



ALGEBRA

Chapter 24

3rd
SECONDARY

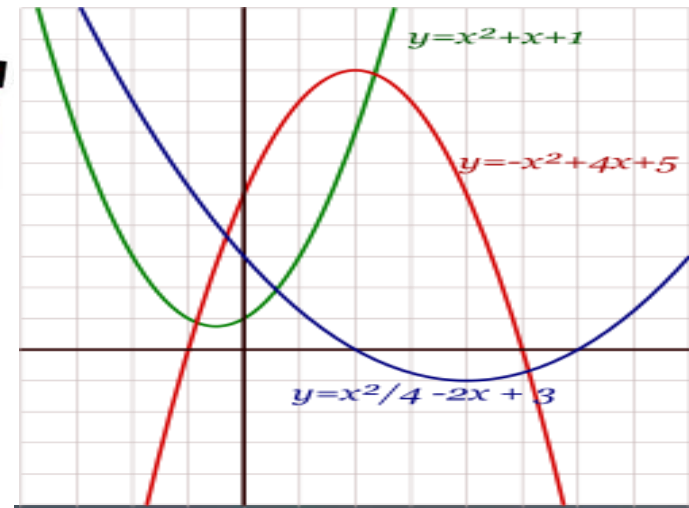
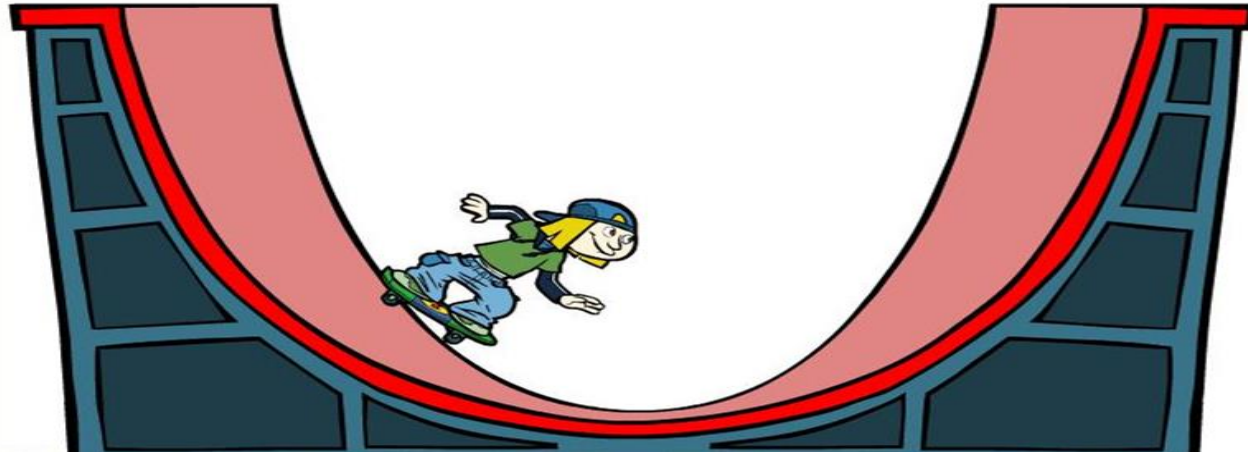
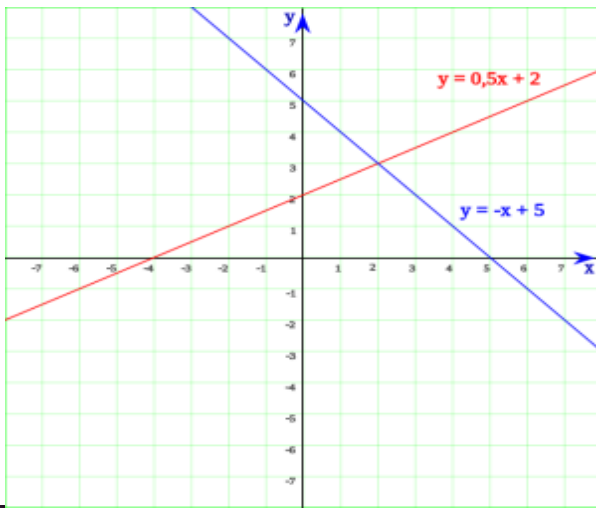
Funciones Especiales II

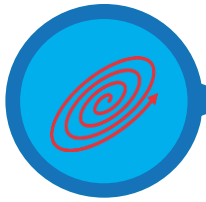


 **SACO OLIVEROS**



FUNCIONES ESPECIALES II





FUNCIÓN LINEAL

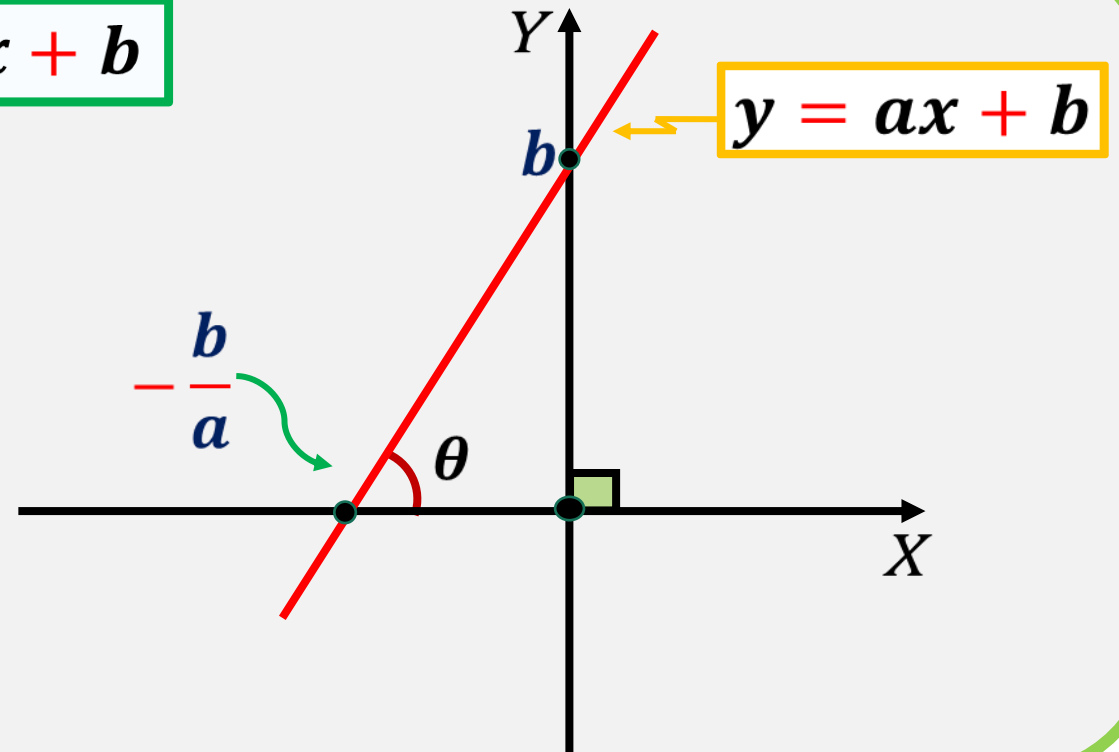


$$f(x) = ax + b$$

$$y = ax + b$$

$$\text{Dom}(f) = \mathbb{R}$$

$$\text{Ran}(f) = \mathbb{R}$$





FUNCIÓN CUADRÁTICA



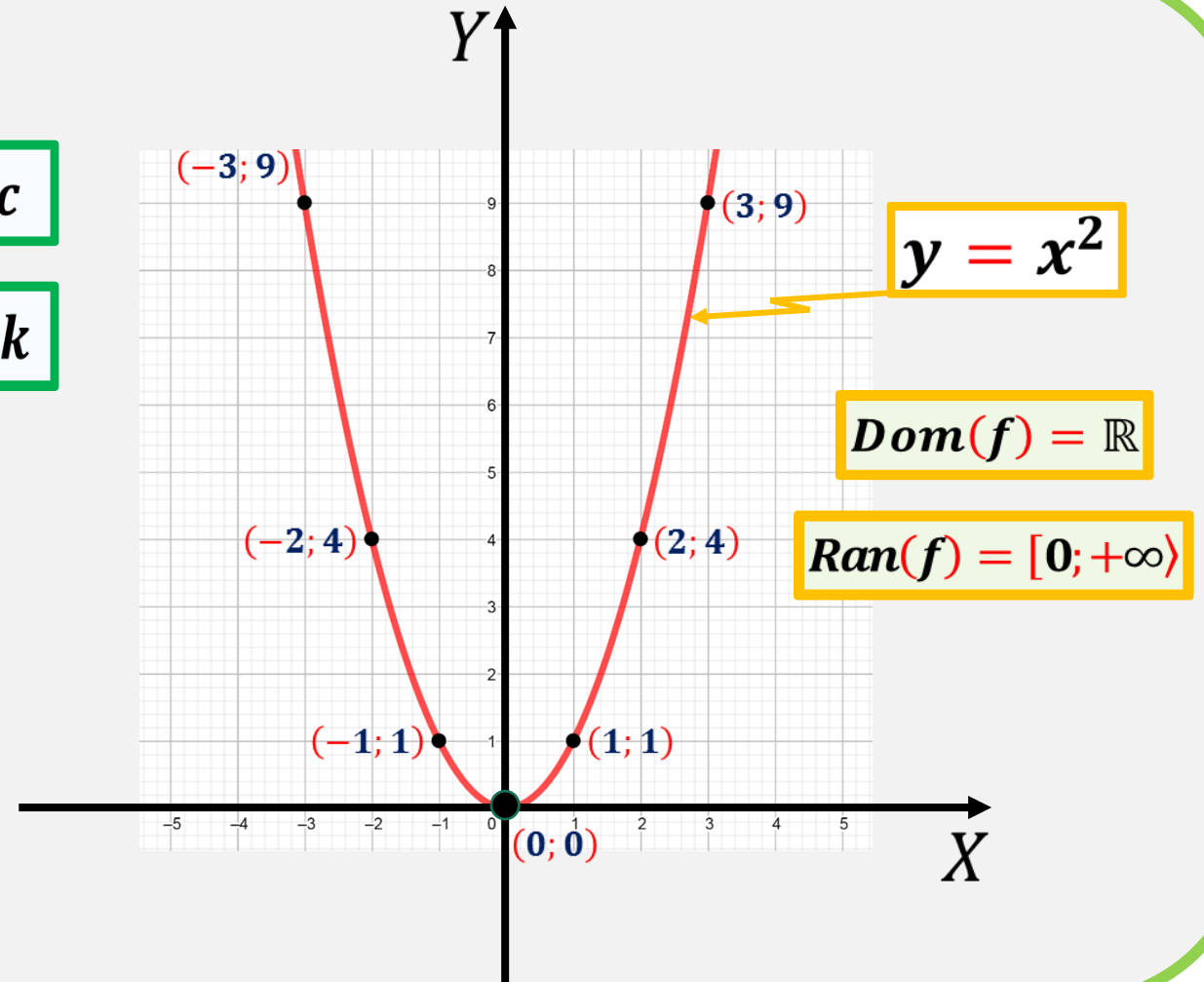
$$y = f(x) = ax^2 + bx + c$$

$$y = f(x) = a(x - h)^2 + k$$

$$\text{Dom}(f) = \mathbb{R}$$

$$\text{Si } a > 0 \longrightarrow \text{Ran}(f) = [k; +\infty)$$

$$\text{Si } a < 0 \longrightarrow \text{Ran}(f) = \langle -\infty; k]$$



**Problema 1**

Sea la función $F(x) = 3x + n$,
donde $F(-2) = 4$. Evalúe
 $F(-3)$.

Resolución:

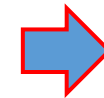
$$F(x) = 3x + n$$

$$F(-2) = 4$$

$$3(-2) + n = 4$$

$$-6 + n = 4$$

$$n = 10$$



$$F(x) = 3x + 10$$

Cálculo de $F(-3)$:

$$F(-3) = 3(-3) + 10$$

$$\therefore F(-3) = 1$$

**Problema 2**

Sea f una función lineal, donde $f(-2) - f(1) = 12$, y $f(2) = 1$. Evalúe $f(3)$.

Resolución: f es función lineal \Rightarrow

$$f(x) = ax + b$$

$$\triangleright \underline{f(-2)} - \underline{f(1)} = 12$$

$$[a(-2) + b] - [a(1) + b] = 12$$

$$-2a + \cancel{b} - a - \cancel{b} = 12$$

$$-3a = 12$$

$$a = -4$$

Nos piden $f(3)$:

$$f(3) = -4(3) + 9$$

$$\triangleright f(2) = 1$$

$$-4(2) + b = 1$$

$$-8 + b = 1$$

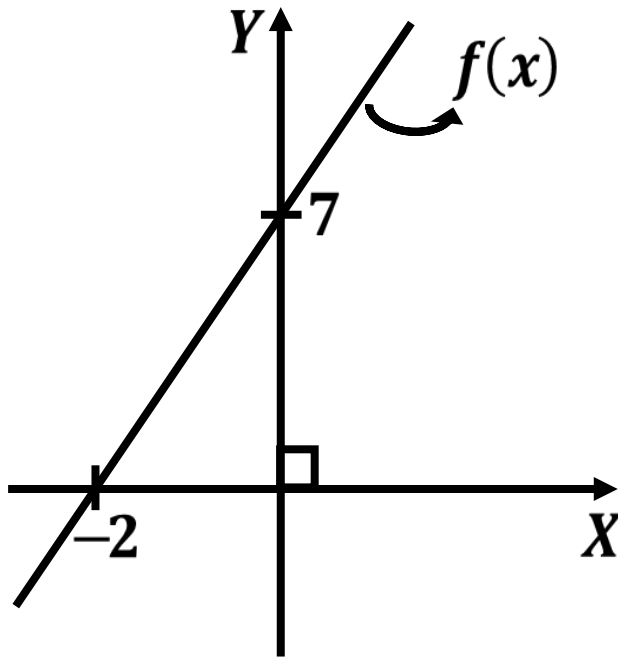
$$b = 9$$

$$\therefore f(3) = -3$$



Problema 3

De la gráfica



evalúe $f(6)$.

Resolución:

Sea: $f(x) = ax + b$

$$\begin{aligned} \triangleright \underline{f(0)} &= a(0) + b \\ 7 &= 0 + b \end{aligned}$$

$$b = 7$$

$$\begin{aligned} \triangleright \underline{f(-2)} &= a(-2) + b \\ 0 &= -2a + 7 \end{aligned}$$

$$a = \frac{7}{2}$$

$$f(x) = \frac{7}{2}x + 7$$

$$\Rightarrow f(6) = \frac{7}{2}(6) + 7$$

$$\therefore f(6) = 28$$

**Problema 4**

Si $H(x) = 3x + 5$, donde $x \in \langle 2; 6 \rangle$, determine su rango.

Resolución:

$$x \in \langle 2; 6 \rangle \longrightarrow 2 < x < 6$$

$$2 < x < 6$$

 $\times 3$

$$6 < 3x < 18$$

 $+5$

$$11 < \underbrace{3x + 5} < 23$$

$$11 < H(x) < 23$$

$$\therefore \text{Ran}(H) = \langle 11; 23 \rangle$$

**Problema 5**

Dada la función

$$G(x) = \begin{cases} x^2 - 3, & x \leq 0 \\ 3x - 1, & x > 0 \end{cases}$$

el valor de $P = G(G(-4)) + G(1)$ representa la edad del profesor Ricardo. ¿Cuánto le falta para cumplir medio siglo?

Resolución:

$$G(x) = \begin{cases} x^2 - 3, & x \leq 0 \\ 3x - 1, & x > 0 \end{cases}$$

$$\begin{array}{l|l} P = G(G(-4)) + G(1) & G(-4) = (-4)^2 - 3 = 13 \\ P = G(13) + 2 & G(1) = 3(1) - 1 = 2 \\ P = 38 + 2 & G(13) = 3(13) - 1 = 38 \end{array}$$

$$P = 40$$

← *Edad del profesor Ricardo*

∴ Para cumplir medio siglo le faltan 10 años.



Problema 6

Calcule el rango en
 $f(x) = x^2 + 4x + 7, x \in \mathbb{R}$

Resolución:

$$f(x) = x^2 + 4x + 7, \quad x \in \mathbb{R}$$

$$f(x) = \underline{x^2 + 4x + 4} + 3$$

$$f(x) = (x + 2)^2 + 3$$

$$(x + 2)^2 \geq 0$$

$$(x + 2)^2 + 3 \geq 3$$

$$f(x) \geq 3$$

$$\therefore \text{Ran}(f) = [3; +\infty)$$

Problema 7

Determine el bosquejo de la gráfica de la función

$$f(x) = x^2 - 10x + 27, x \in \mathbb{R}$$

Resolución:

$$f(x) = x^2 - 10x + 27$$

$$f(x) = \underline{x^2 - 10x + 25} + 2$$

$$f(x) = (x - 5)^2 + 2$$

Vértice:

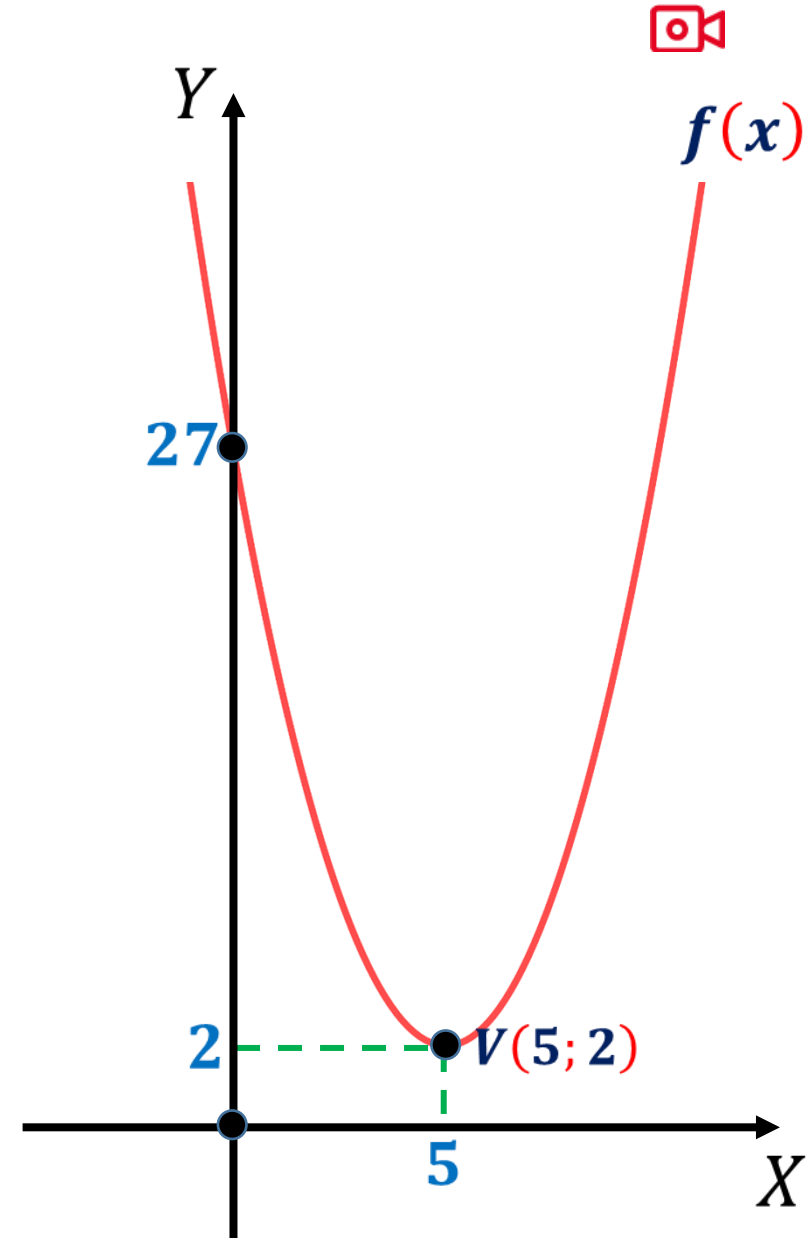
$$x - 5 = 0 \quad \left\{ \begin{array}{l} x = 5 \\ y = 2 \end{array} \right.$$

Intercepto con el eje Y: ($x = 0$)

$$y = x^2 - 10x + 27$$

$$y = (0)^2 - 10(0) + 27$$

$$y = 27$$





Problema 8

Si la gráfica de la función $f(x) = 2x^2 + x + n$ pasa por el punto $(3; 26)$, halle la ordenada de abscisa 2 que pertenece a $f(x)$.

Resolución:

$$f(x) = 2x^2 + x + n$$

$$\triangleright (3; 26) \in f(x) \Rightarrow 26 = 2(3)^2 + 3 + n$$

$$n = 5$$

$$\Rightarrow f(x) = 2x^2 + x + 5$$

$$\triangleright (2; y) \in f(x) \Rightarrow y = 2(2)^2 + 2 + 5$$

$$\therefore y = 15$$