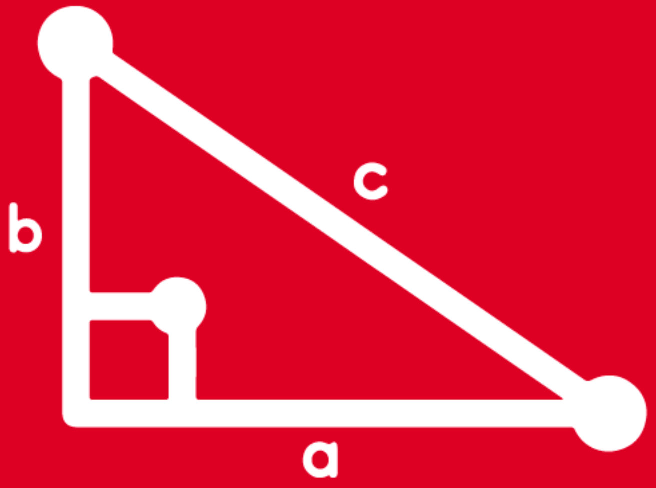
TRIGONOMETRY

Chapter 19
Session 2



4th
SECONDARY

Identidades trigonométricas del ángulo doble

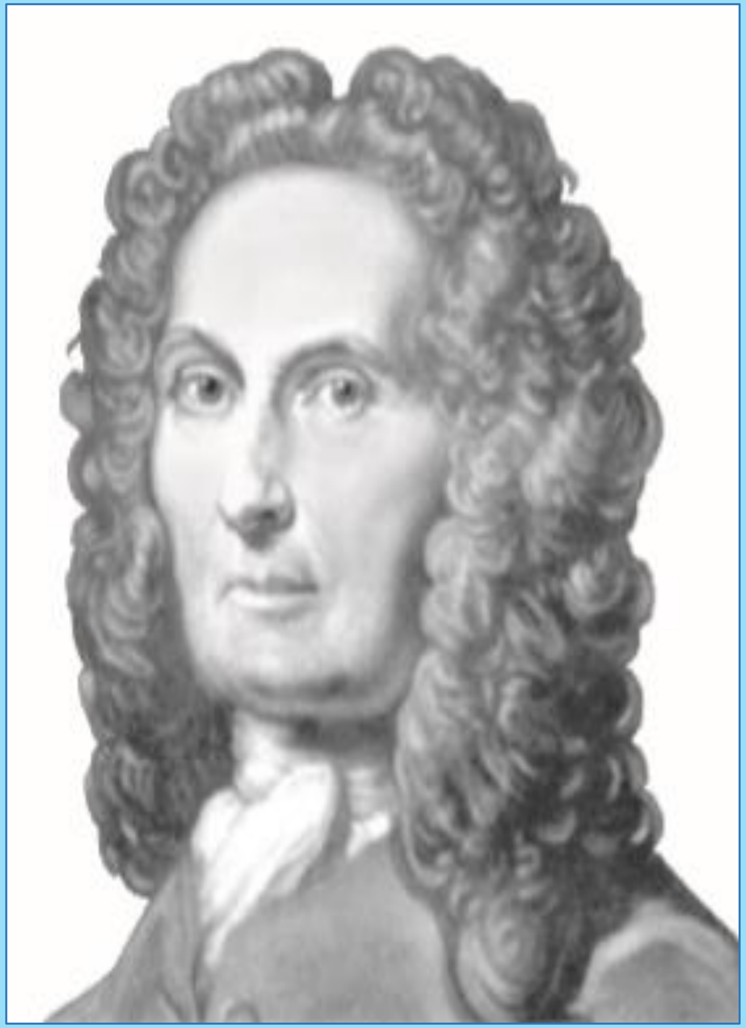
SACO OLIVEROS

ABRAHAM DE MOIVRE

De **Moivre Matemático británico** ,es recordado por la fórmula que ya usó en **1707**.

$$e^{inx} = (cosx + isenx)^n$$

En **1754**, fue elegido miembro de la Academia de Ciencias de Paris. A pesar de su indiscutible categoría científica y su amistad con Newton y Leibniz, de Moivre nunca consiguió una plaza en ninguna universidad. Nunca se casó, era un ferviente cristiano. Fue siempre instructor privado de matemáticas y murió en la pobreza.



IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO DOBLE

$$\text{sen}2\alpha = 2\text{sen}\alpha.\cos\alpha$$

$$\cos 2\alpha = \cos\alpha^2 - \text{sen}^2\alpha$$

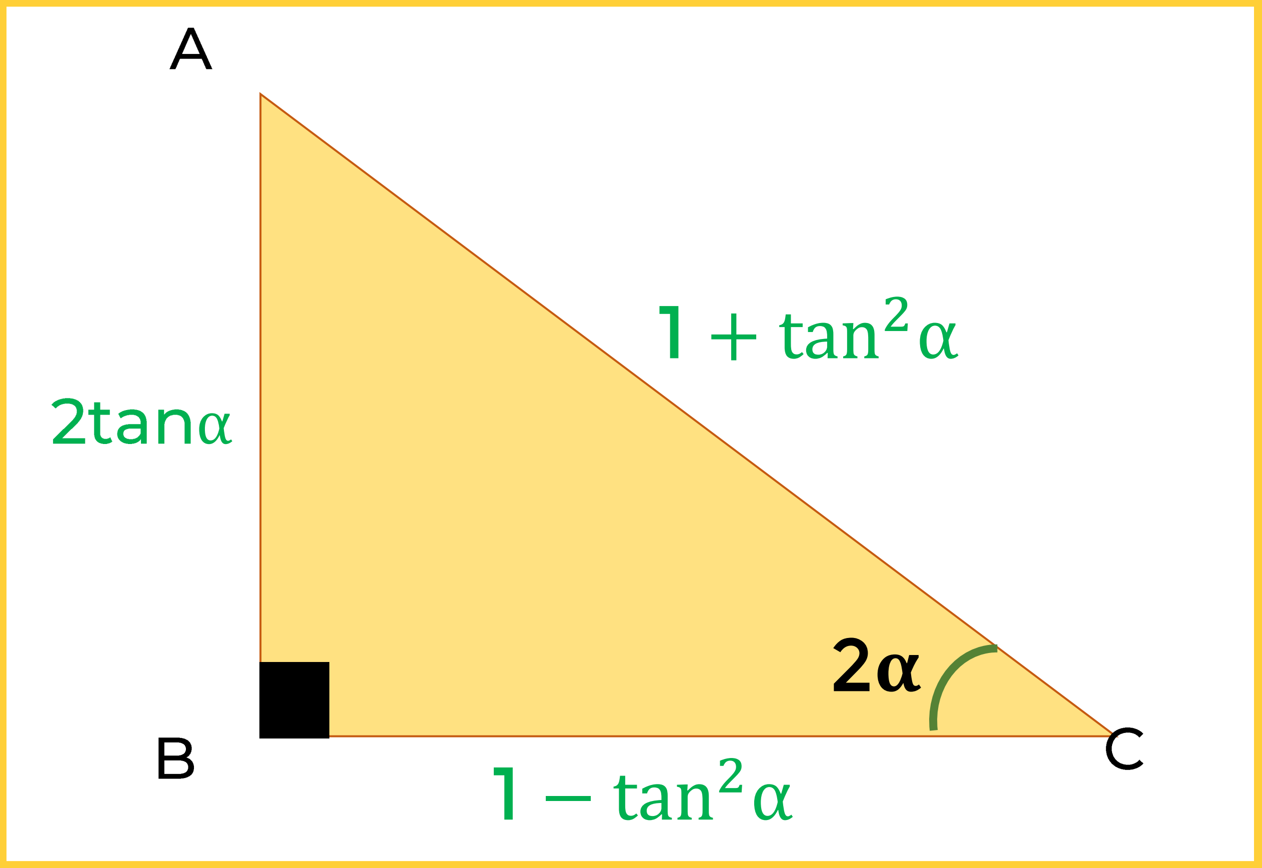
$$\cos 2\alpha = 2cos^2\alpha \ - 1$$

$$\cos 2\alpha = 1 - 2sen^2\alpha$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

HELICO THEORY

TRIÁNGULO DEL ÁNGULO DOBLE



$$\operatorname{sen}2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\cos2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

Además:

$$\sec2\alpha - 1 = \tan2\alpha.\tan\alpha$$
$$\sec2\alpha + 1 = \tan2\alpha.\cot\alpha$$

HELICO THEORY

IDENTIDADES DE DEGRADACION

$$2cos^2\alpha = 1 + \cos 2\alpha$$

$$2sen^2\alpha = 1 - \cos 2\alpha$$

IDENTIDADES DE AUXILIARES

$$\cot\alpha - \tan\alpha = 2cot2\alpha$$

$$\cot\alpha + \tan\alpha = 2\csc 2\alpha$$

$$(sen\alpha + cos\alpha)^2 = 1 + 2\mathrm{sen}\alpha.\cos\alpha$$

$$(sen\alpha - cos\alpha)^2 = 1 - 2\mathrm{sen}\alpha.\cos\alpha$$

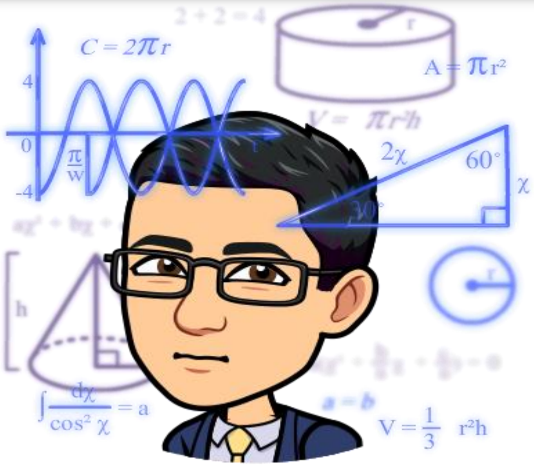
HELICO-PRACTICE 1

Reduzca $M = (\cot x - \tan x)\tan 2x$

Resolución:

Recordar:

$$\cot\alpha - \tan\alpha = 2\cot 2\alpha$$



$$M = \underbrace{(\cot x - \tan x)}_{2\cot 2x}\tan 2x$$

$$M = (2\underbrace{\cot 2x)\tan 2x}_{1}$$

$$\boxed{\therefore M = 2}$$

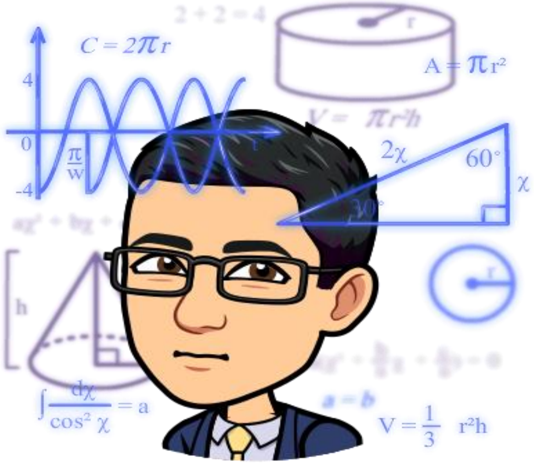
HELICO-PRACTICE 2

Si $\cot\theta + \tan\theta = 3$ **, calcule** $K = \text{sen}2\theta.$

Resolución:

Recordar:

$$\boxed{\cot\alpha + \tan\alpha = 2\csc2\alpha}$$



$$\underbrace{\cot\theta + \tan\theta}_{} = 3$$

$$2\csc2\theta = 3$$

$$\csc2\theta = \frac{3}{2}$$

$$\csc2\theta = \frac{1}{\text{sen}2\theta}$$

$$\frac{3}{2} = \frac{1}{sen2\theta}$$

Piden $K = \text{sen}2\theta$

$$\boxed{\therefore\ \text{sen}2\theta = \frac{2}{3}}$$

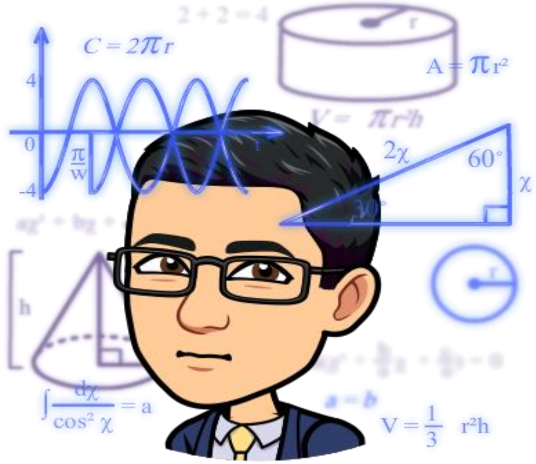
HELICO-PRACTICE 3

Determine el valor de: $E = (\cot 42° + \tan 42°)\cos 6°$

Resolución:

Recordar:

$$\boxed{\cot\alpha + \tan\alpha = 2\csc 2\alpha}$$



$$E = \underbrace{(\cot 42° + \tan 42°)}\cos 6°$$

$$E = [2\csc(2.42°)]\underbrace{\cos 6°}$$

$$E = 2\underbrace{\csc 84°.\,\mathrm{sen}\,84°}_{1}$$

$$\boxed{\therefore E = 2}$$

HELICO-PRACTICE 4

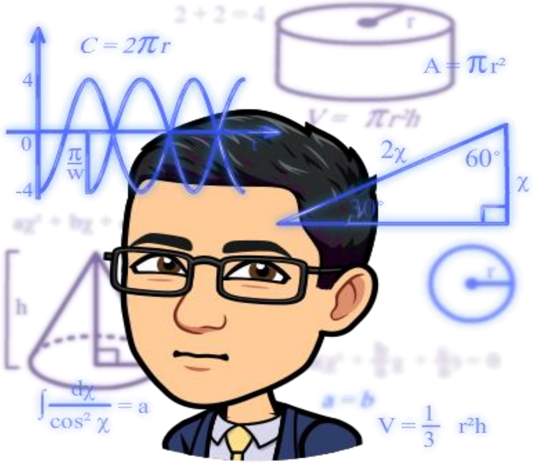
Reducir: $$G = \frac{\text{sen}2\alpha + \text{sen}\alpha}{1 + \cos 2\alpha + \cos\alpha}$$

Resolución:

Recordar:

$$2\cos^2\alpha = 1 + \cos 2\alpha$$

$$\text{sen}2\alpha = 2\text{sen}\alpha.\cos\alpha$$



$$G = \frac{2\text{sen}\alpha.\cos\alpha + \text{sen}\alpha}{2\cos^2\alpha + \cos\alpha}$$

$$G = \frac{\text{sen}\alpha\cancel{(2\cos\alpha + 1)}}{\cos\alpha\cancel{(2\cos\alpha + 1)}}$$

$$G = \frac{\text{sen}\alpha}{\cos\alpha}$$

$$\therefore G = \tan\alpha$$

HELICO-PRACTICE 5

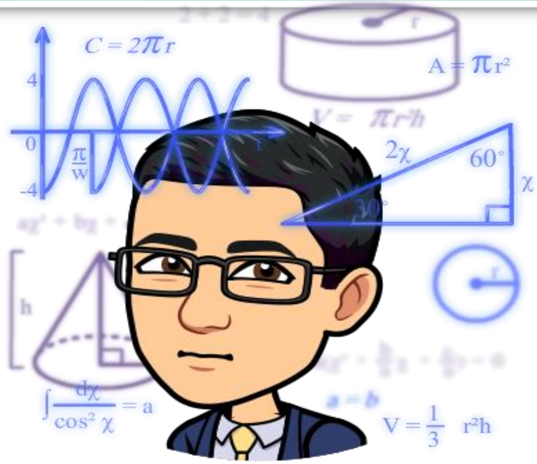
Reduzca: $G = \sqrt{\dfrac{1+\cos 2\theta}{1-\cos 2\theta}}$, $\theta \in IC$

Resolución:

Recordar:

$$2cos^2\alpha = 1 + \cos 2\alpha$$

$$2sen^2\alpha = 1 - \cos 2\alpha$$



$$G = \sqrt{\frac{\mathbf{1+\cos 2\theta}}{\mathbf{1-\cos 2\theta}}}$$

$$G = \sqrt{\frac{\cancel{2}\mathbf{\cos^2\theta}}{\cancel{2}\mathbf{sen^2\theta}}}$$

$$G = \cancel{\sqrt{}}\,\text{cot}^{\cancel{2}}\theta$$

G = $|\cot\theta|$

como $\theta \in IC$

$\Rightarrow$ $|\cot\theta| = \cot\theta$

$$\therefore\ G = \cot\theta$$

HELICO-PRACTICE 6

Reduzca: $$M = \frac{1 + \text{sen}40° - \cos 40°}{1 + \text{sen}40° + \cos 40°}$$

Resolución:

$$M = \frac{1 - \cos 40° + \text{sen}40°}{1 + \cos 40° + \text{sen}40°}$$

$$M = \frac{2\text{sen}^2 20° + 2\text{sen}20° \cos 20°}{2\cos^2 20° + 2\text{sen}20° \cos 20°}$$

$$M = \frac{\cancel{2}\text{sen}20°(\cancel{\text{sen}20° + \cos 20°})}{\cancel{2}\cos 20°(\cancel{\cos 20° + \text{sen}20°})}$$

Recordar:

$$\boxed{2cos^2\alpha = 1 + \cos 2\alpha}$$

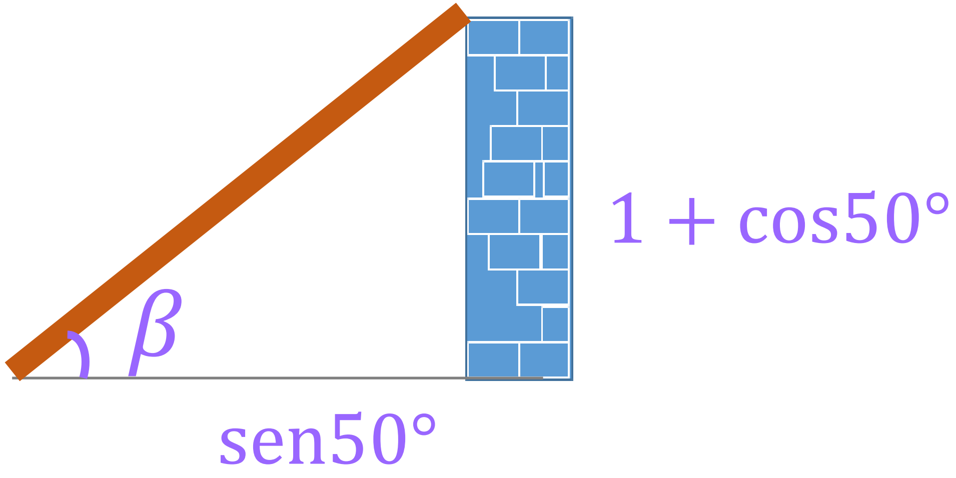
$$\boxed{2sen^2\alpha = 1 - \cos 2\alpha}$$

$$M = \frac{\text{sen}20°}{\cos 20°}$$

$$\boxed{\therefore\ M = \tan 20°}$$

HELICO-PRACTICE 7

Si se tiene una barra metalica que descansa sobre una pared lisa , tal como se muestra en la figura, calcula el valor de β



Resolución:

$$\tan\beta = \frac{1+\cos 50°}{\mathrm{sen}50°}$$

$$\tan\beta = \frac{\cancel{2}\cos^{\cancel{2}}25°}{\cancel{2}\mathrm{sen}25°\cancel{\cos 25°}}$$

$$\tan\beta = \frac{\cos 25°}{\mathrm{sen}25°}$$

$$\tan\beta = \cot 25°$$

Recordar:

$$2\cos^2\alpha = 1+\cos 2\alpha$$

$$2\mathrm{sen}^2\alpha = 1-\cos 2\alpha$$

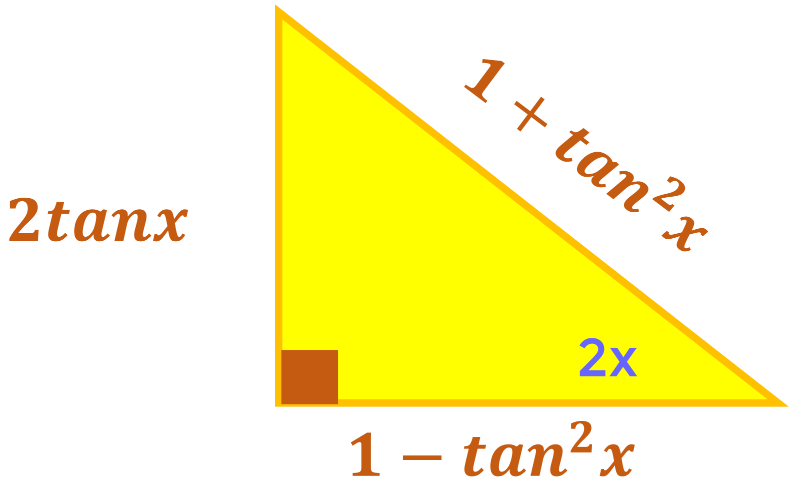
$$\therefore \beta = 65°$$

HELICO-PRACTICE 8

Calcule el valor de: M + N.

$$M = \frac{1 - \tan^2 22°30'}{2\tan 22°30'} \qquad N = \frac{1 + \tan^2 26°30'}{1 - \tan^2 26°30'}$$

Resolución:



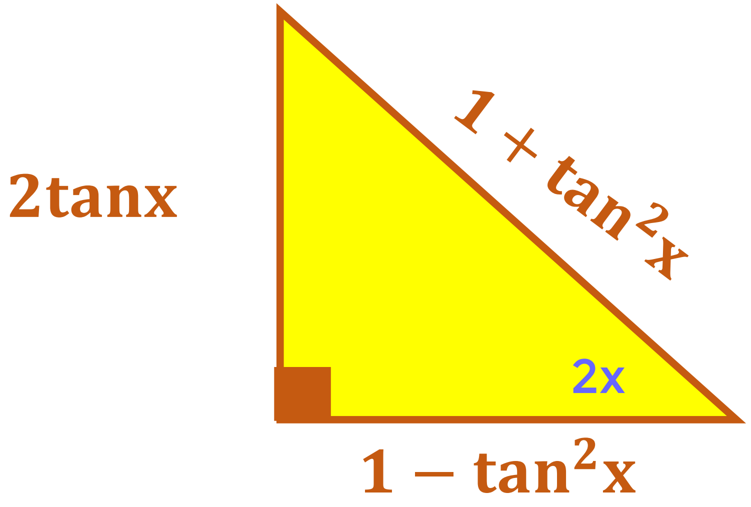
$$\frac{1 - \tan^2 x}{2\tan x} = \cot 2x$$

$$M = \frac{1 - \tan^2 22°30'}{2\tan 22°30'}$$

$$M = \cot 2(22°30')$$

$$M = \cot 45°$$

$$\boxed{M = 1}$$



$$\frac{1+tan^2x}{1-tan^2x} = \sec 2x$$

$$N = \frac{1+\tan^2 26°30'}{1-\tan^2 26°30'}$$

$$N = \sec 2(26°30')$$

$$N = \sec 53°$$

$$\boxed{N = \frac{5}{3}}$$

$$M + N = 1 + \frac{5}{3}$$

$$\boxed{\therefore \mathbf{M + N = \frac{8}{3}}}$$