



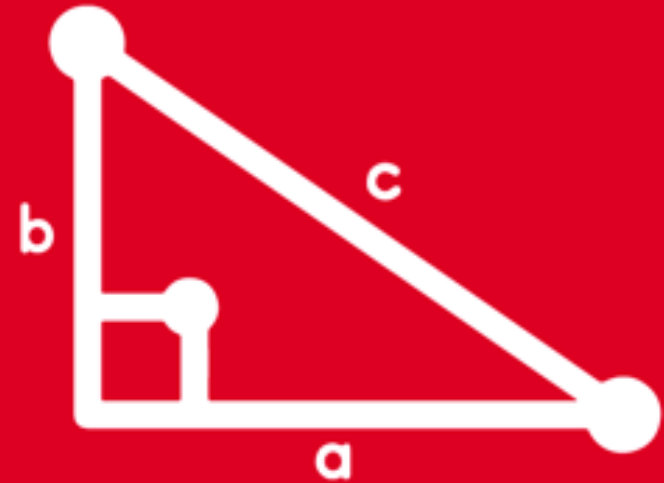
TRIGONOMETRY

Chapter 15

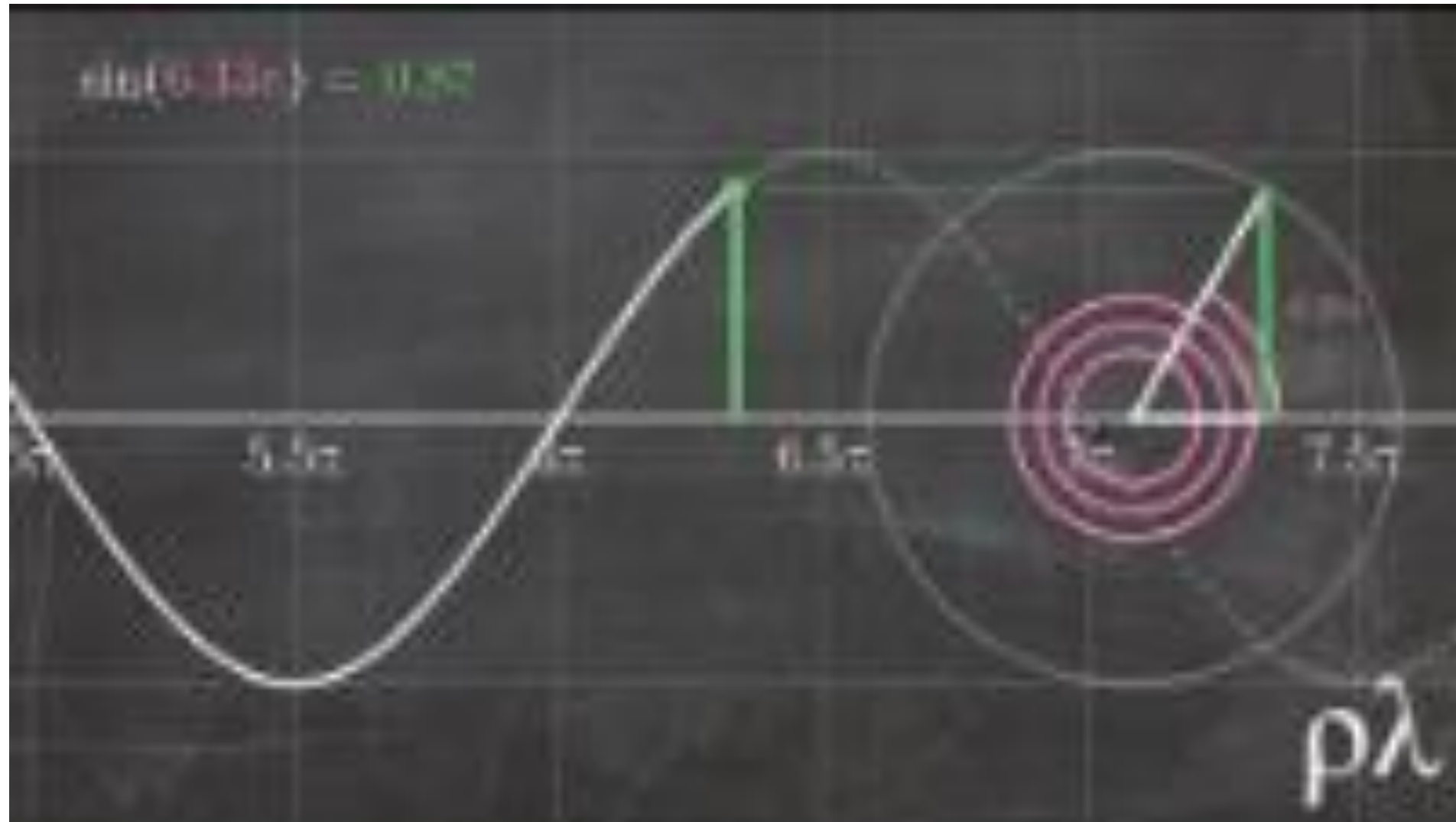
Session 2

4th
SECONDARY

CIRCUNFERENCIA
TRIGONOMETRICA II



 **SACO OLIVEROS**

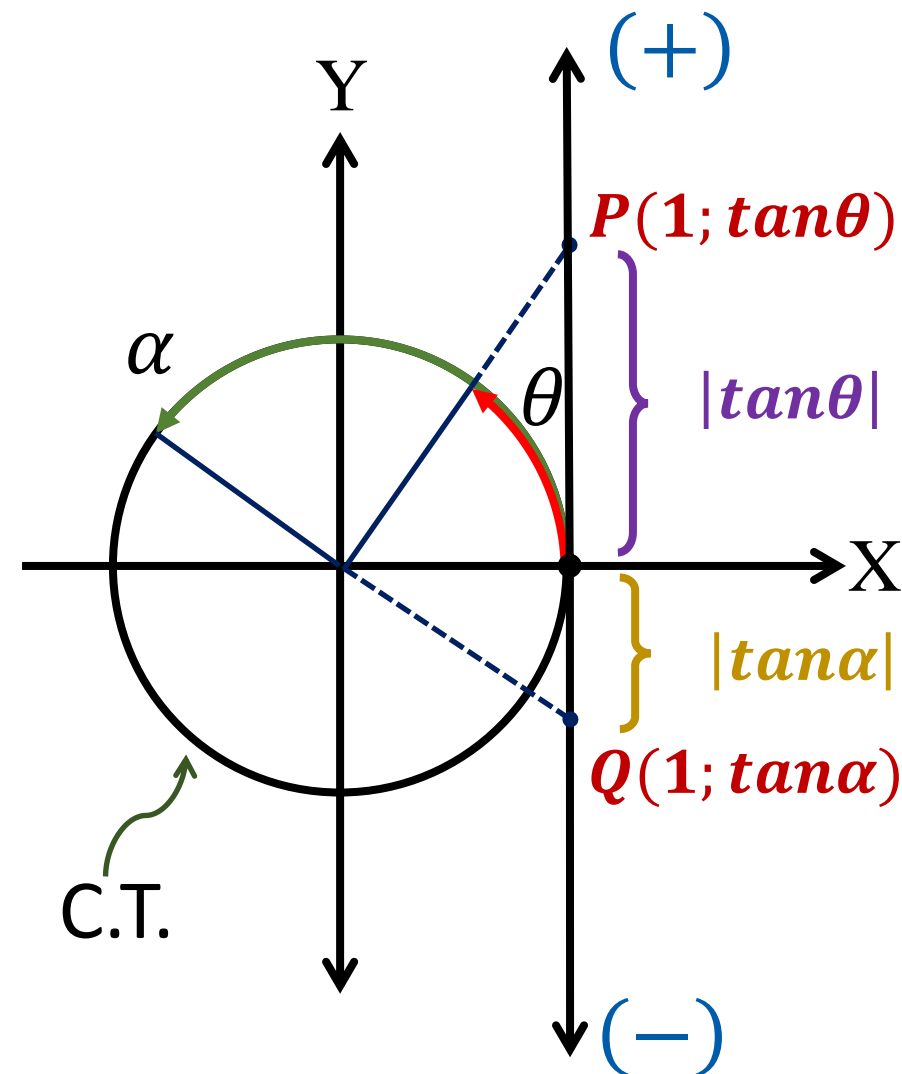


Circunferencia trigonométrica

TANGENTE: Está representada en la C.T. por la ordenada del punto de intersección entre la recta tangente que pasa por el origen de arcos y la prolongación del radio que pasa por el extremo del arco.

$$\tan \theta \in \mathbb{R}$$

$$\theta \in \mathbb{R} - \left\{ (2n + 1) \frac{\pi}{2} \right\}; n \in \mathbb{Z}$$





1. $\alpha \in$

$$\alpha = \frac{\quad}{\quad}$$

RESOLUCIÓN

$$\alpha \in IIC : \tan \alpha < 0$$

$$\rightarrow \frac{2b - 1}{4} < 0$$

$$2b - 1 < 0$$

$$b < \frac{1}{2}$$

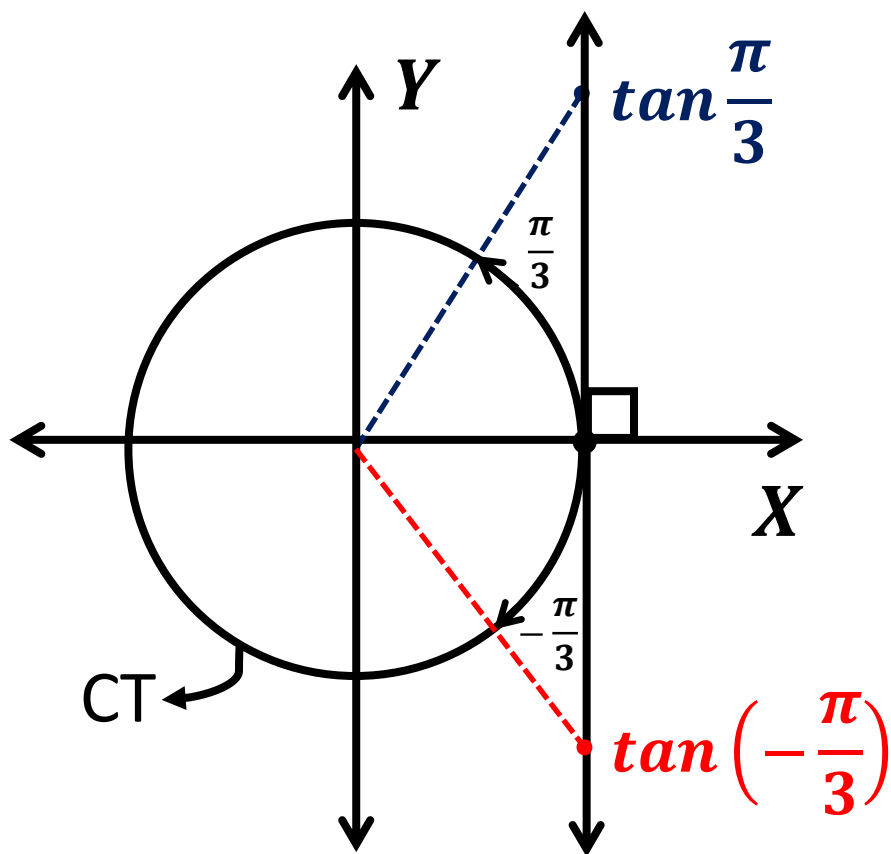


$$\therefore b \in \left\langle -\infty; \frac{1}{2} \right\rangle$$





RESOLUCIÓN



$$\beta \in \left\langle -\frac{\pi}{3} \frac{\pi}{3} \right\rangle$$

$$^2\beta +$$

Del gráfico vemos:

$$\tan\left(-\frac{\pi}{3}\right) < \tan\beta < \tan\frac{\pi}{3}$$

$$-\sqrt{3} < \tan\beta < \sqrt{3}$$

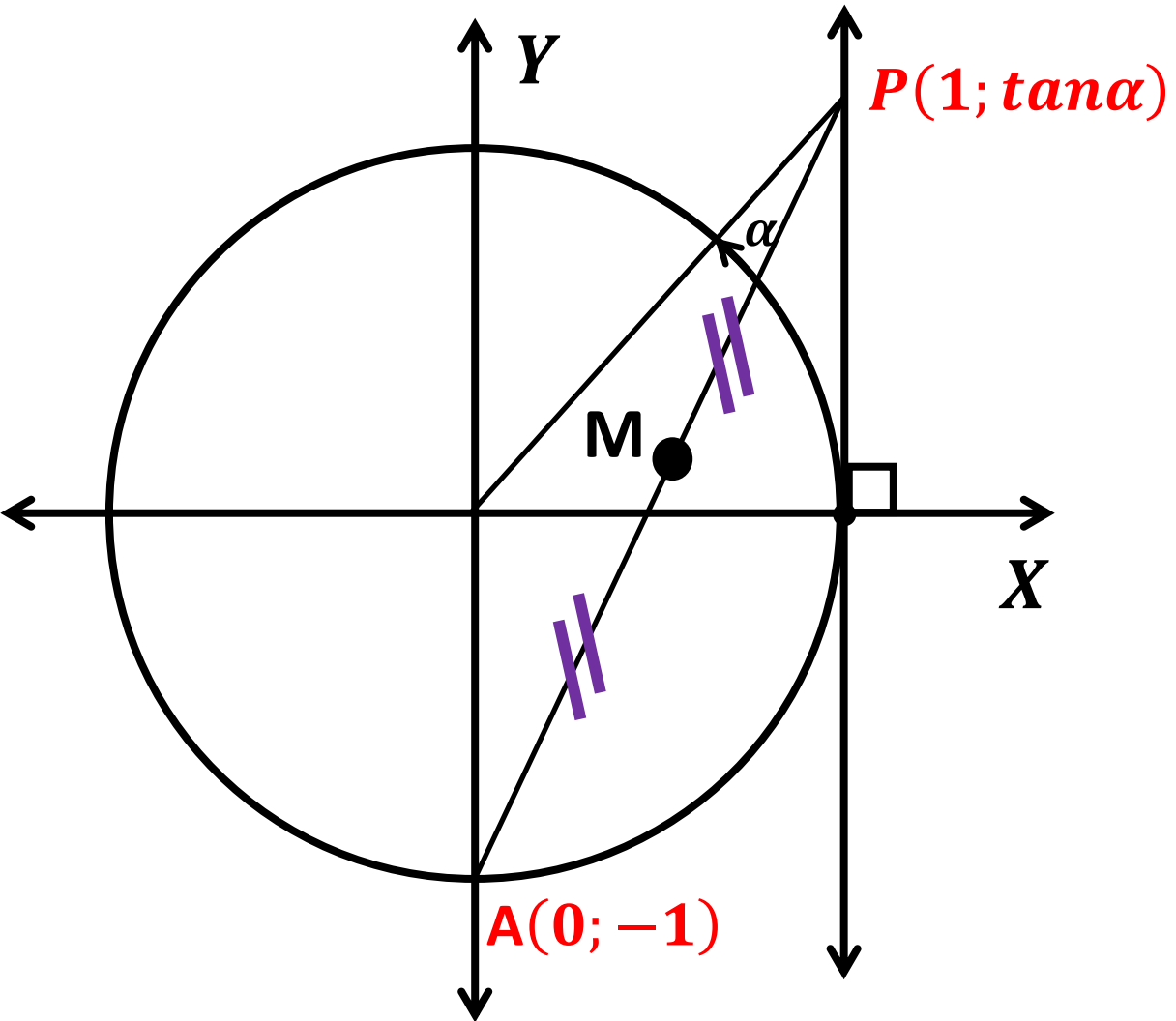
$$0 \leq \tan^2\beta < 3$$

$$0 \leq 2\tan^2\beta < 6$$

$$3 \leq 2\tan^2\beta + 3 < 9$$

$$\therefore M \in [3; 9 >$$



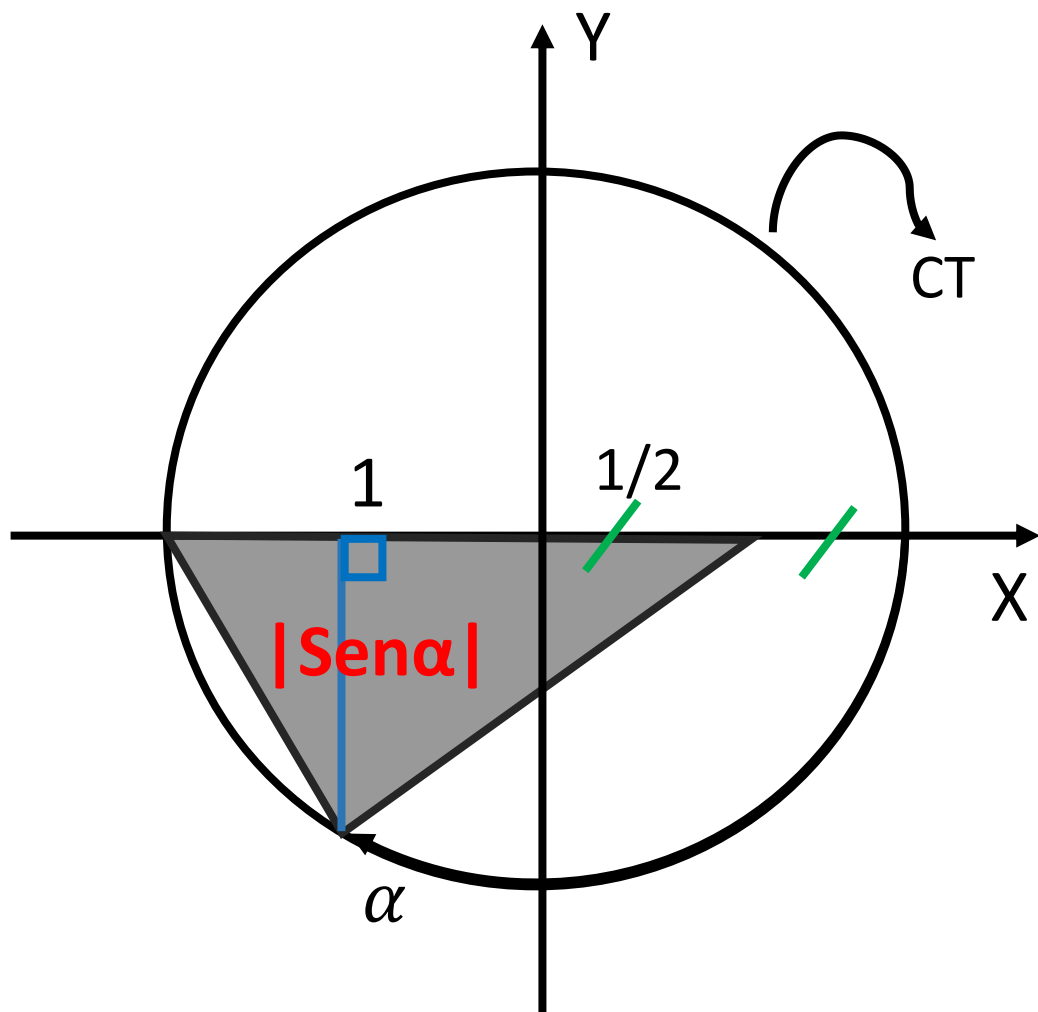
**RESOLUCIÓN**

Calculando el punto medio:

$$M \left(\frac{1 + 0}{2}; \frac{\tan \alpha + (-1)}{2} \right)$$

$$\therefore M \left(\frac{1}{2}; \frac{\tan \alpha - 1}{2} \right)$$





RESOLUCIÓN

Sea S , el área sombreada:

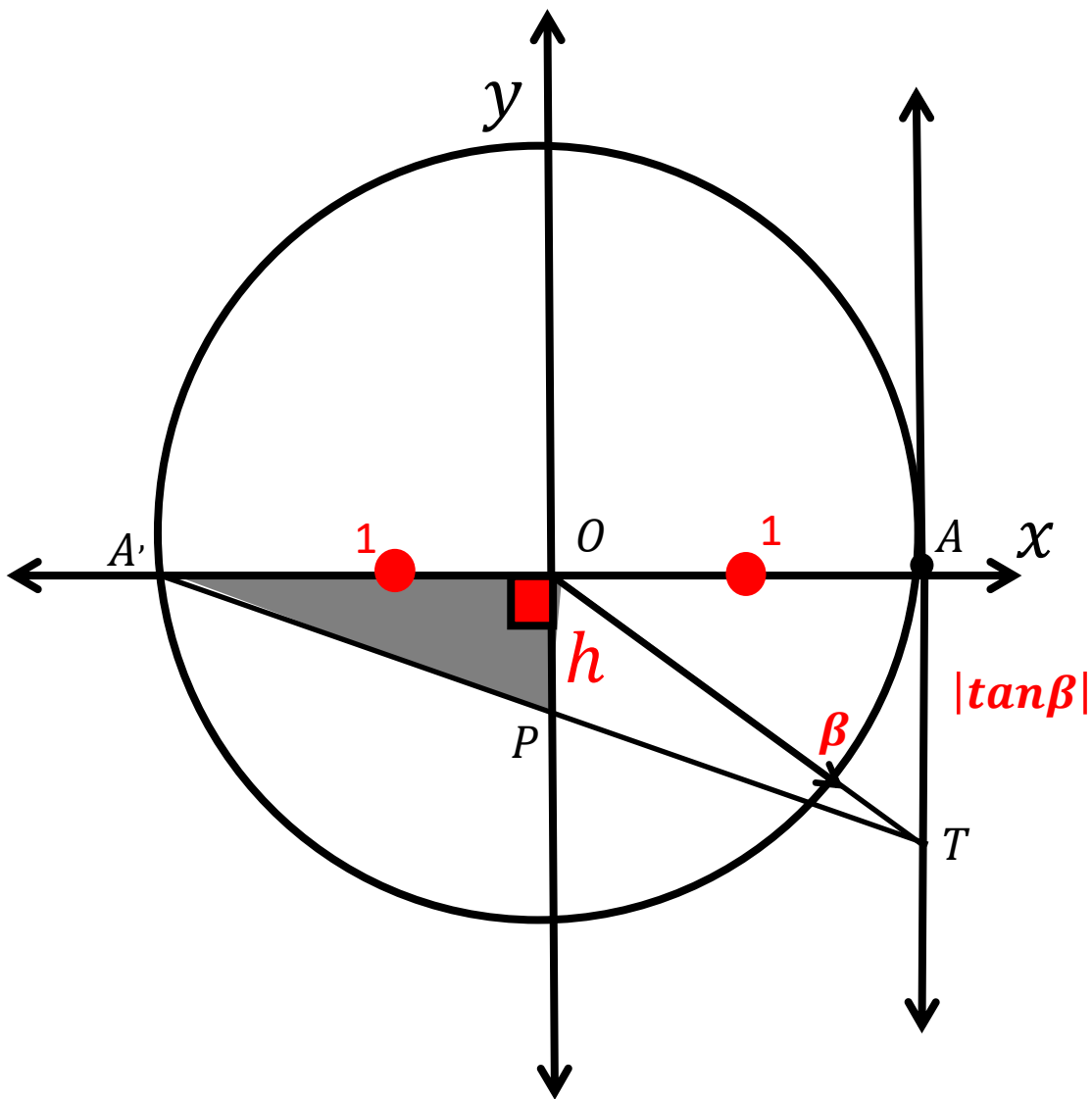
$$S = \frac{\left(1 + \frac{1}{2}\right) |\text{sen} \alpha|}{2}$$

$$S = \frac{3 |\text{sen} \alpha|}{4}$$

Ahora como $\alpha \in \text{IIIC}$  $\text{sen} \alpha: (-)$

$$\therefore S = -\frac{3 \text{sen} \alpha}{4} u^2$$





RESOLUCIÓN

$$A'O = AO = 1 \rightarrow AT = |\tan \beta|$$

$\Delta A'AT$: OP es Base Media

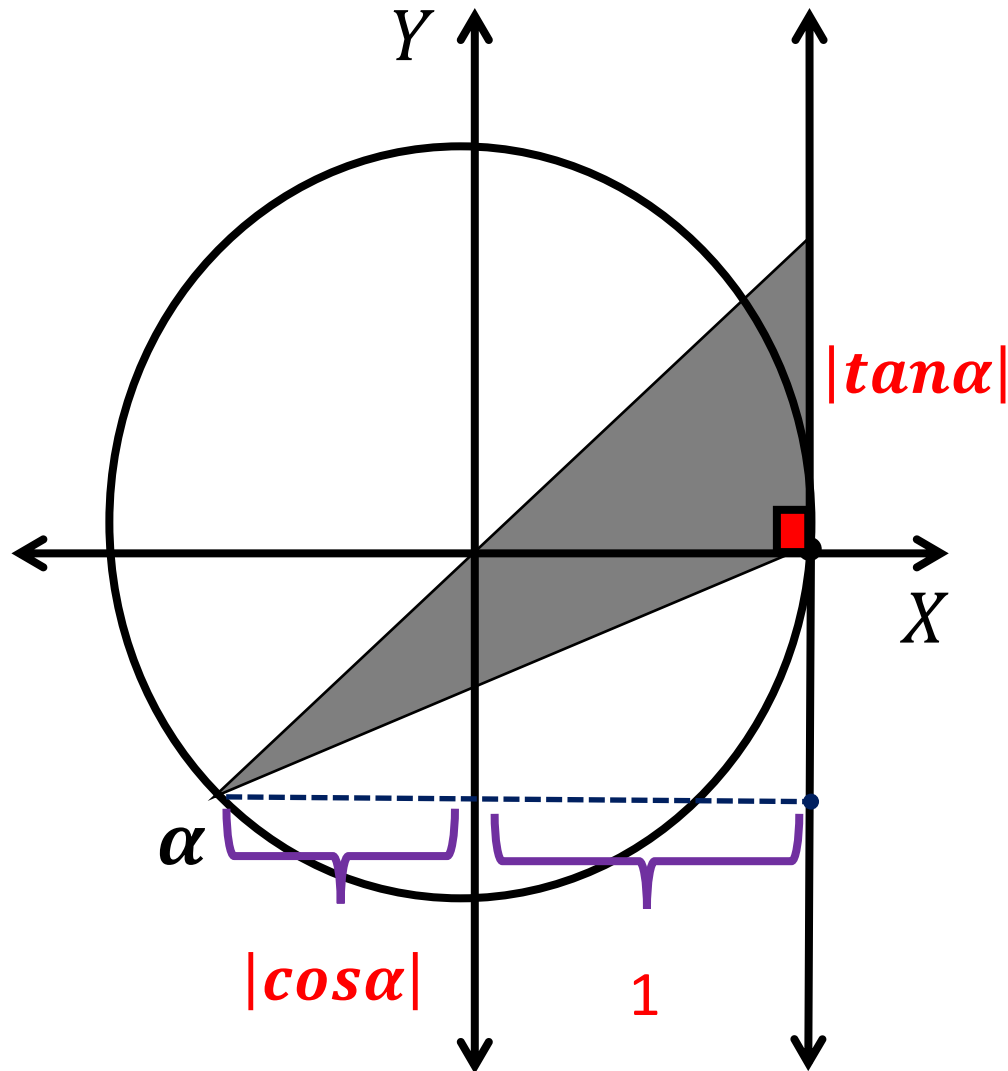
$$OP = h \rightarrow h = \frac{|\tan \beta|}{2}$$

$$Area_{\Delta A'OP} = \frac{(b)(h)}{2} = \frac{(1) \left(\frac{|\tan \beta|}{2} \right)}{2}$$

$$Area = \frac{|\tan \beta|}{4} ; \beta \in IVC$$

$$\Rightarrow |\tan \beta| = -\tan \beta$$

$$\therefore Area = \frac{-\tan \beta}{4}$$



RESOLUCIÓN

$$b = |\tan \alpha| \quad ; \quad h = 1 + |\cos \alpha|$$

$$\alpha \in \text{III}^o \rightarrow |\tan \alpha| = \tan \alpha \quad ; \quad |\cos \alpha| = -\cos \alpha$$

$$Area = \frac{(b)(h)}{2} = \frac{(\tan \alpha)(1 - \cos \alpha)}{2}$$

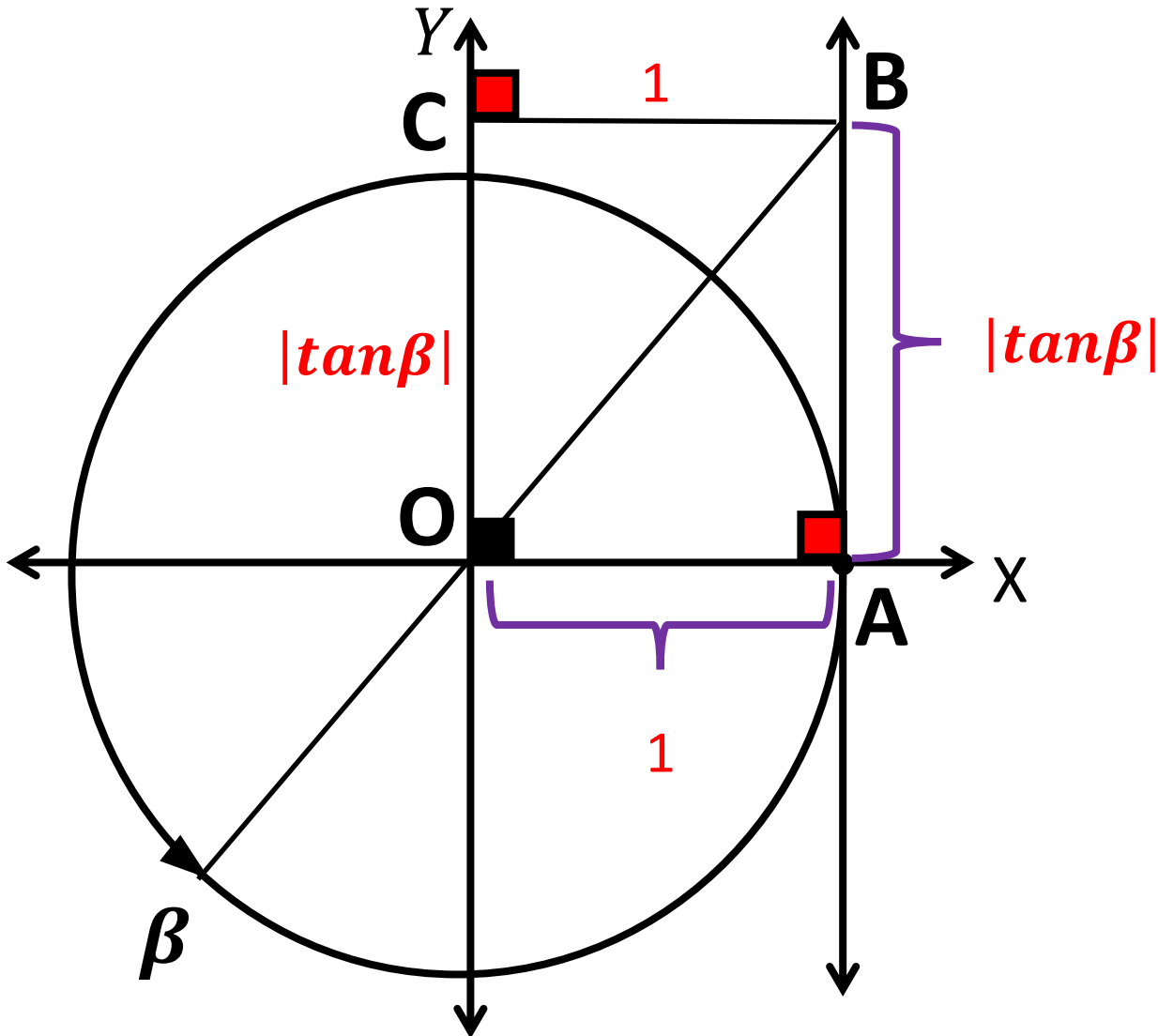
$$Area = \frac{\tan \alpha - \tan \alpha \cos \alpha}{2}$$

$$Area = \frac{\tan \alpha - \left(\frac{\text{sen} \alpha}{\cos \alpha}\right) \cos \alpha}{2}$$

$$Area = \frac{\tan \alpha - \text{sen} \alpha}{2}$$

$$\therefore Area = \frac{1}{2} (\tan \alpha - \text{sen} \alpha)$$





β

RESOLUCIÓN

$$OA = BC = 1 ; AB = |\tan \beta| ; AB = OC$$

$$\beta \in IIC \rightarrow |\tan \beta| = \tan \beta$$

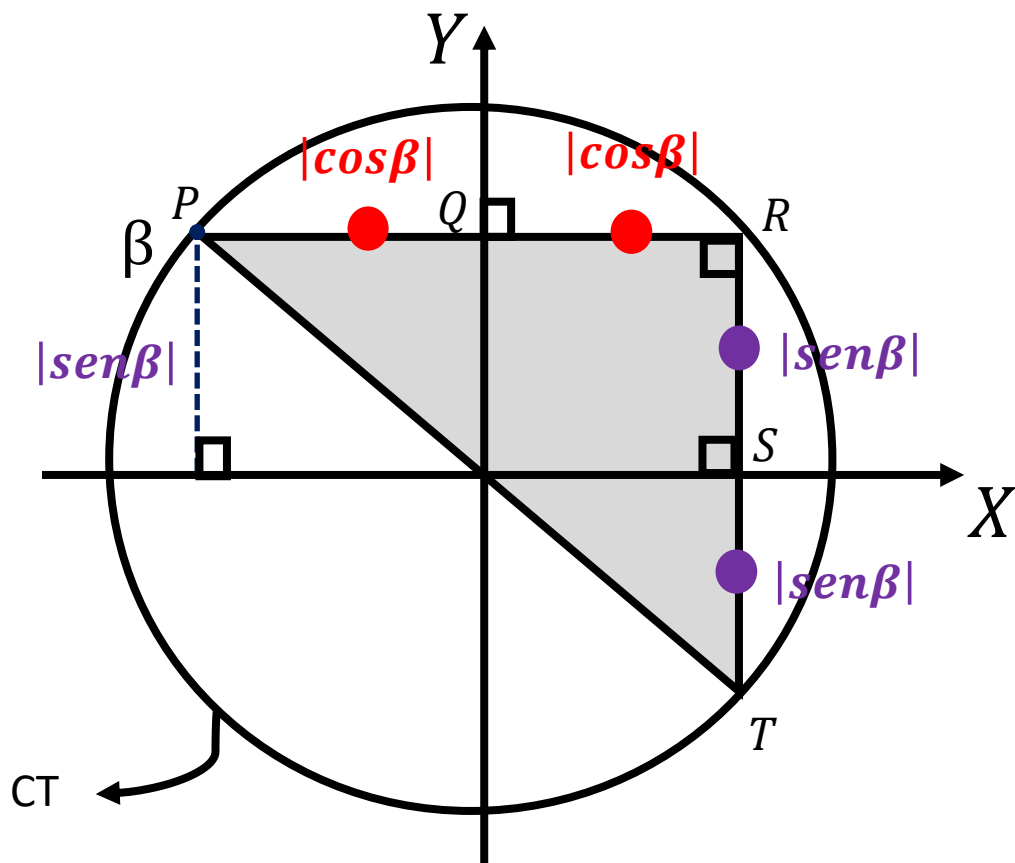
$$\text{Perimetro} = 2OA + 2AB$$

$$\text{Perimetro} = 2(1) + 2(\tan \beta)$$

$$\text{Perimetro} = 2(1 + \tan \beta)$$

$$\therefore \text{Perimetro} = 2(1 + \tan \beta)u$$





Si cada unidad de los ejes X e Y representan 1km.

- ¿Cuál es el perímetro del terreno?
- ¿Cuál es el área del terreno?

RESOLUCIÓN

Propiedades: $PQ = QR$; $RS = ST$

$\beta \in \text{IIC} \rightarrow |\cos \beta| = -\cos \beta$; $|\sen \beta| = \sen \beta$

Perimetro = $PQ + QR + RS + ST$

Perimetro = $2|\sen \beta| + 2|\cos \beta| + 2$

Perimetro = $2\sen \beta - 2\cos \beta + 2$

Perimetro = $2(\sen \beta - \cos \beta + 1)$

$\therefore \text{Perimetro} = 2(\sen \beta - \cos \beta + 1) \text{ km}$

$$\text{Area} = \frac{(b)(h)}{2} \rightarrow \text{Area} = \frac{(2|\cos \beta|)(2|\sen \beta|)}{2}$$

$$\text{Area} = 2(-\cos \beta)(\sen \beta)$$

$\therefore \text{Area} = -2\cos \beta \sen \beta \text{ km}^2$