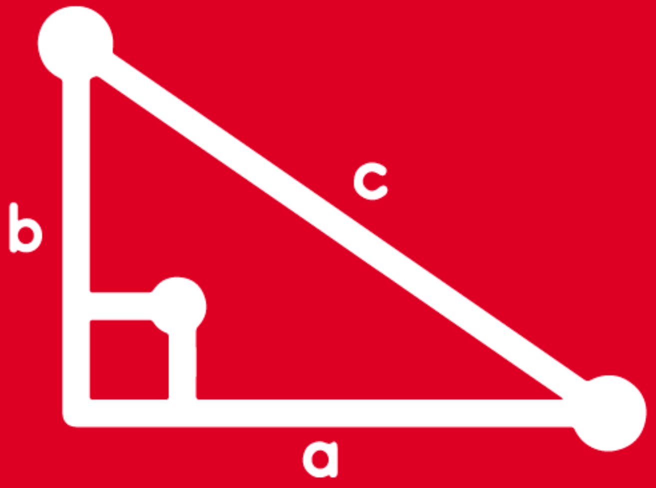
TRIGONOMETRY

Chapter 7, 8 and 9



5th
SECONDARY

REVIEW

SACO OLIVEROS

HELICOREVIEW 1

Siendo θ y β las medidas de dos ángulos cuadrantales diferentes, positivos y menores o iguales a 360°, se cumple que

$$\sqrt{1-\operatorname{sen}\theta}+\sqrt{\operatorname{sen}\theta-1}=1+\cos\beta \;\ldots\ldots\; (*)$$

calcule θ + β.

Resolución:

$$1-\operatorname{sen}\theta \geq 0 \quad \wedge \quad \operatorname{sen}\theta-\mathbf{1} \geq 0$$

$$\underbrace{1 \geq \operatorname{sen}\theta \quad \wedge \quad \operatorname{sen}\theta \geq 1}$$

$$\operatorname{sen}\theta = 1$$

$$\text{como } 0°<\theta \leq 360°$$

$$\Rightarrow \theta = 90°$$

Reemplazando en (*)

$$\underbrace{\cancel{\sqrt{1-\operatorname{sen}\theta}}}_{\mathbf{0}}+\underbrace{\cancel{\sqrt{\operatorname{sen}\theta-1}}}_{\mathbf{0}}=1+\cos\beta$$

Recordar

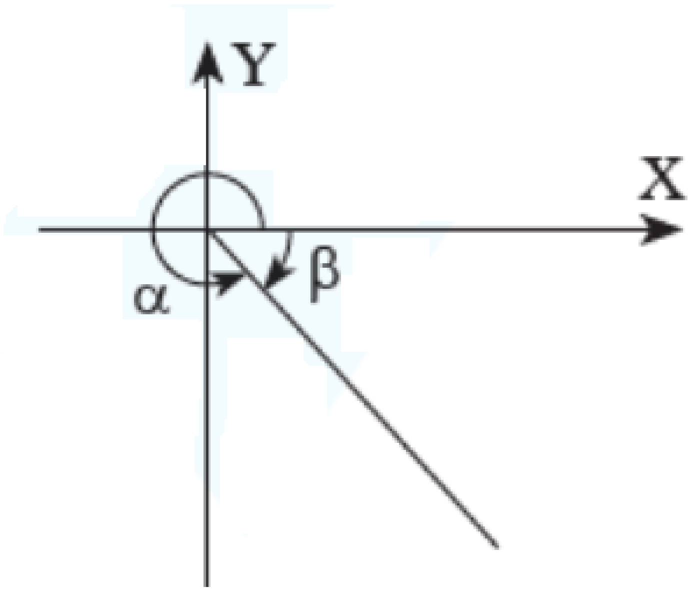
RT \ ∡	0°	90°	180°	270°	360°
sen	0	1	0	–1	0
cos	1	0	–1	0	1

HELICOREVIEW 2

En la figura, se cumple que

$\cot\alpha.\cot\beta + \cos\alpha.\sec\beta = 10$. Calcule $\cot\alpha$

Resolución:



α y $\beta \in$ IVC

Del gráfico se observa que α y β son las medidas de dos ángulos coterminales, se cumple:

$$Rt(\alpha) = Rt(\beta)$$

Recordar

sen, csc (+) | Todas las RT son (+)
tan, cot (+) | cos, sec (+)

Del dato:

$$\cot\alpha.\cot\beta + \cos\alpha.\sec\beta = 10$$

$$\cot\alpha.\cot\alpha + \underbrace{\cos\alpha.\sec\alpha}_{1} = 10$$

$$\cot^2\alpha + 1 = 10$$

$$\cot^2\alpha = 9 \Rightarrow \cot\alpha = \pm\sqrt{9}$$

$$\cot\alpha = \pm 3$$

Como $\alpha \in$ IVC

$$\therefore \boxed{\cot\alpha = -3}$$

HELICOREVIEW 3

Si $\cos\theta > 0$, además $16^{\cot\theta} = 0{,}25$, efectúe

$M = \sqrt{5}\,(\operatorname{sen}\theta - \cos\theta)$

Recordar

sen, csc (+)	Todas las RT son (+)
tan, cot (+)	cos, sec (+)

Resolución:

Del dato:

$$16^{\cot\theta} = \frac{1}{4}$$

$$4^{2\cot\theta} = 4^{-1}$$

$$2\cot\theta = -1$$

$$\cot\theta = -\frac{1}{2}$$

Como $\cos\theta$ es $(+)$ y $\cot\theta$ es $(-)$

$\theta \in \text{IVC}$ ➡ $x(+), y(-), r(+)$

- $\cot\theta = \dfrac{1}{-2} = \dfrac{x}{y}$ ➡ $x = 1\,,\, y = -2$

$r = \sqrt{1^2 + (-2)^2}$ ➡ $r = \sqrt{5}$

$$r = \sqrt{x^2 + y^2}$$

Piden: $M = \sqrt{5}\,(\operatorname{sen}\theta - \cos\theta)$

$$M = \cancel{\sqrt{5}}\left(\frac{-2}{\cancel{\sqrt{5}}} - \frac{1}{\cancel{\sqrt{5}}}\right) = -2 - 1$$

$\therefore$ $\boxed{M = -3}$

HELICOREVIEW 4

Halle el valor de:

$$E = \frac{\operatorname{sen}330°.\cos 120°}{\tan 225°}$$

Resolución:

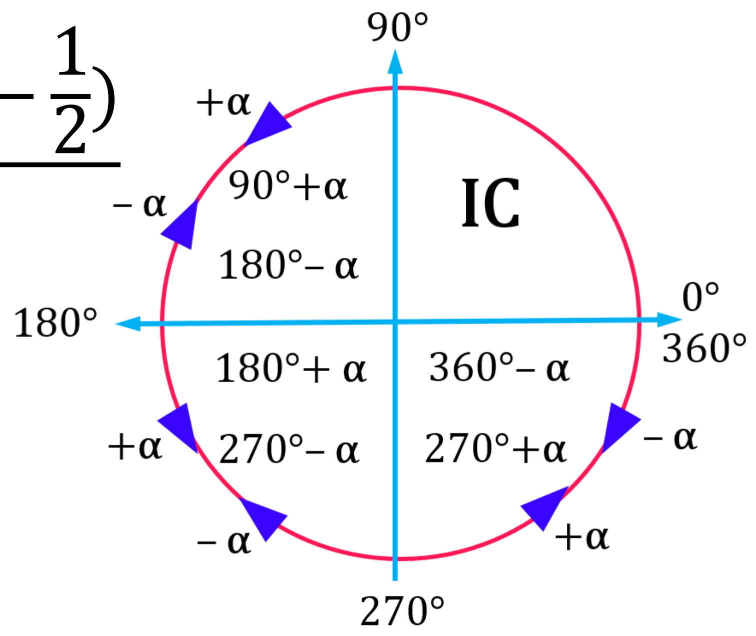
$$\boldsymbol{E} = \frac{\overbrace{Sen\,(360° - 30°)}^{\text{IVC}}\,\overbrace{Cos\,(180° - 60°)}^{\text{IIC}}}{tan\underbrace{(180° + 45°)}_{\text{IIIC}}}$$

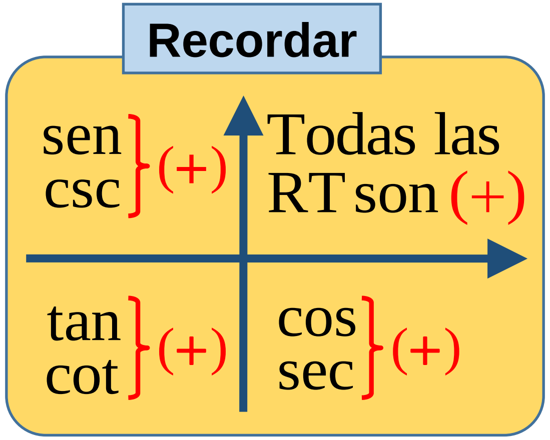
$$\boldsymbol{E} = \frac{(-Sen\,30°)(-Cos\,60°)}{tan45°}$$

$$\boldsymbol{E} = \frac{(-\frac{1}{2})(-\frac{1}{2})}{1}$$

$$E = \frac{\frac{1}{4}}{1}$$

$$\therefore \boxed{E = \frac{1}{4}}$$



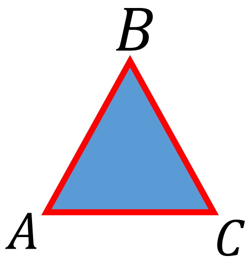


HELICOREVIEW 5

En un triángulo ABC, reduzca $M=\dfrac{\text{sen}(B+C)}{\cos\left(\dfrac{3A+B+C}{2}\right)}$

Resolución:

Del dato:



$$A+B+C=180^\circ$$

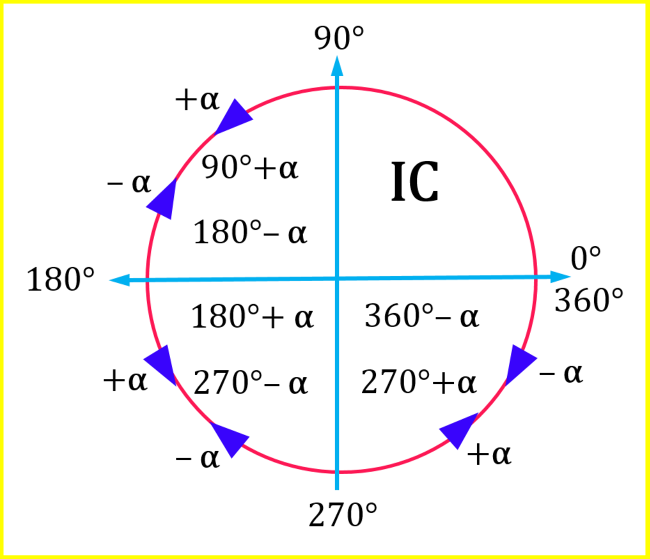
Piden:

$$M=\frac{sen(B+C)}{\cos\left(\dfrac{3A+B+C}{2}\right)}$$

$$M=\frac{sen(B+C)}{\cos\left(\dfrac{3A+B+C}{2}\right)}$$

$$M=\frac{sen(180^\circ-A)}{\cos\left(\dfrac{3A+180^\circ-A}{2}\right)}$$

$$M=\frac{sen(\overbrace{180^\circ-A}^{IIC})}{\cos\left(\underbrace{90^\circ+A}_{IIC}\right)}$$



$$M=\frac{\cancel{senA}}{-\cancel{senA}}$$

$\therefore$ $\boxed{M = -1}$

Recordar:

sen, csc (+) | Todas las RT son (+)

tan, cot (+) | cos, sec (+)

HELICOREVIEW 6

Si α ∈ IVC, además sen(270° + α) = – 0,8, reduzca

T = csc(180° – α) + tan(270° + α)

Resolución:

$$T = \underbrace{\overbrace{\csc(180° - \alpha)}^{IIC}}_{\csc\alpha} + \underbrace{\overbrace{\tan(270° + \alpha)}^{IVC}}_{-\cot\alpha}$$

$$T = \csc\alpha - \cot\alpha \quad ...(*)$$

Del dato:

$$sen(\overbrace{270° + \alpha}^{IVC}) = -0{,}8$$

$$\cancel{-}\cos\alpha = \cancel{-}\frac{4}{5}$$

$$\cos\alpha = \frac{4}{5} = \frac{x}{r}$$

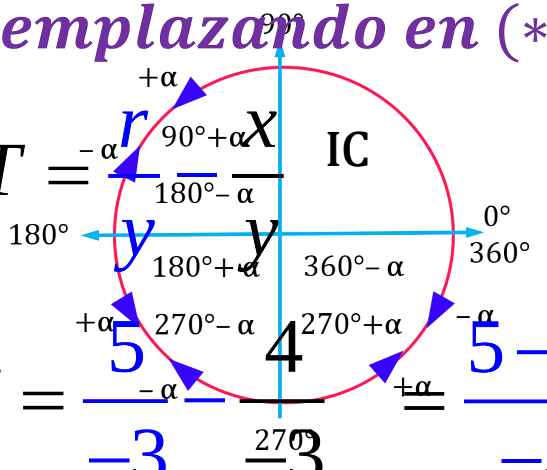
Por radio vector:

$$r = \sqrt{x^2 + y^2}$$

$$5 = \sqrt{4^2 + y^2} \rightarrow y = -3$$

Reemplazando en (*) :

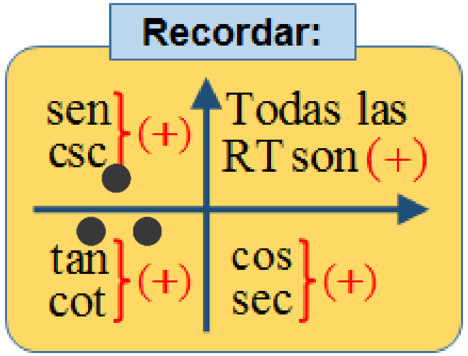
$$T = \frac{r}{y} - \frac{x}{y}$$



$$T = \frac{5}{-3} - \frac{4}{-3} = \frac{5-4}{-3}$$

$$T = \frac{1}{-3}$$

Recordar:



HELICOREVIEW 7

Efectúe

$$P = \frac{\cos 1470° . \operatorname{sen} 1140°}{\cot 3285°}$$

Resolución:

$$\begin{array}{r|l} 1470 & 360 \\ \cline{2-2} 1440 & 4 \\ \hline 30 & \end{array} \qquad \begin{array}{r|l} 1140 & 360 \\ \cline{2-2} 1080 & 3 \\ \hline 60 & \end{array}$$

$$\begin{array}{r|l} 3285 & 360 \\ \cline{2-2} 3240 & 9 \\ \hline 45 & \end{array}$$

Recordar:

$$RT(\cancel{360°k} + x) = RT(x) \; ; k \in Z$$

$$P = \frac{\cos 30° . \operatorname{sen} 60°}{\cot 45°}$$

$$P = \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{1}$$

$$\therefore \boxed{P = \frac{3}{4}}$$

HELICOREVIEW 8

Halle el valor de $\mathrm{E} = sen\left(\dfrac{37\pi}{6}\right) + \cos\left(\dfrac{59\pi}{3}\right)$

Resolución:

Dando forma a los ángulos

$$\mathrm{E} = \mathrm{sen}\left(\frac{36\pi+\pi}{6}\right) + \cos\left(\frac{60\pi-\pi}{3}\right)$$

$$\mathrm{E} = \mathrm{sen}\left(\frac{36\pi}{6}+\frac{\pi}{6}\right) + \cos\left(\frac{60\pi}{3}-\frac{\pi}{3}\right)$$

$$\mathrm{E} = \mathrm{sen}\left(\underset{PAR}{6\pi}+\frac{\pi}{6}\right) + \cos\left(\underset{PAR}{20\pi}-\frac{\pi}{3}\right)$$

$$\mathrm{E} = \underbrace{\mathrm{sen}\overbrace{\left(6\pi+\frac{\pi}{6}\right)}^{IC}}_{sen\frac{\pi}{6}} + \underbrace{\cos\overbrace{\left(20\pi-\frac{\pi}{3}\right)}^{IVC}}_{\cos\frac{\pi}{3}}$$

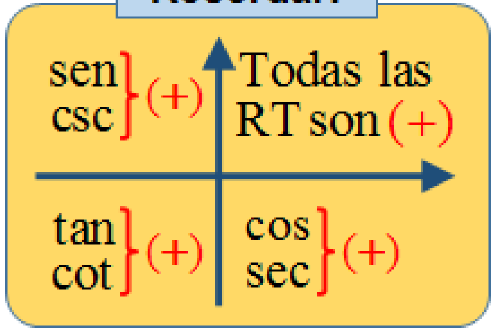
$$\mathrm{E} = \mathrm{sen}\frac{\pi}{6} + \cos\frac{\pi}{3}$$

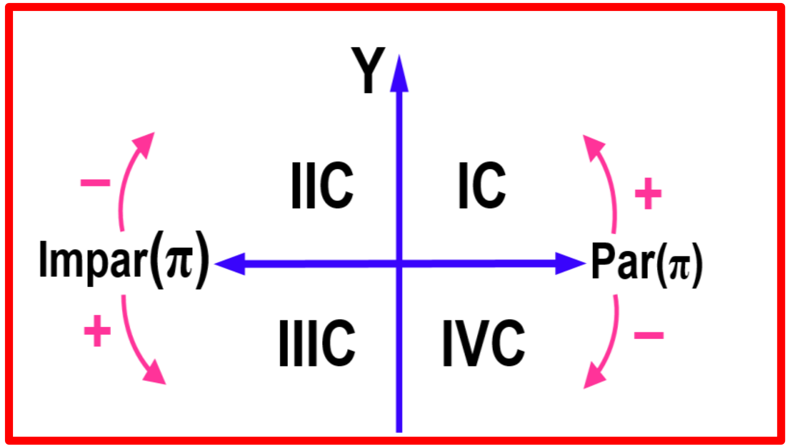
$$\mathrm{E} = \mathrm{sen}30^{\circ} + \cos60^{\circ}$$

$$\mathrm{E} = \frac{1}{2} + \frac{1}{2}$$

$$\therefore \boxed{\mathrm{E} = 1}$$

Recordar:





HELICOREVIEW 9

Efectúe:

$$M=\sec\left(\frac{13\pi}{2}+\theta\right).\tan(22\pi+\theta),\quad \text{si } \cos\theta=\frac{1}{2},\ \text{donde } \theta\in \text{IVC}.$$

Resolución:

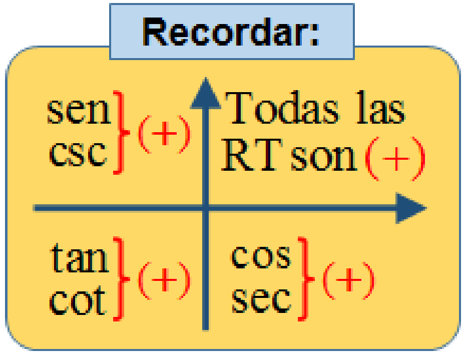
$$M=\sec\left(\overset{\overset{\circ}{4}+1}{13}\frac{\pi}{2}+\theta\right).\tan(\overset{PAR}{22}\pi+\theta)$$

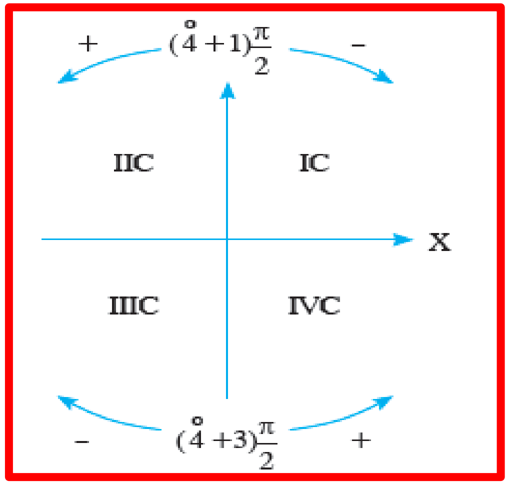
$$M=\underbrace{\sec\overbrace{\left(13\frac{\pi}{2}+\theta\right)}^{IIC}}_{-\csc\theta}.\underbrace{\tan\overbrace{(22\pi+\theta)}^{IC}}_{\tan\theta}$$

$$M=-\csc\theta.\tan\theta$$

$$M=-\frac{r}{\cancel{y}}.\frac{\cancel{y}}{x}$$

$$M=-\frac{r}{x}\ \ldots(*)$$



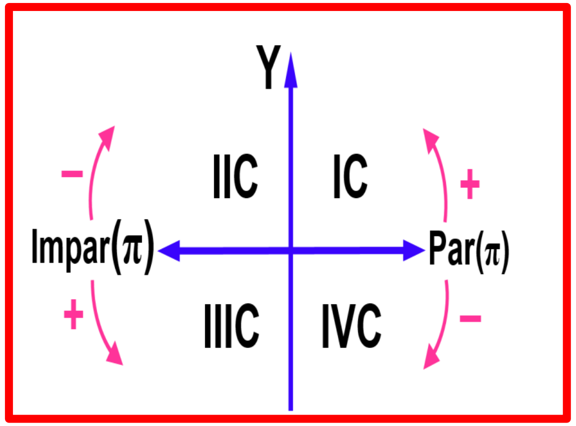


Del dato:

$$\cos\theta=\frac{1}{2}=\frac{x}{r}\quad\rightarrow\frac{r}{x}=2$$

Remplazando en (*)

$$\therefore\ \boxed{M=-2}$$



HELICOREVIEW 10

Se sabe que: $\cot\theta = -0{,}75 \; y \; \cos\theta < 0$

Determine: $M = 5\text{sen}\theta + 3\sec\theta + 1$

Recordar:

$\left.\begin{matrix}\text{sen}\\ \csc\end{matrix}\right\}(+)$ Todas las RT son $(+)$

$\left.\begin{matrix}\tan\\ \cot\end{matrix}\right\}(+)$ $\left.\begin{matrix}\cos\\ \sec\end{matrix}\right\}(+)$

Resolución:

Del dato: $\cot\theta = -\dfrac{75}{100}$

$\Rightarrow \cot\theta = -\dfrac{3}{4} \; y \; \cos\theta < 0$

$\rightarrow \theta \in IIC$

Sabemos:

$x(-), y(+), r(+)$

Se tiene que:

$\cot\theta = \dfrac{-3}{4} = \dfrac{x}{y}$ $\rightarrow x = -3$, $\rightarrow y = 4$

Por radio vector:

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-3)^2 + 4^2}$ $\rightarrow r = 5$

Piden

$M = 5\text{sen}\theta + 3\sec\theta + 1$

$M = 5\left(\dfrac{y}{r}\right) + 3\left(\dfrac{r}{x}\right) + 1$

$M = 5\left(\dfrac{4}{5}\right) + 3\left(\dfrac{5}{-3}\right) + 1$

$M = 4 - 5 + 1$

$\therefore \boxed{M = 0}$