ÁLGEBRA

CHAPTER 23

5th

of Secondary

TEMA:

Logaritmos I

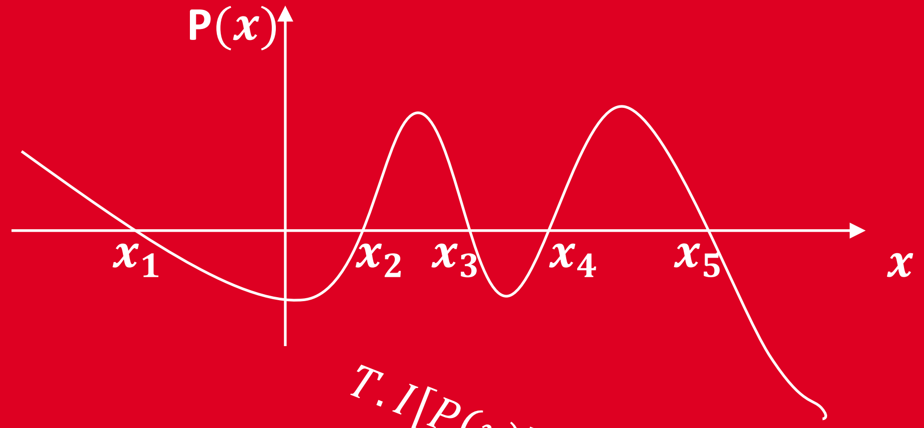
SACO OLIVEROS

$$P(x) \equiv a_0x^n + a_1x^{n-1} + \ \ldots + a_{n-1}x + a_n$$

$$\sum coeficientes\ P(x) = P(1)$$

$$P(x) \equiv 0$$

$$G.A(P)$$



$$T.I[P(x)] = P(0)$$

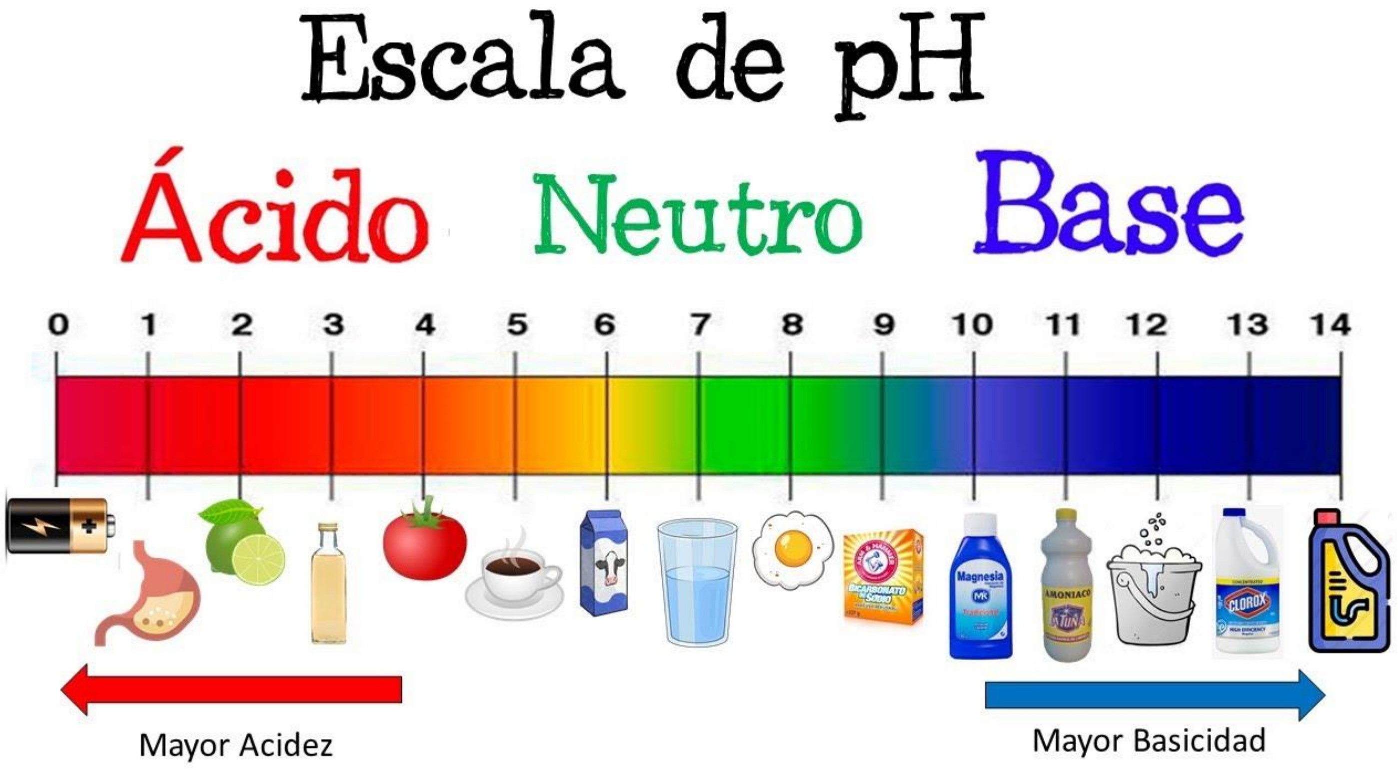
MOTIVATING STRATEGY

SACO OLIVEROS

EL pH Y LOS LOGARITMOS

El pH es la medida de la acidez o alcalinidad de una solución.

$$pH = -\log[H^+]$$



HELICO THEORY

SACO OLIVEROS

LOGARITMOS

DEFINICIÓN

$$\forall a, n \in \mathbb{R}^{+} \wedge \ a \neq 1$$

$$\boxed{\log_a n = L} \quad \text{<->} \quad a^L = n$$

a: base

n: argumento

L: logaritmo

EJEMPLOS

$$\log_3 81 = 4$$

$$\log_2 32 = 5$$

$$\log_{16} 4 = \frac{1}{2}$$

OBSERVACIÓN

$$\boxed{\log_{10} n = \log n}$$

IDENTIDAD FUNDAMENTAL DEL LOGARITMO

$$a^{\log_a n} = n$$

EJEMPLOS

$$7^{\log_7 4} = 4$$

TEOREMAS $\forall x, y \in \mathbb{R}^+$

1) $\log_a(xy) = \log_a x + \log_a y$

$\log_7 5 + \log_7 6 = \log_7 30$

2) $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

$\log_3 20 - \log_3 4 = \log_3 5$

3) $\log_{a^n}(x^m) = \dfrac{m}{n}\log_a x$

$\log_{16} 125 = \log_{2^4}(5^3) = \dfrac{3}{4}\log_2 5$

4) $x^{\log_a y} = y^{\log_a x}$

$3^{\log_4 5} = 5^{\log_4 3}$

5) $\log_a x = \dfrac{1}{\log_x a}$

$\dfrac{1}{\log_2 5} = \log_5 2$

OBSERVACIÓN

$\boxed{\log_a 1 = 0}$

$\log_8 1 = 0$

$\boxed{\log_a a = 1}$

$\log_5 5 = 1$

HELICO PRACTICE

SACO OLIVEROS

1) Calcule $\log_B A$, si: $\log_9 27 = A$ y $\log_{512} 16 = B$

Resolución

$$\log_9 27 = A$$

$$9^A = 27$$

$$\cancel{3}^{2A} = \cancel{3}^3$$

$$\boxed{A = \frac{3}{2}}$$

$$\log_{512} 16 = B$$

$$512^B = 16$$

$$\cancel{2}^{9B} = \cancel{2}^4$$

$$\boxed{B = \frac{4}{9}}$$

Calculemos: $\log_B A$

$$\Rightarrow \log_{\left(\frac{4}{9}\right)} \left(\frac{3}{2}\right) = x$$

$$\left(\frac{4}{9}\right)^x = \frac{3}{2} \Rightarrow \left(\frac{\cancel{2}}{\cancel{3}}\right)^{2x} = \left(\frac{\cancel{2}}{\cancel{3}}\right)^{-1}$$

$$\therefore x = -1/2$$

2) **Si:** $x = \log_9(\log_{64}(\log_3 81))$

Hallar el valor de: $M = 5^{1+2x} + 5^{1-2x}$

Resolución

$$\boxed{\log_3 81 = \log_3(3^4) = 4}$$

$$x = \log_9(log_{64} 4)$$

$$\boxed{\log_{64} 4 = \log_{(4^3)}(4^1) = 1/3}$$

$$x = \log_9 (\frac{1}{3}) = \log_{(3^2)}(3^{-1})$$

$$x = \frac{-1}{2}$$

$$\boxed{2x = -1}$$

$$M = 5^{1+(-1)} + 5^{1-(-1)}$$

$$M = 1 + 25$$

$$\therefore M = 26$$

3) **Si:** $x = \sqrt[5]{3}$ **Reducir:** $\log_x\left[5^{\log_{\sqrt[3]{5}} x} + 27^{\log_3 x} + 8^{\log_2 x}\right]$

Resolución

$$\log_x\left[x^{\log_{\sqrt[3]{5}} 5} + x^{\log_3 27} + x^{\log_2 8}\right]$$

$$\boxed{a^{\log_b c} = c^{\log_b a}}$$

$$\log_{\sqrt[3]{5}} 5 = 3$$

$$\log_3 27 = 3$$

$$\log_2 8 = 3$$

$$\log_x[x^3 + x^3 + x^3] \rightarrow \log_x[3x^3]$$

$$\boxed{x = \sqrt[5]{3} \rightarrow x^5 = 3}$$

$$\rightarrow \log_x[x^5 x^3] \rightarrow \log_x[x^8] = 8$$

$$\therefore Rpta = 8$$

4) Indique la mayor raíz de: $\log_2 x^3 - 2\log_x 2 - 5 = 0$

Resolución

$$3\log_2 x - \frac{2}{\log_2 x} - 5 = 0$$

CAMBIO DE VARIABLE: $a = \log_2 x$

$$3a - \frac{2}{a} - 5 = 0$$

$$3a^2 - 5a - 2 = 0$$

$$(a-2)(3a+1) = 0$$

$a_1 = 2 \qquad a_2 = -\frac{1}{3}$

$a = \log_2 x$

$\log_2 x = 2 \qquad \log_2 x = -\frac{1}{3}$

$x_1 = 4 \qquad x_2 = \sqrt[3]{2^{-1}}$

$\therefore\ Mayor\ raíz = 4$

5) Determine la mayor raíz de x:

$$\log_3 x^{\log_3 x} - \log_3 x^3 - 10 = 0$$

Resolución

$$\log_3 x \cdot \log_3 x - 3\log_3 x - 10 = 0$$

$$\log_3^2 x - 3\log_3 x - 10 = 0$$

$$\log_3 x \qquad -5$$
$$\log_3 x \qquad 2$$

$$(\log_3 x - 5)(\log_3 x + 2) = 0$$

$$\log_3 x - 5 = 0$$

$$\log_3 x = 5 \quad \rightarrow x = 3^5$$

$$\log_3 x + 2 = 0$$

$$\log_3 x = -2 \quad \rightarrow x = 3^{-2}$$

$$\therefore \ Mayor\ raíz = 3^5$$

6) Simplifique

$$T=\log\left(\frac{133}{65}\right)+2\log\left(\frac{13}{7}\right)-\log\left(\frac{143}{90}\right)+\log\left(\frac{77}{171}\right)$$

Resolución

$$T=\log\left(\frac{133}{65}\right)+\log\left(\frac{13}{7}\right)^{2}+\log\left(\frac{143}{90}\right)^{-1}+\log\left(\frac{77}{171}\right)$$

$$T=\log\left(\frac{133}{65}\right)+\log\left(\frac{169}{49}\right)+\log\left(\frac{90}{143}\right)+\log\left(\frac{77}{171}\right)$$

$$T=\log\left(\frac{133}{65}\cdot\frac{169}{49}\cdot\frac{90}{143}\cdot\frac{77}{171}\right)\qquad\rightarrow T=\log\left(\frac{19\cdot 7}{13\cdot 5}\cdot\frac{13\cdot 13}{7\cdot 7}\cdot\frac{9\cdot 5\cdot 2}{13\cdot 11}\cdot\frac{7\cdot 11}{19\cdot 9}\right)$$

$$\boxed{\therefore T=\log 2}$$

7) A qué es igual?

$$P = \frac{1}{1 + \log_3 10e} + \frac{1}{1 + \log_e 30} + \frac{1}{1 + \log_{10} 3e}$$

Resolución

$$P = \frac{1}{\log_3 3 + \log_3 10e} + \frac{1}{\log_e e + \log_e 30} + \frac{1}{\log_{10} 10 + \log_{10} 3e}$$

$$P = \frac{1}{\log_3 30e} + \frac{1}{\log_e 30e} + \frac{1}{\log_{10} 30e}$$

$$P = \log_{30e} 3 + \log_{30e} e + \log_{30e} 10 \qquad \rightarrow P = \log_{30e} 30e$$

$$\therefore P = 1$$

8) La edad de Rubí es 10T años, donde T se calcula como la suma de raíces de la ecuación: $5^{\log_3(2x^2-5x+9)} = 7^{\log_3 5}$

¿Cuál será la edad de Rubí dentro de 3 años?

Resolución

$$\cancel{5}^{\cancel{\log_3}(2x^2-5x+9)} = \cancel{5}^{\cancel{\log_3}7}$$

$$2x^2 - 5x + 9 = 7$$

$$2x^2 - 5x + 2 = 0$$

$$\rightarrow T = \frac{5}{2} \qquad \rightarrow 10T = 25$$

TEOREMA DE CARDANO:

$$ax^2 + bx + c = 0$$

Suma de raíces $= \dfrac{-b}{a}$

$\therefore$ *Edad dentro de 3 años: 28 años*