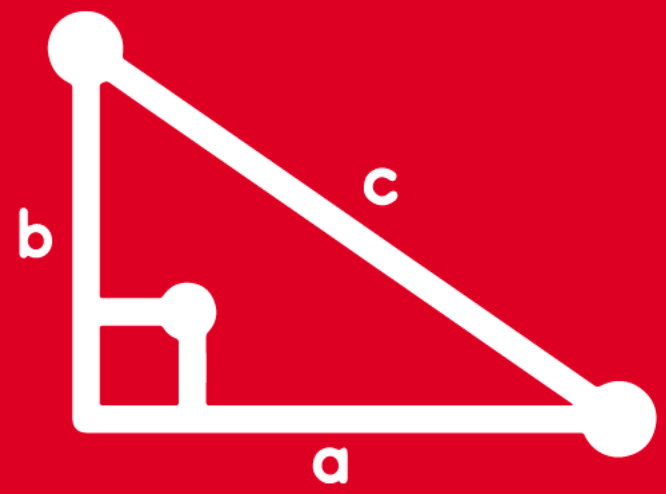
TRIGONOMETRY

Chapter 18

5th
SECONDARY



IDENTIDADES TRIGONOMÉTRICAS DEL ANGULO TRIPLE

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MOTIVATING STRATEGY

Para deducir las identidades del cos(2x), cos(3x), cos(4x), cos(5x) etc; se puede usar la siguiente expresión:

$$\cos(nx) = 2\cos(x)\cos(nx - x) - \cos(nx - 2x)$$

* Para n = 2

$$\Rightarrow \cos(2x) = 2\cos(x)\cos(2x - x) - \cos(2x - 2x)$$

$$\Rightarrow \cos(2x) = 2\cos(x)\cos(x) - \cos(0x)$$

$$\therefore\ \cos(2x) = 2\cos^2(x) - 1$$

* Para n = 3

$$\Rightarrow \cos(3x) = 2\cos(x)\cos(3x - x) - \cos(3x - 2x)$$

$$\Rightarrow \cos(3x) = 2\cos(x)\cos(2x) - \cos(x)$$

$$\Rightarrow \cos(3x) = 2\cos(x)\left[2\cos^2(x) - 1\right] - \cos(x)$$

$$\therefore\ \cos(3x) = 4\cos^3(x) - 3\cos(x)$$

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO TRIPLE

Para el seno :

$$\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3 x$$

Para el coseno :

$$\cos 3x = 4\cos^3 x - 3\cos x$$

Para la tangente :

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

Ejemplos:

- $3\operatorname{sen}10^\circ - 4\operatorname{sen}^3 10^\circ = \operatorname{sen}30^\circ = \frac{1}{2}$
- $4\cos^3 15^\circ - 3\cos 15^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$
- $\frac{3\tan 20^\circ - \tan^3 20^\circ}{1 - 3\tan^2 20^\circ} = \tan 60^\circ = \sqrt{3}$

IDENTIDADES AUXILIARES

1. $\boxed{\operatorname{sen}3x = \operatorname{sen}x(2\cos 2x + 1)}$

2. $\boxed{\cos 3x = \cos x(2\cos 2x - 1)}$

Demostración 1.

$$\text{Sea}: \operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3 x$$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 4\operatorname{sen}^2 x\right)$$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2 \times 2\operatorname{sen}^2 x\right)$$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2 \times (1 - \cos 2x)\right)$$

$$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2 + 2\cos 2x\right)$$

$$\therefore \boxed{\operatorname{sen}3x = \operatorname{sen}x(2\cos 2x + 1)}$$

IDENTIDADES AUXILIARES

3. $\boxed{4\,\text{sen}x\,\text{sen}(60^\circ - x)\,\text{sen}(60^\circ + x) = \text{sen}3x}$

4. $\boxed{4\cos x\cos(60^\circ - x)\cos(60^\circ + x) = \cos 3x}$

5. $\boxed{\tan x\tan(60^\circ - x)\tan(60^\circ + x) = \tan 3x}$

Ejemplo: Calcular $E = 8\,\text{sen}10^\circ\,\text{sen}50^\circ\,\text{sen}70^\circ$

Resolución:

Dando forma: $E = 2 \times \underbrace{4\,\text{sen}10^\circ\,\text{sen}(60^\circ - 10^\circ)\,\text{sen}(60^\circ + 10^\circ)}$ $\Rightarrow$ $E = 2 \times \dfrac{1}{2}$

Usando **Ident Aux 3.** $\longrightarrow$ $\text{sen}(3 \times 10^\circ) = \text{sen}(30^\circ)$

$\therefore \boxed{E = 1}$

HELICO-PRACTICE 1

Reduzca $$E=\frac{4\cos^3 15^\circ-3\cos 15^\circ}{3\operatorname{sen}10^\circ-4\operatorname{sen}^3 10^\circ}$$

Resolución:

Recordar:

$\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3x$

$\cos 3x = 4\cos^3x - 3\cos x$

$$E=\frac{\overbrace{4\cos^3 15^\circ-3\cos 15^\circ}^{\cos 3(15^\circ)}}{\underbrace{3\operatorname{sen}10^\circ-4\operatorname{sen}^3 10^\circ}_{\operatorname{sen}3(10^\circ)}}$$

$$\Rightarrow E=\frac{\cos 45^\circ}{\operatorname{sen}30^\circ}$$

$$\Rightarrow E=\frac{\frac{\sqrt{2}}{\cancel{2}}}{\frac{1}{\cancel{2}}}$$

$$\therefore \boxed{E=\sqrt{2}}$$

HELICO-PRACTICE 2

Si se cumple que: $\operatorname{sen}\theta = \dfrac{1}{3}$; calcule $\operatorname{sen}3\theta$.

Resolución:

Recordar:

$\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3x$

Del dato:

$$\boxed{\operatorname{sen}\theta = \frac{1}{3}}$$

Reemplazando:

$$\operatorname{sen}3\theta = 3\left(\frac{1}{3}\right) - 4\left(\frac{1}{3}\right)^3$$

$$\operatorname{sen}3\theta = 1 - \frac{4}{27}$$

$$\therefore \boxed{\operatorname{sen}3\theta = \frac{23}{27}}$$

HELICO-PRACTICE 3

Simplifique la expresión: $E = \dfrac{\operatorname{sen}3x + \operatorname{sen}x}{\cos^2 x}$

Resolución:

Recordar:

$\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3 x$

Reemplazando sen3x :

$$E = \frac{3\operatorname{sen}x - 4\operatorname{sen}^3 x + \operatorname{sen}x}{\cos^2 x}$$

$$E = \frac{4\operatorname{sen}x - 4\operatorname{sen}^3 x}{\cos^2 x}$$

$$E = \frac{4\operatorname{sen}x(1 - \operatorname{sen}^2 x)}{\cos^2 x}$$

Usar la i

$$\cos^2 x = 1 - \operatorname{sen}^2 x$$

$$E = \frac{4\operatorname{sen}x(\cancel{\cos^2} x)}{\cancel{\cos^2} x}$$

$$\boxed{\therefore E = 4\operatorname{sen}x}$$

HELICO-PRACTICE 4

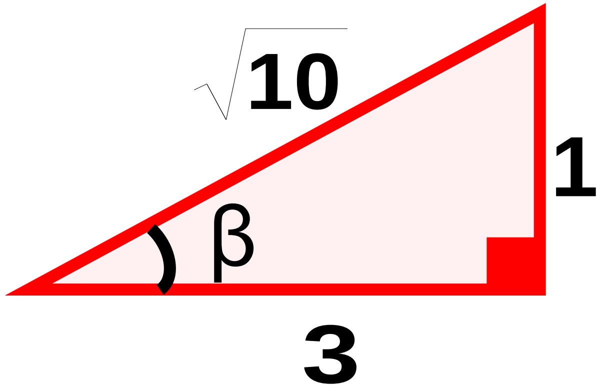
Si un ángulo agudo β, cumple que $\csc\beta = \sqrt{10}$

calcule: tan3β

Resolución:

Dato:

$$\csc\beta = \frac{\sqrt{10}}{1} = \frac{H}{CO} \Rightarrow \tan\beta = \frac{1}{3}$$



Recordar:

$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

Reemplazando en tan3β:

$$\tan 3\beta = \frac{3\left(\frac{1}{3}\right) - \left(\frac{1}{3}\right)^3}{1 - 3\left(\frac{1}{3}\right)^2} = \frac{1 - \frac{1}{27}}{1 - \frac{1}{3}} = \frac{\frac{26}{27}}{\frac{2}{3}}$$

$$\therefore \tan 3\beta = \frac{13}{9}$$

HELICO-PRACTICE 5

De la condición: $\operatorname{sen}x + \cos x = \dfrac{\sqrt{3}}{2}$. Calcule: sen6x

Resolución:

Dato:

$$senx + cosx = \frac{\sqrt{3}}{2}$$

Elevamos al cuadrado:

$$\underbrace{(senx + cosx)^2} = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$1 + sen2x = \frac{3}{4}$$

$$sen2x = -\frac{1}{4}$$

Piden:

$$sen6x = 3sen2x - 4sen^3 2x$$

$$sen6x = 3\left(-\frac{1}{4}\right) - 4\left(-\frac{1}{4}\right)^3$$

$$sen6x = -\frac{3}{4} + \frac{1}{16}$$

$$\boxed{\therefore\ sen6x = -\frac{11}{16}}$$

$$(senx + cosx)^2 = 1 + sen(2x)$$

$$sen3\theta = 3sen\theta - 4sen^3\theta$$

HELICO-PRACTICE 6

De la siguiente identidad: $\dfrac{3sen3x}{senx}+\dfrac{2cos3x}{cosx}=A+Bcos(Cx)$

calcule A + B + C

Resolución:

Dato:

$$\frac{3sen3x}{senx}+\frac{2cos3x}{cosx}=A+Bcos(Cx)$$

$$\frac{3\cancel{senx}(2cos2x+1)}{\cancel{senx}}+\frac{2\cancel{cosx}(2cos2x-1)}{\cancel{cosx}}=A+Bcos(Cx)$$

$$3(2cos2x+1)+2(2cos2x-1)=A+Bcos(Cx)$$

$$6cos2x+3+4cos2x-2=A+Bcos(Cx)$$

$$1+10cos2x+=A+Bcos(Cx)$$

sen3x = senx(2cos2x + 1)

cos3x = cosx(2cos2x – 1)

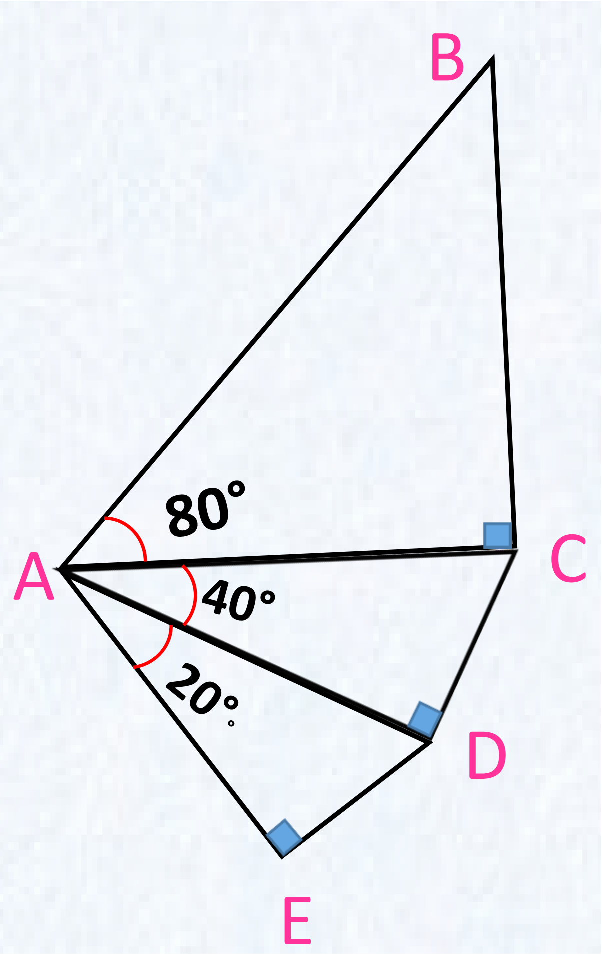
Comparando:

$A=1\,;\ B=10\,;\ C=2$

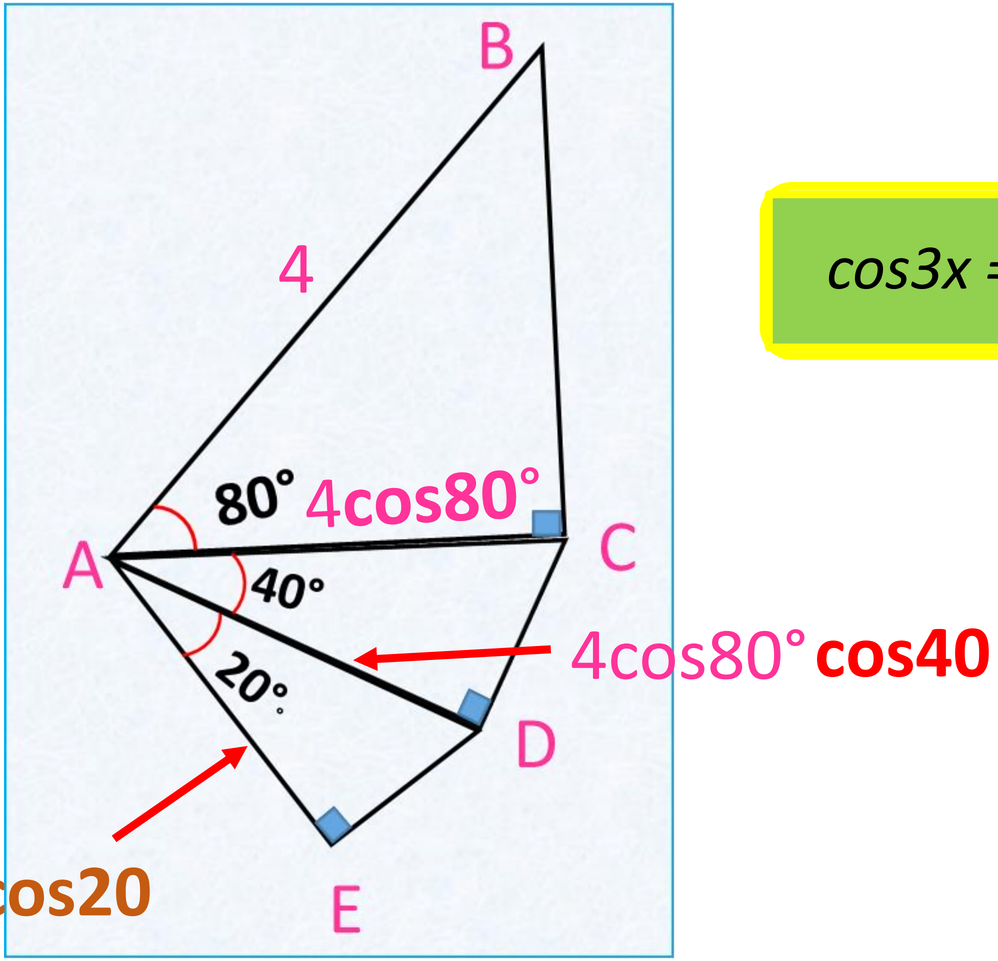
$$\boxed{\therefore\ A+B+C=13}$$

HELICO-PRACTICE 7

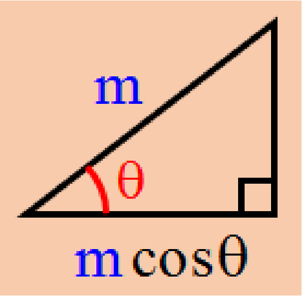
En la figura, AB = 4. Halle AE



Resolución:



Recordar:



$$\cos 3x = 4\cos x.\cos(60° - x).\cos(60° + x)$$

Piden:

$$AE = 4\cos 20° \cos 40° \cos 80°$$

$$AE = \cos 3(20°)$$

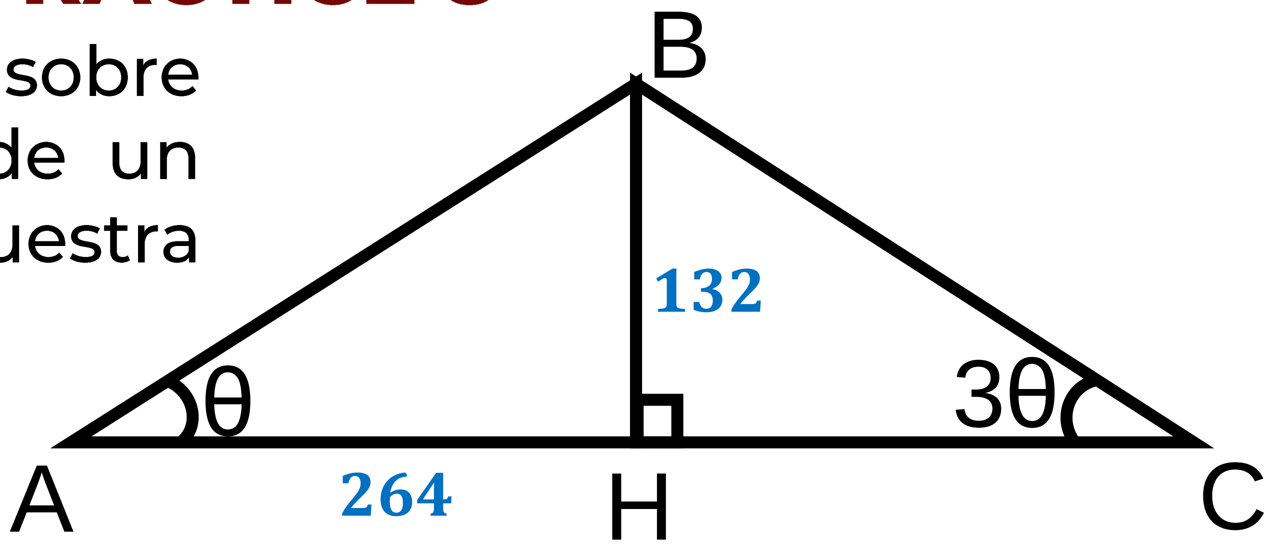
$$AE = \cos 60°$$

$$\therefore\ AE = \frac{1}{2}$$

HELICO-PRACTICE 8

Se construye un centro comercial sobre un terreno que tiene la forma de un triángulo ABC como el que se muestra en la figura.

Si BH = 132m y AH = 264m, ¿cuál es la longitud de HC?



Resolución:

Del gráfico:

$\blacktriangle$AHB: $\tan\theta = \dfrac{\cancel{132}}{\cancel{264}}$

$\Rightarrow \tan\theta = \dfrac{1}{2}$

$\blacktriangle$BHC: $\tan 3\theta = \dfrac{132}{HC}$ ... (*)

Luego: $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$\Rightarrow \tan 3\theta = \dfrac{3\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^3}{1 - 3\left(\frac{1}{2}\right)^2} = \dfrac{\frac{3}{2} - \frac{1}{8}}{1 - \frac{3}{4}} = \dfrac{\frac{11}{8}}{\frac{1}{4}}$

$\Rightarrow \tan 3\theta = \dfrac{11}{2}$

En (*): $\dfrac{11}{2} = \dfrac{132}{HC}$

$\boxed{\therefore\ HC = 24m}$