

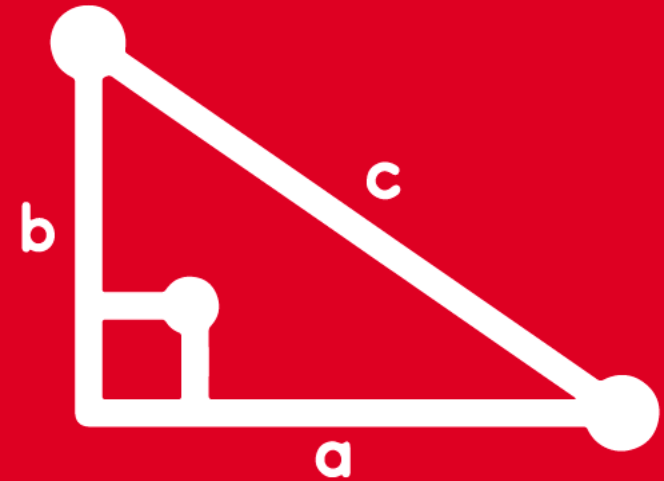


TRIGONOMETRY

Chapter 24

2nd
SECONDARY

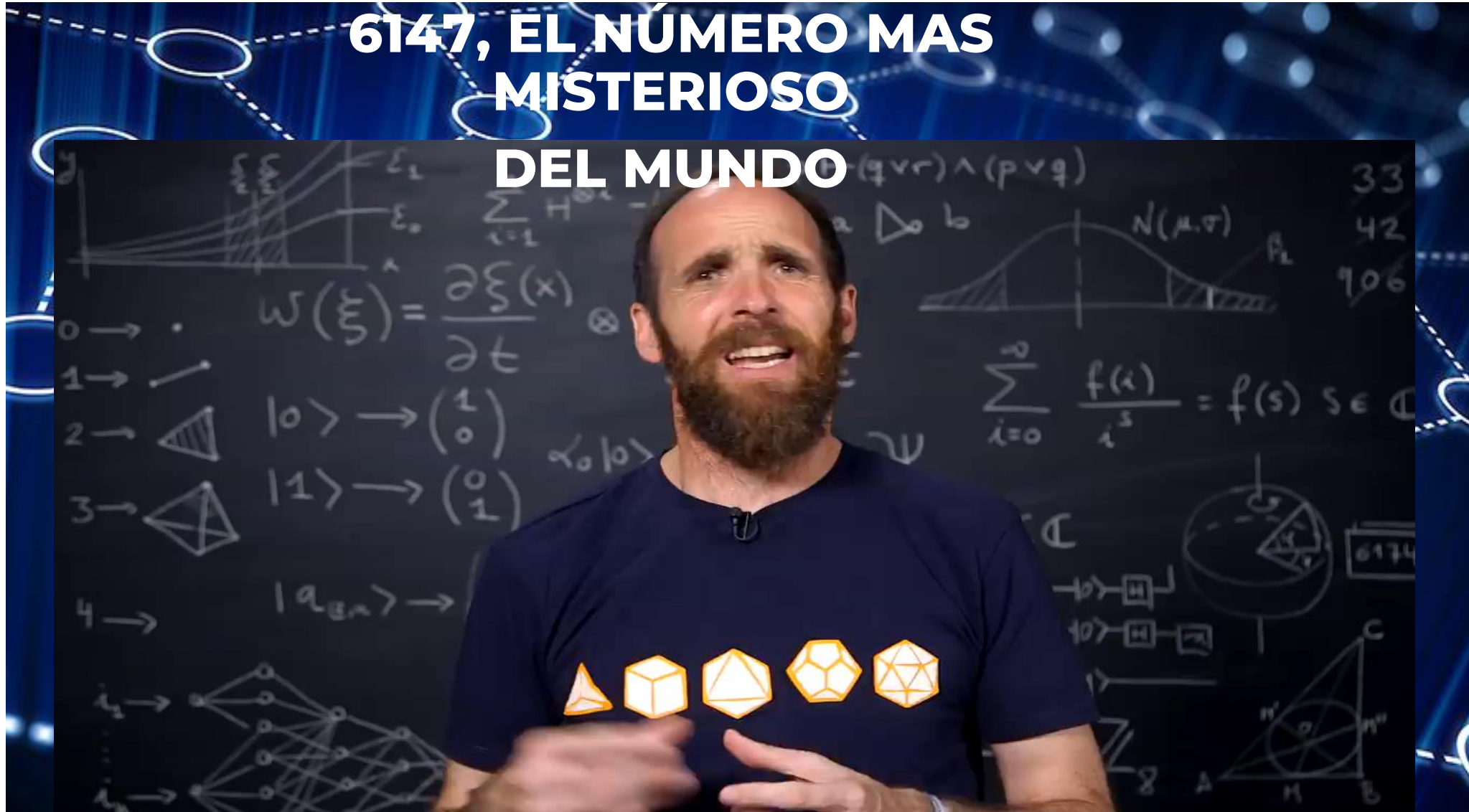
IDENTIDADES TRIGONOMÉTRICAS
FUNDAMENTALES III



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Identidades trigonométricas

¿Qué son las identidades

trigonométricas?

son igualdades entre expresiones que contienen razones trigonométricas de una o mas variables, las cuales se verifican para un conjunto de valores admisibles.

IDENTIDADES TRIGONOMÉTRICAS FUNDAMENTALES

Identidades Recíprocas:

$$\text{sen}x \cdot \text{csc}x = 1 \quad \left\{ \begin{array}{l} \text{sen}x = \frac{1}{\text{csc}x} \\ \text{csc}x = \frac{1}{\text{sen}x} \end{array} \right.$$

$$\text{cos}x \cdot \text{sec}x = 1$$

$$\left\{ \begin{array}{l} \text{cos}x = \frac{1}{\text{sec}x} \\ \text{sec}x = \frac{1}{\text{cos}x} \end{array} \right.$$

$$\text{tan}x \cdot \text{cot}x = 1$$

$$\left\{ \begin{array}{l} \text{tan}x = \frac{1}{\text{cot}x} \\ \text{cot}x = \frac{1}{\text{tan}x} \end{array} \right.$$





Identidades trigonométricas

Identidades por división:

$$\tan x = \frac{\text{sen} x}{\cos x}$$

$$\cot x = \frac{\cos x}{\text{sen} x}$$

Identidades pitagóricas:

$$\text{sen}^2 x + \cos^2 x = 1$$

$$\text{sen}^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \text{sen}^2 x$$

$$\sec^2 x - \tan^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$\csc^2 x - \cot^2 x = 1$$

$$\csc^2 x = 1 + \cot^2 x$$

$$\cot^2 x = \csc^2 x - 1$$





PROBLEMA 1

Simplifique $L = 3 \cdot \cot x \cdot \operatorname{sen} x - \cos x$

Resolución:

$$L = 3 \cdot \cot x \cdot \operatorname{sen} x - \cos x$$

$$L = 3 \cdot \left(\frac{\cos x}{\cancel{\operatorname{sen} x}} \right) \cdot \cancel{\operatorname{sen} x} - \cos x$$

$$\therefore L = 2 \cos x$$



$$\cot x = \frac{\cos x}{\operatorname{sen} x}$$



**PROBLEMA 2**

Reduzca: $B = \frac{\text{sen}x}{\text{csc}x} + \frac{\text{cos}x}{\text{sec}x} + 2$

Resolución:

$$B = \frac{\text{sen}x}{\text{csc}x} + \frac{\text{cos}x}{\text{sec}x} + 2$$

$$B = \frac{\text{sen}x}{\frac{1}{\text{sen}x}} + \frac{\text{cos}x}{\frac{1}{\text{cos}x}} + 2$$

$$B = \underbrace{\text{sen}^2x + \text{cos}^2x}_1 + 2$$

$$\therefore B = 3$$

$$\text{csc}x = \frac{1}{\text{sen}x}$$

$$\text{sec}x = \frac{1}{\text{cos}x}$$

$$\text{sen}^2x + \text{cos}^2x = 1$$





PROBLEMA 3

Reduzca: $Q = (\csc x - \sen x) \cdot \tan x$

Resolución:

$$\begin{aligned}
 Q &= (\csc x - \sen x) \cdot \tan x \\
 Q &= \left(\frac{1}{\sen x} - \sen x \right) \cdot \frac{\sen x}{\cos x} \\
 Q &= \left(\frac{1 - \cancel{\sen^2 x}}{\cancel{\sen x}} \right) \frac{\cancel{\sen x}}{\cos x} \\
 Q &= \frac{\cos^2 x}{\cos x}
 \end{aligned}$$

$$\therefore Q = \cos x$$

$$\csc x = \frac{1}{\sen x}$$

$$\tan x = \frac{\sen x}{\cos x}$$

$$1 - \sen^2 x = \cos^2 x$$





PROBLEMA 4

Reduzca: $P = (\tan x + \cot x) \cdot \cos x$

Resolución:

$$P = (\tan x + \cot x) \cdot \cos x$$

$$P = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) \cdot \cos x$$

$$P = \left(\frac{\sin^2 x + \cos^2 x}{\cancel{\cos x} \cdot \sin x} \right) \cdot \cancel{\cos x}$$

$$P = \frac{1}{\sin x}$$

$$\therefore P = \csc x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{1}{\sin x} = \csc x$$





PROBLEMA 5

Si: $\text{sen}x + \text{csc}x = \frac{5}{2}$;

Efectúe $E = \text{sen}^2x + \text{csc}^2x$

Resolución:

Del dato:

$$\text{sen}x + \text{csc}x = \frac{5}{2}$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\text{sen}x \cdot \text{csc}x = 1$$

Calculamos:

$$\underbrace{(\text{sen}x + \text{csc}x)^2}_{\left(\frac{5}{2}\right)^2} = \underbrace{\text{sen}^2x + \text{csc}^2x}_E + \underbrace{2 \cdot \text{sen}x \cdot \text{csc}x}_{2(1)}$$

$$\frac{25}{4} - 2 = E$$

$$\therefore E = \frac{17}{4}$$



**PROBLEMA 6**

Reduzca: $E = (\text{sen}x + \text{cos}x)^2 + (\text{sen}x - \text{cos}x)^2$

Resolución:

$$(a \pm b)^2 = a^2 + b^2 \pm 2ab$$

$$\text{sen}^2x + \text{cos}^2x = 1$$

$$E = (\text{sen}x + \text{cos}x)^2 + (\text{sen}x - \text{cos}x)^2$$

$$E = \underbrace{\text{sen}^2x + \text{cos}^2x}_1 + \cancel{2 \cdot \text{sen}x \cdot \text{cos}x} + \underbrace{\text{sen}^2x + \text{cos}^2x}_1 - \cancel{2 \cdot \text{sen}x \cdot \text{cos}x}$$

$$\therefore E = 2$$



**PROBLEMA 7**

Si: $\text{sen}x - \text{cos}x = \frac{1}{\sqrt{2}};$

Efectúe $P = \text{sen}x \cdot \text{cos}x$

Resolución:

Del dato:

$$\text{sen}x - \text{cos}x = \frac{1}{\sqrt{2}}$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$\text{sen}^2x + \text{cos}^2x = 1$$

Calculamos:

$$(\text{sen}x - \text{cos}x)^2 = \text{sen}^2x + \text{cos}^2x - 2 \cdot \text{sen}x \cdot \text{cos}x$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 = 1 - 2(P)$$

$$2P = 1 - \frac{1}{2}$$

$$2P = \frac{1}{2}$$

$$\therefore P = \frac{1}{4}$$



**PROBLEMA 8**

Al copiar de la pizarra la expresión $\sec^3 x - \sec x$, Lucas cometió un error y escribió $\csc^3 x - \csc x$. Determine la razón entre lo que estaba escrito en la pizarra y lo que escribió Lucas.

$$\sec^2 x - 1 = \tan^2 x$$

$$\csc^2 x - 1 = \cot^2 x$$

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Resolución:

PIDEN: $M = \frac{\sec^3 x - \sec x}{\csc^3 x - \csc x}$

$$\Rightarrow M = \frac{\sec x \cdot (\sec^2 x - 1)}{\csc x \cdot (\csc^2 x - 1)} \Rightarrow M = \frac{\sec x \cdot (\tan^2 x)}{\csc x \cdot (\cot^2 x)}$$

$$\Rightarrow M = \frac{\left(\frac{1}{\cos x}\right) \left(\frac{\sin^2 x}{\cos^2 x}\right)}{\left(\frac{1}{\sin x}\right) \left(\frac{\cos^2 x}{\sin^2 x}\right)} \Rightarrow M = \frac{\left(\frac{\sin^2 x}{\cos^3 x}\right)}{\left(\frac{\cos^2 x}{\sin^3 x}\right)}$$

$$\Rightarrow M = \frac{\sin^5 x}{\cos^5 x}$$

$$\therefore M = \tan^5 x$$