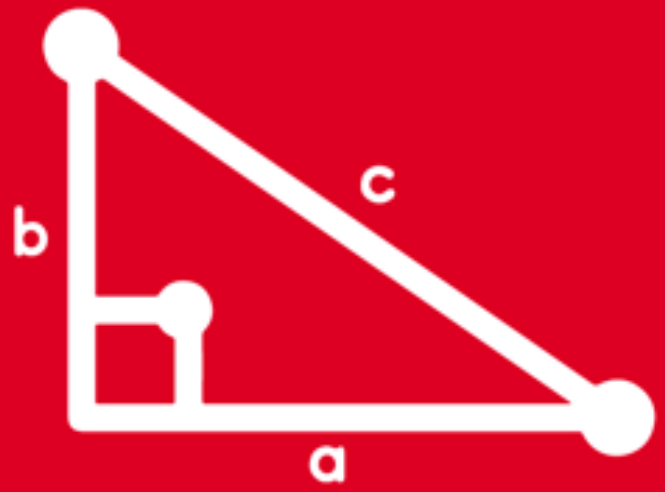
TRIGONOMETRY

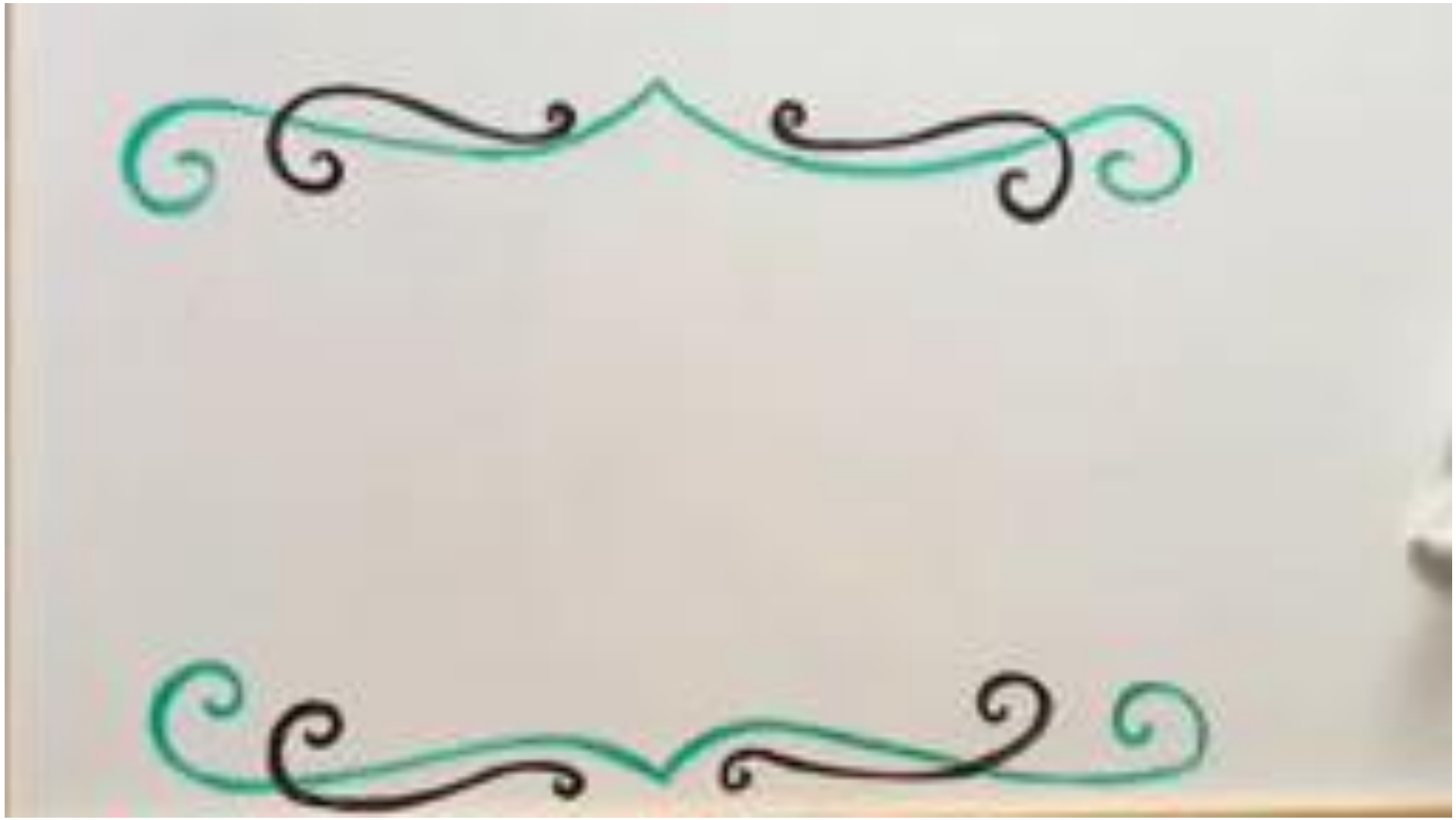
Chapter 16

5th
SECONDARY



IDENTIDADES TRIGONOMÉTRICAS AUXILIARES DEL ÁNGULO DOBLE

SACO OLIVEROS



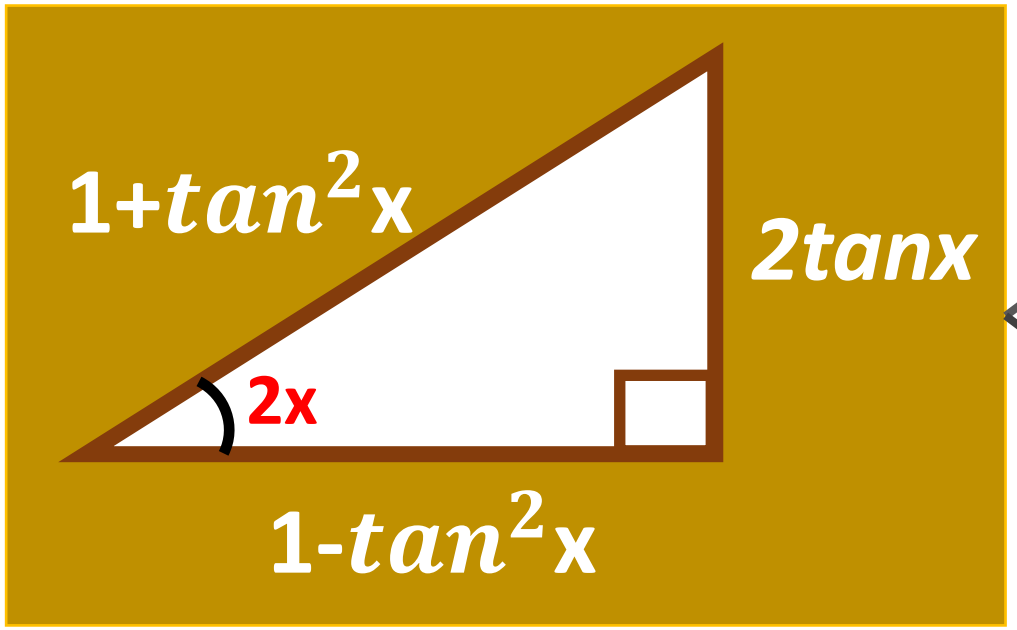
IDENTIDADES TRIGONOMÉTRICAS AUXILIARES DEL ÁNGULO DOBLE

I. IDENTIDADES DE DEGRADACIÓN

$\cos(2x) =$

- $2\cos^2(x) - 1 \rightarrow 2\cos^2(x) = 1 + \cos(2x)$
- $1 - 2\operatorname{sen}^2(x) \rightarrow 2\operatorname{sen}^2(x) = 1 - \cos(2x)$

II. TRIÁNGULO DEL ÁNGULO DOBLE



$$\text{sen } 2x = \frac{2tanx}{1+tan^2x}$$

$$\cos 2x = \frac{1-tan^2x}{1+tan^2x}$$

III. IDENTIDADES AUXILIARES

$$\cot x - \tan x = 2\cot(2x)$$

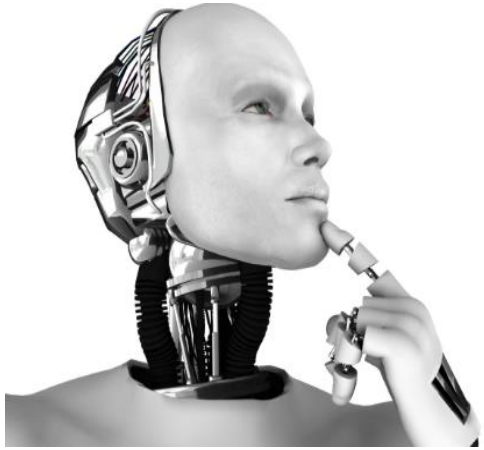
$$\cot x + \tan x = 2\csc(2x)$$

1.

RESOLUCIÓN

Recordar:

$$\cot x + \tan x = 2\csc(2x)$$



$$E = (\underbrace{\cot x + \tan x}_{2\csc 2x})\operatorname{sen} 2x$$

$$E = 2\underbrace{\csc 2x.\operatorname{sen} 2x}_{1}$$

$$\therefore \boxed{E = 2}$$

2.

RESOLUCIÓN

Recordar:

$$\cot x - \tan x = 2\cot(2x)$$

$$\underbrace{\cot x - \tan x} = 16$$

$$2\cot 2x = 16$$

$$\cot 2x = 8$$

$$\therefore \tan 2x = \frac{1}{8}$$

3.

$$\frac{-\qquad \theta+\qquad \theta}{+\qquad \theta+\qquad \theta}=-\qquad\qquad \theta$$

RESOLUCIÓN

$$\frac{\overbrace{1-cos2\theta}^{2sen^2\theta}+\overbrace{sen2\theta}^{2sen\theta cos\theta}}{\underbrace{1+cos2\theta}_{2cos^2\theta}+\underbrace{sen2\theta}_{2sen\theta cos\theta}}=\frac{1}{5}$$

$$\frac{2sen\theta(sen\theta+cos\theta)}{2cos\theta(cos\theta+sen\theta)}=\frac{1}{5}$$

$$\tan\theta=\frac{1}{5}$$

Nos piden: $sen2\theta$

$$sen2\theta=\frac{2\left(\frac{1}{5}\right)}{1+\left(\frac{1}{5}\right)^2}=\frac{1\left(\frac{2}{5}\right)^1}{5\left(\frac{26}{25}\right)^{13}}$$

Recordar:

$$sen2\theta=\frac{2\tan\theta}{1+\tan^2\theta}$$

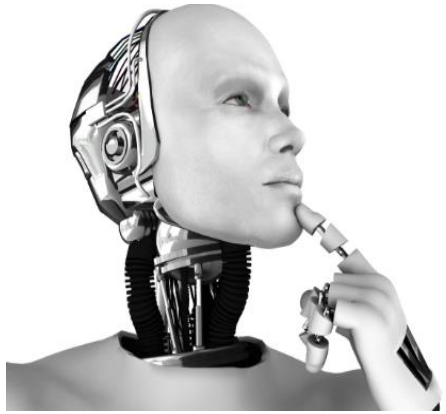
$$\therefore\ sen2\theta=\frac{5}{13}$$

4.

RESOLUCIÓN

Recordar:

$$2\cos^2(x) = 1 + \cos(2x)$$



Debió escribir ➡ $\dfrac{2\cos^2 40° \;\; 1+\cos 80°}{\text{sen}80°}$ ➡ $\dfrac{\cos 40°}{\text{sen}40°}$

escribió ➡ $\dfrac{\text{sen}80°}{2\text{sen}40°\cos 40°}$

$\therefore \; \cot 40°$

5.

$$= \frac{-}{-}\theta + \frac{-}{+}\theta$$

$2\cos^2(x) = 1 + \cos(2x)$

$2\text{sen}^2(x) = 1 - \cos(2x)$

RESOLUCIÓN

$$P = \frac{1-cos4\theta}{1-cos2\theta} + \frac{1-cos4\theta}{1+cos2\theta}$$

$$P = \frac{2sen^2 2\theta}{2sen^2\theta} + \frac{2sen^2 2\theta}{2cos^2\theta}$$

$$P = \frac{(sen2\theta)^2}{sen^2\theta} + \frac{(sen2\theta)^2}{cos^2\theta}$$

$$P = \frac{(2sen\theta cos\theta)^2}{sen^2\theta} + \frac{(2sen\theta cos\theta)^2}{cos^2\theta}$$

$$P = \frac{4sen^2\theta cos^2\theta}{sen^2\theta} + \frac{4sen^2\theta cos^2\theta}{cos^2\theta}$$

$$P = 4cos^2\theta + 4sen^2\theta \Rightarrow P = 4\underbrace{(cos^2\theta + sen^2\theta)}_{1}$$

$\therefore \boldsymbol{P = 4}$

6.

$$= \frac{\left(\frac{\pi}{\quad}\right) + \left(\frac{\pi}{\quad}\right)}{\left(\frac{\pi}{\quad}\right) - \left(\frac{\pi}{\quad}\right)}$$

RESOLUCIÓN

$$E = \frac{cot\left(\frac{\pi}{12}\right) + tan\left(\frac{\pi}{12}\right)}{cot\left(\frac{\pi}{8}\right) - tan\left(\frac{\pi}{8}\right)}$$

$$E = \frac{2csc\left(\frac{\pi}{6}\right)}{2cot\left(\frac{\pi}{4}\right)}$$

$$E = \frac{csc30°}{cot45°}$$

$$E = \frac{2}{1}$$

$$\therefore \boldsymbol{E = 2}$$

cotx + tanx = 2csc(2x)

cotx – tanx = 2cot(2x)

7.

$$= \frac{\pi}{\quad} = \frac{\quad}{+} + \frac{+}{-}$$

$$\text{sen}(2x) = \frac{2\tan x}{1+\tan^2 x}$$

$$\cos(2x) = \frac{1-\tan^2 x}{1+\tan^2 x}$$

RESOLUCIÓN

$$M = \frac{2tanx}{1+tan^2x} + \frac{1+tan^2x}{1-tan^2x}$$

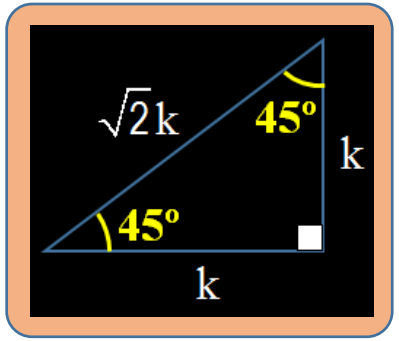
$$M = sen2x + \sec 2x$$

$$M = sen\cancel{2}\left(\frac{\pi}{\cancel{8}}\right) + sec\cancel{2}\left(\frac{\pi}{\cancel{8}}\right)$$

$$M = sen\left(\frac{\pi}{4}\right) + sec\left(\frac{\pi}{4}\right)$$

$$M = sen45° + sec45°$$

$$M = \frac{\sqrt{2}}{2} + \sqrt{2}$$



$$\therefore M = \frac{3\sqrt{2}}{2}$$

8. $= \sqrt{\quad - \sqrt{\quad +} \quad} \qquad \in$

RESOLUCIÓN

$$A = \sqrt{2 - \sqrt{2 + 2cos4x}}$$

$$A = \sqrt{2 - \sqrt{2(1 + \boldsymbol{cos4x})}}$$

$$A = \sqrt{2 - \sqrt{2(\boldsymbol{2cos^2 2x})}}$$

$$A = \sqrt{2 - \sqrt{\boldsymbol{4cos^2 2x}}}$$

$$A = \sqrt{2 - \boldsymbol{2cos2x}}$$

$$A = \sqrt{2(\boldsymbol{1 - cos2x})}$$

$$A = \sqrt{2(\boldsymbol{2sen^2 x})}$$

$$A = \sqrt{\boldsymbol{4sen^2 x}}$$

$2\cos^2(\alpha) = 1 + \cos(2\alpha)$

$2\text{sen}^2(\alpha) = 1 - \cos(2\alpha)$

$\therefore \boldsymbol{A = 2senx}$