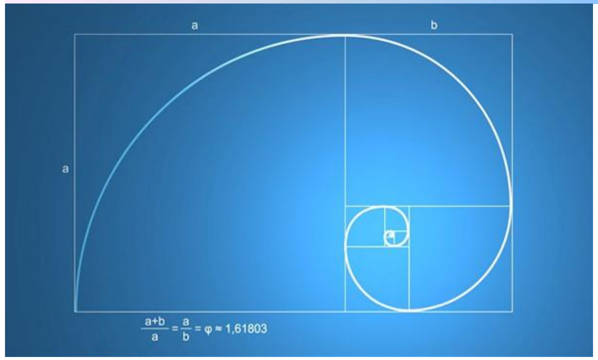
GEOMETRÍA

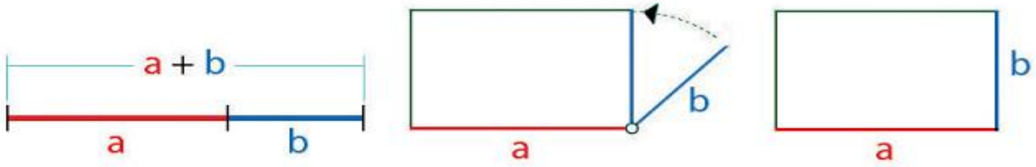
Capítulo 9

4st
SECONDARY

Segmentos proporcionales

SACO OLIVEROS





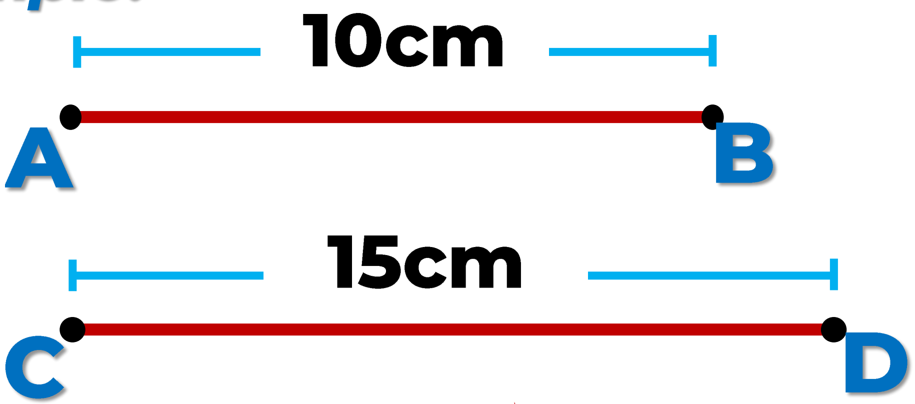
$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = \mathbf{1.61803399...}$$

SEGMENTOS PROPORCIONALES

Razón geométrica de dos segmentos

Es el cociente que se obtiene al dividir las longitudes de dos segmentos que tienen la misma unidad de medida.

Ejemplo:

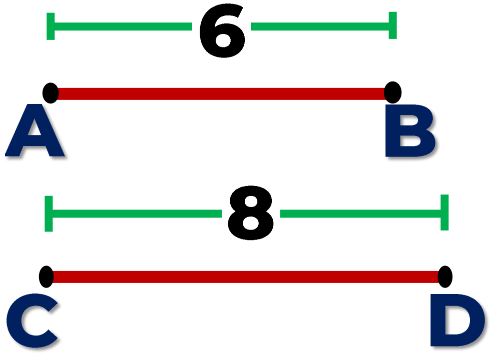


$$\frac{AB}{CD} = \frac{\cancel{10}cm}{\cancel{15}cm} \Rightarrow \frac{AB}{CD} = \frac{2}{3}$$

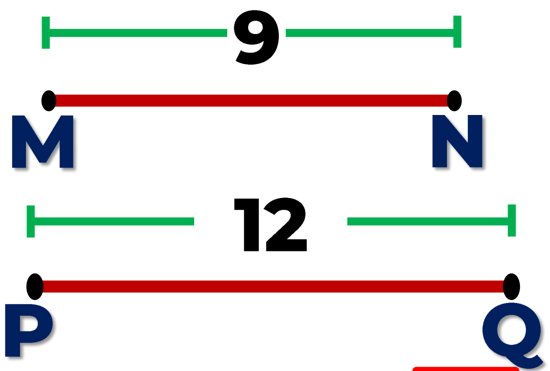
$\frac{2}{3}$: razón geométrica de $\overline{AB}$ y $\overline{CD}$

Segmentos proporcionales

Si la razón geométrica de 2 segmentos es igual a la de otros dos, dichos pares de segmentos son proporcionales.



$$\frac{AB}{CD} = \frac{\cancel{6}}{\cancel{8}} = \boxed{\frac{3}{4}}$$

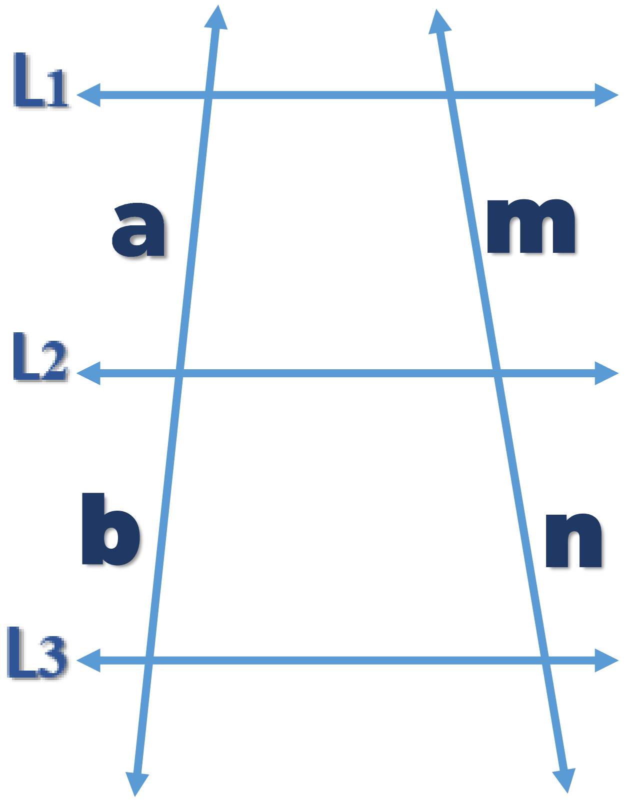


$$\frac{MN}{PQ} = \frac{\cancel{9}}{\cancel{12}} = \boxed{\frac{3}{4}}$$

$$\boxed{\frac{AB}{CD} = \frac{MN}{PQ}} \Rightarrow \text{Son proporcionales}$$

SEGMENTOS PROPORCIONALES

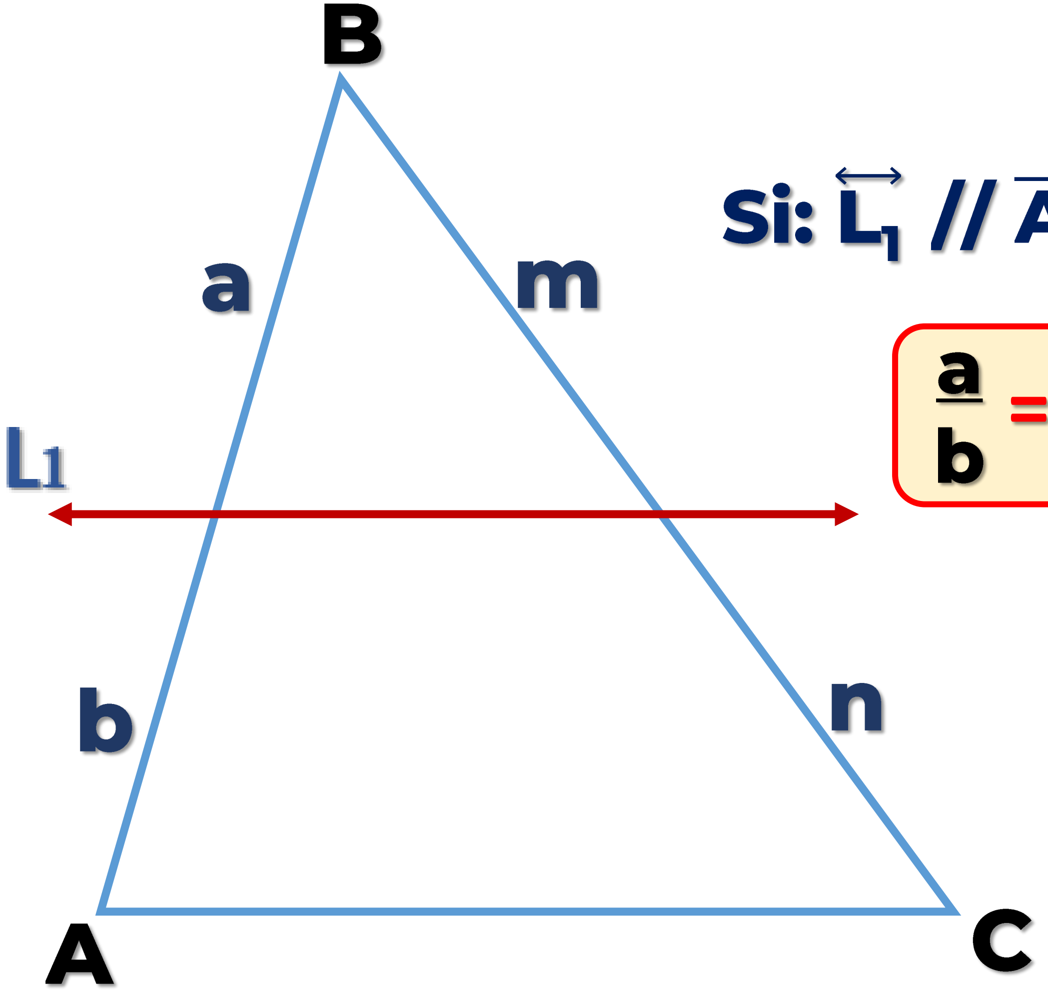
Teorema de Tales



Si: $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2} \parallel \overleftrightarrow{L_3}$

$$\frac{a}{b} = \frac{m}{n}$$

Corolario de Tales

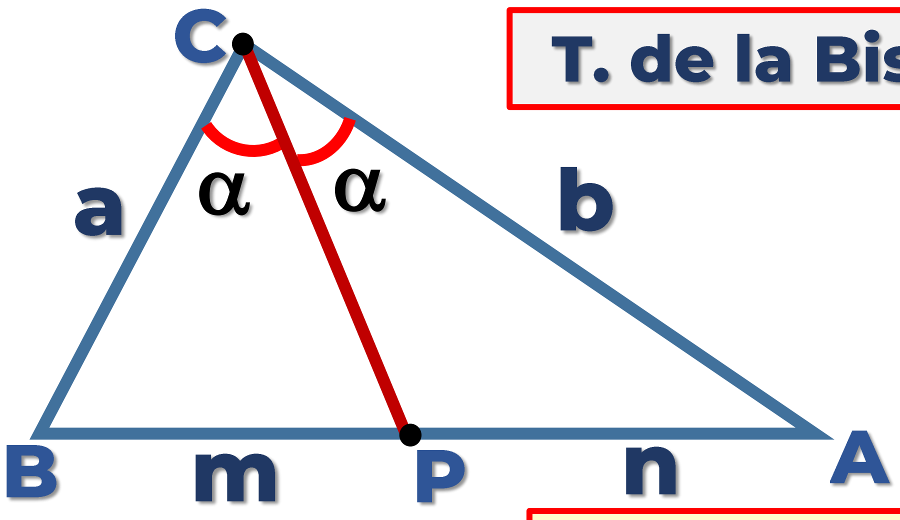


Si: $\overleftrightarrow{L_1} \parallel \overline{AC}$

$$\frac{a}{b} = \frac{m}{n}$$

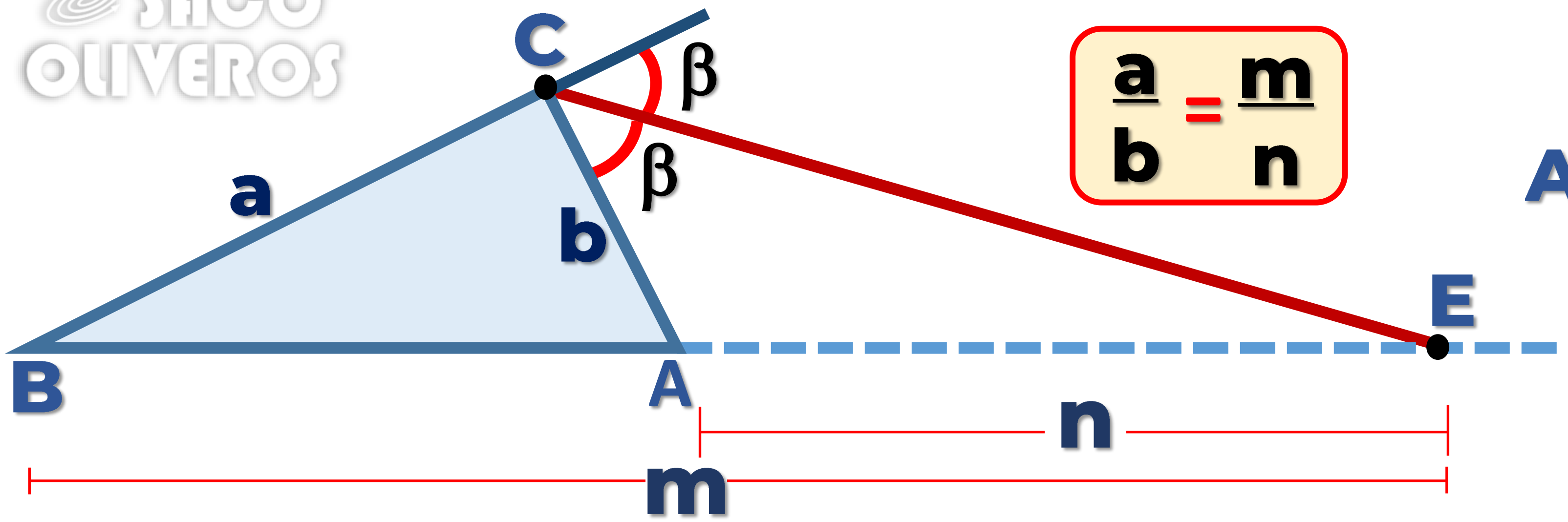
Teorema de la Bisectriz

T. de la Bisectriz Interior



$$\frac{a}{b} = \frac{m}{n}$$

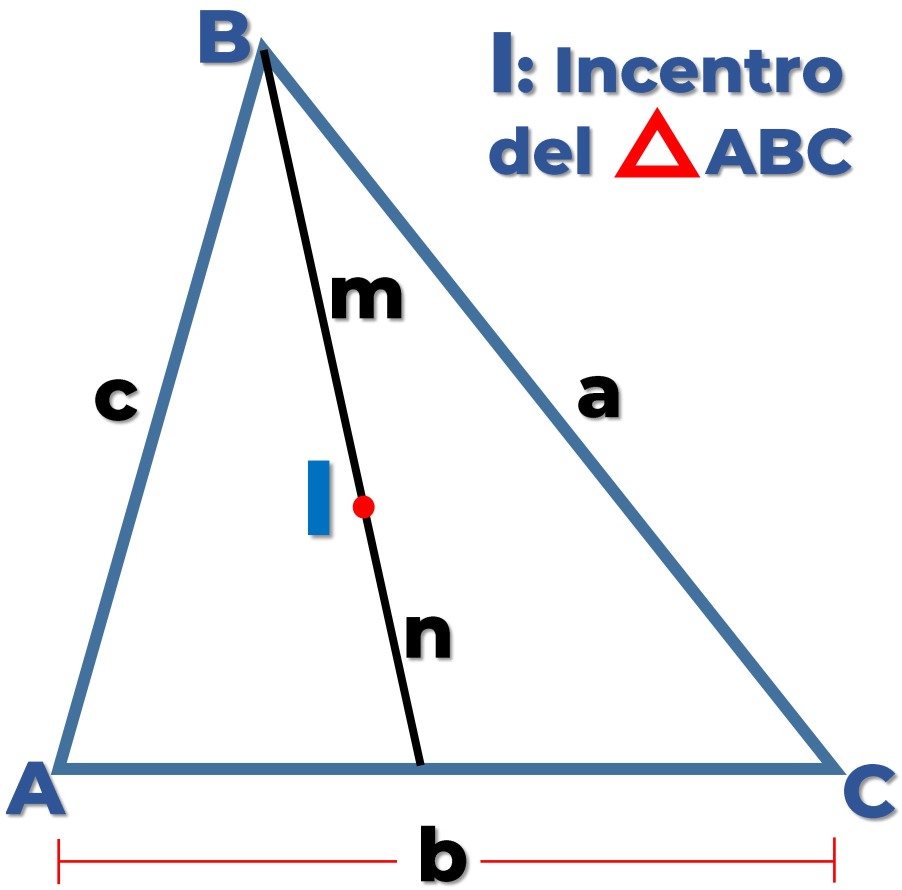
T. de la Bisectriz Exterior



$$\frac{a}{b} = \frac{m}{n}$$

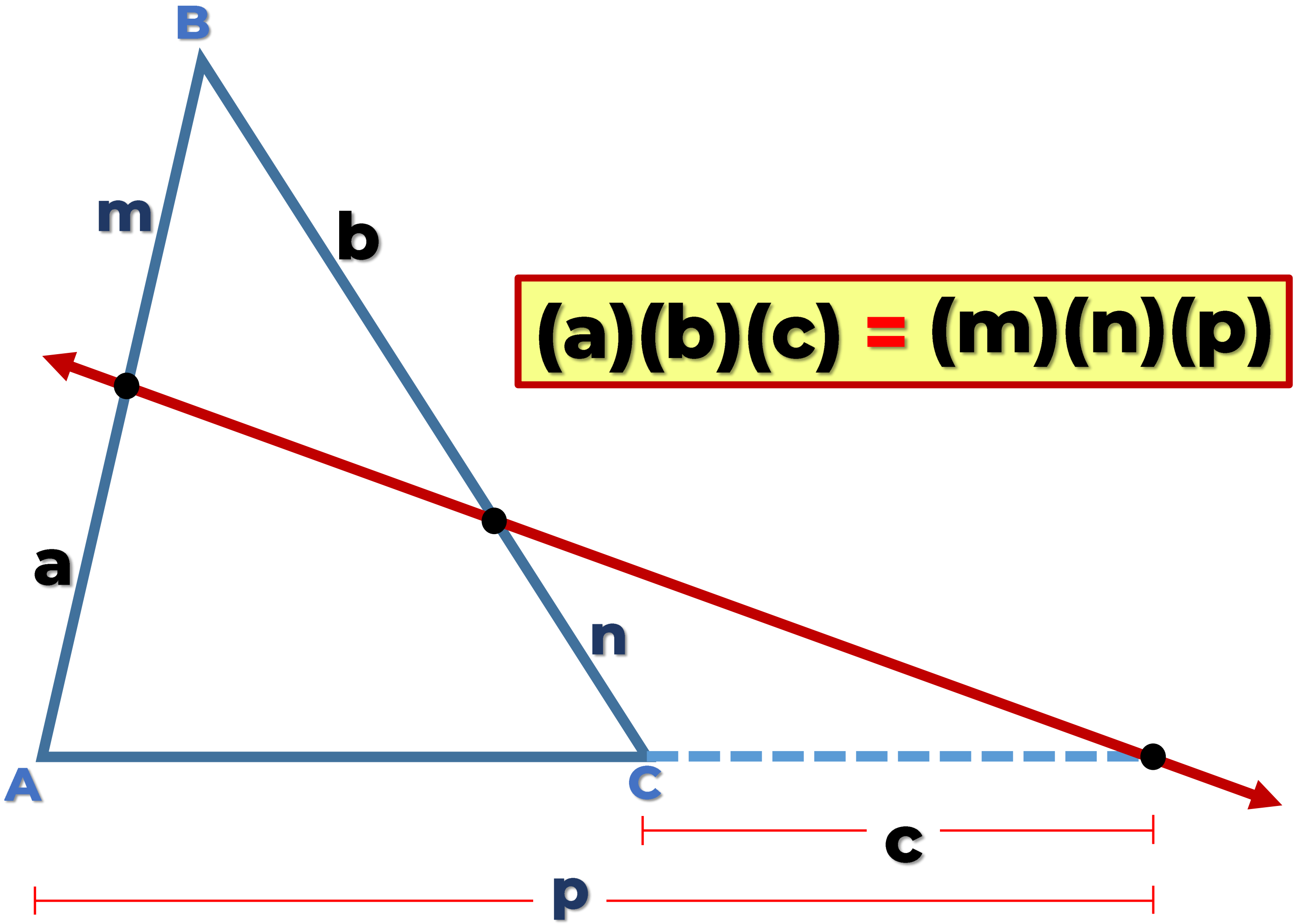
Teorema del Incentro

I: Incentro del △ABC



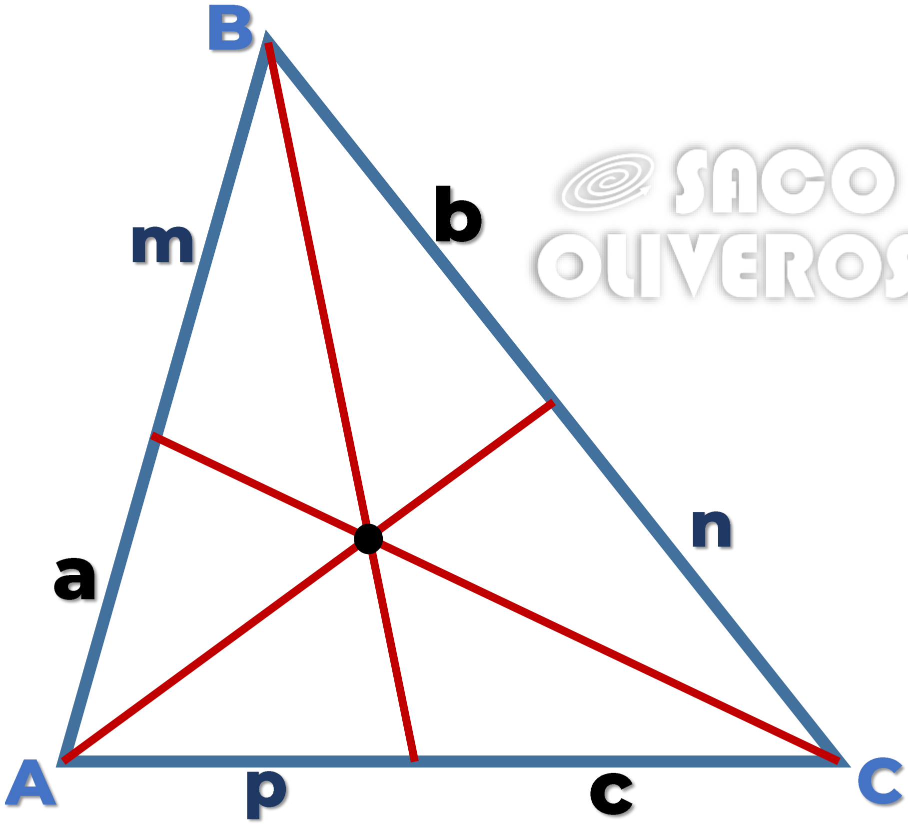
$$\frac{m}{n} = \frac{a + c}{b}$$

Teorema de Menelao



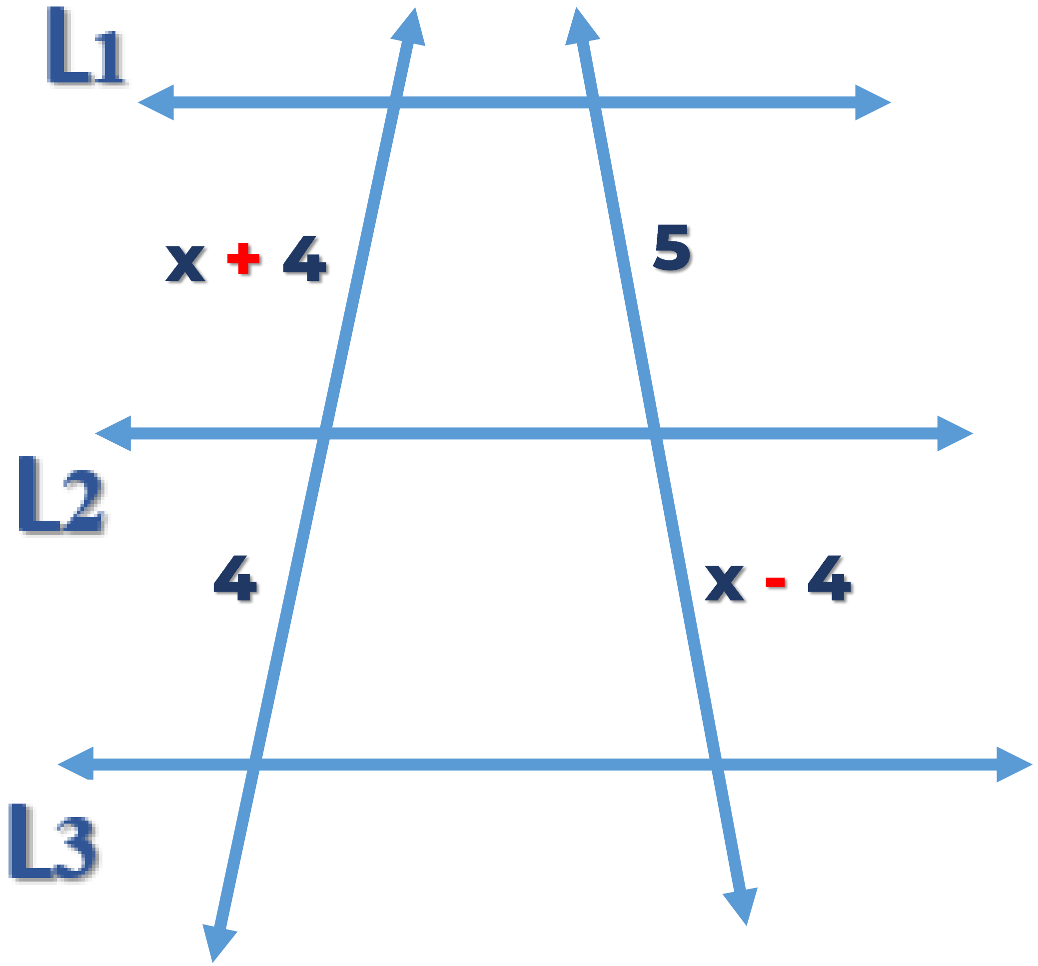
$$(a)(b)(c) = (m)(n)(p)$$

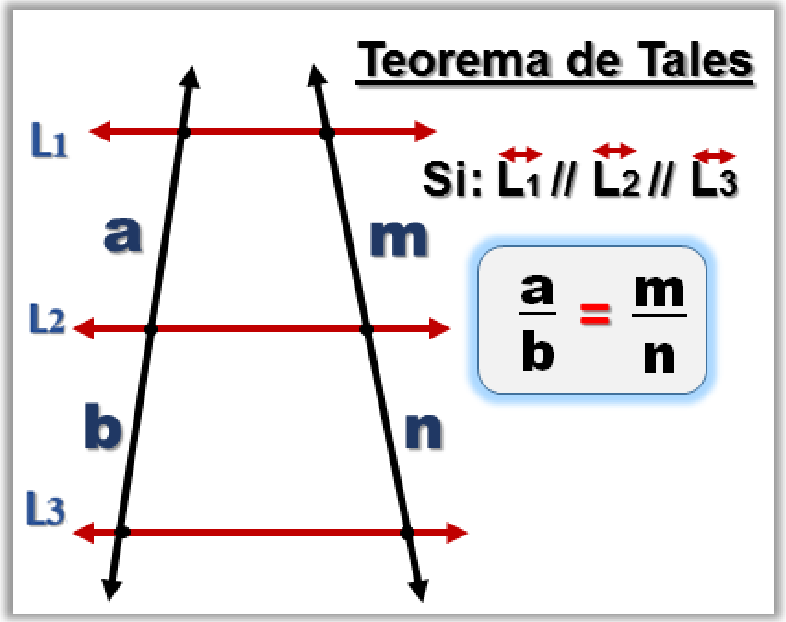
Teorema de Ceva



$$(a)(b)(c) = (m)(n)(p)$$

1. Halle el valor de x si $\overleftrightarrow{L_1} // \overleftrightarrow{L_2} // \overleftrightarrow{L_3}$.





$$\frac{x+4}{4} = \frac{5}{x-4}$$

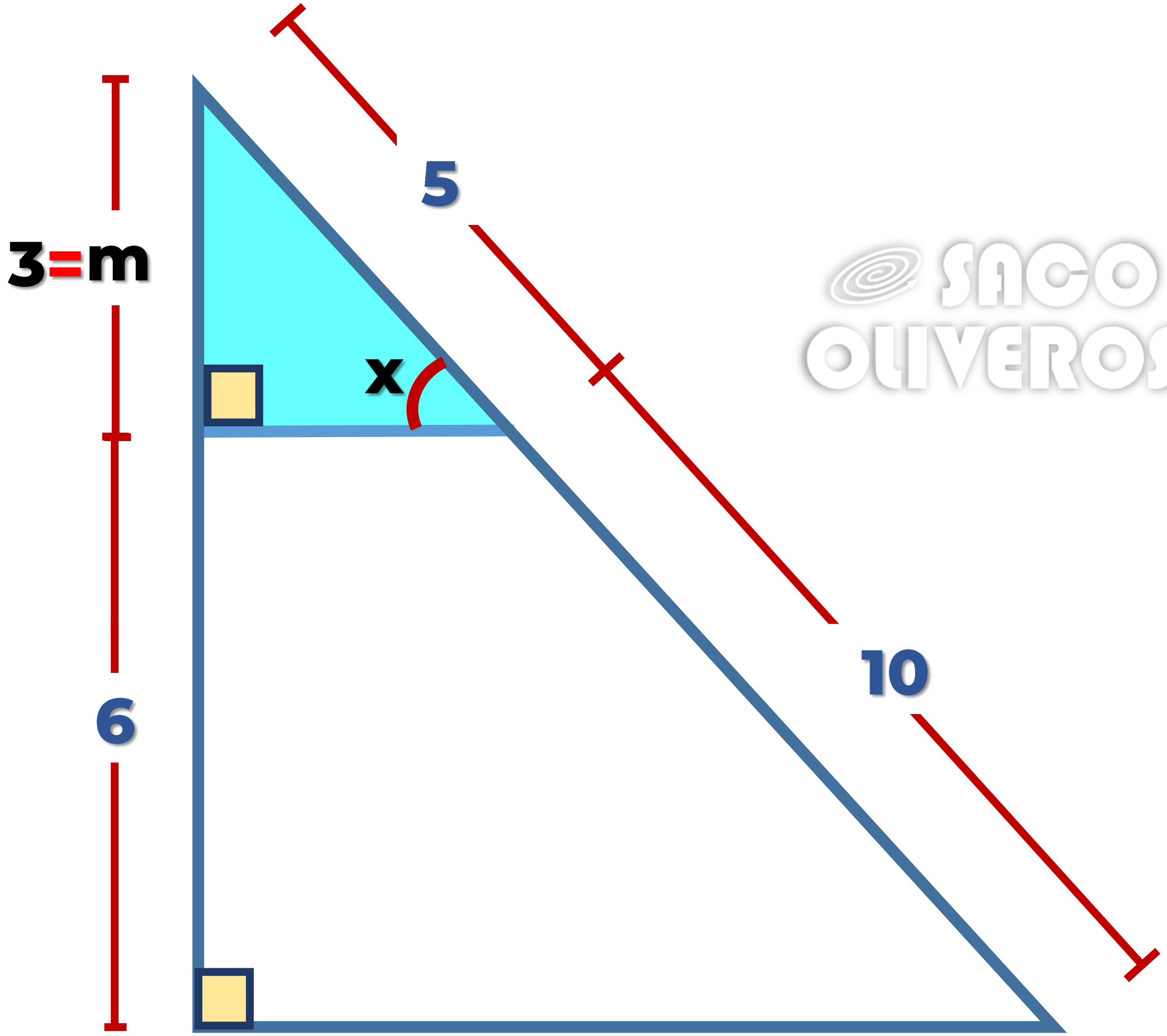
$$(x+4)(x-4) = 20$$

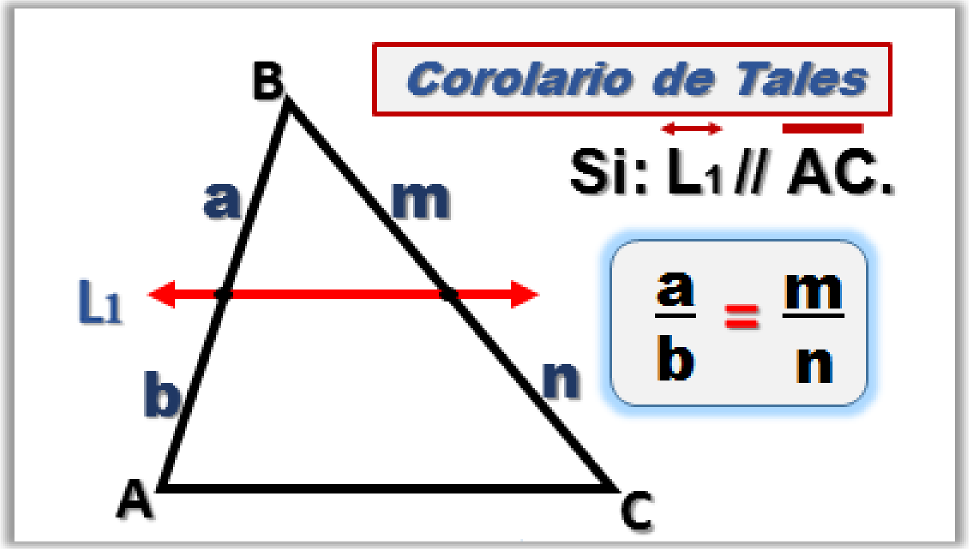
$$x^2 - 4^2 = 20$$

$$x^2 = 36$$

$$\boxed{x = 6}$$

2. En la figura, halle el valor de x.



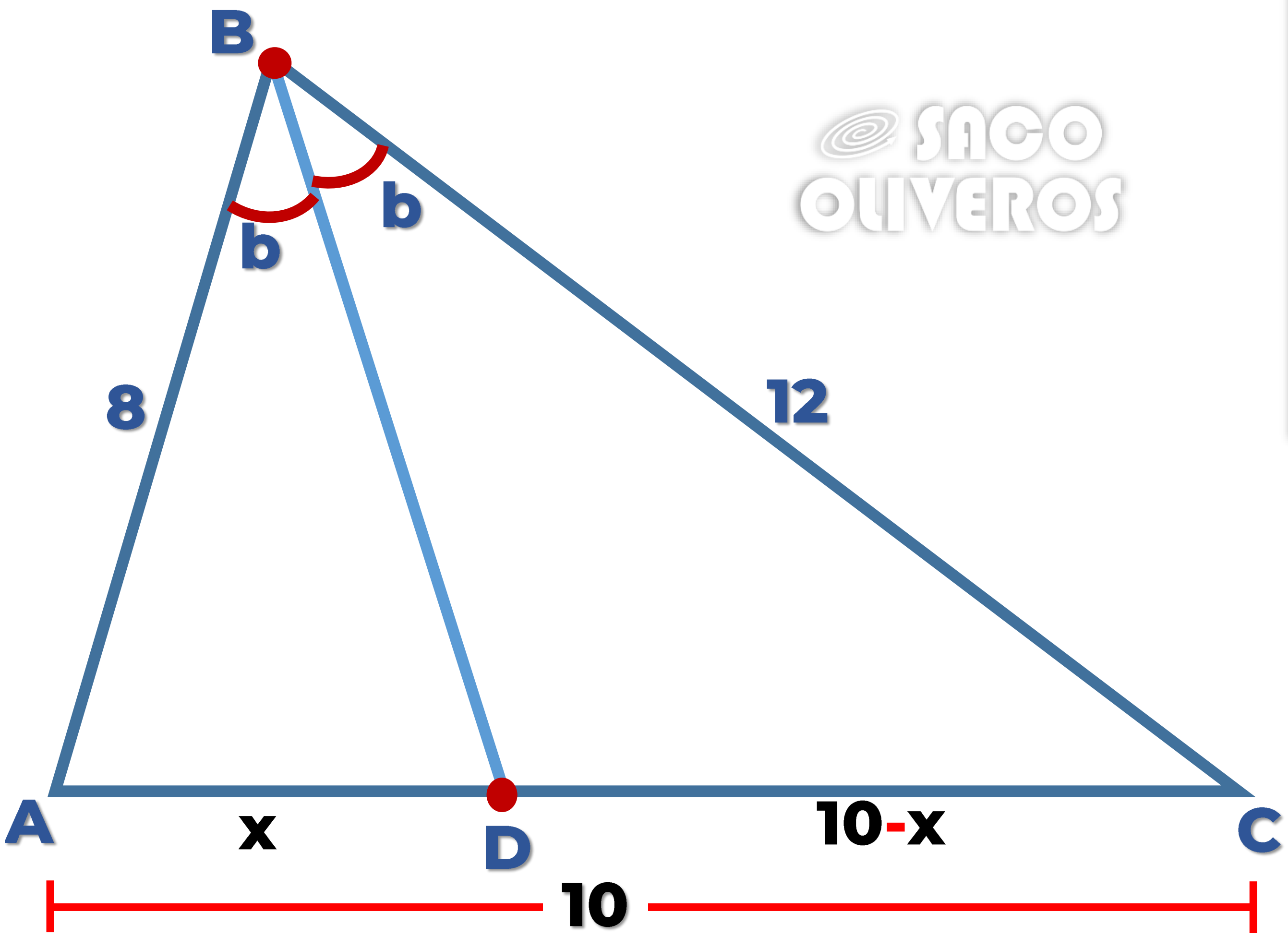


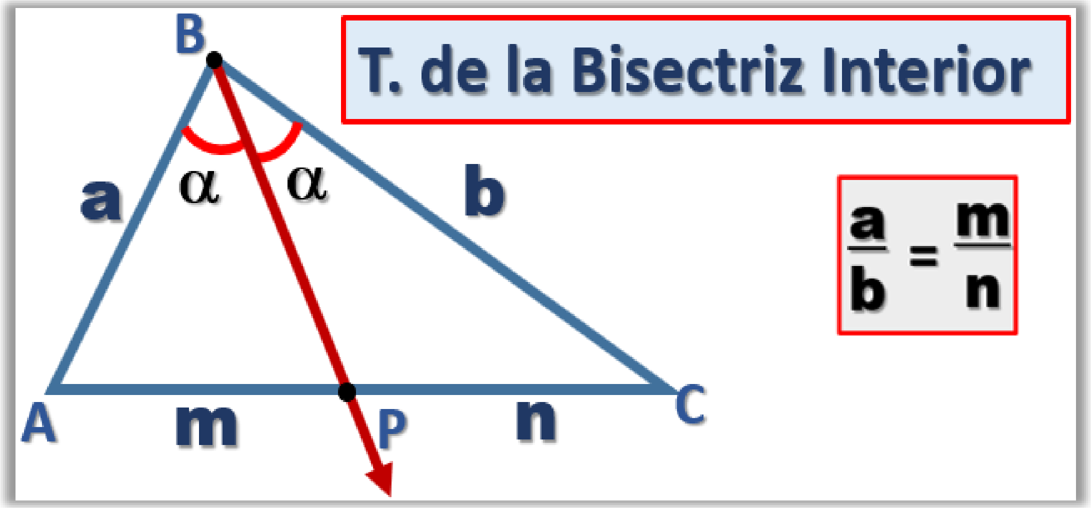
$$\frac{m}{6} = \frac{5}{10} \longrightarrow m = 3$$

- Es $\triangle$ notable de 37° y 53°

$$\boxed{x = 37°}$$

3. En un triángulo ABC, AB = 8, BC = 12 y AC = 10. Luego se traza la bisectriz interior $\overline{BD}$. Halle AD.





$$\frac{a}{b} = \frac{m}{n}$$

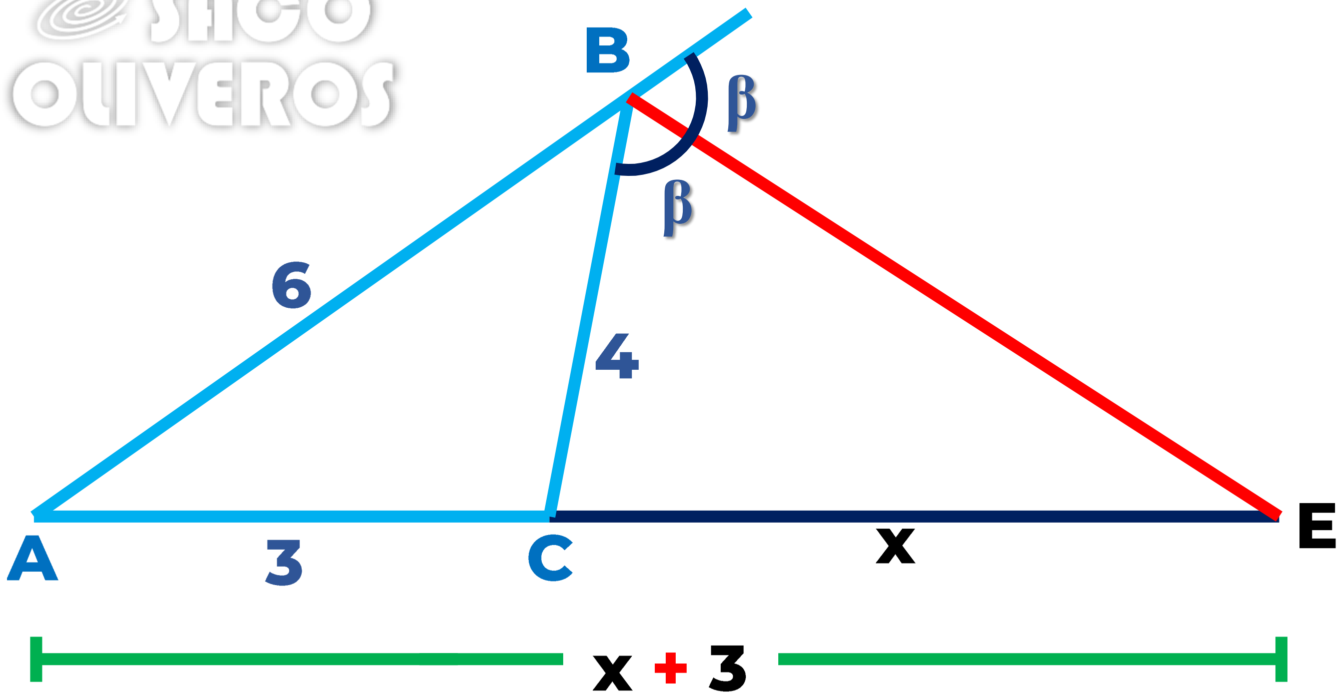
$$\frac{\cancel{8}^{\,2}}{\cancel{12}_{\,3}} = \frac{x}{10 - x}$$

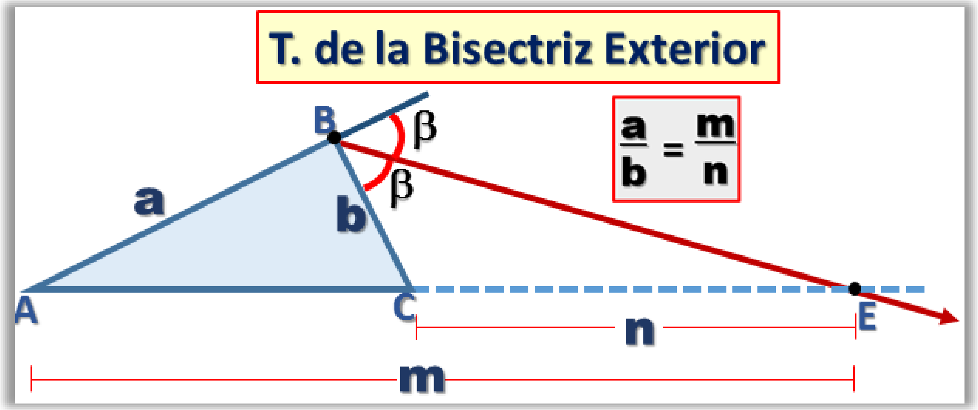
$$20 - 2x = 3x$$

$$20 = 5x$$

$$\boxed{x = 4}$$

4. En un triángulo ABC, AB = 6, BC = 4 y AC = 3. Luego se traza la bisectriz exterior del ángulo exterior en B, la cual interseca a la prolongación de $\overline{AC}$ en E. Halle CE.



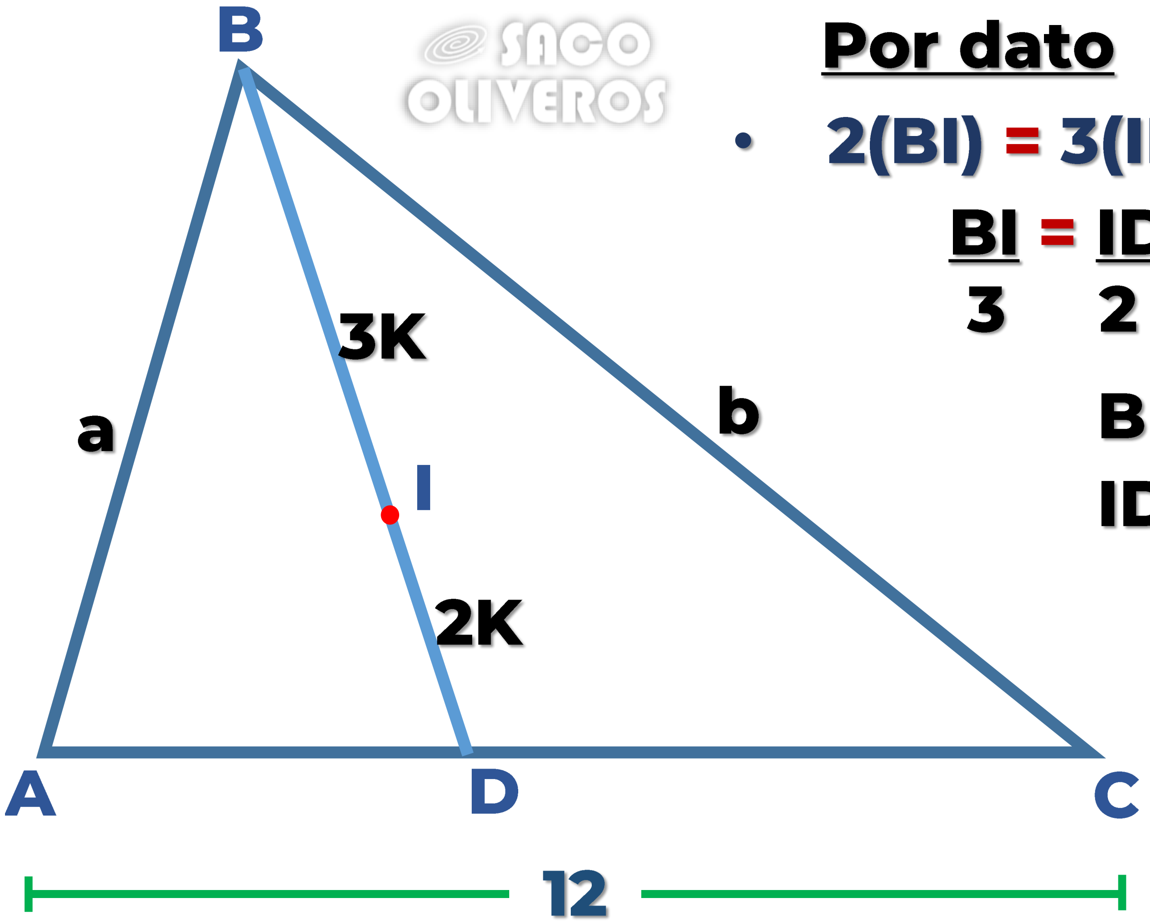


$$\frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{4}}} = \frac{x + 3}{x}$$

$$3x = 2x + 6$$

$$\boxed{x = 6}$$

5. En la figura, I es incentro del triángulo ABC, 2(BI) = 3(ID) y AC = 12. Calcule el perímetro de la región triangular ABC.



Por dato

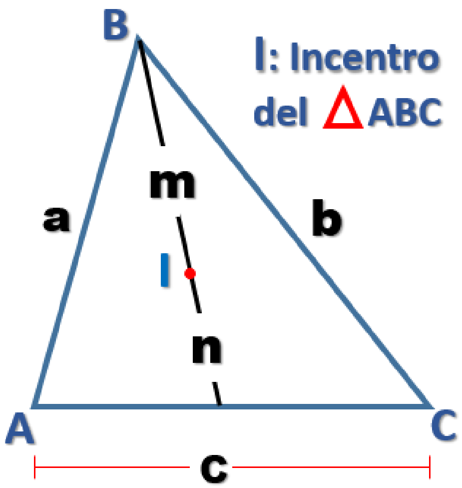
- $2(BI) = 3(ID)$

$$\frac{BI}{3} = \frac{ID}{2} = K$$

$BI = 3K$

$ID = 2K$

Teorema del Incentro



$$\boxed{\frac{m}{n} = \frac{a+b}{c}}$$

$$\frac{3\cancel{k}}{2\cancel{k}} = \frac{a+b}{12}$$

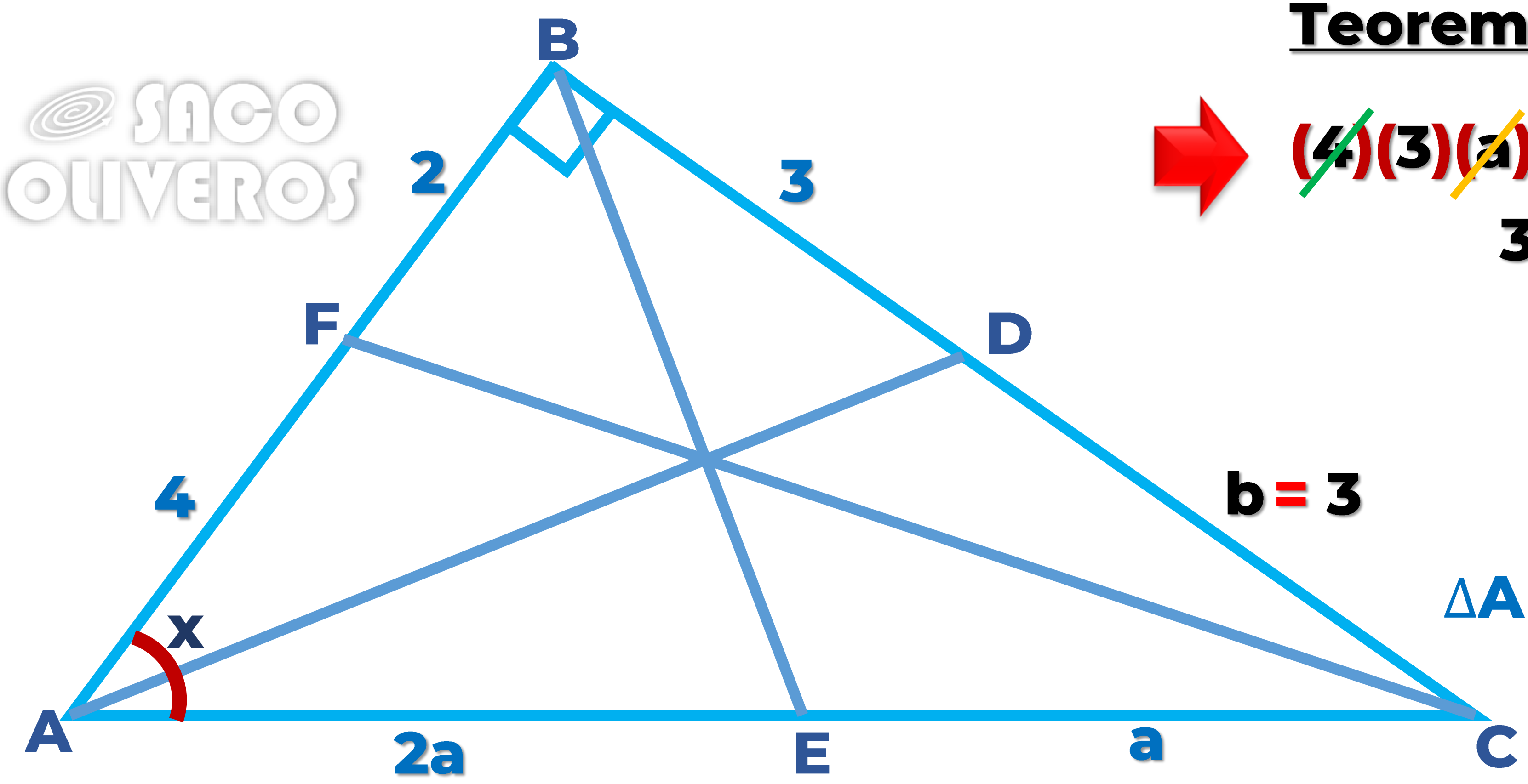
$18 = a + b$

Nos piden

$$2p_{\triangle} = \underbrace{a + b}_{18} + 12$$

$$\boxed{2p_{\triangle} = 30}$$

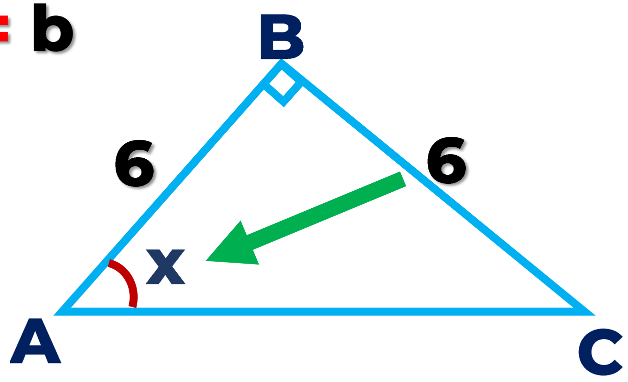
6. En un triángulo rectángulo ABC, recto en B, se trazan las cevianas interiores $\overline{AD}$, $\overline{BE}$ y $\overline{CF}$, las cuales se intersecan en un punto. Si AF = 4, FB = 2, BD = 3 y AE = 2(EC), halle m<BAC.



Teorema de Ceva

$$(\cancel{4})(3)(\cancel{a}) = (\cancel{2})(b)(\cancel{2a})$$

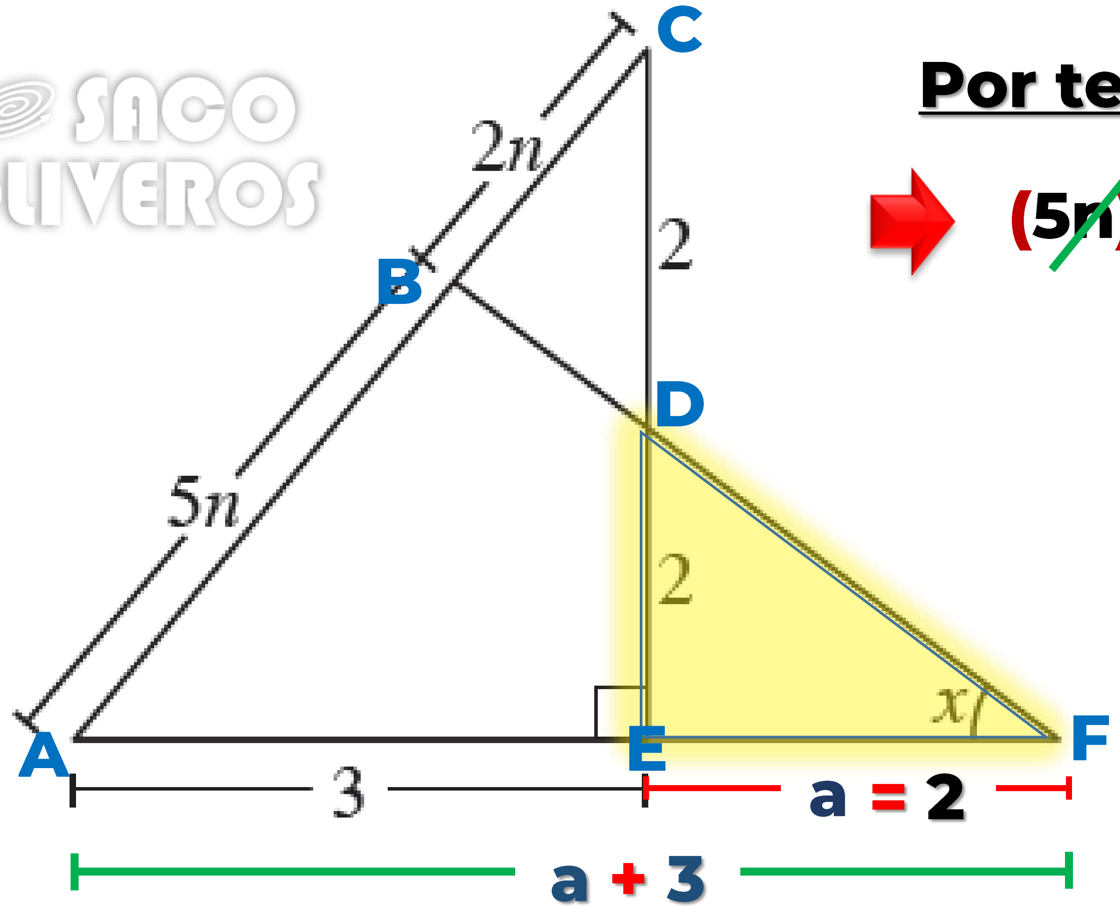
$$3 = b$$



ΔABC: Notable de 45° y 45°

$$\boxed{x = 45°}$$

7. En la figura, halle el valor de x.



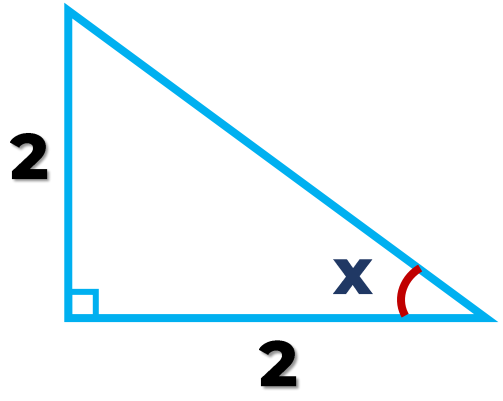
Por teorema de Menelao

$$(5\cancel{n})(\cancel{2})(a) = (\cancel{2}\cancel{n})(2)(a+3)$$

$$5a = 2a + 6$$

$$3a = 6$$

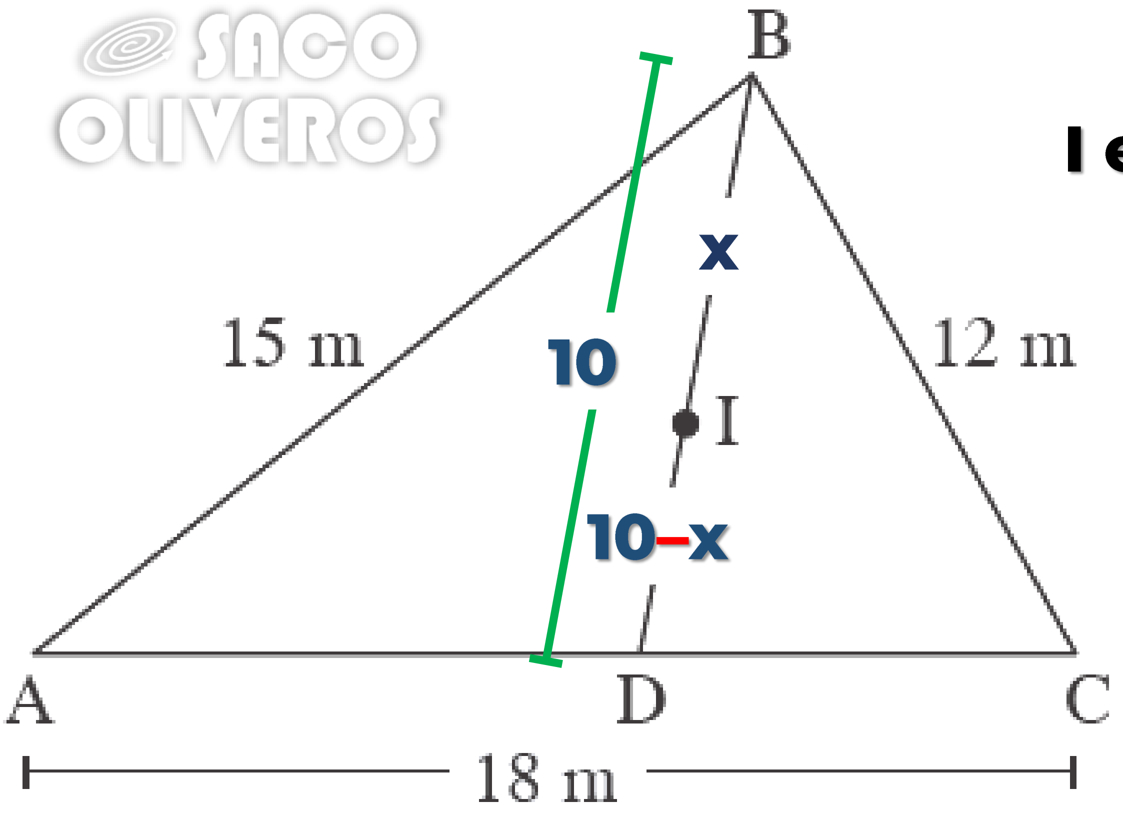
$$a = 2$$



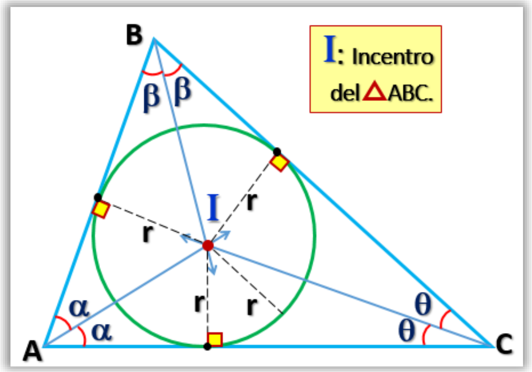
ΔDEF: Notable de 45° y 45°

$$\boxed{x = 45°}$$

8. En la figura se muestra el piso de una piscina donde en el punto I se encuentra el punto de succión del agua, el cual equidista de las paredes de la piscina. Halle la distancia de I a B si BD = 10 m.



I es el incentro △ ABC



Por teorema del Incentro

$$\frac{x}{10-x} = \frac{15+12}{18}$$

$$\frac{x}{10-x} = \frac{\cancel{27}^{\,3}}{\cancel{18}_{\,2}}$$

$$2x = 30 - 3x$$

$$5x = 30$$

$$\boxed{x = 6}$$