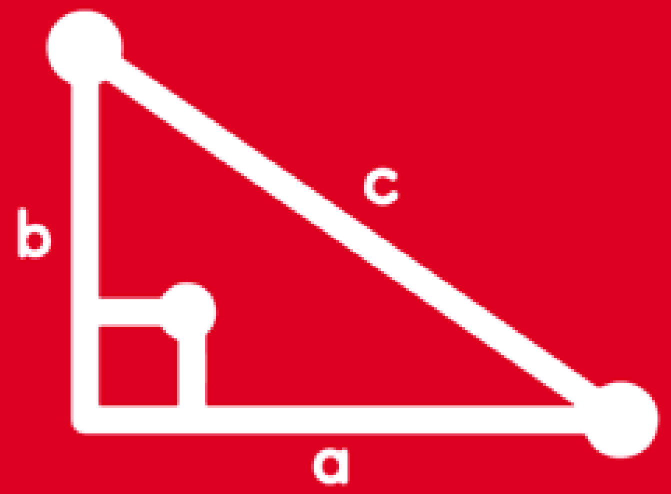
TRIGONOMETRY

Tomo 5 y 6

5th
SECONDARY



Advisory

SACO OLIVEROS

1. Simplifique la expresión: $E = \dfrac{\cos(x+y)}{\text{sen}x\cos y} - \cot x$

RESOLUCIÓN

Recordar:

$\boxed{\cos(x+y) = \cos x.\cos y - \text{sen}x.\text{sen}y}$

$$E = \frac{\cos(x+y)}{\text{sen}x\cos y} - \cot x$$

$$E = \frac{\cos x\cos y - \text{sen}x\,\text{sen}y}{\text{sen}x\cos y} - \cot x$$

$$E = \frac{\cos x\cancel{\cos y}}{\text{sen}x\cancel{\cos y}} - \frac{\cancel{\text{sen}x}\,\text{sen}y}{\cancel{\text{sen}x}\cos y} - \cot x$$

$$E = \frac{\cos x}{\text{sen}x} - \frac{\text{sen}y}{\cos y} - \cot x$$

$$E = \cancel{\cot x} - \tan y - \cancel{\cot x}$$

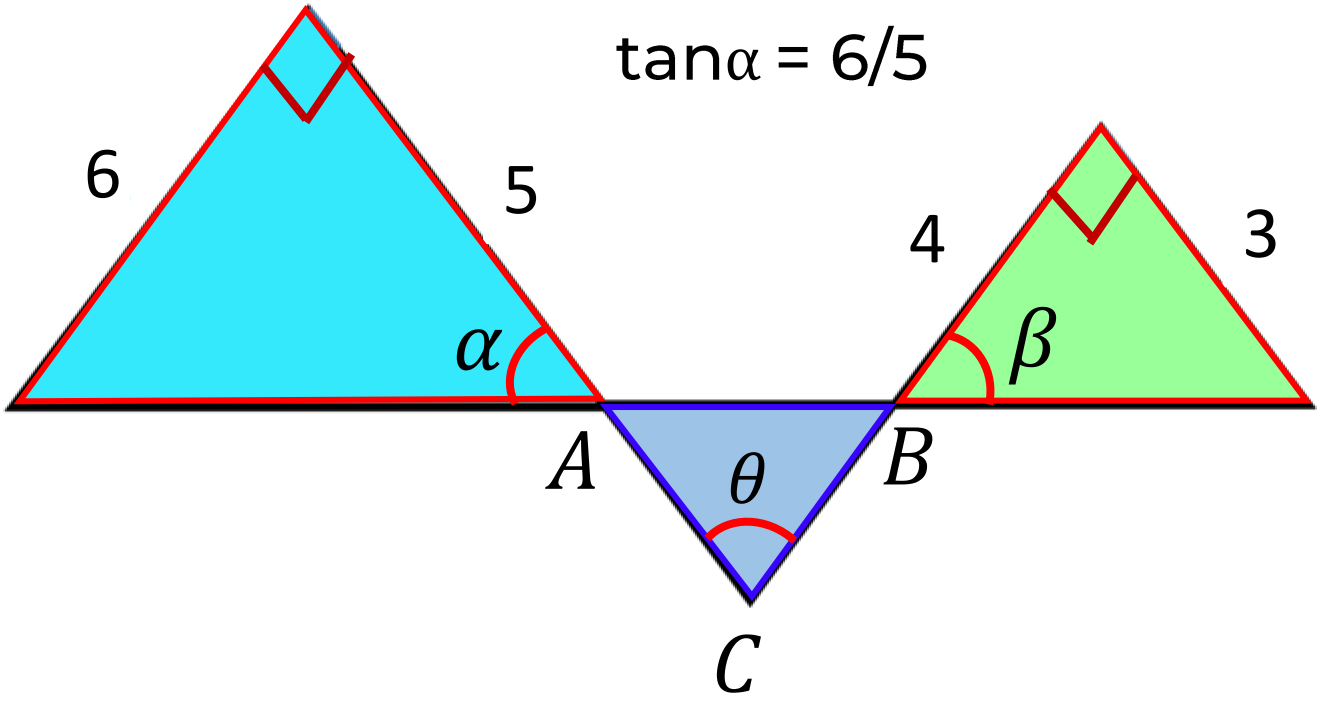
$$\therefore\ E = -\tan y$$

2. Del gráfico, halle el valor de $\tan\theta$.

RESOLUCIÓN

$\tan\beta = 3/4$

$\tan\alpha = 6/5$



Si $x + y + z = 180°$, se cumple:

$\tan x + \tan y + \tan z = \tan x.\tan y.\tan z$

$\triangle ABC$: $\alpha + \beta + \theta = 180°$

$$\tan\alpha + \tan\beta + \tan\theta = \tan\alpha.\tan\beta.\tan\theta$$

$$\frac{6}{5} + \frac{3}{4} + \tan\theta = \frac{6}{5}.\frac{3}{4}.\tan\theta$$

$$\frac{39}{20} + \tan\theta = \frac{18}{20}.\tan\theta$$

$$\frac{39}{\cancel{20}} = -\frac{2}{\cancel{20}}.\tan\theta$$

$\therefore\ \tan\theta = -39/2$

3. Simplifica la expresión:

$$T = \frac{sen^3x - cos^3x}{senx - cosx} + 5senxcosx$$

RESOLUCIÓN

$$T = \frac{sen^3x - cos^3x}{senx - cosx} + 5senxcosx$$

$$T = \frac{\cancel{(senx - cosx)}(sen^2x + senxcosx + cos^2x)}{\cancel{(senx - cosx)}} + 5senxcosx$$

$$T = sen^2x + cos^2x + senxcosx + 5senxcosx$$

$$T = 1 + 3(\underbrace{\mathbf{2senxcosx}}_{sen2x})$$

$\therefore$ T = 1 + 3sen2x

Recordar:

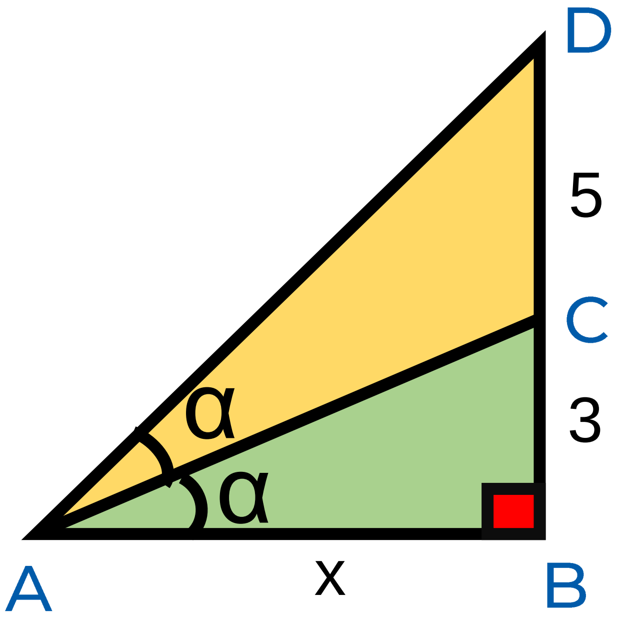
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$sen2x = 2senxcosx$$

$$sen^2x + cos^2x = 1$$

4. Del gráfico, calcule el valor de x.

RESOLUCIÓN



$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

◣ABC: $\tan\alpha = \frac{3}{x}$

◣ABD: $\tan 2\alpha = \frac{8}{x}$

$$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{8}{x}$$

$$\Rightarrow \frac{2\left(\frac{3}{x}\right)}{1-\left(\frac{3}{x}\right)^2} = \frac{8}{x}$$

$$\Rightarrow \frac{\frac{6}{x}}{1-\frac{9}{x^2}} = \frac{8}{x}$$

$$\Rightarrow x\left(\frac{6}{x}\right) = 8\left(1-\frac{9}{x^2}\right)$$

$$\Rightarrow 6 = 8 - \frac{72}{x^2} \qquad \Rightarrow \frac{72}{x^2} = 2$$

$$x^2 = 36$$

$\therefore\ x = 6$

5. De la condición: $\tan x + \cot x = 12$
Calcule: $\text{sen}2x$

RESOLUCIÓN

Recordar: $\cot(x) + \tan(x) = 2\csc(2x)$

$$\csc(x) = \frac{1}{\text{sen}(x)}$$

$$\underbrace{\cot x + \tan x}_{} = 12$$

$$2\csc 2x = 12$$

$$\csc 2x = 6$$

$$\therefore\ \text{sen}2x = \frac{1}{6}$$

6. Reduzca la expresión: $= \dfrac{+ \qquad \theta}{- \qquad \theta} - \dfrac{+ \qquad \theta}{- \qquad \theta}$

Recordar:

$2\cos^2(x) = 1 + \cos(2x)$

$2\text{sen}^2(x) = 1 - \cos(2x)$

$\text{sen}2x = 2\text{sen}x\cos x$

$\csc^2 x - \cot^2 x = 1$

$\cot x = \dfrac{\cos x}{\text{sen}x}$

$\csc x = \dfrac{1}{\text{sen}x}$

RESOLUCIÓN

$$P = \frac{4(1 + \cos 4\theta)}{1 - \cos 8\theta} - \frac{1 + \cos 4\theta}{1 - \cos 4\theta}$$

$$P = \frac{4(\cancel{2}\cos^2 2\theta)}{\cancel{2}\text{sen}^2 4\theta} - \frac{\cancel{2}\cos^2 2\theta}{\cancel{2}\text{sen}^2 2\theta}$$

$$P = \frac{4\cos^2 2\theta}{(\text{sen}4\theta)^2} - \frac{\cos^2 2\theta}{\text{sen}^2 2\theta}$$

$$P = \frac{4\cos^2 2\theta}{(2\text{sen}2\theta\cos 2\theta)^2} - \cot^2 2\theta$$

$$P = \frac{\cancel{4\cos^2 2\theta}}{\cancel{4}\text{sen}^2 2\theta\cancel{\cos^2 2\theta}} - \cot^2 2\theta$$

$$P = \frac{1}{\text{sen}^2 2\theta} - \cot^2 2\theta$$

$$P = \underbrace{\csc^2 2\theta - \cot^2 2\theta}_{\mathbf{1}}$$

$\therefore$ P = 1

7. Reduzca: $= \sqrt{\dfrac{+\sqrt{\dfrac{-}{}}}{}}$

RESOLUCIÓN

Recordar:

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$P = \sqrt{\frac{1+\underbrace{\sqrt{\frac{1-\cos 40^\circ}{2}}}_{\operatorname{sen}20^\circ}}{2}} \;\Rightarrow\; P = \sqrt{\frac{1+\cos 70^\circ}{2}}$$

$\therefore\ P = \cos 35^\circ$

8. Dar el valor de: $= \left(\frac{}{}\right) - \left(\frac{\pi}{}\right)$

RESOLUCIÓN

Recordar:

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

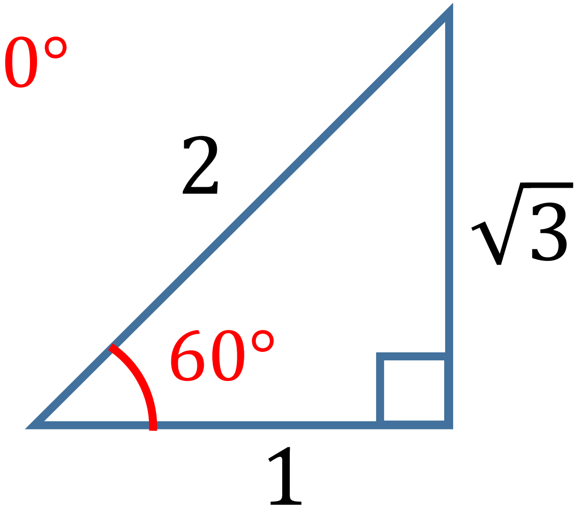
$$\frac{\pi}{3} = 60^\circ$$

$$E = cot\left(\frac{37^\circ}{2}\right) - sec\left(\frac{\pi}{3}\right)$$

$$E = csc(37^\circ) + \cot(37^\circ) - sec60^\circ$$

$$E = \frac{5}{3} + \frac{4}{3} - 2$$

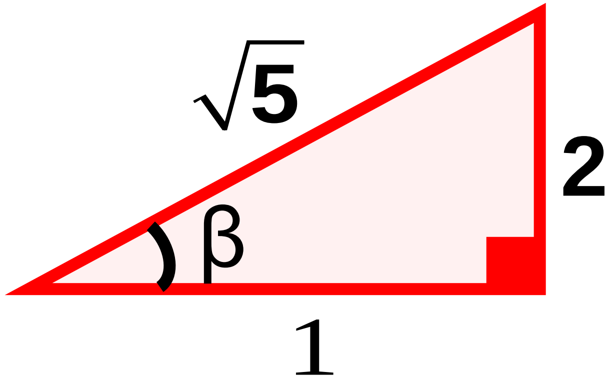
$\therefore$ E = 1



9. Si un ángulo β, cumple que $\beta = \sqrt{}$. Calcule β

RESOLUCIÓN

Dato: $\sec\beta = \dfrac{\sqrt{5}}{1} = \dfrac{\text{Hip}}{\text{CA}}$



$\Rightarrow \tan\beta = 2$

Recordar:

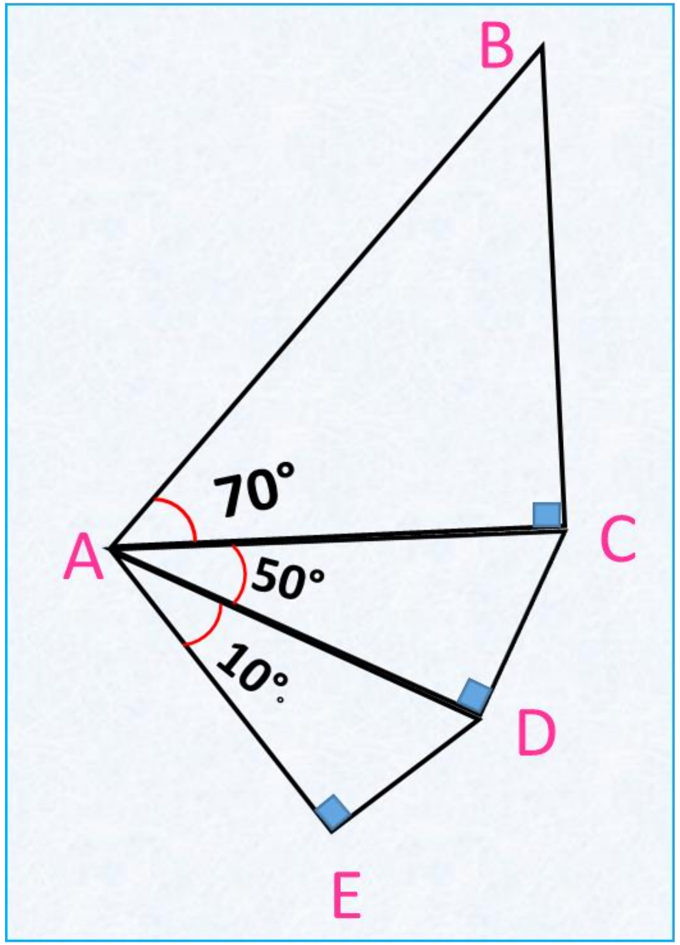
$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

Piden:

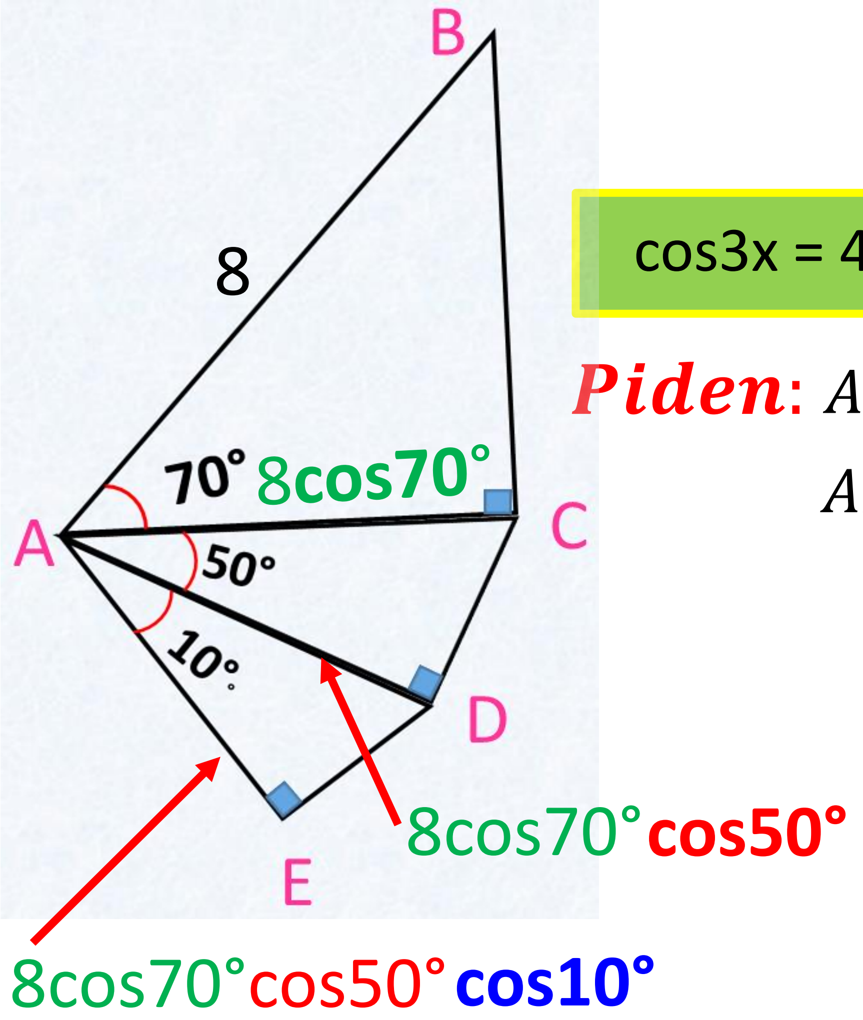
$$\tan 3\beta = \frac{3(2) - (2)^3}{1 - 3(2)^2} = \frac{-2}{-11}$$

$\therefore \quad \beta = —$

10. En la figura, AB = 8.
Halle AE



RESOLUCIÓN



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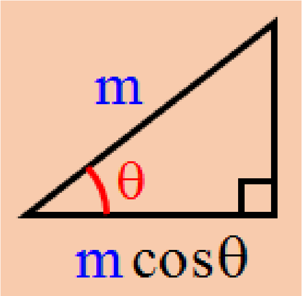

cos3x = 4cosx.cos(60°– x).cos(60°+ x)

Piden: $AE = 8.cos70°cos50°cos10°$

$$AE = 2.\underbrace{4cos10°cos50°cos70°}$$

$$AE = 2cos(3(10°))$$

$$AE = 2\cos 30° = 2.\frac{\sqrt{3}}{2}$$

$$\therefore\ AE = \sqrt{3}$$