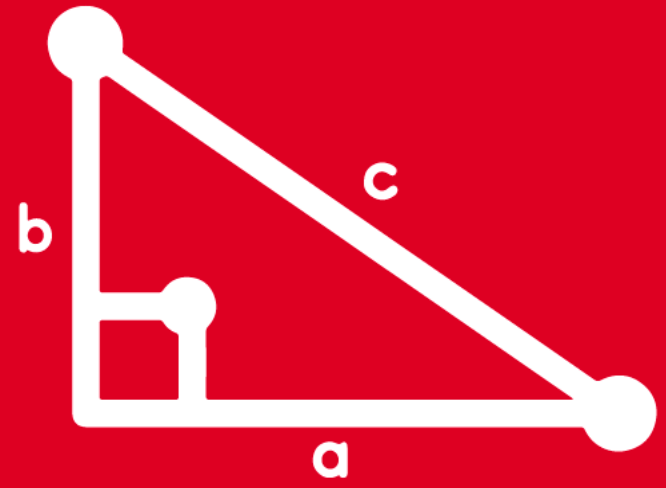
TRIGONOMETRY

Chapter 2

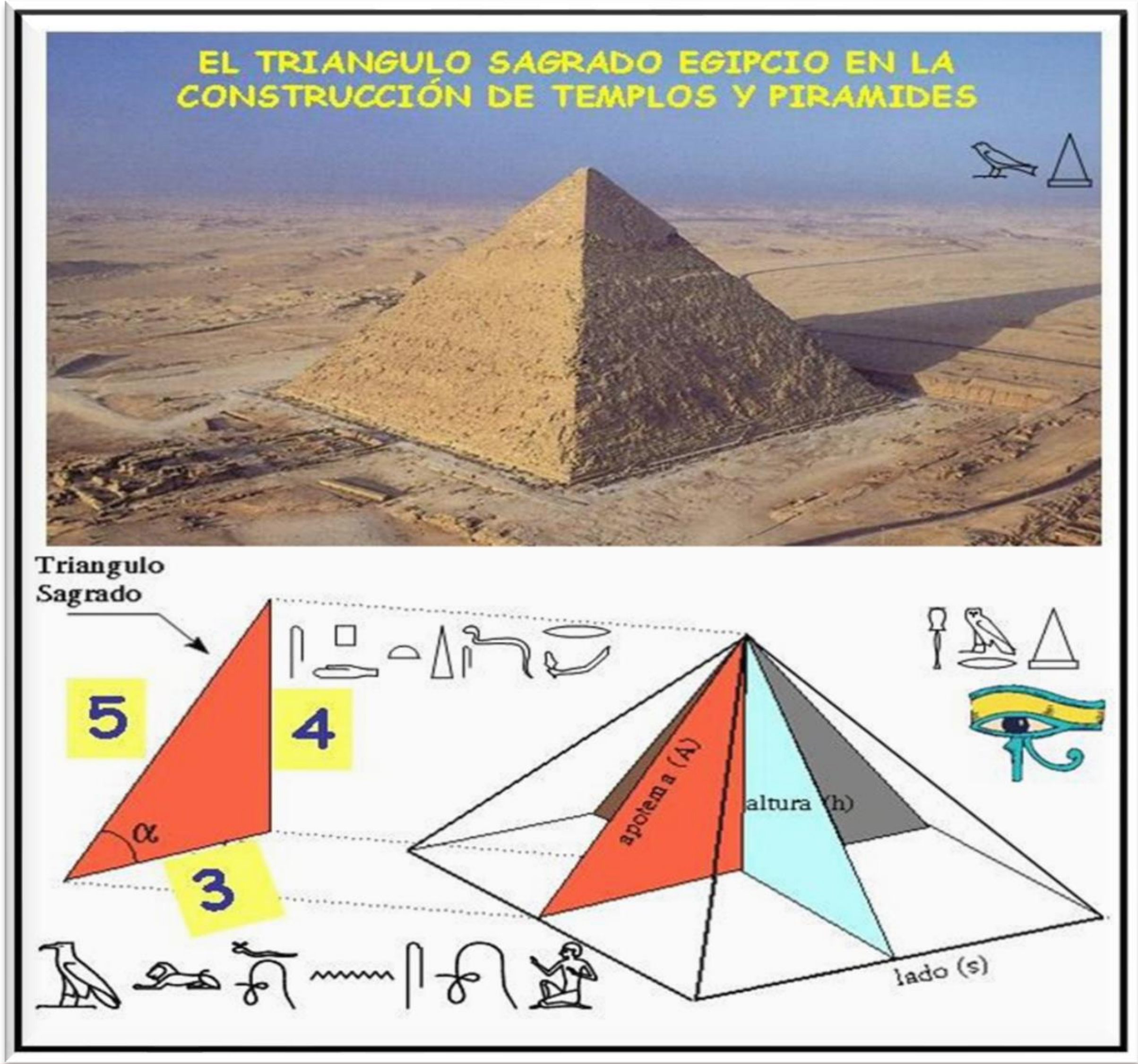
5th
SECONDARY



PROPIEDADES DE LAS RAZONES TRIGONOMÉTRICAS DE ÁNGULOS AGUDOS

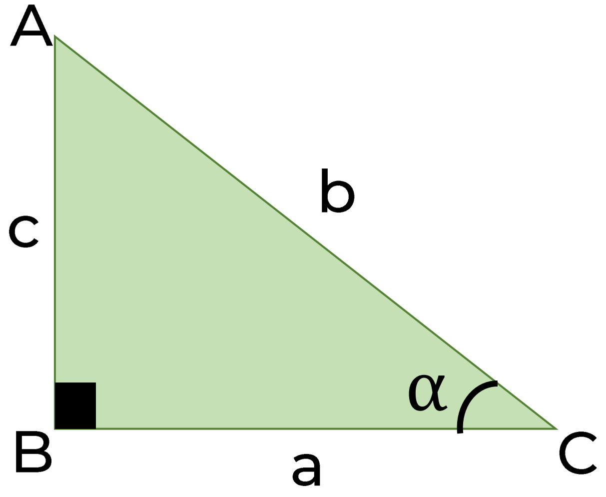
SACO OLIVEROS

MOTIVATING STRATEGY



HELICO THEORY

I) RAZONES TRIGONOMÉTRICAS RECÍPROCAS



De la figura se tiene:

$$\operatorname{sen}\alpha = \frac{c}{b} \quad \wedge \quad \csc\alpha = \frac{b}{c}$$

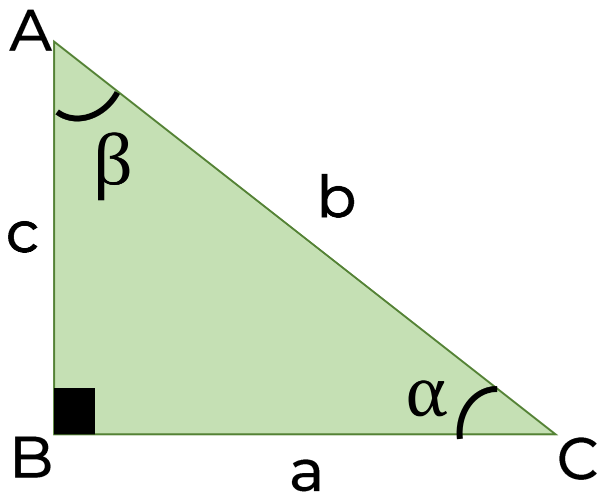
$$\operatorname{sen}\alpha . \csc\alpha = \frac{\cancel{c}}{\cancel{b}} \times \frac{\cancel{b}}{\cancel{c}} = 1$$

$$\boxed{\operatorname{sen}\alpha . \csc\alpha = 1}$$

$$\boxed{\cos\alpha . \sec\alpha = 1}$$

$$\boxed{\tan\alpha . \cot\alpha = 1}$$

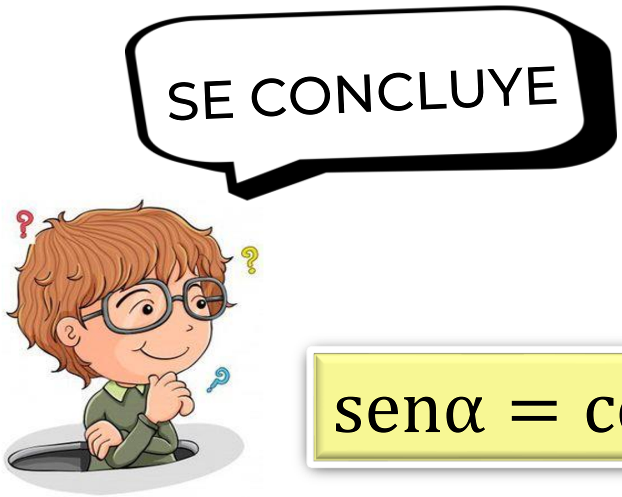
II) RAZONES TRIGONOMÉTRICAS DE ÁNGULOS COMPLEMENTARIOS



De la figura se tiene:

$$\operatorname{sen}\alpha = \frac{c}{b} \quad \wedge \quad \cos\beta = \frac{c}{b}$$

$$\operatorname{sen}\alpha = \cos\beta$$



$$\operatorname{sen}\alpha = \cos\beta$$

$$\tan\alpha = \cot\beta$$

$$\sec\alpha = \csc\beta$$

HELICO-PRACTICE 1

Las edades de Juan e Iván son m y n años respectivamente, si dichos valores se pueden calcular al resolver las siguientes expresiones:

$$\cos(2m+30)°.\sec70° = 1 \;\wedge\; \tan(3n)° = \cot54°$$

a) ¿Cuál es la edad de Juan e Iván?
b) ¿Cuál es la suma de ambas edades?

Resolución:

Usando las RT recíprocas:

$$\underbrace{\cos(2m+30)°.\sec70°}\; = 1 \;......(\mathrm{I})$$

$$(2m+30)\cancel{°} = 70\cancel{°}$$

$$2m+30 = 70$$

$$2m = 40 \Rightarrow \boxed{m = 20}$$

Usando las RT de ángulos complementarios:

$$\underbrace{\tan(3n)° = \cot54°}\;......(\mathrm{II})$$

$$(3n)° + 54° = 90°$$

$$(3n)\cancel{°} = 36\cancel{°} \Rightarrow \boxed{n = 12}$$

Piden:

a) Juan= 20 años

Iván= 12 años

b) 20+12= 32

HELICO-PRACTICE 2

Si α es la medida de un ángulo agudo tal que

$$\tan(45° + 2\alpha)\,.\cot(60° - \alpha) = 1$$

Efectúe $M = (\sec 12\alpha + \tan 9\alpha)^2$

Resolución:

Del dato:

RT recíprocas:

$$\tan(\underbrace{45° + 2\alpha)\,.\cot(60°}_{} - \alpha) = 1$$

$$45° + 2\alpha = 60° - \alpha$$

$$3\alpha = 15°$$

$$\boxed{\alpha = 5°} \quad \text{........(I)}$$

Piden :

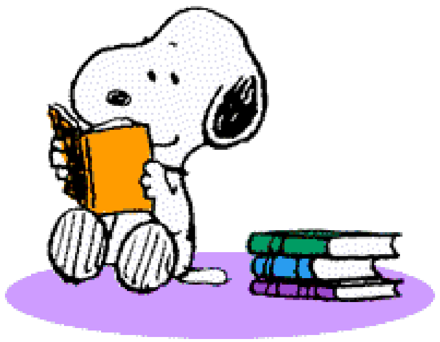
$$M = (\sec 12\alpha + \tan 9\alpha)^2$$

Reemplazando (I) en M

$$M = (\sec 60° + \tan 45°)^2$$

$$M = (2 + 1)^2$$

$$\therefore M = 9$$



HELICO-PRACTICE 3

Siendo α y β la medida de dos ángulos agudos, los cuales cumplen que

$$\text{sen}\alpha - \cos 2\beta = 2\text{sen}30° - 1 \quad \text{........(I)}$$
$$\text{sen}\alpha.\csc 4\beta = \tan 45° \quad \text{..................(II)}$$

Calcule $\tan(\alpha - \beta)$

Resolución:

De (I):

RT de ángulos complementarios:

$$\text{sen}\alpha - \cos 2\beta = \cancel{2}\left(\frac{1}{\cancel{2}}\right) - 1$$

$$\underbrace{\text{sen}\alpha = \cos 2\beta}$$

$$\boxed{\alpha + 2\beta = 90°} \quad \text{.......(*)}$$

De (II):

RT recíprocas:

$$\underbrace{\text{sen}\alpha.\csc 4\beta} = 1$$

$$\boxed{\alpha = 4\beta} \quad \text{...(**)}$$

$$\alpha = 60°$$

Reemplazando(*) en (**)

$$\alpha + 2\beta = 90°$$
$$4\beta + 2\beta = 90°$$
$$6\beta = 90°$$
$$\beta = 15°$$

Piden :

$$\tan(\alpha - \beta)$$
$$\tan(60° - 15°)$$
$$\tan(45°)$$

$$\therefore \tan(\alpha - \beta) = 1$$

HELICO-PRACTICE 4

Determine la medida del ángulo agudo x, que cumple

$$(\tan 10°)^{\operatorname{sen}(20°+x)} = (\cot 80°)^{\cos(x-2°)}$$

Resolución:

$$(\tan 10°)^{\operatorname{sen}(20°+x)} = (\cot 80°)^{\cos(x-2°)}$$

Usando las RT de ángulos complementarios:

$$\cot 80° = \tan 10°$$

$$(\tan 10°)^{\operatorname{sen}(20°+x)} = (\tan 10°)^{\cos(x-2°)}$$

A bases iguales → Exponentes iguales

$$\operatorname{sen}(20° + x) = \cos(x - 2°)$$

$$20° + x + x - 2° = 90°$$

$$2x = 72°$$

$$\therefore x = 36°$$

HELICO-PRACTICE 5

Si θ es la medida de un ángulo agudo y cumple que: $\sec\theta = \dfrac{3\text{sen}70° + \cos20°}{5\text{sen}70° - 2\cos20°}$

Efectúe: $E = \sqrt{7}(\tan\theta + \cot\theta)$

Resolución:

RECORDAR

RT de ángulos complementarios

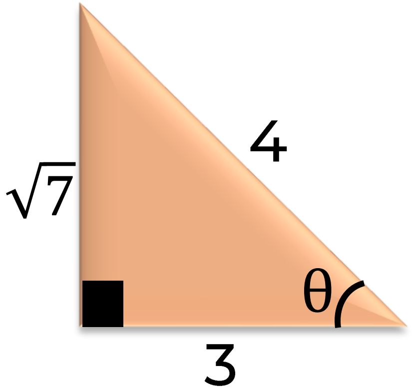
$\text{si } \alpha + \beta = 90°$

$\Rightarrow \text{sen}\alpha = \cos\beta$

Así: $\text{sen}70° = \cos20°$

Vamos a reemplazar:

$$\sec\theta = \frac{3\cos20° + \cos20°}{5\cos20° - 2\cos20°} \Rightarrow \sec\theta = \frac{4\cancel{\cos20°}}{3\cancel{\cos20°}} \Rightarrow \sec\theta = \frac{4}{3}$$



Piden :

$$E = \sqrt{7}(\tan\theta + \cot\theta)$$

$$E = \sqrt{7}\left(\frac{\sqrt{7}}{3} + \frac{3}{\sqrt{7}}\right) = (\sqrt{7})\left(\frac{\sqrt{7}}{3}\right) + (\cancel{\sqrt{7}})\left(\frac{3}{\cancel{\sqrt{7}}}\right)$$

$$E = \frac{7}{3} + 3 \Rightarrow \boxed{\therefore E = \frac{16}{3}}$$

HELICO-PRACTICE 6

Un ángulo agudo cuya medida es θ, cumple que: $(\tan\theta - \cot\theta)^2 = \tan^2\theta + 1$

Reduzca: $F = \dfrac{8\operatorname{sen}\theta + 2}{\tan(\theta + 40°).\cot(2\theta + 10°) + 1}$

Resolución:

Del dato:

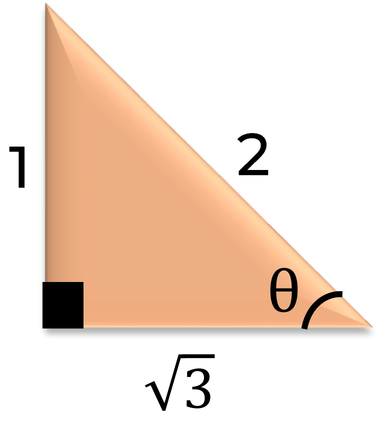
$$\underbrace{(\tan\theta - \cot\theta)^2} = \tan^2\theta + 1$$

$$\cancel{\tan^2\theta} - \underbrace{2\tan\theta.\cot\theta}_{1} + \cot^2\theta = \cancel{\tan^2\theta} + 1$$

$$-2 + \cot^2\theta = 1$$

$$\cot^2\theta = 3$$

$\cot\theta = \sqrt{3}$



$\boxed{\theta = 30°}$

Reemplazando

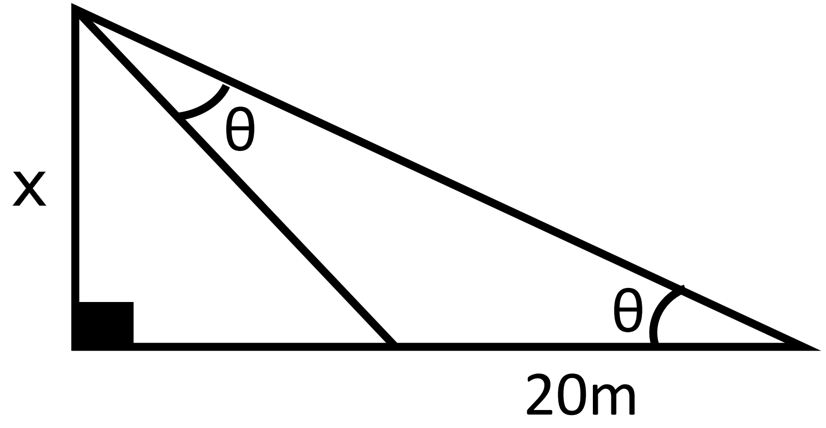
$$F = \frac{8\operatorname{sen}30° + 2}{\underbrace{\tan 70° . \cot 70°}_{1} + 1}$$

$$F = \frac{8\left(\frac{1}{2}\right) + 2}{2}$$

$\therefore F = 3$

HELICO-PRACTICE 7

Halle el valor de x, si en el gráfico se cumple: $\tan(30° - \theta) - \cot(30° + 3\theta) = 0$



Resolución:

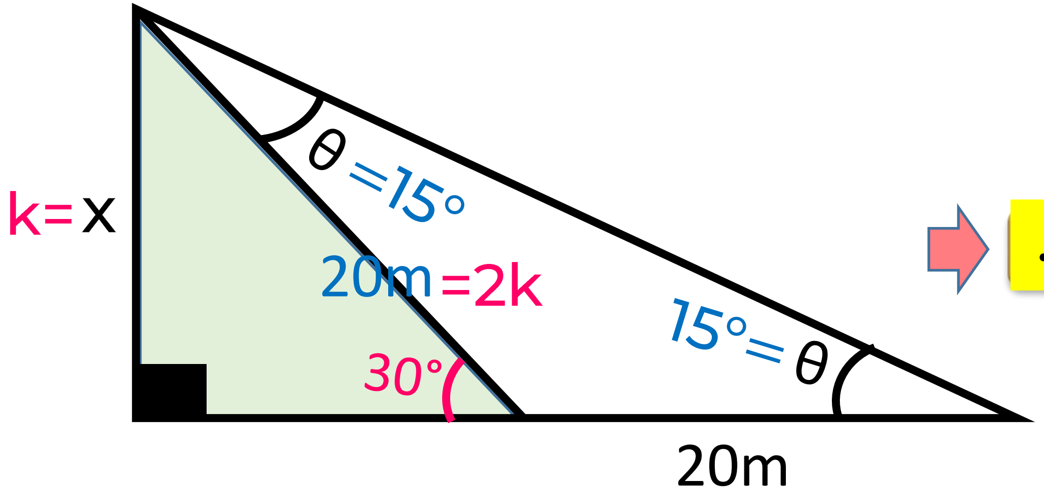
Del dato:

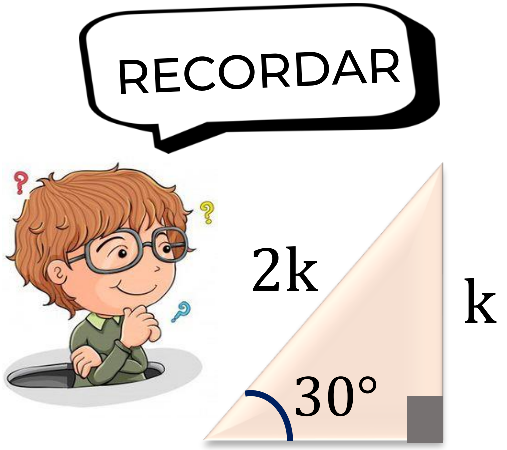
$$\tan(30° - \theta) - \cot(30° + 3\theta) = 0$$

$$\tan(30° - \theta) = \cot(30° + 3\theta)$$

$$(30° - \theta) + (30° + 3\theta) = 90°$$

$$60° + 2\theta = 90° \Rightarrow \boxed{\theta = 15°}$$





$$\therefore x = 10m$$

HELICO-PRACTICE 8

Siendo α y β las medidas de dos ángulos agudos, se cumple que: $\cos\alpha . \csc 2\beta = 1$

calcule $P = \tan^2\left(\dfrac{2\beta+\alpha}{3}\right) + \text{sen}^2\left(\dfrac{2\beta+\alpha}{2}\right)$

Resolución:

Del dato:

$$\cos\alpha . \csc 2\beta = 1$$

Usando las RT de ángulos complementarios

$$\underbrace{\text{sen}(90° - \alpha)}_{(90° - \alpha) = 2\beta} . \csc 2\beta = 1$$

$$(90° - \alpha) = 2\beta$$

$$\boxed{2\beta + \alpha = 90°}$$

Piden :

$$P = \tan^2\left(\frac{2\beta+\alpha}{3}\right) + \text{sen}^2\left(\frac{2\beta+\alpha}{2}\right)$$

$$P = \tan^2\left(\frac{90°}{3}\right) + \text{sen}^2\left(\frac{90°}{2}\right)$$

$$P = \tan^2 30° + \text{sen}^2 45°$$

Reemplazando

$$P = \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$P = \frac{1}{3} + \frac{1}{2}$$

$$\boxed{\therefore P = \frac{5}{6}}$$