GEOMETRÍA

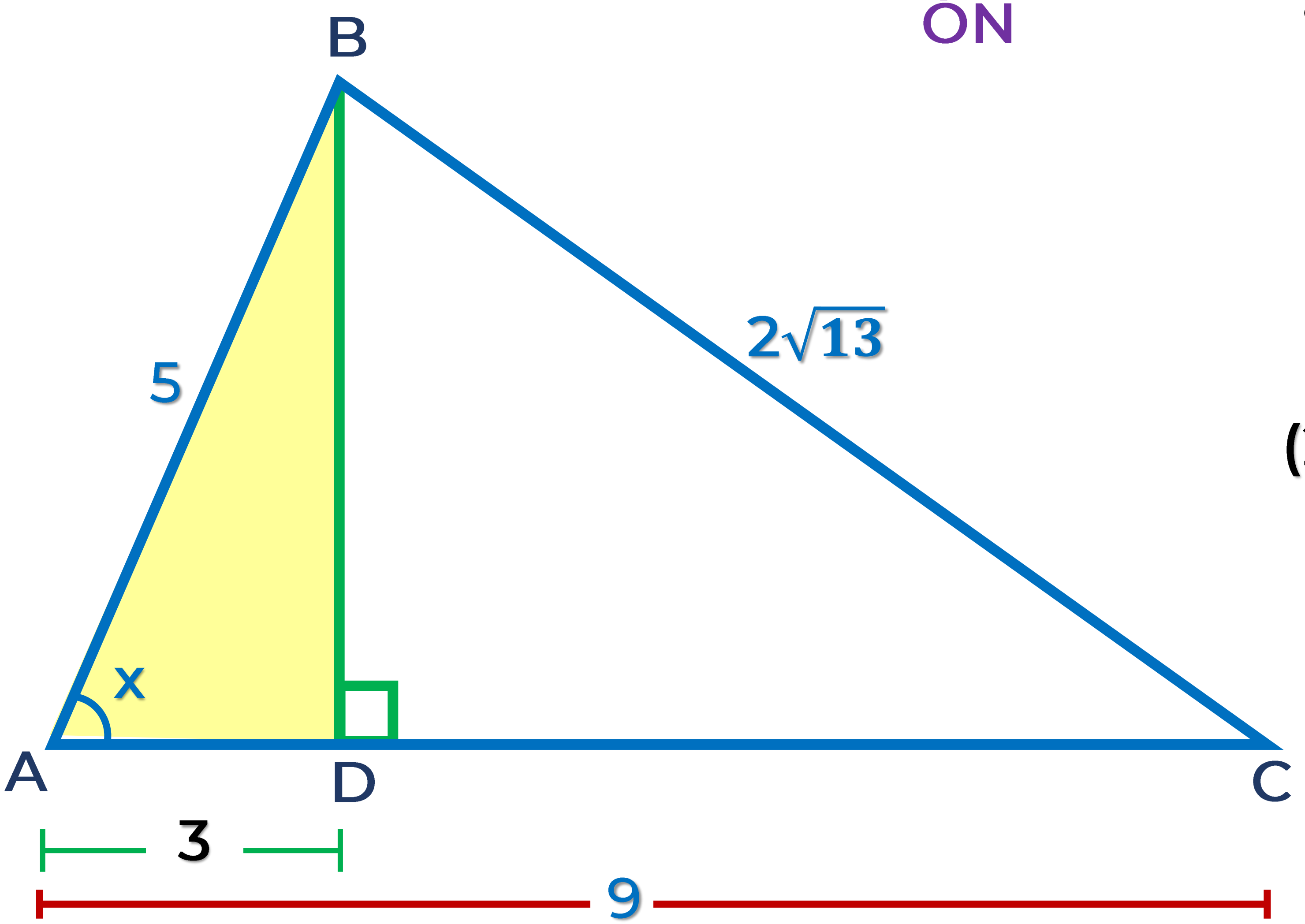
Tomo 4

5th
SECONDARY

RETROALIMENTACIÓN

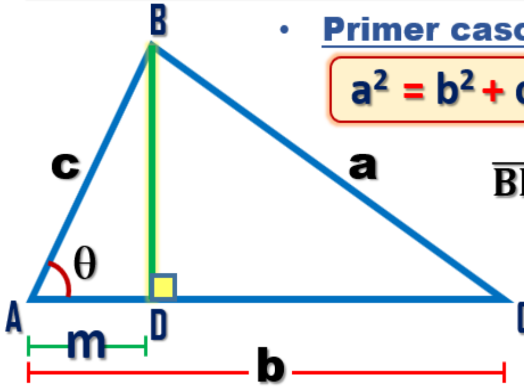
SACO OLIVEROS

1.En la figura, calcule x.



RESOLUCIÓN

- Se traza la altura $\overline{BD}$
- **Teorema de Euclides**



- Primer caso

$$a^2 = b^2 + c^2 - 2bm$$

$\overline{BD}$: Altura

$\theta < 90°$

$$(2\sqrt{13})^2 = 9^2 + 5^2 - 2(9)(m)$$

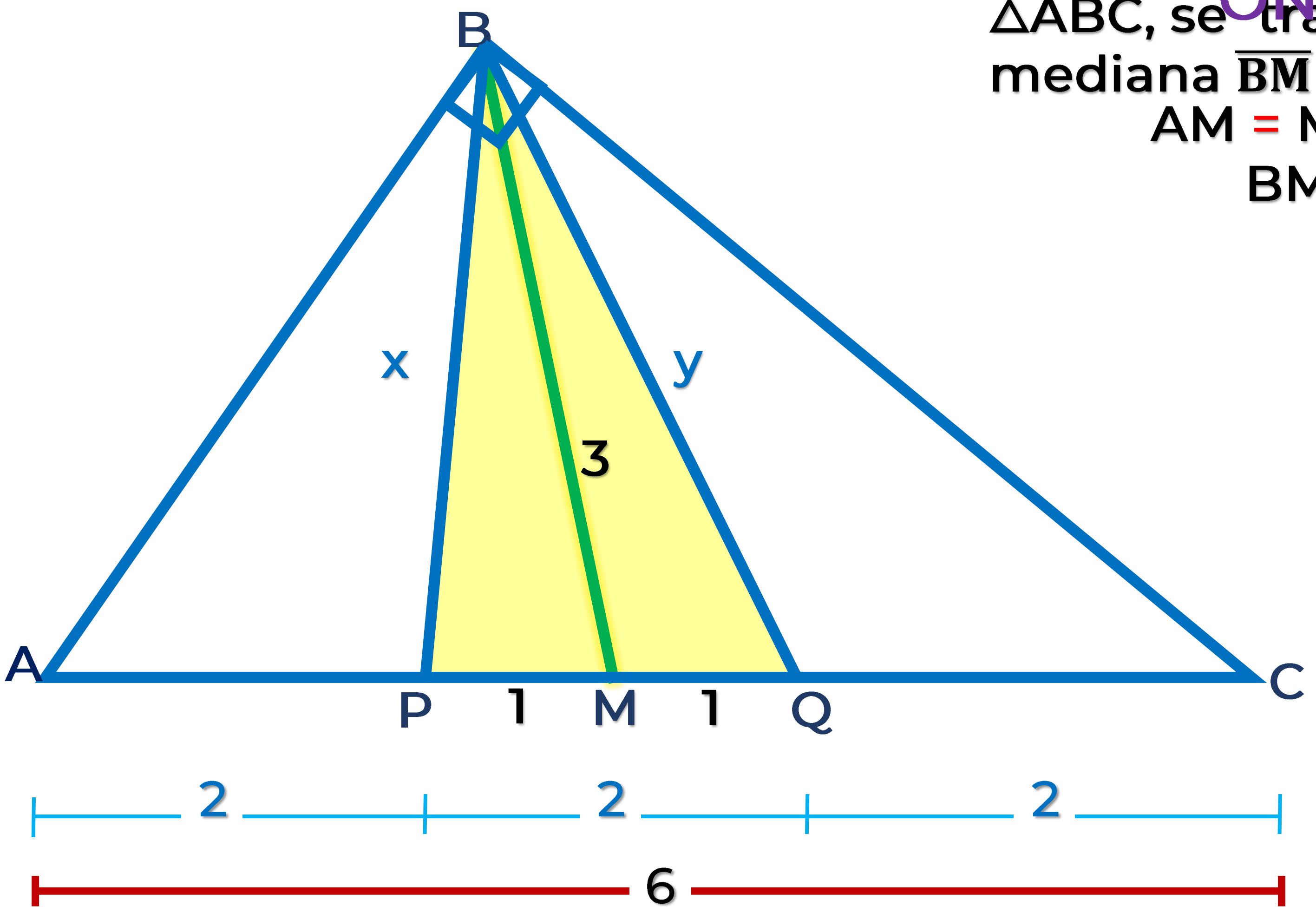
$$52 = 81 + 25 - 18m$$

$$18m = 54 \qquad m = 3$$

⊿ABD aproximado de 37° y 53°

$$\boxed{x = 53°}$$

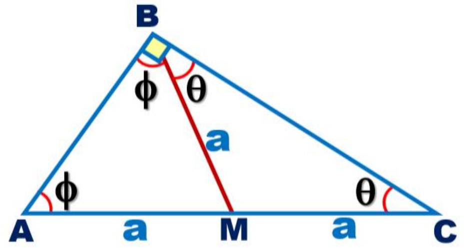
2. En la figura, calcule $x^2 + y^2$.



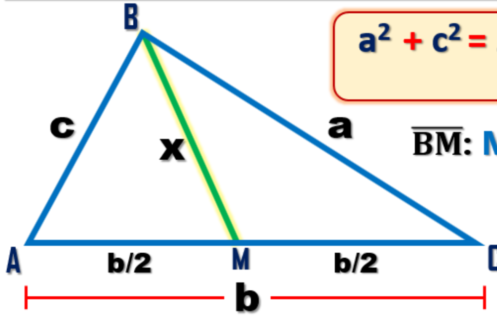
RESOLUCIÓN

$\triangle ABC$, se traza la menor mediana $\overline{BM}$

$AM = MC = BM = 3$



TEOREMA DE LA MEDIANA



$a^2 + c^2 = 2x^2 + \frac{b^2}{2}$

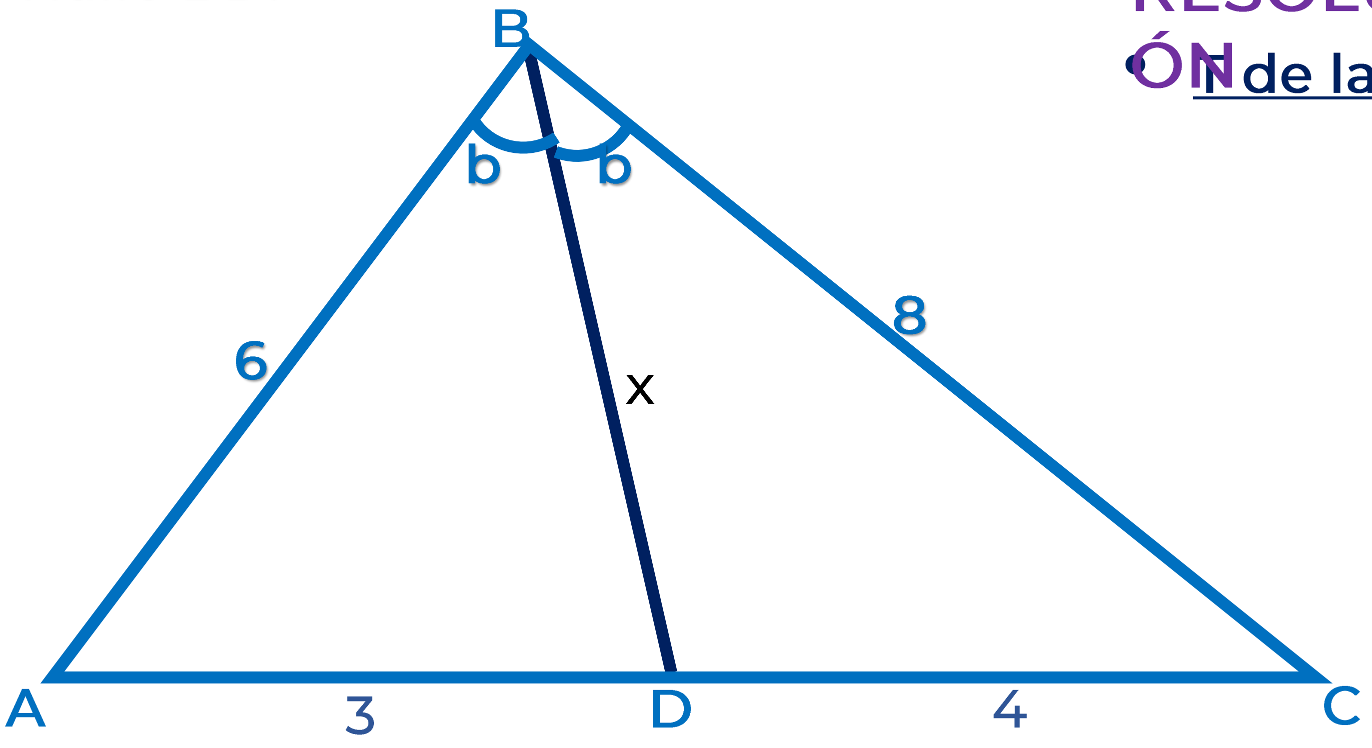
$\overline{BM}$: Mediana

$\triangle PBQ$: $x^2 + y^2 = 2(3)^2 + \frac{(2)^2}{2}$

$x^2 + y^2 = 18 + 2$

$$\boxed{x^2 + y^2 = 20}$$

3. En un triángulo ABC, se traza la bisectriz interior $\overline{BD}$. Si AB = 6, BC = 8 y DC = 4. Halle BD.

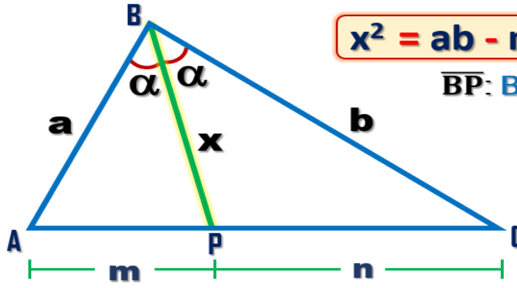


RESOLUCIÓN

- $\overline{BD}$: bisectriz interior.
- <u>T. de la bisectriz interior</u> (Proporcionalidad)

$$\frac{\cancel{6}\;3}{\cancel{8}\;4} = \frac{AD}{4} \qquad AD = 3$$

<u>T. de la longitud de la bisectriz interior</u>



$\boxed{x^2 = ab - mn}$

$\overline{BP}$: Bisectriz interior

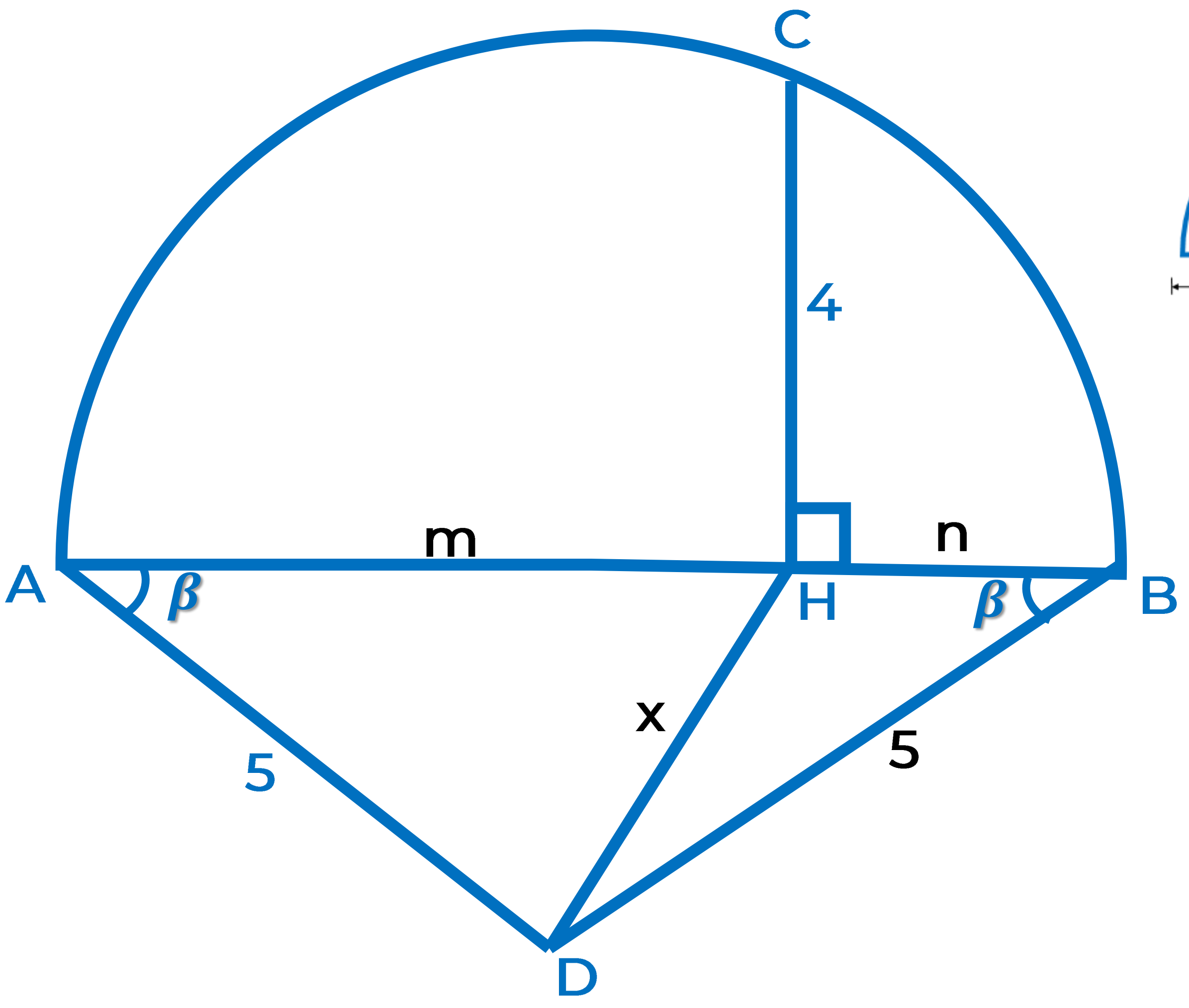
En el $\triangle$ABC: $x^2 = 6.8 - 3.4$

$x^2 = 48 - 12$

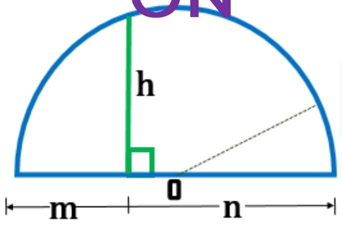
$x^2 = 36$

$\boxed{x = 6}$

4. En la figura, $\overline{AB}$ es diámetro, calcule DH.



RESOLUCIÓN



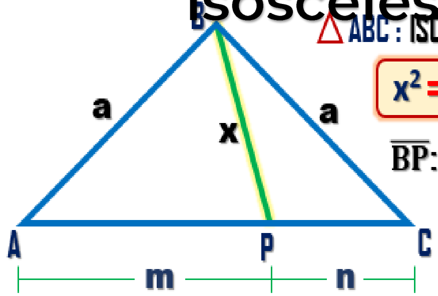
$\boxed{h^2 = mn}$

En la semicircunferencia

- $4^2 = m.n$

$16 = m.n$

Teorema de Stewart (para isósceles)



△ABC : ISÓSCELES

$\boxed{x^2 = a^2 - mn}$

$\overline{BP}$: Ceviana

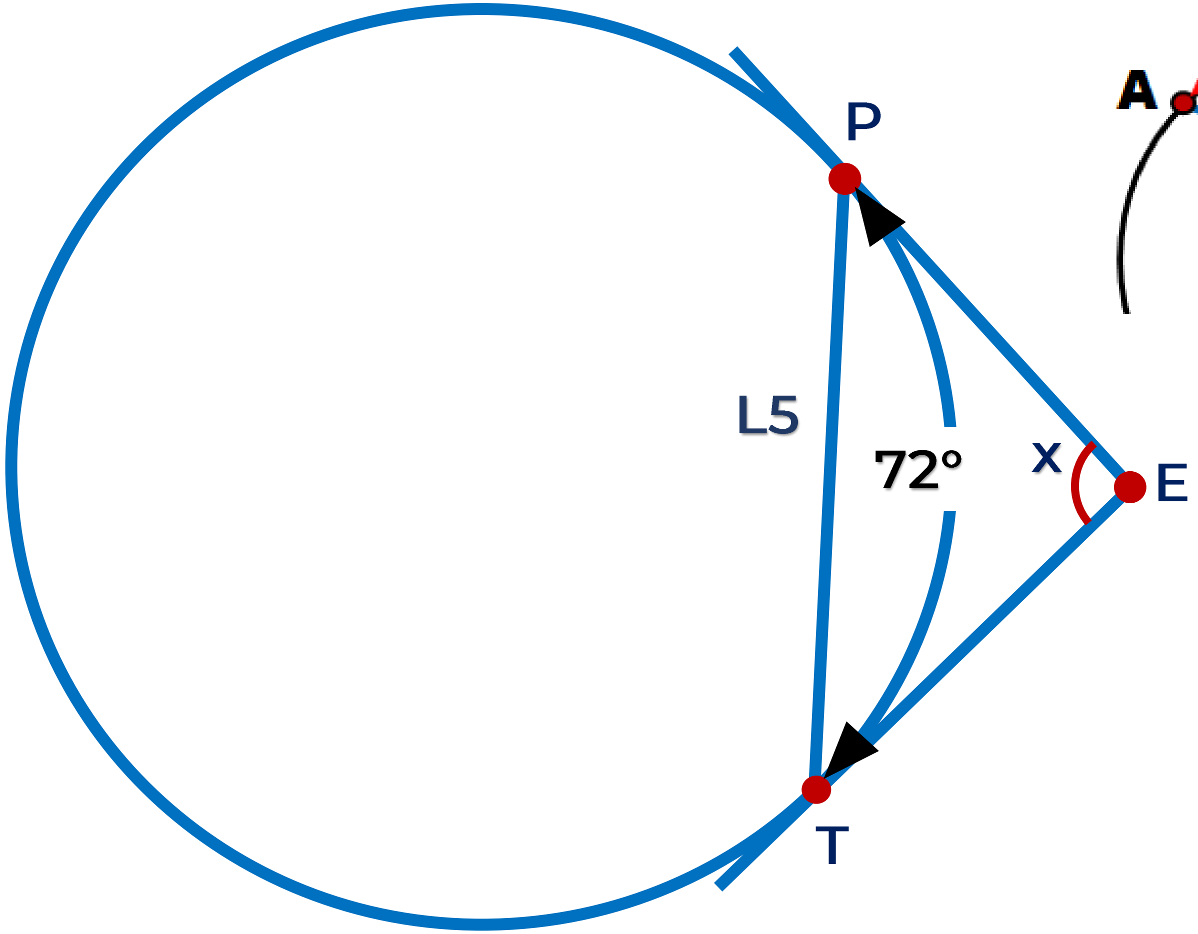
$\triangle ABD: x^2 = 5^2 - \underbrace{m.n}$

$x^2 = 25 - 16$

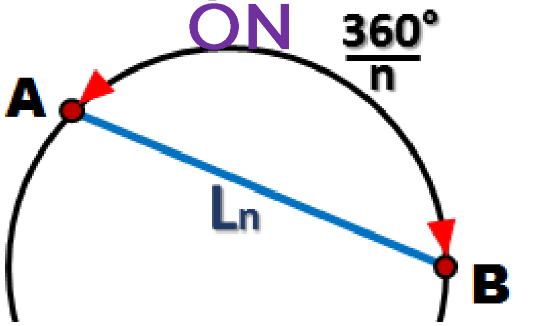
$x^2 = 9$

$\boxed{x = 3}$

5. Desde un punto E exterior a una circunferencia, se trazan los segmentos tangentes $\overline{ET}$ y $\overline{EP}$. Si PT = L5, halle la m<PET.



RESOLUCIÓN

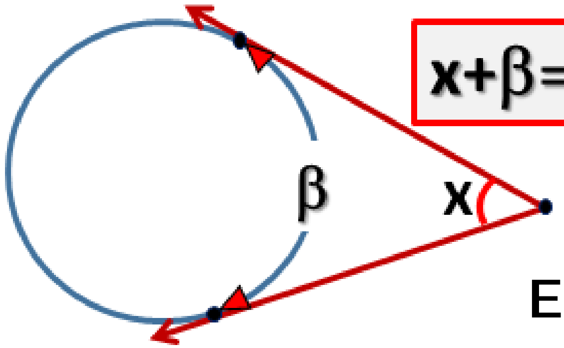


$m\overset{\frown}{AB} = \frac{360°}{n}$

n = 5

• $m\overset{\frown}{AB} = \frac{360}{5}$

$m\overset{\frown}{AB} = 72°$



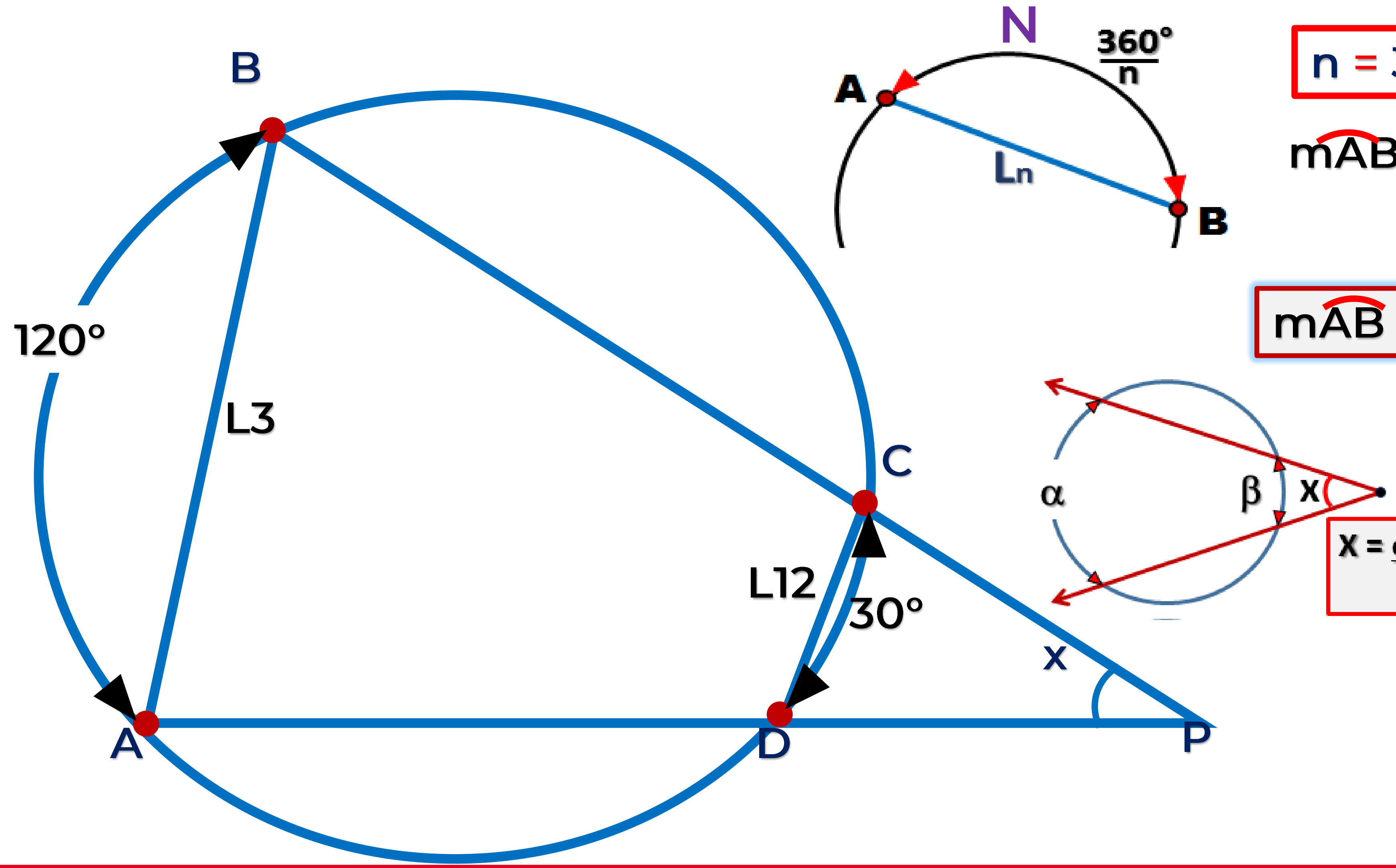
$x + \beta = 180°$

En el problema: x + 72°= 18

$x = 108°$

6. Calcule x, si AB = L3 y CD = L12.

RESOLUCIÓ



$\boxed{n = 3}$

$$m\overset{\frown}{AB} = \frac{360°}{3}$$

$$\boxed{m\overset{\frown}{AB} = 120°}$$

$\boxed{n = 12}$

$$m\overset{\frown}{CD} = \frac{360°}{12}$$

$$\boxed{m\overset{\frown}{CD} = 30°}$$

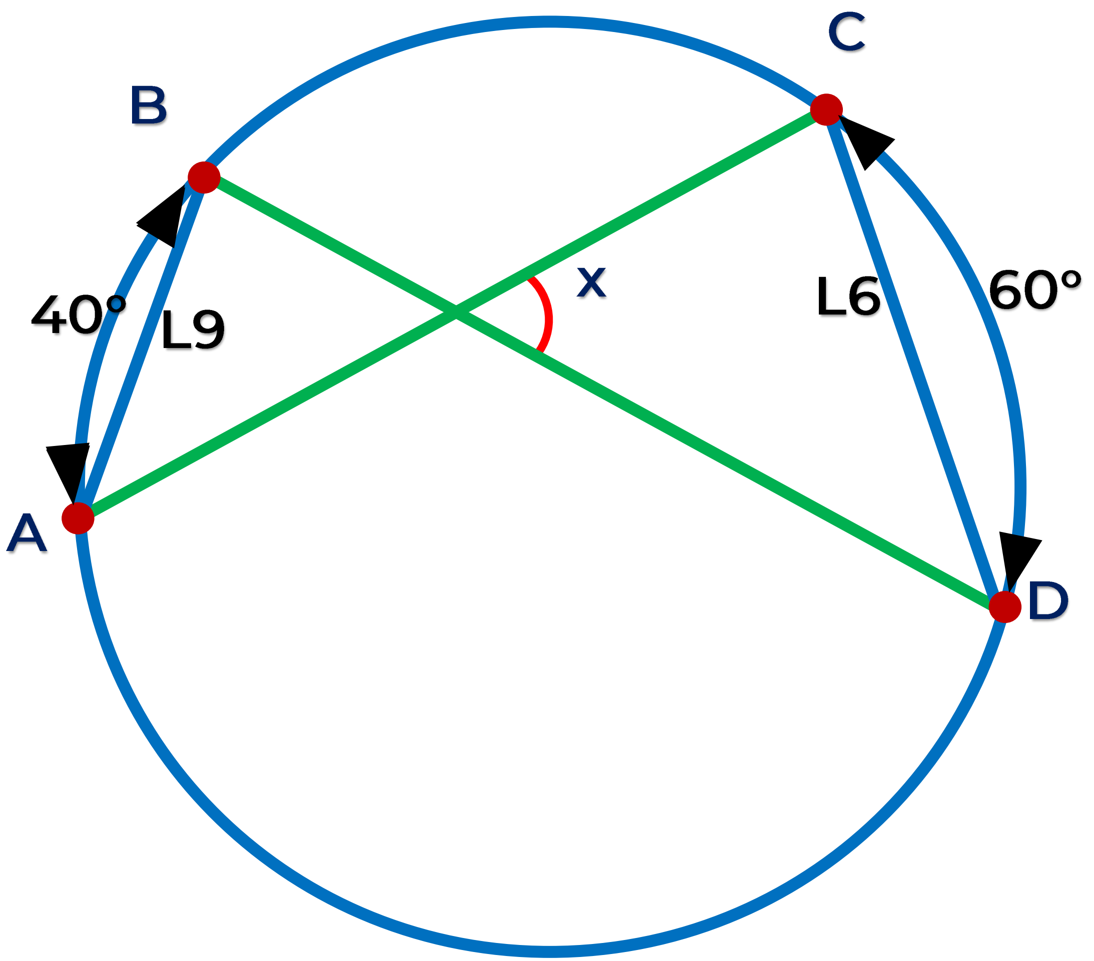
$$\boxed{x = \frac{\alpha - \beta}{2}}$$

Ángulo exterior

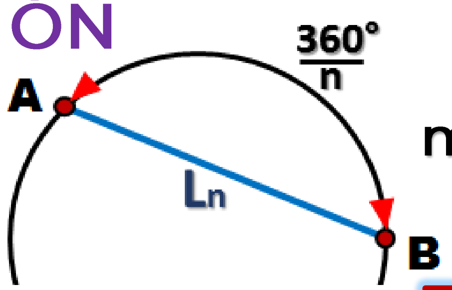
$$x = \frac{120° - 30°}{2}$$

$$\boxed{x = 45°}$$

7. Si AB = L9 y CD = L6, calcule la medida del ángulo que forman $\overline{BD}$ y $\overline{AC}$.



RESOLUCIÓN



$\boxed{n = 9}$

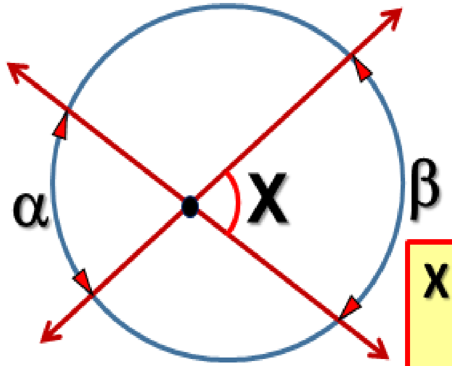
$$m\overset{\frown}{AB} = \frac{360°}{9}$$

$$\boxed{m\overset{\frown}{AB} = 40°}$$

$\boxed{n = 6}$

$$m\overset{\frown}{CD} = \frac{360°}{6}$$

$$\boxed{m\overset{\frown}{CD} = 60°}$$



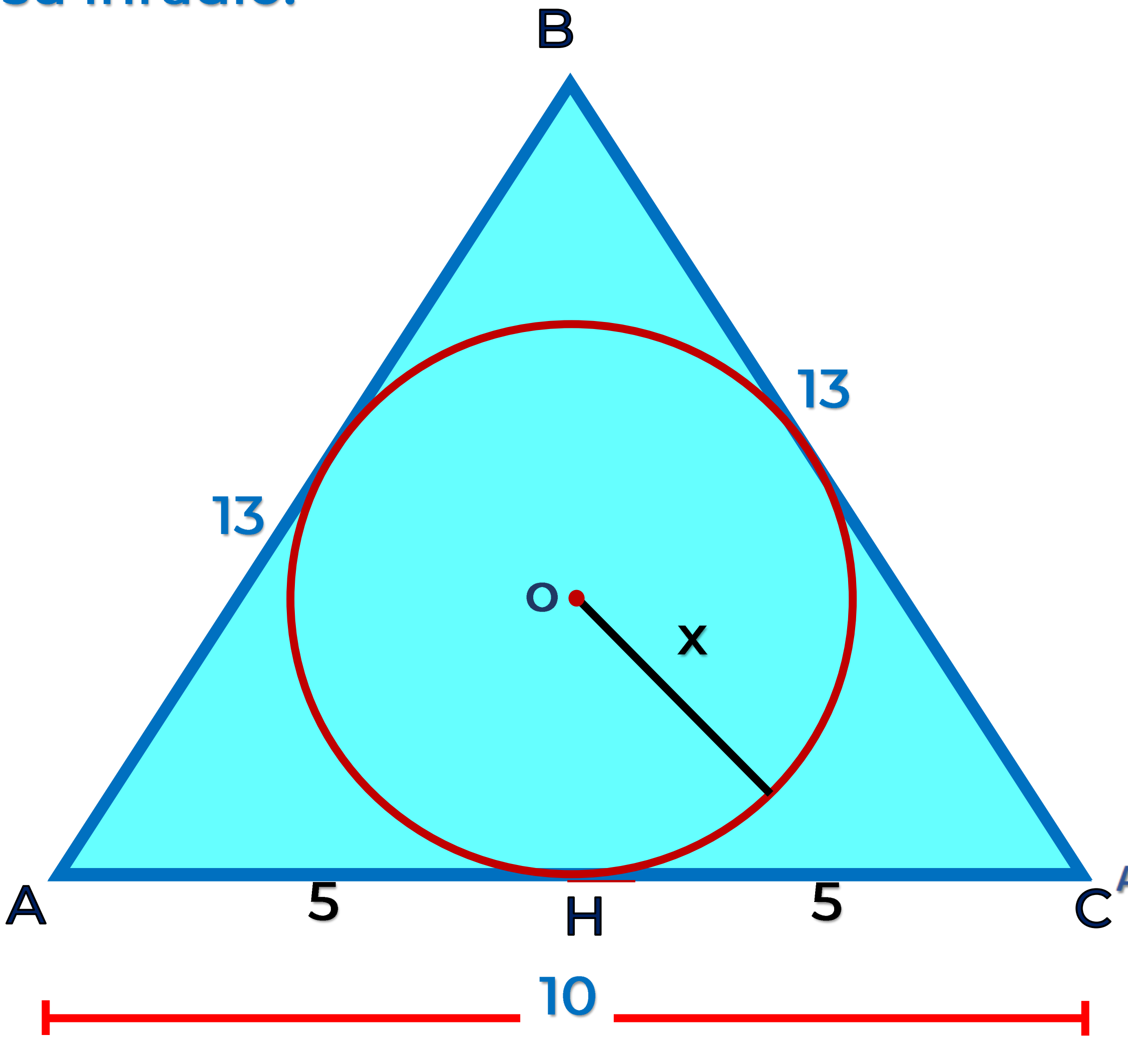
$$\boxed{X = \frac{\alpha+\beta}{2}}$$

Ángulo interior

$$x = \frac{40° + 60°}{2}$$

$$\boxed{x = 50°}$$

8. Las longitudes de los lados del triángulo son: 13; 13 y 10. Calcule la longitud de su inradio.



RESOLUCIÓN

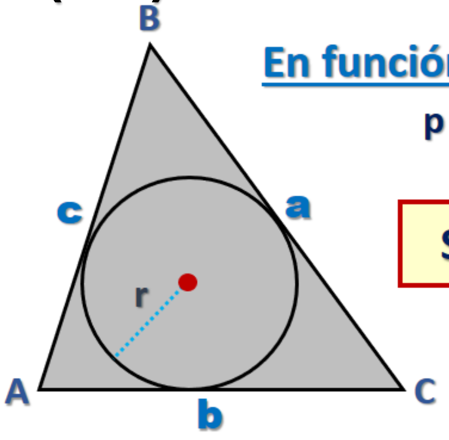
$\triangle ABC$ es isósceles

$\triangle BHC$, T. Pitágoras

$13^2 = (BH)^2 + 5^2$

$144 = (BH)^2$

$12 = BH$



En función al inradio

$p = \frac{a + b + c}{2}$

$\boxed{S_{ABC} = p.r}$

$S(ABC) = \frac{10.12}{2}$

$S(ABC) = 60u^2$

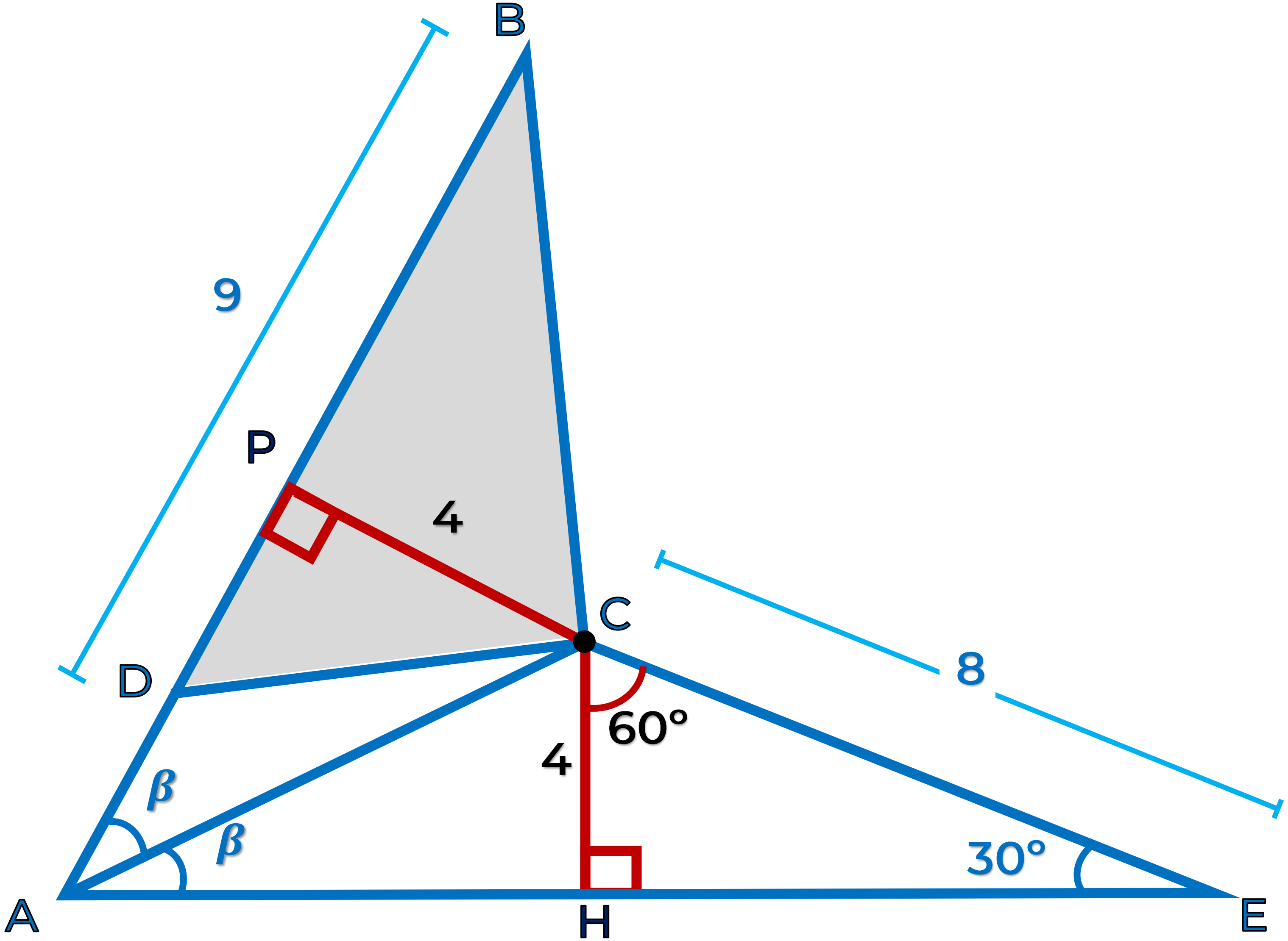
$S_{\triangle ABC} = \frac{(13 + 13 + 10).x}{2}$

$\underbrace{S_{\triangle ABC}}_{60} = (18).x$

$\cancel{60} = \cancel{18}x$

$\boxed{10/3 = x}$

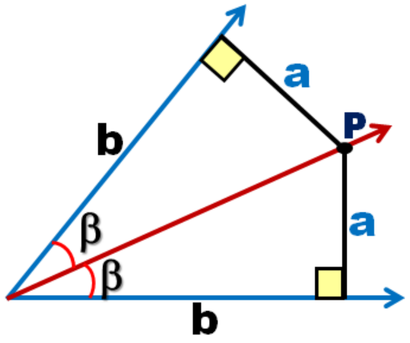
9. En el gráfico, BD = 9 y CE= 8, calcule el área de la región sombreada.



RESOLUCIÓN

- Se traza la altura $\overline{CH}$.

◺CHE es notable de 30° y 60°



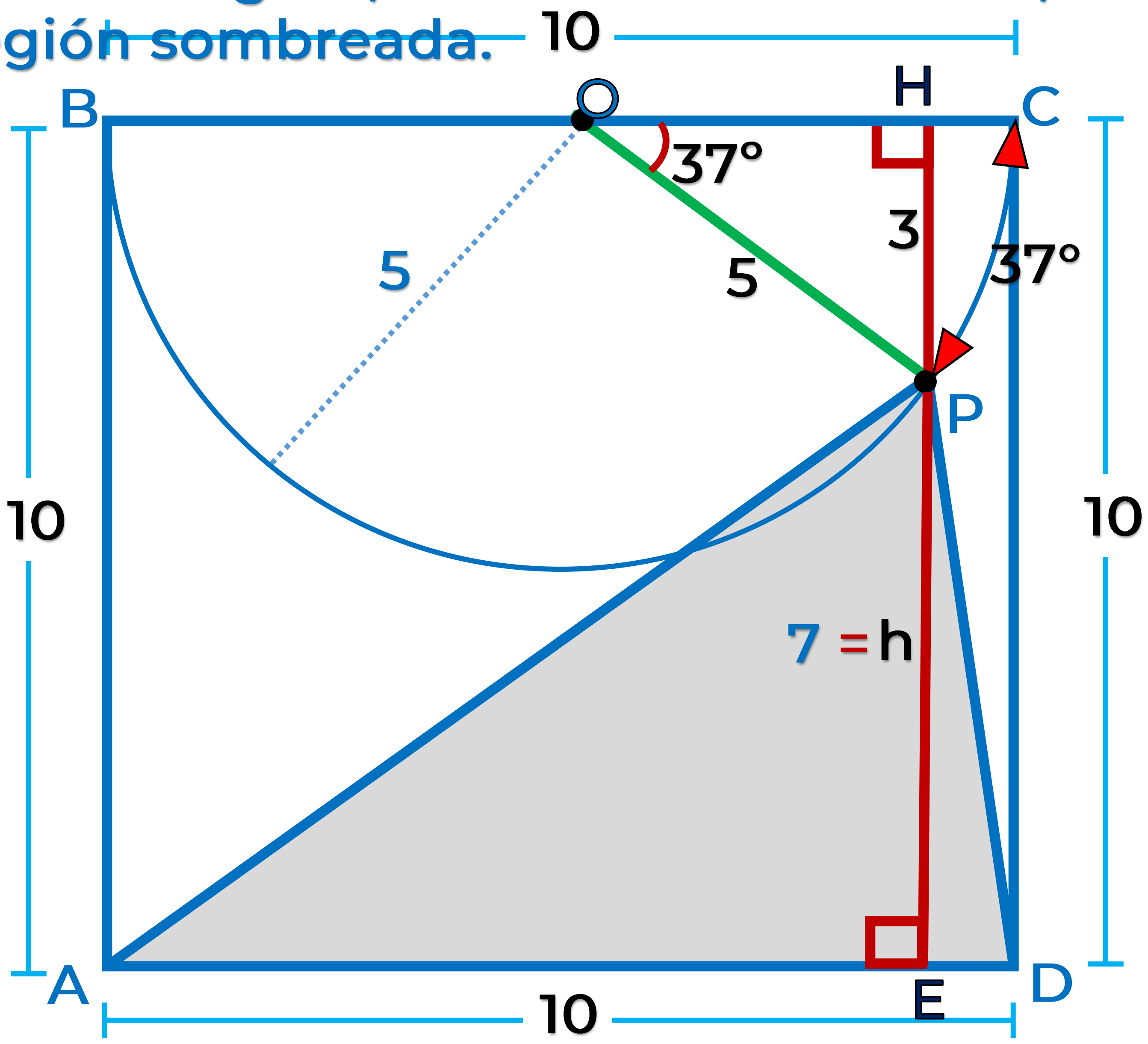
CH = CP = 4

- △ABC, teorema

$$S_{\triangle BCD} = \frac{9 \cdot 4}{2}$$

$$\boxed{S_{\triangle BCD} = 18u^2}$$

10. En la figura, ABCD es un cuadrado, si m$\overset{\frown}{CP}$ = 37°, calcule el área de la región sombreada.



RESOLUCIÓN

- Se traza $\overline{OP}$.
- Se traza $\overline{PH}$ perpendicular a $\overline{BC}$.

⊿OHP es aproximado de 37° y 53°

- Se prolonga $\overline{HP}$ hasta E.

CDEH es rectángulo

HE = CD = 10

h + 3 = 10   h = 7

- Teorema   S(APD) = $\frac{10.7}{2}$

$\boxed{S(APD) = 35u^2}$