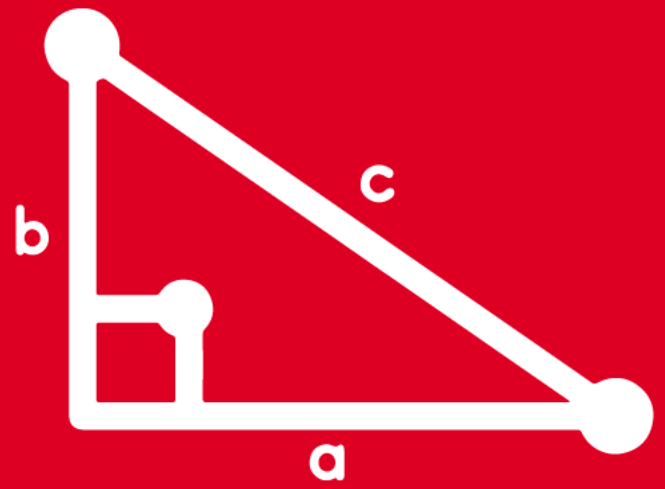
TRIGONOMETRY

Session 2
TOMO 6



4th
SECONDARY

REVIEW

SACO OLIVEROS

1. Si: $csc\alpha + cot\alpha = 4$, calcule $E = 34sen\alpha$

Resolución:

Tenemos:

$$csc\alpha + cot\alpha = 4$$
$$csc\alpha - cot\alpha = \frac{1}{4}$$

(+)

$$2csc\alpha = \frac{17}{4}$$

$$csc\alpha = \frac{17}{8} \Rightarrow sen\alpha = \frac{8}{17}$$

Recordar:

Si: $csc\,x + cot\,x = a$

Entonces:

$$csc\,x - cot\,x = \frac{1}{a}$$

Calculamos: $E = 34sen\alpha$

$$E = \overset{2}{\cancel{34}}\left(\frac{8}{\cancel{17}}\right)$$

$$\therefore \mathbf{E = 16}$$

$sen^2x + cos^2x = 1$

2. Si: senϕ + cosϕ = 1,5
Reduzca: E = senϕ. cosϕ + $\frac{35}{8}$

Resolución:

Dato: $sen\phi + cos\phi = \frac{3}{2}$

ELEVAMOS AL CUADRADO

$$sen^2\phi + cos^2\,\phi + 2sen\phi.cos\phi = \frac{9}{4}$$

$$1 + 2sen\phi.cos\phi = \frac{9}{4}$$

$$2sen\phi.cos\phi = \frac{5}{4}$$

$$sen\phi.cos\phi = \frac{5}{8}$$

Calculamos:

$$E = sen\phi.cos\phi + \frac{35}{8}$$

$$E = \frac{5}{8} + \frac{35}{8} = \frac{40}{8}$$

$$\therefore \mathbf{E = 5}$$

3. Elimine x de las siguientes ecuaciones:

$$senx = \frac{1}{a-b}\ ;\ tanx = \frac{1}{a+b}$$

Resolución:

De los datos:

$$senx = \frac{1}{a-b} \qquad tanx = \frac{1}{a+b}$$

$cscx = a - b$ $\qquad$ $cotx = a + b$

Recordar:

Por identidad pitagórica:

$$1 + cot^2x = csc^2\ x$$

$$1 + cot^2x = csc^2\ x$$

$$1 + (a+b)^2 = (a-b)^2$$

$$1 = (a-b)^2 - (a+b)^2$$

$$1 = a^2 + b^2 - 2ab - (a^2 + b^2 + 2ab)$$

$$1 = \cancel{a^2} + \cancel{b^2} - 2ab - \cancel{a^2} - \cancel{b^2} - 2ab$$

$$\therefore\ 1 = -4ab$$

4. Si: $tan\alpha + cot\alpha = 5$,
reduzca:
$K = sec^2\alpha + csc^2\alpha + 5$

Resolución:

$$tan\alpha + cot\alpha = sec\alpha.csc\alpha$$

$$sec^2\alpha + csc^2\alpha = sec^2\alpha.csc^2\alpha$$

Del dato: $\sec\alpha.csc\alpha = 5$

Al cuadrado: $\sec^2\alpha.\csc^2\alpha = 25$

Por identidad: $\sec^2\alpha + \csc^2\alpha = 25$

Calculamos: $\mathbf{K = \underbrace{sec^2\alpha + csc^2\alpha}_{25} + 5}$

$$K = 25 + 5$$

$$\therefore \mathbf{K = 30}$$

5. Si: $sen\phi + cos\phi = \sqrt{\frac{5}{3}}$

Reduzca: $E = tan\phi + cot\phi$

Resolución:

Dato: $sen\phi + cos\phi = \sqrt{\frac{5}{3}}$

Al cuadrado:

$$\underbrace{sen^2\phi + cos^2\phi}_{1} + 2sen\phi.cos\phi = \frac{5}{3}$$

$$1 + 2sen\phi.cos\phi = \frac{5}{3}$$

$$2sen\phi.cos\phi = \frac{2}{3}$$

$$sen\phi.cos\phi = \frac{1}{3} \Rightarrow sec\phi.csc\phi = 3$$

Calculamos: $\mathbf{E = \underbrace{tan\phi + cot\phi}}$

$$E = sec\phi.csc\phi$$

$$\therefore \mathbf{E = 3}$$

6. Si: $sen^6\alpha + cos^6\alpha = \frac{1}{4}$, reduzca:

$$E = (1 - sen^2\alpha)(1 - cos^2\alpha)$$

Resolución:

Dato: $sen^6\alpha + cos^6\alpha = \frac{1}{4}$

Usamos identidad auxiliar:

$$1 - 3sen^2\alpha\, cos^2\alpha = \frac{1}{4}$$

$$\frac{3}{4} = 3sen^2\alpha\, cos^2\alpha$$

$$\frac{1}{4} = sen^2\alpha\, cos^2\alpha$$

Calculamos:

$$E = (1 - sen^2\alpha)(1 - cos^2\alpha)$$

$$E = 1 - sen^2\alpha + sen^2\alpha.cos^2\alpha - cos^2\alpha$$

$$E = 1 - \underbrace{(sen^2\alpha + cos^2\alpha)}_{1} + \underbrace{sen^2\alpha.cos^2\alpha}_{\frac{1}{4}}$$

$$E = \cancel{1} - \cancel{1} + \frac{1}{4}$$

$$\therefore E = \frac{1}{4}$$

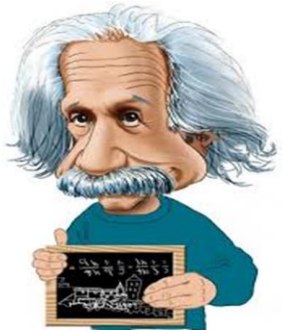
7. Reduzca:

$$M = sen(53° + x).sen(53° - x) - cos^2 x$$

Resolución:

Recordar:

$$sen(x + y).sen(x - y) = sen^2 x - sen^2 y$$



Dato:

$$M = sen(53° + x).sen(53° - x) - cos^2 x$$

Usamos identidad auxiliar:

$$M = sen^2 53° - sen^2 x - cos^2 x$$

$$M = \left(\frac{4}{5}\right)^2 - \underbrace{(sen^2 x + cos^2 x)}_{1}$$

$$M = \frac{16}{25} - 1$$

$$M = \frac{16 - 25}{25}$$

$$\therefore M = -\frac{9}{25}$$

8. En el triángulo ABC se cumple que $tanB = \frac{3}{4}$ y $tanC = 2$; calcular $tanA$

Resolución:

Como ABC es un triángulo, entonces:

$$A + B + C = 180°$$

Recordar:

Si: $A + B + C = 180°$

$tanA + tanB + tanC = tanA.tanB.tanC$

Se cumple:

$$tanA + tanB + tanC = tanA.tanB.tanC$$

$$\Rightarrow tanA + \frac{3}{4} + 2 = tanA.\frac{3}{4}.2$$

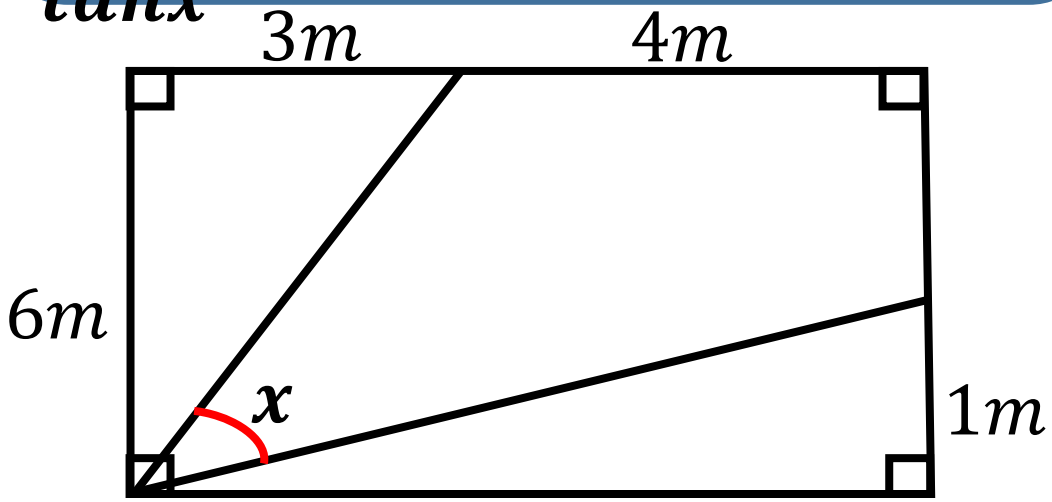
$$tanA + \frac{11}{4} = \frac{6}{4}tanA$$

x 4: $4tanA + 11 = 6tanA$

$$\Rightarrow 11 = 2tanA$$

$$\therefore \boldsymbol{tanA = \frac{11}{2}}$$

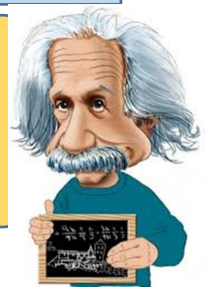
9. A partir del gráfico, determine el valor de $tanx$



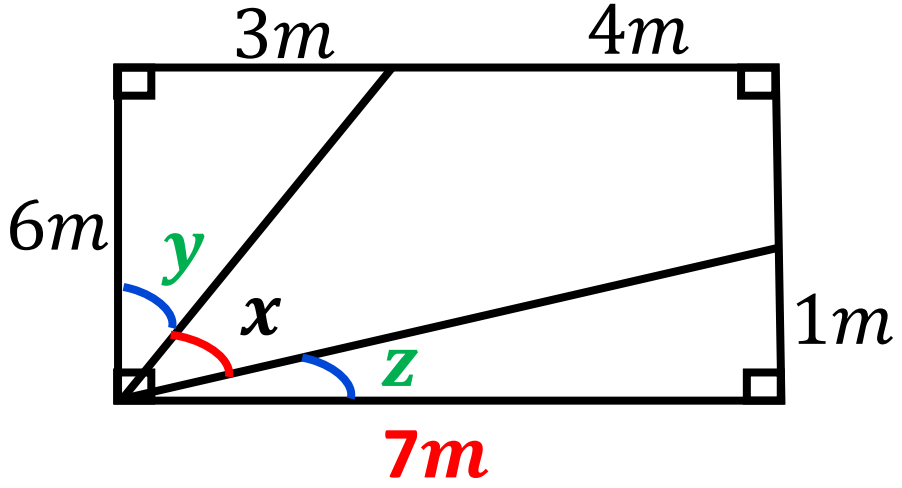
Recordar:

Si: $x + y + z = 90°$

$cotx + coty + cotz = cotx.coty.cotz$



Resolución:



$$cotx + coty + cotz = cotx.coty.cotz$$

$$cotx + \frac{6}{3} + \frac{7}{1} = cotx.\frac{6}{3}.\frac{7}{1}$$

$$cotx + 9 = 14cotx$$

$$\frac{9}{13} = cotx$$

$$\therefore tanx = \frac{13}{9}$$

10. Si la ganancia de un campesino es S/ 10A diarios. Determinar cuánto ganó al mes si trabajó los 30 días.

$A = (1 + sen\alpha)(1 - cos\alpha)$

Dato: $sen\alpha - cos\alpha = \sqrt{5} - 1$

Resolución:

Tenemos:

$$A = (1 + sen\alpha)(1 - cos\alpha)$$

$$2A = 2(1 + sen\alpha)(1 - cos\alpha)$$

Por identidad:

$$2A = (1 + sen\alpha - cos\alpha)^2$$

Usamos dato:

$$2A = (\cancel{1} + \sqrt{5} - \cancel{1})^2$$

$$2A = 5 \quad \Rightarrow \quad 10A = 25$$

Ganancia del campesino

en un mes: $30(10A) = 30(25)$

$\therefore$ Ganancia mensual: S/ **750**