GEOMETRÍA

Capítulo 12

5th
SECONDARY

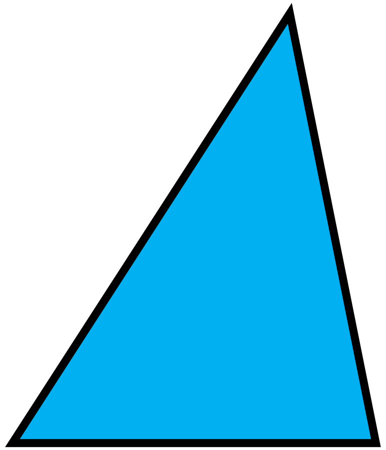
ÁREAS DE REGIONES TRIANGULARES

MOTIVATING | STRATEGY

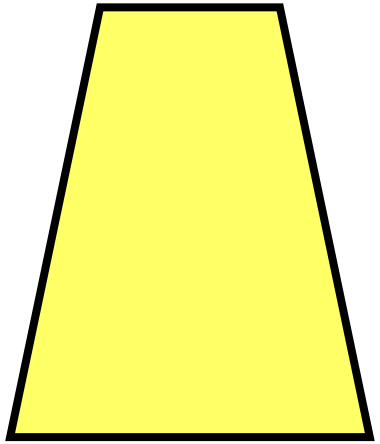
ÁREAS DE REGIONES TRIANGULARES

REGIÓN PLANA.-

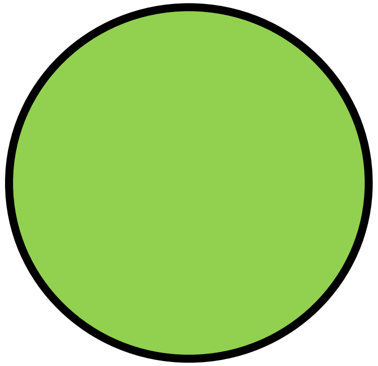
Es una porción del plano limitada por una línea abierta o cerrada.



Región Triangular



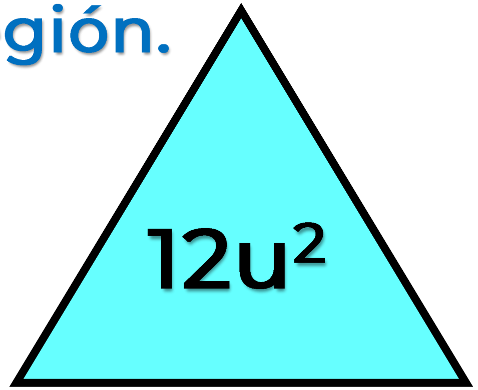
Región Cuadrangular



Región Circular

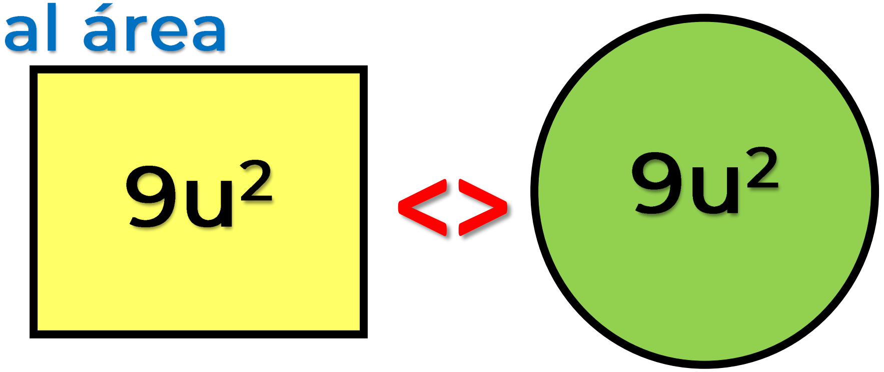
ÁREA.-

Es la medida de una región.

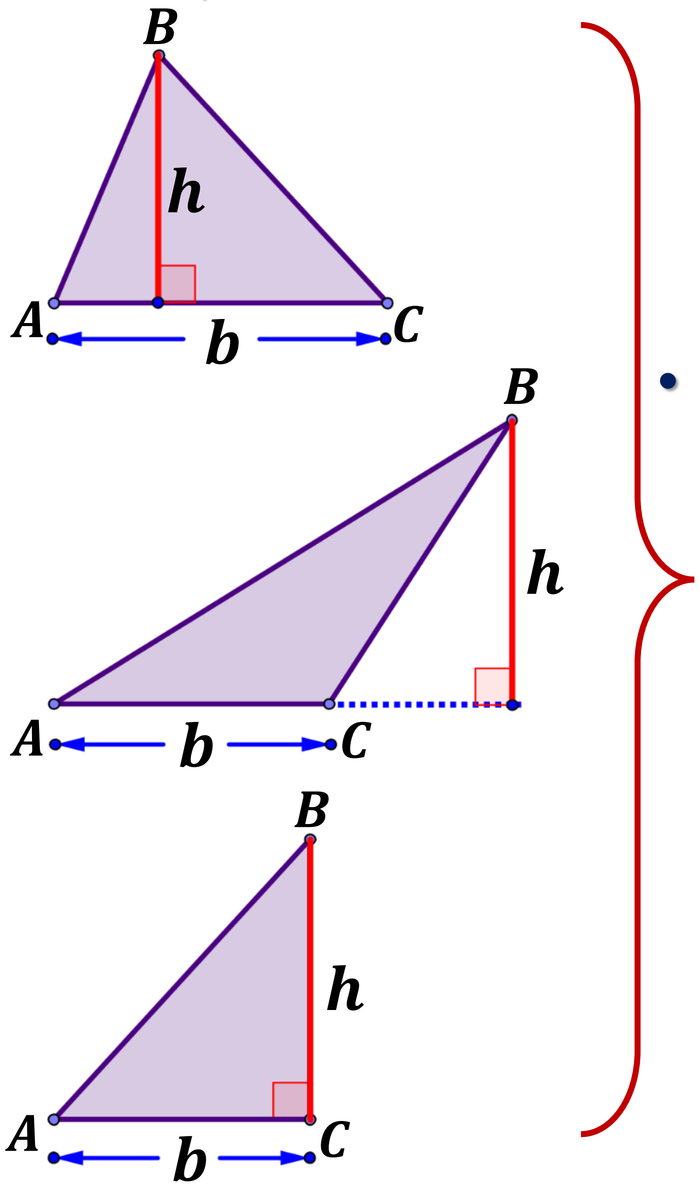


$A_{\triangle} = 12u^2$

REGIONES EQUIVALENTES.- Son Aquellas regiones que tienen igual área



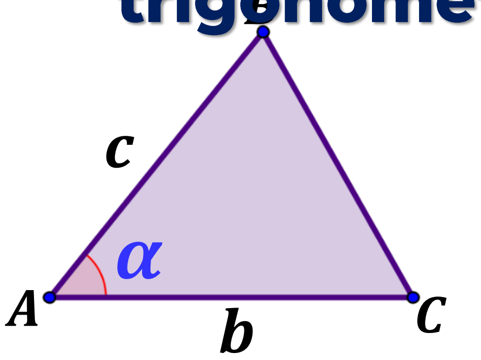
TEOREMAS



- **Teorema básico:**

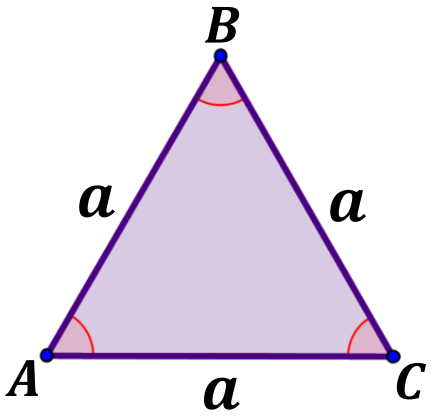
$$S_{ABC} = \frac{bh}{2}$$

- **Teorema trigonométrico:**



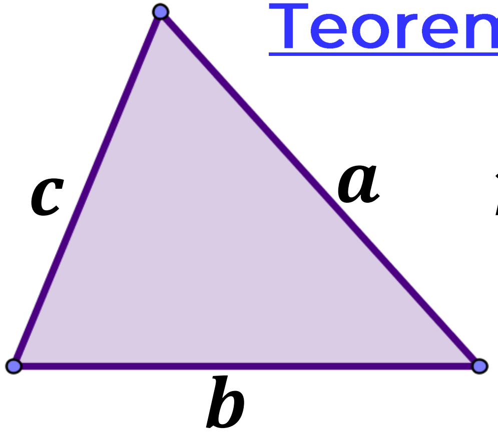
$$S_{ABC} = \frac{bc}{2} \cdot sen\alpha$$

- **Área de una región triangular equilátera:**



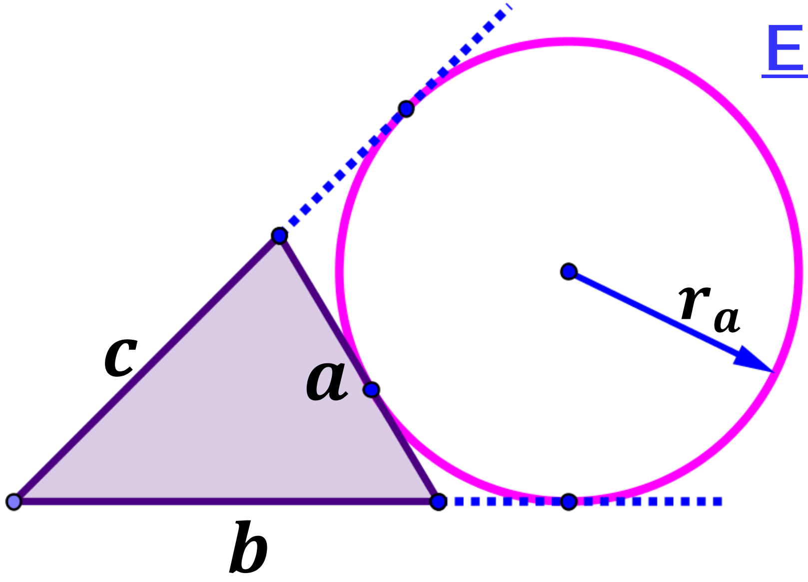
$$S_{ABC} = a^2\frac{\sqrt{3}}{4}$$

Teorema de Herón



$$p=\frac{a+b+c}{2}$$

$$\boxed{S=\sqrt{p(p-a)(p-b)(p-c)}}$$

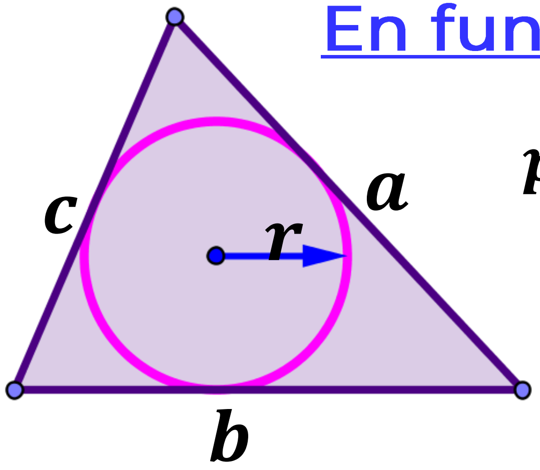


En función al exradio

$$p=\frac{a+b+c}{2}$$

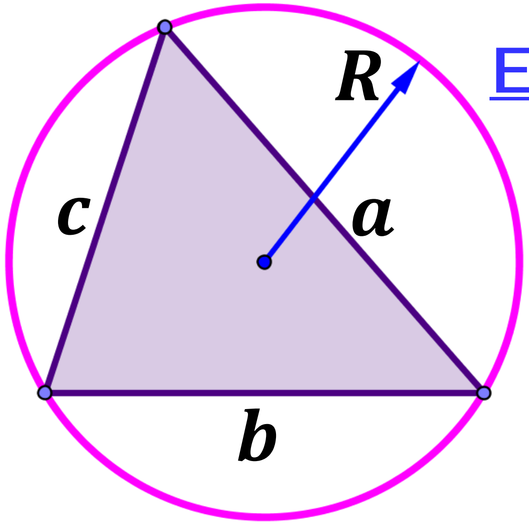
$$\boxed{S=(p-a)\cdot r_a}$$

En función al inradio



$$p=\frac{a+b+c}{2}$$

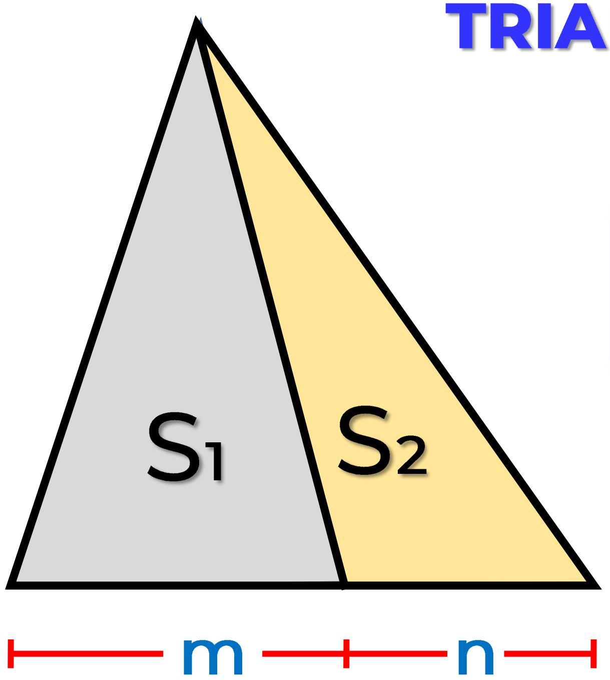
$$\boxed{S=p\cdot r}$$



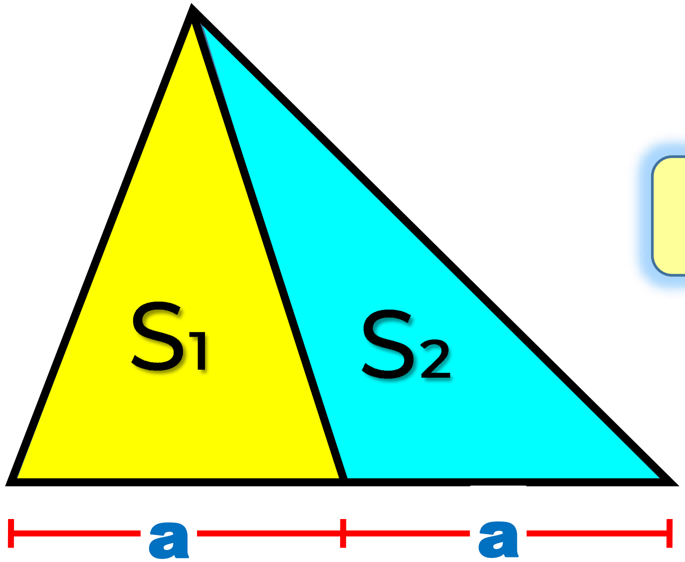
En función al circunradi

$$\boxed{S=\frac{abc}{R}}$$

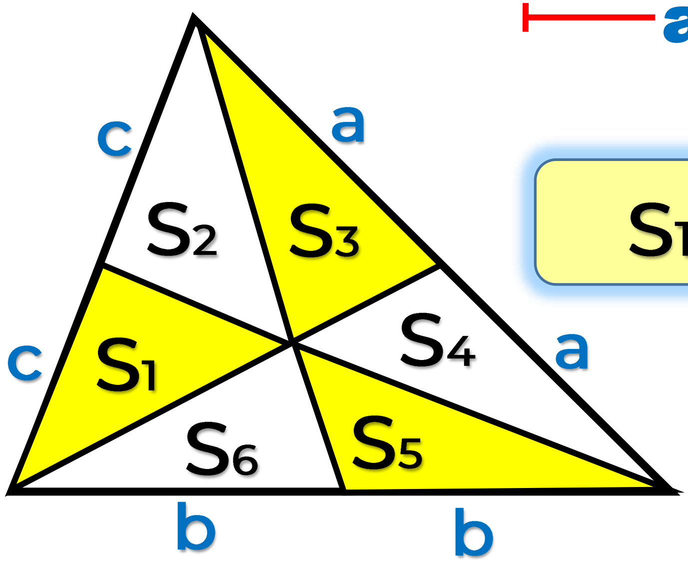
RELACIONES ENTRE ÁREAS DE REGIONES TRIANGULARES



$$\frac{S_1}{S_2} = \frac{m}{n}$$



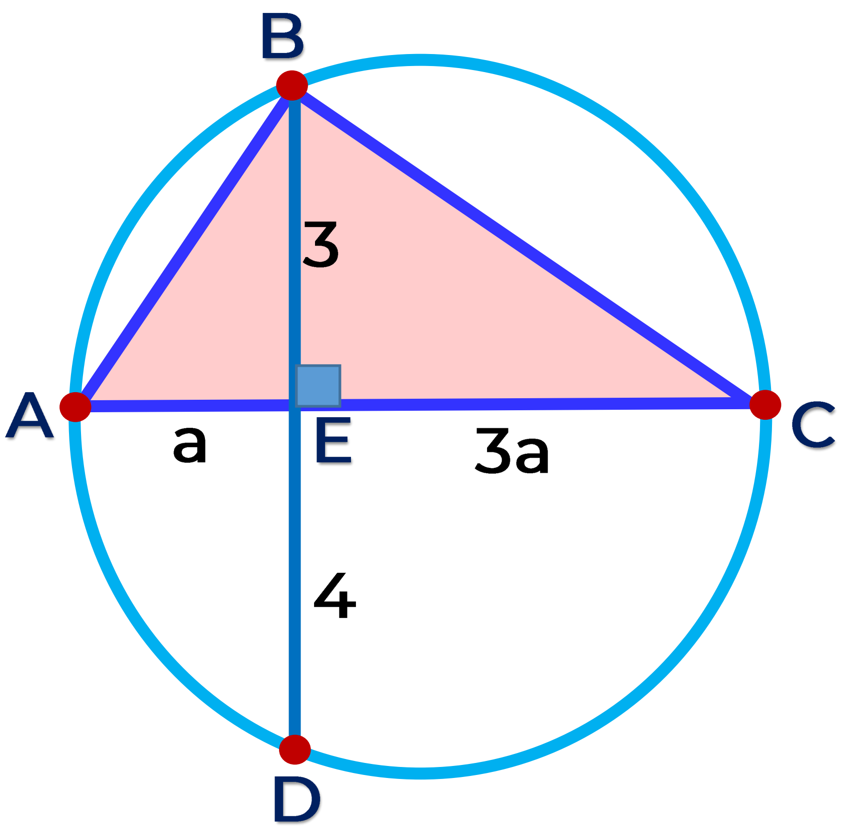
$$S_1 = S_2$$

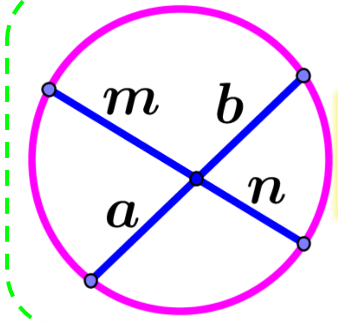


$$S_1 = S_2 = S_3 = S_4 = S_5 = S_6$$

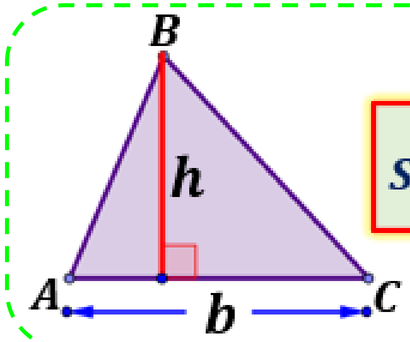
1. Calcule el área de la región triangular ABC si BE = 3, ED = 4 y EC = 3AE.

Resolución





$$ab = mn$$

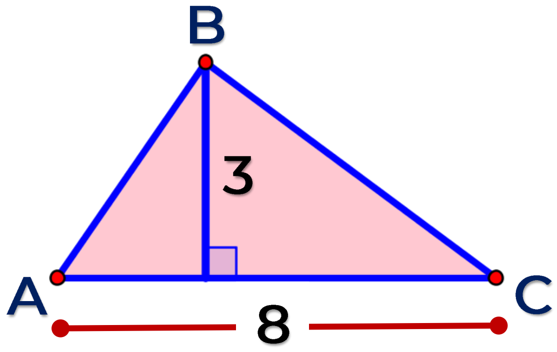


$$S_{ABC} = \frac{bh}{2}$$

$$\cancel{3}a \times a = \cancel{3} \times 4$$

$$a^2 = 4$$

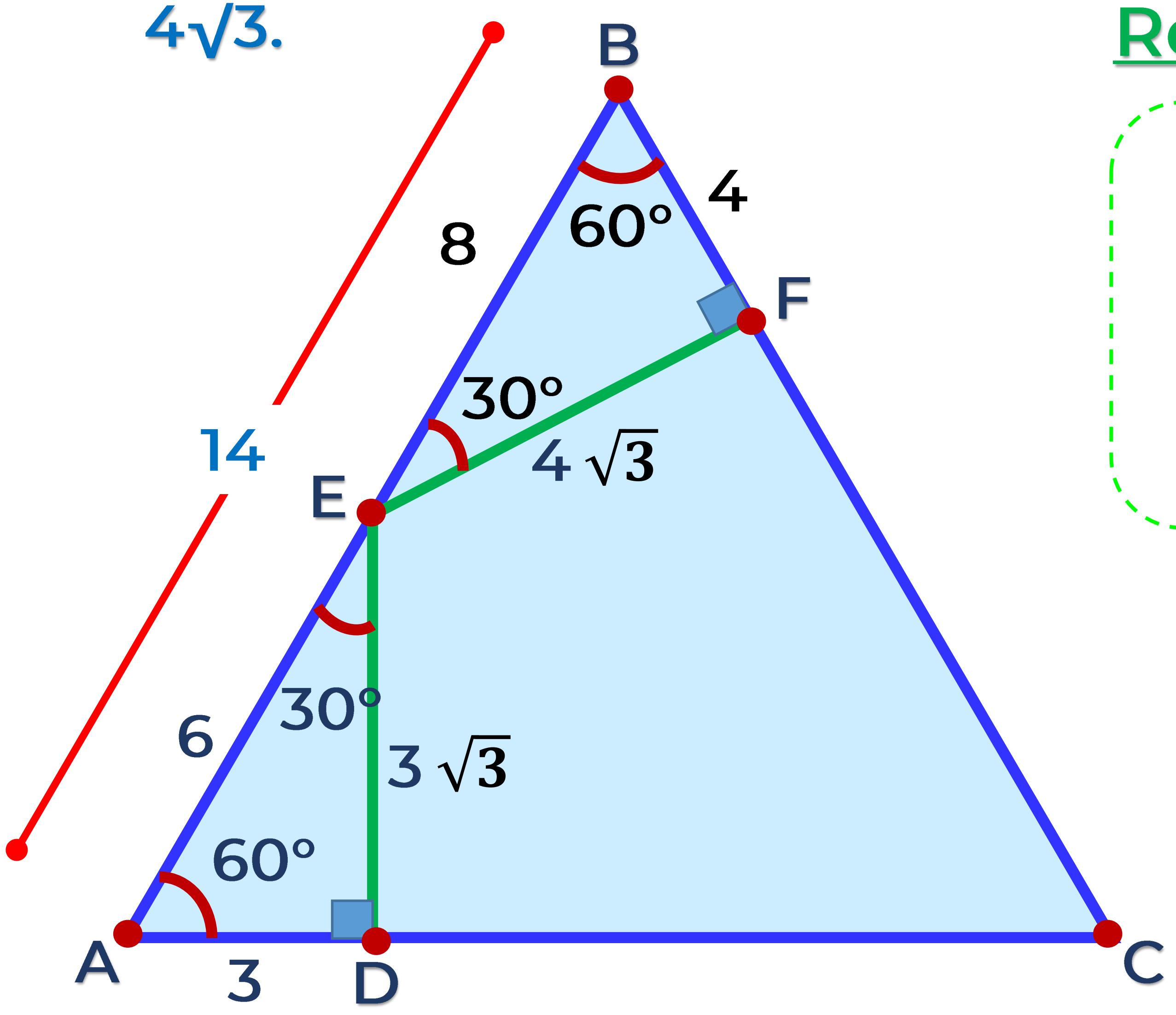
$$a = 2$$



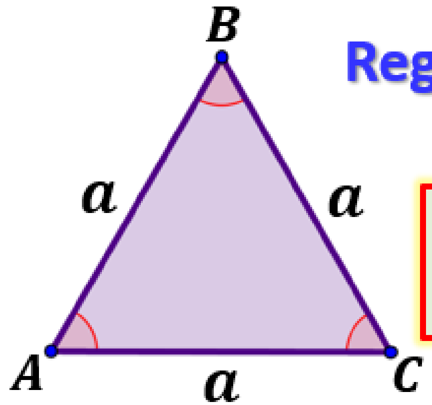
$$S_{ABC} = \frac{8 \times 3}{2}$$

$$\boxed{S_{ABC} = 12u^2}$$

2. Calcule el área de la región triangular equilátera si ED = $3\sqrt{3}$ y EF = $4\sqrt{3}$.



Resolución



Región triangular equilátera

$$S_{ABC} = a^2\frac{\sqrt{3}}{4}$$

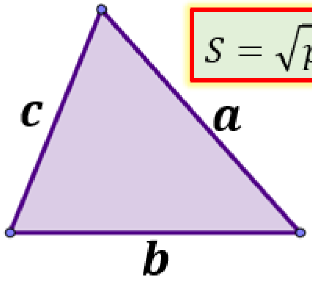
$$S_{ABC} = \frac{14^2\sqrt{3}}{4}$$

$$\boxed{S_{ABC} = 49\sqrt{3}\ u^2}$$

3. Las longitudes de los lados de una región triangular son: 12; 17 y 25. Calcule su área.

Resolución

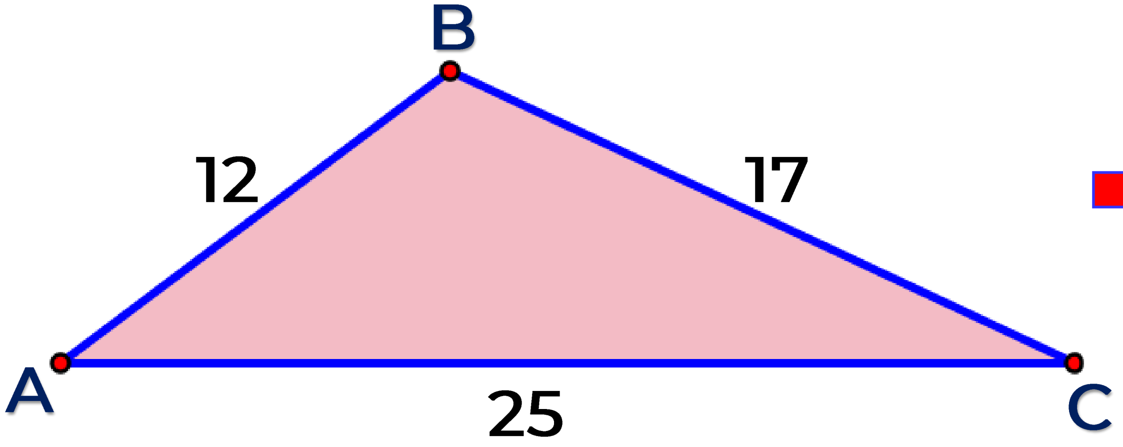
Teorema de Herón



$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

donde:

$$p = \frac{a+b+c}{2}$$



$$p = \frac{12+17+25}{2} = 27$$

$$S_{ABC} = \sqrt{27(27-12)(27-17)(27-25)}$$

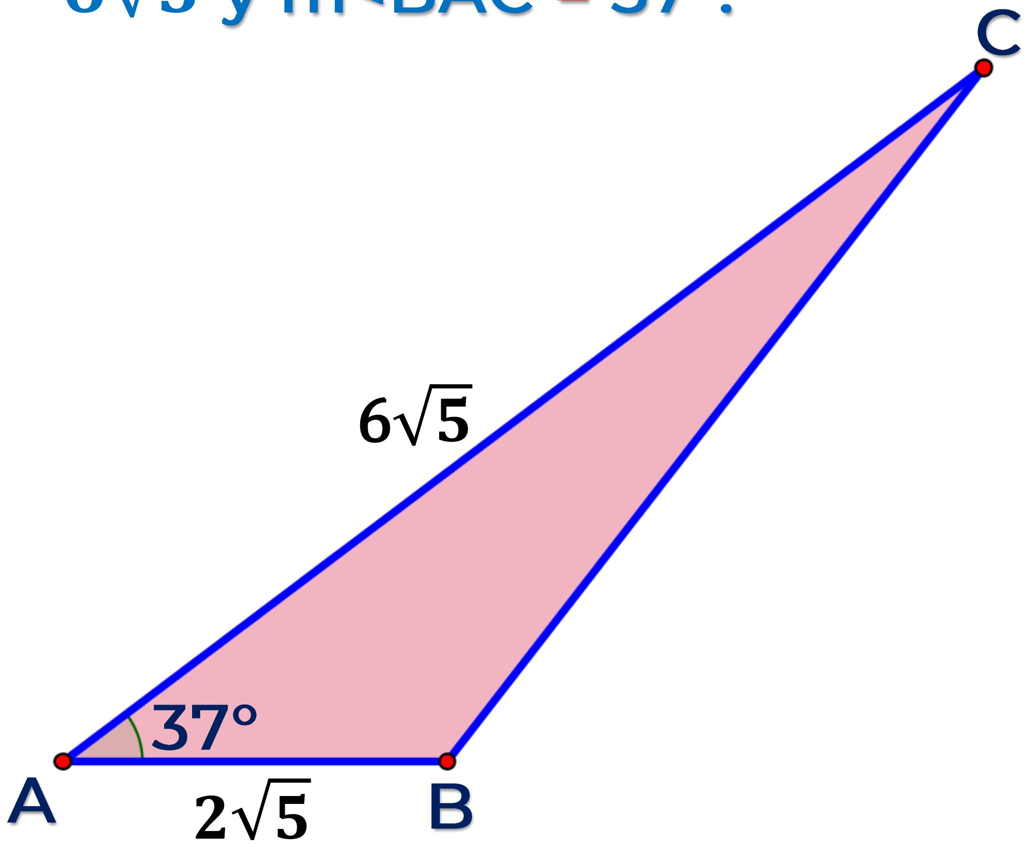
$$S_{ABC} = \sqrt{27(15)(10)(2)}$$

$$S_{ABC} = \sqrt{9\cdot 3\cdot 5\cdot 3\cdot 5\cdot 2\cdot 2}$$

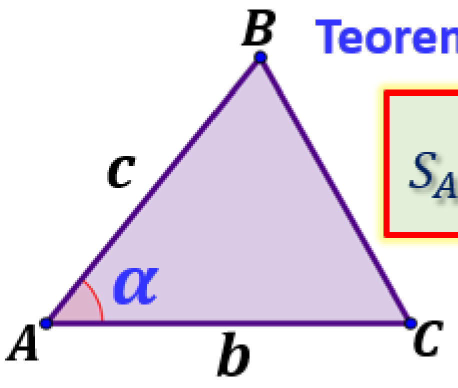
$$S_{ABC} = 3\cdot 3\cdot 5\cdot 2$$

$$\boxed{S_{ABC} = 90\ u^2}$$

4. Calcule el área de una región triangular ABC si AB = $2\sqrt{5}$, AC = $6\sqrt{5}$ y m<BAC = 37°.



Resolución



Teorema trigonométrico

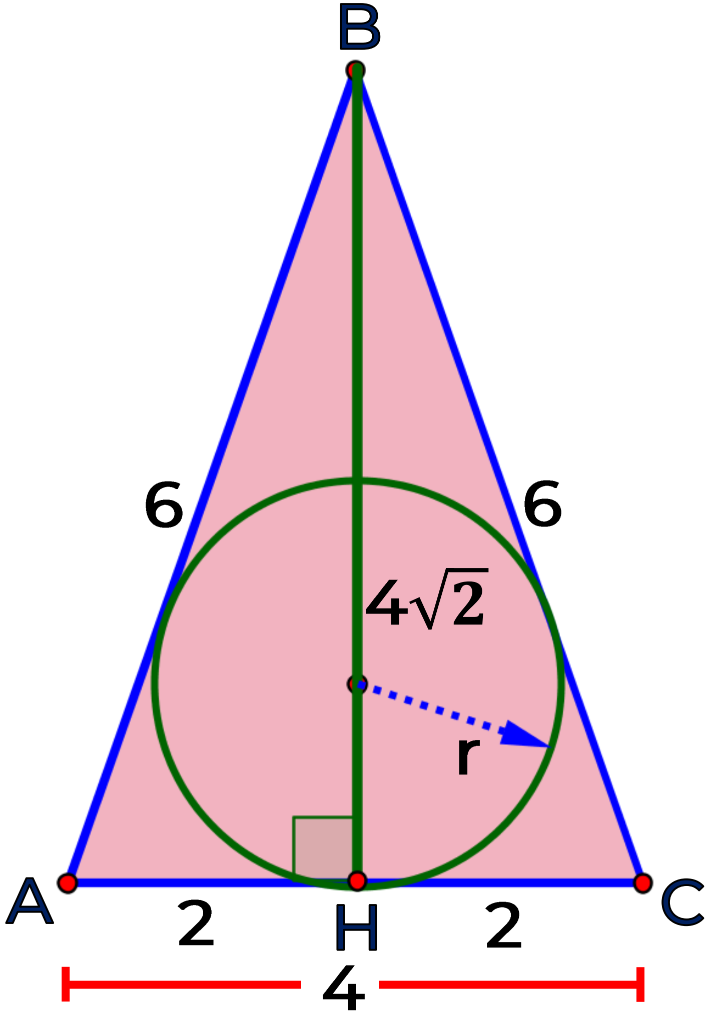
$$\boxed{S_{ABC}=\frac{bc}{2}\cdot sen\alpha}$$

$$S_{ABC}=\frac{\cancel{2}\sqrt{5}\cdot 6\sqrt{5}}{\cancel{2}}sen37°$$

$$S_{ABC}=\cancel{5}\cdot 6\cdot\frac{3}{\cancel{5}}$$

$$\boxed{S_{ABC}=18\ u^2}$$

5. Las longitudes de los lados de un triángulo son: 4; 6 y 6. Halle la longitud de su inradio.

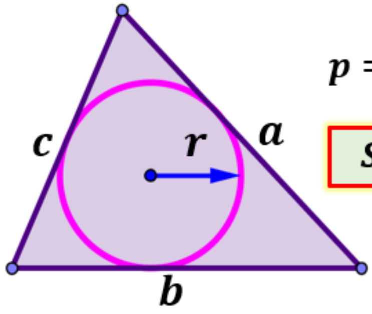


Resolución

- △ABC : Isósceles
- ⊿BCH : T. Pitágoras

$$6^2 = (BH)^2 + 2^2$$

$$4\sqrt{2} = BH$$



$$p = \frac{a+b+c}{2}$$

$$\boxed{S = p \cdot r}$$

$$S_{ABC} = \frac{4 \times 4\sqrt{2}}{2}$$

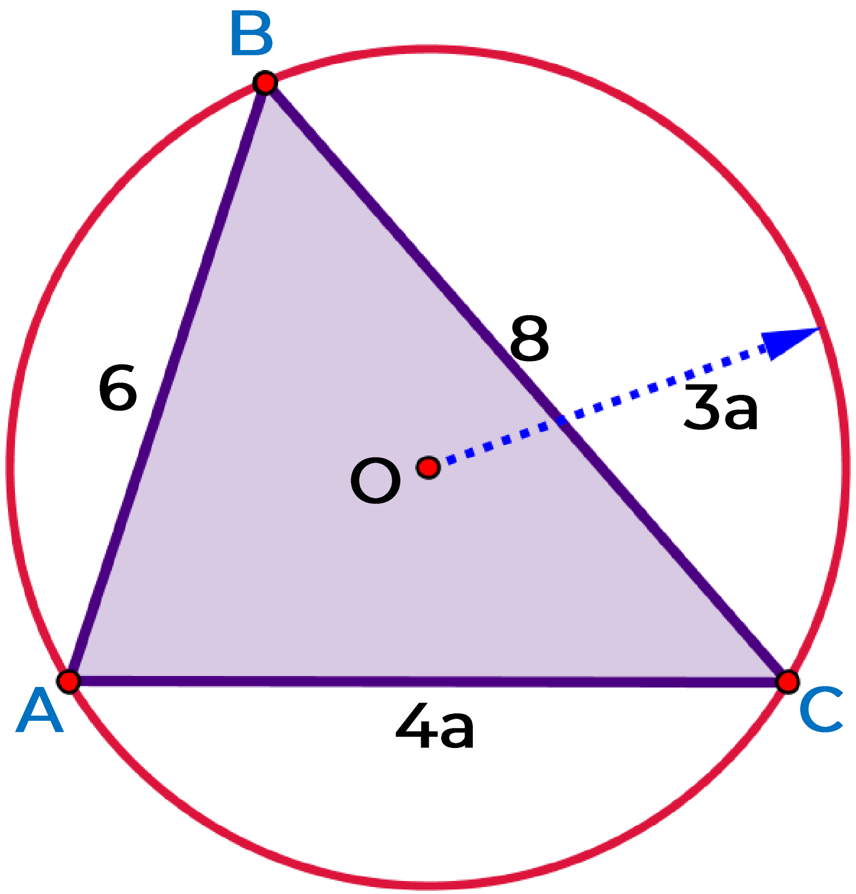
$$\underbrace{S_{ABC}} = 8\sqrt{2}$$

$$p \cdot r = 8\sqrt{2}$$

$$\cancel{8} \cdot r = \cancel{8}\sqrt{2}$$

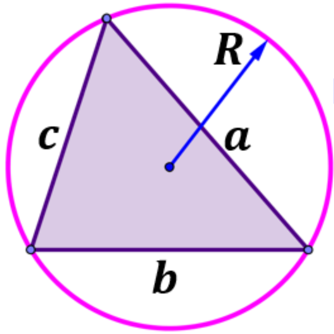
$$\boxed{r = \sqrt{2}}$$

6. Calcule el área de la región triangular ABC, si O es centro.



Resolución

- Piden: S_{ABC}



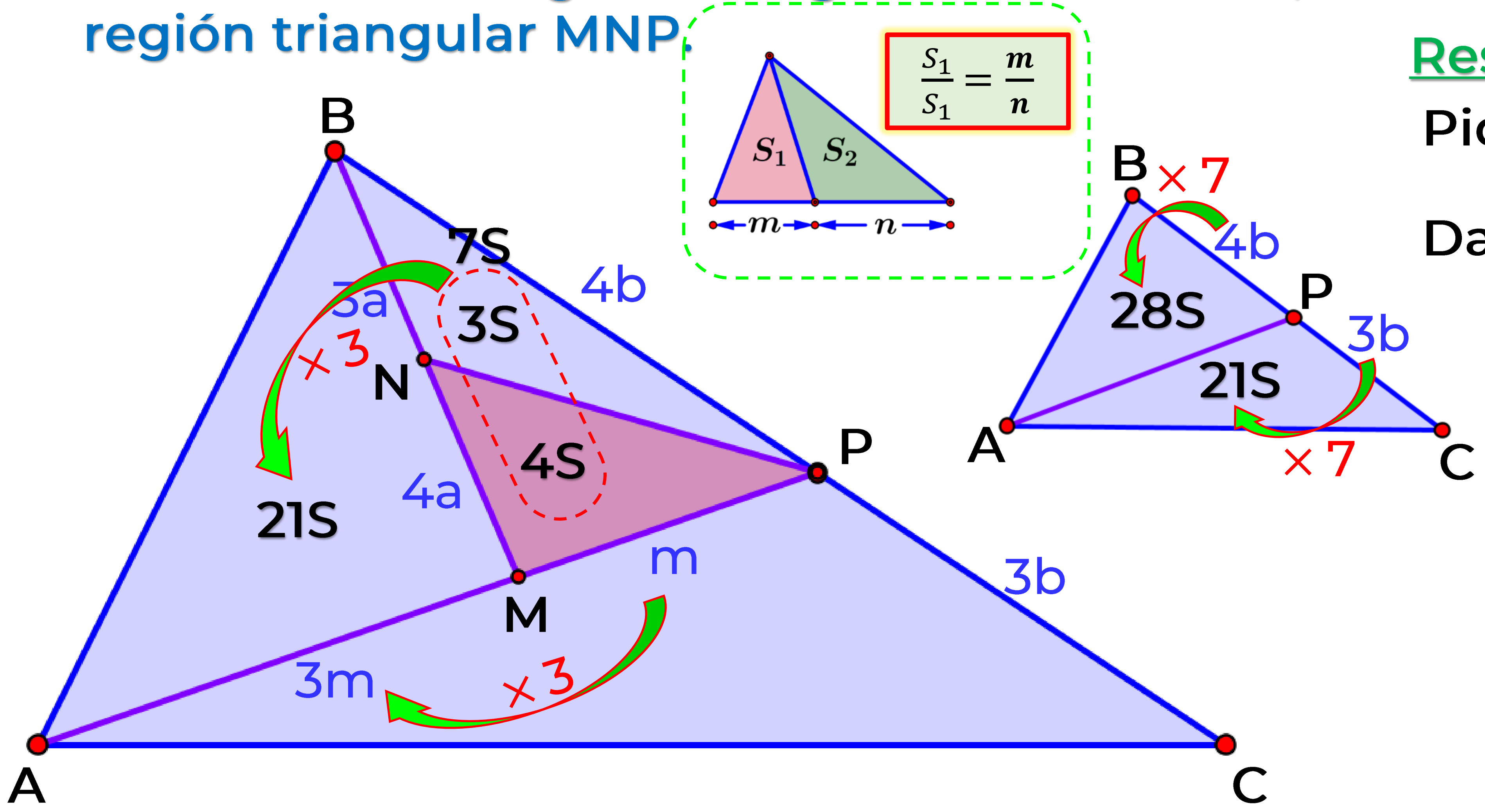
En función del circunradio

$$\boxed{S = \frac{abc}{R}}$$

$$S_{ABC} = \frac{(\overset{2}{\cancel{6}})(8)(4\cancel{a})}{\underset{1}{\cancel{3}}\cancel{a}}$$

$$\boxed{S_{ABC} = 64u^2}$$

7. Si el área de la región triangular ABC es 98u², calcule el área de la región triangular MNP.



$$\frac{S_1}{S_1} = \frac{m}{n}$$

Resolución

Piden: S_{MNP}

Dato: $\underbrace{S_{ABC}}_{49S} = 98$

$49S = 98$

$S = 2$

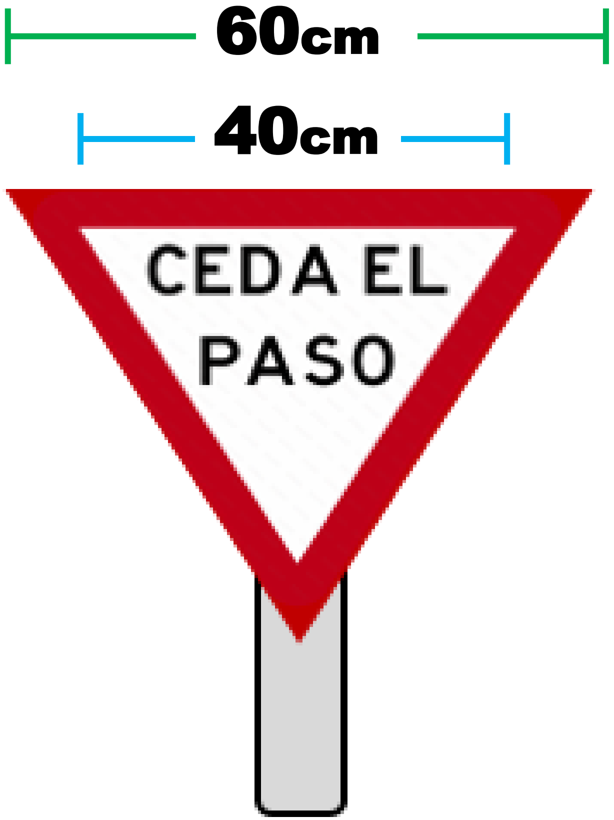
$S_{MNP} = 4S$

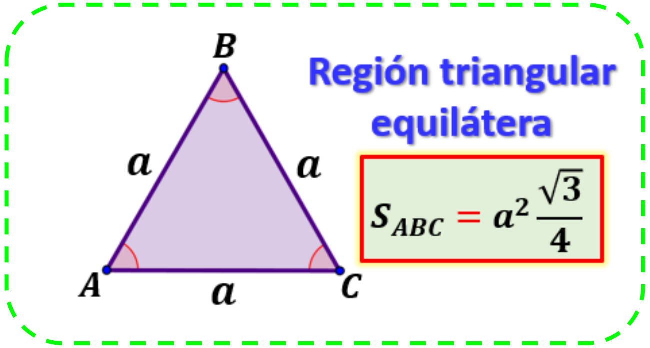
$S_{MNP} = 4(2)$

$\boxed{S_{MNP} = 8\ u^2}$

8. En el gráfico, se muestra una señal de tránsito donde la parte sombreada que se quiere pintar de color rojo, si tiene en sus contornos, dos triángulos equiláteros de lados 60cm y 40cm. Calcule el área de la franja roja.

Resolución





Región triangular equilátera

$$S_{ABC} = a^2\frac{\sqrt{3}}{4}$$

$$S_X = \frac{60^2\sqrt{3}}{4} - \frac{40^2\sqrt{3}}{4}$$

$$S_X = 900\sqrt{3} - 400\sqrt{3}$$

$$\boxed{S_X = 500\sqrt{3}\ cm^2}$$