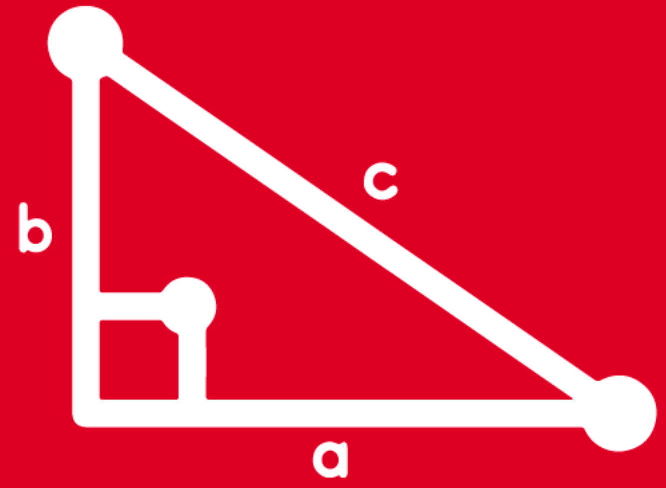
TRIGONOMETRY

Chapter 18

5th
SECONDARY



IDENTIDADES TRIGONOMÉTRICAS DEL ANGULO TRIPLE

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MOTIVATING STRATEGY

Para deducir las identidades del cos(2x), cos(3x), cos(4x), cos(5x) etc; se puede usar la siguiente expresión:

$$\cos(nx) = 2\cos(x)\cos(nx - x) - \cos(nx - 2x)$$

* Para $n = 2$

$$\Rightarrow \cos(2x) = 2\cos(x)\cos(2x - x) - \cos(2x - 2x)$$

$$\Rightarrow \cos(2x) = 2\cos(x)\cos(x) - \cos(0x)$$

$$\therefore\ \cos(2x) = 2\cos^2(x) - 1$$

* Para $n = 3$

$$\Rightarrow \cos(3x) = 2\cos(x)\cos(3x - x) - \cos(3x - 2x)$$

$$\Rightarrow \cos(3x) = 2\cos(x)\cos(2x) - \cos(x)$$

$$\Rightarrow \cos(3x) = 2\cos(x)\left[2\cos^2(x) - 1\right] - \cos(x)$$

$$\therefore\ \cos(3x) = 4\cos^3(x) - 3\cos(x)$$

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO TRIPLE

Para el seno :

$$\boxed{\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3x}$$

Para el coseno :

$$\boxed{\cos 3x = 4\cos^3x - 3\cos x}$$

Para la tangente :

$$\boxed{\tan 3x = \frac{3\tan x - \tan^3x}{1 - 3\tan^2x}}$$

Ejemplos :

- $3\operatorname{sen}10° - 4\operatorname{sen}^3 10° = \operatorname{sen}30° = \boxed{\frac{1}{2}}$
- $4\cos^3 15° - 3\cos 15° = \cos 45° = \boxed{\frac{\sqrt{2}}{2}}$
- $\dfrac{3\tan 20° - \tan^3 20°}{1 - 3\tan^2 20°} = \tan 60° = \boxed{\sqrt{3}}$

IDENTIDADES AUXILIARES

1. $\boxed{\operatorname{sen}3x = \operatorname{sen}x\left(2\cos 2x + 1\right)}$

2. $\boxed{\cos 3x = \cos x\left(2\cos 2x - 1\right)}$

DEMOSTRACIÓN 1.

Sea : $\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3 x$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 4\operatorname{sen}^2 x\right)$$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2 \times 2\operatorname{sen}^2 x\right)$$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2 \times (1 - \cos 2x)\right)$$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2 + 2\cos 2x\right)$$

$$\therefore\ \operatorname{sen}3x = \operatorname{sen}x\left(2\cos 2x + 1\right)$$

IDENTIDADES AUXILIARES

3. $\boxed{4\,\mathrm{sen}x\,\mathrm{sen}(60°-x)\,\mathrm{sen}(60°+x)=\mathrm{sen}3x}$

4. $\boxed{4\cos x\cos(60°-x)\cos(60°+x)=\cos 3x}$

5. $\boxed{\tan x\tan(60°-x)\tan(60°+x)=\tan 3x}$

Ejemplo: Calcular $E=8\,\mathrm{sen}10°\,\mathrm{sen}50°\,\mathrm{sen}70°$

Resolución:

Dando forma: $E=2\times\underbrace{4\,\mathrm{sen}10°\,\mathrm{sen}(60°-10°)\,\mathrm{sen}(60°+10°)}$ ➔ $E=2\times\frac{1}{2}$

Usando Ident Aux 3. ⟶ $\mathrm{sen}(3\times10°)=\mathrm{sen}(30°)$ $\therefore\ \boxed{E=1}$

HELICO-PRACTICE 1

Reduzca $E = \dfrac{4\cos^3 15° - 3\cos 15°}{3\text{sen}10° - 4\text{sen}^3 10°}$

Resolución:

RECORDAR:

$$\boxed{\text{sen}3x = 3\,\text{sen}x - 4\,\text{sen}^3 x}$$

$$\boxed{\cos 3x = 4\cos^3 x - 3\cos x}$$

$$E = \dfrac{\overbrace{4\cos^3 15° - 3\cos 15°}^{\cos 3(15°)}}{\underbrace{3\text{sen}10° - 4\text{sen}^3 10°}_{\text{sen}3(10°)}}$$

$$\Rightarrow E = \dfrac{\cos 45°}{\text{sen}30°}$$

$$\Rightarrow E = \dfrac{\left[\dfrac{\sqrt{2}}{\cancel{2}}\right]}{\left[\dfrac{1}{\cancel{2}}\right]}$$

$$\therefore \boxed{E = \sqrt{2}}$$

HELICO-PRACTICE 2

Si se cumple que: $\operatorname{sen}\theta = \dfrac{1}{3}$; calcule sen3θ.

Resolución:

RECORDAR:

$$\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3 x$$

Del dato:

$$\operatorname{sen}\theta = \frac{1}{3}$$

Reemplazando:

$$\operatorname{sen}3\theta = 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$$

$$\operatorname{sen}3\theta = 1 - \frac{4}{27}$$

$$\therefore\ \boxed{\operatorname{sen}3\theta = \frac{23}{27}}$$

HELICO-PRACTICE 3

Simplifique la expresión: $E=\dfrac{\text{sen}3x+\text{sen}x}{\cos^2 x}$

Resolución:

RECORDAR:

$\text{sen}3x=3\,\text{sen}x-4\,\text{sen}^3x$

Reemplazando **sen3x :**

$$E=\frac{3\text{sen}x-4\text{sen}^3x+\text{sen}x}{\cos^2x}$$

$$E=\frac{4\text{sen}x-4\text{sen}^3x}{\cos^2x}$$

$$E=\frac{4\text{sen}x(1-\text{sen}^2x)}{\cos^2x}$$

Usar la identidad

$\cos^2x=1-\text{sen}^2x$

$$E=\frac{4\text{sen}x(\cancel{\cos^2x})}{\cancel{\cos^2x}}$$

$$\boxed{\therefore E=4\text{sen}x}$$

HELICO-PRACTICE 4

Si un ángulo agudo β, cumple que $\csc\beta = \sqrt{10}$; calcule: tan3β

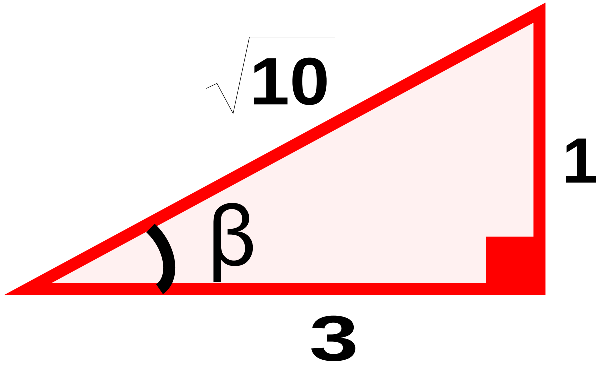
Resolución:

RECORDAR:

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

Dato: $\csc\beta = \sqrt{10}$

$$\csc\beta = \frac{\sqrt{10}}{1} = \frac{H}{CO} \Rightarrow \tan\beta = \frac{1}{3}$$



Reemplazando en tan3β:

$$\tan 3\beta = \frac{3\left(\frac{1}{3}\right) - \left(\frac{1}{3}\right)^3}{1 - 3\left(\frac{1}{3}\right)^2} = \frac{1 - \frac{1}{27}}{1 - \frac{1}{3}} = \frac{\frac{26}{27}}{\frac{2}{3}}$$

$$\therefore \tan 3\beta = \frac{13}{9}$$

HELICO-PRACTICE 5

De la condición: $\text{senx}+\text{cosx}=\dfrac{\sqrt{3}}{2}$. Calcule: sen6x

Resolución:

Dato:

$$\text{senx} + \text{cosx} = \frac{\sqrt{3}}{2}$$

Elevamos al cuadrado:

$$\underbrace{(\textbf{senx} + \textbf{cosx})^2}_{1+\text{sen}2x} = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$1 + \text{sen}2x = \frac{3}{4}$$

$$\text{sen}2x = -\frac{1}{4}$$

Piden:

$$\text{sen}6x = 3\text{sen}2x - 4\text{sen}^3 2x$$

$$\text{sen}6x = 3\left(-\frac{1}{4}\right) - 4\left(-\frac{1}{4}\right)^3$$

$$\text{sen}6x = -\frac{3}{4} + \frac{1}{16}$$

$$\boxed{\therefore\ \text{sen}6x = -\frac{11}{16}}$$

RECORDAR:

$$\boxed{(\text{senx} + \text{cosx})^2 = 1 + \text{sen}(2x)}$$

$$\boxed{\text{sen}3\theta = 3\text{sen}\theta - 4\text{sen}3\theta}$$

HELICO-PRACTICE 6

De la siguiente identidad: $\frac{3sen3x}{senx}+\frac{2cos3x}{cosx}=A+Bcos(Cx)$

calcule A + B + C

Resolución:

Dato:

$$\frac{3sen3x}{senx}+\frac{2cos3x}{cosx}=A+Bcos(Cx)$$

$$\frac{3\cancel{senx}(2cos2x+1)}{\cancel{senx}}+\frac{2\cancel{cosx}(2cos2x-1)}{\cancel{cosx}}=A+Bcos(Cx)$$

$$3(2cos2x+1)+2(2cos2x-1)=A+Bcos(Cx)$$

$$6cos2x+3+4cos2x-2=A+Bcos(Cx)$$

$$1+10cos2x+=A+Bcos(Cx)$$

RECORDAR:

sen3x = senx(2cos2x + 1)

cos3x = cosx(2cos2x – 1)

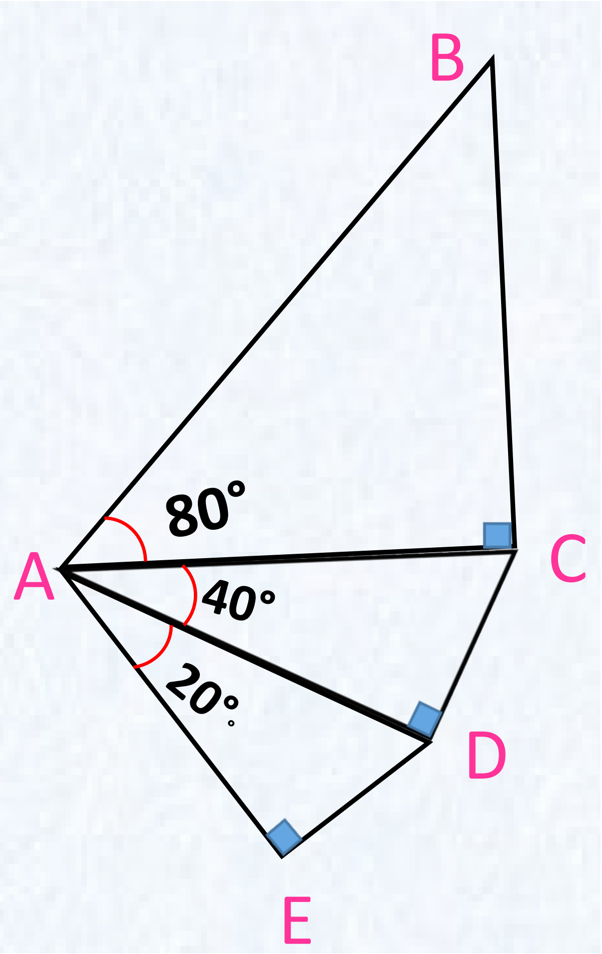
Comparando:

$A=1\,;\ B=10\,;\ C=2$

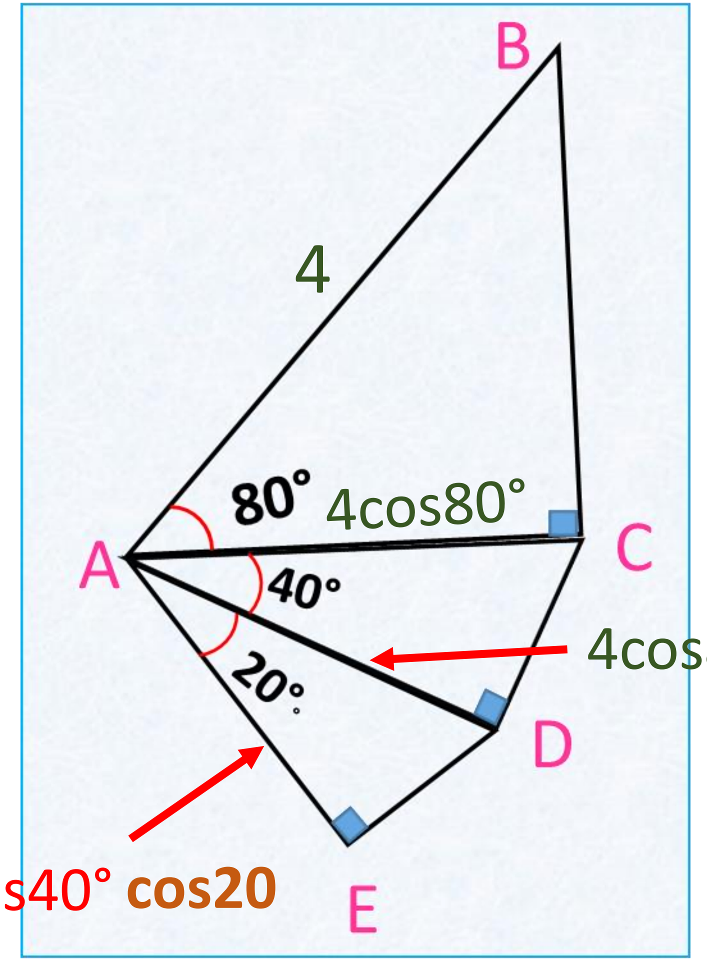
$$\boxed{\therefore\ A+B+C=13}$$

HELICO-PRACTICE 7

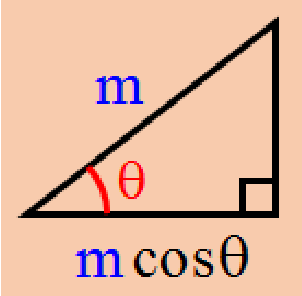
En la figura, AB = 4. Halle AE



Resolución:



Recordar:



$$4\cos x\cos(60°-x)\cos(60°+x)=\cos 3x$$

Piden:

$$AE = 4\cos 20°\cos 40°\cos 80°$$

$$AE = \cos 3(20°)$$

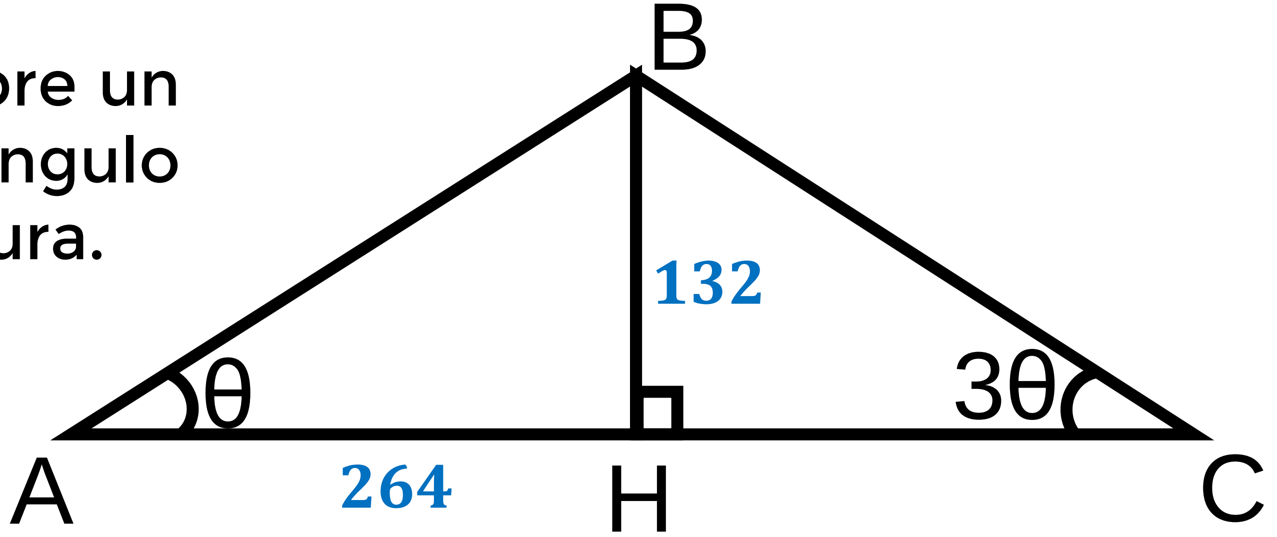
$$AE = \cos 60°$$

$$\therefore\ AE = \frac{1}{2}$$

HELICO-PRACTICE 8

Se construye un centro comercial sobre un terreno que tiene la forma de un triángulo ABC como el que se muestra en la figura.

Si BH = 132m y AH = 264m, ¿cuál es la longitud de HC?



Resolución:

Del gráfico:

$$\blacktriangle AHB:\ \tan\theta = \frac{\cancel{132}}{\cancel{264}}$$

$$\Rightarrow \tan\theta = \frac{1}{2}$$

$$\blacktriangle BHC:\ \tan 3\theta = \frac{132}{HC} \ \dots (*)$$

Luego: $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$$\Rightarrow \tan 3\theta = \frac{3\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^3}{1 - 3\left(\frac{1}{2}\right)^2} = \frac{\frac{3}{2} - \frac{1}{8}}{1 - \frac{3}{4}} = \frac{\frac{11}{8}}{\frac{1}{4}}$$

$$\Rightarrow \tan 3\theta = \frac{11}{2}$$

En $(*)$: $\dfrac{11}{2} = \dfrac{132}{HC}$

$$\boxed{\therefore HC = 24m}$$