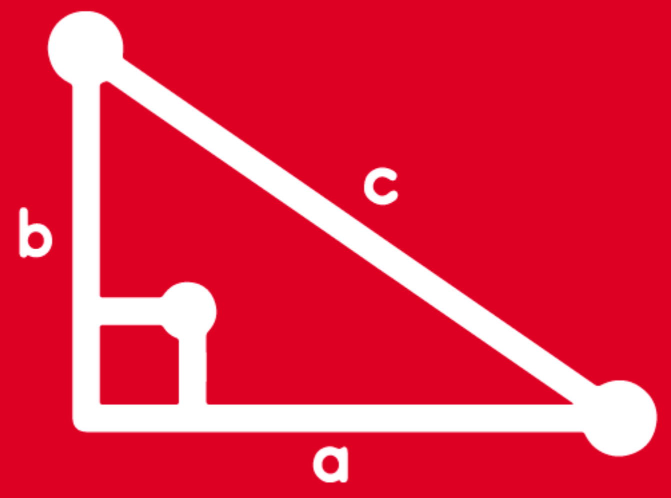
TRIGONOMETRY

Chapter 18

Sessión 1



5th

SECONDARY

Identidades trigonométricas de ángulos compuestos

SACO OLIVEROS

MOTIVATING STRATEGY

¿ A qué es igual $\operatorname{sen} 83^{\circ}$?

¿ A qué es igual $\cos 75^{\circ}$?

¿ A qué es igual $\tan 8^{\circ}$?

Los ángulos 83º , 75º y 8º no son notables !
… pero 30º , 37º , 45º , 53º y 60º si son notables !

Luego:

$$\operatorname{sen} 83^{\circ} = \operatorname{sen}(53^{\circ} + 30^{\circ})$$
$$\cos 75^{\circ} = \cos(30^{\circ} + 45^{\circ})$$
$$\tan 8^{\circ} = \tan(45^{\circ} - 37^{\circ})$$

En este capítulo desarrollaremos las identidades del ángulo compuesto para calcular dichos valores ☺

HELICOTEORÍA

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO (Fundamentales)

Para la suma de dos ángulos

$$\operatorname{sen}(x+y)=\operatorname{sen}x.\cos y+\cos x.\operatorname{sen}y$$

$$\cos(x+y)=\cos x.\cos y-\operatorname{sen}x.\operatorname{sen}y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$$

Para la resta de dos ángulos

$$\operatorname{sen}(x-y)=\operatorname{sen}x.\cos y-\cos x.\operatorname{sen}y$$

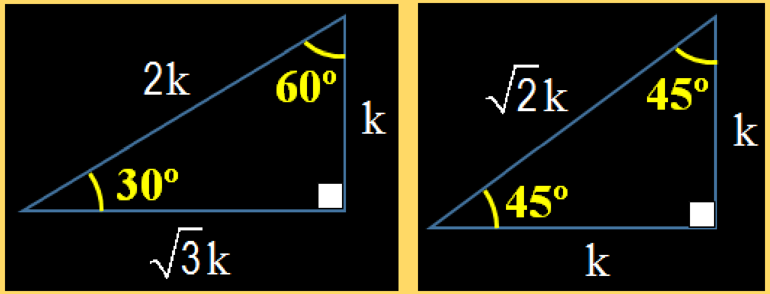
$$\cos(x-y)=\cos x.\cos y+\operatorname{sen}x.\operatorname{sen}y$$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x.\tan y}$$

HELICOTEORÍA

Ejemplo: Calcular sen15°

Recordar:



Resolución:

$$\text{sen}15^\circ = \text{sen}(45^\circ - 30^\circ)$$

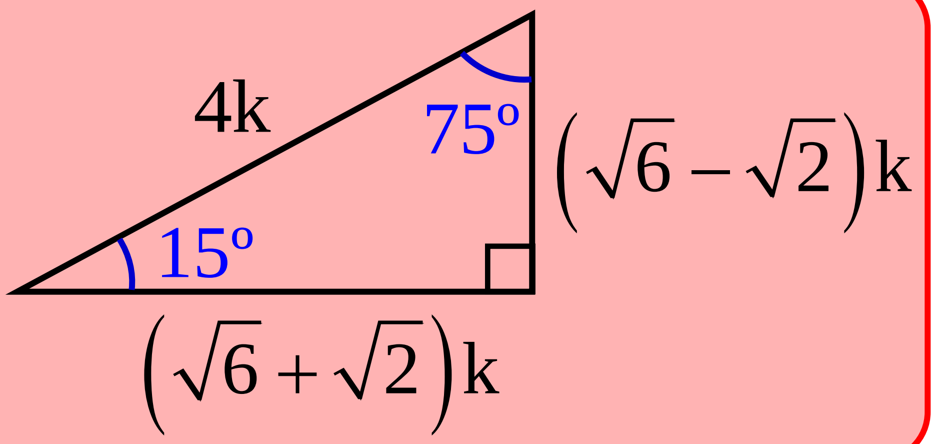
$$\text{sen}15^\circ = \text{sen}45^\circ.\cos 30^\circ - \cos 45^\circ.\text{sen}30^\circ$$

$$\text{sen}15^\circ = \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}.\frac{1}{2}$$

$$\text{sen}15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Conclusión

◣15° y 75°



PROBLEMA 1

Calcule $cos16°$

Resolución:

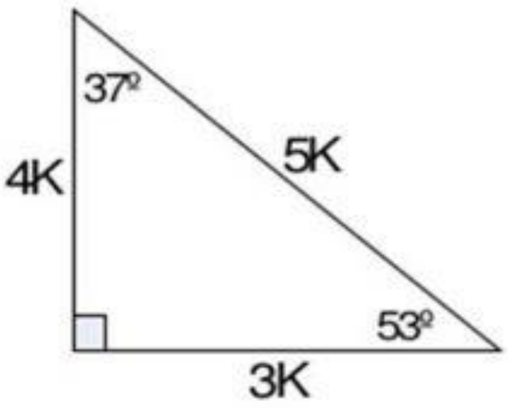
$$\cos(x-y) = cosx.cosy + senx.seny$$

$$cos16° = \cos(53° - 37°)$$

$$cos16° = \underbrace{cos53°}.\underbrace{cos37°} + \underbrace{sen53°}.\underbrace{sen37°}$$

$$cos16° = \underbrace{\frac{3}{5} \cdot \frac{4}{5}} + \underbrace{\frac{4}{5} \cdot \frac{3}{5}}$$

$$cos16° = \frac{12}{25} + \frac{12}{25}$$



$$\therefore cos16° = \frac{24}{25}$$

PROBLEMA 2

Reducir $R = \sqrt{2}cos(x - 45°) - senx$

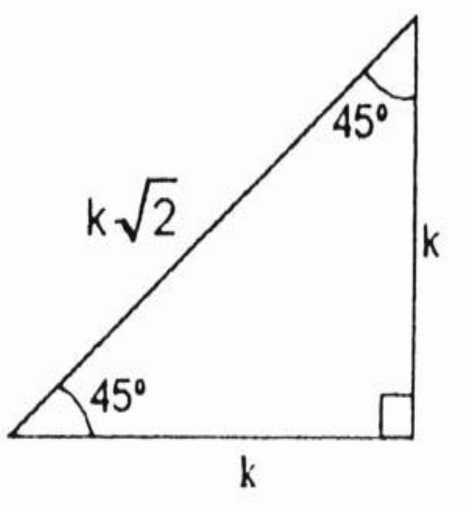
Resolución:

$$\cos(x - y) = cosx.\,cosy + senx.\,seny$$

$$R = \sqrt{2}\cos(x - 45°) - \text{senx}$$

$$R = \sqrt{2}\ [\text{cosx}.\underbrace{\cos 45°} + \text{senx}.\underbrace{\text{sen}45°}] - \text{senx}$$

$$R = \cancel{\sqrt{2}}[\ \text{cosx}\ .\ \frac{1}{\cancel{\sqrt{2}}}\ + \text{senx}\ .\ \frac{1}{\cancel{\sqrt{2}}}\ ] - \text{senx}$$



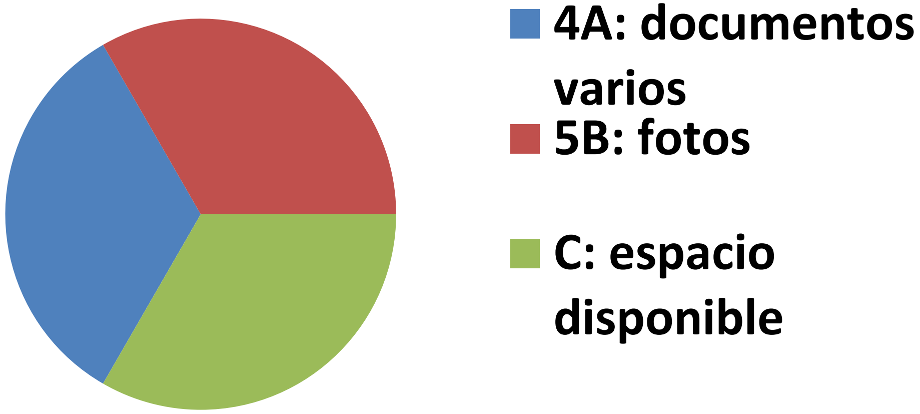
$$R = \ \text{cosx} + \cancel{\text{senx}} - \cancel{\text{senx}}$$

$$\boxed{\therefore R = cosx}$$

PROBLEMA 3

Observe el siguiente diagrama y determine el espacio disponible del USB

distribucion del almacenamiento de una memoria de 8 GB



Resolución:

$$A = \underbrace{\mathbf{sen25°.cos5° + cos25°.sen5°}}_{\mathbf{sen(25° + 5°)}}$$

$$A = \text{sen}30° = \frac{\mathbf{1}}{\mathbf{2}}$$

$$B = \frac{\boldsymbol{tan55° - tan10°}}{\boldsymbol{1 + tan55°.tan10°}} = \mathbf{tan(55° - 10°)}$$

$$\Rightarrow B = \tan 45° = \mathbf{1}$$

Piden:

$$\boldsymbol{C} = \mathbf{8} - [\overset{\mathbf{2}}{\cancel{4}}\left(\frac{\mathbf{1}}{\cancel{\mathbf{2}}}\right) + 5(\mathbf{1})]$$

$$\boxed{\therefore \boldsymbol{C = 1GB}}$$

PROBLEMA 4

Calcule el valor de x si

$$senx.cos(2x-10°)+cosx.sen(2x-10°)=cos40°$$

Donde $x \in \langle 0°;90° \rangle$

Resolución:

$$senx.cosy+cosx.seny=sen(x+y)$$

$$\underbrace{senx.\cos(2x-10°)+cosx.sen(2x-10°)}_{sen[(x)+(2x-10°)]}=cos40°$$

$$\Rightarrow \text{sen}(3\text{x}-10°)=\cos 40°$$

$*$ **Por RT. complementarios:** $3x-10°+40°=90°$

$$3x=60°$$

$$\therefore x=20°$$

PROBLEMA 5

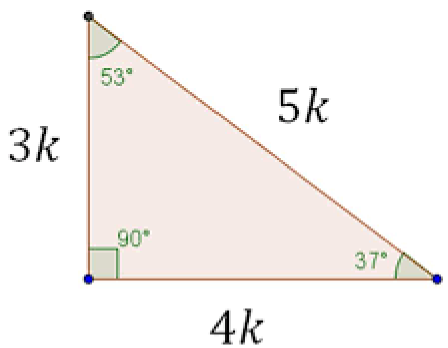
Si $\tan\theta = \frac{5}{12}$ **; calcule** $\tan(37° + \theta)$

Resolución:

$$\tan(37° + \theta) = \frac{\tan 37° + \tan\theta}{1 - \tan 37° . \tan\theta}$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$$

$$\tan(37° + \theta) = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{\cancel{3}}{4} . \frac{5}{\cancel{12}_{4}}} = \frac{\frac{14}{12}}{1 - \frac{5}{16}} = \frac{\frac{7}{6}}{\frac{11}{16}}$$



$$\Rightarrow \tan(37° + \theta) = \frac{7 \times \cancel{16}^{8}}{\cancel{6}_{3} \times 11}$$

$$\boxed{\therefore \tan(37° + \theta) = \frac{56}{33}}$$

PROBLEMA 6

Si $tan(x+y)=\frac{1}{3}$ y $tan(x-y)=2$; calcule $tan2y$

Resolución:

$Consideramos:$

$$x+y=m \quad \rightarrow \quad \tan(m)=\frac{1}{3}$$

$$x-y=n \quad \rightarrow \quad \tan(n)=2$$

$Además:$

$$\underbrace{(x+y)}_{m}-\underbrace{(x-y)}_{n}=2y$$

$$\tan(m-n)=tan2y$$

$$tan(m-n)=\frac{tan(m)-tan(n)}{1+tan(m).tan(n)}$$

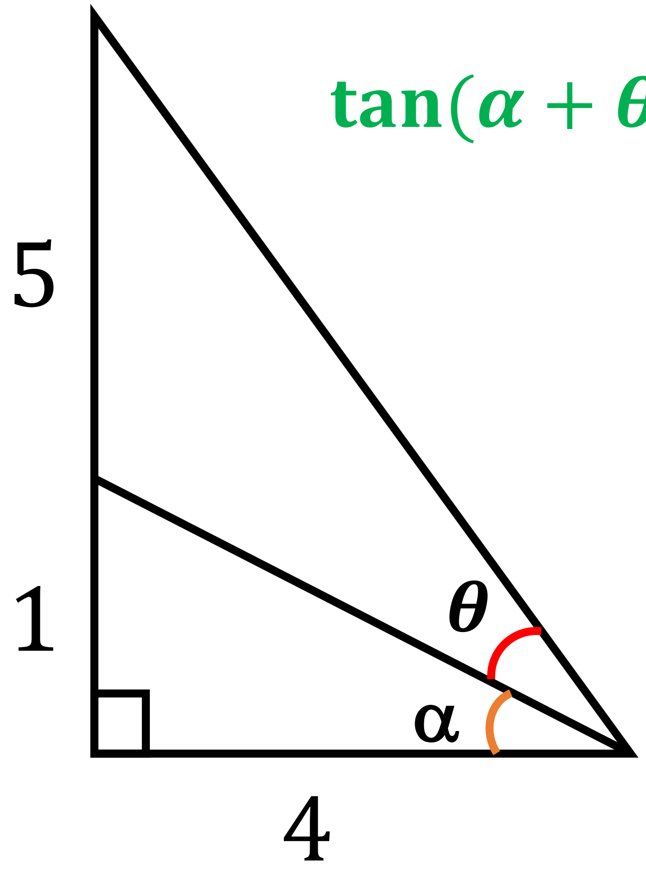
$$tan2y=\frac{\frac{1}{3}-2}{1+\frac{1}{3}.2}$$

$$tan2y=\frac{-\frac{\cancel{5}}{\cancel{3}}}{\frac{\cancel{5}}{\cancel{3}}}$$

$$\boxed{\therefore\ tan2y=-1}$$

PROBLEMA 7

A partir del gráfico, determine el valor de $tan\theta$



$$\tan(\alpha+\theta)=\frac{6}{4}=\frac{3}{2}$$

$$tan\alpha=\frac{1}{4}$$

Resolución:

Recordamos: $\tan(\alpha+\theta)=\dfrac{\tan\alpha+\tan\theta}{1-\tan\alpha.\tan\theta}$

$$\frac{3}{2}=\frac{\frac{1}{4}+\tan\theta}{1-\frac{1}{4}.\tan\theta} \Rightarrow \frac{3}{2}=\frac{\frac{1+4tan\theta}{\cancel{4}}}{\frac{4-tan\theta}{\cancel{4}}}$$

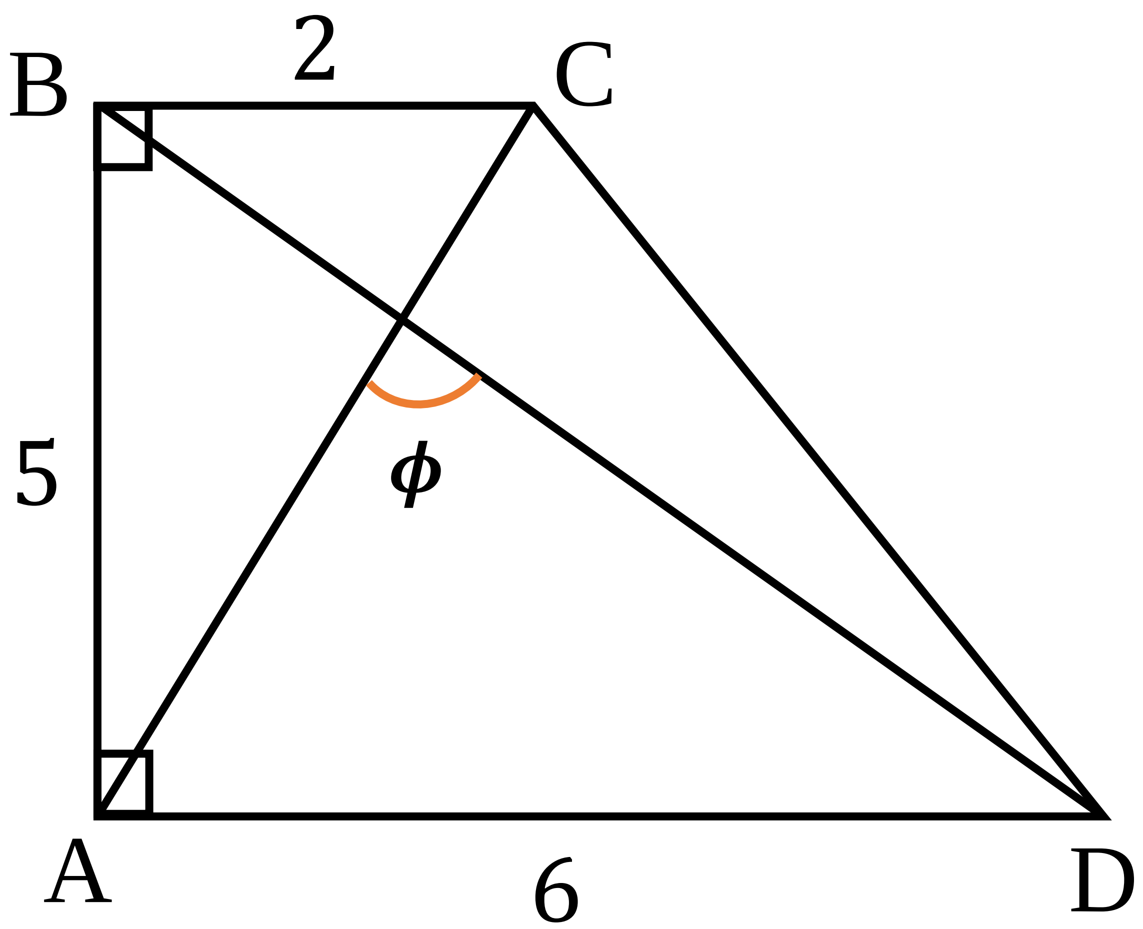
$$\frac{3}{2}=\frac{1+4tan\theta}{4-tan\theta} \Rightarrow 12-3\tan\theta=2+8\tan\theta$$

$$10=11\tan\theta$$

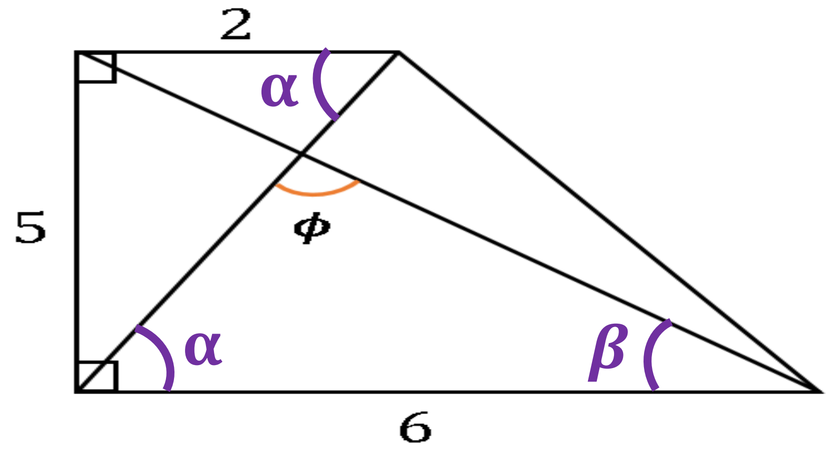
$$\boxed{\therefore\ tan\theta=\frac{10}{11}}$$

PROBLEMA 8

En el trapecio $ABCD$ mostrado, determine el valor de $tan\phi$



Resolución:



$$tan\alpha = \frac{5}{2}$$

$$tan\beta = \frac{5}{6}$$

Observamos: $\alpha + \beta + \emptyset = 180°$

$$tan\alpha + tan\beta + tan\emptyset = tan\alpha . tan\beta . tan\emptyset$$

$$\frac{5}{2} + \frac{5}{6} + tan\emptyset = \frac{5}{2} . \frac{5}{6} . tan\emptyset$$

$$\frac{40}{12} + \tan\emptyset = \frac{25}{12} . \tan\emptyset \quad \ldots \times (12)$$

$$40 + 12tan\emptyset = 25tan\emptyset$$

$$\Rightarrow 40 = 13tan\emptyset$$

$$\therefore tan\emptyset = \frac{40}{13}$$