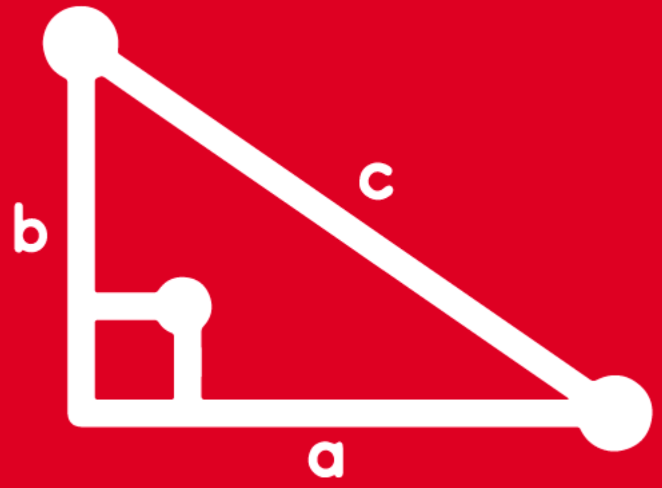
TRIGONOMETRY

Chapter 18
Session 01

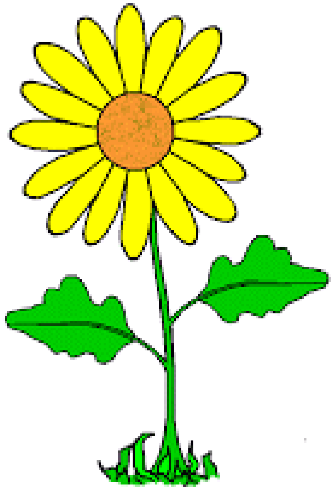


4th
SECONDARY

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO

SACO OLIVEROS


Trabajar duro
te llevará a la cima,
disfrutar el camino
te llevará más lejos



IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO (FUNDAMENTALES)

Para la suma de dos ángulos:

$$\operatorname{sen}(x+y)=\operatorname{sen}x.\cos y+\cos x.\operatorname{sen}y$$

$$\cos(x+y)=\cos x.\cos y-\operatorname{sen}x.\operatorname{sen}y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$$

Para la resta de dos ángulos:

$$\operatorname{sen}(x-y)=\operatorname{sen}x.\cos y-\cos x.\operatorname{sen}y$$

$$\cos(x-y)=\cos x.\cos y+\operatorname{sen}x.\operatorname{sen}y$$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x.\tan y}$$

PREGUNTA 1

Calcule $\cos 16°$

$$\cos(x - y) = \cos x . \cos y + \operatorname{sen} x . \operatorname{sen} y$$

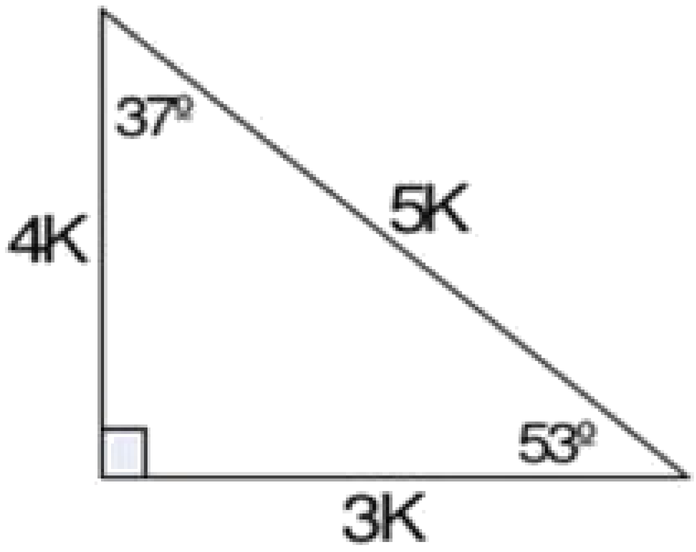
Resolución:

$$\cos 16° = \cos(53° - 37°)$$

$$\cos 16° = \underbrace{\cos 53°}_{} . \underbrace{\cos 37°}_{} + \underbrace{\operatorname{sen} 53°}_{} . \underbrace{\operatorname{sen} 37°}_{}$$

$$\cos 16° = \underbrace{\frac{3}{5} . \frac{4}{5}}_{} + \underbrace{\frac{4}{5} . \frac{3}{5}}_{}$$

$$\cos 16° = \frac{12}{25} + \frac{12}{25}$$



$$\therefore \cos 16° = \frac{24}{25}$$

PREGUNTA 2

Reducir $R = \sqrt{2}\cos(x - 45°) - \text{sen}x$

Resolución:

$$\cos(x - y) = \cos x . \cos y + \text{sen}x . \text{sen}y$$

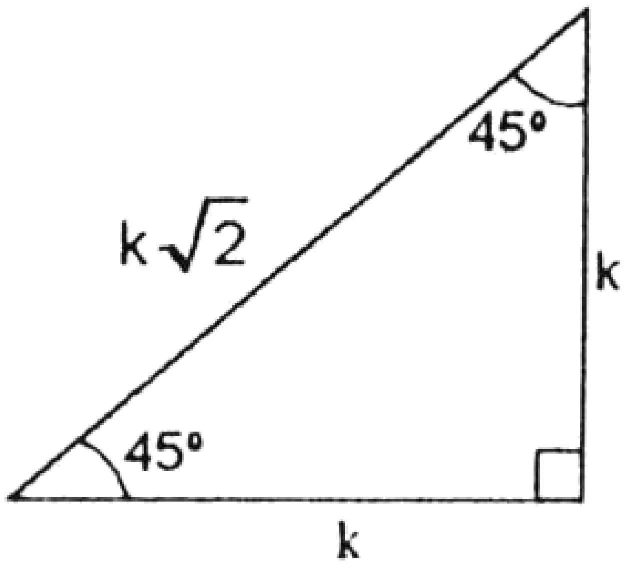
$$R = \sqrt{2}\cos(x - 45°) - \text{sen}x$$

$$R = \sqrt{2}\ [\cos x . \underbrace{\cos 45°} + \text{sen}x . \underbrace{\text{sen}45°}] - \text{sen}x$$

$$R = \cancel{\sqrt{2}}\,[\cos x . \frac{1}{\cancel{\sqrt{2}}} + \text{sen}x . \frac{1}{\cancel{\sqrt{2}}}] - \text{sen}x$$

$$R = \cos x + \cancel{\text{sen}x} - \cancel{\text{sen}x}$$

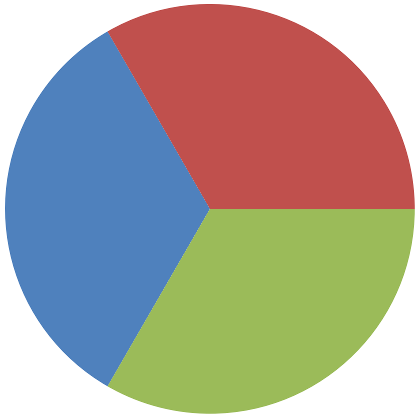
$$\boxed{\therefore R = \cos x}$$



PREGUNTA 3

Observe el siguiente diagrama y determine el espacio disponible del USB

Distribución del almacenamiento de una memoria de 8 GB



DATOS:

$$A = \text{sen}25°.\cos5° + \cos25°.\text{sen}5°$$

$$B = \frac{tan55° - tan10°}{1 + tan55°.tan10°}$$

Resolución

:

$$A = \underbrace{\text{sen}25°.\cos5° + \cos25°.\text{sen}5°}$$

$$\text{sen}(25° + 5°) = \text{sen}30° = \frac{1}{2}$$

$$B = \frac{tan55° - tan10°}{1 + tan55°.tan10°} = \tan(55° - 10°)$$

$$\Rightarrow B = \tan45° = 1$$

Piden

:

$$C = 8 - [\cancel{4}^{2}\left(\frac{1}{\cancel{2}}\right) + 5\,(1)]$$

$$\boxed{\therefore C = 1GB}$$

PREGUNTA 4

Calcule el valor de x, si: $\operatorname{sen}x.\cos(2x-10°)+\cos x.\operatorname{sen}(2x-10°)=\cos 40°$

Donde $x\in\langle 0°;90°\rangle$

Resolución:

$$\operatorname{sen}A.\cos B+\cos A.\operatorname{sen}B=\operatorname{sen}(A+B)$$

$$\underbrace{\operatorname{sen}x.\cos(2x-10°)+\cos x.\operatorname{sen}(2x-10°)}_{\operatorname{sen}[(x)+(2x-10°)]}=\cos 40°$$

$$\Rightarrow \operatorname{sen}(3x-10°)=\cos 40°$$

$*$ Por RT. complementarios: $3x-10°+40°=90°$

$$3x=60°$$

$$\therefore x=20°$$

PREGUNTA 5

Si $\tan\theta = \frac{5}{12}$; calcule $\tan(37° + \theta)$

Resolución:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$$

$$\tan(37° + \theta) = \frac{\tan 37° + \tan\theta}{1 - \tan 37° . \tan\theta}$$

$$\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{\cancel{3}}{4} . \frac{5}{\cancel{12}_{4}}} = \frac{\frac{14}{12}}{1 - \frac{5}{16}} = \frac{\frac{7}{6}}{\frac{11}{16}}$$

$$\Rightarrow \tan(37° + \theta) = \frac{7 \times \cancel{16}^{8}}{\cancel{6}_{3} \times 11}$$

$$\boxed{\therefore \tan(37° + \theta) = \frac{56}{33}}$$

PREGUNTA 6

Si $\tan(x+y)=\dfrac{1}{3}$ y $\tan(x-y)=2$; calcule $\tan 2y$

Resolución:

$$\tan(m-n)=\frac{\tan(m)-\tan(n)}{1+\tan(m).\tan(n)}$$

Consideramos:

$$x+y=m \quad \rightarrow \quad \tan(m)=\frac{1}{3}$$

$$x-y=n \quad \rightarrow \quad \tan(n)=2$$

Además:

$$\underbrace{(x+y)}_{m}-\underbrace{(x-y)}_{n}=2y$$

entonces: $\tan(m-n)=\tan 2y$

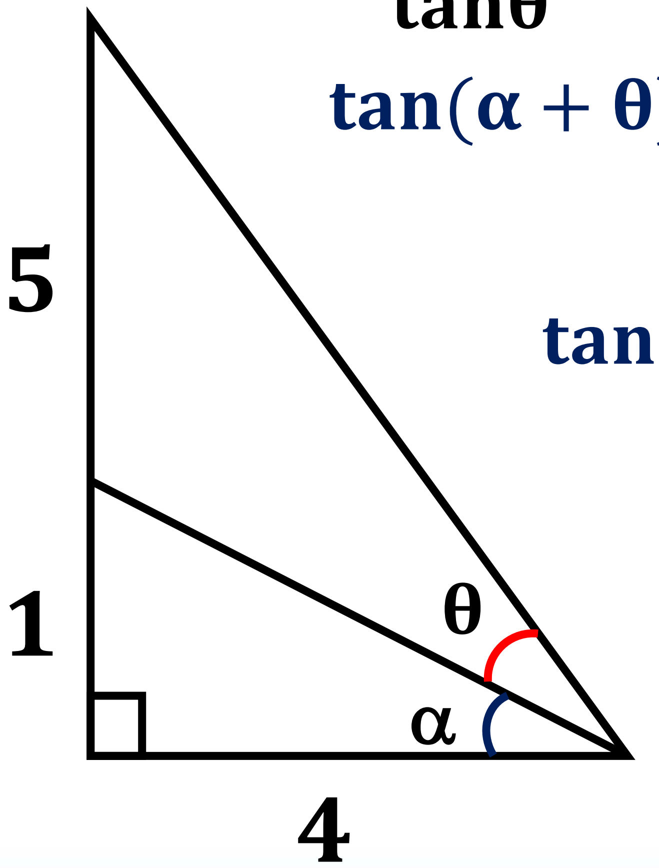
$$\tan 2y=\frac{\frac{1}{3}-2}{1+\frac{1}{3}.2}$$

$$\tan 2y=\frac{-\frac{5}{\cancel{3}}}{\frac{5}{\cancel{3}}}$$

$$\boxed{\therefore\ \tan 2y=-1}$$

PREGUNTA 7

A partir del gráfico, determine el valor de $\tan\theta$



$$\tan(\alpha+\theta)=\frac{6}{4}=\frac{3}{2}$$

$$\tan\alpha=\frac{1}{4}$$

Resolución:

Recordamos:

$$\tan(\alpha+\theta)=\frac{\tan\alpha+\tan\theta}{1-\tan\alpha.\tan\theta}$$

$$\frac{3}{2}=\frac{\frac{1}{4}+\tan\theta}{1-\frac{1}{4}.\tan\theta}\Rightarrow\frac{3}{2}=\frac{\frac{1+4\tan\theta}{\cancel{4}}}{\frac{4-\tan\theta}{\cancel{4}}}$$

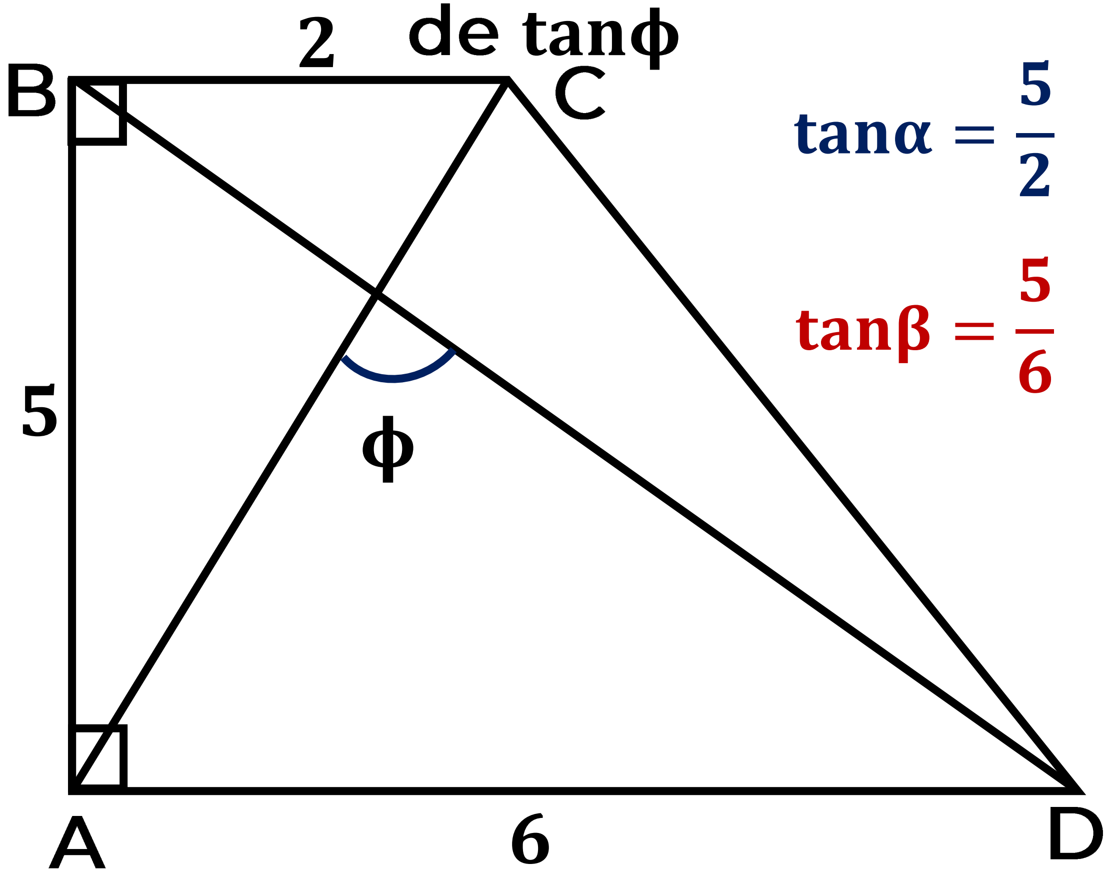
$$\frac{3}{2}=\frac{1+4\tan\theta}{4-\tan\theta}\Rightarrow 12-3\tan\theta=2+8\tan\theta$$

$$10=11\tan\theta$$

$$\boxed{\therefore\ \tan\theta=\frac{10}{11}}$$

PREGUNTA 8

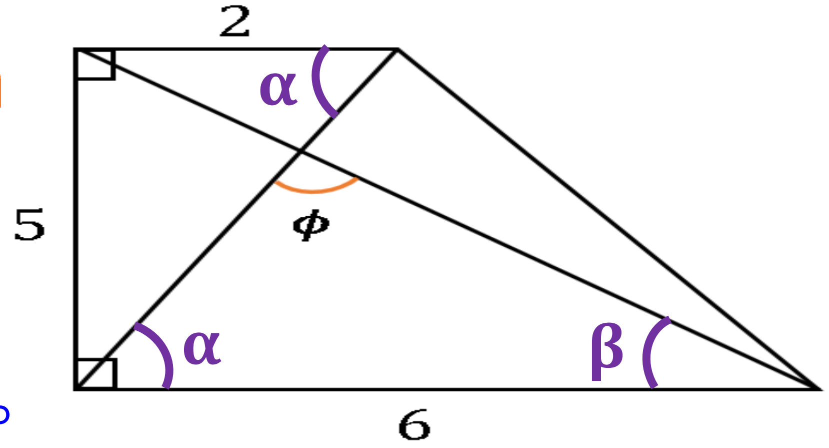
En el trapecio ABCD mostrado, determine el valor de tanϕ



$\tan\alpha = \frac{5}{2}$

$\tan\beta = \frac{5}{6}$

Resolución:



Observamos:

$\alpha + \beta + \emptyset = 180°$

$\Rightarrow \tan\alpha + \tan\beta + \tan\emptyset = \tan\alpha.\tan\beta.\tan\emptyset$

$$\frac{5}{2} + \frac{5}{6} + \tan\emptyset = \frac{5}{2}.\frac{5}{6}.\tan\emptyset$$

$$\frac{40}{12} + \tan\emptyset = \frac{25}{12}.\tan\emptyset \quad \ldots \times (12)$$

$$40 + 12\tan\emptyset = 25\tan\emptyset$$

$$\Rightarrow 40 = 13\tan\emptyset$$

$$\therefore \tan\phi = \frac{40}{13}$$