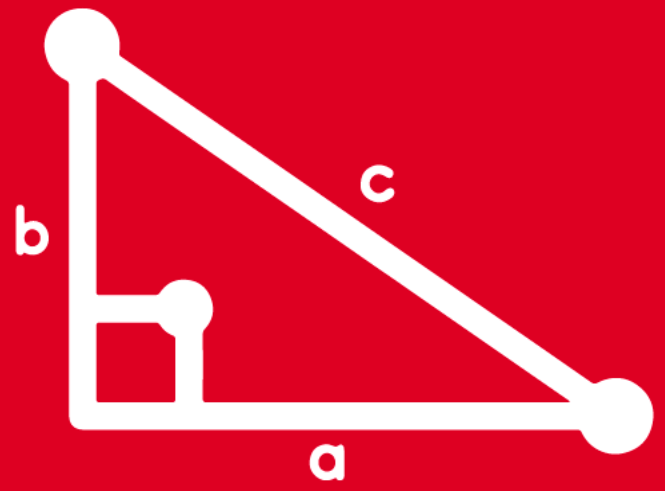
# TRIGONOMETRY

## Chapter 11
## Session I




**4th**
**SECONDARY**

## Reducción al primer Cuadrante I

SACO OLIVEROS

# Sistema de Radar :

El radar es un sistema electrónico que permite detectar objetos y determinar la distancia y su velocidad , ello lo realiza proyectando ondas de radio que son reflejadas por el objeto y recibidas de nuevo por la antena.

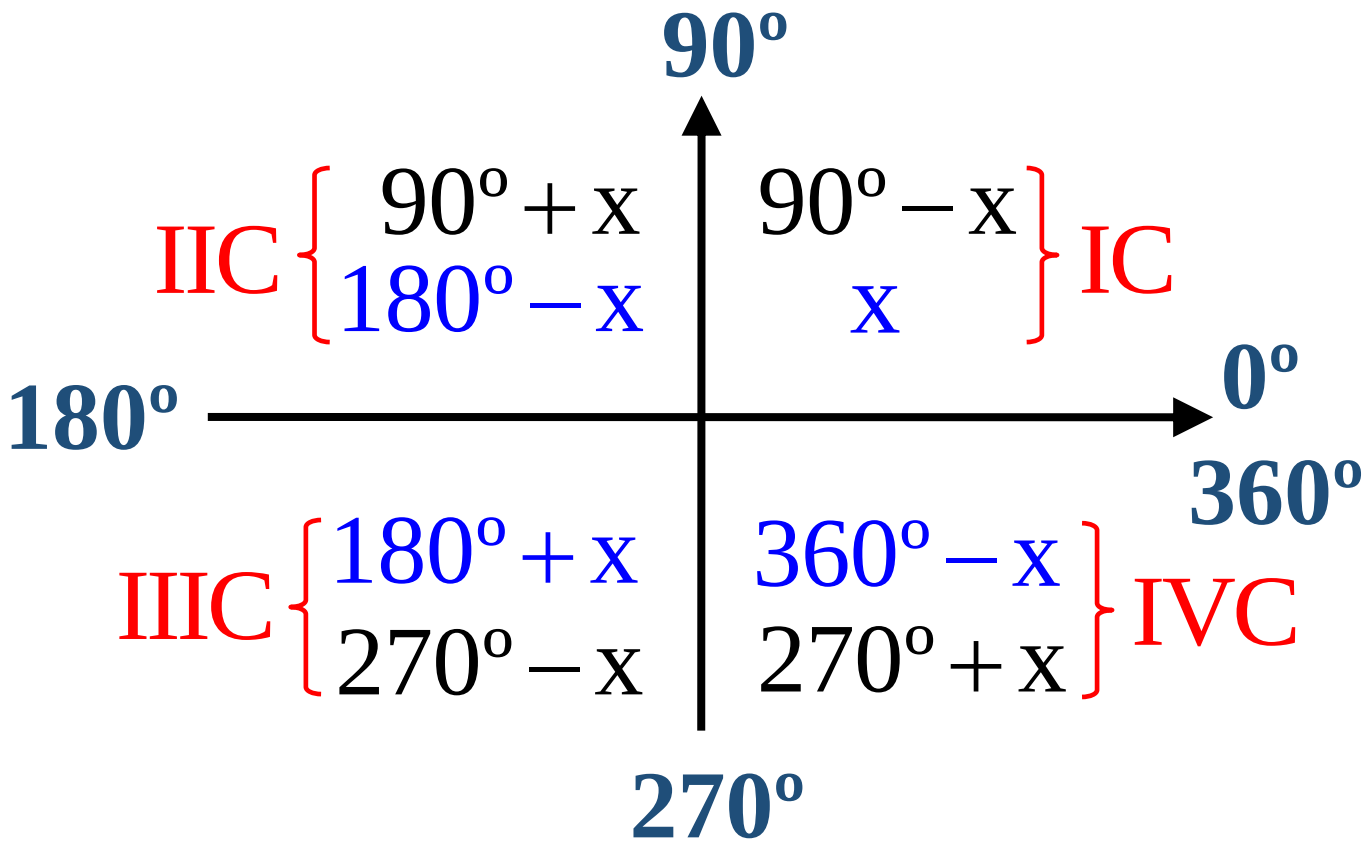
La antena de radar gira ( 360° ) en un mismo sentido a velocidad constante mostrando la señal en la pantalla.

Transmisor / Receptor

Pantalla de radar

# REDUCCIÓN AL PRIMER CUADRANTE

## 1° CASO : Para ángulos positivos menores a una vuelta

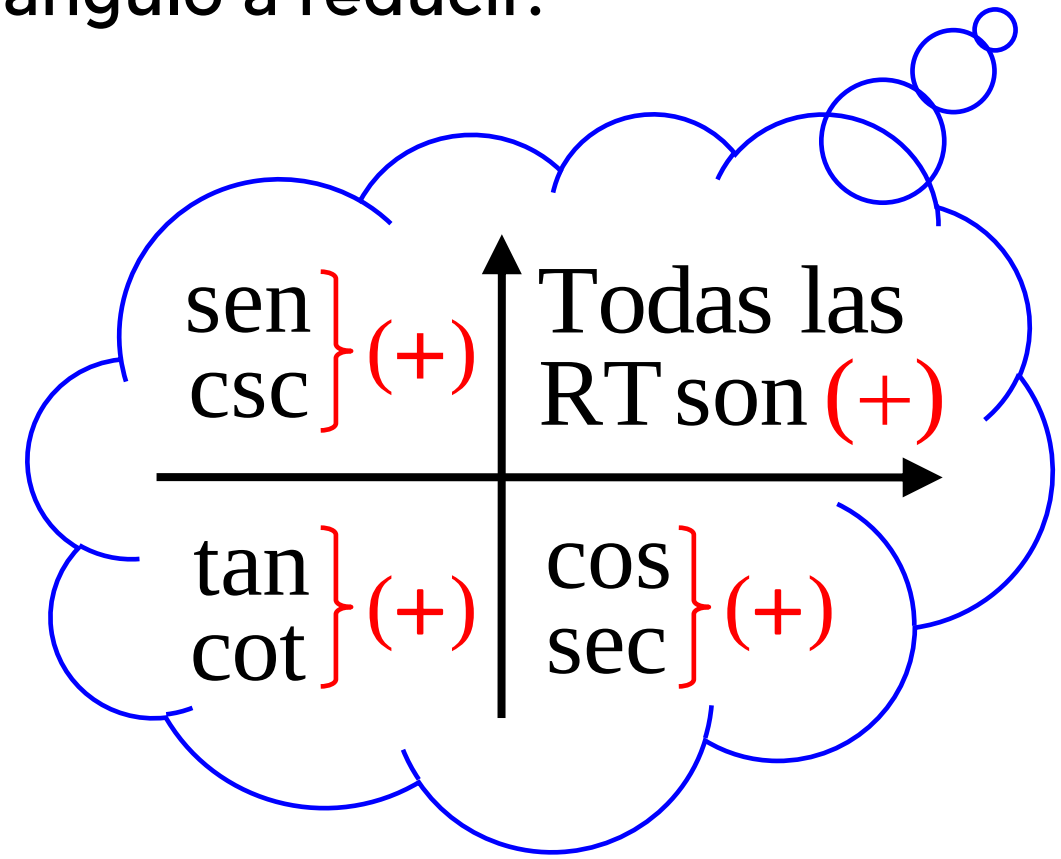



$$RT\begin{pmatrix}180^\circ \pm x\\ 360^\circ - x\end{pmatrix} = (\pm)RT(x)$$

$$RT\begin{pmatrix}90^\circ \pm x\\ 270^\circ \pm x\end{pmatrix} = (\pm)CoRT(x)$$

sen $\Leftrightarrow$ cos
tan $\Leftrightarrow$ cot
sec $\Leftrightarrow$ csc

Donde el signo $(\pm)$ del segundo miembro depende de la RT y el cuadrante al cual pertenece el ángulo a reducir.




**Ejemplos:** Reducir al IC

- $\text{sen}(\underbrace{180^\circ - x}_{\text{IIC}}) = \underbrace{+}_{\uparrow}\ \text{sen}(x)$
- $\tan(\underbrace{270^\circ + x}_{\text{IVC}}) = \underbrace{-}_{\uparrow}\ \cot(x)$
- $\cos(240^\circ) = \cos(\underbrace{180^\circ + 60^\circ}_{\text{IIIC}})$

$$\cos(240^\circ) = \underbrace{-}_{\downarrow}\ \cos(60^\circ)$$

$$\cos(240^\circ) = -1/2$$

## 2° CASO: Para ángulos negativos

$\text{sen}(-x) = -\text{sen}(x)$ $\cos(-x) = \cos(x)$ $\tan(-x) = -\tan(x)$

$\csc(-x) = -\csc(x)$ $\sec(-x) = \sec(x)$ $\cot(-x) = -\cot(x)$

## Ejemplos: Reducir al IC

- $\text{sen}(-30^\circ) = -\text{sen}(30^\circ) = -\frac{1}{2}$

- $\cos(-45^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2}$

**1.** Efectúe: $\mathbf{P = (sen240° + tan150°).cos330°}$

**RESOLUCIÓN:**

$$P = [sen(180° + 60°) + tan(180° - 30°)].cos(360° - 30°)$$

$$\underset{IIIC}{} \qquad \underset{IIC}{} \qquad \underset{IVC}{}$$

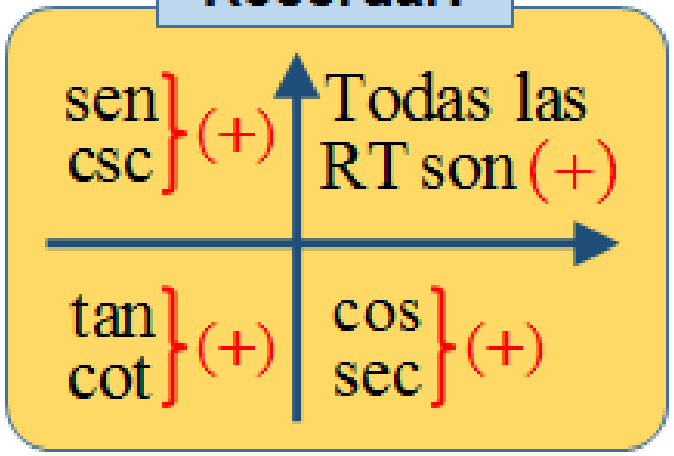
$$P = [(-sen60°) + (-tan30°)].(cos30°)$$

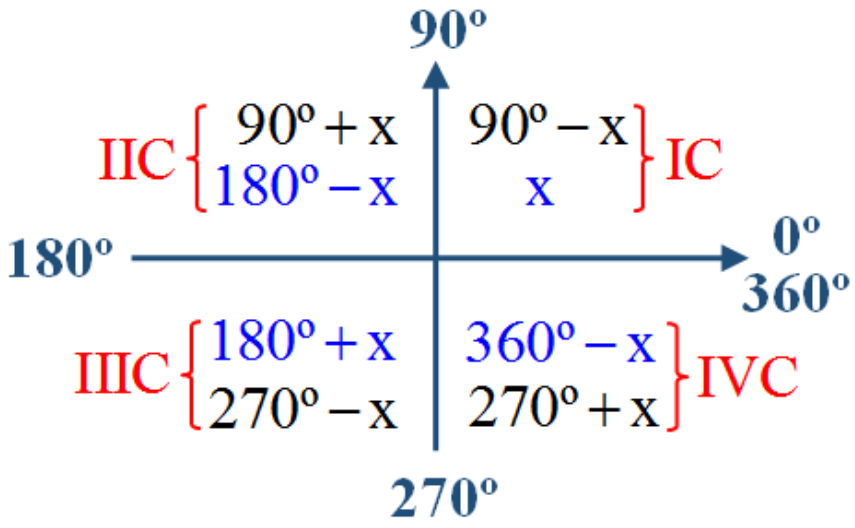
$$P = \left(\frac{-\sqrt{3}}{2} + \frac{-\sqrt{3}}{3}\right).\frac{\sqrt{3}}{2} = \left(\frac{-5\sqrt{3}}{6}\right).\frac{\sqrt{3}}{2}$$

$$P = -\frac{5 \times \cancel{3}^{1}}{\cancel{6}_{2} \times 2}$$

$$\therefore \boldsymbol{P = -\frac{5}{4}}$$

**Recordar:**



**2.** Reduzca: $Q = \dfrac{4\tan 130° + \cot 40°}{\tan 310°}$

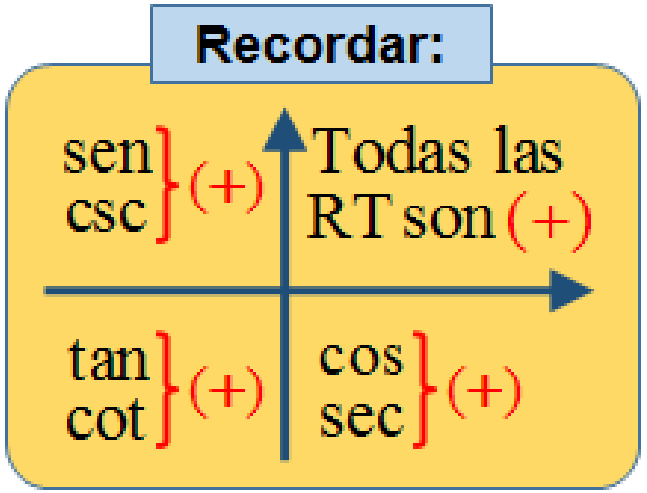
**RESOLUCIÓN:**

$$Q = \frac{4\tan(\overbrace{180° - 50°}^{IIC}) + \cot(\overbrace{90° - 50°}^{IC})}{\tan(\underbrace{360° - 50°}_{IVC})}$$

$$Q = \frac{(-4\tan 50°) + (\tan 50°)}{-\tan 50°}$$

$$Q = \frac{-3\tan 50°}{-\tan 50°} \Rightarrow \therefore Q = 3$$

**Recordar:**




$$RT\left(\begin{matrix}180° \\ 360°\end{matrix} \pm \alpha\right) = \pm RT(\alpha)$$

$$RT\left(\begin{matrix}90° \\ 270°\end{matrix} \pm \alpha\right) = \pm CO - RT(\alpha)$$

**3.** Simplifique: $\mathbf{T = \sqrt{3}tan(-60°) + 5cos(-37°)}$

**RESOLUCIÓN:**

$$T = \sqrt{3}tan(-60°) + 5cos(-37°)$$

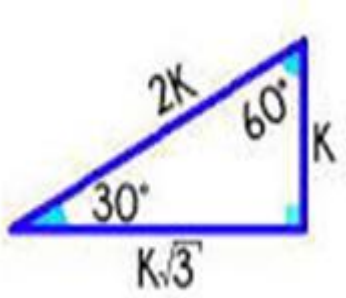
$$T = \sqrt{3}\,(-tan60°) + 5\,(cos37°)$$

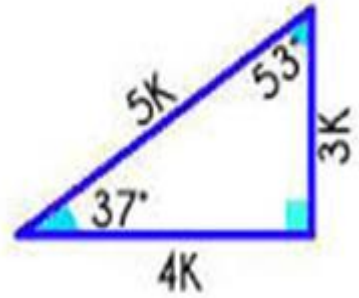
$$T = \sqrt{3}\,(-\sqrt{3}) + \cancel{5}\,\left(\frac{4}{\cancel{5}}\right)$$

$$T = -3 + 4$$

$$\therefore T = 1$$

| | |
|---|---|
| $sen(-x) = -senx$ | $csc(-x) = -cscx$ |
| $cos(-x) = cosx$ | $sec(-x) = secx$ |
| $tan(-x) = -tanx$ | $cot(-x) = -cotx$ |



**4.** Iván quiere pasar un álbum de fotos de 350 MB a su memoria usb, pero no hay suficiente espacio disponible. Si bien no quiere eliminar ninguna de las fotos, no le importará eliminar un álbum de música. El tamaño de los álbumes de música que Iván tiene almacenados en su usb es el siguiente:

| Álbum | Álbum 1 | Álbum 2 | Álbum 3 |
|---|---|---|---|
| Tamaño | A | B | C |

(A, B y C en MB), donde:

$$A = 100sec^2\left(\frac{4\pi}{3}\right);\ B = 140sen^2\left(\frac{5\pi}{6}\right) \text{ y } C = 135sec^2\left(\frac{3\pi}{4}\right)$$

¿Qué álbum deberá ser eliminado para obtener el espacio que necesita?

## RESOLUCIÓN:

$$A = 100sec^2\left(\frac{4\pi}{3}\right) = 100sec^2 240° = 100sec^2(180° + 60°)$$

*IIIC*

$$= 100.(-sec60°)^2 = 100.(-2)^2 = \boxed{400}$$

Usamos el equivalente :

$$\pi \text{rad} = 180°$$

$$B = 140sen^2\left(\frac{5\pi}{6}\right) = 140sen^2 150° = 140sen^2(180° - 30°)$$

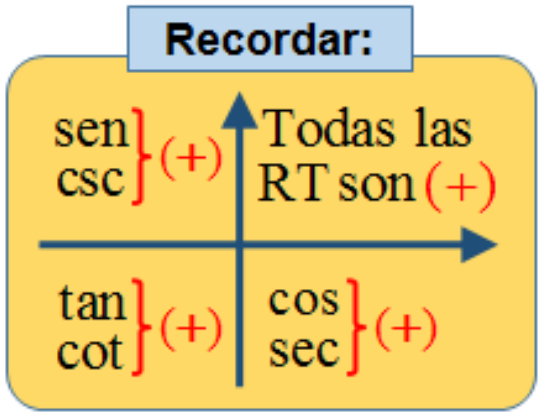
*IIC*

$$= 140(sen^2 30) = 140.\left(\frac{1}{2}\right)^2 = \cancel{140}.\left(\frac{1}{\cancel{4}}\right) = 35$$

$$C = 135sec^2\left(\frac{3\pi}{4}\right) = 135sec^2 135° = 135sec^2(180° - 45°)$$

*IIC*

$$= 135(-sec45°)^2 = 135.\left(-\sqrt{2}\right)^2 = 270$$



**5.** Reduzca: $\mathbf{M = \frac{sen\,(270^\circ + x)}{\cos(180^\circ + x)} - \frac{2\tan(360^\circ - x)}{\cot(90^\circ + x)}}$

**RESOLUCIÓN:**

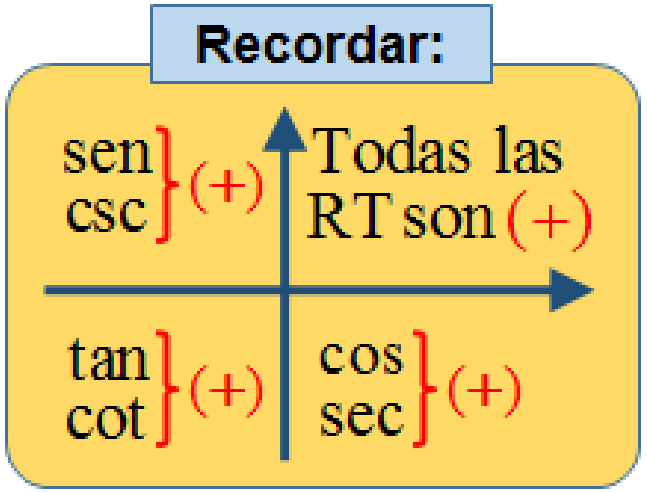
$$M = \frac{\overbrace{sen\,(270^\circ + x)}^{IVC}}{\underbrace{cos(180^\circ + x)}_{IIIC}} - \frac{2\overbrace{tan(360^\circ - x)}^{IVC}}{\underbrace{cot(90^\circ + x)}_{IIC}}$$

$$M = \frac{-\cancel{\cos x}}{-\cancel{\cos x}} - \frac{2(-\cancel{\tan x})}{-\cancel{\tan x}}$$

$$M = 1 - (2)$$

$$\therefore M = -1$$

**Recordar:**




$$RT\left(\begin{matrix}180^\circ \\ 360^\circ\end{matrix} \pm \alpha\right) = \pm RT(\alpha)$$

$$RT\left(\begin{matrix}90^\circ \\ 270^\circ\end{matrix} \pm \alpha\right) = \pm CO - RT(\alpha)$$

**6.** Reduzca: $\mathbf{K} = \dfrac{sen(90^\circ+x).cos(360^\circ-x).csc(270^\circ+x)}{\cos(180^\circ-x)}$

**RESOLUCIÓN:**

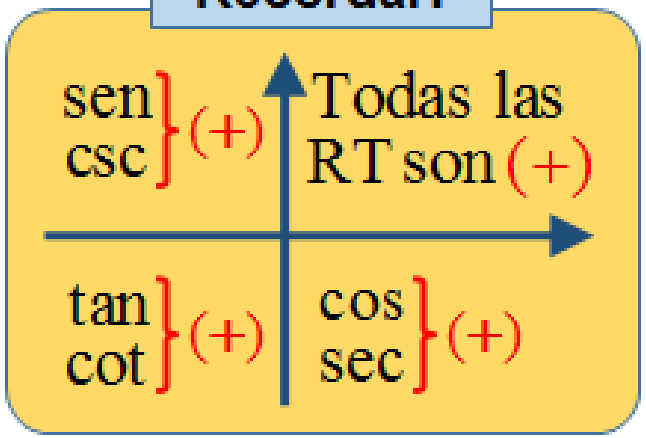
$$K = \frac{\overbrace{sen(90^\circ + x)}^{IIC}.\overbrace{cos(360^\circ - x)}^{IVC}.\overbrace{csc(270^\circ + x)}^{IVC}}{\underbrace{cos(180^\circ - x)}_{IIC}}$$

$$K = \frac{(\cancel{cosx})\,(cosx)(\cancel{-secx})}{(\cancel{-cosx})}$$

$$K = (cosx)(secx)$$

$$\therefore K = 1$$

**Recordar:**




$$RT\left(\begin{matrix}180^\circ\\360^\circ\end{matrix} \pm \alpha\right) = \pm RT(\alpha)$$

$$RT\left(\begin{matrix}90^\circ\\270^\circ\end{matrix} \pm \alpha\right) = \pm CO - RT(\alpha)$$

**7.** Si $\alpha + \beta = 270^\circ$, reduzca: $\mathbf{H} = \frac{\text{sen}\alpha}{\cos\beta} + \tan\alpha . tan\beta$

**RESOLUCIÓN:**

*Del dato:*

$$\alpha + \beta = 270^\circ \Rightarrow \boxed{\beta = 270^\circ - \alpha}$$

*Piden:*

$$H = \frac{sen\alpha}{\cos(270^\circ - \alpha)} + \tan\alpha . \tan(270^\circ - \alpha)$$

IIIC

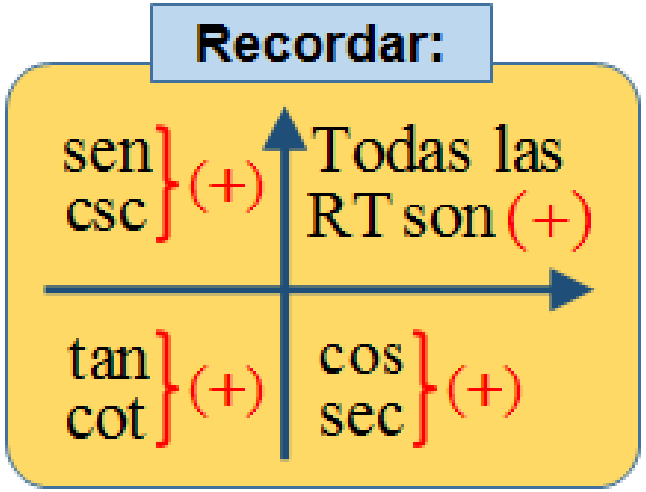
IIIC

$$H = \frac{sen\alpha}{-sen\alpha} + \tan\alpha . (cot\alpha)$$

$-1$ $\quad$ $1$

$$\therefore H = 0$$

**Recordar:**




$$RT\left(\begin{matrix}180^\circ \\ 360^\circ\end{matrix} \pm \alpha\right) = \pm RT(\alpha)$$

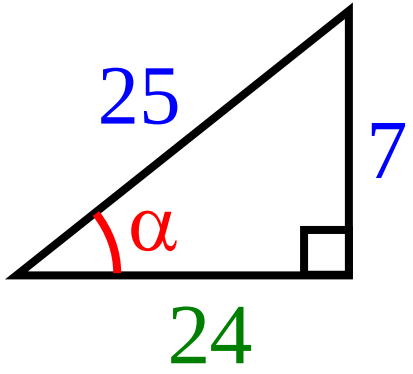
$$RT\left(\begin{matrix}90^\circ \\ 270^\circ\end{matrix} \pm \alpha\right) = \pm CO - RT(\alpha)$$

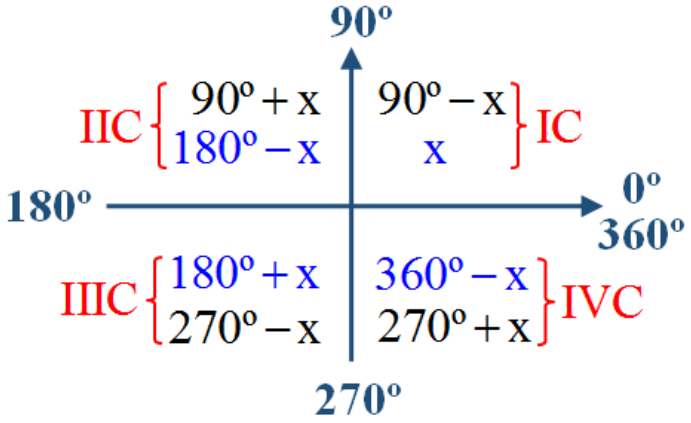
**8.** Si α es ángulo agudo, además: $sen\alpha = \frac{7}{25}$

Calcule: $P = sec(180° + \alpha) - cot(90° + \alpha)$

**RESOLUCIÓN:**

$$sen\alpha = \frac{7}{25} = \frac{CO}{H}$$

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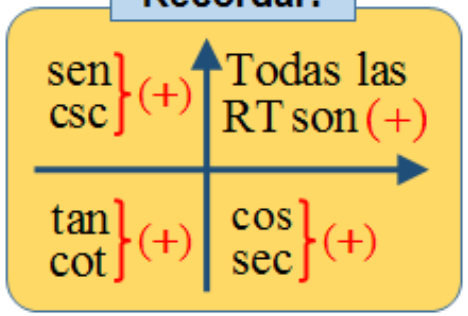


*Piden*:

$$P = sec\underbrace{(180° + \alpha)}_{IIIC} - cot\underbrace{(90° + \alpha)}_{IIC}$$

$$P = -sec\alpha - (-tan\alpha)$$

$$P = -\left(\frac{25}{24}\right) - \left(-\frac{7}{24}\right) = -\frac{25}{24} + \frac{7}{24}$$

$$P = -\frac{18}{24}$$

$$\therefore P = -\frac{3}{4}$$