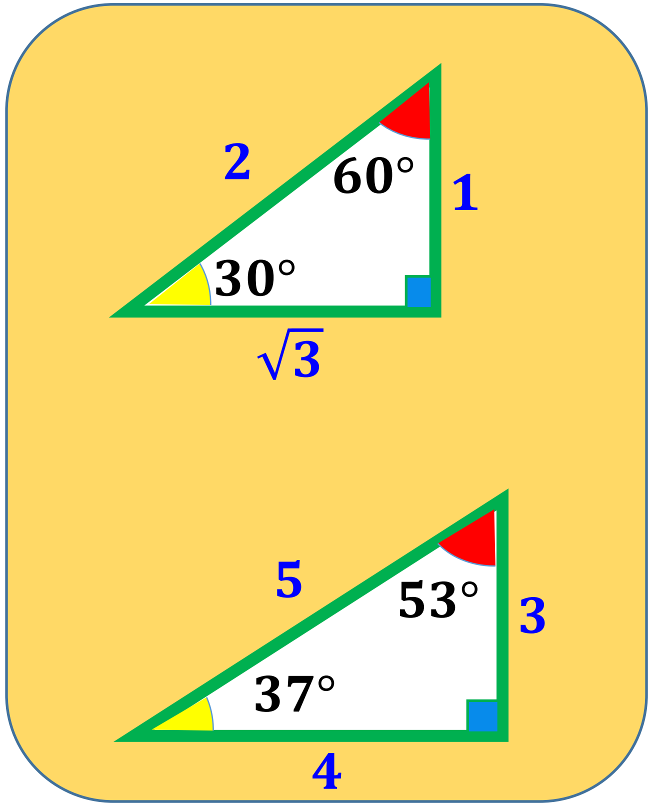
Triángulo aproximado.



RT ∢	30°	60°	37°	53°	45°
sen	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{\sqrt{2}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{\sqrt{2}}{2}$
tan	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{3}{4}$	$\frac{4}{3}$	1
cot	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	$\frac{4}{3}$	$\frac{3}{4}$	1
sec	$\frac{2\sqrt{3}}{3}$	2	$\frac{5}{4}$	$\frac{5}{3}$	$\sqrt{2}$
csc	2	$\frac{2\sqrt{3}}{3}$	$\frac{5}{3}$	$\frac{5}{4}$	$\sqrt{2}$

1. Efectúe: $P=\left(5\operatorname{sen}37°+\sqrt{3}\tan 60°+\cot^2 30°\right)^{\cos 60°}$

RESOLUCIÓN



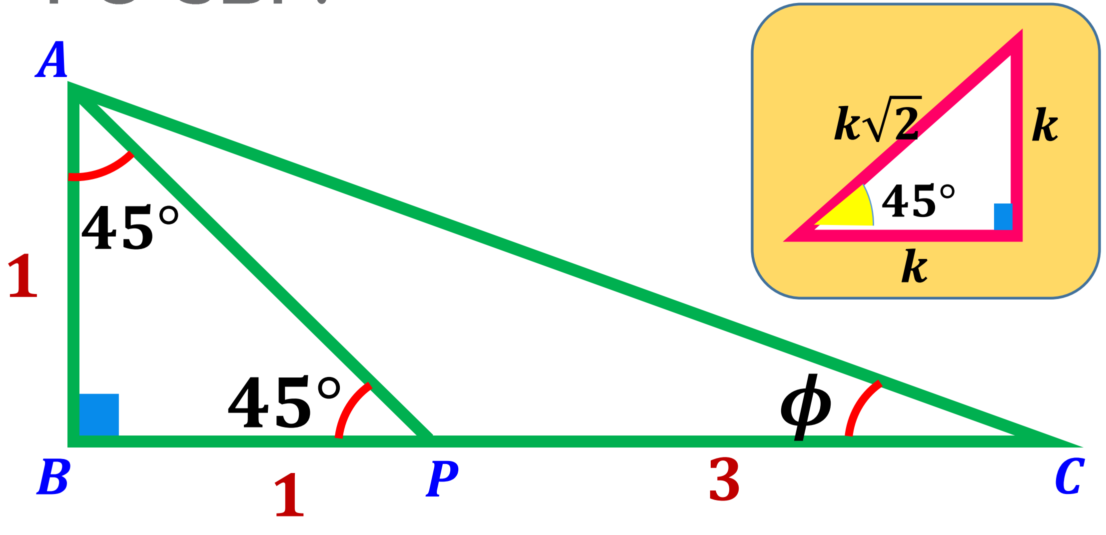
$$P=\left(\cancel{5}\times\left(\frac{3}{\cancel{5}}\right)+\sqrt{3}\times\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2\right)^{1/2}$$

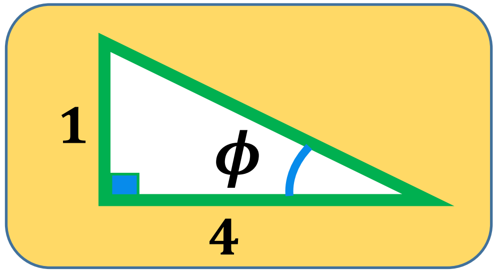
$$P=(3+3+3)^{1/2}$$

$$P=\sqrt{9}$$

$$\boxed{\therefore P=3}$$

2. Del gráfico, calcule $\cot\phi$ si $PC=3BP$.





RESOLUCIÓN

Del dato:

$1PC=3BP \Rightarrow \dfrac{PC}{BP}=\dfrac{3}{1}$

En el ΔABP (Notable de 45°)

$AB = BP$

Pero

$BP=1 \Rightarrow AB=1$

Piden:

$\cot\phi = \dfrac{4}{1}$

$\boxed{\therefore \cot\phi = 4}$

3. Si $\tan\alpha = \dfrac{\sec 45° + \cos 45°}{\tan^2 60°}$,

donde α es un ángulo agudo, efectúe:

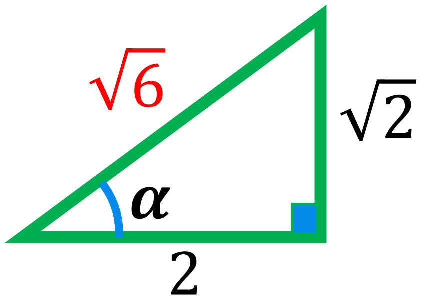
$$K = \sqrt{6}\sec\alpha + 6\,\text{sen}^2\alpha$$

RESOLUCIÓN

Del dato: $tan\alpha = \dfrac{sec45° + cos45°}{tan^2 60°}$

$$tan\alpha = \frac{(\sqrt{2}) + \left(\dfrac{\sqrt{2}}{2}\right)}{(\sqrt{3})^2} = \frac{\dfrac{\cancel{3}\sqrt{2}}{2}}{\dfrac{\cancel{3}}{1}}$$

$$tan\alpha = \frac{\sqrt{2}}{2} = \frac{CO}{CA}$$



$(H)^2 = (\sqrt{2})^2 + (2)^2$

$(H)^2 = 6$

$H = \sqrt{6}$

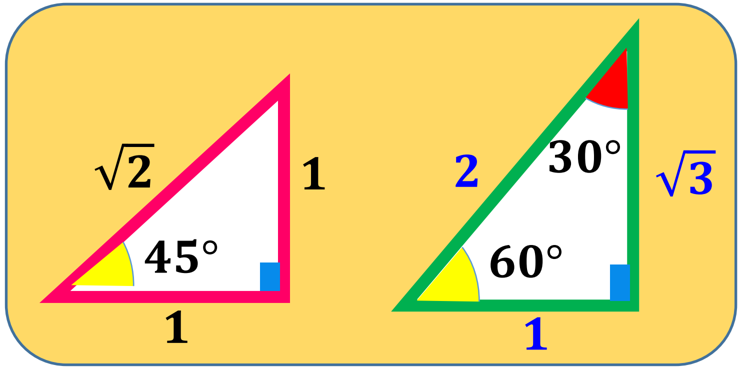
Piden: $K = \sqrt{6}seca + 6sen^2\alpha$

$$K = \sqrt{6} \times \left(\frac{\sqrt{6}}{2}\right) + 6 \times \left(\frac{\sqrt{2}}{\sqrt{6}}\right)^2$$

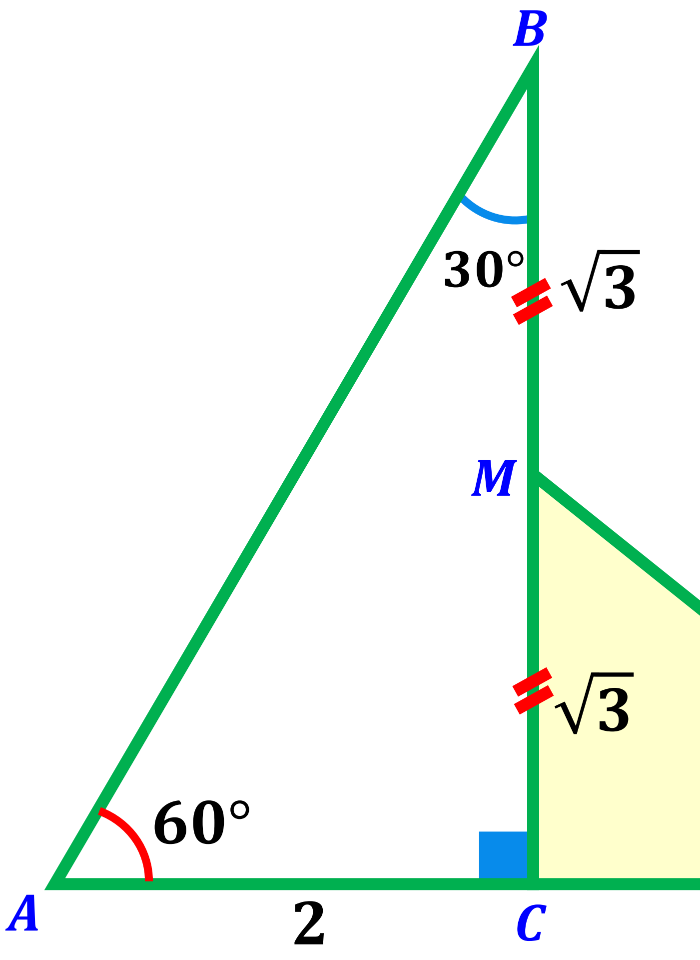
$$K = \frac{6}{2} + \frac{\cancel{6} \times 2}{\cancel{6}}$$

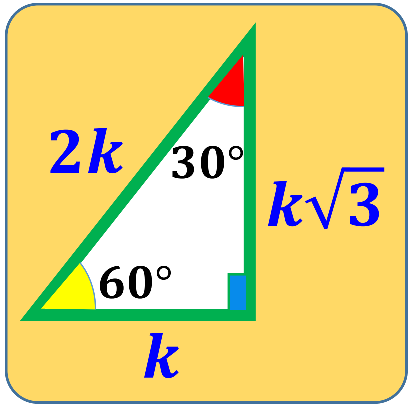
$$K = 3 + 2$$

$$\boxed{\therefore\ K = 5}$$



4. Del gráfico, calcule tanθ, si BM = MC y AC=CD





RESOLUCIÓN

Sea: $BM = MC = \sqrt{3}$

En el ΔACB (Notable de 30° y 60°)

$BC = k\sqrt{3}$ ➔ $2\sqrt{3} = k\sqrt{3}$

➔ $k = 2$

Pero

$AC = k$ ➔ $AC = 2$

Del dato:

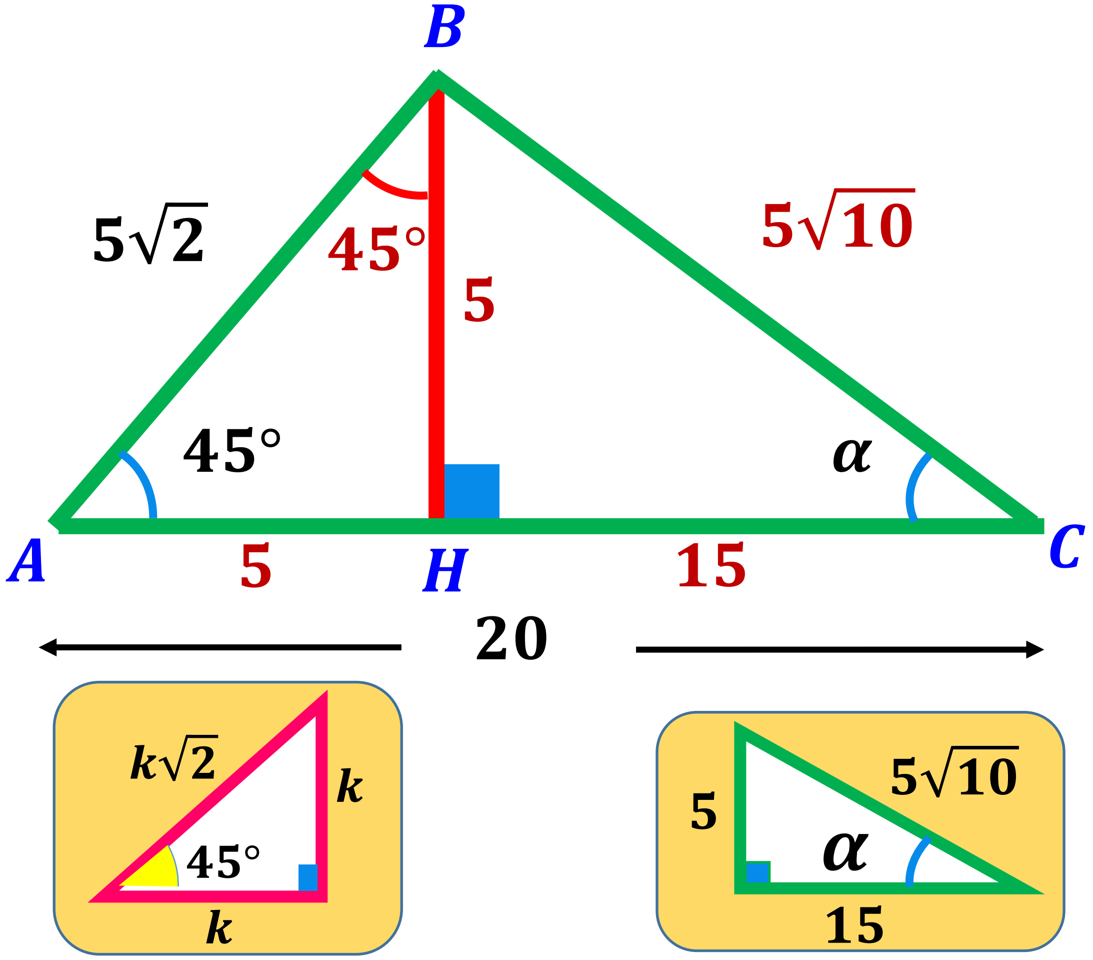
$AC = CD = 2$

Piden: $tan\,\theta$

$$\therefore tan\,\theta = \frac{\sqrt{3}}{2}$$

5. Del gráfico, efectúe:

$E = \sqrt{10}\,\text{sen}\alpha + \cot\alpha$



RESOLUCIÓN

En el ΔAHB (Notable de 45°)

$$\boldsymbol{AB = k\sqrt{2}} \Rightarrow 5\sqrt{2} = k\sqrt{2}$$

$$\Rightarrow k = 5$$

Pero: $AH = HB = k \Rightarrow \boldsymbol{AH = HB = 5}$

En el ΔBHC (Por el Teorema de Pitágoras)

$$(BC)^2 = (5)^2 + (15)^2$$

$$(BC)^2 = 250 \Rightarrow \boldsymbol{BC = 5\sqrt{10}}$$

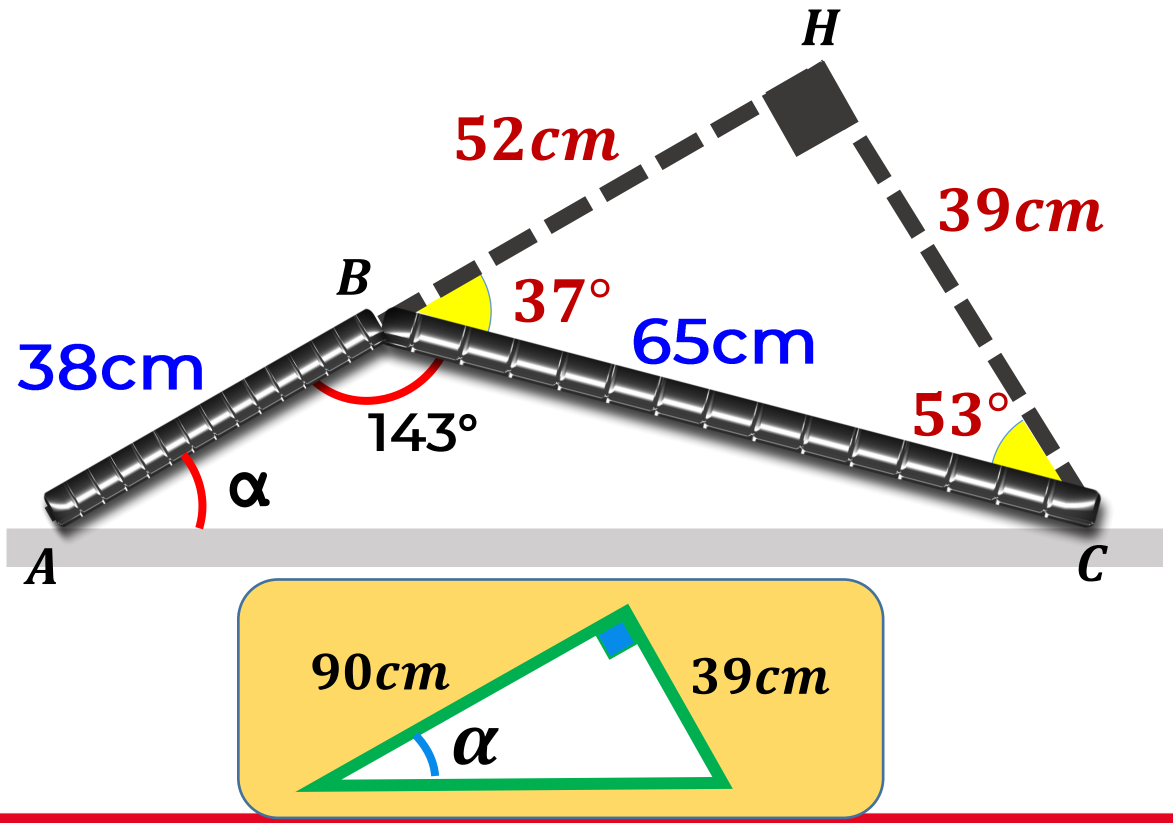
Piden: $\boldsymbol{E = \sqrt{10}sen\alpha + cot\alpha}$

$$E = \cancel{\sqrt{10}} \times \left(\frac{5}{5\cancel{\sqrt{10}}}\right) + \frac{15}{5}$$

$$E = 1 + 3$$

$$\boxed{\therefore E = 4}$$

6. Dos barras metálicas se encuentran apoyadas, tal como se muestra en la figura. Si el ángulo que forman las barras en su punto de apoyo es de 143°, calcule 13 cot α.



RESOLUCIÓN

En el ΔBHC (Notable de 37° y 53°)

$HC = 3k$; $HB = 4k$; $BC = 5k$

Pero

$$BC = 65cm$$

$$5k = 65cm \Rightarrow k = 13cm$$

Luego:

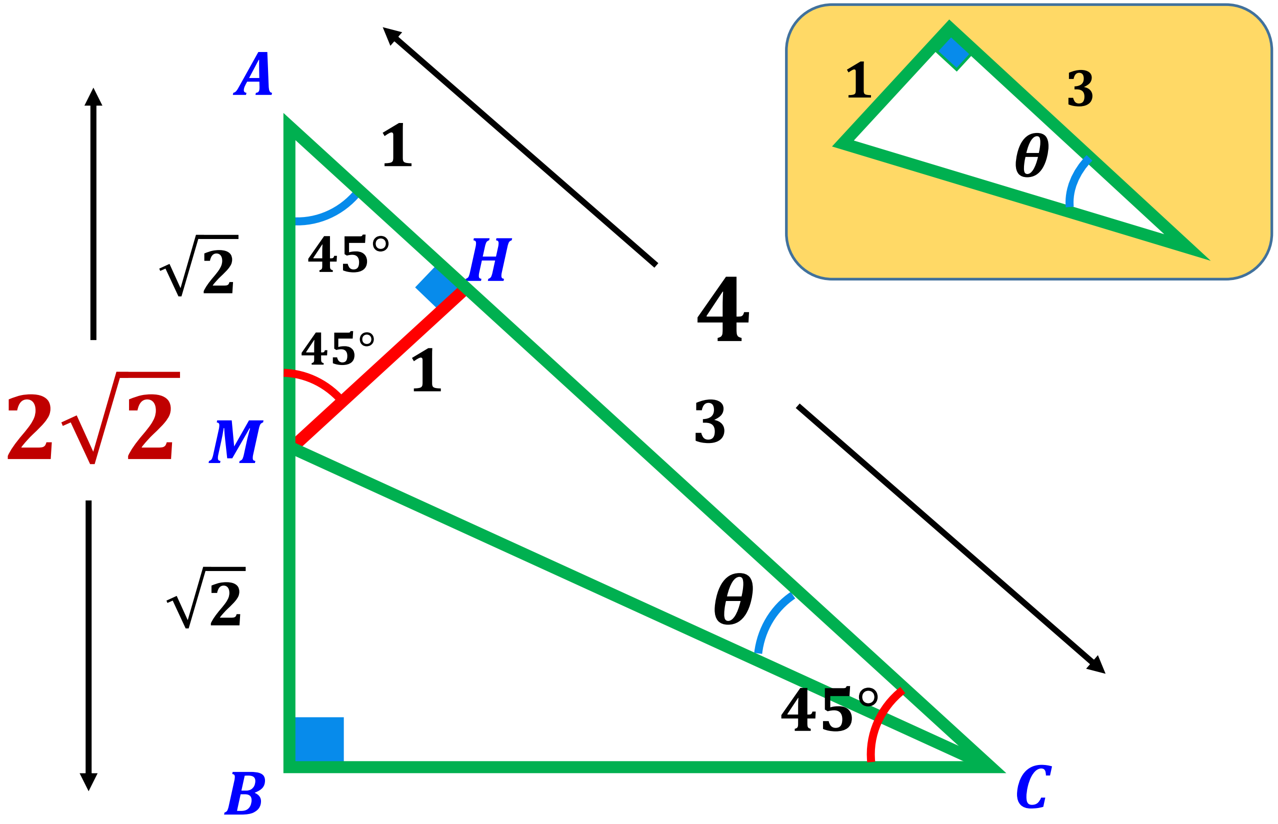
$$HB = 4(13cm) = 52cm$$

$$HC = 3(13cm) = 39cm$$

Piden: $13\cot\alpha = \overset{1}{\cancel{13}} \times \left(\dfrac{90}{\underset{3}{\cancel{39}}}\right)$

$$\boxed{\therefore 13\cot\alpha = 30}$$

7. Del gráfico, calcule $\tan\theta$ si AM = MB.



RESOLUCIÓN

Sea: $AM = MB = \sqrt{2}$

En el ΔABC (Triángulo Notable de 45°)

$AB = a$; $AC = a\sqrt{2}$

Pero: $AB = 2\sqrt{2}$ ➔ $a = 2\sqrt{2}$

Luego:

$AC = a\sqrt{2} = (2\sqrt{2})\sqrt{2}$ ➔ $AC = 4$

En el ΔAHM (Triángulo Notable de 45°)

$AM = k\sqrt{2}$ ➔ $\sqrt{2} = k\sqrt{2}$

➔ $k = 1$

Luego: $AH = MH = k = 1$

Piden: $\tan\theta$

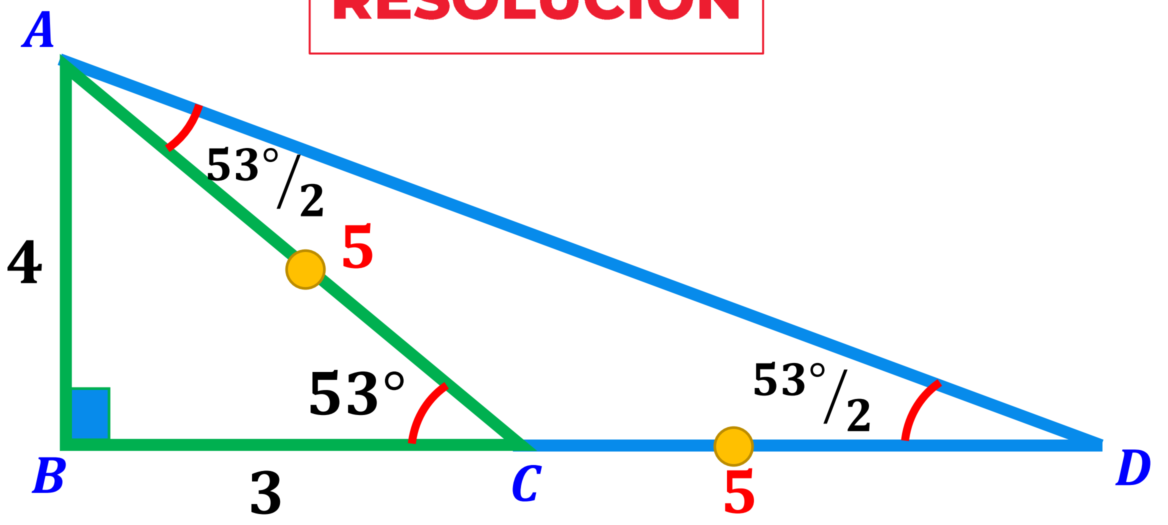
$$\boxed{\therefore \tan\theta = \frac{1}{3}}$$

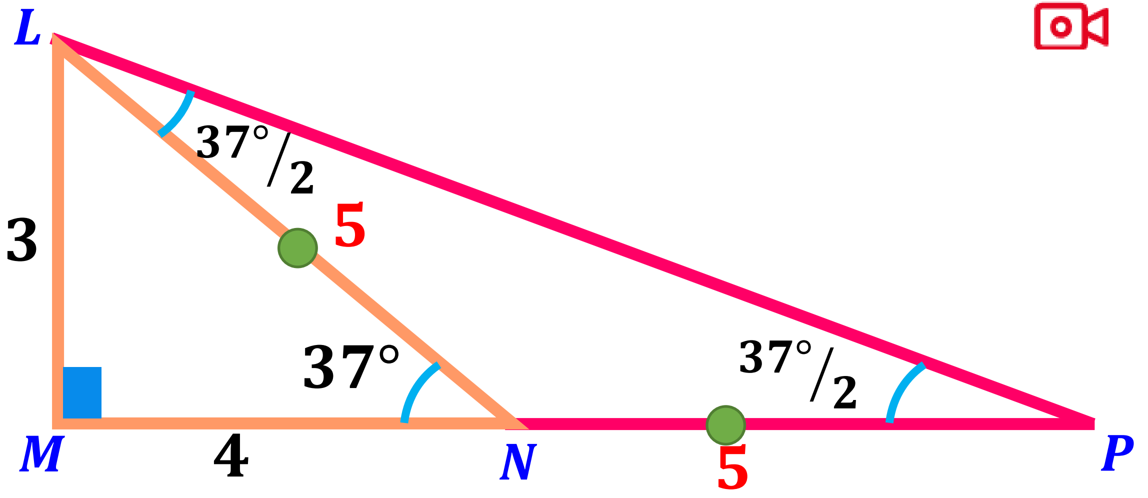
8. En una reunión familiar se encuentra dos primos, Rodrigo y Saúl. Se sabe que la edad de Rodrigo duplica la de Saúl, además, la edad de Saúl está dada por:

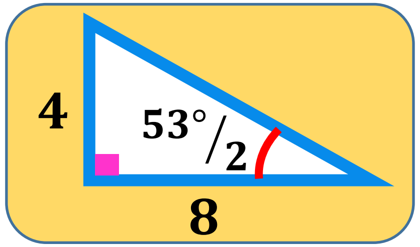
$$S = 6\left[\tan\left(\frac{37°}{2}\right) + \cot\left(\frac{53°}{2}\right)\right]$$

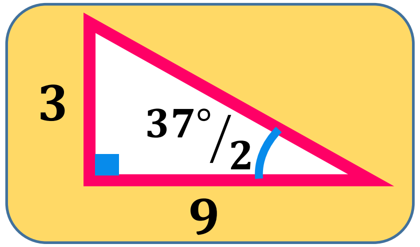
¿Cuál es la edad de Rodrigo?

RESOLUCIÓN









Del dato: $S = 6\left[\tan\left(\frac{37°}{2}\right) + \cot\left(\frac{53°}{2}\right)\right]$

$$S = 6\left[\frac{3}{9} + \frac{4}{8}\right] = 2 + 3 = 5$$

∴ La edad de Rodrigo es 10 años