ALGEBRA

Chapter 14

3th

SECONDARY

Radicación

SACO OLIVEROS

MOTIVATING STRATEGY

¿Puedes ordenar de menor a mayor las siguientes expresiones?

$$\sqrt{5} \; ; \; \sqrt[3]{3} \; ; \; \sqrt[6]{2}$$

$$\sqrt[6]{2} \quad ; \quad \sqrt[6]{9} \quad ; \quad \sqrt[6]{125}$$

HELICO THEORY

RADICACIÓN

Es la **operación** *matemática en la cual, dada una variable real "x" y un número natural "n", existe un tercer número "r" llamado raíz, siempre que:*

$$\boxed{\sqrt[n]{x} = r \iff r^n = x}$$

n: *índice*

$(n \in \mathbb{N} \;\; ; \;\; n \geq 2)$

PROPIEDADES

1) $\sqrt[n]{a.b} = \sqrt[n]{a}.\sqrt[n]{b}$

2) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

3) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m.n]{a}$

¡Recuerda!

$\sqrt[2n]{(+)} = (+)$

$\sqrt[2n]{(-)} = \nexists \text{ en } \mathbb{R}$

$\sqrt[2n+1]{(+)} = (+)$

$\sqrt[2n+1]{(-)} = (-)$

Extraer un factor de un radical:

Ejemplo:

$* \quad \sqrt{180} = \sqrt{36.5}$

$\sqrt{180} = \sqrt{36}.\sqrt{5}$

$\boxed{\sqrt{180} = 6\sqrt{5}}$

Ejemplo:

$* \quad \sqrt[5]{a^{10}b^{15}c^{2}} = \sqrt[5]{a^{10}}.\sqrt[5]{b^{15}}\ \sqrt[5]{c^{2}}$

$\boxed{\sqrt[5]{a^{10}b^{15}c^{2}} = a^{2}\quad \sqrt[5]{c^{2}}}$

CLASIFICACIÓN DE LOS RADICALES:

- **Radicales Heterogéneos:**

Ejm.: $\sqrt[3]{2}$; $\sqrt[5]{7}$; $\sqrt{5}$

- **Radicales Homogéneos:**

Ejm.: $\sqrt[5]{9}$; $\sqrt[5]{8}$; $\sqrt[5]{7}$

- **Radicales Semejantes:**

Ejm.: $5\sqrt[3]{2}$; $6\sqrt[3]{2}$; $2\sqrt[3]{2}$

HOMOGENIZACIÓN DE RADICALES:

$\sqrt[4]{2}$; $\sqrt{5}$; $\sqrt[6]{3}$

$$mcm(4;2;6) = 12$$

$\sqrt[4.3]{2^{1.3}}$; $\sqrt[2.6]{5^{1.6}}$; $\sqrt[6.2]{3^{1.2}}$

$\sqrt[12]{8}$; $\sqrt[12]{15625}$; $\sqrt[12]{9}$

TRANSFORMACIÓN DE RADICALES DOBLES A RADICALES SIMPLES

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A+C}{2}} \pm \sqrt{\frac{A-C}{2}}$$

$$C = \sqrt{A^2 - B}$$

Ejemplo.: *Transforme a radicales simples* $\sqrt{3+\sqrt{5}}$

Resolución: $C = \sqrt{3^2 - 5} = \sqrt{4} = 2$

$$\sqrt{3+\sqrt{5}} = \sqrt{\frac{3+2}{2}} + \sqrt{\frac{3-2}{2}} = \sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}$$

Método práctico:

$$\sqrt{A \pm \sqrt{B}} = \sqrt{(x+y) \pm 2\sqrt{x.y}} = \sqrt{x} \pm \sqrt{y} \qquad (x > y)$$

Ejemplo. *Transforme a radicales simples* $\sqrt{5+\sqrt{24}}$

Resolución:

$$\sqrt{5+\sqrt{24}} = \sqrt{5+\sqrt{4.6}}$$

$$= \sqrt{\underbrace{5}_{3+2} + 2\underbrace{\sqrt{6}}_{3\times 2}}$$

$$\boxed{\sqrt{5+\sqrt{24}} = \sqrt{3}+\sqrt{2}}$$

RACIONALIZACIÓN

Caso I:

$$\boxed{\frac{N}{\sqrt[n]{a^m}}}$$

$$\frac{N}{\sqrt[n]{a^m}} = \frac{N}{\sqrt[n]{a^m}} \times \frac{\sqrt[n]{a^{n-m}}}{\sqrt[n]{a^{n-m}}}$$

$$\frac{N}{\sqrt[n]{a^m}} = \frac{N\sqrt[n]{a^{n-m}}}{a}$$

Ejemplo. Racionalizar $\frac{12}{\sqrt[3]{2}}$

Resolución:

$$\frac{12}{\sqrt[3]{2}} = \frac{12}{\sqrt[3]{2}} \times \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$$

$$= \frac{12.\sqrt[3]{4}}{2}$$

$$\boxed{\frac{12}{\sqrt[3]{2}} = 6\sqrt[3]{4}}$$

Caso II:

$$\boxed{\frac{N}{\sqrt{a} \pm \sqrt{b}}}$$

$$\frac{N}{\sqrt{a} \pm \sqrt{b}} = \frac{N}{\sqrt{a} \pm \sqrt{b}} \cdot \frac{\sqrt{a} \mp \sqrt{b}}{\sqrt{a} \mp \sqrt{b}}$$

$$\frac{N}{\sqrt{a} \pm \sqrt{b}} = \frac{N\left(\sqrt{a} \mp \sqrt{b}\right)}{a - b}$$

Ejemplo. Racionalizar $\frac{7}{\sqrt{5}+\sqrt{2}}$

Resolución:

$$\frac{7}{\sqrt{5} + \sqrt{2}} = \frac{7}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$= \frac{7\left(\sqrt{5} - \sqrt{2}\right)}{5 - 2}$$

$$\boxed{\frac{7}{\sqrt{5} + \sqrt{2}} = \frac{7\left(\sqrt{5} - \sqrt{2}\right)}{3}}$$

HELICO PRACTICE

Problema 1

Efectúe

$$M = \frac{\sqrt{8}+\sqrt{32}+\sqrt{128}}{\sqrt{50}-\sqrt{18}}$$

Resolución:

$$M = \frac{\sqrt{8}+\sqrt{32}+\sqrt{128}}{\sqrt{50}-\sqrt{18}}$$

$$M = \frac{\sqrt{4.2}+\sqrt{16.2}+\sqrt{64.2}}{\sqrt{25.2}-\sqrt{9.2}}$$

$$M = \frac{2\sqrt{2}+4\sqrt{2}+8\sqrt{2}}{5\sqrt{2}-3\sqrt{2}}$$

$$M = \frac{14\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}}$$

$$\boxed{\therefore\ M = 7}$$

Problema 2

Simplifique:

$$E=\sqrt[n]{\sqrt{2}+1}.\sqrt[2n]{3-2\sqrt{2}}$$

Resolución:

$$E=\sqrt[2n]{\left(\sqrt{2}+1\right)^{2}}.\sqrt[2n]{3-2\sqrt{2}}$$

$$E=\sqrt[2n]{3+2\sqrt{2}}.\sqrt[2n]{3-2\sqrt{2}}$$

$$E=\sqrt[2n]{\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)}$$

$$E=\sqrt[2n]{3^{2}-\left(2\sqrt{2}\right)^{2}}$$

$$E=\sqrt[2n]{9-4.2}$$

$$E=\sqrt[2n]{1}$$

$$\boxed{\therefore\quad E=1}$$

Problema 3

Calcule:

$$M = \frac{5\sqrt{32} - \sqrt{50} + \sqrt{2}}{4\sqrt{8} - \sqrt{2}}$$

Resolución:

$$M = \frac{5\sqrt{32} - \sqrt{50} + \sqrt{2}}{4\sqrt{8} - \sqrt{2}}$$

$$M = \frac{5\sqrt{16}\sqrt{2} - \sqrt{25}\sqrt{2} + \sqrt{2}}{4\sqrt{4}\sqrt{2} - \sqrt{2}}$$

$$M = \frac{5 . 4\sqrt{2} - 5\sqrt{2} + \sqrt{2}}{4 . 2\sqrt{2} - \sqrt{2}}$$

$$M = \frac{20\sqrt{2} - 5\sqrt{2} + \sqrt{2}}{8\sqrt{2} - \sqrt{2}}$$

$$M = \frac{16\cancel{\sqrt{2}}}{7\cancel{\sqrt{2}}}$$

$$\boxed{\therefore \quad M = \frac{16}{7}}$$

Problema 4

Reduzca

$$P=\sqrt{5+2\sqrt{6}}-\sqrt{7+2\sqrt{10}}+\sqrt{8-2\sqrt{15}}$$

Resolución:

$$P=\sqrt{5+2\sqrt{6}}-\sqrt{7+2\sqrt{10}}+\sqrt{8-2\sqrt{15}}$$

$3+2 \quad 3\times 2 \quad 5+2 \quad 5\times 2 \quad 5+3 \quad 5\times 3$

$$P=\sqrt{3}+\sqrt{2}-\left(\sqrt{5}+\sqrt{2}\right)+\sqrt{5}-\sqrt{3}$$

$$P=\cancel{\sqrt{3}}+\cancel{\sqrt{2}}-\cancel{\sqrt{5}}-\cancel{\sqrt{2}}+\cancel{\sqrt{5}}-\cancel{\sqrt{3}}$$

$$\boxed{\therefore\ P=0}$$

Problema 5

*El valor **reducido** de*

$$E=\left(\sqrt{9-2\sqrt{20}}+\sqrt{7-2\sqrt{12}}+\sqrt{3}\right).\sqrt{5}$$

es el costo por kilo de naranjas. ¿Cuánto costará 60 kilos de naranjas?

Resolución:

$$E=\left(\sqrt{9-2\sqrt{20}}+\sqrt{7-2\sqrt{12}}+\sqrt{3}\right).\sqrt{5}$$

$5+4 \quad 5\times4 \quad 4+3 \quad 4\times3$

$$E=\left(\sqrt{5}-\cancel{\sqrt{4}}+\cancel{\sqrt{4}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}\right).\sqrt{5}$$

$$E=\left(\sqrt{5}\right).\sqrt{5}$$

$\boxed{E=5}$ → $1\ kg$ *de naranjas cuesta* $S/.\,5$

∴ *60 Kg de naranjas costarán* $S/.\,300$

Problema 6

Determine *el valor* ***racionalizado*** *de*

$$\frac{5}{\sqrt[7]{5}}$$

Resolución:

$$\frac{5}{\sqrt[7]{5}} = \frac{5}{\sqrt[7]{5}} \times \frac{\sqrt[7]{5^{7-1}}}{\sqrt[7]{5^{7-1}}}$$

$$\frac{5}{\sqrt[7]{5}} = \frac{5\sqrt[7]{5^6}}{\sqrt[7]{5}.\sqrt[7]{5^6}}$$

$$\frac{5}{\sqrt[7]{5}} = \frac{5\sqrt[7]{5^6}}{\sqrt[7]{5^7}}$$

$$\frac{5}{\sqrt[7]{5}} = \frac{\cancel{5}\sqrt[7]{5^6}}{\cancel{5}}$$

$$\therefore \quad \boxed{\frac{5}{\sqrt[7]{5}} = \sqrt[7]{5^6}}$$

Problema 7

Obtenga el denominador después de racionalizar y reducir la expresión

$$F = \frac{7}{\sqrt{7}} + \frac{14}{\sqrt{14}} - \frac{11}{\sqrt{11}}$$

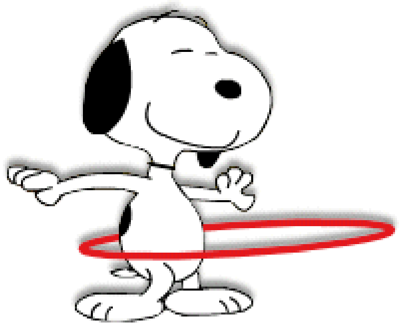
Resolución:

$$F = \frac{7}{\sqrt{7}} + \frac{14}{\sqrt{14}} - \frac{11}{\sqrt{11}}$$

$$F = \frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} + \frac{14}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} - \frac{11}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}$$

$$F = \frac{\cancel{7}\sqrt{7}}{\cancel{7}} + \frac{\cancel{14}\sqrt{14}}{\cancel{14}} - \frac{\cancel{11}\sqrt{11}}{\cancel{11}}$$

$$\boxed{F = \sqrt{7} + \sqrt{14} - \sqrt{11}}$$



$\therefore$ *El denominador es* 1

Problema 8

Efectúe

$$Q=\frac{4}{\sqrt{7}-\sqrt{3}}+\frac{2}{3+\sqrt{7}}-\sqrt{3}$$

Resolución:

$$Q=\frac{4}{\sqrt{7}-\sqrt{3}}+\frac{2}{3+\sqrt{7}}-\sqrt{3}$$

$$Q=\frac{4}{(\sqrt{7}-\sqrt{3})}\times\frac{(\sqrt{7}+\sqrt{3})}{(\sqrt{7}+\sqrt{3})}+\frac{2}{(3+\sqrt{7})}\times\frac{(3-\sqrt{7})}{(3-\sqrt{7})}-\sqrt{3}$$

$$Q=\frac{4(\sqrt{7}+\sqrt{3})}{7-3}+\frac{2(3-\sqrt{7})}{9-7}-\sqrt{3}$$

$$Q=\frac{\cancel{4}(\sqrt{7}+\sqrt{3})}{\cancel{4}}+\frac{\cancel{2}(3-\sqrt{7})}{\cancel{2}}-\sqrt{3}$$

$$Q=\cancel{\sqrt{7}}+\cancel{\sqrt{3}}+3-\cancel{\sqrt{7}}-\cancel{\sqrt{3}}$$

$$\boxed{\therefore\quad Q=3}$$


Que tengas un
lindo dia
jolmarg