



TRIGONOMETRY

Tomo 7 y 8

5th
SECONDARY

ADVISORY



 **SACO OLIVEROS**



HELICOPRACTICE 1

Reduzca la expresión: $K = \frac{\text{sen}11x + \text{sen}7x + \text{sen}3x}{\text{cos}11x + \text{cos}7x + \text{cos}3x}$

Resolución:

Ordenando numerador y denominador:

$$K = \frac{\text{sen}11x + \text{sen}3x + \text{sen}7x}{\text{cos}11x + \text{cos}3x + \text{cos}7x}$$

$$K = \frac{2\text{sen}(7x) \cdot \text{cos}(4x) + \text{sen}7x}{2\text{cos}(7x) \cdot \text{cos}(4x) + \text{cos}7x}$$

$$K = \frac{\text{sen}7x \cdot (\cancel{2\text{cos}4x + 1})}{\text{cos}7x \cdot (\cancel{2\text{cos}4x + 1})}$$

$$\text{sen}A + \text{sen}B = 2\text{sen}\left(\frac{A+B}{2}\right) \cdot \text{cos}\left(\frac{A-B}{2}\right)$$

$$\text{cos}A + \text{cos}B = 2\text{cos}\left(\frac{A+B}{2}\right) \cdot \text{cos}\left(\frac{A-B}{2}\right)$$

$$\therefore K = \tan 7x$$



HELICOPRACTICE 2

Determine el rango de la función: $f(x) = 4\cos 3x + 5$

Resolución:

Se cumple que: $-1 \leq \cos 3x \leq 1$

Ahora le damos la forma de la función f :

$$-1 \leq \cos 3x \leq 1 \quad \dots \times 4$$

$$-4 \leq 4 \cos 3x \leq 4 \quad \dots + 5$$

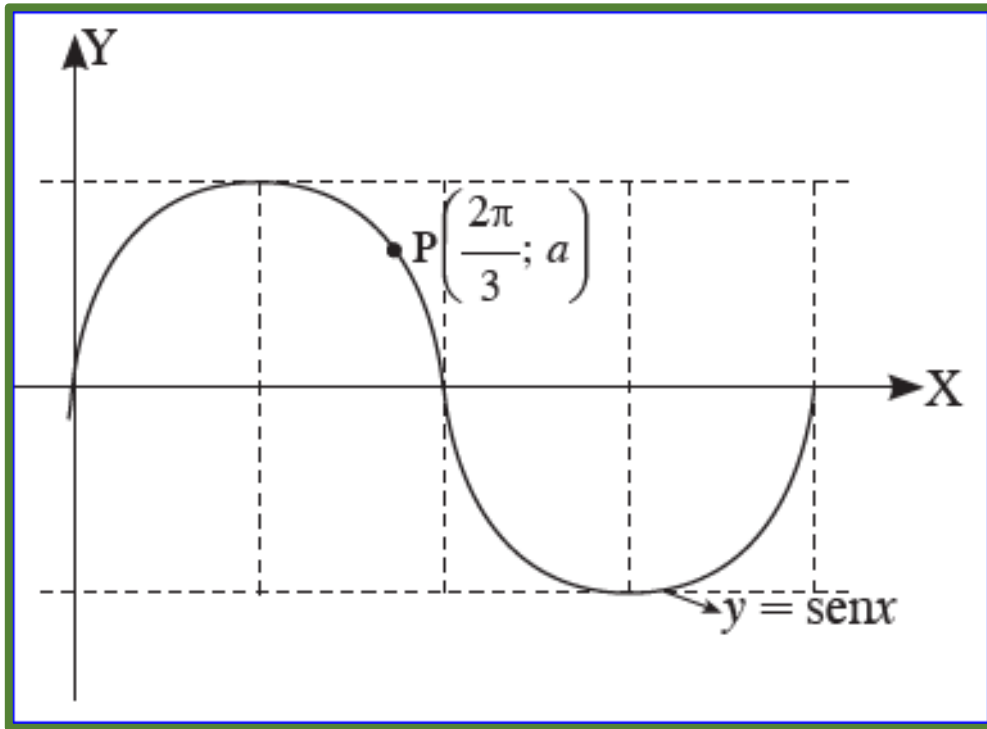
$$1 \leq \underbrace{4 \cos 3x + 5}_{f(x)} \leq 9$$

$$\therefore \text{Ranf} = [1; 9]$$



HELICOPRACTICE 3

Del gráfico, halle el valor de a



Resolución:

Sea $f(x) = y = \text{sen } x$

Vemos que: $P\left(\frac{2\pi}{3}; a\right) \in f$

$$\Rightarrow a = \text{sen}\left(\frac{2\pi}{3}\right)$$

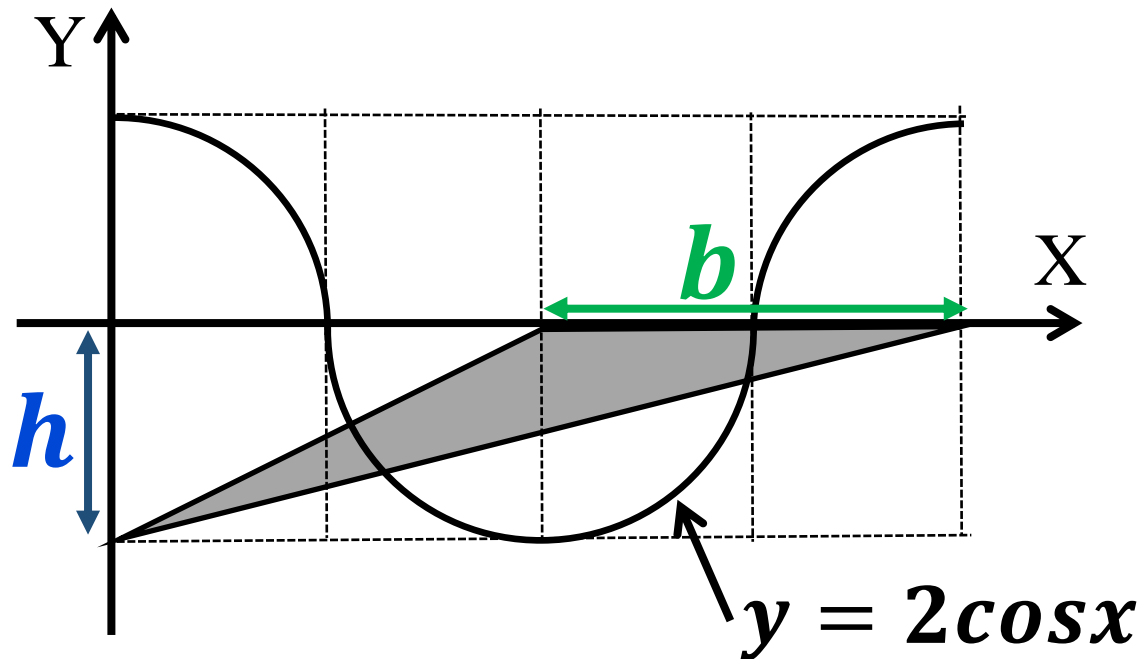
$$\text{Así: } a = \text{sen}\left(\underbrace{\pi - \frac{\pi}{3}}_{\text{IIC}}\right) = + \text{sen}\left(\frac{\pi}{3}\right)$$

$$\therefore a = \frac{\sqrt{3}}{2}$$



HELICOPRACTICE 4

El siguiente gráfico muestra las ondas emitidas por un teléfono móvil. Calcule el área de la región triangular sombreada.



Resolución:

Del gráfico notamos:

$$h = A \quad b = \frac{T}{2} = \frac{\cancel{2\pi}}{\cancel{2}}$$

$$h = 2 \quad b = \pi$$



Recordar:

$$S = \frac{b \cdot h}{2} \Rightarrow S = \frac{\pi \cdot \cancel{2}}{\cancel{2}}$$

$$\therefore S = \pi u^2$$



HELICOPRACTICE 5

Halle el valor de: $T = \tan(\arccos(\frac{1}{2})) + \sin(\arctan(\sqrt{3}))$

Resolución:

$$T = \tan(\underbrace{\arccos(\frac{1}{2})}_{\alpha}) + \sin(\underbrace{\arctan(\sqrt{3})}_{\theta})$$

- $\alpha = \arccos\left(\frac{1}{2}\right) \Rightarrow \cos\alpha = \frac{1}{2} \rightarrow \alpha = \frac{\pi}{3}$
- $\theta = \arctan(\sqrt{3}) \Rightarrow \tan\theta = \sqrt{3} \rightarrow \theta = \frac{\pi}{3}$

Reemplazando:

$$T = \tan\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)$$

$$\rightarrow T = \sqrt{3} + \frac{\sqrt{3}}{2}$$

$$\therefore T = \frac{3\sqrt{3}}{2}$$

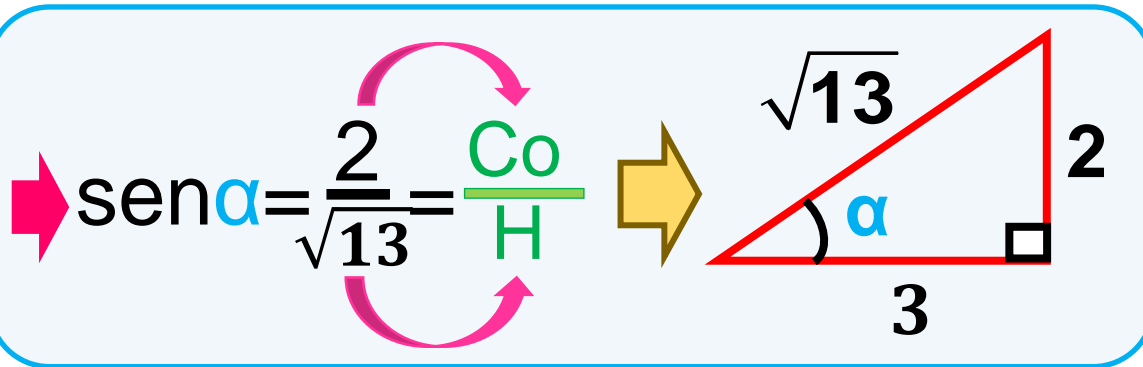


HELICOPRACTICE 6

Halle el valor de: $M = \tan\left[\frac{\pi}{4} - \arcsen\left(\frac{2}{\sqrt{13}}\right)\right]$

Resolución:

$$M = \tan\left[\frac{\pi}{4} - \underbrace{\arcsen\left(\frac{2}{\sqrt{13}}\right)}_{\alpha}\right]$$



Entonces: $M = \tan\left[\frac{\pi}{4} - \alpha\right]$

$$\Rightarrow M = \frac{\tan\frac{\pi}{4} - \tan\alpha}{1 + \tan\frac{\pi}{4} \cdot \tan\alpha}$$

$$\Rightarrow M = \frac{1 - \frac{2}{3}}{1 + 1 \cdot \frac{2}{3}} = \frac{\frac{1}{3}}{\frac{5}{3}} = \frac{1}{5}$$

∴ $M = \frac{1}{5}$



HELICOPRACTICE 7

Indique la menor solución positiva de:

$$\cos^2 x - \sin^2 x = 0,5$$

Resolución:

$$\underbrace{\cos^2 x - \sin^2 x}_{\cos 2x} = \underbrace{0,5}_{\frac{1}{2}}$$

$$\Rightarrow \cos 2x = \frac{1}{2} \dots \text{ETE}$$



Recuerda:

$$\cos 60^\circ = \frac{1}{2}$$

Luego: $2x = \frac{\pi}{3}$



La menor solución positiva es $x = \frac{\pi}{6}$



HELICOPRACTICE 8

Hallar la solución general de: $2\cos 2x - \tan 60^\circ = 0$

Resolución:

$$2\cos 2x - \tan 60^\circ = 0$$

$$2\cos 2x - \sqrt{3} = 0$$

Luego: $\cos 2x = \frac{\sqrt{3}}{2}$... ETE

➡ $V_p = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

La solución general para el coseno:

$$X_g = 2k\pi \pm V_p ; k \in \mathbb{Z}$$

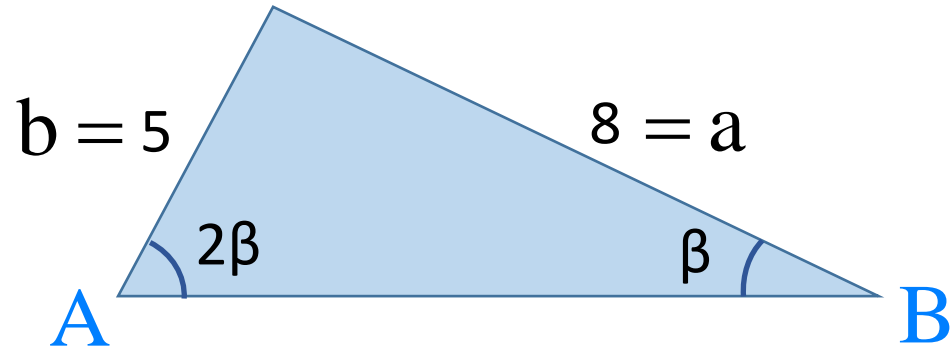
➡ $2x = 2k\pi \pm \frac{\pi}{6} ; k \in \mathbb{Z}$

∴ $x = k\pi \pm \frac{\pi}{12} ; k \in \mathbb{Z}$



HELICOPRACTICE 9

De la figura, calcule: $E = \csc(\beta - 7^\circ)$



Resolución:

$$\frac{a}{\text{sen}A} = \frac{b}{\text{sen}B} \Rightarrow \frac{8}{\text{sen}2\beta} = \frac{5}{\text{sen}\beta}$$

Usando Identidad Ángulo doble :

$$\frac{8}{2\cancel{\text{sen}\beta}\cos\beta} = \frac{5}{\cancel{\text{sen}\beta}}$$

$$\Rightarrow \frac{8}{2\cos\beta} = 5 \Rightarrow \cos\beta = \frac{4}{5} \rightarrow \beta = 37^\circ$$

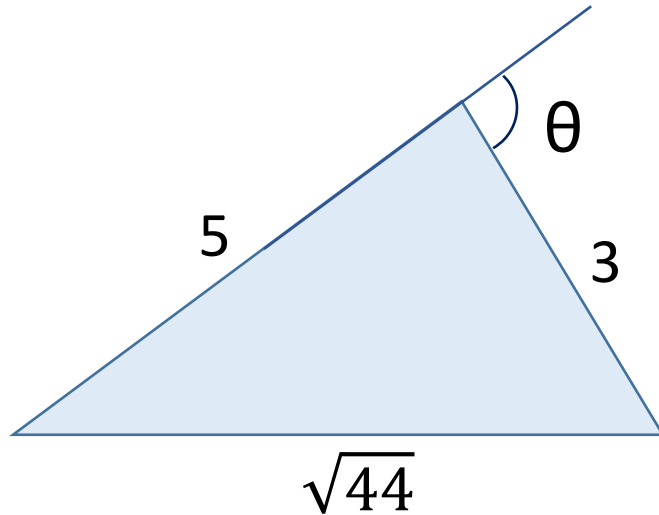
Piden: $E = \csc(\beta - 7^\circ)$

$$\Rightarrow E = \csc(37^\circ - 7^\circ) = \csc 30^\circ$$

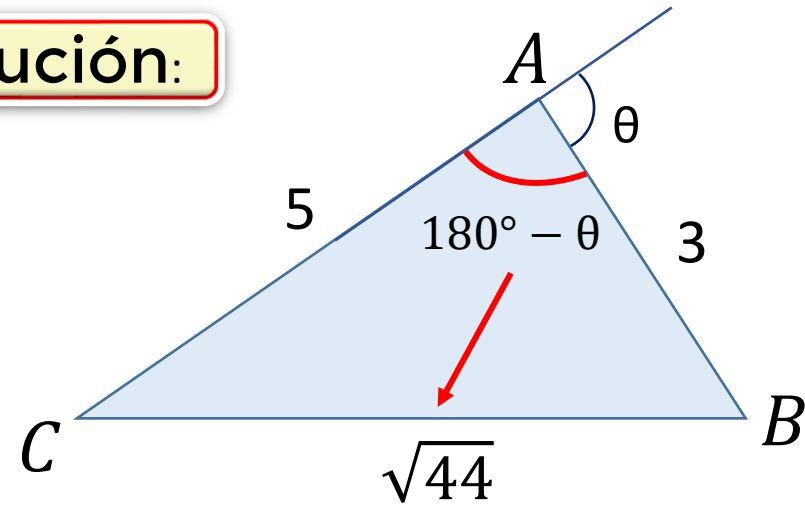
$$\therefore \boxed{E = 2}$$

HELICOPRACTICE 10

Del gráfico, calcule $\sec\theta$.



Resolución:



Ley de Cosenos: $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$$\sqrt{44}^2 = 5^2 + 3^2 - 2(5)(3)\cos(180^\circ - \theta)$$

$$44 = 25 + 9 - 30(-\cos\theta)$$

$$44 = 34 + 30\cos\theta$$

$$10 = 30\cos\theta \Rightarrow \cos\theta = \frac{1}{3}$$

$$\therefore \sec\theta = 3$$