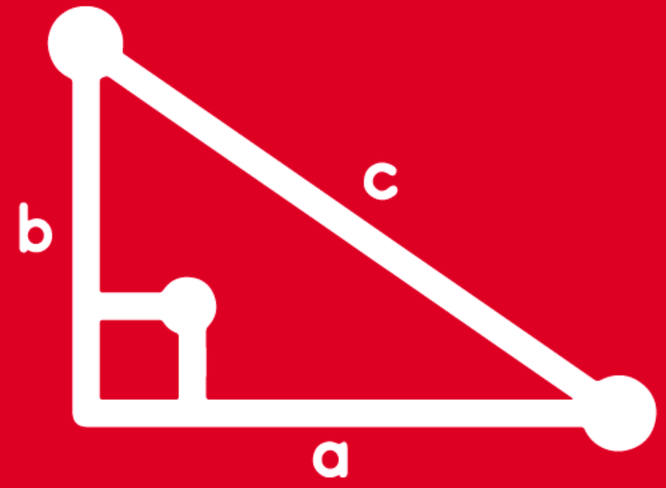
TRIGONOMETRY

Tomo 7 y 8
Session II



4nd
SECONDARY

Advisory

SACO OLIVEROS

1) Si cotθ + tanθ = 5 ;

calcule K = sec²2θ

RESOLUCIÓN:

Recordar:

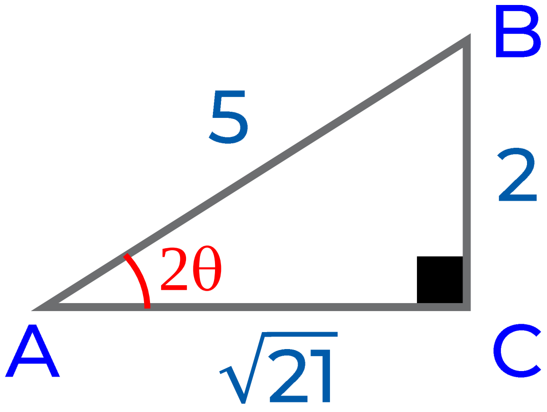
$$\boxed{\cot\theta + \tan\theta = 2\csc 2\theta}$$

Tenemos como ***dato***:

$$\underbrace{\cot\theta + \tan\theta}_{2\csc 2\theta} = 5$$

$$2\csc 2\theta = 5$$

$$\Rightarrow \csc 2\theta = \frac{5}{2}$$



Teorema de pitágoras: AC $= \sqrt{21}$

Nos piden:

$$K = \sec^2 2\theta = \frac{5^2}{\sqrt{21}^2}$$

$$\therefore \boxed{K = \frac{25}{21}}$$

2) Reducir:

$$G = \frac{1 - \cos 2\alpha + \operatorname{sen}\alpha}{\operatorname{sen}2\alpha + \cos\alpha}$$

RESOLUCIÓN:

Recordamos

$$2sen^2x = 1 - cos2x$$

$$\operatorname{sen}2x = 2\, senx\, cosx$$

Tenemos :

$$G = \frac{1 - cos2\alpha + sen\alpha}{sen2\alpha + cos\alpha}$$

$$G = \frac{2sen^2\alpha + sen\alpha}{2\ \operatorname{sen}\alpha\ \cos\alpha + cos\alpha}$$

$$G = \frac{\operatorname{sen}\alpha\,(\cancel{2sen\alpha + 1})}{cos\alpha\,(\cancel{2sen\alpha + 1})}$$

$$G = \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

$$\therefore\ \boxed{G = tan\alpha}$$

3) Si $8sen\alpha - 2 = 0$;

calcule: $E = 32sen3\alpha$

RESOLUCIÓN:

Del dato tenemos: $8sen\alpha - 2 = 0$

$$8sen\alpha = 2$$

$$sen\alpha = \frac{1}{4}$$

Recordar:

$$\boxed{\boldsymbol{sen3\alpha = 3sen\alpha - 4sen^3\alpha}}$$

Así tenemos :

$$sen3\alpha = 3\left(\frac{1}{4}\right) - 4\left(\frac{1}{4}\right)^3$$

$$sen3\alpha = \frac{3}{4} - \frac{1}{16} \Rightarrow sen3\alpha = \frac{11}{16}$$

Nos piden: $\mathrm{E} = 32\mathrm{sen}3\alpha$

$$\mathrm{E} = \overset{2}{\cancel{32}} \cdot \frac{11}{\underset{1}{\cancel{16}}}$$

$$\therefore \boxed{\boldsymbol{E = 22}}$$

4)Si $\tan\theta = \dfrac{2}{3}$; calcule $\tan 3\theta$

RESOLUCIÓN:

Recordar:

$$\boldsymbol{\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}}$$

$$\tan 3\theta = \frac{3.\left(\frac{2}{3}\right) - \left(\frac{2}{3}\right)^3}{1 - 3\left(\frac{2}{3}\right)^2}$$

$$\tan 3\theta = \frac{2 - \left(\frac{8}{27}\right)}{1 - \left(\frac{4}{3}\right)} = \frac{\frac{46}{27}}{\frac{-1}{3}}$$

$$\tan 3\theta = \frac{46 \,.\, \overset{1}{\cancel{3}}}{\underset{9}{\cancel{27}}\,(-1)}$$

$$\therefore \quad \boxed{\boldsymbol{\tan 3\theta = -\frac{46}{9}}}$$

5) Reducir:

E = 8sen20°cos10°− 4cos80°

RESOLUCIÓN:

Recordar

$$2\,\mathrm{sen}x\,\cos y = \mathrm{sen}(x+y) + \mathrm{sen}(x-y)$$

Tenemos:

$$E = 8sen20°cos10° - 4cos80°$$

$$E = 4(\underbrace{2\mathrm{sen}20°\cos10°}\ - \cos80°)$$

$$E = 4(\mathrm{sen}30° + \cancel{\mathrm{sen}10°} \cancel{- \cos80°})$$

$$E = 4\left(\frac{1}{2}\right)$$

$$\therefore \boxed{E = 2}$$

6) Calcule $T_1 + T_2$, siendo T_1 y T_2 los periodos de las funciones f(x) y g(x) respectivamente, donde:

$$f(x) = 6\cos(5x)$$

$$g(x) = 8\operatorname{sen}\left(\frac{2x}{3}\right)$$

RESOLUCIÓN:

Recordar: $\boxed{T = \frac{2\pi}{B}}$

Para f y g, tenemos:

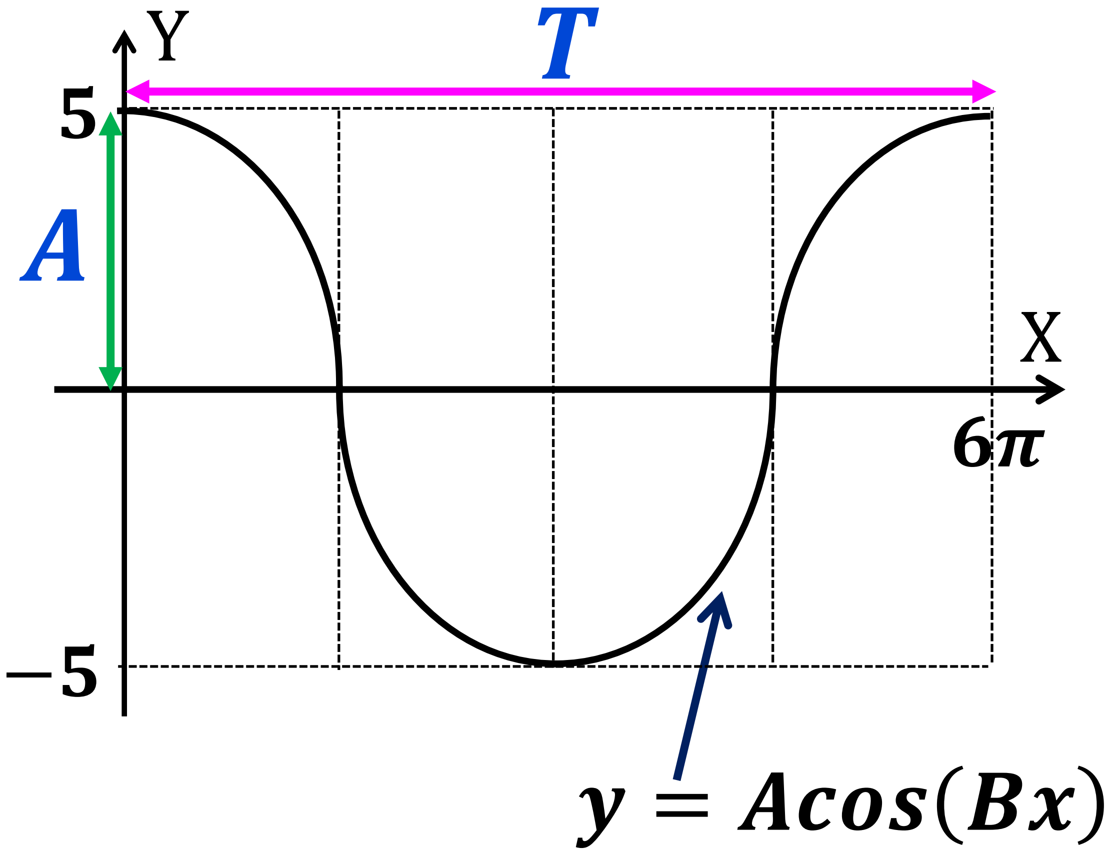
$$T_1 = \frac{2\pi}{5} \quad y \quad T_2 = \frac{\cancel{2}\pi}{\frac{\cancel{2}}{3}} = 3\pi$$

Nos piden:

$$T_1 + T_2 = \frac{2\pi}{5} + 3\pi$$

$$\therefore \boxed{T_1 + T_2 = \frac{17\pi}{5}}$$

7)Del gráfico; calcule A + B



RESOLUCIÓN:

$$\textit{Del gráfico: } A = 5 \ \ y \ \ T = 6\pi$$

$$\textit{Periodo: } \frac{2\cancel{\pi}}{B} = 6\cancel{\pi} \Rightarrow \frac{1}{3} = B$$

$$\textit{Nos piden: } A + B = 5 + \frac{1}{3}$$

$$\therefore \boxed{A + B = \frac{16}{3}}$$

8) En un triángulo ABC:

m∠C = 74° y a = 3b.

Calcule: $\tan\left(\frac{A-B}{2}\right)$

RESOLUCIÓN:

Del dato: $\boxed{a = 3b}$

Teorema de tangentes

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

$$A + B + \underbrace{C}_{74°} = 180° \Rightarrow A + B = 106°$$

Reemplazando valores:

$$\frac{3b-b}{3b+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{106°}{2}\right)}$$

$$\Rightarrow \frac{1}{2}\tan(53°) = \tan\left(\frac{A-B}{2}\right)$$

Así: $\frac{1}{2}\cdot\frac{4}{3} = \tan\left(\frac{A-B}{2}\right)$

$$\therefore \boxed{\tan\left(\frac{A-B}{2}\right) = \frac{2}{3}}$$

9)En un triángulo ABC de lados a, b y c y circunradio R; simplifique:

M=2RsenA+c.cos(A+C)+b.cos(A+B)

RESOLUCIÓN:

Recordar: $2R\mathrm{sen}A = a$

$a = c.\cos B + b.\cos C$

Tenemos:

$$M = 2R\mathrm{sen}A + c.\cos(A + C) + b.\cos(A + B)$$

Dato: A+B+C = 180°

$$\mathbf{M} = 2R\mathrm{sen}A + c.\cos(180° - B) + b.\cos(180° - C)$$

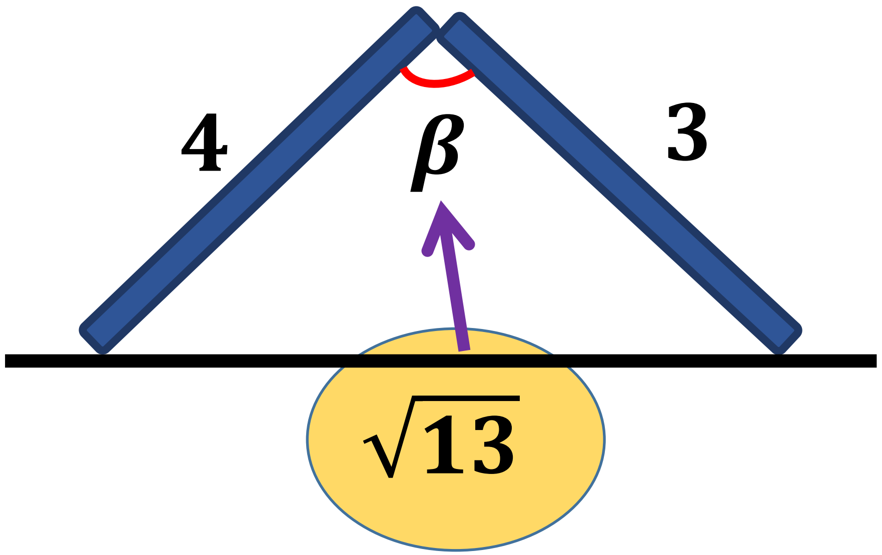
$$M = a - c.\cos(B) - b.\cos(C)$$

$$M = a - \underbrace{(c.\cos B + b.\cos C)}_{\mathbf{a}}$$

$$M = a - a$$

$$\therefore \boxed{\mathbf{M = 0}}$$

10)Dos barras metálicas se encuentran apoyadas, tal como se muestra en la figura. Si el ángulo que forman las barras en su punto de apoyo es β, calcule $\boldsymbol{sec\beta}$.



RESOLUCIÓN:

Ley de cosenos:

$$\boxed{b^2 = a^2 + c^2 - 2ac.\cos B}$$

$$\sqrt{13}^2 = 4^2 + 3^2 - 2.4.3\cos\beta$$

$$13 = 16 + 9 - 24\cos\beta$$

$$24\cos\beta = 25 - 13$$

$$24\cos\beta = 12$$

$$\cos\beta = \frac{1}{2}$$

$$\therefore \boxed{\boldsymbol{sec\beta = 2}}$$