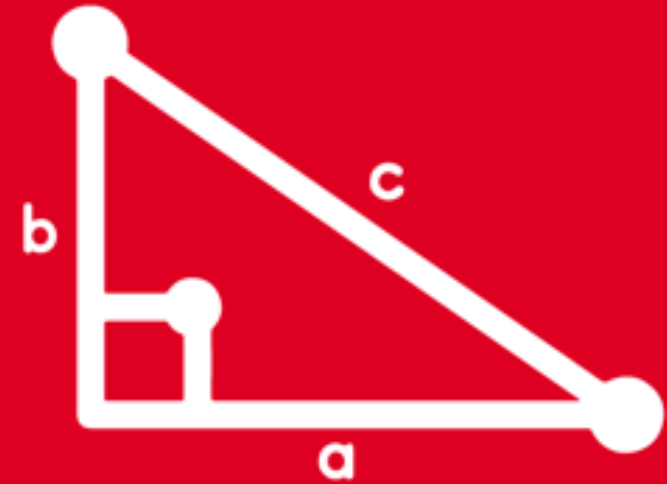




TRIGONOMETRY

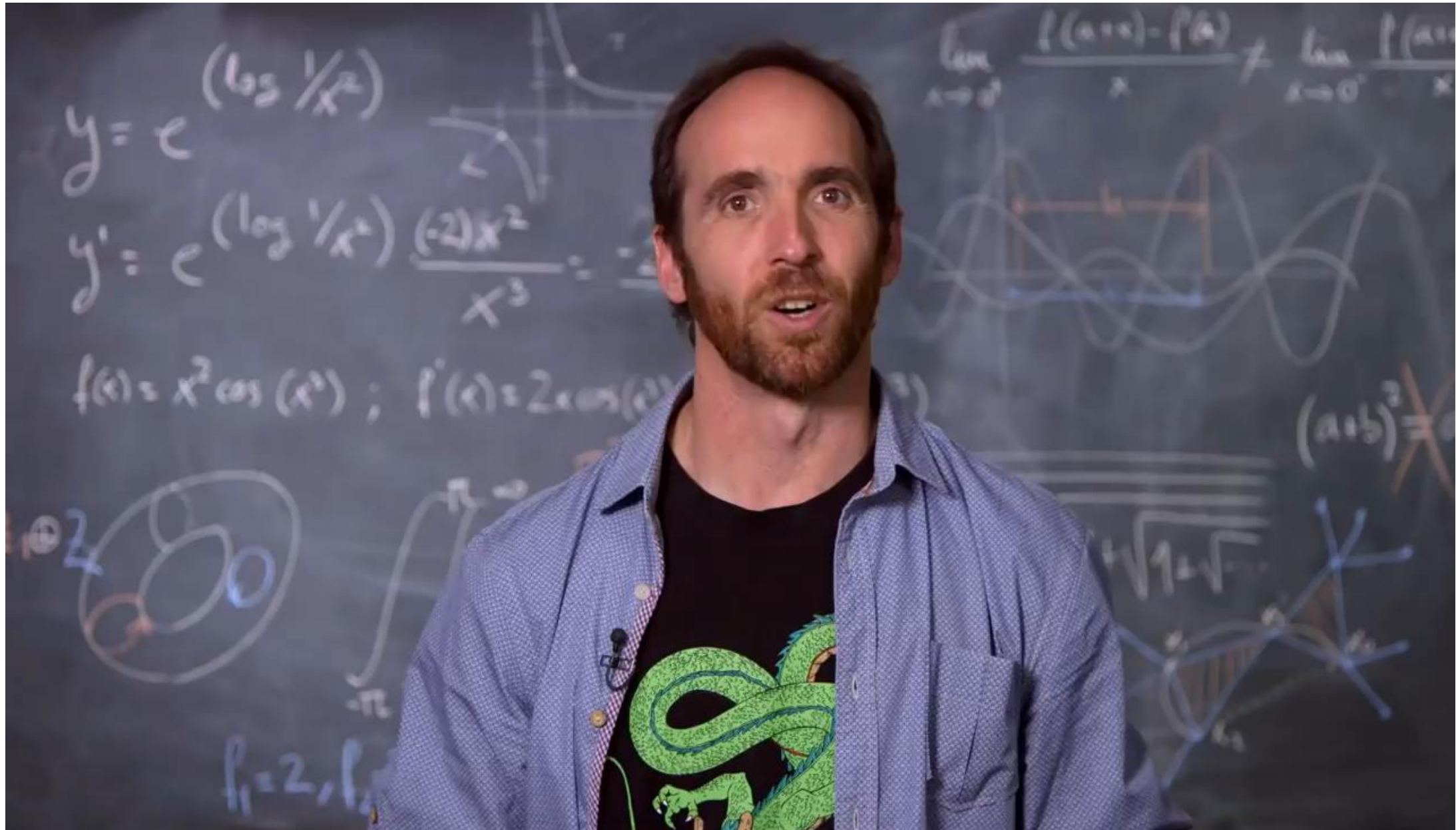
Chapter 17

5th
SECONDARY



IDENTIDADES TRIGONOMÉTRICAS
DEL ÁNGULO MITAD

 **SACO OLIVEROS**



IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO MITAD

I. IDENTIDADES BÁSICAS

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1-\cos x}{2}}$$

$$\operatorname{cos}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1+\cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1-\cos x}{1+\cos x}}$$

Observación: El signo \pm depende del cuadrante de $\left(\frac{x}{2}\right)$



II. IDENTIDADES AUXILIARES



$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

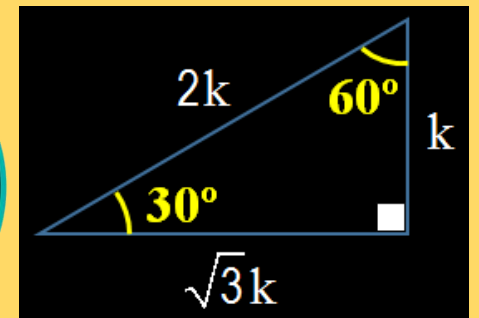
$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

Ejemplo:

$$\tan 15^\circ = \csc 30^\circ - \cot 30^\circ$$

$$\therefore \tan 15^\circ = 2 - \sqrt{3}$$

Recordar:





1.

$$= \sqrt{\frac{+}{-}}$$

RESOLUCIÓN

RECORDAR

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1+\cos x}{2}}$$



$$H = \sqrt{\frac{1+\cos 100^\circ}{2}} - \frac{\cancel{2\sin 40^\circ} \cos 40^\circ}{\cancel{2\cos 40^\circ}}$$

Diagram showing the simplification of the expression. The first term is a square root of $\frac{1+\cos 100^\circ}{2}$, with a purple arrow pointing to $\cos 50^\circ$ above it. The second term is a fraction where the numerator is $\cancel{2\sin 40^\circ} \cos 40^\circ$ and the denominator is $\cancel{2\cos 40^\circ}$. A green arrow points to $\sin 80^\circ$ in the numerator, which is circled in green.

$$\Rightarrow H = \cos 50^\circ - \sin 40^\circ \quad \therefore H = 0$$





2.

RESOLUCIÓN

Recordar:

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$



$$= \sqrt{1 - \sqrt{\frac{1 + \cos 80^\circ}{2}}}$$

$$P = \sqrt{\frac{1 - \sqrt{\frac{1 + \cos 80^\circ}{2}}}{2}}$$

$\cos 40^\circ$

$$\Rightarrow P = \sqrt{\frac{1 - \cos 40^\circ}{2}}$$

$$\therefore P = \operatorname{sen} 20^\circ$$





3.

 θ $\theta =$

$$\left(\frac{\theta}{2} \right)$$

RESOLUCIÓN

Recordar:

$$\text{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$



Dato: $360^\circ < \theta < 450^\circ \rightarrow$ Si $180^\circ < \frac{\theta}{2} < 225^\circ \in \text{IIIC}$

$$\text{sen}\left(\frac{\theta}{2}\right) = \overset{\text{IIIC}}{-} \sqrt{\frac{1 - \cos \theta}{2}} = -\sqrt{\frac{1 - \frac{1}{2}}{2}} = -\sqrt{\frac{\frac{1}{2}}{2}} = -\sqrt{\frac{1}{4}}$$

$$\therefore \text{sen}\left(\frac{\theta}{2}\right) = -\frac{1}{2}$$



4. Del gráfico, calcule $\tan\left(\frac{\theta}{2}\right)$

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

RESOLUCIÓN

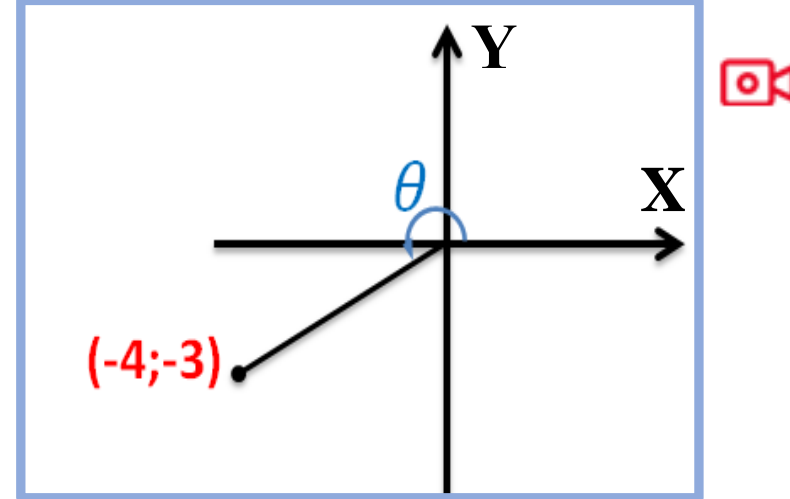
Del gráfico se observa: $180^\circ < \theta < 270^\circ$
 $\in \text{IIC}$

$$\Rightarrow 90^\circ < \frac{\theta}{2} < 135^\circ$$

Además: $x = -4$; $y = -3$

$$r = \sqrt{(-4)^2 + (-3)^2} = 5$$

$$\Rightarrow \cos \theta = \frac{x}{r} = -\frac{4}{5}$$



$$\tan\left(\frac{\theta}{2}\right) = \overset{\text{IIC}}{-} \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = - \sqrt{\frac{1 - \left(-\frac{4}{5}\right)}{1 + \left(-\frac{4}{5}\right)}}$$

$$\tan\left(\frac{\theta}{2}\right) = - \sqrt{\frac{9}{1}} = -\sqrt{9} \quad \therefore \tan\left(\frac{\theta}{2}\right) = -3$$



5.

$$\theta = \theta$$

$$\theta$$

$$\circ$$

$$\left(\frac{\theta}{2} \right)$$

Recordar:

$$\text{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}}$$

Dato: $270^\circ < \theta < 360^\circ$

$$\rightarrow 135^\circ < \frac{\theta}{2} < 180^\circ \quad \text{IIC}$$

RESOLUCIÓN

$$2\text{sen}2\theta = 3\text{sen}\theta$$

$$2(2\cancel{\text{sen}\theta}\cos\theta) = 3\cancel{\text{sen}\theta}$$

$$4\cos\theta = 3 \rightarrow \cos\theta = \frac{3}{4}$$

$$\text{IIC} \rightarrow \text{sen}\left(\frac{\theta}{2}\right) = + \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\text{sen}\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \frac{3}{4}}{2}}$$

$$\text{sen}\left(\frac{\theta}{2}\right) = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

$$\therefore 2\text{sen}\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}$$



6.

$$= \left(\frac{\pi}{-} \right) - \left(\frac{\pi}{-} \right)$$

Recordar:

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$



RESOLUCIÓN

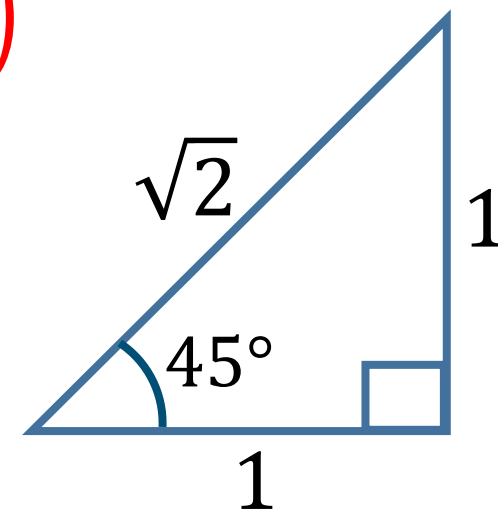
$$E = \cot\left(\frac{\pi}{8}\right) - \sec\left(\frac{\pi}{4}\right)$$

$$E = \csc\left(\frac{\pi}{4}\right) + \cot\left(\frac{\pi}{4}\right) - \sec\left(\frac{\pi}{4}\right)$$

$$E = \cancel{\sqrt{2}} + 1 - \cancel{\sqrt{2}}$$

$$\therefore E = 1$$

$$\frac{\pi}{4} = 45^\circ$$





7. Reduzca: $E = \frac{\tan\left(\frac{x}{2}\right) + \cot x}{\cot x - \cot\left(\frac{x}{2}\right)}$

RESOLUCIÓN

$$E = \frac{\tan\left(\frac{x}{2}\right) + \cot x}{\cot x - \cot\left(\frac{x}{2}\right)}$$

$$E = \frac{\cancel{cscx} - \cancel{cotx} + \cancel{cotx}}{\cancel{cotx} - (\cancel{cscx} + \cancel{cotx})}$$

$$E = \frac{\cancel{cscx}}{-\cancel{cscx}}$$

∴

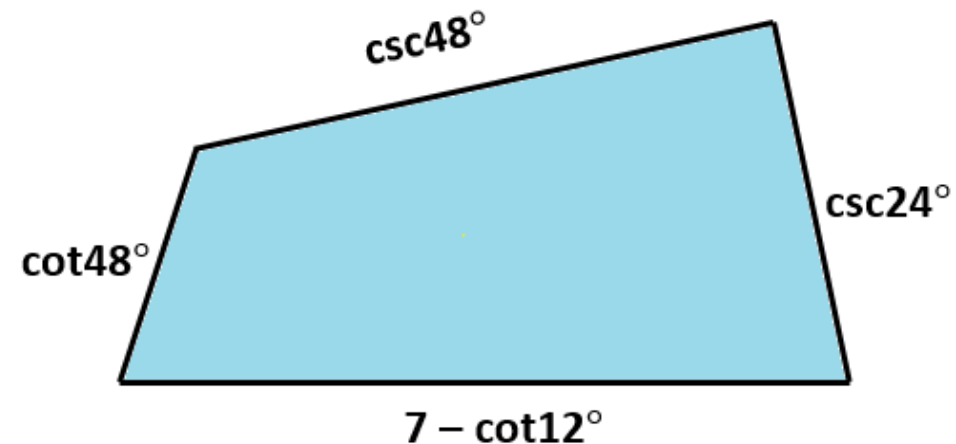
$$E = -1$$

Recordar:

$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

8. El contorno de la mesa en la sala de espera de una clínica dental tiene las siguientes dimensiones. (en metros)
¿Cuál es el perímetro de dicho contorno?



Recordar:

$$\csc x + \cot x = \cot\left(\frac{x}{2}\right)$$



RESOLUCIÓN

$$(2p) \text{ (quadrilateral)} = \underbrace{\cot 48^\circ + \csc 48^\circ}_{\text{using identity}} + \csc 24^\circ + 7 - \cot 12^\circ$$

$$(2p) \text{ (quadrilateral)} = \underbrace{\cot 24^\circ + \csc 24^\circ}_{\text{using identity}} + 7 - \cot 12^\circ$$

$$(2p) \text{ (quadrilateral)} = \cancel{\cot 12^\circ} + 7 - \cancel{\cot 12^\circ}$$

$$\therefore (2p) \text{ (quadrilateral)} = 7m$$