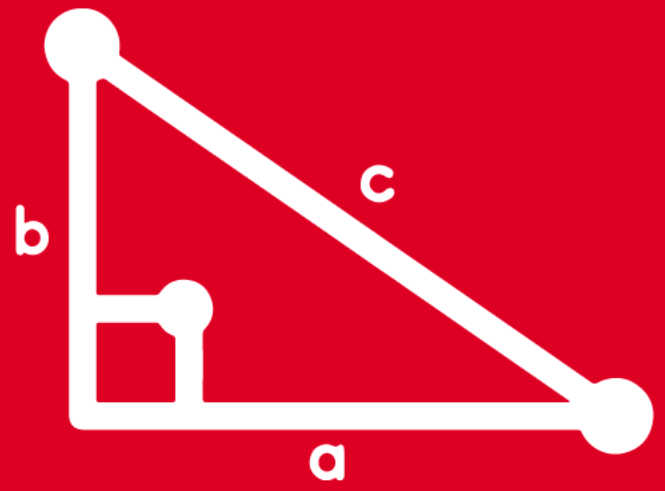
TRIGONOMETRY

Chapter 17

3rd
SECONDARY

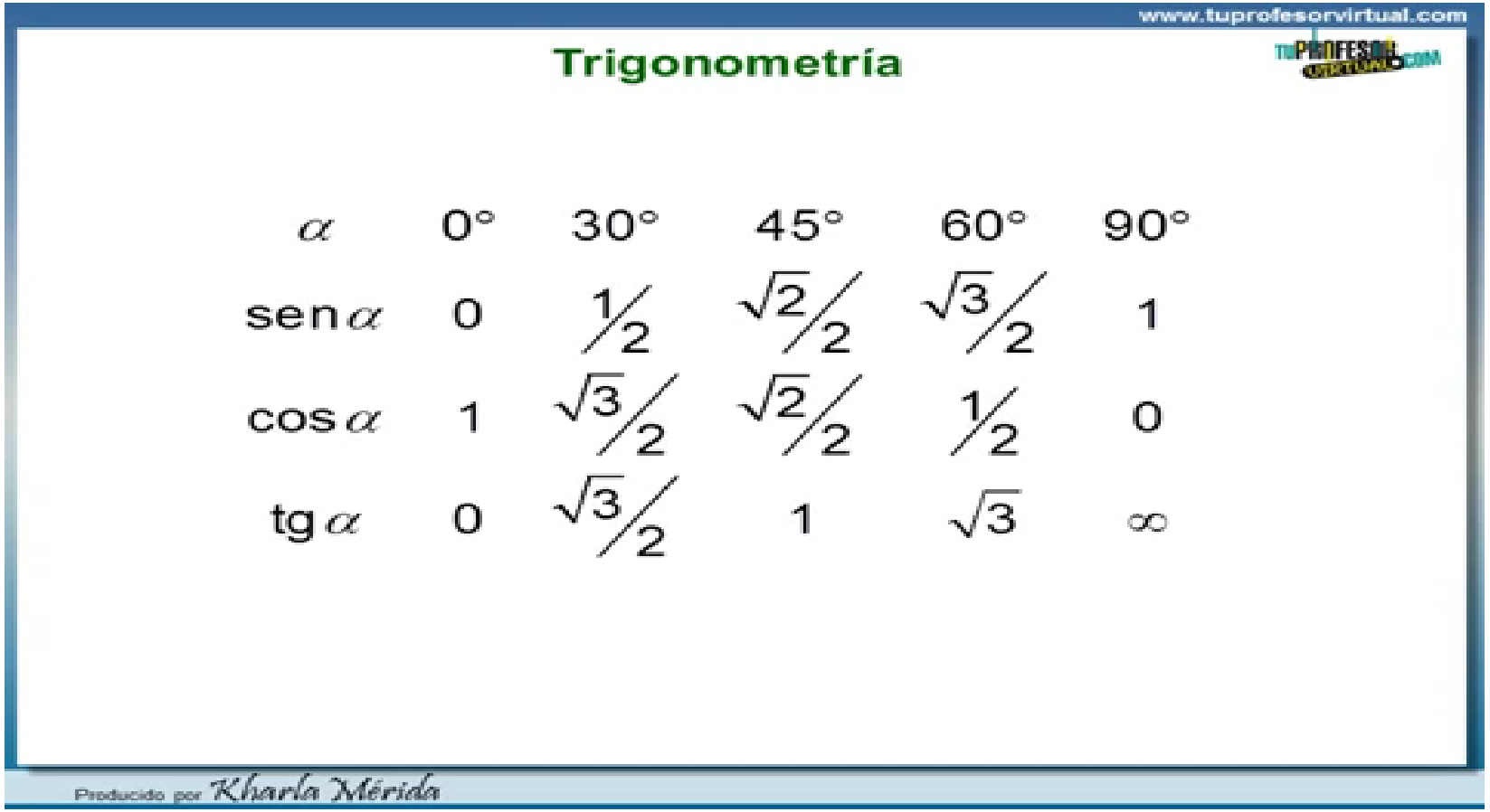


REDUCCIÓN AL PRIMER CUADRANTE II

SACO OLIVEROS

MOTIVATING | STRATEGY

¿ CÓMO REPRESENTAMOS EL SENO Y EL COSENO DE UN ÁNGULO NEGATIVO ?



α	0°	30°	45°	60°	90°
sen α	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos α	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg α	0	$\frac{\sqrt{3}}{2}$	1	$\sqrt{3}$	∞

REDUCCIÓN AL PRIMER CUADRANTE II

3er CASO : PARA ÁNGULOS MAYORES A UNA VUELTA

Si a un ángulo positivo α mayor de una vuelta, se le elimina de su medida el número entero de vueltas que contiene, entonces los valores de sus razones trigonométricas no varían, es decir :

$$\begin{array}{c|c} \alpha & 360^0 \\ \hline (\theta) & n \end{array}$$

$$RT(\alpha) = RT(360^\circ n + \theta) = RT(\theta)$$

$$n \in z^+$$

$$0^0 < \theta < 360^0$$

Nota : "n" indica el número entero positivo de vueltas contenidas en el ángulo, que podemos eliminar.

Ejemplo :

$$\tan 765° = \tan(\cancel{360°}\cdot 2 + 45°) = \tan 45° = 1$$

$$\begin{array}{r|l} 765° & 360° \\ \cline{2-2} 720° & 2 \\ \cline{1-1} (45°) & \end{array}$$

4to CASO : PARA ARCOS NUMÉRICOS CON FACTOR π

A) Para arcos fraccionarios de la forma $\frac{a\pi}{b}$; donde a > 2b

$$\begin{array}{c|c} a & 2b \\ \cline{2-2} (r) & q \end{array}$$

$$RT\left(\frac{a\,\pi}{b}\right) = RT\left(\frac{r\,\pi}{b}\right)$$

Ejemplo :

$$\csc\left(\frac{33\pi}{4}\right) = \csc\left(\frac{1\pi}{4}\right) = \sqrt{2}$$

$$\begin{array}{r|l} 33 & 8 \\ \cline{2-2} 32 & 4 \\ \cline{1-1} (1) & \end{array}$$

4to CASO : PARA ARCOS NUMÉRICOS CON FACTOR π

B) Para arcos enteros de la forma $n\pi$; donde $n \in \mathbb{Z}$

$$RT\left(par.\pi \pm \theta\right) = RT\left(\pm \theta\right)$$

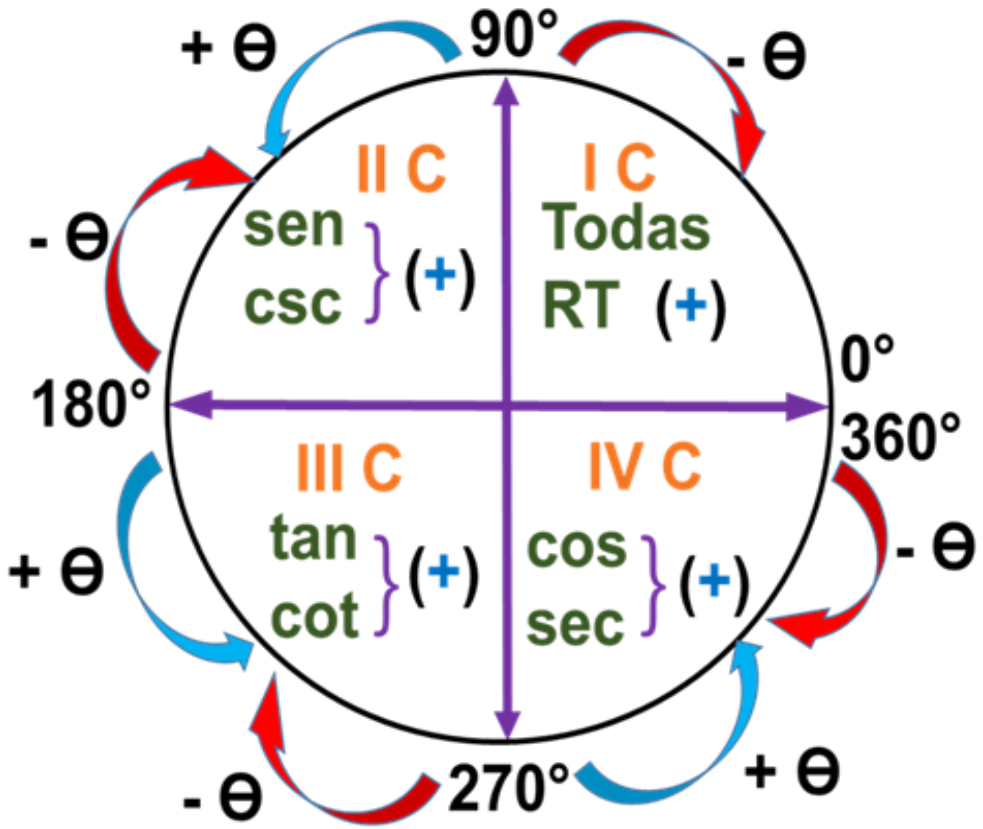
$$RT\left(impar.\pi \pm \theta\right) = RT\left(\pi \pm \theta\right)$$

Ejemplos :

$$\cot\left(\underbrace{6\pi}_{par} - \frac{\pi}{3}\right) = \cot\left(-\frac{\pi}{3}\right) = -\cot\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$$

$$\operatorname{sen}\left(\underbrace{9\pi}_{impar} - \frac{\pi}{6}\right) = \operatorname{sen}\underbrace{\left(\pi - \frac{\pi}{6}\right)}_{II\,C} = \operatorname{sen}\frac{\pi}{6} = \frac{1}{2}$$

DEBEMOS RECORDAR



$$RT\left\{\begin{matrix}180° \pm \theta \\ 360° - \theta\end{matrix}\right\} = \pm RT(\theta)$$

$$RT\left\{\begin{matrix}90° \pm \theta \\ 270° \pm \theta\end{matrix}\right\} = \pm Co\_RT(\theta)$$

$$\cos(-\alpha) = \cos\alpha$$
$$\sec(-\alpha) = \sec\alpha$$

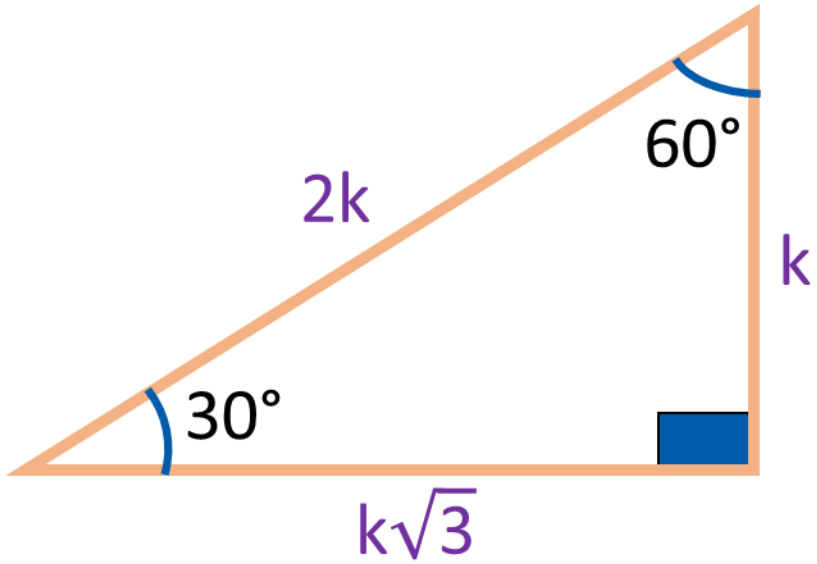
$$\operatorname{sen}(-\alpha) = -\operatorname{sen}\alpha$$
$$\tan(-\alpha) = -\tan\alpha$$
$$\cot(-\alpha) = -\cot\alpha$$
$$\csc(-\alpha) = -\csc\alpha$$

1) Calcule cos1110°

RESOLUCIÓN:

$$\begin{array}{r|l} 1110° & 360° \\ \cline{2-2} 1080° & 3 \\ \cline{1-1} (30°) & \end{array}$$

$$\cos\theta = \frac{CA}{H}$$



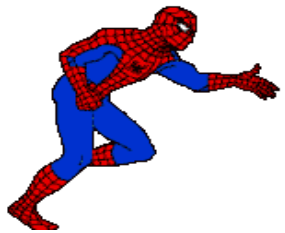
Piden : $\cos 1110° = \cos 30°$

$$\therefore \cos 1110° = \frac{\sqrt{3}}{2}$$

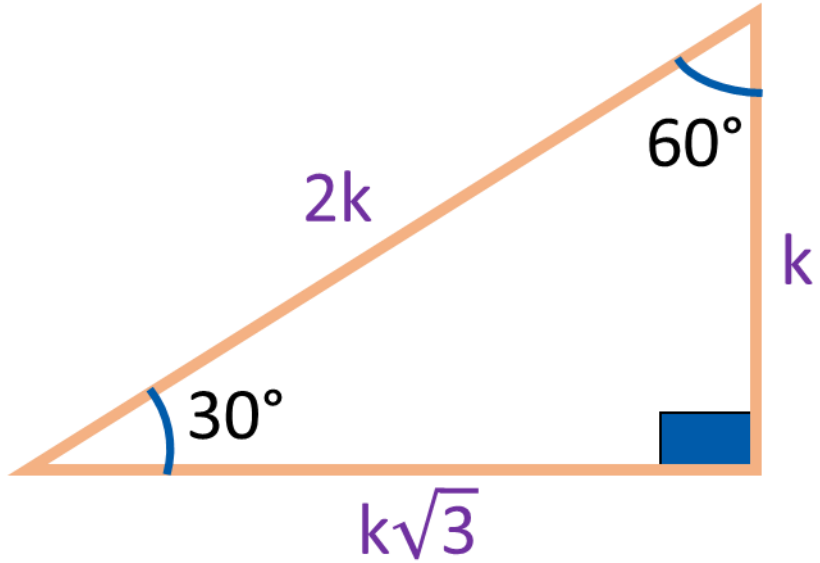
2) Calcule sen4020°

RESOLUCIÓN:

$$\begin{array}{r|l} 4020^\circ & 360^\circ \\ \cline{2-2} 3960^\circ & 11 \\ \cline{1-1} (\ 60^\circ\ ) & \end{array}$$



$$\text{sen}\theta = \frac{CO}{H}$$



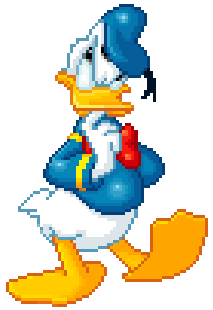
Piden : sen4020° = sen60°

$$\therefore \text{sen}4020^\circ = \frac{\sqrt{3}}{2}$$

3) Reduzca
E = cos780°. sec1485°

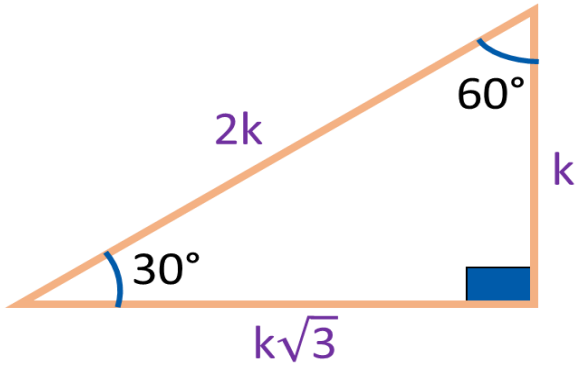
RESOLUCIÓN:

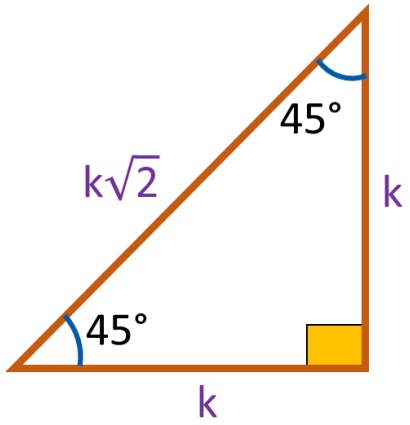
$$\begin{array}{r|l} 780° & 360° \\ 720° & 2 \\ \hline (\ 60°) & \end{array} \qquad \begin{array}{r|l} 1485° & 360° \\ 1440° & 4 \\ \hline (\ 45°\ ) & \end{array}$$



$$\cos\theta = \frac{CA}{H}$$

$$\sec\theta = \frac{H}{CA}$$





Piden : E = cos780°. sec1485°

E = cos60° . sec45°

$$E = \left(\frac{1}{2}\right)\left(\sqrt{2}\right)$$

$$\therefore E = \frac{\sqrt{2}}{2}$$

4) Reduzca

$A = \operatorname{sen}(24\pi + x)$

$B = \tan(12\pi - x)$

RESOLUCIÓN:

Recordemos que :

$$RT\,(\,par.\,\pi \pm \theta\,) = RT(\,\pm\theta\,)$$

$$RT\,(\,impar.\,\pi \pm \theta\,) = RT(\pi \pm \theta\,)$$

$$\tan(-\alpha) = -\tan\alpha$$

Luego :

$A = \operatorname{sen}(\underbrace{24\pi}_{par} + x)$

$\therefore A = \operatorname{sen} x$

$B = \tan(\underbrace{12\pi}_{par} - x)$

$B = \tan(- x)$

$\therefore B = -\tan x$

5) Reduzca

a) $\operatorname{sen}\left(\frac{13\pi}{2}+x\right)$

b) $\tan\left(\frac{23\pi}{2}+x\right)$

RESOLUCIÓN:

Recordemos que :

$$RT\left(\frac{a\,\pi}{b}\right)=RT\left(\frac{r\,\pi}{b}\right)$$

$$\begin{array}{c|c} a & 2b \\ (r) & q \end{array}$$

$$RT\left\{\begin{matrix}90°\pm\theta\\270°\pm\theta\end{matrix}\right\}=\pm\,Co\_RT(\theta)$$

Luego :

a) $\operatorname{sen}\left(\frac{13\pi}{2}+x\right)=\operatorname{sen}\left(\frac{1\pi}{2}+x\right)$ (II C)

$$\begin{array}{c|c} 13 & 4 \\ \hline 12 & 3 \\ \hline (1) & \end{array}$$

$= \cos x$

b) $\tan\left(\frac{23\pi}{2}+x\right)=\tan\left(\frac{3\pi}{2}+x\right)$ (IV C)

$$\begin{array}{c|c} 23 & 4 \\ \hline 20 & 5 \\ \hline (3) & \end{array}$$

$= -\cot x$

6) Reduzca

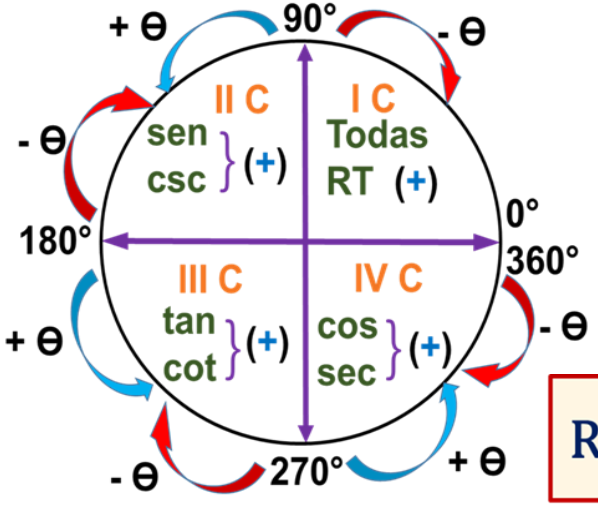
$$E = \cos(35\pi + x) \cdot \sec(23\pi + x)$$

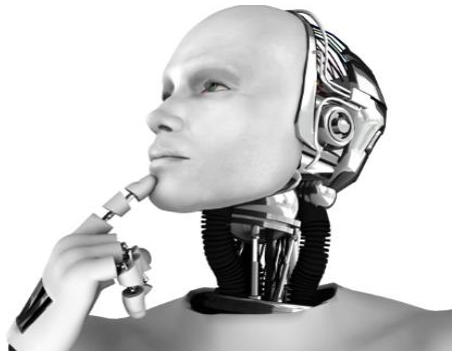
RESOLUCIÓN:

Recordemos que :

$$RT(\text{par}.\pi \pm \theta) = RT(\pm\theta)$$

$$RT(\text{impar}.\pi \pm \theta) = RT(\pi \pm \theta)$$





$$RT\left\{\begin{matrix}180° \pm \theta \\ 360° - \theta\end{matrix}\right\} = \pm RT(\theta)$$

Luego :

$$E = \cos(\underbrace{35\pi}_{\text{impar}} + x) \cdot \sec(\underbrace{23\pi}_{\text{impar}} + x)$$

$$E = \cos(\underbrace{\pi + x}_{\text{III C}}) \cdot \sec(\underbrace{\pi + x}_{\text{III C}})$$

$$E = (-\cos x)(-\sec x)$$

$$E = \cos x \cdot \sec x$$

$$\therefore E = 1$$

7) Reduzca

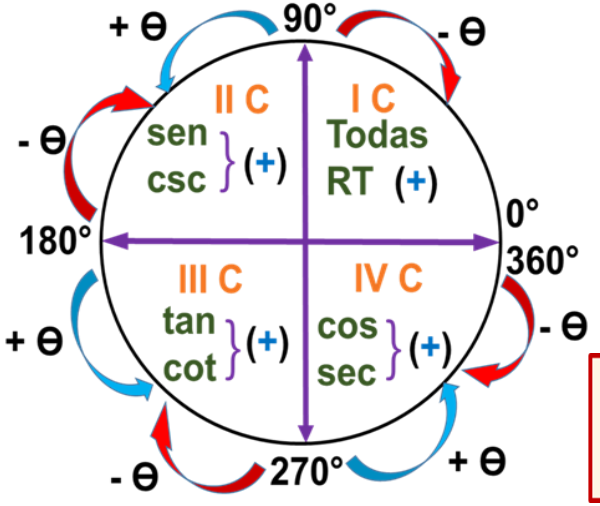
$$T = \tan\left(\frac{25\pi}{2} + x\right) - 3\tan\left(\frac{17\pi}{2} + x\right)$$

RESOLUCIÓN:

Recordemos que :

$$RT\left(\frac{a\,\pi}{b}\right) = RT\left(\frac{r\,\pi}{b}\right)$$

$$\begin{array}{r|l} a & 2b \\ (r) & q \end{array}$$



$$RT\left\{\begin{matrix} 90° \pm \theta \\ 270° \pm \theta \end{matrix}\right\} = \pm\, Co\_RT(\theta)$$

Luego :

$$\begin{array}{r|l} 25 & 4 \\ 24 & 6 \\ \hline (1) & \end{array} \qquad \begin{array}{r|l} 17 & 4 \\ 16 & 4 \\ \hline (1) & \end{array}$$

$$T = \tan\left(\frac{1\pi}{2} + x\right) - 3\tan\left(\frac{1\pi}{2} + x\right)$$

$$T = -2\tan\underbrace{\left(\frac{\pi}{2} + x\right)}_{\text{II C}}$$

$$T = -2\,(-\cot x)$$

$$\therefore\ T = 2\cot x$$

8) El gasto diario de Jhon en pasajes es de S/. A ¿ Cuál será el gasto total a la semana ?. Para calcular dicho valor deberás reducir lo siguiente:

$$A = \frac{\operatorname{sen}(42\pi + x)}{\operatorname{sen}(31\pi + x)} + \frac{\tan(\frac{21\pi}{2} - x)}{\tan(\frac{39\pi}{2} + x)} + 3$$

RESOLUCIÓN

par

impar

$$A = \frac{\operatorname{sen}(42\pi + x)}{\operatorname{sen}(31\pi + x)} + \frac{\tan(\frac{21\pi}{2} - x)}{\tan(\frac{39\pi}{2} + x)} + 3 = \frac{\operatorname{sen}x}{\operatorname{sen}(\pi + x)} + \frac{\tan(\frac{1\pi}{2} - x)}{\tan(\frac{3\pi}{2} + x)} + 3$$

I C

III C

IV C

$$A = \frac{\operatorname{sen}x}{-\operatorname{sen}x} + \frac{\cot x}{-\cot x} + 3 = -1 - 1 + 3 = 1$$

Piden : $7A = 7(1) = 7$

∴ Gasto semanal = S/. 7

$$\begin{array}{r|l} 21 & 4 \\ \hline 20 & 5 \\ \hline (1) & \end{array}$$

$$\begin{array}{r|l} 39 & 4 \\ \hline 36 & 9 \\ \hline (3) & \end{array}$$

COLEGIOS
SACO OLIVEROS APEIRON
SISTEMA HELICOIDAL

MUCHAS GRACIAS POR TU ATENCIÓN

Tu curso amigo
TRIGONOMETRÍA