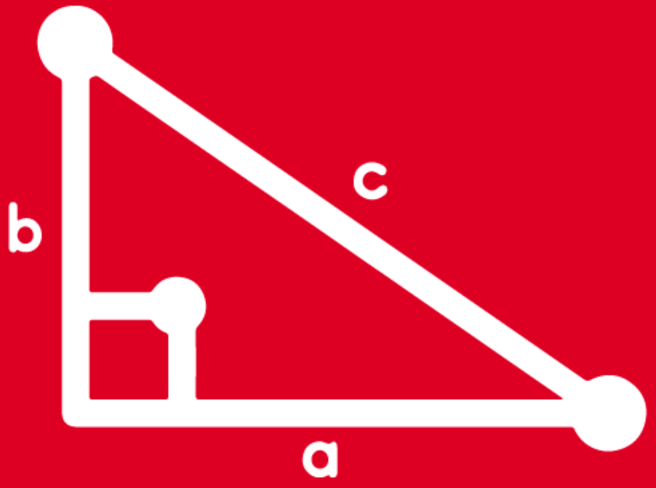
TRIGONOMETRY

SESION 2
TOMO 4



4th
SECONDARY

Feedback

SACO OLIVEROS

PROBLEMA 1

Determine el valor de θ coterminal a $150°$, donde $\theta \in \langle 2800°; 3100° \rangle$

RESOLUCIÓN

Como θ y $150°$

son **coterminales** entonces:

$$\theta - 150° = 360°k$$

$$\boxed{\theta = 360°k + 150°}$$

Pero: $2800° < \theta < 3100°$

$$2800° < 360°k + 150° < 3100°$$

$$(Restar\ 150°)$$

$$2650° < 360°k < 2950°$$

$$(Dividir\ entre\ 360°)$$

$$7{,}361 < k < 8{,}194$$

$$k = 8$$

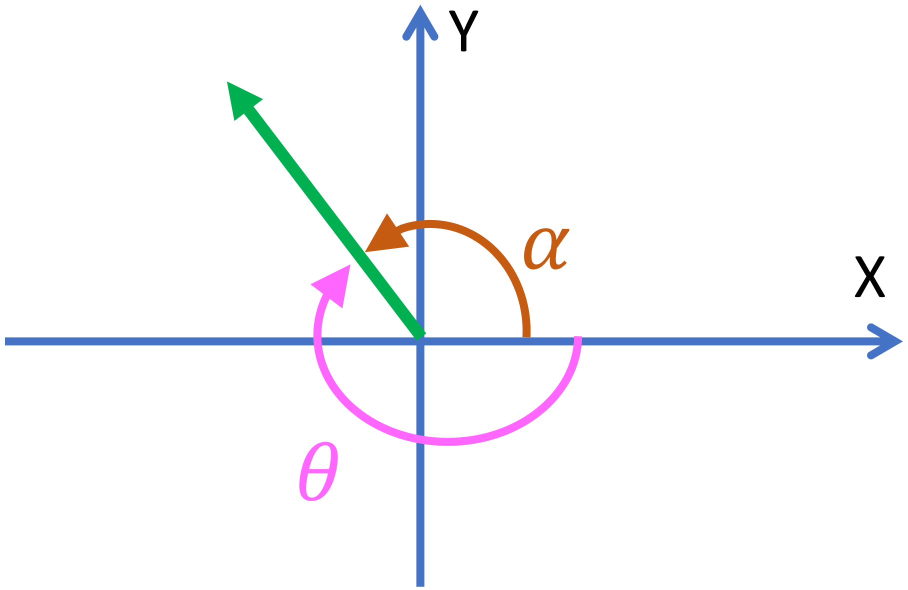
$$\Rightarrow \theta = 360°(8) + 150°$$

$$\therefore \theta = 3030°$$

PROBLEMA 2

De acuerdo al gráfico, reduzca:

$$E = \frac{4\cos\alpha.\sec\theta + 4\tan^2\alpha}{\csc\theta.\operatorname{sen}\alpha \ + \tan^2\theta}$$



RESOLUCIÓN

Si α y θ son ángulos coterminales

$$\operatorname{sen}\alpha = \operatorname{sen}\theta$$
$$\cos\alpha = \cos\theta$$
$$\tan\alpha = \tan\theta$$

Piden: $E = \dfrac{4\cos\alpha.\sec\theta + 4\tan^2\alpha}{\csc\theta.\operatorname{sen}\alpha + \tan^2\theta}$

$$E = \frac{4\cancel{\cos\theta.\sec\theta} + 4\tan^2\theta}{\cancel{\csc\theta.\operatorname{sen}\theta} + \tan^2\theta}$$

$$E = \frac{4 + 4\tan^2\theta}{1 + \tan^2\theta} = \frac{4\cancel{(1 + \tan^2\theta)}}{\cancel{1 + \tan^2\theta}}$$

$$\therefore E = 4$$

PROBLEMA 3

Siendo α y β ángulos coterminales y $\cot\alpha = -\frac{3}{2}$ $\alpha \in IIC$. Calcule: secβ

RESOLUCIÓN

$$\cot\alpha = -\frac{3}{2} = \frac{x}{y}$$

Como $\alpha \in IIC$ se tiene que: $x < 0\,;\, y > 0$

Entonces : $x = -3\,;\; y = 2$

Radio vector: $\boxed{r = \sqrt{x^2 + y^2}}$

$$r = \sqrt{(-3)^2 + (2)^2}$$

$$r = \sqrt{9 + 4} \Rightarrow \boxed{r = \sqrt{13}}$$

Luego : $\sec\alpha = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$

Como α y β son coterminales entonces: $RT(\beta) = RT(\alpha)$

$$\Rightarrow \sec\beta = \sec\alpha$$

$$\boxed{\therefore\ \sec\beta = -\frac{\sqrt{13}}{3}}$$

PROBLEMA 4

Efectúe: $$A = \frac{sec(-120^{\circ}) + 3csc(-217^{\circ})}{cot315^{\circ}}$$

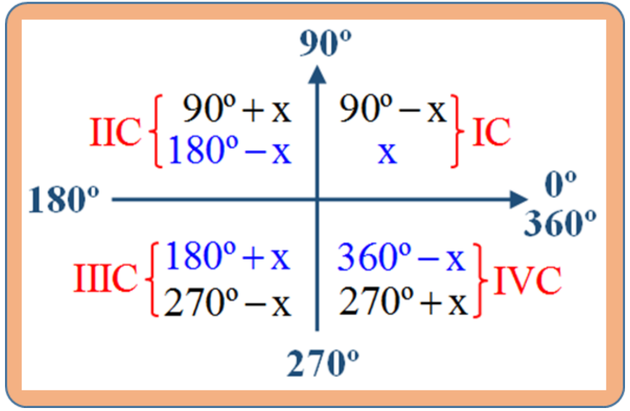
RESOLUCIÓN

Recordar:

$$sec(-x) = sec(x)$$

$$csc(-x) = -csc(x)$$

Recordar:

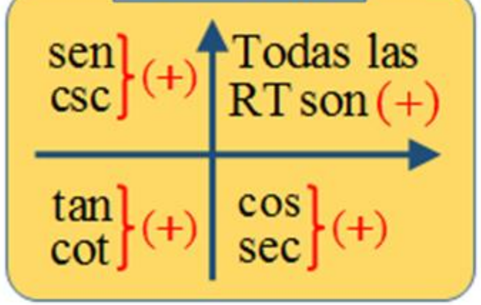


$$A = \frac{sec120^{\circ} + 3(-csc217^{\circ})}{cot315^{\circ}}$$

$$A = \frac{sec(\overbrace{180^{\circ} - 60^{\circ}}^{IIC}) - 3csc(\overbrace{180^{\circ} + 37^{\circ}}^{IIIC})}{cot(\underbrace{360^{\circ} - 45^{\circ}}_{IVC})}$$

$$A = \frac{(-sec60^{\circ}) - 3(-csc37^{\circ})}{-\,cot45^{\circ}}$$

Recordar:



$$A = \frac{-2 + \cancel{3}\left(\frac{5}{\cancel{3}}\right)}{-1}$$

$$A = \frac{3}{-1}$$

$$\therefore\ \boldsymbol{A = -3}$$

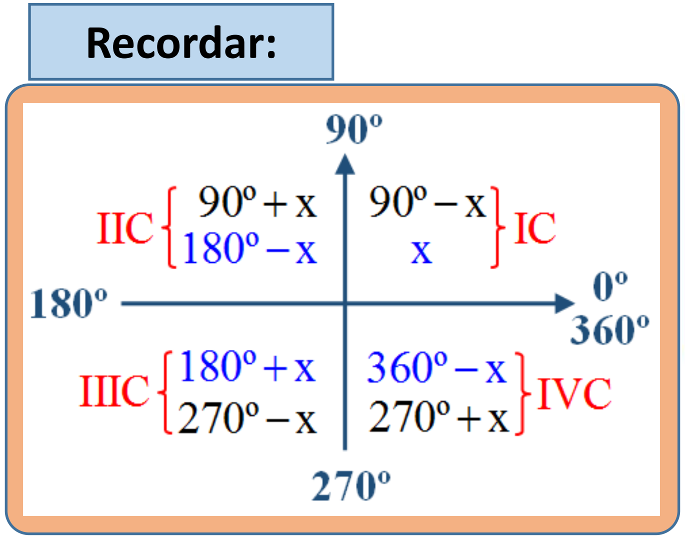
PROBLEMA 5

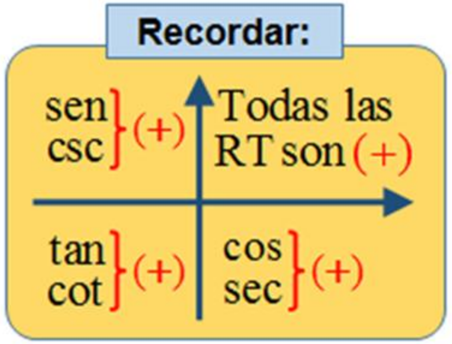
Reduzca

$$M=\frac{sen(\frac{3\pi}{2}-x)}{cos(\pi+x)}+\frac{cot(360°-x)}{tan(90°+x)}$$

RESOLUCIÓN

$$M=\frac{\overbrace{sen(\frac{3\pi}{2}-x)}^{IIIC}}{\underbrace{cos(\pi+x)}_{IIIC}}+\frac{\overbrace{cot(360°-x)}^{IVC}}{\underbrace{tan(90°+x)}_{IIC}}$$





$$M=\frac{-cosx}{-cosx}+\frac{-cotx}{-cotx}$$

$$M=1+1$$

$$\therefore \mathbf{M=2}$$

PROBLEMA 6

Si $\alpha + \phi = \frac{3\pi}{2}$, *reduzca*:

$$B = \frac{sen\alpha}{\cos\phi} + \cot\alpha . \cot\phi$$

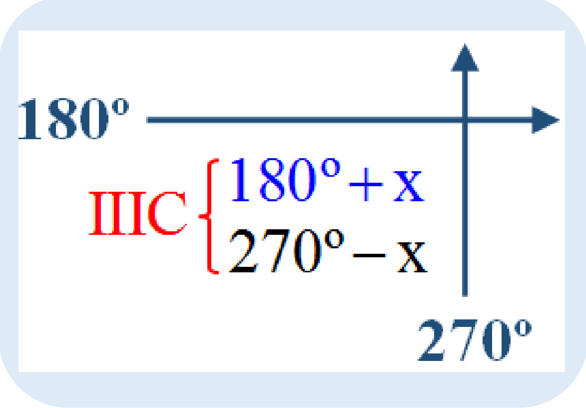
RESOLUCIÓN

$$\alpha + \phi = \frac{3\pi}{2} \Rightarrow \alpha = \frac{3\pi}{2} - \phi$$

$$B = \frac{sen\alpha}{\cos\phi} + \cot\alpha . \cot\phi$$

$$B = \frac{\overbrace{sen\left(\frac{3\pi}{2} - \phi\right)}^{IIIC}}{\cos\phi} + \cot\overbrace{\left(\frac{3\pi}{2} - \phi\right)}^{IIIC} . \cot\phi$$

$$B = \cancel{\frac{-\cos\phi}{\cos\phi}} + \underbrace{\tan\phi . \cot\phi}_{1}$$



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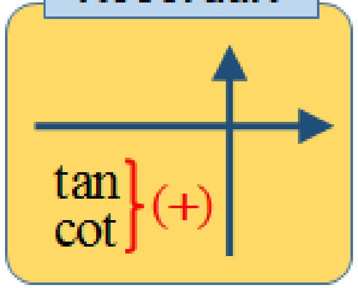


$$\Rightarrow B = -1 + 1$$

$$\boxed{\therefore B = 0}$$

PROBLEMA 7

Efectúe

$$G = \cot 870°.\sec 1215°$$

RESOLUCIÓN

cot30° = $\sqrt{3}$

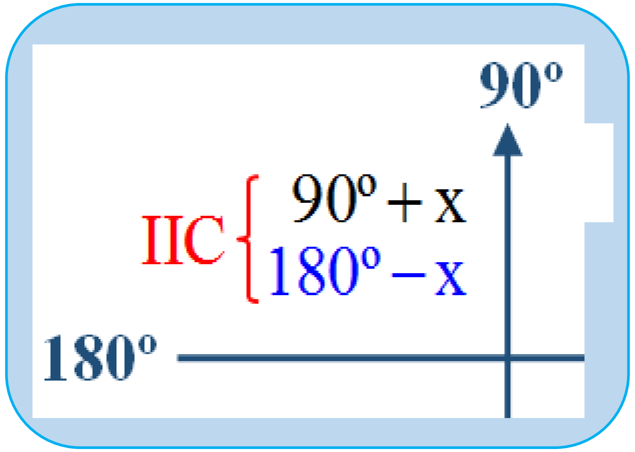
sec45° = $\sqrt{2}$

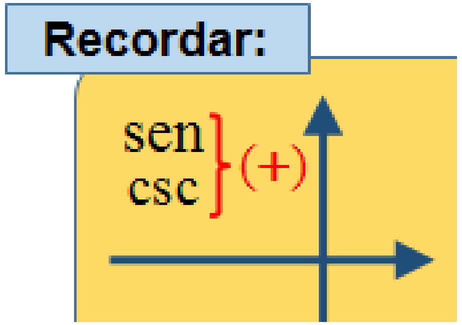
$$\begin{array}{r|l} 870° & 360° \\ \hline 720° & 2 \\ \hline 150° & \end{array}$$

$$\begin{array}{r|l} 1215° & 360° \\ \hline 1080° & 3 \\ \hline 135° & \end{array}$$

Eliminando vueltas:

$$G = \cot 150°.\sec 135°$$





$$G = \cot\underbrace{(180° - 30°)}_{IIC}.\sec\underbrace{(180 - 45°)}_{IIC}$$

$$G = (-\cot 30°)(-\sec 45°)$$

$$G = (-\sqrt{3})(-\sqrt{2})$$

$$\therefore\ G = \sqrt{6}$$

PROBLEMA 8

En un triángulo ABC, reduzca:

$$K = \frac{5csc(3A + 2B + 2C)}{csc(B + C)}$$

RESOLUCIÓN

Del dato: A + B + C = 180°

➡ B + C = 180° - A

También: 2A + 2B + 2C = 2(180°)

➡ 2A + 2B + 2C = 360°

Nos piden:

$$K = \frac{5csc(3A + 2B + 2C)}{csc(B + C)}$$

$$K = \frac{5csc(A + 2A + 2B + 2C)}{csc(B + C)}$$

$$K = \frac{5csc(\overbrace{360° + A}^{IC})}{csc(\underbrace{180° - A}_{IIC})} = \frac{5\cancel{cscA}}{\cancel{cscA}}$$

$$\therefore \boxed{K = 5}$$

PROBLEMA 9

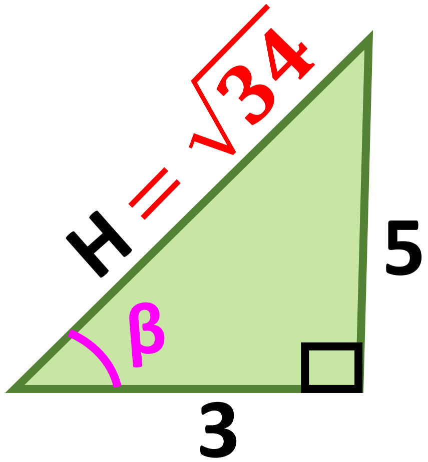
Si $tan\beta = \frac{5}{3}$, donde β es un ángulo agudo, reduzca:

$$K = cos(51\pi - \beta).cos\left(73\frac{\pi}{2} + \beta\right)$$

RESOLUCIÓN

Del dato:

$$tan\beta = \frac{5}{3}$$

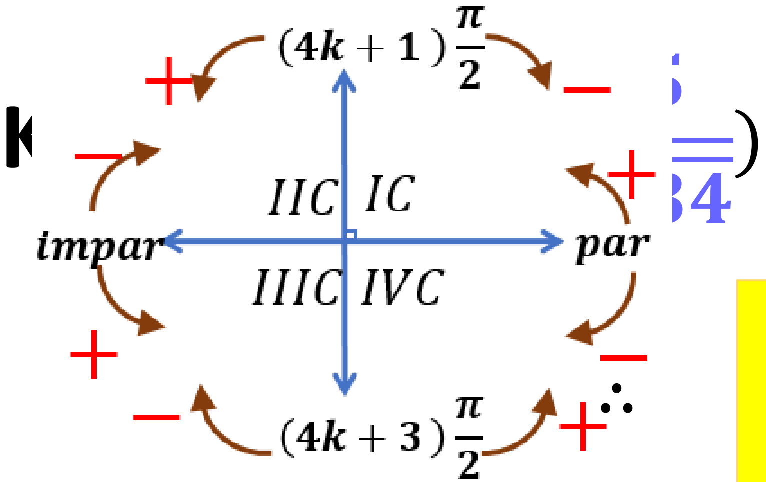


Nos piden:

$$K = cos(\underbrace{\overset{\overset{\text{IMPAR}}{\uparrow}}{51}\pi - \beta}_{IIC}).cos\left(\underbrace{\overset{\overset{4k+1}{\uparrow}}{73}\frac{\pi}{2} + \beta}_{IIC}\right)$$

$$K = (-cos\beta)(-sen\beta)$$

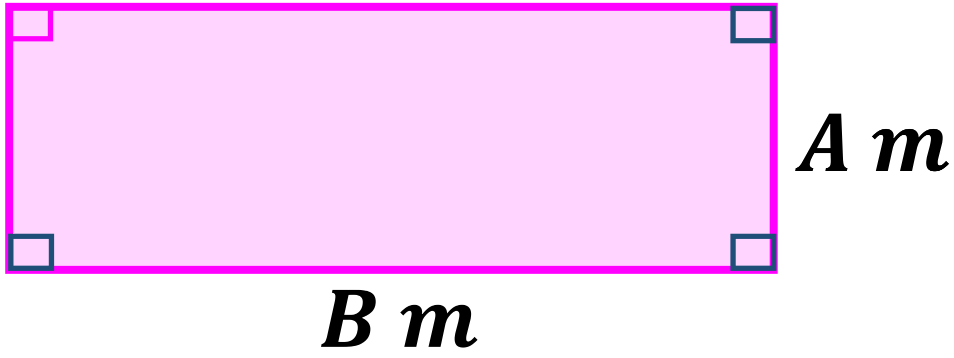
Del triángulo reemplazamos:



$$\boxed{K = \frac{15}{34}}$$

PROBLEMA 10

Juan desea cercar su jardín con una malla metálica. Las dimensiones del jardín (ancho y largo) son las siguientes:



$\boldsymbol{A = 8sen3\alpha + 2sen9\alpha}$
$\boldsymbol{B = 12cos12\alpha - 2sec6\alpha}$

Si se sabe que $\boldsymbol{\alpha = 30°}$, ¿cuál es el perímetro del jardín?

RESOLUCIÓN

$$A = 8sen3\alpha + 2sen9\alpha$$
$$A = 8sen3(30°) + 2sen9(30°)$$
$$A = 8sen90° + 2sen270°$$
$$A = 8(1) + 2(-1) = 6$$
$$B = 12cos12\alpha - 2sec6\alpha$$
$$B = 12cos12(30°) - 2sec6(30°)$$
$$B = 12cos360° - 2sec180°$$
$$B = 12(1) - 2(-1) = 14$$

Piden: $\boldsymbol{2p = 2A + 2B}$

$\Rightarrow 2p = 2(6) + 2(14)$

$\boldsymbol{\therefore\ 2p = 40m}$