



ALGEBRA

Chapter 13

3th
SECONDARY

FACTORIZACION II



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¡RECORDEMOS!

➤ $P(x) = ax^2 - a - bx^2 + b$

➡ $P(x) = (a - b)(x - 1)(x + 1)$

➤ $P(a) = ma^4 - m$

➡ $P(a) = m(a - 1)(a + 1)(a^2 + 1)$

FACTORIZACIÓN



Es el proceso transformar un polinomio en una multiplicación indicada de dos o más factores primos o irreducibles.

Ejemplo:

$$P(x) = x^2 - 81 = (x - 9)(x + 9)$$

factorización

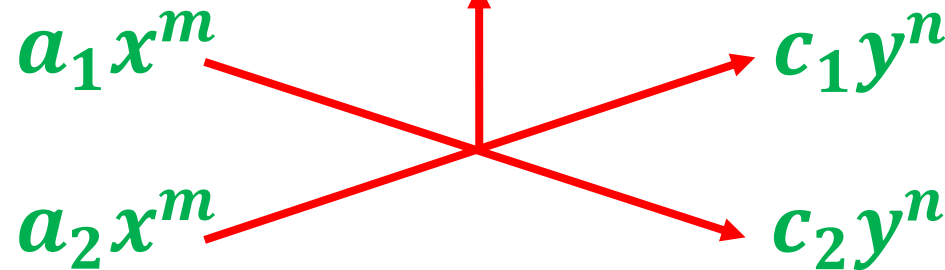
Factores primos: $x - 9$ y $x + 9$



I. CRITERIO DEL ASPA SIMPLE:

$$P(x, y) = Ax^{2m} + Bx^m y^n + Cy^{2m}$$

$$\{A, B, C\} \subset \mathbb{Z}$$



$$P(x, y) = (a_1 x^m + c_1 y^n)(a_2 x^m + c_2 y^n)$$

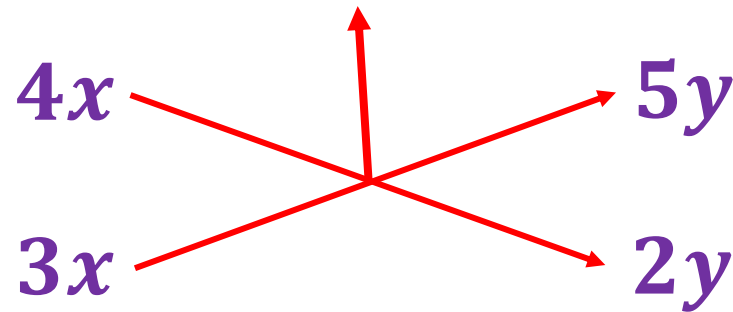


Ejemplo:

Factorice $P(x, y) = 10x^2 + 23xy + 10y^2$

Resolución:

$$P(x, y) = 12x^2 + \underbrace{23xy}_{\substack{4x \quad 5y \\ 3x \quad 2y}} + 10y^2$$

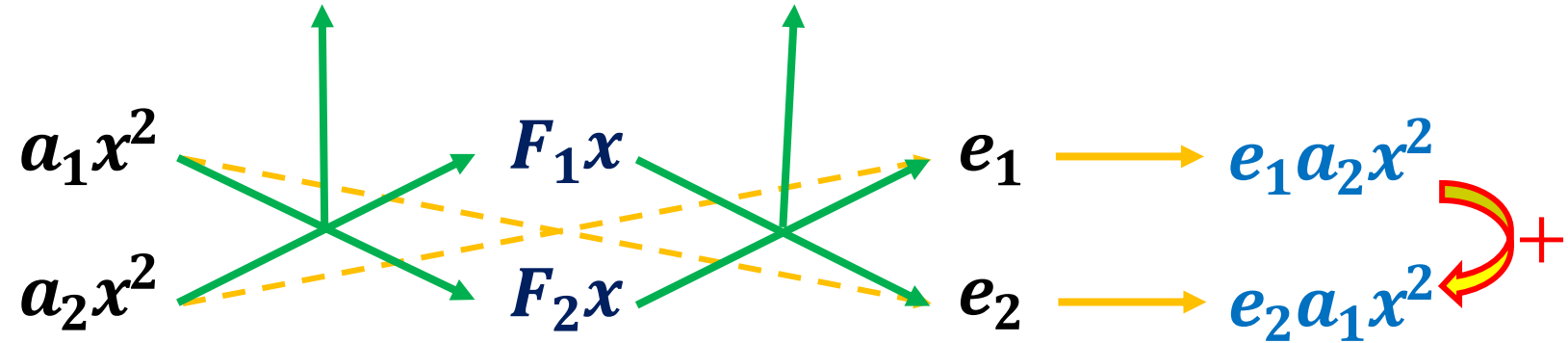


$$P(x, y) = (4x + 5y)(3x + 2y)$$



II. CRITERIO DEL ASPA DOBLE ESPECIAL: TC

$$P(x) = Ax^4 + \underbrace{Bx^3} + \underbrace{Cx^2} + \underbrace{Dx} + E$$



$$FALTA = T.C - S.T = Fx^2$$

$$S.T = Ex^2$$



$$P(x) = (a_1x^2 + F_1x + e_1)(a_2x^2 + F_2x + e_2)$$



Ejemplo: Factorice $P(x) = 12x^4 + 44x^3 + 11x^2 - 36x + 9$

Resolución:

$$P(x) = 12x^4 + \underbrace{44x^3}_{TC} + \underbrace{11x^2}_{TC} - \underbrace{36x}_{TC} + 9$$

Diagram illustrating the cross-multiplication process for factoring the quadratic part of the polynomial. The terms are arranged as follows:

- Left side: $6x^2$ and $2x^2$
- Middle: $7x$ and $5x$
- Right side: -3 and -3

Connections (solid green arrows):

- $6x^2$ connects to -3 (resulting in $-6x^2$)
- $2x^2$ connects to -3 (resulting in $-18x^2$)
- $7x$ connects to -3 (resulting in $-21x$)
- $5x$ connects to -3 (resulting in $-15x$)

Connections (dashed yellow arrows):

- $6x^2$ connects to $5x$ (resulting in $30x^2$)
- $2x^2$ connects to $7x$ (resulting in $14x^2$)

Resulting terms on the right:

- $-6x^2$ and $-18x^2$ (summing to $-24x^2$)
- $-21x$ and $-15x$ (summing to $-36x$)

The final result shown is:

$$ST = -24x^2$$

$$FALTA = 11x^2 - (-24x^2) = 35x^2$$

$$P(x) = (6x^2 + 7x - 3)(2x^2 + 5x - 3)$$

$$P(x) = (3x - 1)(2x + 3)(2x - 1)(x + 3)$$



III. CRITERIO DE LOS DIVISORES BINÓMICOS:

$$P(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n, a_0 \neq 0$$

Si $x = \alpha$ es una raíz de $P(x)$  $P(\alpha) = 0$

Luego, $(x - \alpha)$ es un factor de $P(x)$

Calculamos los otros factores de $P(x)$
usando la regla de RUFFINI

$$\frac{P(x)}{x - \alpha}$$

Posibles ceros o raíces:

$$PC = \pm \left\{ \frac{\text{div}(a_n)}{\text{div}(a_0)} \right\}$$



Ejemplo: Factorice $P(x) = x^3 - x^2 - 2x - 12$

Resolución:

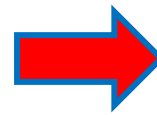
Posibles ceros o raíces:

$$PC = \pm \left\{ \frac{\text{div}(12)}{\text{div}(1)} \right\}$$

$$PC = \pm \left\{ \frac{\{1; 2; 3; 4; 6; 12\}}{1} \right\}$$

$$PC = \pm \{1; 2; 3; 4; 6; 12\}$$

| | | | | |
|---------|---|----|----|-----|
| | 1 | -1 | -2 | -12 |
| $x = 3$ | ↓ | 3 | 6 | 12 |
| × | 1 | 2 | 4 | 0 |



$$P(x) = (x - 3)(x^2 + 2x + 4)$$

Problema 1

**Indique un factor primo,
luego de factorizar**

$$P(x, y) = 15x^2 + 7xy - 2y^2$$

Resolución:



$$P(x, y) = 15x^2 + \underline{7xy} - 2y^2$$

$$\begin{array}{ccc} 3x & & +2y \\ & \nearrow & \searrow \\ 5x & & -y \end{array}$$

$$P(x, y) = (3x + 2y)(5x - y)$$

Factores primos:

$$(3x + 2y) \quad y \quad (5x - y)$$

Problema 2

Determine el número de factores primos en

$$M(x) = 20x^4 + 31x^2 - 9$$

Resolución:



$$M(x) = 20x^4 + \underline{31x^2} - 9$$

$$\begin{array}{ccc} 4x^2 & & -1 \\ & \nearrow & \searrow \\ 5x^2 & & +9 \end{array}$$

$$M(x) = \underline{(4x^2 - 1)}(5x^2 + 9)$$

$$M(x) = (2x + 1)(2x - 1)(5x^2 + 9)$$

$\therefore M(x)$ tiene 3 factores primos

Problema 3

Calcule la suma de los términos independientes de los factores primos de

$$P(x) = 25x^4 - 109x^2 + 36$$

Resolución:



$$P(x) = 25x^4 - 109x^2 + 36$$

$$25x^2 \quad -9$$

$$x^2 \quad -4$$

$$P(x) = (25x^2 - 9)(x^2 - 4)$$

$$P(x) = (5x + 3)(5x - 3)(x + 2)(x - 2)$$

$$\sum TI = 3 - 3 + 2 - 2$$

$$\therefore \sum TI = 0$$

Resolución:



Problema 4

Un factor primo de

$$P(x) = x^3 + 3x^2 + 5x + 3$$

luego de factorizarlo
es

$$PC = \pm \left\{ \frac{\text{div}(3)}{\text{div}(1)} \right\}$$

$$PC = \pm \left\{ \frac{\{1; 3\}}{1} \right\}$$

$$PC = \pm \{1; 3\}$$

$$P(x) = x^3 + 3x^2 + 5x + 3$$

| | | | | |
|----------|---|----|----|----|
| | 1 | 3 | 5 | 3 |
| $x = -1$ | ↓ | -1 | -2 | -3 |
| × | 1 | 2 | 3 | 0 |

$$P(x) = (x + 1)(x^2 + 2x + 3)$$

Factores primos:

$$(x + 1) \quad y \quad (x^2 + 2x + 3)$$

Resolución:

$$P(x) = x^3 - x^2 - x - 2$$

$$PC = \pm \left\{ \frac{\text{div}(2)}{\text{div}(1)} \right\}$$

$$PC = \pm \left\{ \frac{\{1; 2\}}{1} \right\}$$

$$PC = \pm \{1; 2\}$$

**Factores
Primos:**

$$(x - 2)$$

$$\Rightarrow \sum \text{Coef} = 1 - 2 = \boxed{-1}$$

$$(x^2 + x + 1)$$

$$\Rightarrow \sum \text{Coef} = 1 + 1 + 1 = \boxed{3} \text{ Mayor}$$

\therefore La temperatura de Lima es 15°C

| | | | | |
|---------|---|----|----|----|
| | 1 | -1 | -1 | -2 |
| $x = 2$ | ↓ | 2 | 2 | 2 |
| × | 1 | 1 | 1 | 0 |

$$P(x) = (x - 2)(x^2 + x + 1)$$

Problema 5

Luego de factorizar

$$P(x) = x^3 - x^2 - x - 2$$

el quíntuplo del valor de la mayor suma de coeficientes de sus factores primos representa la temperatura en $^\circ\text{C}$ del medio ambiente de la ciudad de Lima. ¿Cuál es la temperatura?

Problema 6

Un factor primo de

$$P(x) = x^4 + 7x^3 + 14x^2 + 7x + 1$$

es

Resolución:

$$P(x) = x^4 + 7x^3 + \overset{TC}{14x^2} + 7x + 1$$

$$x^4 + 7x^3 + 14x^2 + 7x + 1$$

$$x^4 + 7x^3 + 4x^2 + 3x^2 + 7x + 1$$

$$x^4 + 4x^2 + 3x^2 + 4x + 3 + 1$$

$$x^2 + 4x + 1 \quad x^2 + 3x + 1$$

$$FALTA = 14x^2 - 2x^2 = 12x^2$$

$$S.T = +2x^2$$



$$P(x) = (x^2 + 4x + 1)(x^2 + 3x + 1)$$

Factores primos:

$$(x^2 + 4x + 1) \text{ y } (x^2 + 3x + 1)$$

Problema 7

Factorice e indique el término independiente de un factor primo.

$$P(x) = x^4 - 8x^3 + 15x^2 - 38x + 15$$

Resolución:

$$P(x) = x^4 - 8x^3 + 15x^2 - 38x + 15$$

TC

$$S.T = +8x^2$$

$$FALTA = 15x^2 - 8x^2 = 7x^2$$

$$P(x) = (x^2 - x + 5)(x^2 - 7x + 3)$$

Términos independientes de los Factores primos:

$$TI_1 = 5 \quad y \quad TI_2 = 3$$

Problema 8

Calcule el número de factores primos luego de factorizar

$$P(x) = 2x^4 + 5x^3 + 10x^2 + 9x + 6$$

Resolución:

$$P(x) = 2x^4 + 5x^3 + 10x^2 + 9x + 6$$

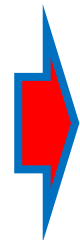
TC

$$2x^2 + 3x + 3 \quad x^2 + x + 2$$

$$3x^2 + 4x^2 = 7x^2$$

$$S.T = +7x^2$$

$$FALTA = 10x^2 - 7x^2 = 3x^2$$



$$P(x) = (2x^2 + 3x + 3)(x^2 + x + 2)$$

$\therefore P(x)$ tiene 2 factores primos.

