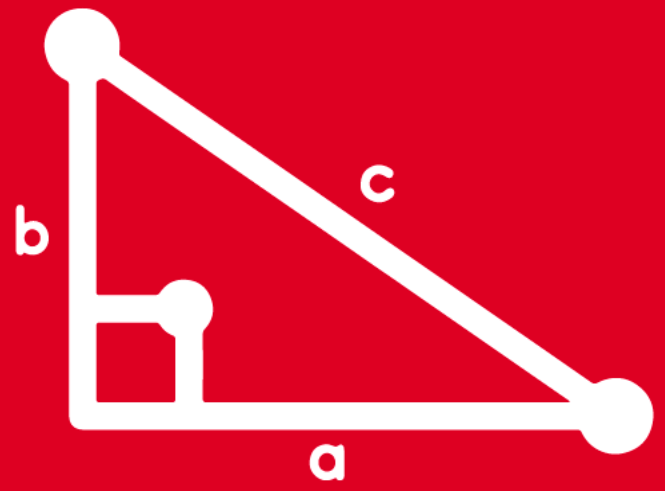
TRIGONOMETRY

Chapter 17
Session 2



4th
SECONDARY

Identidades Trigonométricas Auxiliares

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VideoScribe

Identidades Auxiliares

$$\tan x + \cot x = \sec x . \csc x$$

$$\sec^2 x + \csc^2 x = \sec^2 x . \csc^2 x$$

$$\operatorname{sen}^4 x + \cos^4 x = 1 - 2\operatorname{sen}^2 x . \cos^2 x$$

$$\operatorname{sen}^6 x + \cos^6 x = 1 - 3\operatorname{sen}^2 x . \cos^2 x$$

Identidades Auxiliares

$$(1 \pm \text{senx} \pm \text{cosx})^2 = 2(1 \pm \text{senx})(1 \pm \text{cosx})$$

$$\frac{\text{cosx}}{1 + \text{senx}} = \frac{1 - \text{senx}}{\text{cosx}}$$

$$\frac{\text{cosx}}{1 - \text{senx}} = \frac{1 + \text{senx}}{\text{cosx}}$$

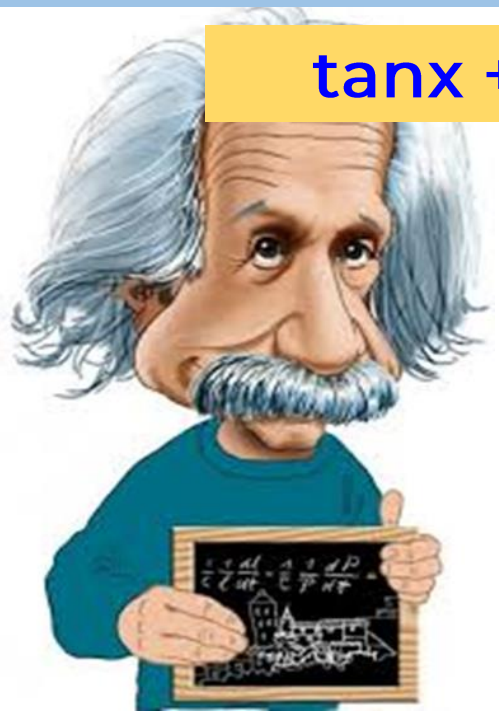
$$\frac{\text{senx}}{1 + \text{cosx}} = \frac{1 - \text{cosx}}{\text{senx}}$$

$$\frac{\text{senx}}{1 - \text{cosx}} = \frac{1 + \text{cosx}}{\text{senx}}$$

1. Si: tanx + cotx = 6
Reduzca: E = $sen^4x + cos^4x$

$sen^4x + cos^4x = 1 - 2sen^2x.cos^2x$

$tanx + cotx = secx.cscx$



RESOLUCIÓN

Por dato: $\boldsymbol{tanx + cotx = 6}$

$$secx.cscx = 6$$

Invirtiendo: $$senx.cosx = \frac{1}{6}$$

Nos piden: $\boldsymbol{E = sen^4x + cos^4x}$

$$E = 1 - 2sen^2x.cos^2x$$

$$E = 1 - 2\,(senx.cosx)^2$$

$$E = 1 - 2\left(\frac{1}{6}\right)^2 = 1 - 2\left(\frac{1}{36}\right)$$

$$\therefore \boldsymbol{E = \frac{17}{18}}$$

2. Si $\sec^2\theta + \csc^2\theta = 36$, donde $\theta \in IC$, reduzca: $E = \tan\theta + \cot\theta - 2$

$\tan x + \cot x = \sec x.\csc x$

$\sec^2 x + \csc^2 x = \sec^2 x.\csc^2 x$

RESOLUCIÓN

$\boldsymbol{Por\ dato:}\quad \boldsymbol{\sec^2\theta + \csc^2\theta = 36}$

$$\sec^2\theta.\csc^2\theta = 36$$

$$\sec\theta.\csc\theta = 6$$

Nos piden: $\mathbf{E = \tan\theta + \cot\theta - 2}$

$$E = \sec\theta.\csc\theta - 2$$

$$E = 6 - 2$$

$$\therefore \boldsymbol{E = 4}$$

3. Si tanα + cotα = 4, reduzca:
$K = \sec^2\alpha + \csc^2\alpha + 1$

$\sec^2 x + \csc^2 x = \sec^2 x.\csc^2 x$

$\tan x + \cot x = \sec x.\csc x$

RESOLUCIÓN

Por dato: $\tan\alpha + \cot\alpha = 4$

$$\sec\alpha.\csc\alpha = 4$$

Nos piden: $K = \sec^2\alpha + \csc^2\alpha + 1$

$$K = \sec^2\alpha \,.\, \csc^2\alpha + 1$$

$$K = (\sec\alpha.\csc\alpha)^2 + 1$$

$$K = (4)^2 + 1$$

$$\therefore K = 17$$

4. Si $sen\phi + cos\phi = \sqrt{\frac{3}{2}}$

reduzca: $G = tan\phi + cot\phi$

RESOLUCIÓN

Por dato: $\boldsymbol{sen\phi + cos\phi = \sqrt{\frac{3}{2}}}$

$$(sen\phi + cos\phi)^2 = \left(\sqrt{\frac{3}{2}}\right)^2$$

$$\underbrace{sen^2\phi + cos^2\phi}_{1} + 2sen\phi.cos\phi = \frac{3}{2}$$

$$1 + 2sen\phi.cos\phi = \frac{3}{2}$$

$$2sen\phi.cos\phi = \frac{1}{2}$$

$$sen\phi.cos\phi = \frac{1}{4}$$

$$sec\phi\,.csc\phi = 4$$

Nos piden: $\boldsymbol{G = tan\phi + cot\phi}$

$$G = sec\phi\,.csc\phi$$

$$G = 4$$

$$\therefore \boldsymbol{G = 4}$$

5. Si $\text{sen}^6\alpha + \cos^6\alpha = \frac{1}{3}$

reduzca $E = (1 + \text{sen}^2\alpha)(1 + \cos^2\alpha)$

RESOLUCIÓN

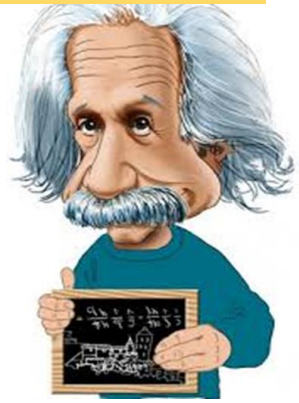
$$\textbf{Por dato: } sen^6\alpha + cos^6\alpha = \frac{1}{3}$$

$$1 - 3sen^2\alpha cos^2\alpha = \frac{1}{3}$$

$$\frac{2}{3} = 3sen^2\alpha cos^2\alpha$$

$$\frac{2}{9} = sen^2\alpha cos^2\alpha$$

$$sen^6x + cos^6x = 1 - 3sen^2x.cos^2x$$



Nos piden:

$$E = (1 + sen^2\alpha)(1 + cos^2\alpha)$$

$$E = 1 + cos^2\alpha + sen^2\alpha + sen^2\alpha cos^2\alpha$$

$$E = 1 + \quad 1 \quad + \frac{2}{9}$$

$$\therefore E = \frac{20}{9}$$

6. Si tanx + cotx = 5
reduzca: P = secx − cscx

RESOLUCIÓN

Tenemos: $\boldsymbol{tanx + cotx = 5}$

$$secx \,.\, cscx = 5$$

$$sec^2x.csc^2x = 25$$

$$\boxed{sec^2x + csc^2x = 25}$$

Nos piden:

$$\boldsymbol{P = secx - cscx}$$ . . . . . Al cuadrado.

$$P^2 = sec^2x + csc^2x - 2secx\,.\,cscx$$

$$P^2 = \quad 25 \quad - \; 2\,(5)$$

$$P^2 = \quad 15$$

$$\therefore \boldsymbol{P = \sqrt{15}}$$

7. Si $\text{sen}\alpha - \cos\alpha = \frac{\sqrt{3}}{3} - 1$

Efectúe $E = (1+\text{sen}\alpha)(1-\cos\alpha)$

$$(1+\text{sen}x-\cos x)^2 = 2(1+\text{sen}x)(1-\cos x)$$

RESOLUCIÓN

Tenemos: $E = (1 + sen\alpha)(1 - cos\alpha)$

$$2E = 2(1 + sen\alpha)(1 - cos\alpha)$$

$$2E = (1 + sen\alpha - cos\alpha)^2$$

$$2E = (1 + \frac{\sqrt{3}}{3} - 1)^2$$

$$2E = \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{3}{9}$$

$$2E = \frac{1}{3}$$

$$\therefore E = \frac{1}{6}$$

8. El gasto diario de Kelly en pasaje es s/Bcotx.¿Cuál será el gasto total de la semana? Para ello resuelva lo siguiente:

$$B = \left(\frac{cosx}{1+senx} + \frac{cosx}{1-senx}\right) senx$$

$$\frac{cosx}{1+senx} = \frac{1-senx}{cosx}$$

$$\frac{cosx}{1-senx} = \frac{1+senx}{cosx}$$

RESOLUCIÓN

Por dato:

$$\boldsymbol{B = \left(\frac{cosx}{1+senx} + \frac{cosx}{1-senx}\right) senx}$$

$$B = \left(\frac{1-\cancel{senx}}{cosx} + \frac{1+\cancel{senx}}{cosx}\right) senx$$

$$B = \left(\frac{2}{cosx}\right) senx \Rightarrow B = 2\ tanx$$

El gasto diario será: $\boldsymbol{B}.cotx$

$$= 2\underbrace{tanx\,.\,cotx}_{1} = 2\ soles$$

∴ *El gasto semanal*: 14 soles