ALGEBRA

Chapter 11

4th
SECONDARY

NUMEROS COMPLEJOS

SACO OLIVEROS

HELICO

MOTIVATING

SACO OLIVEROS

¿Puedes multiplicar mentalmente el siguiente números complejos y dar la respuesta en menos de 10 segundos

$$z_1 = 7 + i$$

$$z_2 = 7 - i$$

Rpta. 50

HELICO THEORY

CHAPTHER 01

SACO OLIVEROS

NÚMEROS COMPLEJOS

I) UNIDAD IMAGINARIA

$$\boxed{i^2 = -1} \quad y \quad \boxed{i = \sqrt{-1}}$$

- $\sqrt{-9} = \sqrt{9}.\underbrace{\sqrt{-1}}_{i} = 3i$

- $\sqrt{-25} = \sqrt{25}.\underbrace{\sqrt{-1}} = 5i$

POTENCIAS DE LA UNIDAD IMAGINARIA

$i^1 = i$	$i^5 = i$	$i^9 = i$
$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{12} = 1$

Teorema:

$$i^{4k} = 1 \qquad i^{4k+1} = i$$

$$i^{4k+2} = -1 \qquad i^{4k+3} = -i$$

Ejemplo

- $i^{23} = i^{20+3} = i^{3} = -i$

- $i^{201} = i^{200+1} = i^{1} = i$

Teorema:

$$i + i^{2} + i^{3} + i^{4} + \cdots + i^{4k} = 0$$

Ejemplo

$$i + i^{2} + i^{3} + i^{4} + \cdots + i^{200} = 0$$

Definición:

$a, b \epsilon R$

$$z = (a; b) = a + bi \ / i = \sqrt{-1}$$

Donde:

- *Parte real:* $Re_{(z)} = a$
- *Parte imaginaria:* $Im_{(z)} = b$

Ejemplo:

- $z = (3; 2) = 3 + 2i$

$$Re_{(z)} = 3$$

$$Im_{(z)} = 2$$

Definiciones:

Sea: $z = a + \mathrm{bi}, a, b \epsilon R; entonces\ se\ define$

1. $complejo\ conjugado\ (\bar{z})$: $\bar{z} = a - bi$

2. Complejo opuesto (z^*): $z^* = -a - bi = Op(z)$

Ejemplo

$z = 3 - 4i \longrightarrow \bar{z} = 3 + 4i$

$z = 3 - 4i \longrightarrow z^* = -3 + 4i = Op(z)$

Operaciones con Números complejos

Adición y sustracción

Ejemplo:

- $\left.\begin{aligned} z_1 &= 2+4i \\ z_2 &= 3+2i \end{aligned}\right\} +$

$$z_1 + z_2 = 5 + 6i$$

- $\left.\begin{aligned} z_1 &= 2+4i \\ z_2 &= 3+2i \end{aligned}\right\} -$

$$z_1 - z_2 = -1 + 2i$$

Multiplicación

Sea: $z_1 = 2 + 4i$

$z_2 = 3 + 2i$

$$z_1 . z_2 = (2+4i)(3+2i)$$

$$z_1 . z_2 = 6 + 4i + 12i + \underbrace{8i^2}_{\boxed{-8}}$$

$$z_1 . z_2 = -2 + 16i$$

$$(a+bi)(a-bi)=a^2+b^2$$

División:

$$z=\frac{2+4i}{3-2i}$$

$$z=\frac{(2+4i)(3+2i)}{(3-2i)(3+2i)}$$

- $(2+4i)(3+2i)=-2+16i$
- $(3-2i)(3+2i)=3^2+2^2=13$

$$z=\frac{-2+16i}{13}=\frac{-2}{13}+\frac{16}{13}i$$

PROPIEDADES:

$$\boxed{\frac{1+i}{1-i}=i} \quad \boxed{\frac{1-i}{1+i}=-i}$$

$$\boxed{(1+i)^2=2i}$$

$$\boxed{(1-i)^2=-2i}$$

$$\boxed{(1\mp i)^4=-4}$$

$a, b, m, n \epsilon R$ con $m, n \neq 0$

$$\frac{a+bi}{n+mi} \rightarrow \mathbb{C}.\ imaginario\ puro$$

$$se\ cumple: \frac{a}{m}=-\frac{b}{n}$$

$$\frac{a+bi}{n+mi} \rightarrow complejo\ real$$

$$se\ cumple: \frac{a}{n}=\frac{b}{m}$$

HELICO PRACTICE

CHAPTHER 01

SACO OLIVEROS

PROBLEMA 1

Simplifique:

$$A=\frac{i^{428}+i^{817}+3i^{721}+i^{342}+2i^{755}}{i^{221}+4i^{436}+i^{473}-2i^{469}}$$

Resolución

$$k \text{ es } ENTERO$$

$$A=\frac{i^{4k}+i^{4k+1}+3i^{4k+1}+i^{4k+2}+2i^{4k+3}}{i^{4k+1}+4i^{4k}+i^{4k+1}-2i^{4k+1}}$$

$$A=\frac{1+i+3i+(-1)+2(-i)}{i+4(1)+i-2i}=\frac{2i}{4}=\frac{i}{2}$$

PROBLEMA 2

Sean los números complejos:

$$z_1 = 5 + 7i \qquad z_2 = 8 - 4i$$

$$Calcule: \; Op(z_1) + \overline{z_2} - 2\overline{z_1}$$

Resolución

$z_1 = 5 + 7i$ ⇒ $\overline{z_1} = 5 - 7i$ ⇒ $0p(z_1) = -5 - 7i$

$z_2 = 8 - 4i$ ⇒ $\overline{z_2} = 8 + 4i$

$$Luego: \; -5 - 7i + 8 + 4i - 2(5 - 7i)$$

⇒ $-7 + 11i$

PROBLEMA 3

Sean:

$$z_1 = -7 + 2i \qquad z_2 = 4 - 3i$$

$$Calcule:\ z_1.z_2 + Op(z_2) + \overline{z_1}$$

Resolución

$z_1 = -7 + 2i$ $\Rightarrow$ $\overline{z_1} = -7 - 2i$

$z_2 = 4 - 3i$ $\Rightarrow$ $0p(z_2) = -4 + 3i$

$$z_1.z_2 = (-7 + 2i)(4 - 3i) = -28 + 21i + 8i - 6\overset{-1}{i^2}$$

$z_1.z_2 = -22 + 29i$

$\Rightarrow -22 + 29i - 4 + 3i - 7 - 2i = \boxed{-33 + 30i}$

PROBLEMA 4

Si: $\frac{5+2i}{3+4i} = a + bi \quad Calcule: \frac{b}{a}$

Resolución

$$\frac{(5+2i)}{(3+4i)} \times \frac{(3-4i)}{(3-4i)} = \frac{(15-20i+6i-8\overset{-1}{i^2})}{9-16i^2} = \frac{23-14i}{25}$$

$$\Rightarrow \frac{23}{25} - \frac{14}{25} i = a + bi$$

$$\Rightarrow \boxed{\frac{b}{a} = \frac{-14}{23}}$$

PROBLEMA 5

Sean los números complejos:

$$z_1 = -3 + 2i \qquad z_2 = 5i - \overline{z_1}$$

Calcule $Re(z_1 . z_2)$

Resolución

$$z_1 = -3 + 2i \rightarrow \overline{z_1} = -3 - 2i$$

$$z_2 = 5i - \overline{z_1}$$

$$z_2 = 5i - (-3 - 2i)$$

$$z_2 = 5i + 3 + 2i$$

$$\boxed{z_2 = 3 + 7i}$$

Hallando $Re(z_1 . z_2)$:

$$z_1 . z_2 = (-3 + 2i)(3 + 7i)$$

$$z_1 . z_2 = -9 - 21i + 6i - 14$$

$$\boxed{z_1 . z_2 = -23 - 15i}$$

$$\boxed{\therefore\ Re(z_1 . z_2) = -23}$$

PROBLEMA 6

La edad de Carlos hace 15 años coincide con la parte imaginaria de $z_1.\overline{z_2}$ *, donde:* $z_1 = 4 - 3i$; $z_2 = -7 - \overline{z_1}$
¿Qué edad tiene Carlos?

Resolución

$$z_1 = 4 - 3i \quad \Rightarrow \quad \overline{z_1} = 4 + 3i$$

$$z_2 = -7 - \overline{z_1}$$

$$z_2 = -7 - (4 + 3i)$$

$$z_2 = -7 - 4 - 3i$$

$$z_2 = -11 - 3i$$

$$\Rightarrow \boxed{\overline{z_2} = -11 + 3i}$$

Hallando $Imag(z_1.\overline{z_2})$:

$$z_1.\overline{z_2} = (4 - 3i)(-11 + 3i)$$

$$z_1.\overline{z_2} = -44 + 12i + 33i + 9$$

$$\boxed{z_1.\overline{z_2} = -35 + 45i}$$

$$\boxed{Imag(z_1.z_2) = 45}$$

Rpta: 60 años

PROBLEMA 7

Al reducir $T = \dfrac{(1+i)^{12}+(1-i)^{4}}{17}$ *, calcule* T^2+1

Resolución

Recordar:

$(1+i)^2 = 2i$

$(1-i)^2 = -2i$

$i^2 = -1$

$i^6 = -1$

$$T = \frac{(1+i)^{12}+(1-i)^{4}}{17}$$

$$T = \frac{\left[(1+i)^2\right]^6+\left[(1-i)^2\right]^2}{17}$$

$$T = \frac{[2i]^6+[-2i]^2}{17}$$

$$T = \frac{2^6 . i^6+(-2)^2 . i^2}{17}$$

$$T = \frac{64(-1)+4(-1)}{17}$$

$$T = \frac{-68}{17}$$

$$\boxed{T = -4}$$

$$\boxed{\therefore \quad T^2+1 = 17}$$

PROBLEMA 8

Reduce $T=\left(\frac{1+i}{1-i}\right)^5+\left(\frac{1-i}{1+i}\right)^9$

Resolución

Recordar:

$$\boxed{\frac{1+i}{1-i}=i}$$

$$\boxed{\frac{1-i}{1+i}=-i}$$

$$\boxed{i^5=i}$$

$$\boxed{i^9=i}$$

$$T=\left(\frac{1+i}{1-i}\right)^5+\left(\frac{1-i}{1+i}\right)^9$$

$$T=(i)^5+(-i)^9$$

$$T=i+(-1)^9.(i)^9$$

$$T=i+(-1)(i)$$

$$T=i-i$$

$$\boxed{\therefore\quad T=0}$$