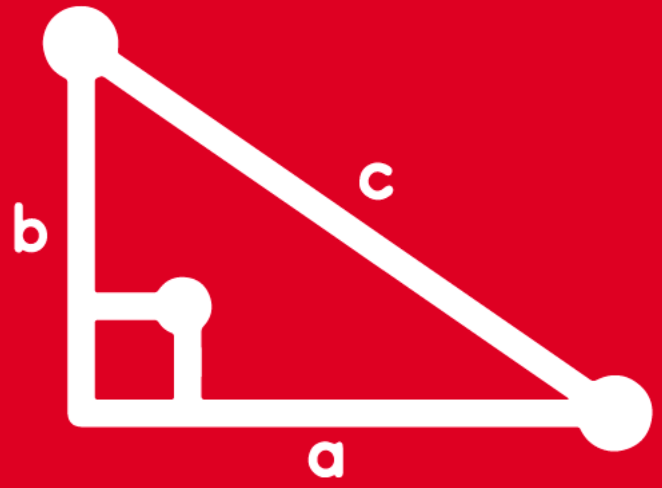
TRIGONOMETRY

Chapter 12
Sesión II

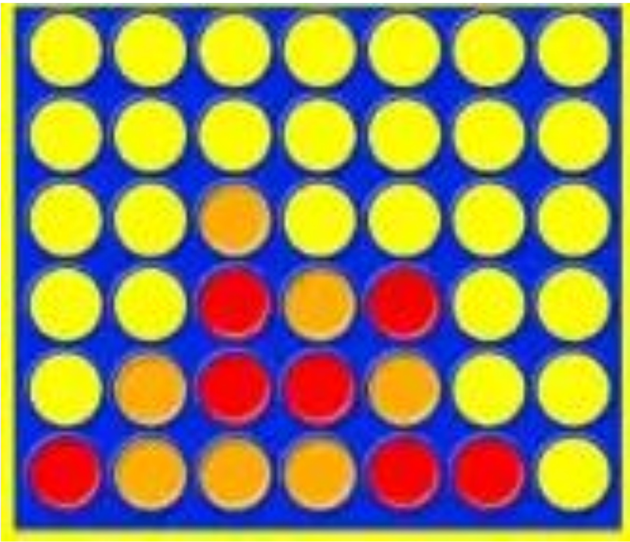


4th
SECONDARY

REDUCCIÓN AL PRIMER CUADRANTE II

SACO OLIVEROS

CUATRO EN RAYA TRIGONOMÉTRICO



Saberes previos al tema, se presenta este juego de tablero llamado“cuatro en raya” donde para poder ocupar una casilla del tablero los jugadores deben averiguar el valor de ciertas razones trigonométricas o la expresión simplificada de una fórmula, el turno decide el docente. Gana el que acierta 4 lugares cualesquiera.

$\frac{1}{tgx}$	$tg30°$	$sen^2x + cos^2x$	$cotg\,45°$	$cos(\frac{\pi}{4})$	$sen\,225°$
$cos\,120°$	tg^2x+1	$sen\,270°$	$1-sen^2x$	$tg\,315°$	$\frac{1}{cotg\,x}$
$tg\frac{3\pi}{4}$	$cos(-x)$	$\frac{1}{cotg\,30°}$	$sen\frac{\pi}{4}$	$sen360°$	$cos\,270°$
$cotg^2x+1$	$1-cos^2x$	$sen(-30°)$	$sen\,180°$	$cos\,xtgx$	$sen\,135°$
$cos\frac{3\pi}{4}$	$cotgx\,tgx$	$sen\,210°$	$cos\frac{3\pi}{2}$	$sen(-x)$	$\frac{cos\,x}{sen\,x}$
$cos\,90°$	$senxcotgx$	$cos\,225°$	$sen\frac{5\pi}{4}$	$sec\,30°$	$sen\,240°$

Reducción al primer cuadrante II

Para ángulos mayores a una vuelta:
Si $\theta > 360°$; entonces:

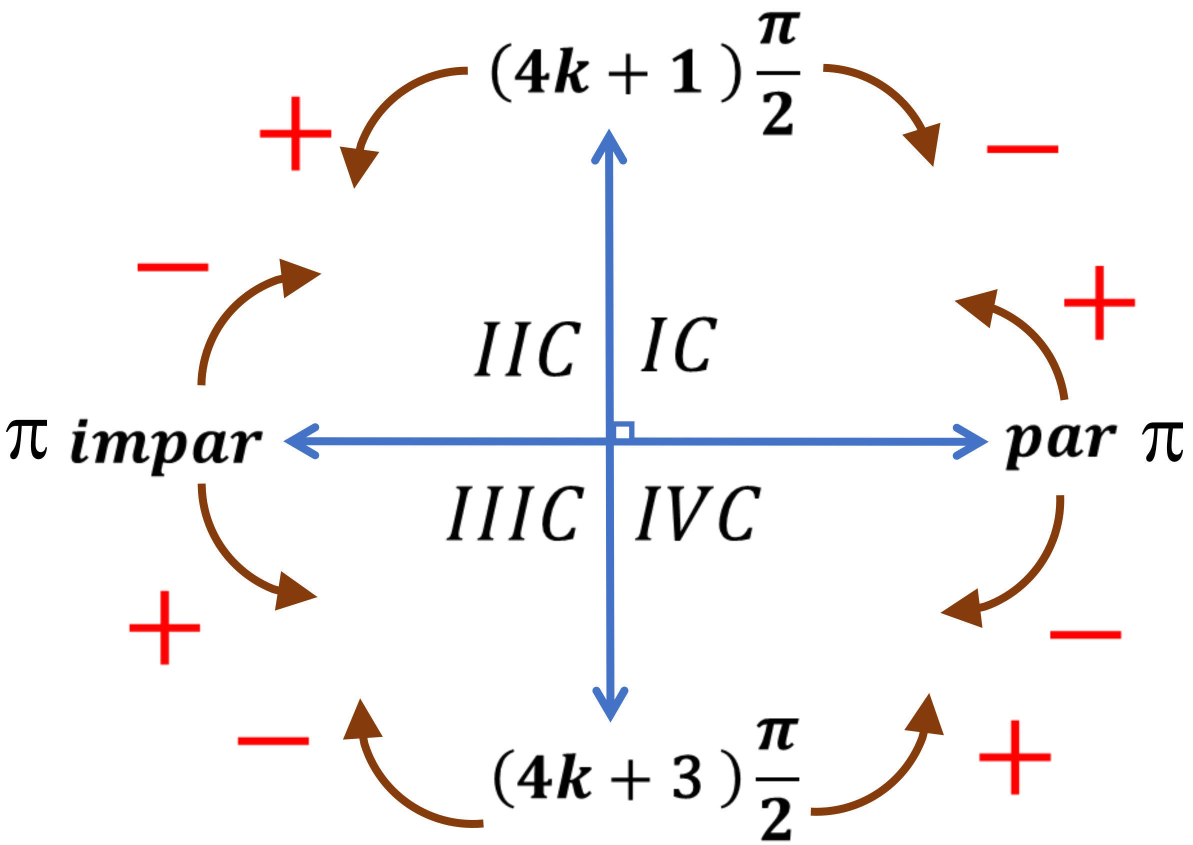
$$\boxed{\theta = 360k + \beta}$$

Donde β es un ángulo menor a una vuelta y $k \in Z - \{0\}$

$$\boxed{RT(\theta) = RT(\cancel{360°k} + \beta)}$$

$$\boxed{RT(\theta) = RT(\beta)}$$

Para ángulos expresados en radianes:



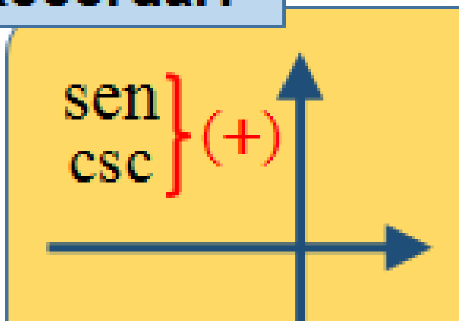
1) Efectúe G = tan 1920°. cot 36 135°

tan60° = √3
cot45° = 1

RESOLUCIÓN

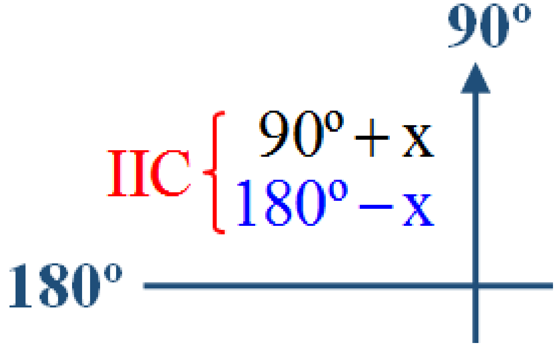
$$\begin{array}{l|l} 1920° & 360° \\ \hline 1800° & 5 \\ \hline 120° & \end{array} \qquad \begin{array}{l|l} 36135° & 360° \\ \hline 36000° & 100 \\ \hline 135° & \end{array}$$

Recordar:



$$G = tan\,120°.\,cot\,135°$$

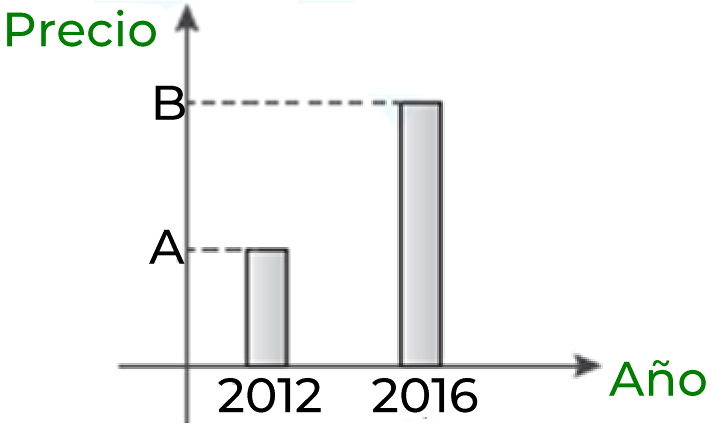
$$G = tan(\underbrace{180° - 60°}_{IIC}).\,cot(\underbrace{180 - 45°}_{IIC})$$



$$G = (-\overset{\sqrt{3}}{tan60°})\ (-\overset{1}{cot45°})$$

$$\therefore\ G = \sqrt{3}$$

2) En el siguiente gráfico se muestra el precio del dólar en los años 2012 y 2016. Indique la diferencia de ambos precios en los últimos 4 años. Donde:



$$A = 2\csc\left(161\frac{\pi}{4}\right) = 2\csc\left(1\frac{\pi}{4}\right) = 2\csc 45° = 2\sqrt{2} = 2{,}82$$

$$B = 2\cot\left(37\frac{\pi}{6}\right) = 2\cot\left(1\frac{\pi}{6}\right) = 2\cot 30° = 2\sqrt{3} = 3{,}46$$

Considere: $\sqrt{2} \cong 1{,}41$ y $\sqrt{3} \cong 1{,}73$

RESOLUCIÓN

$$\begin{array}{r|l} 161 & 8 \\ \hline 160 & 20 \\ \hline 1 & \end{array} \leftarrow 2\text{x}4 \qquad \begin{array}{r|l} 37 & 12 \\ \hline 36 & 3 \\ \hline 1 & \end{array} \leftarrow 2\text{x}6$$

Recordar:
$\cot 30° = \sqrt{3}$
$\csc 45° = \sqrt{2}$

$$\text{Diferencia} = 3{,}46 - 2{,}82 = \boxed{S/0{,}64}$$

3) Simplifique:

$$Q = 4\text{sen } 3630° + \frac{\tan\left(\frac{39\pi}{2} - x\right)}{\cot(15\pi - x)}$$

RESOLUCIÓN

$$\begin{array}{r|l} 3630° & 360° \\ \hline 3600° & 10 \\ \hline 30° & \end{array}$$

$$\tan\Big(\underbrace{\overset{4k+3}{\frac{39\pi}{2}} - x}_{IIIC}\Big) = +\cot x$$

$$\cot(\underbrace{\overset{IMPAR}{15}\pi - x}_{IIC}) = -\cot x$$

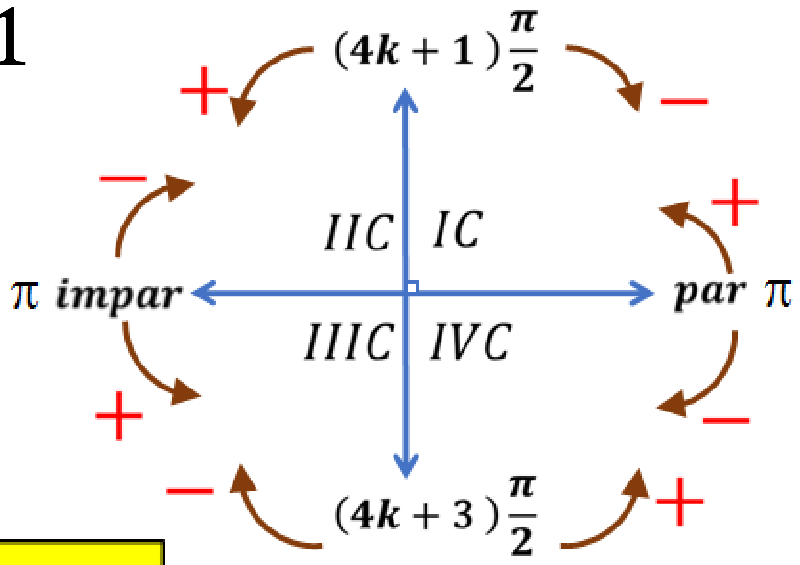
$$Q = 4\text{sen } 30° + \frac{\cot x}{-\cot x}$$

$$Q = \cancel{4}\left(\frac{1}{\cancel{2}}\right) + (-1)$$

Recordar:

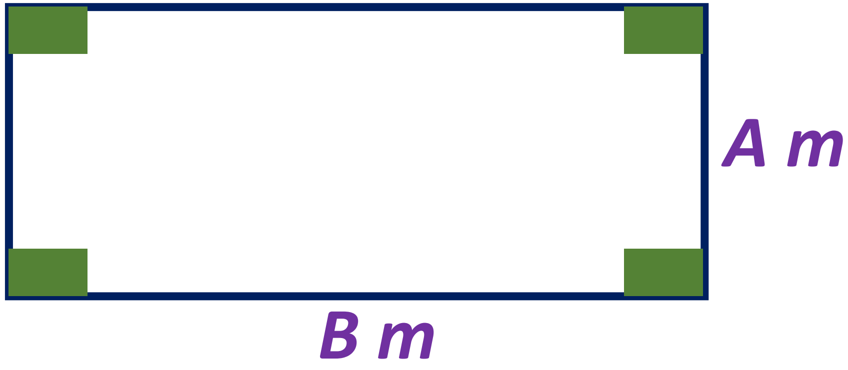
sen, csc (+)	Todas las RT son (+)
tan, cot (+)	cos, sec (+)

$$Q = 2 - 1$$



$$\boxed{Q = 1}$$

4) Daniela desea comprar un terreno. Dicho terreno tiene las siguientes dimensiones:



Donde:

$$A = \frac{8\text{sen}(80\pi + x)}{\cos\left(75\frac{\pi}{2} + x\right)}$$

$$B = 10\,\frac{\cot(71\pi + x)}{\tan\left(61\frac{\pi}{2} - x\right)}$$

Si cada m^2 tiene un valor de \$900 ¿Cuánto invertirá por su compra?

RESOLUCIÓN

$$A = \frac{8sen\overbrace{(80\pi + x)}^{IC}}{\cos\underbrace{\left(75\frac{\pi}{2} + x\right)}_{IVC}} \Rightarrow A = \frac{8.\,se\cancel{nx}}{se\cancel{nx}} = 8$$

$$B = 10\,\frac{cot\overbrace{(71\pi + x)}^{IIIC}}{\tan\underbrace{\left(61\frac{\pi}{2} - x\right)}_{IC}} \Rightarrow B = 10\,\frac{\cancel{cotx}}{\cancel{cotx}} = 10$$

S = (8m)(10m) ➡ S = 80 m^2

Inversión = (80)(\$900) = $\boxed{\$72\,000}$

5) Si $x + y = 17\frac{\pi}{2}$ reduzca:

$$M = \frac{\text{sen}x}{\cos y} + \frac{\tan x}{\cot y}$$

RESOLUCIÓN

Del dato: $$x = \underbrace{\overset{\substack{4k+1\\ \uparrow}}{17}\frac{\pi}{2} - y}_{IC}$$

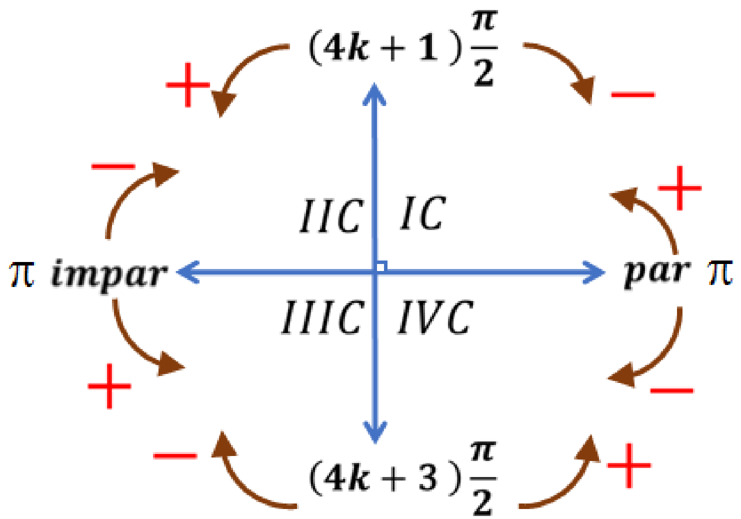
$$M = \frac{\text{sen}x}{\cos y} + \frac{\tan x}{\cot y}$$

$$M = \frac{\text{sen}\overbrace{\left(17\frac{\pi}{2} - y\right)}^{IC}}{\cos y} + \frac{\tan\overbrace{\left(17\frac{\pi}{2} - y\right)}^{IC}}{\cot y}$$

$$M = \frac{\cancel{\cos y}}{\cancel{\cos y}} + \frac{\cancel{\cot y}}{\cancel{\cot y}}$$

$$M = 1 + 1$$

$$\therefore \boxed{M = 2}$$



6) En un triángulo ABC, reduzca:

$$K = \frac{\tan(3A + 3B + 4C)}{\tan(A + B)}$$

RESOLUCIÓN

Del dato: $A + B + C = 180°$

$A + B = 180° - C$

$3A + 3B + 3C = 3(180°)$

$3A + 3B + 3C = \cancel{2(180°)} + 180°$

Nos piden: $K = \dfrac{\tan(3A + 3B + 4C)}{\tan(A + B)}$

$$K = \frac{\tan(3A + 3B + 3C + C)}{\tan(A + B)}$$

$$K = \frac{\overbrace{\tan(180° + C)}^{IIIC}}{\underbrace{\tan(180° - C)}_{IIC}} = \frac{\cancel{\tan C}}{-\cancel{\tan C}}$$

$\therefore$ $\boxed{K = -1}$

Recordar:

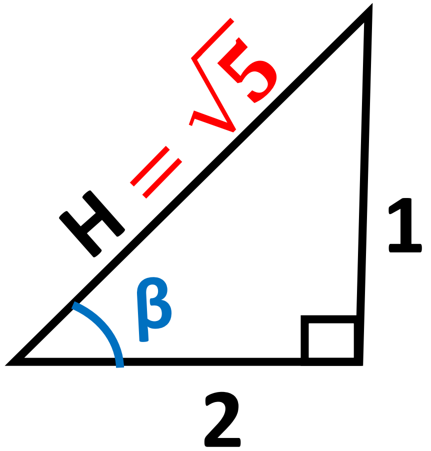
sen, csc (+) | Todas las RT son (+)
tan, cot (+) | cos, sec (+)

7) Si $\tan\beta = \dfrac{1}{2}$, donde β es un ángulo agudo, reduzca:

$$K = \operatorname{sen}(15\pi-\beta).\operatorname{sen}\left(13\frac{\pi}{2}+\beta\right)$$

RESOLUCIÓN

Del dato: $\tan\beta = \dfrac{1}{2}$

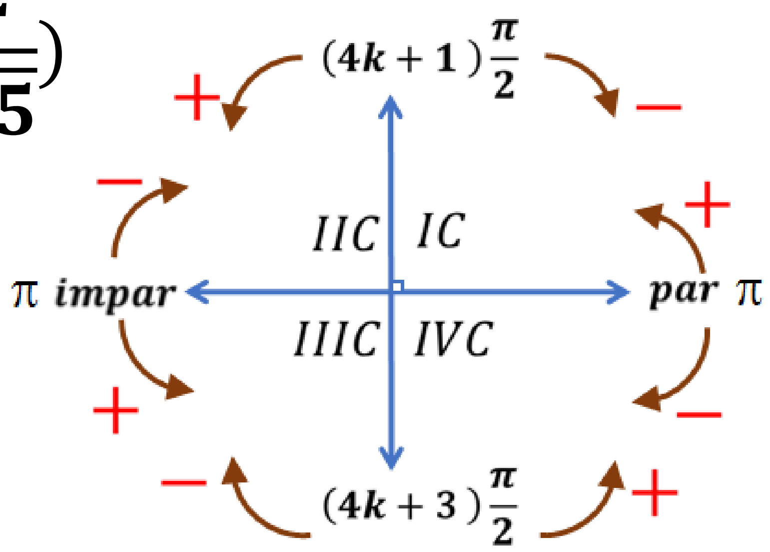


$$K = \operatorname{sen}(\underbrace{\overset{\substack{\text{IMPAR}\\ \uparrow}}{15}\pi-\beta}_{IIC}).\operatorname{sen}\left(\underbrace{\overset{\substack{4k+1\\ \uparrow}}{13}\frac{\pi}{2}+\beta}_{IIC}\right)$$

$$K = (\operatorname{sen}\beta)(\cos\beta)$$

$$K = \left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$$

$$\therefore\ \boxed{K = \frac{2}{5}}$$



8) Si $\sec\left(17\frac{\pi}{2}+\alpha\right)=-\frac{13}{5}$, donde α es un ángulo agudo, efectúe:

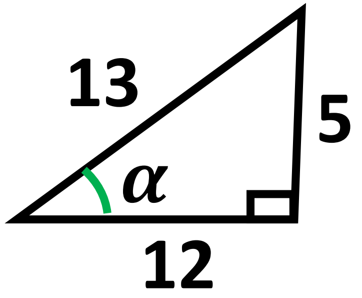
$$E=\csc\left(19\frac{\pi}{2}+\alpha\right)-\tan(36\pi-\alpha)$$

RESOLUCIÓN

Del dato: $\sec\underbrace{\left(\overset{\overset{4k+1}{\uparrow}}{17}\frac{\pi}{2}+\alpha\right)}_{IIC}=-\frac{13}{5}$

$$-\csc\alpha=-\frac{13}{5}$$

$$\Rightarrow \csc\alpha=\frac{13}{5}$$



Nos piden

$$E=\csc\underbrace{\left(\overset{\overset{4k+3}{\uparrow}}{19}\frac{\pi}{2}+\alpha\right)}_{IVC}-\tan\underbrace{(\overset{\overset{PAR}{\uparrow}}{36}\pi-\alpha)}_{IVC}$$

$$E=(-\sec\alpha)-(-\tan\alpha)$$

$$E=-\sec\alpha+\tan\alpha$$

$$E=-\frac{13}{12}+\frac{5}{12}$$

$$E=-\frac{8}{12}$$

$$\therefore\ \boxed{E=-\frac{2}{3}}$$