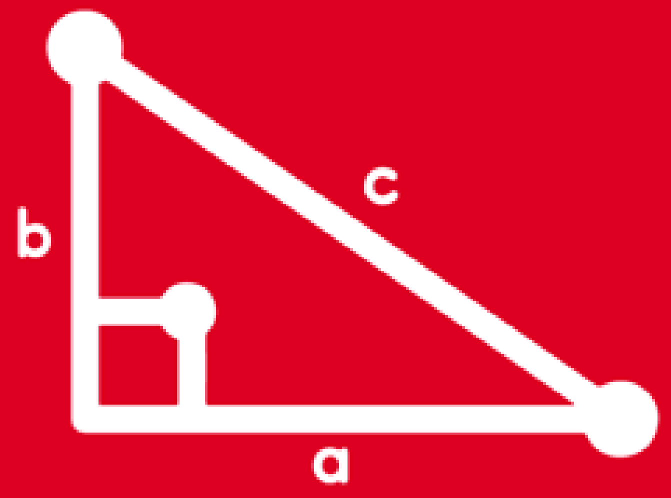
TRIGONOMETRY

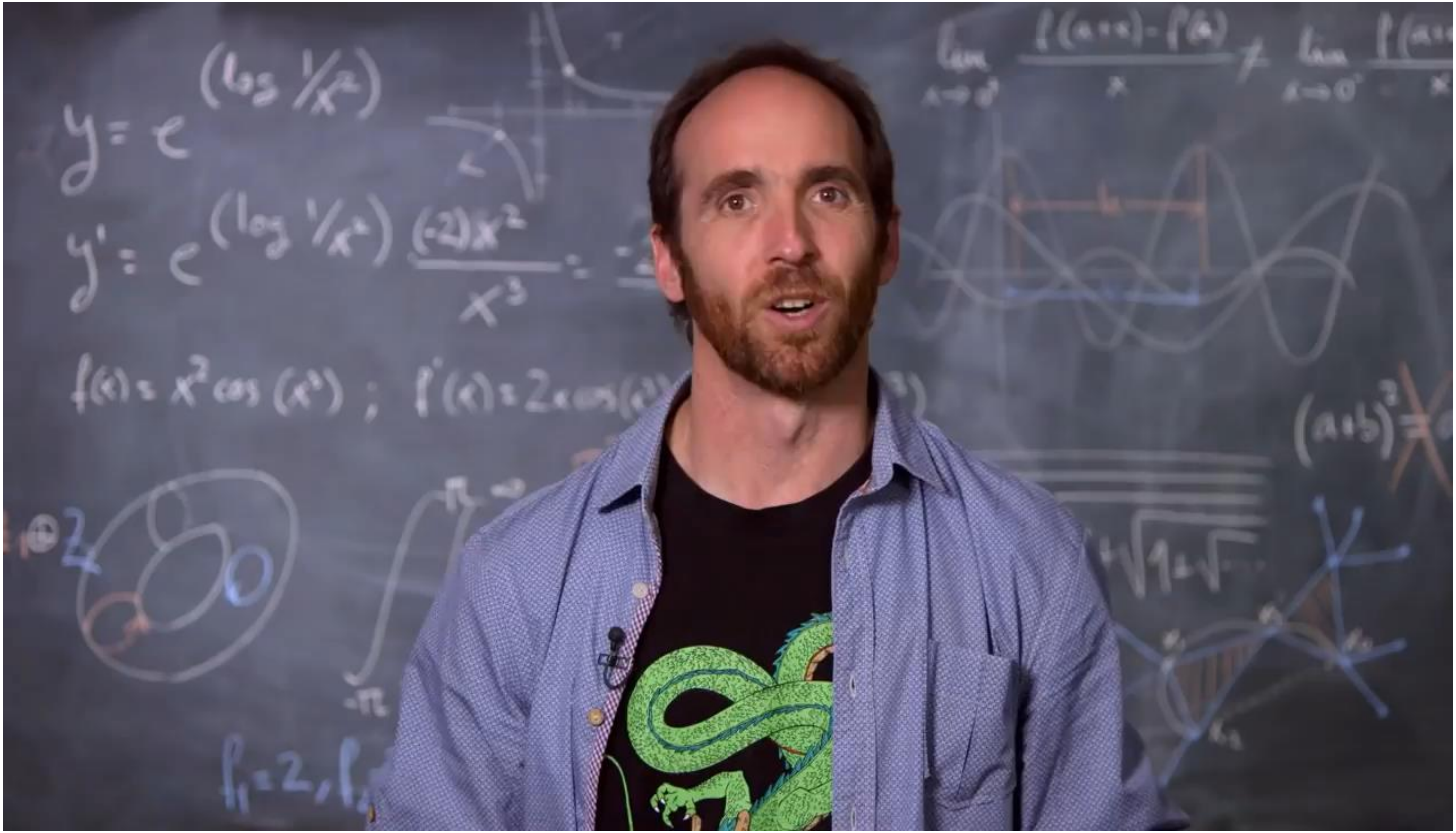
Chapter 17



5th
SECONDARY

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO MITAD

SACO OLIVEROS



IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO MITAD

I. IDENTIDADES BÁSICAS

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

Observación: El signo $\pm$ depende del cuadrante de $\left(\frac{x}{2}\right)$

II. IDENTIDADES AUXILIARES

$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$$

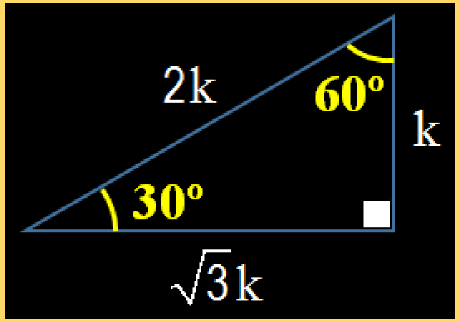
$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

Ejemplo:

$$tan15° = csc30° - cot30°$$

$$\therefore \ tan15° = 2 - \sqrt{3}$$

Recordar:



1.

$$= \sqrt{\frac{\quad + \quad ^\circ}{\quad}} - \frac{\quad ^\circ}{\quad ^\circ}$$

RESOLUCIÓN

RECORDAR

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+cosx}{2}}$$

$$H = \overbrace{\sqrt{\frac{1+\cos 100^\circ}{2}}}^{\cos 50^\circ} - \frac{\overbrace{\text{sen}80^\circ}^{\cancel{2}\text{sen}40^\circ\cancel{\cos 40^\circ}}}{\cancel{2}\cancel{\cos 40^\circ}}$$

$$H = \cos 50^\circ - \overbrace{\text{sen}40^\circ}^{\cos 50^\circ}$$

$$\therefore \boxed{H = 0}$$

2.

RESOLUCIÓN

Recordar:

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$P=\sqrt{\frac{1-\overbrace{\sqrt{\frac{1+\cos 80^\circ}{2}}}^{\cos 40^\circ}}{2}} \Rightarrow P=\sqrt{\frac{1-\cos 40^\circ}{2}}$$

$$\therefore\ \boxed{P = \operatorname{sen}20^\circ}$$

3.

θ

$\theta =$

$\left(\frac{\theta}{}\right)$

RESOLUCIÓN

Recordar:

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

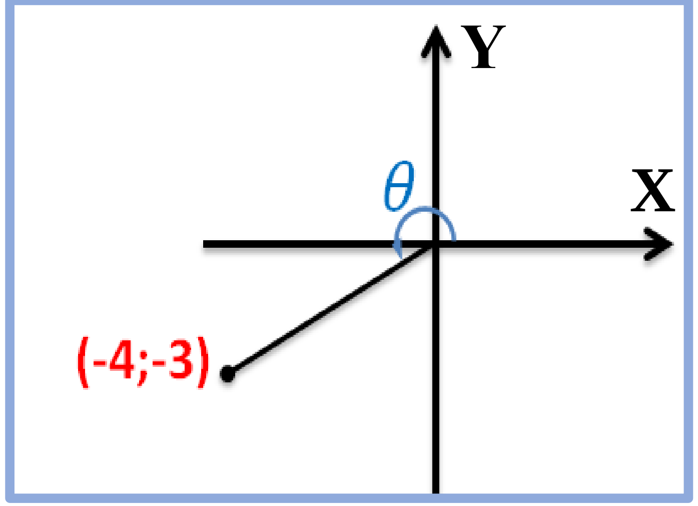
Dato: $360° < \theta < 450° \Rightarrow$ Si $180° < \frac{\theta}{2} < 225°$ $\in$ IIIC

IIIC

$$\operatorname{sen}\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-\cos\theta}{2}} = -\sqrt{\frac{1-\frac{1}{2}}{2}} = -\sqrt{\frac{\frac{1}{2}}{2}} = -\sqrt{\frac{1}{4}}$$

$$\therefore \boxed{\operatorname{sen}\left(\frac{\theta}{2}\right) = -\frac{1}{2}}$$

4. Del gráfico, calcule $\tan\left(\frac{\theta}{2}\right)$



$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

RESOLUCIÓN

Del gráfico se observa: $180° < \theta < 270°$ $\in$ IIC

$\Rightarrow 90° < \frac{\theta}{2} < 135°$

Además: x = - 4 ; y = -3

$$r = \sqrt{(-4)^2 + (-3)^2} = 5$$

$$\Rightarrow \cos\theta = \frac{x}{r} = -\frac{4}{5}$$

$$\tan\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = -\sqrt{\frac{1-\left(-\frac{4}{5}\right)}{1+\left(-\frac{4}{5}\right)}}$$

IIC

$$\tan\left(\frac{\theta}{2}\right) = -\sqrt{\frac{\frac{9}{\cancel{5}}}{\frac{1}{\cancel{5}}}} = -\sqrt{9}$$

$$\therefore \boxed{\tan\left(\frac{\theta}{2}\right) = -3}$$

5. $\theta =$ θ θ $°$

$\left(\frac{\theta}{\ }\right)$

Recordar:

$$\text{sen}\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

Dato: $270° < \theta < 360°$

$\Rightarrow 135° < \frac{\theta}{2} < 180°$ $\in$ IIC

RESOLUCIÓN

$$2\text{sen}2\theta = 3\text{sen}\theta$$

$$2(2\cancel{\text{sen}}\theta\cos\theta) = 3\cancel{\text{sen}}\theta$$

$$4\cos\theta = 3 \Rightarrow \cos\theta = \frac{3}{4}$$

IIC

$$\Rightarrow \text{sen}\left(\frac{\theta}{2}\right) = +\sqrt{\frac{1-\cos\theta}{2}}$$

$$\text{sen}\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\frac{3}{4}}{2}}$$

$$\text{sen}\left(\frac{\theta}{2}\right) = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

$$\therefore \boxed{2\text{sen}\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}}$$

6.

$= \left(\frac{\pi}{\ }\right) - \left(\frac{\pi}{\ }\right)$

Recordar:

$$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

RESOLUCIÓN

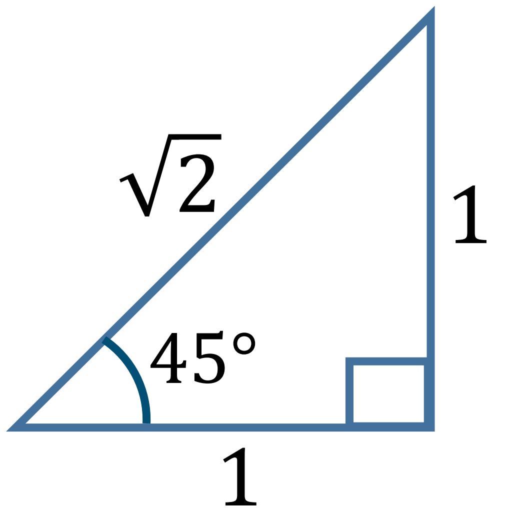
$$\frac{\pi}{4} = 45°$$

$$E = \cot\left(\frac{\pi}{8}\right) - \sec\left(\frac{\pi}{4}\right)$$

$$E = \csc\left(\frac{\pi}{4}\right) + \cot\left(\frac{\pi}{4}\right) - \sec\left(\frac{\pi}{4}\right)$$

$$E = \cancel{\sqrt{2}} + 1 - \cancel{\sqrt{2}}$$

$$\therefore \boxed{E = 1}$$



7. Reduzca: $E = \dfrac{\tan\left(\frac{x}{2}\right) + \cot x}{\cot x - \cot\left(\frac{x}{2}\right)}$

RESOLUCIÓN

$$E = \frac{\tan\left(\frac{x}{2}\right) + \cot x}{\cot x - \cot\left(\frac{x}{2}\right)}$$

$$E = \frac{\csc x - \cancel{\cot x} + \cancel{\cot x}}{\cancel{\cot x} - (\csc x + \cancel{\cot x})}$$

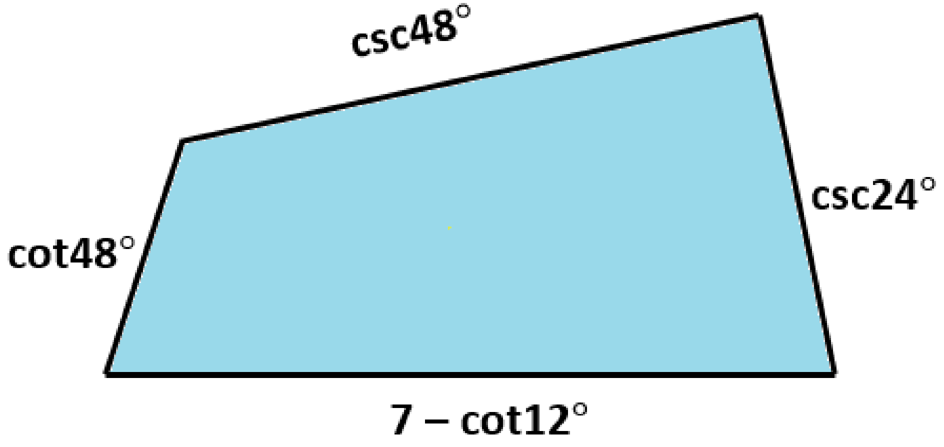
$$E = \frac{\cancel{\csc x}}{-\cancel{\csc x}}$$

$$\therefore \boxed{E = -1}$$

Recordar:

$\tan\left(\frac{x}{2}\right) = \csc x - \cot x$
$\cot\left(\frac{x}{2}\right) = \csc x + \cot x$

8. El contorno de la mesa en la sala de espera de una clínica dental tiene las siguientes dimensiones. (en metros) ¿Cuál es el perímetro de dicho contorno?



Recordar:

$$\csc x + \cot x = \cot\left(\frac{x}{2}\right)$$

RESOLUCIÓN

$$(2p) = \underbrace{cot48° + csc48°} + csc24° + 7 - cot12°$$

$$(2p) = \underbrace{cot24° + csc24°} + 7 - cot12°$$

$$(2p) = \cancel{cot12°} + 7 - \cancel{cot12°}$$

$$\therefore \boxed{(2p) = 7m}$$