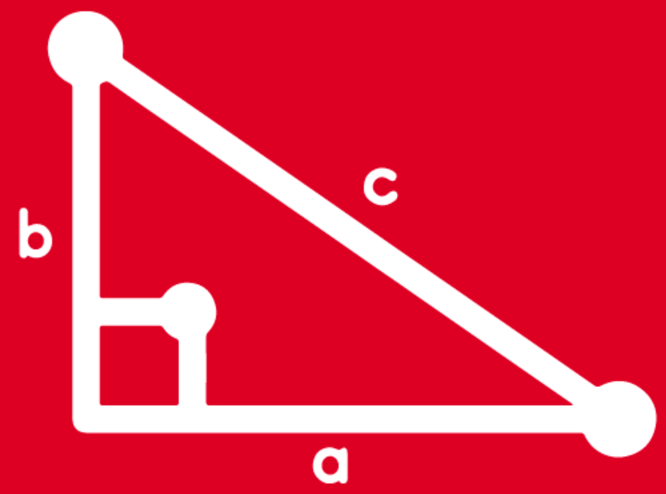
TRIGONOMETRY

Chapter 23

3rd
SECONDARY



IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO DOBLE

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HELICO-MOTIVACIÓN

¿ QUÉ HACEN LOS “DOBLES” DE LOS ACTORES FAMOSOS ?

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO DOBLE

Se obtienen a partir de las identidades del ángulo compuesto cuando $\beta = \alpha$

$$\text{sen}(\alpha + \beta) = \text{sen}\alpha \cos\beta + \cos\alpha\, \text{sen}\beta$$

$$\text{sen}(\alpha + \alpha) = \text{sen}\alpha \cos\alpha + \cos\alpha\, \text{sen}\alpha$$

$$\text{sen}(2\alpha) = 2\text{sen}\alpha \cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \operatorname{sen}^2\alpha$$

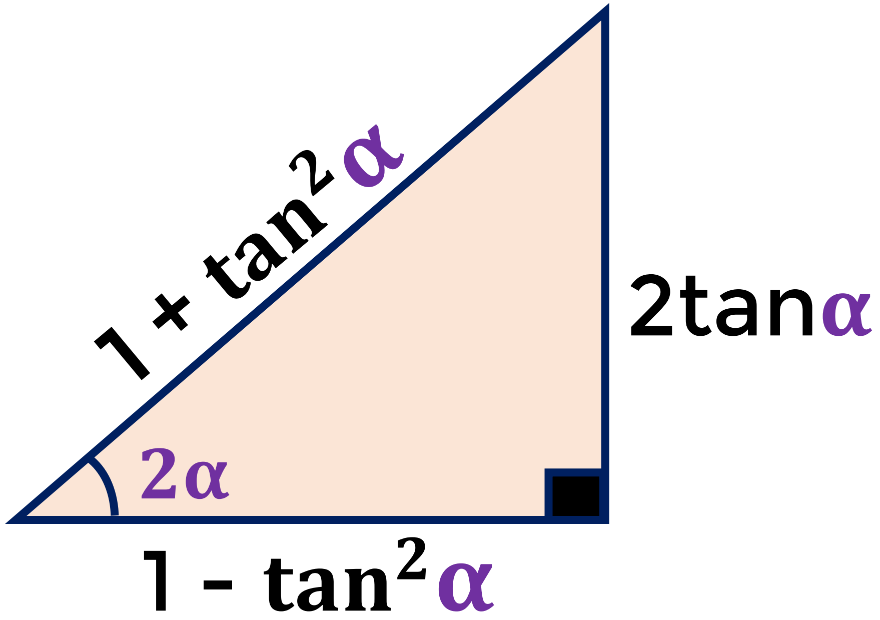
Además utilizando identidad pitagórica :

$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

$$\cos(2\alpha) = 1 - 2\operatorname{sen}^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

TRIÁNGULO PRÁCTICO DEL ÁNGULO DOBLE



Se obtiene:

$$\operatorname{sen}2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$$

$$\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

HELICO-PRACTICA

1) Siendo α un ángulo agudo, tal que $\tan\alpha = \frac{3}{5}$, calcule sen2α

Resolución:

$$\text{sen}2\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$

$$\text{sen}2\alpha = \frac{2\left(\frac{3}{5}\right)}{1 + \left(\frac{3}{5}\right)^2} = \frac{\left(\frac{6}{5}\right)}{1 + \frac{9}{25}} = \frac{\frac{6}{5}}{\frac{34}{25}}$$

$$\text{sen}2\alpha = \frac{(6)(25)}{(5)(34)}$$

$$\therefore\ \text{sen}\alpha = \frac{15}{17}$$

Recordar

$$\text{sen}2\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$

HELICO-PRACTICE

2) Siendo β un ángulo agudo, tal que $\tan\beta = \frac{1}{5}$,
calcule cos2β.

Resolución:

$$\cos 2\beta = \frac{1 - \left(\frac{1}{5}\right)^2}{1 + \left(\frac{1}{5}\right)^2} = \frac{1 - \frac{1}{25}}{1 + \frac{1}{25}} = \frac{\frac{24}{25}}{\frac{26}{25}}$$

$$\cos 2\beta = \frac{24}{26}$$

$$\therefore \cos\beta = \frac{12}{13}$$

Recordar

$$\cos 2\beta = \frac{1 - \tan^2\beta}{1 + \tan^2\beta}$$

HELICO-PRACTICE

3) Si θ es un ángulo agudo, tal que $\cos\theta = \frac{2}{\sqrt{5}}$,
calcule $\cos 2\theta$.

Resolución:

$$\cos 2\theta = 2\left(\frac{2}{\sqrt{5}}\right)^2 - 1$$

$$\cos 2\theta = 2\left(\frac{4}{5}\right) - 1$$

$$\cos 2\theta = \frac{8}{5} - 1$$

$$\therefore \cos 2\theta = \frac{3}{5}$$

Recordar

$$\cos 2\theta = 2\cos^2\theta - 1$$

HELICO-PRACTICE

4) Siendo x un ángulo agudo y $\csc x = \sqrt{17}$, calcule tan2x

Resolución:

$$\csc x = \frac{\sqrt{17}}{1} = \frac{H}{CO}$$

- Utilizando el teorema de Pitágoras

$$(CA)^2 + (1)^2 = (\sqrt{17})^2$$

$$\boxed{CA = 4}$$

- Obtenemos : $\tan x = \frac{1}{4}$

- Piden :

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\tan 2x = \frac{2\left(\frac{1}{4}\right)}{1-\left(\frac{1}{4}\right)^2}$$

$$\tan 2x = \frac{\frac{1}{2}}{\frac{15}{16}}$$

$$\therefore \tan 2x = \frac{8}{15}$$

HELICO-PRACTICE

5) Calcule M + N si :

M = 2 sen15° cos15°

N = $\cos^2 18°30' - \text{sen}^2 18°30'$

Resolución:

M = 2sen15°cos15°

M = sen2(15°)

M = sen30°

M = $\left(\frac{1}{2}\right)$

Recordar

sen(2α) = 2senα cosα

N = $\cos^2 18°30' - \text{sen}^2 18°30'$

N = cos2(18°30')

N = cos37°

N = $\frac{4}{5}$

Recordar

$\cos(2\alpha) = \cos^2\alpha - \text{sen}^2\alpha$

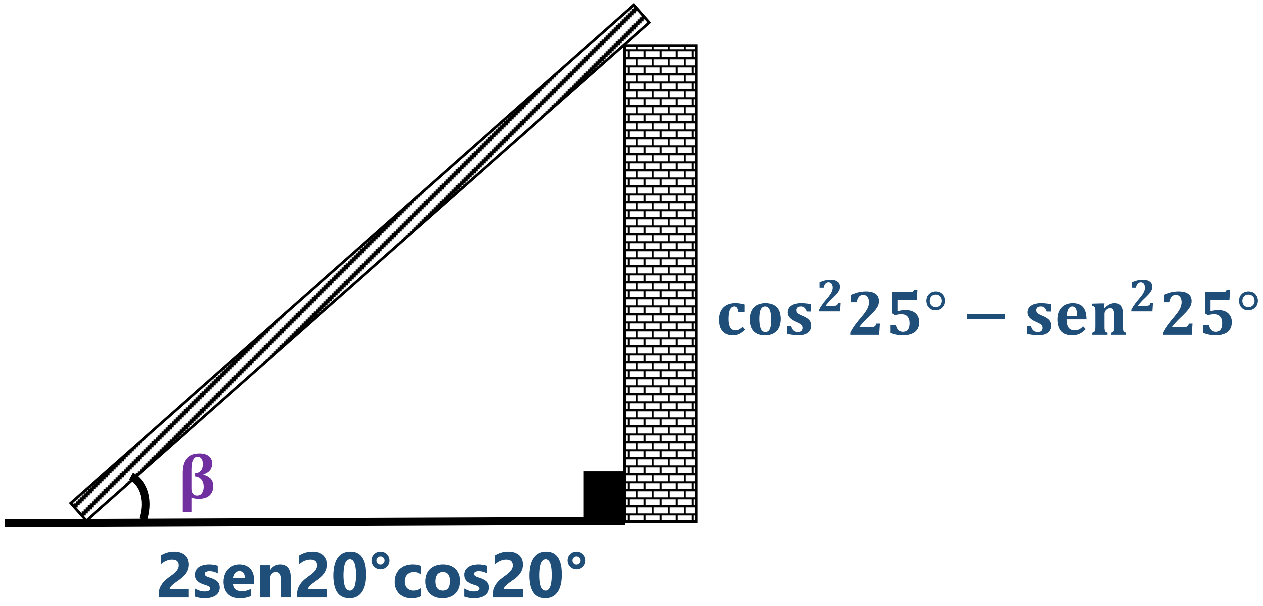
❖ Piden :

M + N = $\frac{1}{2} + \frac{4}{5}$

$\therefore$ M + N = $\frac{13}{10}$

HELICO-PRACTICE

6) Una barra metálica se encuentra apoyada sobre una pared, tal como se muestra en la figura. Calcule tanβ



Resolución:

$$\tan\beta = \frac{\cos^2 25° - \operatorname{sen}^2 25°}{2\operatorname{sen}20°.\cos 20°}$$

$$\tan\beta = \frac{\cos 2(25°)}{\operatorname{sen}2(20°)}$$

$$\tan\beta = \frac{\cos 50°}{\operatorname{sen}40°} = \frac{\cancel{\cos 50°}}{\cancel{\cos 50°}}$$

$\therefore \tan\beta = 1$

HELICO-PRACTICE

7) Reduzca $F = 4\text{sen}10°\cos 10°\cos 20°\cos 40°$

Resolución:

Recordar

$\text{sen}(2\alpha) = 2\text{sen}\alpha \cos\alpha$

$F = 2.2\text{sen}10°\cos 10°\cos 20°\cos 40°$

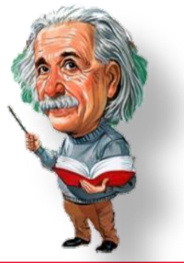
$F = \underbrace{2.\text{sen}20°\cos 20°}\cos 40°$

$F = \text{sen}40°\cos 40°$ … x2

$2F = \underbrace{2\text{sen}40°\cos 40°}$

$2F = \text{sen}80°$

$\therefore\ F = \dfrac{\text{sen}80°}{2}$

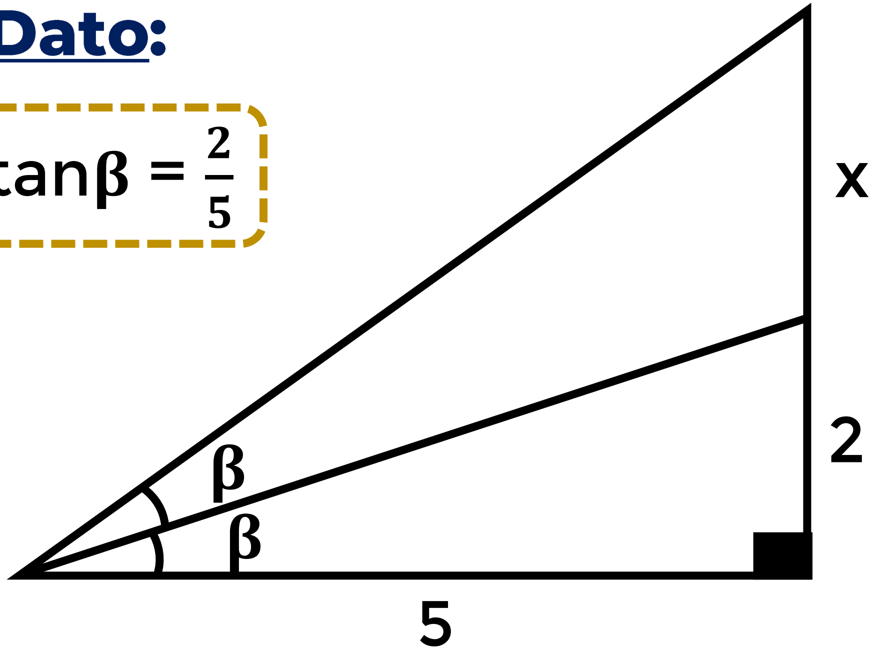


HELICO-PRACTICE

8) Del gráfico, halle el valor de x

Dato:

$\boxed{\tan\beta = \frac{2}{5}}$



Resolución:

❖ Del gráfico se observa :

$$\underbrace{\tan 2\beta} = \frac{x+2}{5}$$

$$\frac{2\tan\beta}{1-\tan^2\beta} = \frac{x+2}{5}$$

$$\frac{2(\frac{2}{5})}{1-(\frac{2}{5})^2} = \frac{x+2}{5} \quad \Rightarrow \frac{20}{21} = \frac{x+2}{5}$$

$$\therefore\ x = \frac{58}{21}$$

MUCHAS GRACIAS POR TU ATENCIÓN

Tu curso amigo

TRIGONOMETRÍA