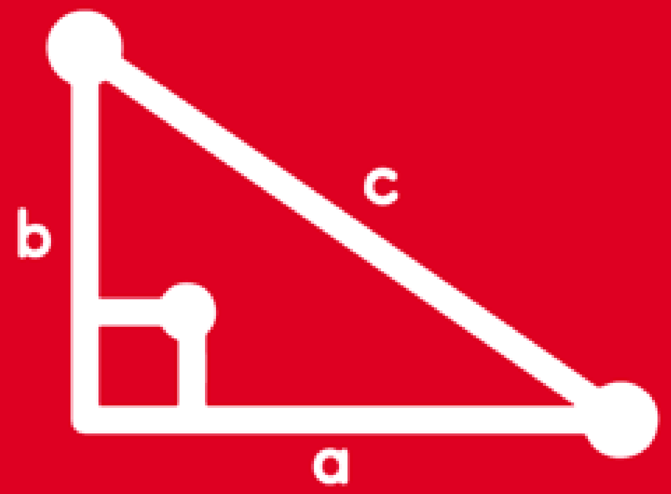
TRIGONOMETRY

Chapter 14



5th
SECONDARY

IDENTIDADES TRIGONOMÉTRICAS AUXILIARES DEL ÁNGULO COMPUESTO

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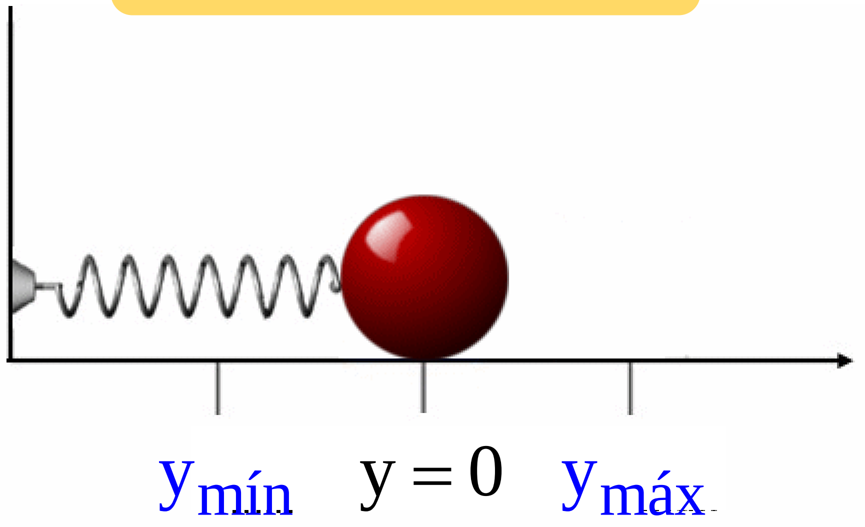
Los fenómenos periódicos son aquellos que se **repiten** en el tiempo de forma idéntica.

Así tenemos fenómenos periódicos como el movimiento de rotación de la tierra , el sonido , la corriente alterna , la luz , las mareas , los latidos del corazón entre otros.

Para un mejor estudio de estos fenómenos , se usan a **las funciones trigonométricas seno y coseno**.

Ejemplo: La elongación de un resorte **y** (cm), se puede modelar por la ecuación:

$$y = 3\operatorname{sen}x + 4\cos x$$



¿Puedes calcular la máxima elongación del resorte?

Rpta: **5 cm**

IDENTIDADES TRIGONOMÉTRICAS AUXILIARES

1. $\text{sen}(x+y).\text{sen}(x-y)=\text{sen}^2x-\text{sen}^2y$

2. $\cos(x+y).\cos(x-y)=\cos^2x-\text{sen}^2y$

Ejemplo: Reducir la expresión

$$E=\text{sen}(30^o+\alpha).\text{sen}(30^o-\alpha)+\text{sen}^2\alpha$$

Resolución:

$$E=\underbrace{\text{sen}(30^o+\alpha).\text{sen}(30^o-\alpha)}+\text{sen}^2\alpha$$

IA 1.

$$E=\text{sen}^2 30^o-\cancel{\text{sen}^2\alpha}+\cancel{\text{sen}^2\alpha} \Rightarrow E=(1/2)^2$$

$$\therefore \boxed{E=1/4}$$

3. $\boxed{\tan x + \tan y + \tan(x+y).\tan x.\tan y = \tan(x+y)}$

4. $\boxed{\tan x - \tan y - \tan(x-y).\tan x.\tan y = \tan(x-y)}$

Ejemplo:

Calcule $E = \tan 20^\circ + \tan 17^\circ + \frac{3}{4}.\tan 20^\circ.\tan 17^\circ$

Resolución:

$$E = \tan 20^\circ + \tan 17^\circ + \frac{3}{4}.\tan 20^\circ.\tan 17^\circ$$

$$E = \underbrace{\tan 20^\circ + \tan 17^\circ + \tan(20^\circ + 17^\circ).\tan 20^\circ.\tan 17^\circ}$$

IA 3.

$$\rightarrow E = \tan(20^\circ + 17^\circ) \Rightarrow E = \tan(37^\circ)$$

Recuerda :

$$\tan(37^\circ) = \frac{3}{4}$$

$$\therefore \boxed{E = 3/4}$$

5. Para x variable :

$$\underbrace{-\sqrt{a^2+b^2}}_{\text{mínimo}} \leq a.\text{senx} + b.\cos x \leq \underbrace{\sqrt{a^2+b^2}}_{\text{máximo}}$$

Ejemplos:

$$-5 \leq 3\text{senx} + 4\cos x \leq 5$$

$$-2 \leq \sqrt{3}\,\text{senx} + \cos x \leq 2$$

$$-13 \leq 12\text{senx} - 5\cos x \leq 13$$

Importante: $-\sqrt{2} \leq \text{senx} \pm \cos x \leq \sqrt{2}$

6. Si: $A+B+C=180^\circ$

$\Rightarrow \tan A+\tan B+\tan C=\tan A.\tan B.\tan C$

$\Rightarrow \cot A.\cot B+\cot B.\cot C+\cot C.\cot A=1$

7. Si: $x+y+z=90^\circ$

$\Rightarrow \cot x+\cot y+\cot z=\cot x.\cot y.\cot z$

$\Rightarrow \tan x.\tan y+\tan y.\tan z+\tan z.\tan x=1$

Ejemplo: Calcule

$$E=\frac{\tan 40^\circ+\tan 60^\circ+\tan 80^\circ}{\tan 40^\circ.\tan 60^\circ.\tan 80^\circ}$$

Resolución:

$$E=\frac{\tan 40^\circ+\tan 60^\circ+\tan 80^\circ}{\tan 40^\circ.\tan 60^\circ.\tan 80^\circ} \quad \ldots (*)$$

OBS: $40^\circ+60^\circ+80^\circ=180^\circ$

Usando **IA 6.** en **(*)**, tenemos:

$$E=\frac{\tan 40^\circ.\tan 60^\circ.\tan 80^\circ}{\tan 40^\circ.\tan 60^\circ.\tan 80^\circ}$$

$$\therefore\ \boxed{E=1}$$

1. $= \quad + \quad ° \quad - \quad ° \quad + \quad ^2$

RESOLUCIÓN

$$\boxed{\operatorname{sen}(x+y).\operatorname{sen}(x-y) = \operatorname{sen}^2x - \operatorname{sen}^2y}$$

$$F = \operatorname{sen}(x + 45°).\operatorname{sen}(x - 45°) + \cos^2x$$

$$F = \operatorname{sen}^2x - \operatorname{sen}^2 45° + \cos^2x$$

$$F = \underbrace{\operatorname{sen}^2x + \cos^2x}_{1} - \underbrace{\operatorname{sen}^2 45°}_{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$F = 1 - \left(\frac{1}{2}\right)$$

$$\therefore \boxed{F = \left(\frac{1}{2}\right)}$$

2.

$+ \qquad - \quad = \quad$ ....(I)

$+ \qquad - \quad = \quad$ ....(II)

2

RESOLUCIÓN

$\text{sen}(x+y)\cdot\text{sen}(x-y) = \text{sen}^2x - \text{sen}^2y$

$\cos(x+y)\cdot\cos(x-y) = \cos^2x - \text{sen}^2y$

De (I): $\text{sen}^2x - \text{sen}^2y = a$

De (II): $\cos^2x - \text{sen}^2y = b$ $+$

$\text{sen}^2x + \cos^2x = 1$

$1 - 2\text{sen}^2y = a + b$

$1 - 2(\underbrace{1 - \cos^2y}) = a + b$

$2\cos^2y - 1 = a + b$

$$\therefore \boxed{\cos^2 y = \frac{a+b+1}{2}}$$

3. $= \sqrt{\ } + \qquad ° \sqrt{\ } + \qquad °$

RESOLUCIÓN

$$\boxed{\tan x + \tan y + \tan(x+y)\cdot\tan x\cdot\tan y = \tan(x+y)}$$

$$J = (\sqrt{3} + \tan 13°)(\sqrt{3} + \tan 17°).$$

$$J = \sqrt{3}^{\,2} + \sqrt{3}\tan 17° + \sqrt{3}\tan 13° + \tan 13°.\tan 17° \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$J = 3 + \sqrt{3}\left[\tan 17° + \tan 13° + \overbrace{\frac{1}{\sqrt{3}}}^{\tan 30°}\tan 17°.\tan 13°\right]$$

$$J = 3 + \sqrt{3}\,[\tan 30°] \qquad J = 3 + \cancel{\sqrt{3}}\left(\frac{1}{\cancel{\sqrt{3}}}\right)$$

$$\therefore \boxed{J = 4}$$

4.

$= \quad + \sqrt{} \quad ° - \quad +$

RESOLUCIÓN

$F = 2senx + \sqrt{2}\,(\cos45°\cos x + sen45°senx) + 3\cos x$

$F = 2senx + \cancel{\sqrt{2}}.\dfrac{1}{\cancel{\sqrt{2}}}.\cos x + \cancel{\sqrt{2}}.\dfrac{1}{\cancel{\sqrt{2}}}.senx + 3\cos x$

$F = 2senx + \cos x + senx + 3\cos x$

$$\underbrace{-\sqrt{a^2+b^2}}_{\text{mínimo}} \leq asenx + b\cos x \leq \underbrace{\sqrt{a^2+b^2}}_{\text{máximo}}$$

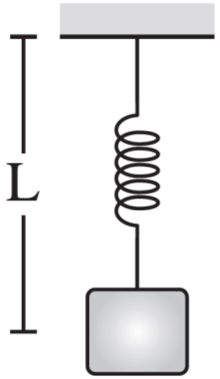
$F = 3senx + 4\cos x$

Nos piden **Fmín** : $F_{mín} = -\sqrt{3^2+4^2}$

$\therefore \boxed{F_{mín} = -5}$

5.

= +



RESOLUCIÓN

$$L = 4\text{sen}t + 6\cos t$$

$$\underbrace{-\sqrt{a^2+b^2}}_{\text{mínimo}} \leq a\,\text{senx} + b\cos x \leq \underbrace{\sqrt{a^2+b^2}}_{\text{máximo}}$$

$$L_{\text{máximo}} = \sqrt{4^2 + 6^2}$$

$$L_{\text{máximo}} = \sqrt{16 + 36}$$

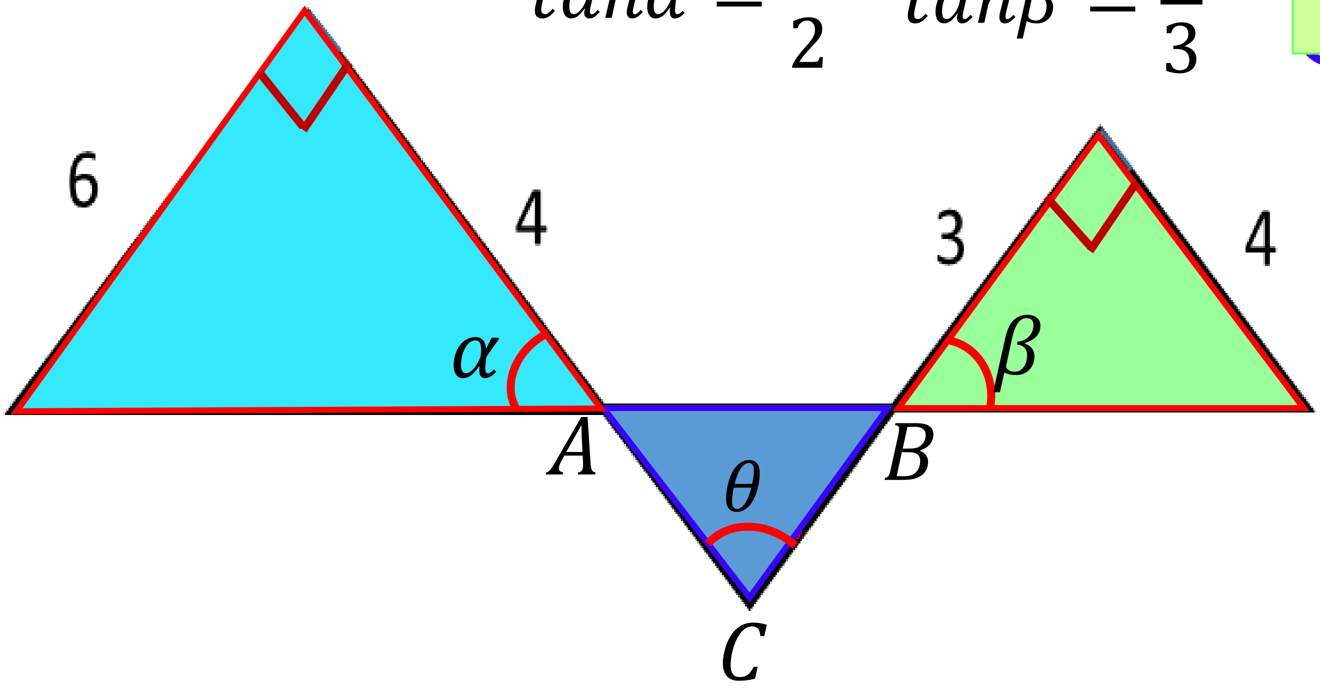
$$L_{\text{máximo}} = \sqrt{52}$$

$$\therefore \boxed{L_{\text{máximo}} = 2\sqrt{13}\text{ cm}}$$

6.

θ

RESOLUCIÓN



$$tan\alpha = \frac{3}{2} \quad tan\beta = \frac{4}{3}$$

Si $x + y + z = 180°$, se cumple:
$tanx + tany + tanz = tanx.tany.tanz$

$$\frac{3}{2} + \frac{4}{3} + tan\theta = \frac{\cancel{3}}{2}.\frac{4}{\cancel{3}}.tan\theta$$

$$\frac{17}{6} + tan\theta = 2.tan\theta$$

$\triangle ABC: \alpha + \beta + \theta = 180°$

$$tan\alpha + tan\beta + tan\theta = tan\alpha tan\beta tan\theta$$

$$\therefore \boxed{tan\theta = \frac{17}{6}}$$

7.

$$= \frac{\circ + \quad \circ + \quad \circ}{\circ \quad \circ}$$

RESOLUCIÓN

Si $x + y + z = 180°$, se cumple:

$\tan x + \tan y + \tan z = \tan x . \tan y . \tan z$

$$P = \frac{\tan 40° + \tan 60° + \tan 80°}{\cot 10° \cot 50°}$$

Se observa que:

$$\boxed{40° + 60° + 80° = 180°}$$

Entonces:

$$\tan 40° + \tan 60° + \tan 80° = \tan 40° \tan 60° \tan 80°$$

$$P = \frac{\tan 40° \tan 60° \tan 80°}{\cot 10° \cot 50°}$$

$$P = \frac{\cancel{\tan 40°} \tan 60° \cancel{\tan 80°}}{\cancel{\tan 80°} \cancel{\tan 40°}}$$

$$P = \tan 60°$$

$$\therefore \boxed{P = \sqrt{3}}$$

8.

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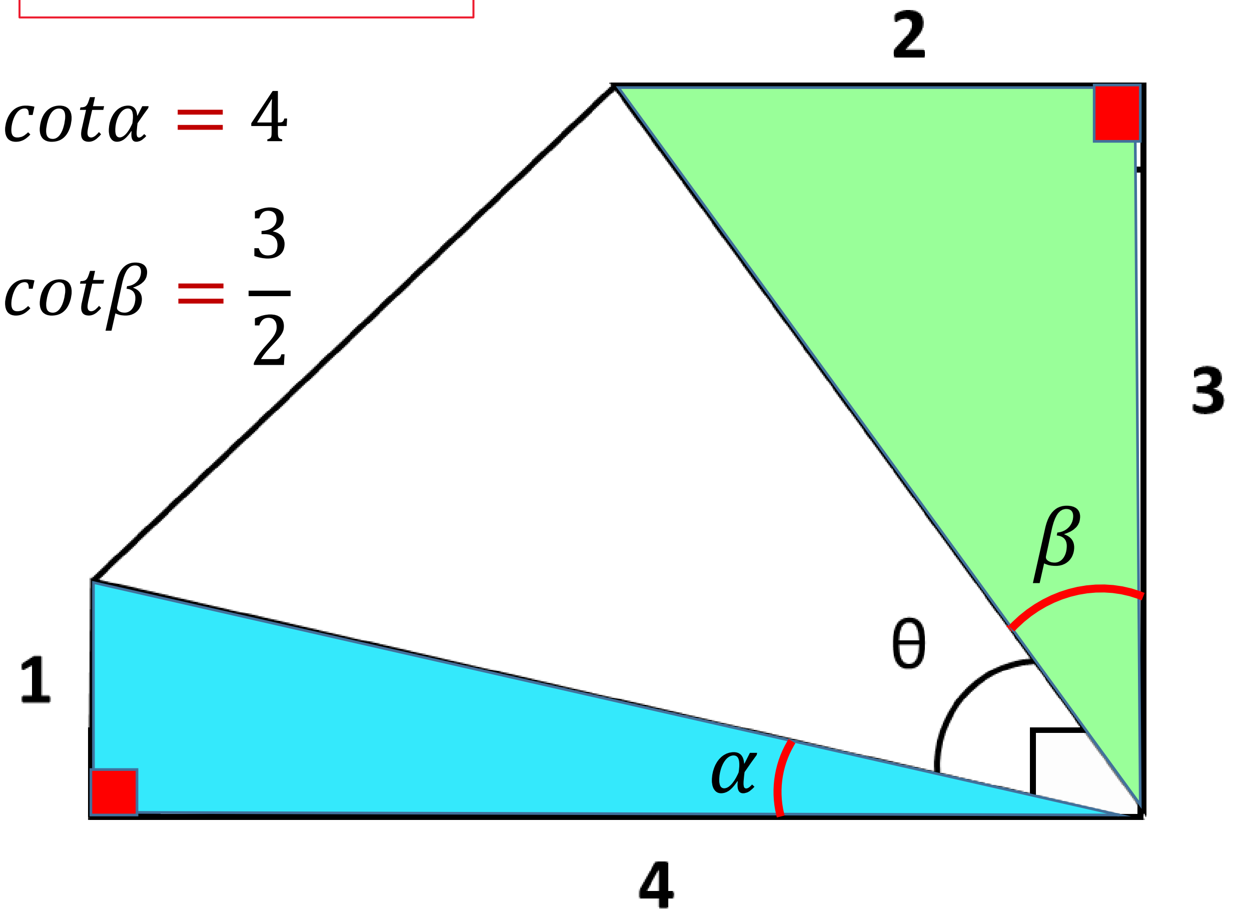
RESOLUCIÓN

$$\cot\alpha = 4$$

$$\cot\beta = \frac{3}{2}$$



Si $x + y + z = 90°$, se cumple:

$\cot x + \cot y + \cot z = \cot x.\cot y.\cot z$

Se observa que: $\alpha + \beta + \theta = 90°$

Entonces:

$$\cot\alpha + \cot\beta + \cot\theta = \cot\alpha\cot\beta\cot\theta$$

$$\Rightarrow 4 + \frac{3}{2} + \cot\theta = 4.\frac{3}{2}.\cot\theta$$

$$\frac{11}{2} + \cot\theta = 6\cot\theta$$

$$\frac{11}{2} = 5\cot\theta \qquad \therefore \boxed{\cot\theta = \frac{11}{10}}$$