



TRIGONOMETRY

Chapter 16, 17 and 18

5th
SECONDARY

REVIEW





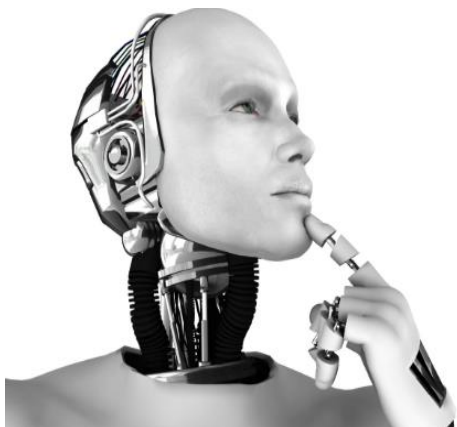
HELICOREVIEW 1

Simplifique la expresión $E = 3(\cot x - \tan x)\tan 2x$

Resolución:

Recordar:

$$\cot x - \tan x = 2\cot(2x)$$



$$E = 3 \underbrace{(\cot x - \tan x)}_{2\cot 2x} \tan 2x$$

$$E = 6 \underbrace{\cot 2x \cdot \tan 2x}_1$$



$$E = 6$$



HELICOREVIEW 2

Al copiar de la pizarra la expresión $1 + \cos 40^\circ$, un estudiante cometió un error y escribió $\sin 40^\circ$. Calcule la razón entre lo que estaba escrito en la pizarra y lo que copió el estudiante.

Resolución:

Recordar:

$$2\cos^2(x) = 1 + \cos(2x)$$



Debió escribir

escribió

$$\begin{array}{lcl}
 \cancel{2\cos^2 20^\circ} & \xrightarrow{\text{Debió escribir}} & 1 + \cos 40^\circ \\
 & \xrightarrow{\text{escribió}} & \sin 40^\circ \\
 \cancel{2\sin 20^\circ \cos 20^\circ} & & \\
 & & \xrightarrow{\quad} \frac{\cos 20^\circ}{\sin 20^\circ} \\
 & & \therefore \cot 20^\circ
 \end{array}$$



HELICOREVIEW 3

Simplifique $M = \sqrt{2 + \sqrt{2 - 2\cos 80^\circ}}$

Resolución:

$$M = \sqrt{2 + \sqrt{2 - 2\cos 80^\circ}}$$

$$M = \sqrt{2 + \sqrt{2(1 - \cos 80^\circ)}}$$

$$M = \sqrt{2 + \sqrt{2(2\sin^2 40^\circ)}}$$

$$M = \sqrt{2 + \sqrt{4\sin^2 40^\circ}}$$

$$M = \sqrt{2 + 2\sin 40^\circ}$$

$$M = \sqrt{2(1 + \sin 40^\circ)}$$

$$M = \sqrt{2(1 + \cos 50^\circ)}$$

$$M = \sqrt{2(2\cos^2 25^\circ)}$$

$$M = \sqrt{4\cos^2 25^\circ}$$



$$M = 2\cos 25^\circ$$

$$2\sin^2(x) = 1 - \cos(2x)$$

$$2\cos^2(x) = 1 + \cos(2x)$$



HELICOREVIEW 4

Reduzca $H = \sqrt{\frac{1 + \cos 140^\circ}{2}} + \frac{\sin 40^\circ}{2 \cos 20^\circ}$

Resolución:

RECORDAR

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$



$$H = \sqrt{\frac{1 + \cos 140^\circ}{2}} + \frac{\sin 40^\circ}{2 \cos 20^\circ}$$

Diagram showing the simplification process with annotations:

- A purple oval highlights the square root term $\sqrt{\frac{1 + \cos 140^\circ}{2}}$. A purple arrow points to it with the label $\cos 70^\circ$.
- A green oval highlights the $\sin 40^\circ$ term in the numerator. A green arrow points to it with the label $2 \sin 20^\circ \cos 20^\circ$ (crossed out).
- The denominator $2 \cos 20^\circ$ is crossed out with a red line.

$$\Rightarrow H = \cos 70^\circ + \sin 20^\circ$$

Diagram showing the final simplified expression with annotations:

- A blue arrow points to $\sin 20^\circ$ with the label $\cos 70^\circ$.



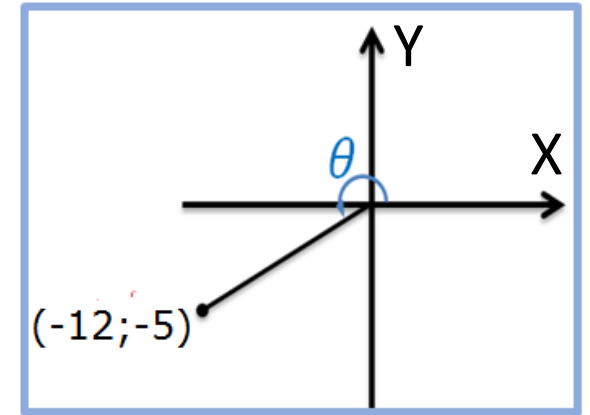
$$H = 2 \cos 70^\circ$$

HELICOREVIEW 5



Del gráfico, calcule: $\tan\left(\frac{\theta}{2}\right)$

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$



Resolución:

Del gráfico se observa: $180^\circ < \theta < 270^\circ$

$\rightarrow 90^\circ < \frac{\theta}{2} < 135^\circ$
 $\in \text{IIC}$

Además: $x = -12$; $y = -5$

$$r = \sqrt{(-12)^2 + (-5)^2} = 13$$

$$\cos \theta = \frac{x}{r} = -\frac{12}{13}$$

$$\tan\left(\frac{\theta}{2}\right) = \overset{\text{IIC}}{-} \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = - \sqrt{\frac{1 - \left(-\frac{12}{13}\right)}{1 + \left(-\frac{12}{13}\right)}}$$

$$\tan\left(\frac{\theta}{2}\right) = - \sqrt{\frac{\frac{25}{13}}{\frac{1}{13}}} = -\sqrt{25}$$



$$\tan\left(\frac{\theta}{2}\right) = -5$$



HELICOREVIEW 6

Reduzca la expresión: $P = \frac{\cot\left(\frac{x}{2}\right) - \csc x}{\csc x - \tan\left(\frac{x}{2}\right)}$

Resolución:

$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$



$$P = \frac{\cot\left(\frac{x}{2}\right) - \csc x}{\csc x - \tan\left(\frac{x}{2}\right)}$$

$$P = \frac{\cancel{\csc x} + \cot x - \cancel{\csc x}}{\cancel{\csc x} - (\cancel{\csc x} - \cot x)}$$

$$P = \frac{\cancel{\cot x}}{\cancel{\cot x}}$$

$$\therefore P = 1$$



HELICOREVIEW 7

Reduzca: $T = \frac{4\cos^3 20^\circ - 3\cos 20^\circ}{3\sin 15^\circ - 4\sin^3 15^\circ}$

Resolución:

Recordar:

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$



$$T = \frac{\cos 3(20^\circ)}{3\sin 15^\circ - 4\sin^3 15^\circ}$$

$$T = \frac{\cos 60^\circ}{\sin 45^\circ}$$

$$T = \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}$$

$$\therefore T = \frac{\sqrt{2}}{2}$$



HELICOREVIEW 8

De la condición: $\text{sen}x - \text{cos}x = \frac{\sqrt{2}}{2}$; calcule $\text{sen}6x$.

Resolución:

Dato:

$$\text{sen}x - \text{cos}x = \frac{\sqrt{2}}{2}$$

Elevamos al cuadrado:

$$(\text{sen}x - \text{cos}x)^2 = \left(\frac{\sqrt{2}}{2}\right)^2$$

$$1 - \text{sen}2x = \frac{2}{4}$$

$$\text{sen}2x = \frac{1}{2}$$

Piden:

$$\text{sen}6x = 3\text{sen}2x - 4\text{sen}^3 2x$$

$$\text{sen}6x = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$\text{sen}6x = \frac{3}{2} - \frac{1}{2}$$



$$\text{sen}6x = 1$$

$$(\text{sen}x - \text{cos}x)^2 = 1 - \text{sen}(2x)$$

$$\text{sen}3\alpha = 3\text{sen}\alpha - 4\text{sen}^3\alpha$$





HELICOREVIEW 9

De la siguiente identidad: $\frac{3\text{sen}3x}{\text{sen}x} - \frac{2\text{cos}3x}{\text{cos}x} = M + N\text{cos}(Px)$

Calcule: $M + N + P$

Resolución:

Dato:

$$\frac{3\text{sen}3x}{\text{sen}x} - \frac{2\text{cos}3x}{\text{cos}x} = M + N\text{cos}(Px)$$

$$\frac{3\cancel{\text{sen}x}(2\text{cos}2x + 1)}{\cancel{\text{sen}x}} - \frac{2\cancel{\text{cos}x}(2\text{cos}2x - 1)}{\cancel{\text{cos}x}} = M + N\text{cos}(Px)$$

$$3(2\text{cos}2x + 1) - 2(2\text{cos}2x - 1) = M + N\text{cos}(Px)$$

$$6\text{cos}2x + 3 - 4\text{cos}2x + 2 = M + N\text{cos}(Px)$$

$$5 + 2\text{cos}2x = M + N\text{cos}(Px)$$

$$\text{sen}3x = \text{sen}x(2\text{cos}2x + 1)$$

$$\text{cos}3x = \text{cos}x(2\text{cos}2x - 1)$$

Comparando:

$$M = 5 ; N = 2 ; P = 2$$



$$M + N + P = 9$$



HELICOREVIEW 10

Un científico observa el movimiento de una mariposa en el aire y ve que en un instante de tiempo t , la altura en metros respecto al suelo está dado por la siguiente expresión: $h(t)=16\text{sen}t\cos 2t\cos 4t\cos 8t$, si t está en segundos. ¿A qué altura se encuentra para $t = \frac{\pi}{30}$ seg?

Resolución:

$$h(t)=16\text{sen}t\cos 2t\cos 4t\cos 8t$$

$$h(t).\text{cost}=8.\underbrace{2\text{sen}t\text{cost}}_{\text{sen}2t}\cos 2t\cos 4t\cos 8t$$

$$h(t).\text{cost}=4.\underbrace{2\text{sen}2t\cos 2t}_{\text{sen}4t}\cos 4t\cos 8t$$

$$h(t).\text{cost}=2.\underbrace{2\text{sen}4t\cos 4t}_{\text{sen}8t}\cos 8t$$

$$h(t).\text{cost}=2.\text{sen}8t\cos 8t$$

$$h(t).\text{cost}=\text{sen}16t$$

$$h\left(\frac{\pi}{30}\right).\text{cos}6^\circ = \text{sen}96^\circ \quad \text{IIC}$$

$$h\left(\frac{\pi}{30}\right).\text{cos}6^\circ = \text{sen}(90^\circ + 6^\circ)$$

$$h\left(\frac{\pi}{30}\right).\text{cos}6^\circ = \text{cos}6^\circ$$

$$\frac{\pi}{30} = 6^\circ$$

$$\therefore h\left(\frac{\pi}{30}\right) = 1\text{m}$$