



TRIGONOMETRY

Chapter 17

Session 2

4th
SECONDARY

Identidades Trigonométricas
Auxiliares



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Identidades Auxiliares

$$\tan x + \cot x = \sec x \cdot \csc x$$

$$\sec^2 x + \csc^2 x = \sec^2 x \cdot \csc^2 x$$

$$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cdot \cos^2 x$$

$$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cdot \cos^2 x$$





Identidades Auxiliares

$$(1 \pm \operatorname{sen} x \pm \operatorname{cos} x)^2 = 2(1 \pm \operatorname{sen} x)(1 \pm \operatorname{cos} x)$$

$$\frac{\operatorname{cos} x}{1 + \operatorname{sen} x} = \frac{1 - \operatorname{sen} x}{\operatorname{cos} x}$$

$$\frac{\operatorname{cos} x}{1 - \operatorname{sen} x} = \frac{1 + \operatorname{sen} x}{\operatorname{cos} x}$$

$$\frac{\operatorname{sen} x}{1 + \operatorname{cos} x} = \frac{1 - \operatorname{cos} x}{\operatorname{sen} x}$$

$$\frac{\operatorname{sen} x}{1 - \operatorname{cos} x} = \frac{1 + \operatorname{cos} x}{\operatorname{sen} x}$$

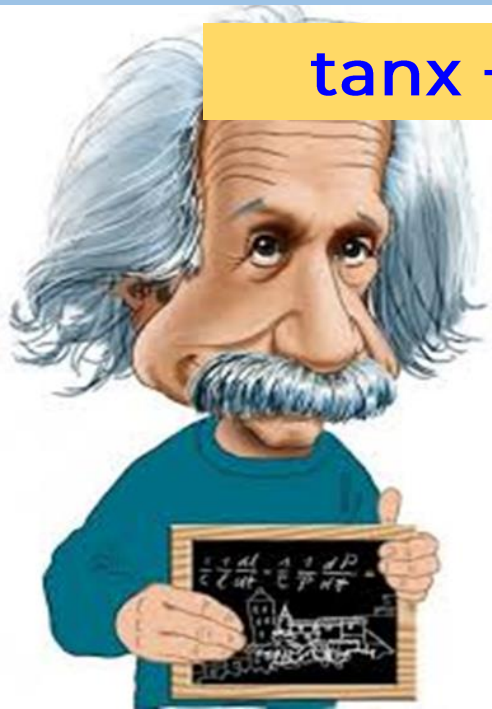




1. Si: $\tan x + \cot x = 6$
 Reduzca: $E = \sin^4 x + \cos^4 x$

$$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x \cdot \cos^2 x$$

$$\tan x + \cot x = \sec x \cdot \csc x$$



RESOLUCIÓN

Por dato: $\tan x + \cot x = 6$

$$\sec x \cdot \csc x = 6$$

Invirtiendo: $\sin x \cdot \cos x = \frac{1}{6}$

Nos piden: $E = \sin^4 x + \cos^4 x$

$$E = 1 - 2\sin^2 x \cdot \cos^2 x$$

$$E = 1 - 2(\sin x \cdot \cos x)^2$$

$$E = 1 - 2\left(\frac{1}{6}\right)^2 = 1 - 2\left(\frac{1}{36}\right)$$

$$\therefore E = \frac{17}{18}$$



2. Si $\sec^2\theta + \csc^2\theta = 36$; donde $\theta \in \text{IC}$,
 reduzca: $E = \tan\theta + \cot\theta - 2$

$$\tan x + \cot x = \sec x \cdot \csc x$$

$$\sec^2 x + \csc^2 x = \sec^2 x \cdot \csc^2 x$$



RESOLUCIÓN

Por dato: $\sec^2\theta + \csc^2\theta = 36$

$$\sec^2\theta \cdot \csc^2\theta = 36$$

$$\sec\theta \cdot \csc\theta = 6$$

Nos piden: $E = \tan\theta + \cot\theta - 2$

$$E = \sec\theta \cdot \csc\theta - 2$$

$$E = 6 - 2$$

$$\therefore E = 4$$





3. Si $\tan\alpha + \cot\alpha = 4$, reduzca:
 $K = \sec^2\alpha + \csc^2\alpha + 1$

$$\sec^2x + \csc^2x = \sec^2x \cdot \csc^2x$$

$$\tan x + \cot x = \sec x \cdot \csc x$$



RESOLUCIÓN

Por dato: $\tan\alpha + \cot\alpha = 4$

$$\sec\alpha \cdot \csc\alpha = 4$$

Nos piden: $K = \sec^2\alpha + \csc^2\alpha + 1$

$$K = \sec^2\alpha \cdot \csc^2\alpha + 1$$

$$K = (\sec\alpha \cdot \csc\alpha)^2 + 1$$

$$K = (4)^2 + 1$$

$$\therefore K = 17$$





4. Si $\text{sen}\phi + \text{cos}\phi = \sqrt{\frac{3}{2}}$
 reduzca: $G = \text{tan}\phi + \text{cot}\phi$

RESOLUCIÓN

Por dato: $\text{sen}\phi + \text{cos}\phi = \sqrt{\frac{3}{2}}$

$$(\text{sen}\phi + \text{cos}\phi)^2 = \left(\sqrt{\frac{3}{2}}\right)^2$$

$$\underbrace{\text{sen}^2\phi + \text{cos}^2\phi}_1 + 2\text{sen}\phi.\text{cos}\phi = \frac{3}{2}$$

$$1 + 2\text{sen}\phi.\text{cos}\phi = \frac{3}{2}$$

$$2\text{sen}\phi.\text{cos}\phi = \frac{1}{2}$$

$$\text{sen}\phi.\text{cos}\phi = \frac{1}{4}$$

$$\text{sec}\phi.\text{csc}\phi = 4$$

Nos piden: $G = \text{tan}\phi + \text{cot}\phi$

$$G = \text{sec}\phi.\text{csc}\phi$$

$$G = 4$$

$$\therefore G = 4$$





5. Si $\sin^6 \alpha + \cos^6 \alpha = \frac{1}{3}$
 reduzca $E = (1 + \sin^2 \alpha)(1 + \cos^2 \alpha)$

RESOLUCIÓN

Por dato: $\sin^6 \alpha + \cos^6 \alpha = \frac{1}{3}$

$$1 - 3\sin^2 \alpha \cos^2 \alpha = \frac{1}{3}$$

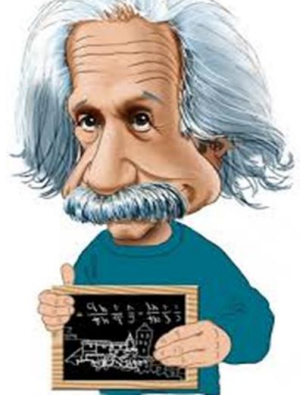
$$\frac{2}{3} = 3\sin^2 \alpha \cos^2 \alpha$$

$$\frac{2}{9} = \sin^2 \alpha \cos^2 \alpha$$

$$\sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x$$

Nos piden:

$$E = (1 + \sin^2 \alpha)(1 + \cos^2 \alpha)$$



$$E = 1 + \underbrace{\cos^2 \alpha + \sin^2 \alpha}_{1} + \sin^2 \alpha \cos^2 \alpha$$

$$E = 1 + 1 + \frac{2}{9}$$

$$\therefore E = \frac{20}{9}$$





6. Si $\tan x + \cot x = 5$
 reduzca: $P = \sec x - \csc x$

RESOLUCIÓN

Tenemos: $\tan x + \cot x = 5$

$$\sec x \cdot \csc x = 5$$

$$\sec^2 x \cdot \csc^2 x = 25$$

$$\sec^2 x + \csc^2 x = 25$$

Nos piden:

$$P = \sec x - \csc x \quad \dots \text{Al cuadrado.}$$

$$P^2 = \sec^2 x + \csc^2 x - 2\sec x \cdot \csc x$$

$$P^2 = 25 - 2(5)$$

$$P^2 = 15$$

$$\therefore P = \sqrt{15}$$



7. Si $\operatorname{sen} \alpha - \operatorname{cos} \alpha = \frac{\sqrt{3}}{3} - 1$
Efectúe $E = (1 + \operatorname{sen} \alpha)(1 - \operatorname{cos} \alpha)$

$$(1 + \operatorname{sen} x - \operatorname{cos} x)^2 = 2(1 + \operatorname{sen} x)(1 - \operatorname{cos} x)$$



RESOLUCIÓN

Tenemos: $E = (1 + \operatorname{sen} \alpha)(1 - \operatorname{cos} \alpha)$

$$2E = 2(1 + \operatorname{sen} \alpha)(1 - \operatorname{cos} \alpha)$$

$$2E = (1 + \operatorname{sen} \alpha - \operatorname{cos} \alpha)^2$$

$$2E = \left(1 + \frac{\sqrt{3}}{3} - 1\right)^2$$

$$2E = \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{3}{9}$$

$$2E = \frac{1}{3}$$

$$\therefore E = \frac{1}{6}$$





- 8.** El gasto diario de Kelly en pasaje es S/ $B \cot x$. ¿Cuál será el gasto total de la semana? Para ello resuelva lo siguiente:

$$B = \left(\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} \right) \sin x$$

$$\frac{\cos x}{1 + \sin x} = \frac{1 - \sin x}{\cos x}$$

$$\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$$



RESOLUCIÓN

Por dato:

$$B = \left(\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} \right) \sin x$$

$$B = \left(\frac{1 - \cancel{\sin x}}{\cos x} + \frac{1 + \cancel{\sin x}}{\cos x} \right) \sin x$$

$$B = \left(\frac{2}{\cos x} \right) \sin x \quad \Rightarrow \quad B = 2 \tan x$$

El gasto diario será: $B \cdot \cot x$

$$= \underbrace{2 \tan x \cdot \cot x}_1 = 2 \text{ soles}$$

\therefore El gasto semanal: S/14