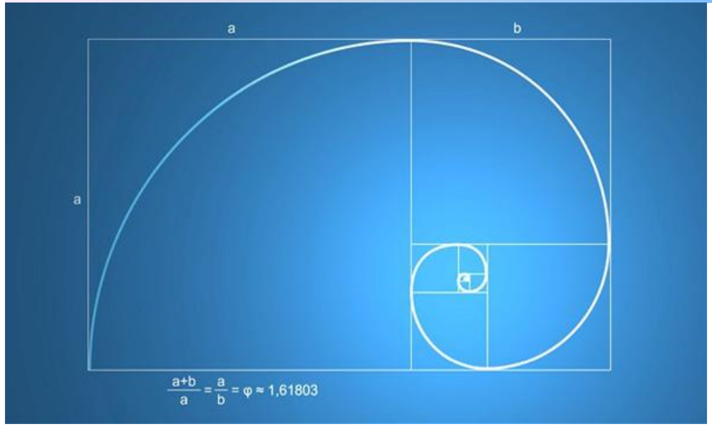
GEOMETRÍA

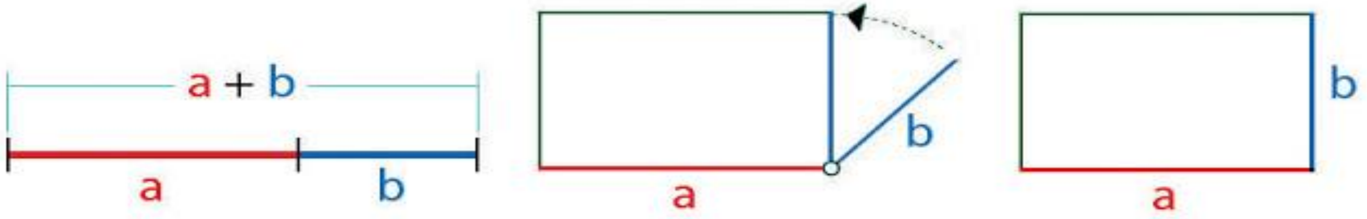
Capítulo 7

5th
SECONDARY

SEGMENTOS PROPORCIONALES

SACO OLIVEROS





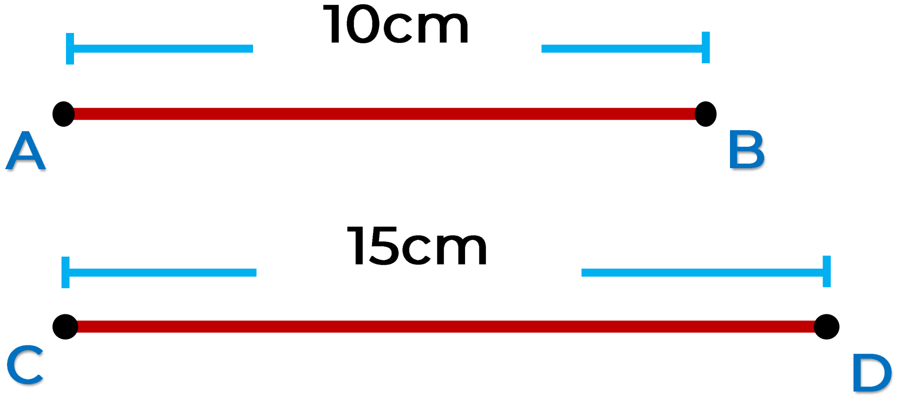
$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399\ldots$$

SEGMENTOS PROPORCIONALES

Razón geométrica de dos segmentos

Es el cociente que se obtiene al dividir las longitudes de dos segmentos que tienen la misma unidad de medida.

Ejemplo:

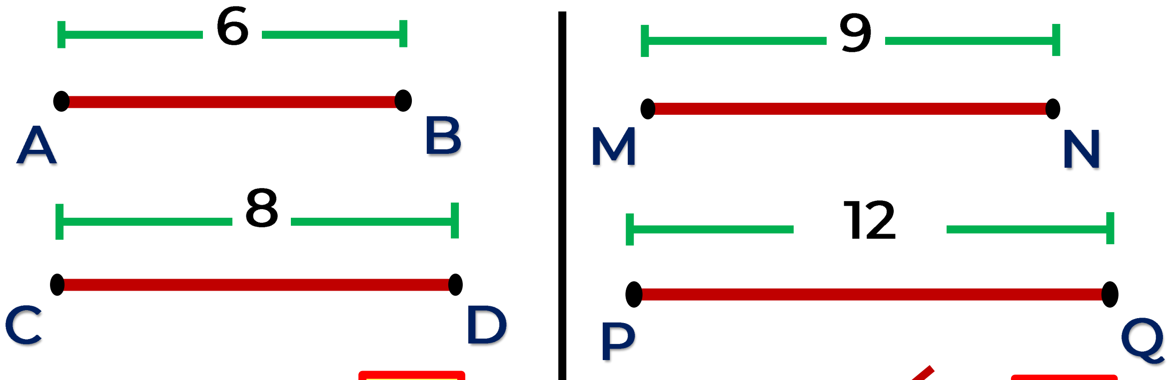


$$\frac{AB}{CD} = \frac{\cancel{10cm}}{\cancel{15cm}} \Rightarrow \frac{AB}{CD} = \frac{2}{3}$$

$\frac{2}{3}$: razón geométrica de $\overline{AB}$ y $\overline{CD}$

Segmentos proporcionales

Si la razón geométrica de 2 segmentos es igual a la de otros dos, dichos pares de segmentos son proporcionales.

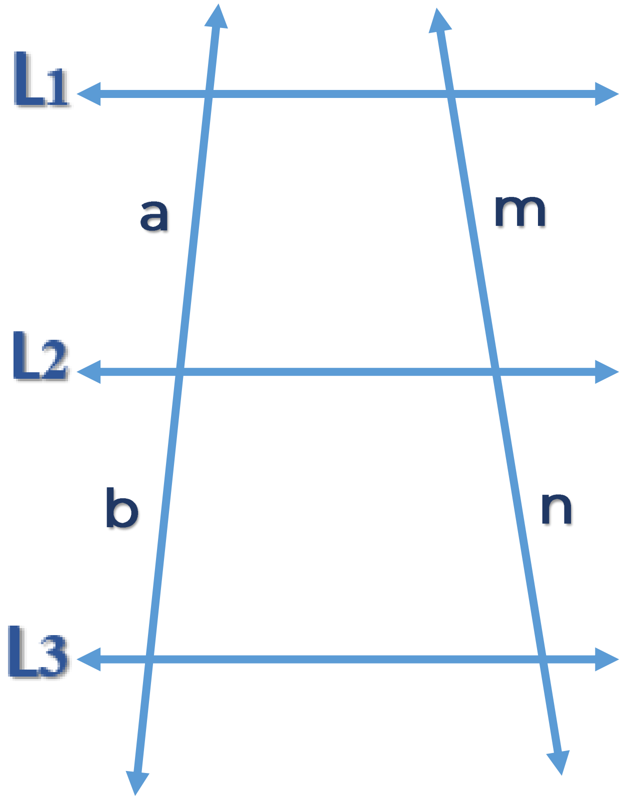


$$\frac{AB}{CD} = \frac{\cancel{6}}{\cancel{8}} = \boxed{\frac{3}{4}}$$

$$\frac{MN}{PQ} = \frac{\cancel{9}}{\cancel{12}} = \boxed{\frac{3}{4}}$$

$$\boxed{\frac{AB}{CD} = \frac{MN}{PQ}}$$ ➡ Son proporcionales

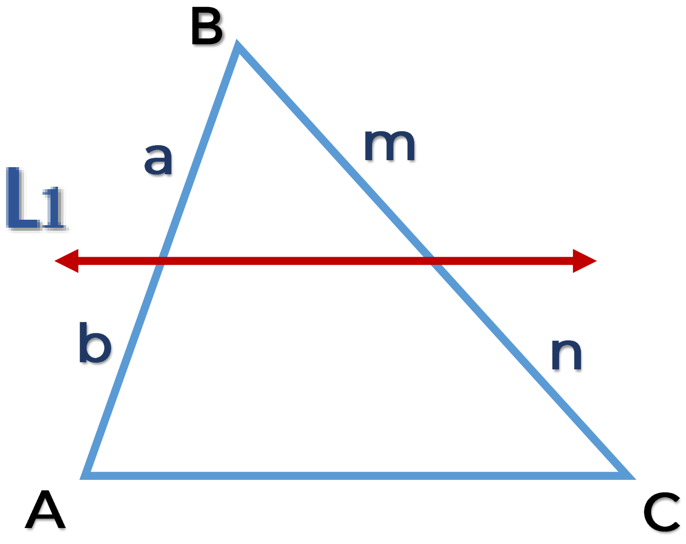
Teorema de Tales



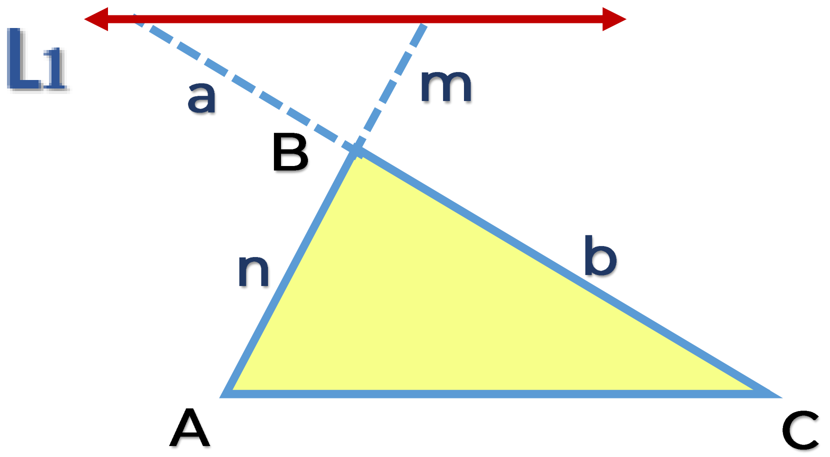
Si: $\overleftrightarrow{L1} \parallel \overleftrightarrow{L2} \parallel \overleftrightarrow{L3}$

$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

Corolario de Tales

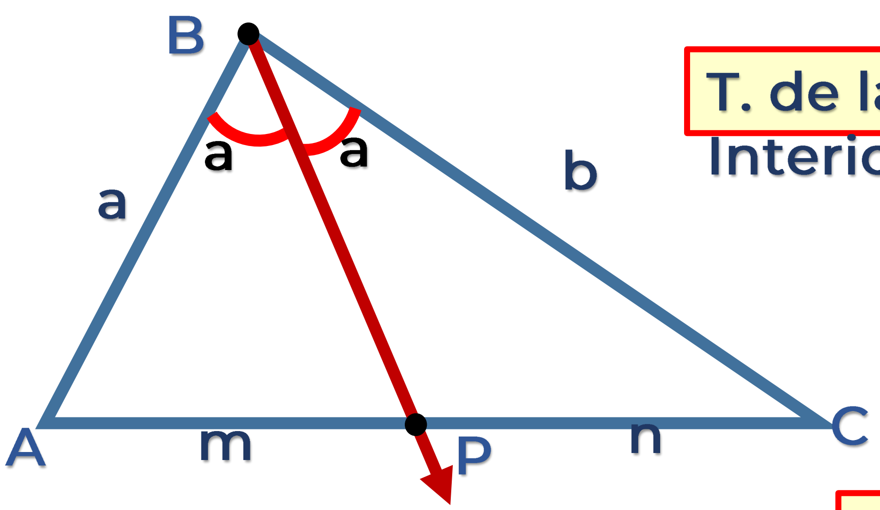


Si: $\overleftrightarrow{L1} \parallel \overline{AC}$.



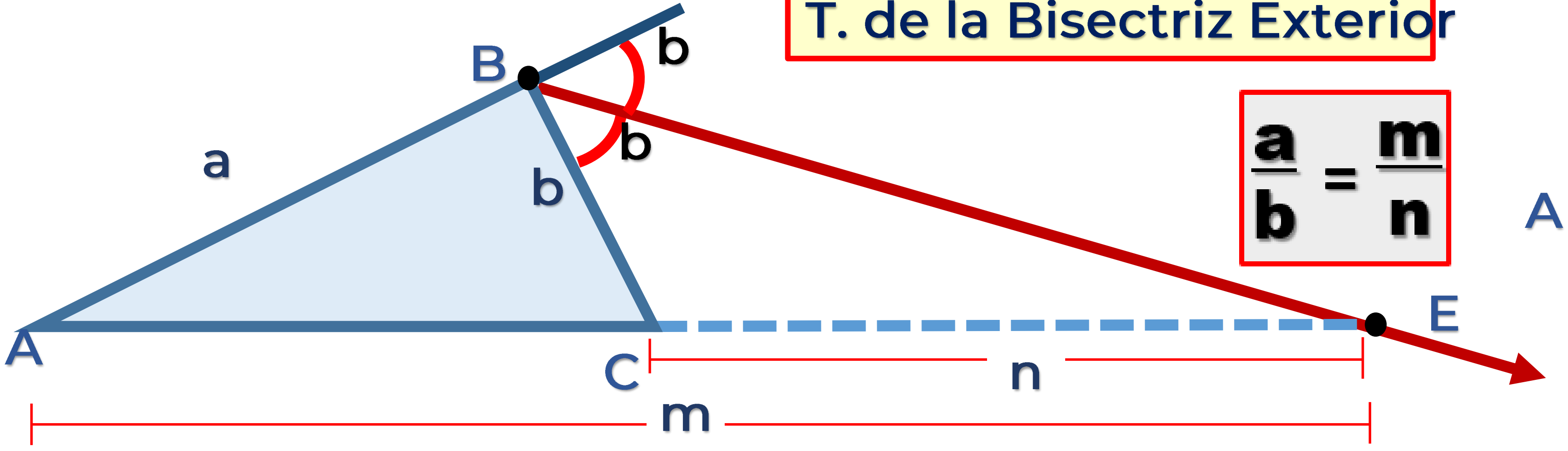
$$\boxed{\frac{a}{b} = \frac{m}{n}}$$

Teorema de la Bisectriz



T. de la Bisectriz Interior

$$\frac{a}{b} = \frac{m}{n}$$

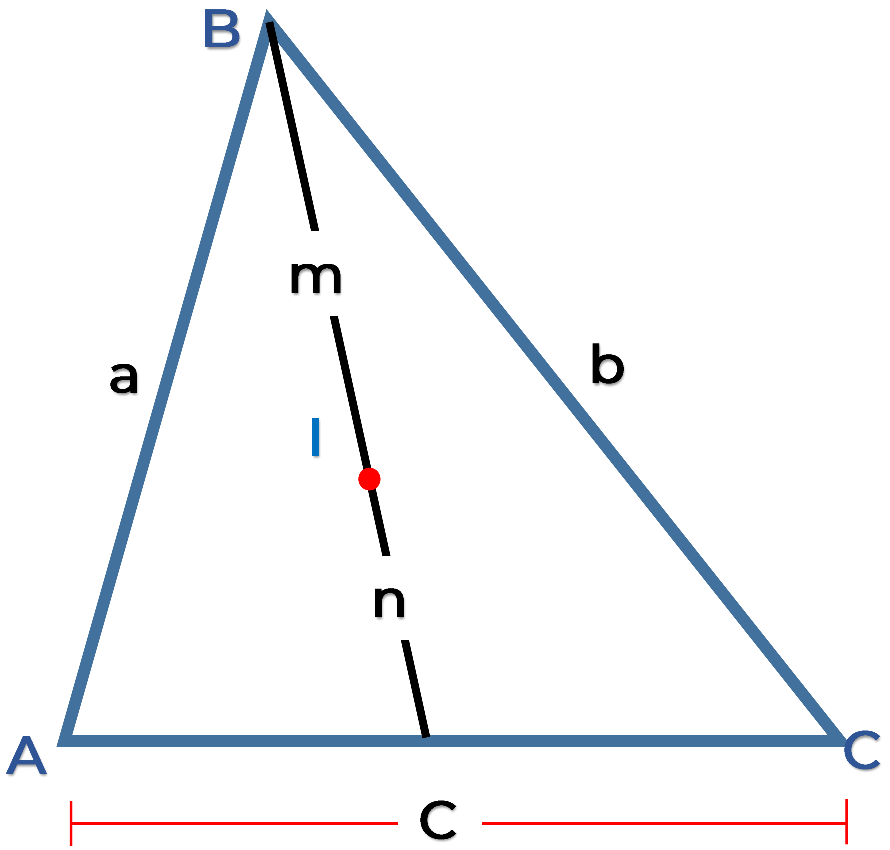


T. de la Bisectriz Exterior

$$\frac{a}{b} = \frac{m}{n}$$

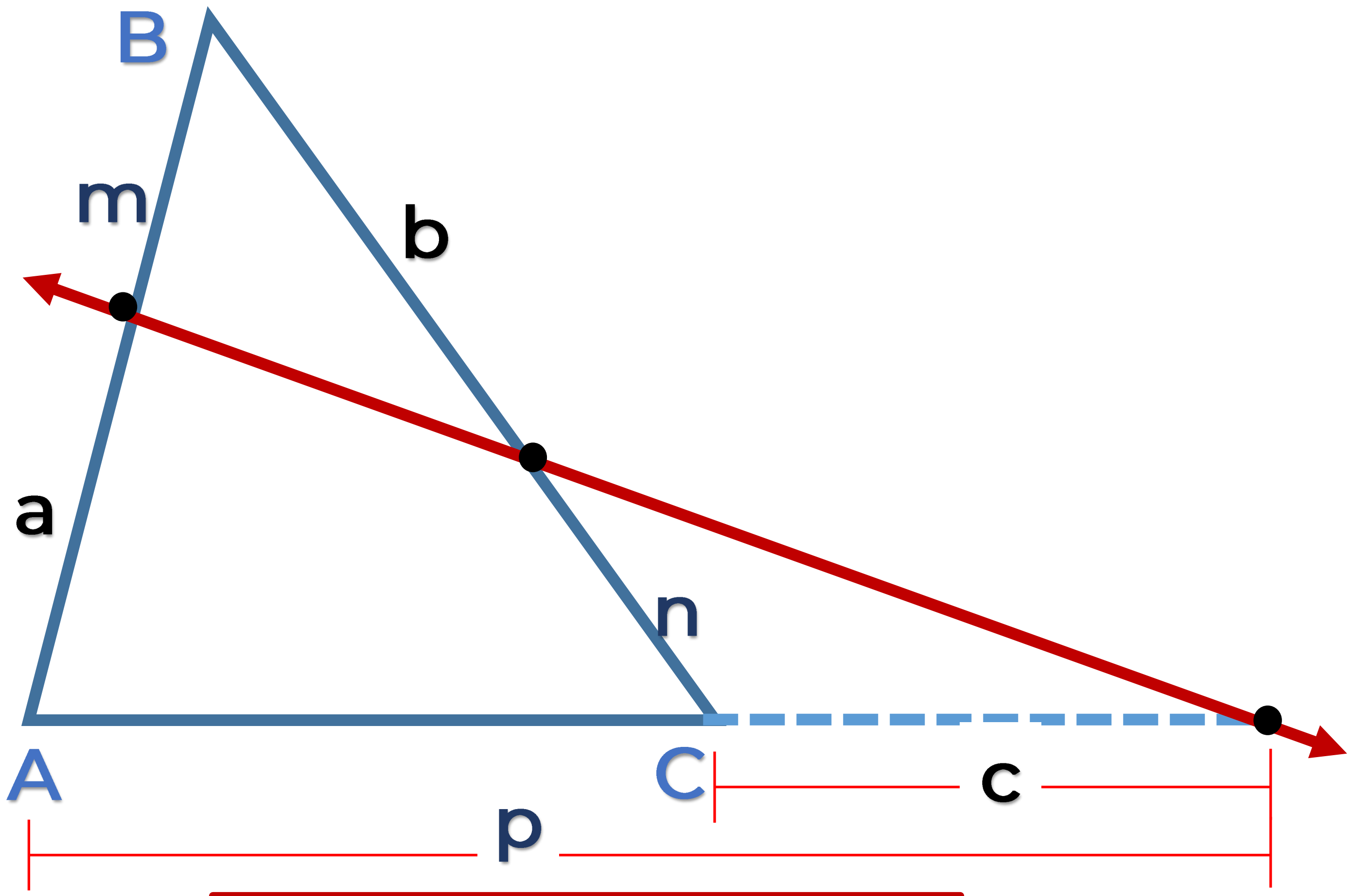
Teorema del Incentro

I: Incentro del △ ABC



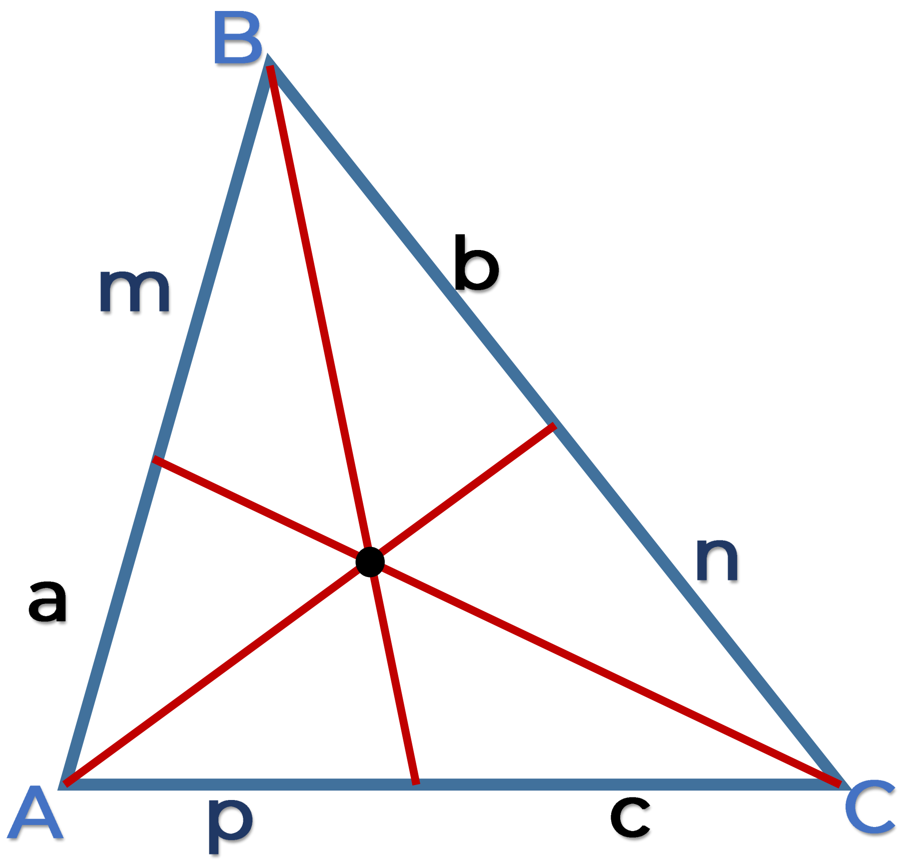
$$\frac{m}{n} = \frac{a+b}{c}$$

Teorema de Menelao



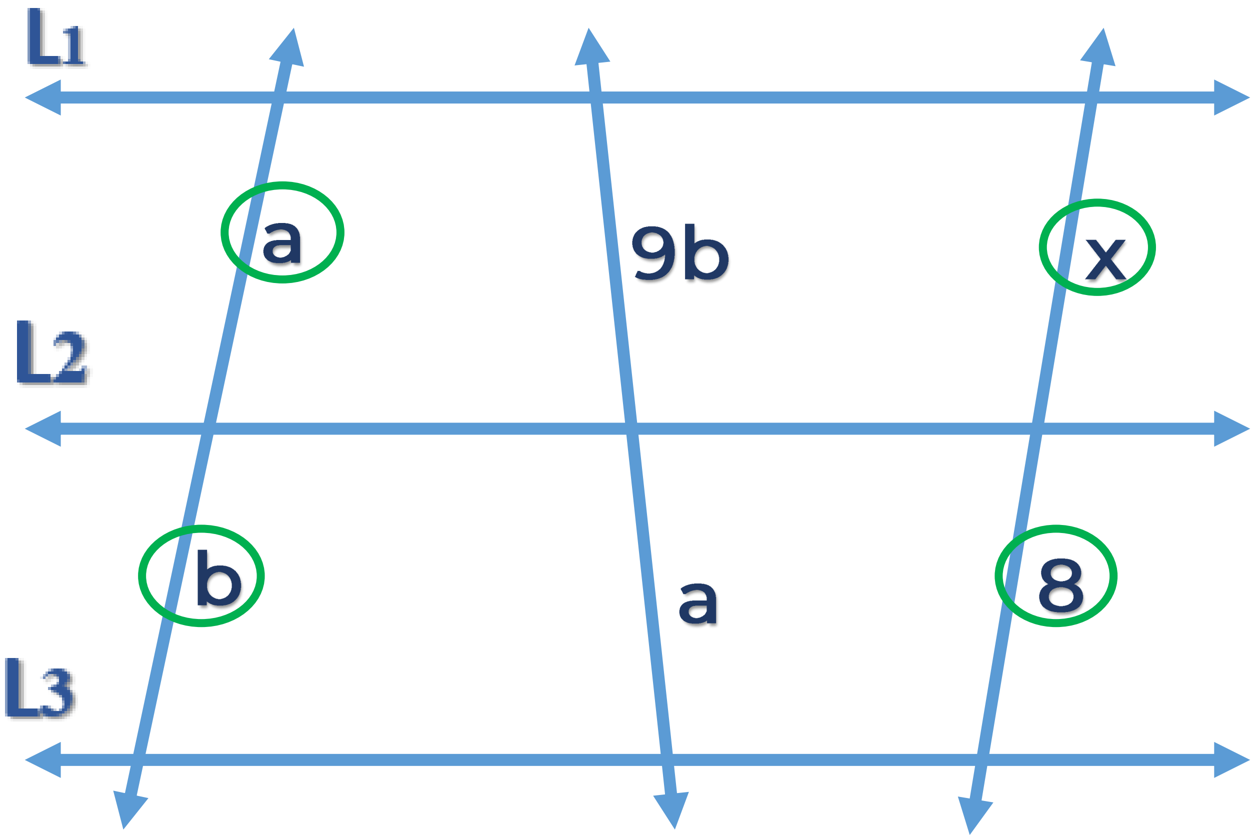
$(a)(b)(c) = (m)(n)(p)$

Teorema de Ceva

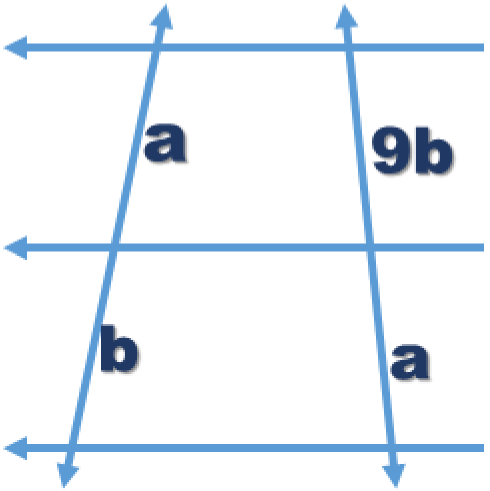


$(a)(b)(c) = (m)(n)(p)$

1. Halle el valor de x, si $L_1 \parallel L_2 \parallel L_3$.



TEOREMA DE TALES



$$\frac{a}{b} = \frac{9b}{a}$$

$$a^2 = 9b^2$$

$$\boxed{a = 3b}$$

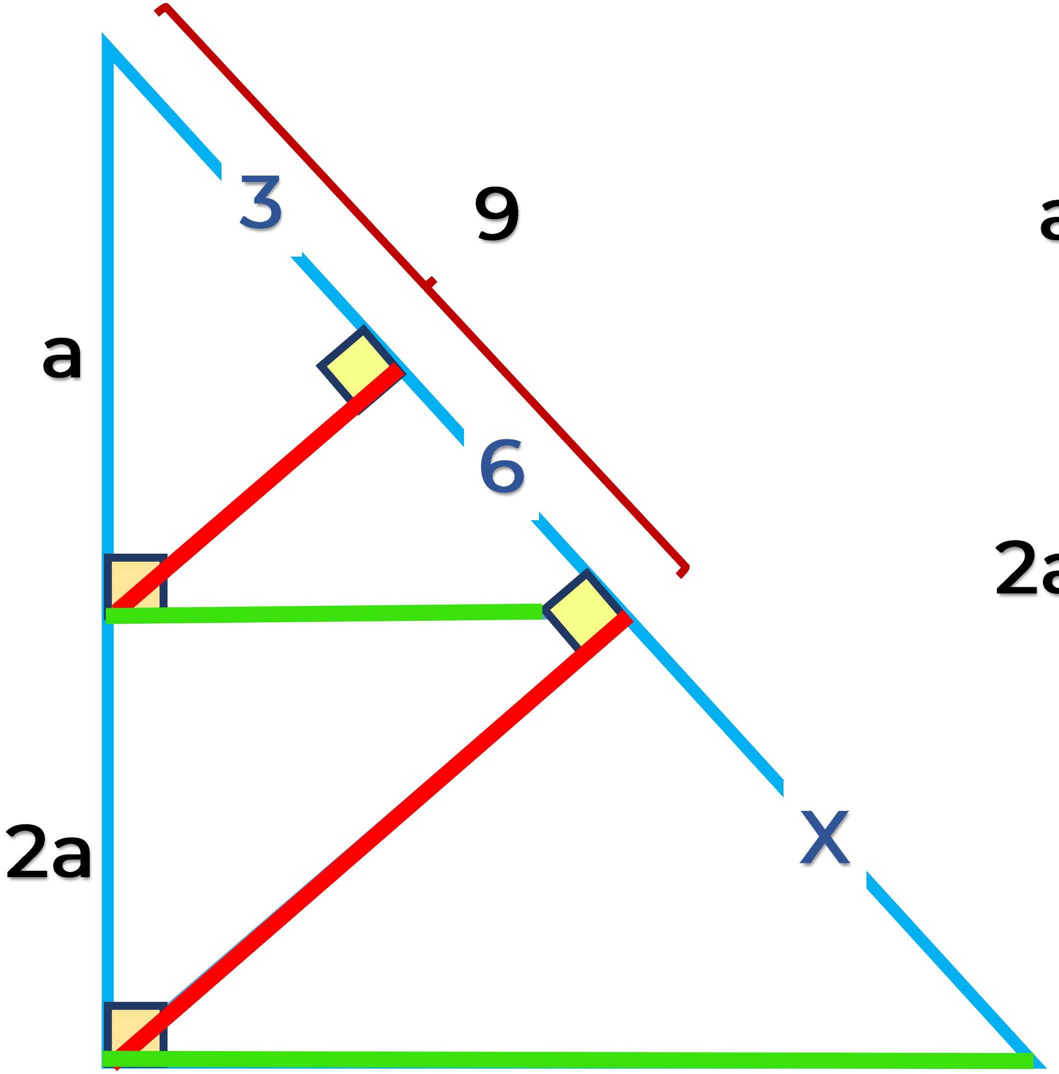
$$\frac{a}{b} = \frac{x}{8}$$

$$\frac{3\cancel{b}}{\cancel{b}} = \frac{x}{8}$$

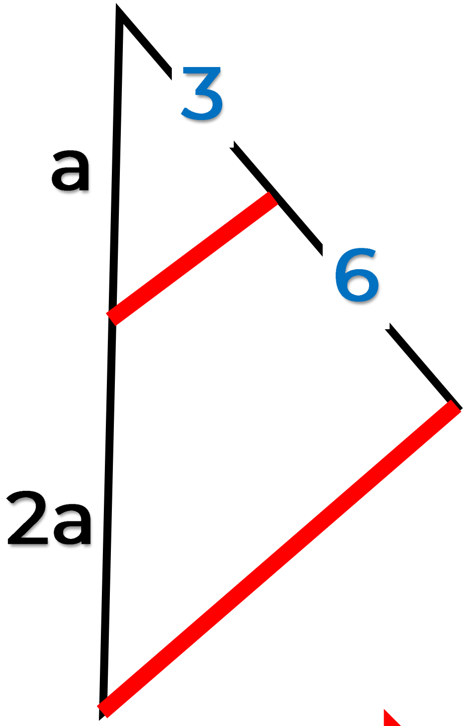
$$3(8) = x$$

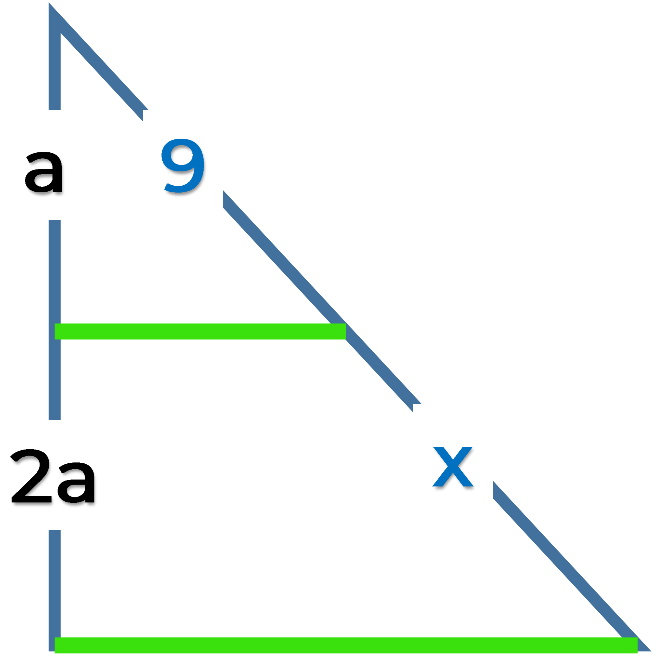
$$\boxed{x = 24}$$

2. Halle el valor de X.



COROLARIO DE TALES

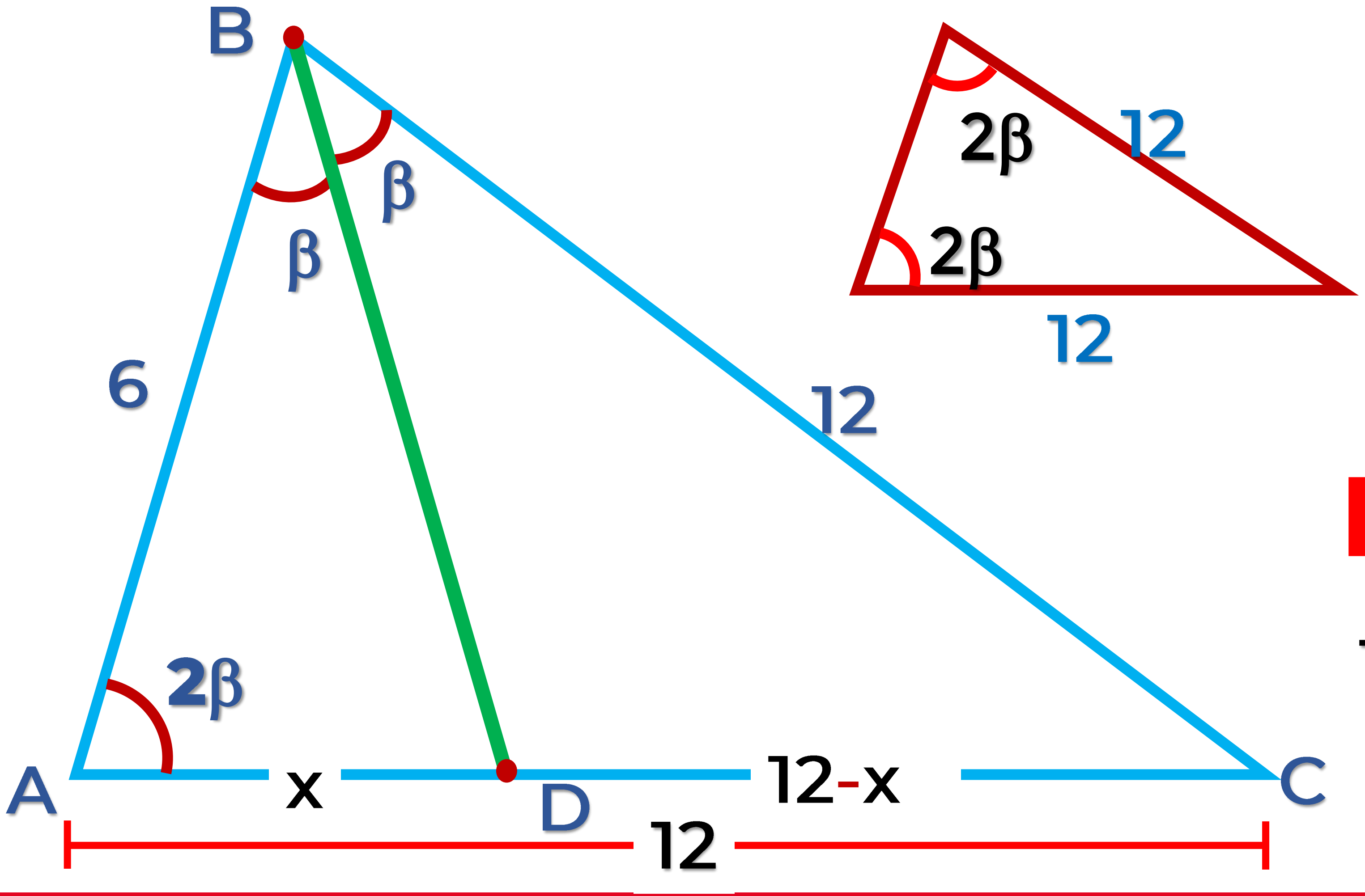




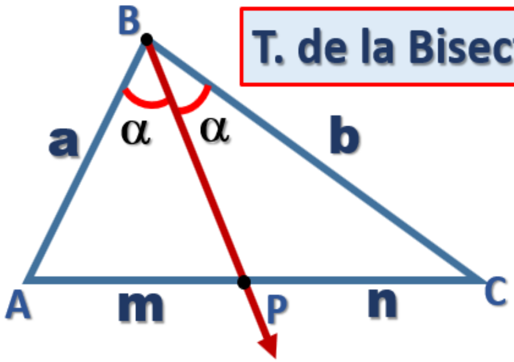
$$\frac{\cancel{a}}{2\cancel{a}} = \frac{9}{x}$$

$\boxed{x = 18}$

3. En un triángulo ABC, donde AB = 6 y BC = 12, se traza la bisectriz interior BD. Halle AD, si m∡BAD = m ∡ ABC.



T. de la Bisectriz Interior



$$\frac{a}{b} = \frac{m}{n}$$

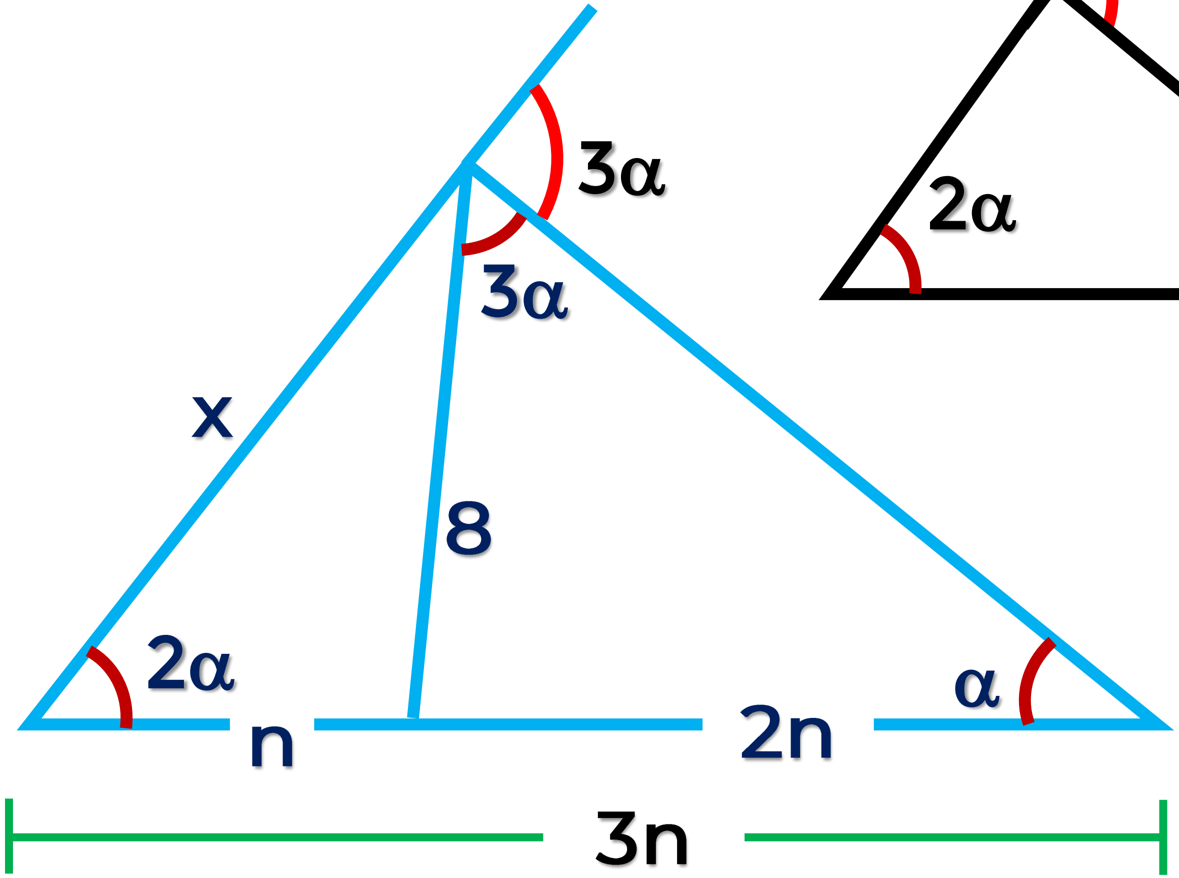
$$\frac{\cancel{6}^{\,1}}{\cancel{12}_{\,2}} = \frac{x}{12 - x}$$

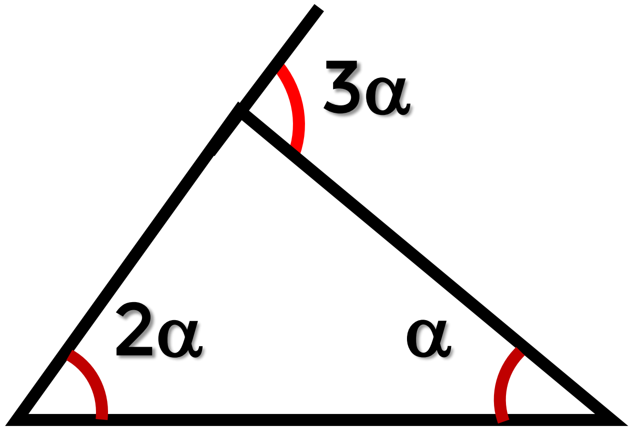
$$12 - x = 2x$$

$$12 = 3x$$

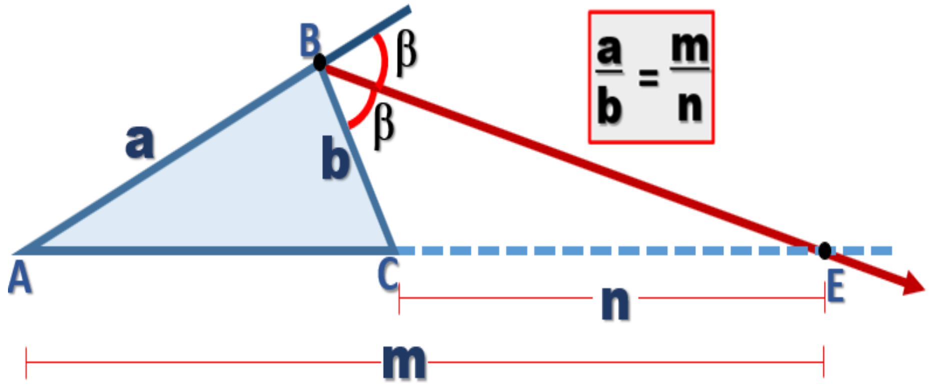
$$\boxed{x = 4}$$

4. Halle el valor de X.





T. de la Bisectriz Exterior

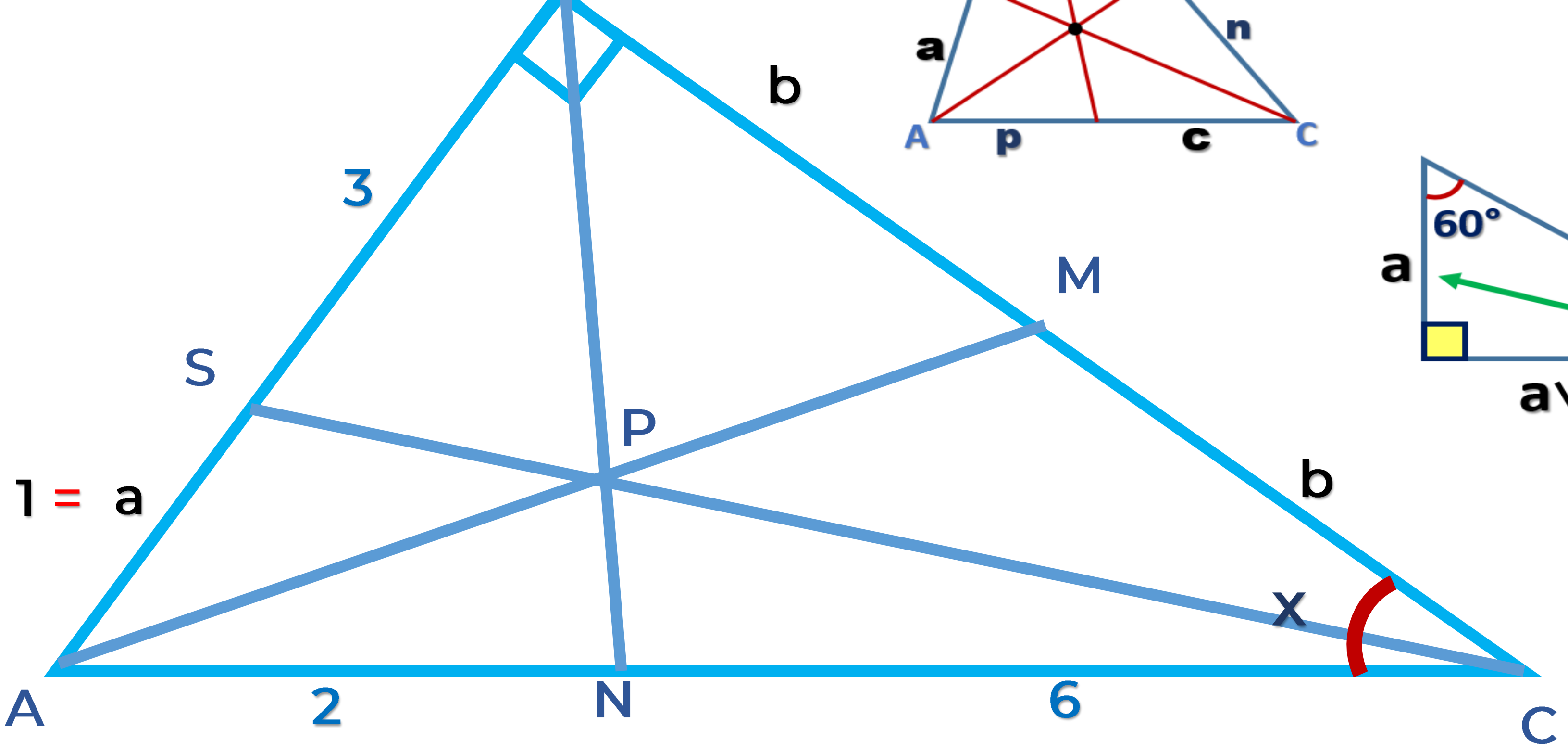


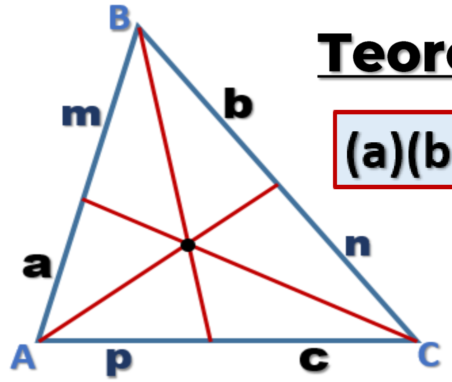
$$\frac{x}{8} = \frac{3\cancel{n}}{2\cancel{n}}$$

$$2x = 24$$

$$\boxed{x = 12}$$

5. En un triángulo rectángulo ABC, recto en B, la mediana $\overline{AM}$ y las cevianas interiores $\overline{BN}$ y $\overline{CS}$ se intersecan en P. Si SB = 3, AN = 2 y NC = 6, halle m∡BCA.



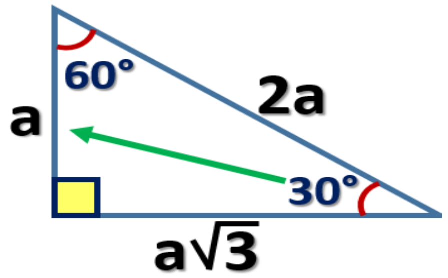


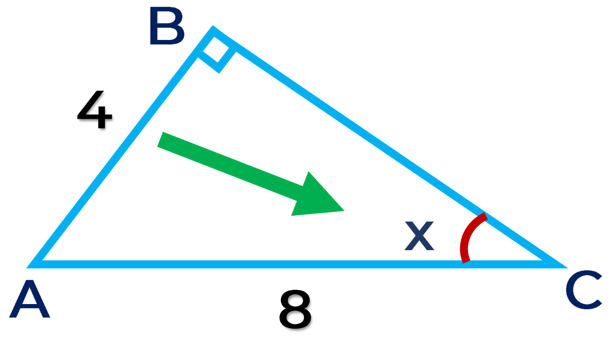
Teorema de Ceva

$$\boxed{(a)(b)(c) = (m)(n)(p)}$$

$$(a)\cancel{(b)}(6) = (3)\cancel{(b)}(2)$$

$$a = 1$$

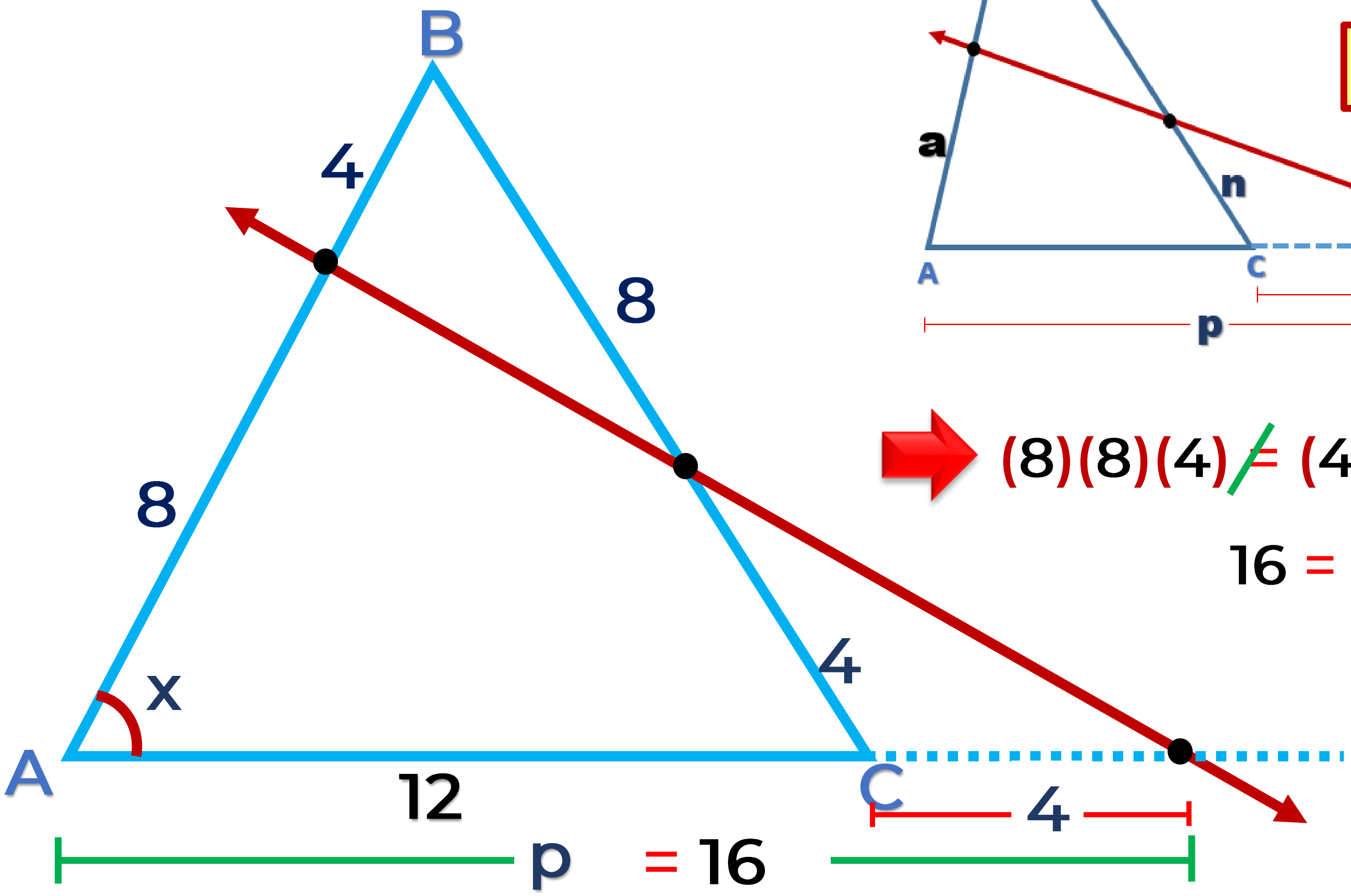


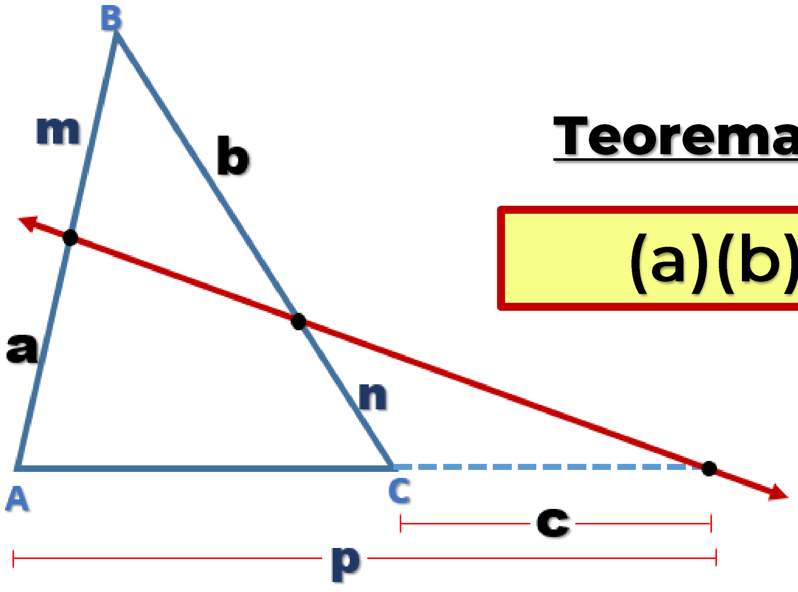


ΔABC: Notable de 30° y 60°

$$\boxed{x = 30°}$$

6. Halle el valor de *x*.



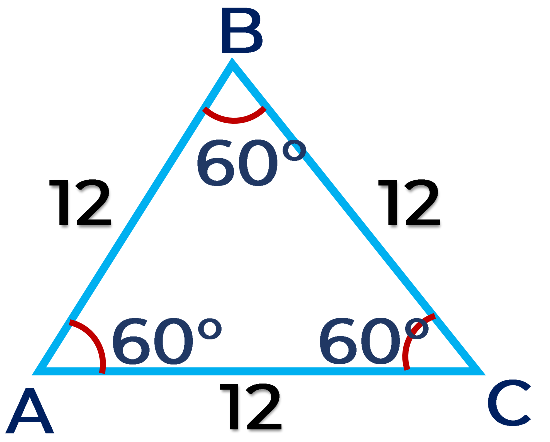


Teorema de Menelao

$$\boxed{(a)(b)(c) = (m)(n)(p)}$$

$$(8)(8)\cancel{(4)} = (4)\cancel{(4)}(p)$$

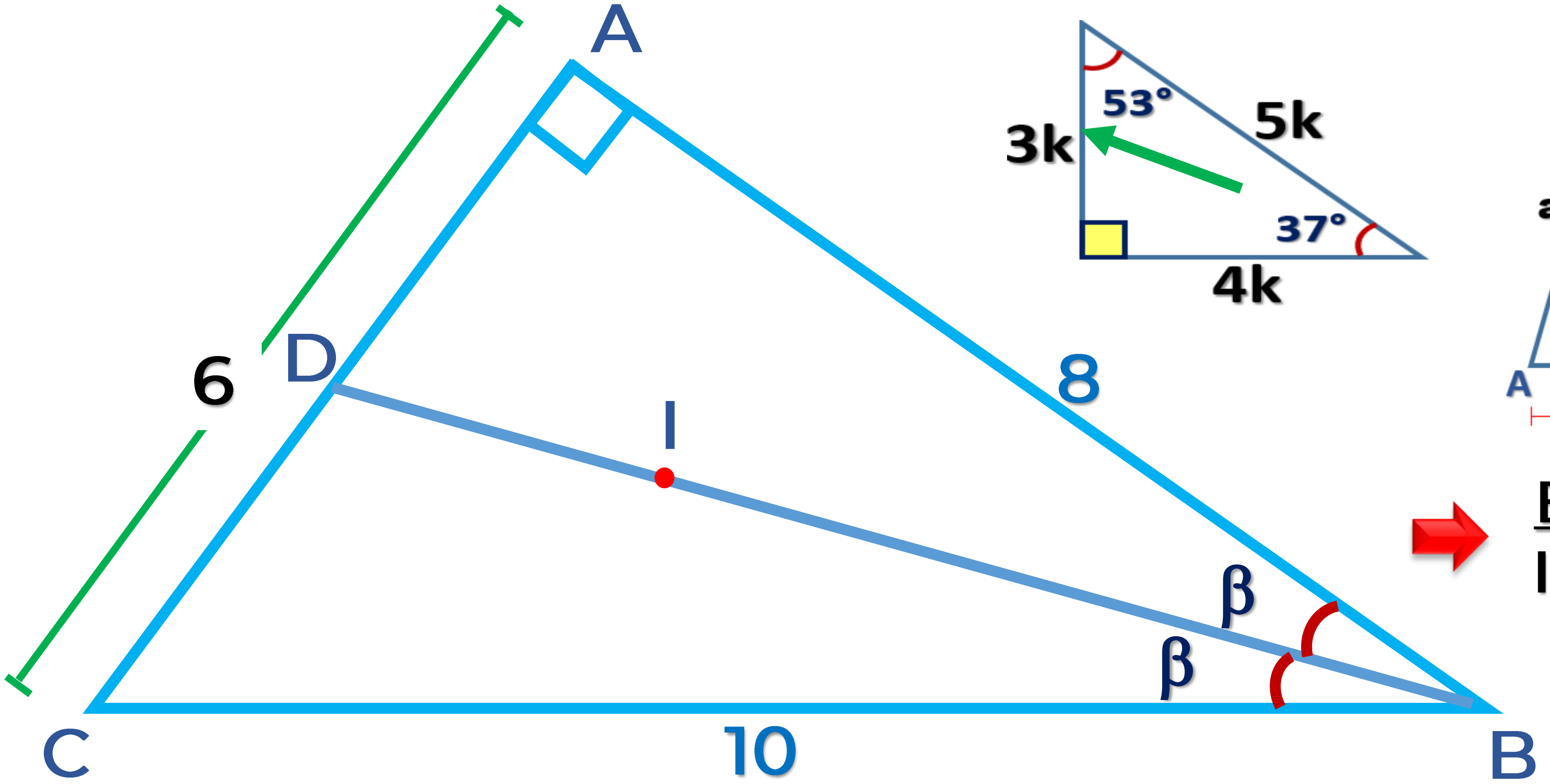
$$16 = p$$



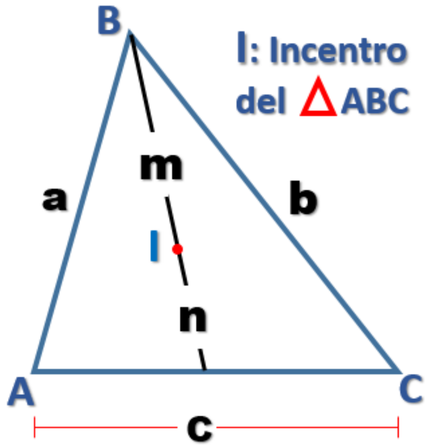
El ΔABC: EQUILÁTERO

$$\boxed{x = 60°}$$

7. En un triángulo rectángulo ABC, recto en A, se traza la bisectriz interior $\overline{BD}$. Halle (BI/ID) si AB = 8, BC = 10 y, además, I es incentro del triángulo ABC.



Teorema del Incentro

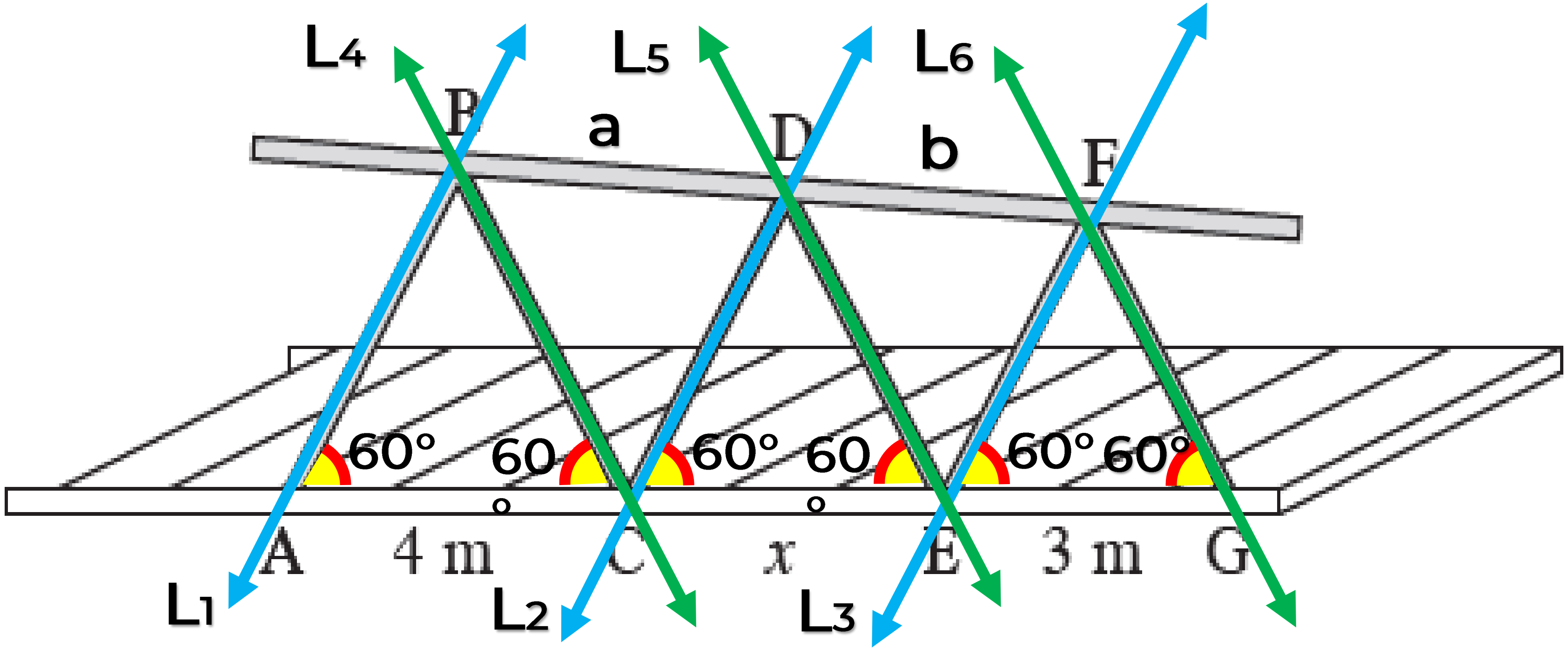


$$\frac{m}{n} = \frac{a+b}{c}$$

$$\frac{BI}{ID} = \frac{8+10}{6} = \frac{18}{6}$$

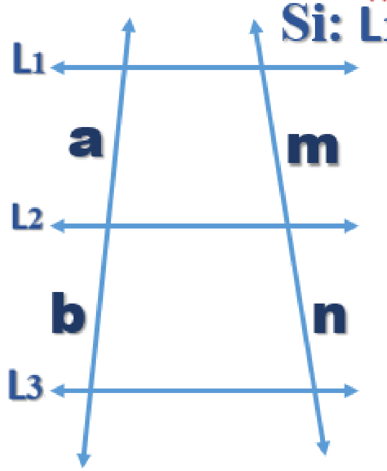
$$\frac{BI}{ID} = 3$$

8. Los triángulos ABC, CDE y EFG son equiláteros. Halle el valor de x.



Teorema de Tales

Si: $\overleftrightarrow{L_1} // \overleftrightarrow{L_2} // \overleftrightarrow{L_3}$



$$\boxed{\frac{a}{b}=\frac{m}{n}}$$

$\overleftrightarrow{L_1} // \overleftrightarrow{L_2} // \overleftrightarrow{L_3}$

$\overleftrightarrow{L_4} // \overleftrightarrow{L_5} // \overleftrightarrow{L_6}$

$$\Rightarrow \frac{a}{b} = \frac{4}{x} \quad \text{........} \quad (1)$$

$$\frac{a}{b} = \frac{x}{3} \quad \text{........} \quad (2)$$

Igualando 1 y 2

$$\frac{4}{x} = \frac{x}{3}$$

$$12 = x^2$$

$$\boxed{x = 2\sqrt{3}}$$