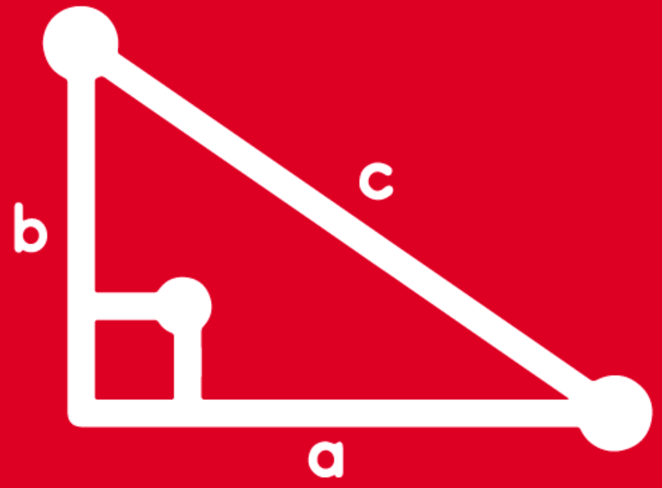
TRIGONOMETRY

Chapter 19
Session 1



4th
SECONDARY

Identidades trigonométricas del ángulo doble

SACO OLIVEROS

HISTORIA Y APLICACIONES DE LA TRIGONOMETRÍA

Identidad trigonométrica del ángulo doble

Para el seno:

$$\boxed{\text{sen}2x = 2\text{sen}x\cos x}$$

Para el coseno :

$$\boxed{\cos 2x = \cos^2 x - \text{sen}^2 x}$$

$$\boxed{\cos 2x = 1 - 2\text{sen}^2 x}$$

$$\boxed{\cos 2x = 2\cos^2 x - 1}$$

Ejemplos

- $\text{sen}20° = 2\text{sen}10°\cos 10°$
- $\cos 6\alpha = \cos^2 3\alpha - \text{sen}^2 3\alpha$
- $2\text{sen}^2 15° = 1 - \cos 30°$

Identidades de degradación

$$\boxed{2\text{sen}^2 x = 1 - \cos 2x}$$

$$\boxed{2\cos^2 x = 1 + \cos 2x}$$

Identidad trigonométrica del ángulo doble

Para la tangente :

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Ejemplo: Si $\tan x = 2$; calcule: $\tan 2x$

Resolución :

Dato : $\tan x = 2$

Piden : $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$$\Rightarrow \tan 2x = \frac{2(2)}{1-(2)^2}$$

$$\therefore \tan 2x = -\frac{4}{3}$$

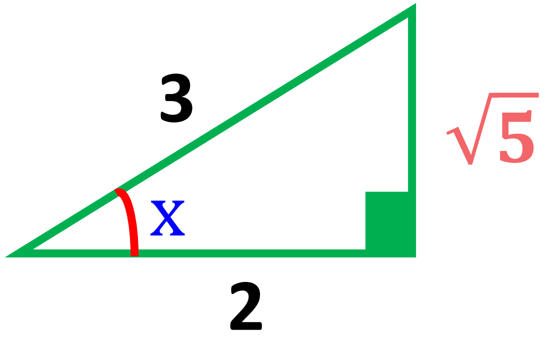
PROBLEMA 1

Si $\cos x = \frac{2}{3}$, donde $0° < x < 90°$ calcule sen2x

Resolución:

Del dato:

$$\cos x = \frac{2}{3} = \frac{C.A.}{H.}$$



$$\boxed{\operatorname{sen} x = \frac{\sqrt{5}}{3}}$$

Piden: $\operatorname{sen} 2x = 2\operatorname{sen} x\cos x$

$$\operatorname{sen} 2x = 2 \cdot \left(\frac{\sqrt{5}}{3}\right) \cdot \left(\frac{2}{3}\right)$$

$$\boxed{\therefore \operatorname{sen} 2x = \frac{4\sqrt{5}}{9}}$$

PROBLEMA 3

Si $\operatorname{sen}\beta - \cos\beta = \frac{1}{5}$, calcule $\operatorname{sen}2\beta$

$(a-b)^2 = a^2 + b^2 - 2ab$

$\operatorname{sen}^2 x + \cos^2 x = 1$

Resolución:

Del dato: $\operatorname{sen}\beta - \cos\beta = \frac{1}{5} \ldots (\ \ )^2$

$$(\operatorname{sen}\beta - \cos\beta)^2 = \left(\frac{1}{5}\right)^2$$

$$\underbrace{\operatorname{sen}^2\beta + \cos^2\beta}_{1} - 2\operatorname{sen}\beta\cos\beta = \frac{1}{25}$$

$$1 - 2\operatorname{sen}\beta.\cos\beta = \frac{1}{25}$$

$$1 - \frac{1}{25} = \underbrace{2\operatorname{sen}\beta\cos\beta}$$

$$\frac{24}{25} = \operatorname{sen}2\beta$$

$$\boxed{\therefore \operatorname{sen}2\beta = \frac{24}{25}}$$

PROBLEMA 4

Si $\cos 2\theta = \dfrac{3}{4}$, calcule $\operatorname{sen}\theta$, si $\theta \in$ IC.

Resolución:

Del dato: $\cos 2\theta = \dfrac{3}{4}$

Por identidades de degradación:

$$\boxed{2\operatorname{sen}^2\theta = 1 - \cos 2\theta}$$

$$2\operatorname{sen}^2\theta = 1 - \frac{3}{4}$$

$$2\operatorname{sen}^2\theta = \frac{1}{4} \Rightarrow \operatorname{sen}^2\theta = \frac{1}{8}$$

Como $\theta \in$ IC $\Rightarrow$ $\operatorname{sen}\theta = \dfrac{1}{\sqrt{8}}$

Racionalizando:

$$\operatorname{sen}\theta = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{\therefore\ \operatorname{sen}\theta = \frac{\sqrt{2}}{4}}$$

PROBLEMA 5

Simplifique:

$$K = \frac{\text{sen}2x + \text{sen}x}{2\cos x + 1}$$

Resolución:

$$\boxed{\text{sen}2x = 2.\text{sen}x.\cos x}$$

$$K = \frac{\text{sen}2x + \text{sen}x}{2\cos x + 1}$$

$$K = \frac{2.\text{sen}x.\cos x + \text{sen}x}{2\cos x + 1}$$

$$K = \frac{\text{sen}x\,\cancel{(2\cos x + 1)}}{\cancel{(2\cos x + 1)}}$$

$$\boxed{\therefore\ K = \text{sen}x}$$

PROBLEMA 6

Si $x = \frac{\pi}{16}$, determine el valor de: $P = 8 \cdot \text{senx} \cdot \cos^3 x - 8 \cdot \text{sen}^3 x \cdot \text{cosx}$

Resolución:

$2.\text{senx}.\text{cosx} = \text{sen}2x$

$\cos^2 x - \text{sen}^2 x = \cos 2x$

$P = 8 \cdot \text{senx} \cdot \cos^3 x - 8\,\text{sen}^3 x \cdot \text{cosx}$

$P = 8 \cdot \text{senx} \cdot \text{cosx}\,(\cos^2 x - \text{sen}^2 x)$

$P = 4 \cdot \underbrace{2 \cdot \text{senx.cosx}} \, \underbrace{(\cos^2 x - \text{sen}^2 x)}$

$P = 2 \cdot \underbrace{2 \quad \text{sen}2x \quad \cos 2x}$

$\Rightarrow P = 2\ \text{sen}4x$

Piden :

$\frac{\pi}{4} = 45°$

$P = 2 \cdot \text{sen}\left(4 \cdot \frac{\pi}{16}\right) = 2 \cdot \text{sen}\left(\frac{\pi}{4}\right)$

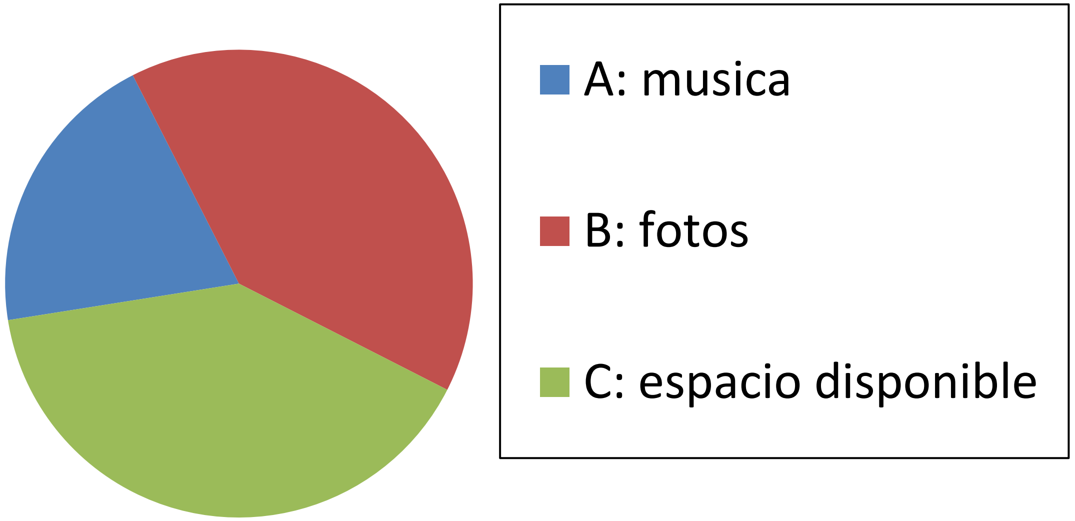
$P = 2 \cdot \underbrace{\text{sen}(45°)}$

$P = \cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}}$

$\boxed{\therefore P = \sqrt{2}}$

PROBLEMA 7

Observe el siguiente diagrama y determine el espacio disponible del USB de 16 GB.



Donde :

$$A = \frac{8\tan 22°30'}{1 - \tan^2 22°30'} \quad , \quad B = 10\left(\cos 18°30' + \operatorname{sen} 18°30'\right)\left(\cos 18°30' - \operatorname{sen} 18°30'\right)$$

Resolución:

$$A = \frac{8\cdot\tan 22°30'}{1-\tan^2 22°30'} = 4\underbrace{\left(\frac{2\tan 22°30'}{1-\tan^2 22°30'}\right)}_{\tan 2(22°30')} = 4\cdot\tan 45° = 4\cdot(1)$$

$$\boxed{A = 4\text{ GB}}$$

$$B = 10\underbrace{\cancel{(\cos 18°30' + \operatorname{sen} 18°30')(\cos 18°30' - \operatorname{sen} 18°30')}}_{\cos^2 18°30' - \operatorname{sen}^2 18°30'}$$

$$\underbrace{\cos^2 18°30' - \operatorname{sen}^2 18°30'}_{\cos 2(18°30')}$$

$$B = 10\cdot\cos 37° = 10\cdot\left(\frac{4}{5}\right)$$

$$\boxed{B = 8\text{ GB}}$$

Piden :

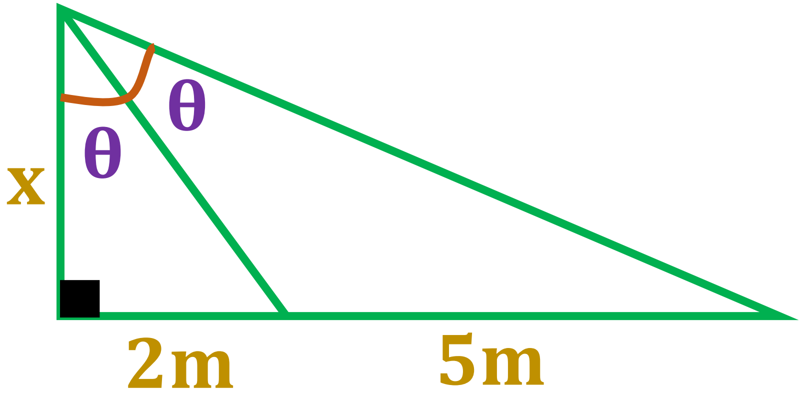
$$C = 16 - (A + B)$$

$$C = 16 - (4 + 8)$$

$$\boxed{C = 4\text{ GB}}$$

PROBLEMA 8

A partir del gráfico, determine el valor de x.



Resolución:

Del gráfico: $\tan\theta = \frac{2}{x}$; $\tan 2\theta = \frac{7}{x}$

$$\tan 2\theta = \frac{2 \cdot \tan\theta}{1 - \tan^2\theta}$$

Reemplazando:

$$\frac{7}{x} = \frac{2 \cdot \left(\frac{2}{x}\right)}{1 - \left(\frac{2}{x}\right)^2} \Rightarrow \frac{7}{x} = \frac{\frac{4}{x}}{\frac{x^2 - 4}{x^2}}$$

$$\frac{7}{\cancel{x}} = \frac{4x^2}{\cancel{x}(x^2 - 4)} \Rightarrow 7x^2 - 28 = 4x^2$$

$$\Rightarrow 3x^2 = 28 \Rightarrow x^2 = \frac{28}{3} \Rightarrow x = \sqrt{\frac{28}{3}}$$

$$\Rightarrow x = \frac{2\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{\therefore x = \frac{2\sqrt{21}}{3} m}$$