



TRIGONOMETRY

Chapter 16

Session 1

4th
SECONDARY



Identidades trigonométricas
fundamentales



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ECUACIONES E IDENTIDADES

ECUACIONES Y SISTEMAS

ECUACIONES E IDENTIDADES

$$5x-2=3(x+4)$$

$$2(x-3)=2x-6$$





Identidad trigonométrica

Definición:

Es una igualdad que contiene expresiones trigonométricas y que se verifica para todo valor admisible de la(s) variable(s).

Expresiones Trigonométricas:

Es toda expresión matemática donde la(s) variable(s) está (n) afectada(s) por operadores trigonométricos (sen, cos, tan, cot, sec, csc).

Identidades Fundamentales:

Llamadas también identidades trigonométricas básicas; son aquellas que se obtienen luego de relacionar las líneas trigonométricas en la circunferencia trigonométrica. Se clasifican en:





Identidad trigonométrica

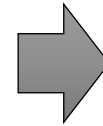
1. Identidades Por División

$$\tan x = \frac{\text{sen} x}{\cos x}$$

$$\cot x = \frac{\cos x}{\text{sen} x}$$

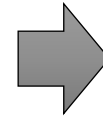
2. Identidades Recíprocas :

$$\text{sen} x \cdot \csc x = 1$$



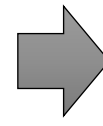
$$\csc x = \frac{1}{\text{sen} x}$$

$$\cos x \cdot \sec x = 1$$



$$\sec x = \frac{1}{\cos x}$$

$$\tan x \cdot \cot x = 1$$



$$\cot x = \frac{1}{\tan x}$$





Identidad trigonométrica

3. Identidades Pitagóricas:

$$\boxed{\text{sen}^2 x + \text{cos}^2 x = 1}$$

$$\text{sen}^2 x = 1 - \text{cos}^2 x$$

$$\text{cos}^2 x = 1 - \text{sen}^2 x$$

$$\boxed{\text{sec}^2 x - \text{tan}^2 x = 1}$$

$$\text{sec}^2 x = \text{tan}^2 x + 1$$

$$\text{tan}^2 x = \text{sec}^2 x - 1$$

$$\boxed{\text{csc}^2 x - \text{cot}^2 x = 1}$$

$$\text{csc}^2 x = \text{cot}^2 x + 1$$

$$\text{cot}^2 x = \text{csc}^2 x - 1$$





PROBLEMA 1

Simplifique : $E = \sec x \cdot \sen x + \csc x \cdot \cos x - \tan x$

Resolución:

$$E = \sec x \cdot \sen x + \csc x \cdot \cos x - \tan x$$

$$E = \frac{1}{\cos x} \cdot \sen x + \frac{1}{\sen x} \cdot \cos x - \tan x$$

$$E = \cancel{\tan x} + \cot x - \cancel{\tan x}$$

$$\therefore E = \cot x$$

Recordar las identidades:

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sen x}$$

$$\tan x = \frac{\sen x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sen x}$$





PROBLEMA 2

Simplifique: $E = \cot x - \frac{\csc x}{\sec x}$

Resolución:

$$E = \cot x - \frac{\csc x}{\sec x}$$

↓

$$E = \frac{\cos x}{\sin x} - \frac{\frac{1}{\sin x}}{\frac{1}{\cos x}}$$



$$E = \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x}$$

$$\therefore E = 0$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$





PROBLEMA 3

Reduzca: $P = \frac{1 + \cot x}{\csc x} - \cos x$

^{cosx}
Resolución:

$$P = \frac{1 + \cot x}{\csc x} - \cos x$$

$$P = \frac{1 + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}} - \cos x$$

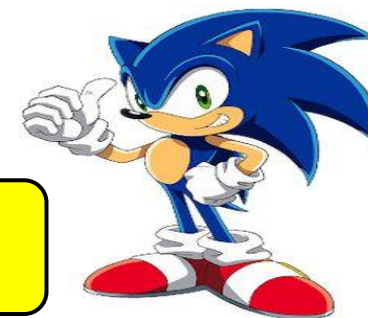
$$P = \frac{\frac{\sin x + \cos x}{\sin x}}{\frac{1}{\sin x}} - \cos x$$

$$P = \sin x + \cancel{\cos x} - \cancel{\cos x}$$

$$\therefore P = \sin x$$

$$\frac{\cot x}{\cos x}$$

$$\frac{\csc x}{1} = \frac{1}{\sin x}$$





PROBLEMA 4

Simplifique: $N =$

$$\frac{\tan \alpha + \tan^3 \alpha}{\cot \alpha + \cot^3 \alpha}$$

Resolución:

N

$$= \frac{\tan \alpha + \tan^3 \alpha}{\cot \alpha + \cot^3 \alpha}$$

$$N = \frac{\tan \alpha (1 + \tan^2 \alpha)}{\cot \alpha (1 + \cot^2 \alpha)}$$

$$N = \frac{\tan \alpha (\sec^2 \alpha)}{\cot \alpha (\csc^2 \alpha)}$$

$$N = \frac{\frac{\text{sen} \alpha}{\text{cos} \alpha} \cdot \frac{1}{\text{cos}^2 \alpha}}{\frac{\text{cos} \alpha}{\text{sen} \alpha} \cdot \frac{1}{\text{sen}^2 \alpha}}$$

$$N = \frac{\text{sen}^4 \alpha}{\text{cos}^4 \alpha} \rightarrow N = \underbrace{\left(\frac{\text{sen} \alpha}{\text{cos} \alpha} \right)^4}_{\tan \alpha}$$

$$1 + \tan^2 \alpha = \sec^2 \alpha$$

$$1 + \cot^2 \alpha =$$

$$\frac{\csc^2 \alpha}{\sec^2 \alpha} = \frac{1}{1}$$

$$= \frac{\csc^2 \alpha}{1} = \frac{1}{\text{sen}^2 \alpha}$$

$$\therefore N = \tan^4 \alpha$$





PROBLEMA 5

Simplifique: $E = (\text{sen}x + \text{cos}x \cdot \text{cot}x)$

$\text{sen}x$

Resolución:

$$E = (\text{sen}x + \text{cos}x \cdot \text{cot}x) \text{sen}x$$

$$E = \text{sen}x \cdot \text{sen}x + \text{cos}x \cdot \text{cot}x \cdot \text{sen}x$$

$$E = \text{sen}^2x + \text{cos}x \cdot \frac{\text{cos}x}{\text{sen}x} \cdot \text{sen}x$$

$$\Rightarrow E = \text{sen}^2x + \text{cos}^2x$$

$$\therefore E = 1$$

$$\text{cot}x = \frac{\text{cos}x}{\text{sen}x}$$

$$\text{sen}^2x + \text{cos}^2x = 1$$





PROBLEMA 6

Simplifique:

$$Q = \frac{1}{\operatorname{sen}^2 x} + \frac{1}{\cos^2 x} - \frac{1}{\tan^2 x} - \frac{1}{\cot^2 x}$$

Resolución:

$$Q = \frac{1}{\operatorname{sen}^2 x} + \frac{1}{\cos^2 x} - \frac{1}{\tan^2 x} - \frac{1}{\cot^2 x}$$

$$Q = \csc^2 x + \sec^2 x - \cot^2 x - \tan^2 x$$

$$Q = \underbrace{\csc^2 x - \cot^2 x}_1 + \underbrace{\sec^2 x - \tan^2 x}_1$$

$$\therefore Q = 2$$

$\frac{1}{\operatorname{sen}^2 x} =$	$\frac{1}{\tan^2 x} =$
$\frac{1}{\cos^2 x} =$	$\frac{1}{\cot^2 x} =$
$\csc^2 x - \cot^2 x = 1$	
$\sec^2 x - \tan^2 x = 1$	





PROBLEMA 7

Si $x \in \text{IIIC}$, simplifique la expresión:

$$E = \frac{\sqrt{(1-\text{sen } x)(1+\text{sen } x)}}{\cos x} + \csc^2 x$$

$$(a-b)(a+b) = a^2 - b^2$$

$$1 - \text{sen}^2 x =$$

$$\sqrt{a^2} = |a|$$

$$\csc^2 x - 1 =$$

Obs: Como $x \in \text{IIIC}$: $\cos x = (-)$

$$\rightarrow |\cos x| = -\cos x$$

Resolución:

$$E = \frac{\sqrt{1 - \text{sen}^2 x}}{\cos x} + \csc^2 x$$

$$E = \frac{\sqrt{\cos^2 x}}{\cos x} + \csc^2 x$$

$$E = \frac{|\cos x|}{\cos x} + \csc^2 x$$

$$E = \frac{-\cancel{\cos x}}{\cancel{\cos x}} + \csc^2 x$$

$$E = -1 + \csc^2 x$$

$$\therefore E = \cot^2 x$$





PROBLEMA 8

Al copiar de la pizarra la expresión $\sec x - \tan x - 1$; un estudiante cometió un error y escribió $\csc x - \cot x - 1$.

Calcule la razón entre lo que estaba escrito en la pizarra y lo que copió el alumno.

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Resolución:

Piden: $E = \frac{\sec x - \tan x - 1}{\csc x - \cot x - 1}$

$$E = \frac{\frac{1}{\cos x} - \frac{\sin x}{\cos x} - \frac{\cos x}{\cos x}}{\frac{1}{\sin x} - \frac{\cos x}{\sin x} - \frac{\sin x}{\sin x}}$$

$$E = \frac{\frac{1 - \sin x - \cos x}{\cos x}}{\frac{1 - \cos x - \sin x}{\sin x}}$$

$$E = \frac{\sin x}{\cos x}$$

$$\therefore E = \tan x$$