ALGEBRA

Chapter
09

4th

FACTORIAL Y NÚMERO COMBINATORIO

SACO OLIVEROS

HELICO

MOTIVATING

SACO OLIVEROS

SABIAS QUE

El hombre-computadora Horacio Uhler, en la década de 1950, calculó el valor de 450! (factorial de 450) sin la ayuda de ordenadores. Encontró que tenía exactamente 1.001 dígitos, por lo que lo bautizó como el "factorial de las mil y una noches".

17.333.687.331.126.326.593.447.131.461.045.793.996.778.112.652.0
90.510.155.692.075.095.553.330.016.834.367.506.046.750.882.9
04.387.106.145.811.284.518.424.097.858.618.583.806.301.650.208
.347.296.181.351.667.570.171.918.700.422.280.962.237.272.230.6
63.528.084.038.062.312.369.342.674.135.036.610.101.508.838.22
0.494.970.929.739.011.636.793.766.165.023.730.853.896.403.901
.590.836.144.149.594.432.684.204.513.784.716.402.303.182.604.0
94.683.993.315.061.302.563.918.385.303.341.510.606.761.462.420
.205.820.006.936.352.095.967.417.183.191.538.725.617.509.521.3
80.556.781.309.195.429.800.229.273.803.342.553.558.164.591.99
6.298.912.368.598.547.771.179.158.461.351.340.068.905.647.127.6
58.164.836.377.126.303.774.923.360.078.072.307.462.008.554.3
55.068.361.448.126.606.281.145.760.960.499.187.813.428.397.924
.840.592.504.537.849.487.425.060.488.481.036.571.447.957.046.
788.635.742.936.714.615.176.219.148.469.743.102.979.949.740.714
.485.104.716.169.664.052.397.392.602.848.408.694.007.408.998.
901.127.492.905.171.514.473.431.386.633.392.492.040.661.522.692
.303.043.813.960.541.966.093.224.243.809.225.137.268.851.717.9
04.303.214.058.238.447.936.111.678.568.236.973.036.238.404.62
6.507.890.688.000.000.000.000.000.000.000.000.000.000.000.0
00.000.000.000.000.000.000.000.000.000.000.000.000.000.00
0.000.000.000.000.000.000.000.000.000.000.000.000.000.000

HELICO THEORY

CHAPTHE

R 09

SACO OLIVEROS

FACTORIAL

DEFINICIÓN

Sea $n \in N$ (además del cero), denotado por n! ; se define como:

$$n! = \begin{cases} \mathbf{0}, \ si\ n = 0 \ \vee \ n = 1 \\ \mathbf{1x2x3 \ldots\ldots n},\ \mathrm{si}\ \ n \in N \ \wedge \ n \geq 2 \end{cases}$$

Ejemplos:

$5! = 5\,(4)(3)(2)(1) = 120$

$3! = 3(2)(1) = 6$

$14! = \underbrace{14\,(13)(12)(11)(10)\ 9!}_{\text{Degradación de factorial}} = 240240.\ 9!$

Propiedades

$$\boxed{n! + (n+1)! = n!(n+2)}$$

$5! + 6! = 5!\,(5+2) = 5!\,(7)$

$$\boxed{n! + (n+1)! + (n+2)! = n!(n+2)^2}$$

$4! + 5! + 6! = 4!\,(4+2)^2 = 5!\,(36)$

$$\boxed{(n+1)! - n! = n!\,(n)}$$

$5! - 4! = 4!\,(4)$

NÚMERO COMBINATORIO

DEFINICIÓN

El número combinatorio denotado por C_k^n representa el número total de combinaciones que se pueden realizar con n elementos tomados de k en k .

$$\boxed{C_k^n = \frac{n!}{k!.(n-k)!}} \quad (n, k \in \mathrm{N} \wedge n \geq k)$$

Ejemplo:

$$C_2^7 = \frac{7!}{2!.(7-2)!} = \frac{7!}{2!.(5)!} = \frac{7(\cancel{6}).\cancel{5!}}{\cancel{2}(1).\cancel{5!}} = \boxed{21}$$

Caso Práctico:

$$C_2^7 = \frac{7(6)}{2(1)} = \boxed{21}$$

Propiedades

$$\boxed{C_k^n = C_{n-k}^n}$$

Ejemplo: $C_2^7 = C_{7-2}^7 = C_5^7$

Si: $\boxed{C_k^n = C_p^n}$ ⇒ $\boxed{k = p \quad \vee \quad n = k + p}$

Ejemplo: Si: $C_{10}^{15} = C_p^{15}$

⇒ $p = 10 \quad \vee \quad 15 = p + 10$

$$\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}$$

Ejemplo: $C_4^{12} + C_5^{12} = C_5^{12+1} = C_5^{13}$

$$\boxed{C_k^n = \frac{n}{k} C_{k-1}^{n-1}}$$

Ejemplo:

$$C_9^{15} = \frac{\cancel{15}}{\cancel{9}} C_{9-1}^{15-1} = \frac{5}{3} C_8^{14}$$

HELICO PRACTICE

CHAPTHE
R 09

SACO OLIVEROS

1. Reduzca

$$P=\left(\frac{32!+33!}{34!}\right)\left(\frac{67!}{66!+65!}\right)$$

RESOLUCIÓN

$$n!+(n+1)!=n!(n+2)$$

$$P=\left(\frac{\overbrace{32!+33!}}{34!}\right)\left(\frac{67!}{\underbrace{66!+65!}}\right)$$

$$n!+(n+1)!=n!(n+2)$$

$$P=\left(\frac{\cancel{32!}\,(\cancel{34})}{\cancel{34}(33)\,\cancel{32!}}\right)\left(\frac{\cancel{67}(66)\,\cancel{65!}}{\cancel{65!}\,(\cancel{67})}\right)$$

$$P=\left(\frac{\cancel{66}}{\cancel{33}}\right)$$

$$\boxed{P=2}$$

2. Halle el valor de "x" en:

$$\frac{(x+4)!\,(x+2)!}{(x+3)!+(x+2)!}=720$$

RESOLUCIÓN

Degradación de factorial

$$\frac{\underbrace{(x+4)!}\,(x+2)!}{\underbrace{(x+3)!+(x+2)!}}=720$$

$$\boxed{n!+(n+1)!=n!(n+2)}$$

$$\frac{\cancel{(x+4)}(x+3)!.\,\cancel{(x+2)!}}{\cancel{(x+2)!}\,\cancel{(x+4)}}=720$$

$$(x+3)!=\underbrace{720}_{6!}$$

$$(x+3)!=6!$$

$$\boxed{x=3}$$

3. Halle el valor de x, si se cumple:

$$\frac{(x+2)!+(x+3)!+(x+4)!}{(x+3)!-(x+2)!}=\frac{25}{x+2}$$

RESOLUCIÓN

$$\boxed{n!+(n+1)!+(n+2)!=n!(n+2)^2}$$

$$\frac{\overbrace{(x+2)!+(x+3)!+(x+4)!}}{\underbrace{(x+3)!-(x+2)!}}=\frac{25}{x+2}$$

$$\boxed{(n+1)!-n!=n!\,(n)}$$

$$\Rightarrow \frac{\cancel{(x+2)!}\,(x+4)^2}{\cancel{(x+2)!}\,\cancel{(x+2)}}=\frac{25}{\cancel{x+2}}$$

$$\Rightarrow (x+4)^2=25$$

$$\boxed{x=1}$$

4. Sabiendo que:

$$\frac{8!}{(a!)(b!)} = 14$$

calcule: $a + b$

RESOLUCIÓN

Degradación de factorial

$$\frac{8!}{(a!)(b!)} = 14$$

$$\frac{\overset{4}{\cancel{8}}\,(\overset{1}{\cancel{7}})(\overset{3\times 2}{6})\,5!}{\underset{\underset{1}{\cancel{2}}}{\cancel{14}}} = (a!)(b!)$$

$$\underbrace{4 \times 3 \times 2 \times 1}_{4!} \times 5! = (a!)(b!)$$

$$4! \times 5! = (a!)(b!)$$

$\boxed{a = 4}$ $\wedge$ $\boxed{b = 5}$

$\vee$

$\boxed{a = 5}$ $\wedge$ $\boxed{b = 4}$

Nos piden

$$a + b$$

$$\boxed{a + b = 9}$$

5. Pedro le regala a su esposa una licuadora marca OSTER, cuyo precio fue el valor de 2T soles, donde T está dado por:

$$T = C_5^8 + C_6^8 + C_7^9 + C_8^{10} + C_2^{11}$$

¿Cuánto le costó la licuadora a Pedro?

RESOLUCIÓN

$$T = \underbrace{C_5^8 + C_6^8} + C_7^9 + C_8^{10} + C_2^{11}$$

$$\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}$$

$$T = \underbrace{C_6^9 + C_7^9} + C_8^{10} + C_2^{11}$$

$$\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}$$

$$T = \underbrace{C_7^{10} + C_8^{10}} + C_2^{11}$$

$$\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}$$

$$T = C_8^{11} + \underbrace{C_2^{11}}$$

$$\boxed{C_k^n = C_{n-k}^n}$$

$$T = \underbrace{C_8^{11} + C_9^{11}}$$

$$\boxed{C_k^n + C_{k+1}^n = C_{k+1}^{n+1}}$$

$$T = \underbrace{C_9^{12}}$$

$$\boxed{C_k^n = \frac{n!}{k!.(n-k)!}}$$

$$T = C_9^{12} = \frac{12!}{9!.(3)!} = \frac{\overset{2}{\cancel{12}}(11)(10)\cancel{9!}}{\cancel{9!}.(\cancel{3})(\cancel{2})(1)} = 220$$

$$\boxed{El\ costo\ de\ la\ licuadora = S/.440}$$

6. Halle el valor de “n” en:

$$3C_3^{2n} = 44C_2^n$$

RESOLUCIÓN

Caso Práctico:

$$\cancel{3}\left\{\frac{\cancel{2}n(2n-1)(2n-2)}{(\cancel{3})(\cancel{2})(1)}\right\} = \overset{22}{\cancel{44}}\left\{\frac{n(n-1)}{(\cancel{2})(1)}\right\}$$

$$\Rightarrow \cancel{n}(2n-1)\cancel{2}\cancel{(n-1)} = \overset{11}{\cancel{22}}\cancel{n}\cancel{(n-1)}$$

$$\Rightarrow (2n-1) = 11$$

$$\boxed{n = 6}$$

7. Halle el valor de M en:

$$M = \frac{3C_2^{11} - 5C_9^{11} + 7C_2^{11}}{C_9^{11}}$$

RESOLUCIÓN

$$\boxed{C_k^n = C_{n-k}^n}$$

$$M = \frac{3C_2^{11} - 5C_9^{11} + 7C_2^{11}}{C_9^{11}}$$

$$\boxed{C_k^n = C_{n-k}^n}$$

$$M = \frac{3C_2^{11} - 5C_2^{11} + 7C_2^{11}}{C_2^{11}}$$

$$M = \frac{5\cancel{C_2^{11}}}{\cancel{C_2^{11}}}$$

$$\boxed{M = 5}$$

8. Calcule n^n

$$\frac{C_2^n + C_3^{n+1}}{C_4^{n+2}} = \frac{7}{5}$$

RESOLUCIÓN

$$\boxed{C_k^n = \frac{n}{k} C_{k-1}^{n-1}}$$

$$\frac{C_2^n + \overbrace{C_3^{n+1}}}{\underbrace{C_4^{n+2}}} = \frac{7}{5}$$

$$\boxed{C_k^n = \frac{n}{k} C_{k-1}^{n-1}}$$

$$\Rightarrow \quad \frac{C_2^n + \frac{(n+1)}{3} C_2^n}{\frac{(n+2)}{4} \underbrace{C_3^{n+1}}} = \frac{7}{5}$$

$$\boxed{C_k^n = \frac{n}{k} C_{k-1}^{n-1}}$$

$$\Rightarrow \quad \frac{\frac{(n+4)}{\cancel{3}} \cancel{C_2^n}}{\frac{(n+2)}{4} \frac{(n+1)}{\cancel{3}} \cancel{C_2^n}} = \frac{7}{5} \quad \Rightarrow \quad \frac{(n+4)}{(n+2)(n+1)} = \frac{7}{20}$$

$$\Rightarrow \quad \frac{(n+4)}{(n+2)(n+1)} = \frac{(3+4)}{(3+2)(3+1)}$$

$$\Rightarrow \quad \boxed{n = 3}$$

$$\boxed{n^n = 27}$$