TRIGONOMETRY

Chapter 09

2th
SECONDARY

Razones trigonométricas recíprocas

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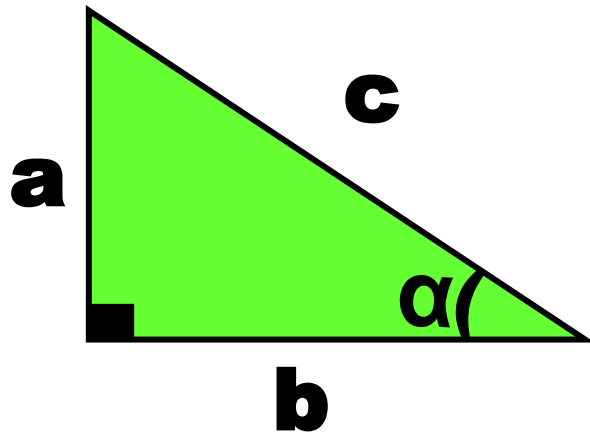
HELICOMOTIVACIÓN

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PROPIEDADES DE LAS R.T DE ÁNGULOS AGUDOS I

1. RAZONES TRIGONOMÉTRICAS RECÍPROCAS

Recordando:

DE LA FIGURA SE TIENE:

$$\text{sen}\alpha = \frac{a}{c} \quad ; \quad \csc\alpha = \frac{c}{a}$$

➡️ $$\text{sen}\alpha \cdot \csc\alpha = \frac{a}{c} \times \frac{c}{a} = 1$$
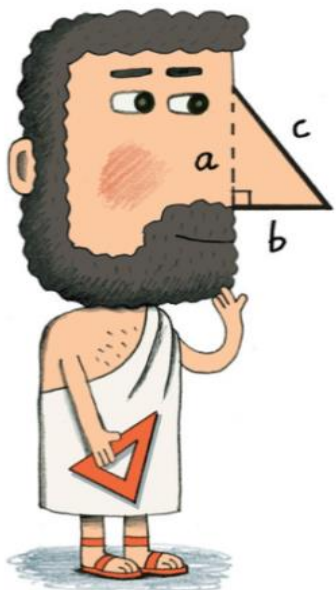
SE CONCLUYE:

$$\text{sen}\alpha \cdot \csc\alpha = 1$$

$$\cos\alpha \cdot \sec\alpha = 1$$

$$\tan\alpha \cdot \cot\alpha = 1$$

OBSERVACIONES:

Si: sen α . csc β = 1 ➡ **α = β**

Si: cos α . sec β = 1 ➡ **α = β**

Si: tan α . cot β = 1 ➡ **α = β**

1. **Escriba verdadero (V) o falso (F) según corresponda:**

a) $sen\,23° \cdot csc\,23° = 1$ **(V)**

c) $tan\,20° \cdot cot\,20° = 1$ **(V)**

b) $cos\,15° \cdot sen\,15° = 1$ **(F)**

d) $cos\,3\alpha \cdot sec\,3\alpha = 1$ **(V)**

RESOLUCIÓN:

a) $sen\,23° \cdot csc\,23° = 1$

c) $tan\,20° \cdot cot\,20° = 1$

b) $cos\,15° \cdot sen\,15° = 1$

d) $cos\,3\alpha \cdot sec\,3\alpha = 1$

Remember:

$$sen\,\alpha \cdot csc\,\alpha = 1$$

Remember:

$$cos\,\theta \cdot sec\,\theta = 1$$

Remember:

$$tan\,\beta \cdot cot\,\beta = 1$$

2. Si $\cos\theta = \dfrac{2}{7}$ y $\cot\alpha = \dfrac{2}{9}$, efectúe: $M = \sec\theta + \tan\alpha$

RESOLUCIÓN:

Del dato:

$$cos\,\theta = \frac{2}{7} \implies sec\,\theta = \frac{7}{2}$$

$$cot\,\alpha = \frac{2}{9} \implies tan\,\alpha = \frac{9}{2}$$

Remember:

$$cos\,\theta = \frac{a}{b} \implies sec\,\theta = \frac{b}{a}$$

Piden: $M = \sec\theta + \tan\alpha$
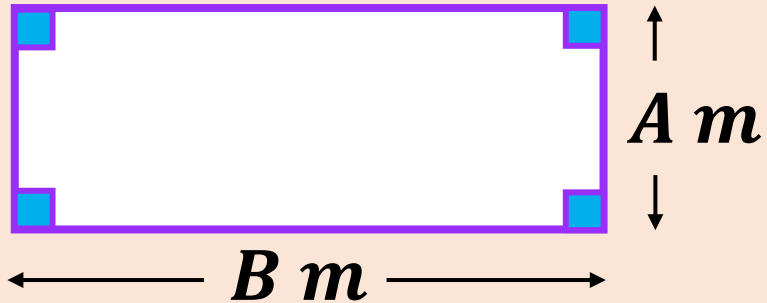
$$M = \frac{7}{2} + \frac{9}{2}$$

$$M = \frac{16}{2} \qquad \therefore\ M = 8$$

Remember:

$$tan\,\theta = \frac{a}{b} \implies cot\,\theta = \frac{b}{a}$$

3. Sabrina ha heredado un terreno rectangular, cuyas dimensiones son las siguientes:

$A\ m$

$B\ m$

Donde:

$$A = \dfrac{5\cos 19° \cdot \sec 19° + 3\tan\alpha \cdot \cot\alpha}{\sin 15° \cdot \csc 15°}$$

¿Cuál es el área del terreno? $B = 6\tan 10° \cdot \cot 10° + 9\sin\beta \cdot \csc\beta$

RESOLUCIÓN:

$$A = \dfrac{5\,\underset{1}{\underbrace{\cos 19° \cdot \sec 19°}} + 3\,\underset{1}{\underbrace{\tan\alpha \cdot \cot\alpha}}}{\underset{1}{\underbrace{\sin 15° \cdot \csc 15°}}}$$

$$A = \dfrac{5 + 3}{1} \quad\Rightarrow\quad \boxed{A = 8}$$

$$B = 6\,\underset{1}{\underbrace{\tan 10° \cdot \cot 10°}} + 9\,\underset{1}{\underbrace{\sin\beta \cdot \csc\beta}}$$

$$B = 6 + 9 \quad\Rightarrow\quad \boxed{B = 15}$$

Piden:

$$A_{\blacksquare} = (8)(15)$$

$$\therefore A_{\blacksquare} = 120m^2$$

4. **Efectúe:** $E = \dfrac{a}{b}$ **Si** $sen\ 18° \cdot csc\ a = 1$

$$tan\ 9° \cdot cot\ b = 1$$

RESOLUCIÓN:

Del dato:

$$sen\ 18° \cdot csc\ a = 1$$

➡ $a = 18°$

$$tan\ 9° \cdot cot\ b = 1$$

➡ $b = 9°$

Remember:

$$sen\ \alpha \cdot csc\ \alpha = 1$$

Piden: $E = \dfrac{a}{b}$

$$E = \dfrac{18}{9}$$

$$\therefore\ E = 2$$

Remember:

$$tan\ \beta \cdot cot\ \beta = 1$$

5. **Halle el valor de x, si** $cos\,3x \cdot sec(x + 40°) = 1$

RESOLUCIÓN:

Del dato:

$$cos\,3x \cdot sec(x + 40°) = 1$$

$$3x = x + 40°$$

$$2x = 40°$$

Remember:

$$cos\,\alpha \cdot sec\,\alpha = 1$$

$$\therefore x = 20°$$

6. **Halle el valor de x, si** $tan(2x + 20°) \cdot cot(x + 65°) = 1$

RESOLUCIÓN:

Del dato:

$$tan(2x + 20°) \cdot cot(x + 65°) = 1$$

➡ $$2x + 20° = x + 65°$$

Remember:

$$tan\,\alpha \cdot cot\,\alpha = 1$$

$$\therefore x = 45°$$

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7. **Sabiendo que** $sen\,7x \cdot csc(x + 36°) - 1 = 0$

Determine: $A = sen\,5x \cdot csc(4x + 6°)$

RESOLUCIÓN:

Del dato:

$$sen\,7x \cdot csc(x + 36°) - 1 = 0$$

$$sen\,7x \cdot csc(x + 36°) = 1$$

➡ $7x = x + 36°$

$$6x = 36°$$

$$x = 6°$$

Piden:

$$A = sen\,5x \cdot csc(4x + 6°)$$

$$A = sen\,5(6°) \cdot csc(4(6°) + 6°)$$

$$A = sen\,30° \cdot csc(30°)$$

$$\therefore A = 1$$

Remember:

$$sen\,\alpha \cdot csc\,\alpha = 1$$

8. **Si** $tan(a + b) \cdot cot \, 50° = 1$
$cos(a - b) \cdot sec \, 30° = 1$

Efectúe:

$K = sen^2(a + 20°) + sen \, 3b$

RESOLUCIÓN:

Del dato:

$tan(a + b) \cdot cot \, 50° = 1$

$\Rightarrow \quad a + b = 50°$

$cos(a - b) \cdot sec \, 30° = 1$

$\Rightarrow \quad a - b = 30°$

Tenemos:

$a + b = 50°$
$a - b = 30°$ $\Big\downarrow +$
_____
$2a = 80°$

$a = 40° \implies b = 10°$

Piden: $K = sen^2(60°) + sen(30°)$

$K = \left(\dfrac{\sqrt{3}}{2}\right)^2 + \dfrac{1}{2}$

$K = \dfrac{3}{4} \times \dfrac{1}{2} = \dfrac{6 + 4}{8}$

$K = \dfrac{10}{8}$

$\therefore K = \dfrac{5}{4}$