



TRIGONOMETRY

Chapter 13, 14 and 15

5th
SECONDARY

REVIEW



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Halle el valor de $\text{sen}16^\circ$ y $\text{cos}67^\circ$.

Resolución:

Como: $16^\circ = 53^\circ - 37^\circ \Rightarrow \text{sen}(16^\circ) = \text{sen}(53^\circ - 37^\circ)$

$$\Rightarrow \text{sen}16^\circ = \underbrace{\text{sen}53^\circ}_{\left(\frac{4}{5}\right)} \underbrace{\text{cos}37^\circ}_{\left(\frac{4}{5}\right)} - \underbrace{\text{cos}53^\circ}_{\left(\frac{3}{5}\right)} \underbrace{\text{sen}37^\circ}_{\left(\frac{3}{5}\right)}$$

$$\therefore \text{sen}16^\circ = \boxed{\frac{7}{25}}$$

Como: $67^\circ = 37^\circ + 30^\circ \Rightarrow \text{cos}(67^\circ) = \text{cos}(37^\circ + 30^\circ)$

$$\Rightarrow \text{cos}67^\circ = \underbrace{\text{cos}37^\circ}_{\left(\frac{4}{5}\right)} \underbrace{\text{cos}30^\circ}_{\left(\frac{\sqrt{3}}{2}\right)} - \underbrace{\text{sen}37^\circ}_{\left(\frac{3}{5}\right)} \underbrace{\text{sen}30^\circ}_{\left(\frac{1}{2}\right)}$$

$$\therefore \text{cos}67^\circ = \boxed{\frac{4\sqrt{3} - 3}{10}}$$

HELICOPRACTICE 2



Si se cumple que $3\text{sen}(x + 45^\circ) = \sqrt{2}$, calcule $\text{sen}x \cos x$

Resolución:

$$\text{sen}(x + y) = \text{sen}x.\text{cos}y + \text{cos}x.\text{sen}y$$

$$3\left[\text{sen}x \underbrace{\text{cos}45^\circ}_{\frac{\sqrt{2}}{2}} + \text{cos}x \underbrace{\text{sen}45^\circ}_{\frac{\sqrt{2}}{2}}\right] = \sqrt{2} \quad \Rightarrow \quad \frac{3}{2} \cancel{\sqrt{2}} [\text{sen}x + \text{cos}x] = \cancel{\sqrt{2}}$$

$$\frac{\sqrt{2}}{2}$$

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$$\left\{ \text{sen}x + \text{cos}x = \frac{2}{3} \right\}^2 \Rightarrow 1 + 2\text{sen}x \text{cos}x = \frac{4}{9} \Rightarrow 2\text{sen}x \text{cos}x = -\frac{5}{9}$$



$$\text{sen}x \text{cos}x = -\frac{5}{18}$$

HELICOPRACTICE 3



Siendo $\alpha + \beta = 60^\circ$, calcule el valor de $M = (\cos \alpha + \cos \beta)^2 + (\operatorname{sen} \alpha - \operatorname{sen} \beta)^2$

Resolución:

$$M = (\cos \alpha + \cos \beta)^2 + (\operatorname{sen} \alpha - \operatorname{sen} \beta)^2$$

$$M = \cos^2 \alpha + 2\cos \alpha \cos \beta + \cos^2 \beta + \operatorname{sen}^2 \alpha - 2\operatorname{sen} \alpha \operatorname{sen} \beta + \operatorname{sen}^2 \beta$$

$$M = \underbrace{\operatorname{sen}^2 \alpha + \cos^2 \alpha}_1 + 2 \underbrace{(\cos \alpha \cos \beta - \operatorname{sen} \alpha \operatorname{sen} \beta)}_{\cos(\alpha + \beta)} + \underbrace{\cos^2 \beta + \operatorname{sen}^2 \beta}_1$$

$$M = 2 + 2\cos 60^\circ$$

$$M = 2 + \cancel{2} \left(\frac{1}{\cancel{2}} \right)$$

\therefore

$$M = 3$$

HELICOPRACTICE 4



Reduzca $T = \cos(x + 30^\circ) \cdot \cos(x - 30^\circ) - \text{sen}^2 x$

Resolución:

$$\cos(x + y) \cdot \cos(x - y) = \cos^2 x - \text{sen}^2 y$$

$$T = \cos(x + 30^\circ) \cdot \cos(x - 30^\circ) + \text{sen}^2 x$$

$$T = \cos^2 x - \text{sen}^2 30^\circ + \text{sen}^2 x$$

$$\Rightarrow T = \underbrace{\text{sen}^2 x + \cos^2 x}_1 - \underbrace{\text{sen}^2 30^\circ}_{\left(\frac{1}{2}\right)^2} \Rightarrow T = 1 - \left(\frac{1}{4}\right)$$

$$\therefore T = \left(\frac{3}{4}\right)$$

HELICOPRACTICE 5



Calcule el máximo valor de

$$E = 13\text{sen}x + \sqrt{2}\text{sen}(45^\circ - x) + 4\cos x$$

Resolución:

$$E = 13\text{sen}x + \sqrt{2} (\text{sen}45^\circ\cos x - \cos45^\circ\text{sen}x) + 4\cos x$$

$$E = 13\text{sen}x + \cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}} \cdot \cos x - \cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}} \cdot \text{sen}x + 4\cos x$$

$$E = 13\text{sen}x + \cos x - \text{sen}x + 4\cos x$$

$$E = 12\text{sen}x + 5\cos x$$

$$\underbrace{-\sqrt{a^2 + b^2}}_{\text{mínimo}} \leq a.\text{sen}x + b.\cos x \leq \underbrace{\sqrt{a^2 + b^2}}_{\text{máximo}}$$

Nos piden: $E_{\text{máx}} = \sqrt{12^2 + 5^2}$



$$E_{\text{máx}} = 13$$

HELICOPRACTICE 6



Reduzca: $M = \frac{2 \tan 50^\circ + \tan 80^\circ}{\cot 40^\circ \cdot \cot 10^\circ}$

Resolución:

$$M = \frac{\tan 50^\circ + \tan 50^\circ + \tan 80^\circ}{\cot 40^\circ \cot 10^\circ}$$

Se observa que:

$$50^\circ + 50^\circ + 80^\circ = 180^\circ$$

Entonces:

$$\tan 50^\circ + \tan 50^\circ + \tan 80^\circ = \tan 50^\circ \tan 50^\circ \tan 80^\circ$$

Si $x + y + z = 180^\circ$, se cumple:
 $\tan x + \tan y + \tan z = \tan x \cdot \tan y \cdot \tan z$

$$M = \frac{\tan 50^\circ \tan 50^\circ \tan 80^\circ}{\cot 40^\circ \cot 10^\circ}$$

$$M = \frac{\cancel{\tan 50^\circ} \tan 50^\circ \cancel{\tan 80^\circ}}{\tan \cancel{50^\circ} \tan \cancel{80^\circ}}$$

$$M = \tan 50^\circ$$



$$M = \tan 50^\circ$$



Si $\tan \alpha = -\frac{1}{2}$ y $\alpha \in \text{IIc}$, calcule $\text{sen} 2\alpha$

Resolución:

$$\tan \alpha = -\frac{1}{2} = \frac{y}{x} \quad \Rightarrow \quad \begin{array}{l} \text{Como } \alpha \in \text{IIc} \\ x = -2 ; \quad y = 1 \end{array}$$

Por Radio Vector:

$$\begin{aligned} r &= \sqrt{(-2)^2 + (1)^2} \\ r &= \sqrt{5} \end{aligned}$$

Piden:

$$\begin{aligned} \text{sen} 2\alpha &= 2 \underbrace{\text{sen} \alpha}_{\frac{y}{r}} \cdot \underbrace{\text{cos} \alpha}_{\frac{x}{r}} \quad \Rightarrow \quad \text{sen} 2\alpha = 2 \left(\frac{1}{\sqrt{5}} \right) \left(\frac{-2}{\sqrt{5}} \right) \\ &\quad \Rightarrow \quad \text{sen} 2\alpha = -\frac{4}{5} \end{aligned}$$



$$\text{sen} 2\alpha = -\frac{4}{5}$$

HELICOPRACTICE 9



Calcule $\text{sen}2x$ si se cumple que $\cos\left(\frac{\pi}{4} + x\right) = \frac{1}{\sqrt{5}}$

Resolución:

Dato: $\cos(45^\circ + x) = \frac{1}{\sqrt{5}}$

Usar identidad:

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \text{sen}\alpha \cdot \text{sen}\beta$$

$$\Rightarrow \cos 45^\circ \cos x - \text{sen} 45^\circ \text{sen} x = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \text{sen} x = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} (\cos x - \text{sen} x) = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \cos x - \text{sen} x = \frac{\sqrt{2}}{\sqrt{5}}$$

Elevando al cuadrado:

$$\underbrace{\cos^2 x + \text{sen}^2 x}_1 - \underbrace{2\cos x \text{sen} x}_{\text{sen} 2x} = \frac{2}{5}$$



$$\text{sen} 2x = \frac{3}{5}$$



Simplificar la expresión: $\cos^4 8^\circ - 6\sin^2 8^\circ \cos^2 8^\circ + \sin^4 8^\circ$

Resolución:

$$E = \cos^4 8^\circ - 6\sin^2 8^\circ \cos^2 8^\circ + \sin^4 8^\circ$$

$$E = \underbrace{\sin^4 8^\circ + \cos^4 8^\circ} - 6\sin^2 8^\circ \cos^2 8^\circ$$

$$E = 1 - 2\sin^2 8^\circ \cos^2 8^\circ - 6\sin^2 8^\circ \cos^2 8^\circ$$

$$E = 1 - 8\sin^2 8^\circ \cos^2 8^\circ$$

$$E = 1 - 2(\underbrace{4\sin^2 8^\circ \cos^2 8^\circ})$$

$$E = 1 - 2(\sin 16^\circ)^2$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta)$$

$$E = 1 - 2\sin^2 16^\circ$$

$$E = \cos 32^\circ$$



$$E = \cos 32^\circ$$