



# TRIGONOMETRY

**TOMO VII**  
**Sesión 1**

**4th**  
SECONDARY

**Feedback**

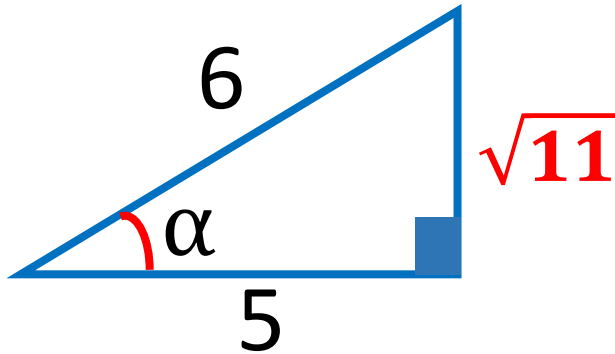




1. Si  $\cos\alpha = \frac{5}{6}$ , donde  $0 < \alpha < 90^\circ$ , calcule  $\sin 2\alpha$ .

### Resolución

Del dato:  $\cos\alpha = \frac{5}{6} = \frac{\text{CA}}{\text{H}}$



Recuerda:

$$\sin 2x = 2\sin x \cdot \cos x$$

Del gráfico:  $\sin\alpha = \frac{\sqrt{11}}{6}$

Piden  $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$

$$\sin 2\alpha = 2 \left( \frac{\sqrt{11}}{6} \right) \left( \frac{5}{6} \right)$$

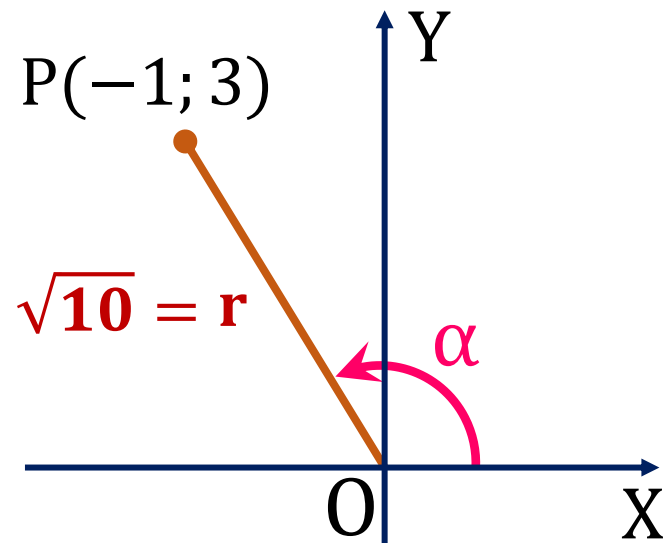
$$\therefore \sin 2\alpha = \frac{5\sqrt{11}}{18}$$



**2.** Si el punto  $P(-1; 3)$  pertenece al lado final de un ángulo en posición normal  $\alpha$ , calcule  $5\cos 2\alpha$ .

### Resolución

Graficando, según la condición:



Calculamos  $r$ :

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-1)^2 + (3)^2}$$

$$r = \sqrt{10}$$

Recuerda:  $\cos 2\alpha = \cos^2 \alpha - \sen^2 \alpha$

Así:

$$\sen \alpha = \frac{3}{\sqrt{10}}$$

$$\cos \alpha = \frac{-1}{\sqrt{10}}$$

$$\rightarrow \cos 2\alpha = \left(\frac{-1}{\sqrt{10}}\right)^2 - \left(\frac{3}{\sqrt{10}}\right)^2$$

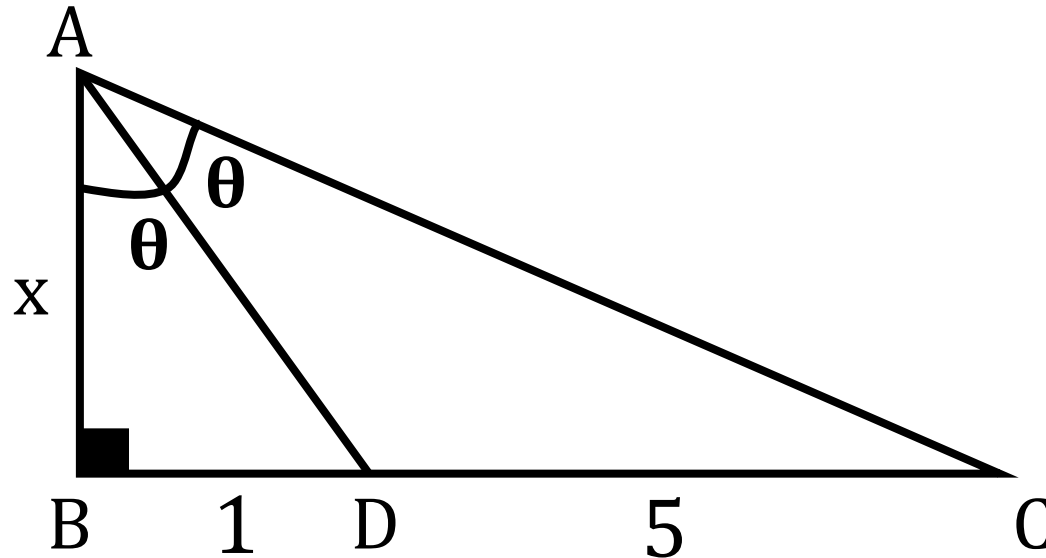
$$\cos 2\alpha = \frac{1}{10} - \frac{9}{10} = \frac{-8}{10}$$

$$\cos 2\alpha = \frac{-4}{5}$$

$$\therefore 5\cos 2\alpha = -4$$



**3.** A partir del gráfico, determine el valor de  $2x$ .



### Resolución

$$\Delta ABD: \tan \theta = \frac{1}{x} \quad \Delta ABC: \tan 2\theta = \frac{6}{x}$$

Aplicamos:

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\frac{6}{x} = \frac{2 \left( \frac{1}{x} \right)}{1 - \left( \frac{1}{x} \right)^2}$$

$$\frac{6}{x} = \frac{\frac{2}{x}}{\frac{x^2 - 1}{x^2}}$$

$$\frac{3}{x} = \frac{1}{x(x^2 - 1)}$$

Tenemos:

$$3x^2 - 3 = x^2$$

$$2x^2 = 3$$

$$x^2 = \frac{3}{2}$$

$$x = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\therefore 2x = \sqrt{6}$$



**4.** Si  $\cos \alpha = \frac{1}{9}$  y  $\alpha \in \langle 0; \frac{\pi}{2} \rangle$ , calcule  $\sin \left( \frac{\alpha}{2} \right)$ .

### Resolución

Recordamos:  $\sin \left( \frac{\alpha}{2} \right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$

Del dato:  $0 < \alpha < \frac{\pi}{2}$   
 $\div 2$   
 $0 < \underbrace{\frac{\alpha}{2}}_{\text{IC}} < \frac{\pi}{4} \rightarrow \sin \left( \frac{\alpha}{2} \right) : (+)$

$$\rightarrow \sin \left( \frac{\alpha}{2} \right) = \sqrt{\frac{1 - \frac{1}{9}}{2}}$$

$$\sin \left( \frac{\alpha}{2} \right) = \sqrt{\frac{\frac{8}{9}}{2}}$$

$$\sin \left( \frac{\alpha}{2} \right) = \sqrt{\frac{4}{9}}$$

$$\therefore \sin \left( \frac{\alpha}{2} \right) = \frac{2}{3}$$



## 5. Reduzca

$$K = \sqrt{\frac{1 - \sin 35^\circ}{1 + \sin 35^\circ}}$$

### Resolución

Recordamos:

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Por RT complementarias:

$$\sin 35^\circ = \cos 55^\circ$$

Reemplazando:

$$K = \sqrt{\frac{1 - \cos 55^\circ}{1 + \cos 55^\circ}} = \tan\left(\frac{55^\circ}{2}\right)$$

$$\rightarrow K = \tan 27,5^\circ$$

$$K = \tan(27^\circ + 0,5^\circ) \quad \text{a minutos}$$

$$K = \tan(27^\circ + 0,5 \times 60')$$

$$\therefore K = \tan 27^\circ 30'$$



**6.** Si  $\tan x = \sec(-26^\circ) + \cot 64^\circ$  calcule el valor de  $x$  siendo este un ángulo agudo.

### Resolución

Del dato:  $\tan x = \sec(\cancel{-26^\circ}) + \cot 64^\circ$

$$\tan x = \sec 26^\circ + \cot 64^\circ$$

¡CO-razón!

$$\tan x = \csc 64^\circ + \cot 64^\circ$$

$$\tan x = \cot\left(\frac{64^\circ}{2}\right)$$

$$\rightarrow \tan x = \cot 32^\circ$$

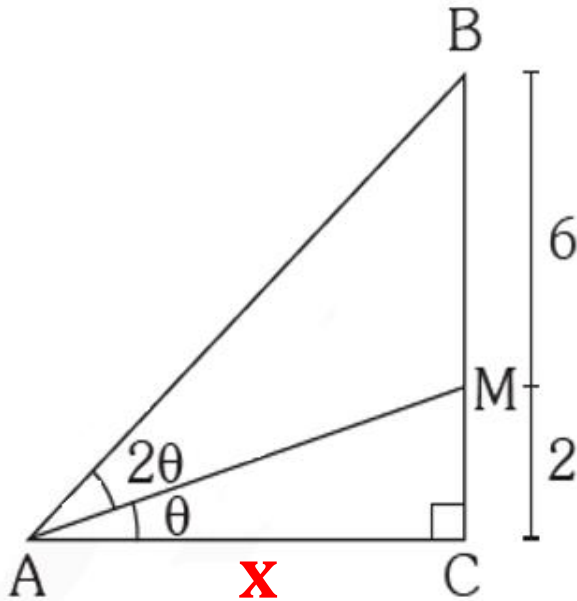
Por RT complementarias:

$$x + 32^\circ = 90^\circ$$

$$\therefore \boxed{x = 58^\circ}$$



**7.** A partir del gráfico, determine AC.



Recordamos

s

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

### Resolución

$$\Delta ABC: \tan 3\theta = \frac{8}{x} \quad \Delta ACM: \tan \theta = \frac{2}{x}$$

Reemplazando:

$$\frac{8}{x} = \frac{3 \left( \frac{2}{x} \right) - \left( \frac{2}{x} \right)^3}{1 - 3 \left( \frac{2}{x} \right)^2}$$

Cambio de variable:  $\frac{2}{x} = a$

$$\rightarrow 4a = \frac{3a - a^3}{1 - 3a^2}$$





## 7. Continuación

De la anterior:

$$4a = \frac{3a - a^3}{1 - 3a^2}$$

$$4a - 12a^3 = 3a - a^3$$

$$a = 11a^3$$

$$\frac{1}{11} = a^2$$

Pero:  $a = \frac{2}{x}$

$$\rightarrow \frac{1}{11} = \frac{4}{x^2}$$

$$x^2 = 44$$

$\therefore$

$$x = 2\sqrt{11}$$



## 8. Reduzca

$$T = \frac{\cos 11x + \cos 9x + \cos 7x + \cos 5x}{\cos 3x + \cos x}$$

### Resolución

Recordamos:

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

Aplicando la IT en el numerador:

$$T = \frac{2\cos 8x \cdot \cos 3x + 2\cos 8x \cdot \cos x}{\cos 3x + \cos x}$$

Factorizando "2cos8x":

$$T = \frac{2\cos 8x (\cos 3x + \cos x)}{\cancel{\cos 3x + \cos x}}$$

$$\therefore \boxed{T = 2\cos 8x}$$



## 9. Efectúe

$$A = \frac{2\text{sen}20^\circ + \text{sen}40^\circ}{\text{sen}50^\circ}$$

### Resolución

Descomponiendo “2sen20°”

$$A = \frac{\text{sen}40^\circ + \text{sen}20^\circ + \text{sen}20^\circ}{\text{sen}50^\circ}$$

Recordamos:

$$\text{sen}A + \text{sen}B = 2\text{sen}\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

Aplicando la IT en el numerador:

$$A = \frac{\cancel{2\text{sen}30^\circ}^1 \cdot \text{sen}80^\circ \cdot \text{cos}10^\circ + \text{sen}20^\circ}{\text{sen}50^\circ}$$

$$A = \frac{\cancel{2\text{sen}50^\circ} \cdot \text{cos}30^\circ}{\cancel{\text{sen}50^\circ}}$$

$$A = \cancel{2\left(\frac{\sqrt{3}}{2}\right)}$$

∴

$$A = \sqrt{3}$$



- 10.** Gerald va al mercado y compra  $(3A)$  kg de fresa,  $(2B)$  kg de naranjas y  $(C)$  kg de manzanas. Si  $\text{sen}11x \cdot \cos3x - \text{sen}9x \cdot \cos5x = A \text{sen}(Bx) \cdot \cos(Cx)$ . Determine la cantidad total de frutas que compró Gerald.

### Resolución

Dando forma al 1er miembro:

$$\frac{2\text{sen}11x \cdot \cos3x - 2\text{sen}9x \cdot \cos5x}{2} = \frac{\text{sen}14x + \text{sen}8x - (\text{sen}14x + \text{sen}4x)}{2}$$

$$= \frac{\cancel{\text{sen}14x} + \text{sen}8x - \cancel{\text{sen}14x} - \text{sen}4x}{2}$$

$2\text{sen}2x \cdot \cos6x$

$$A \text{sen}(Bx) \cdot \cos(Cx) = \text{sen}2x \cdot \cos6x \quad \left. \begin{array}{l} A = 1 \\ B = 2 \\ C = 6 \end{array} \right\}$$

$$\rightarrow \text{Cantidad total} = 3(1) + 2(2) + 6$$

$$\therefore \text{Cantidad total} = 13 \text{ kg}$$