



TRIGONOMETRY

Chapter 13

5th
SECONDARY

IDENTIDADES TRIGONOMÉTRICAS
DE ÁNGULOS COMPUESTOS



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IDENTIDADES TRIGONOMÉTRICAS DEL

ÁNGULO COMPUESTO (Fundamentales)

Para la suma de dos ángulos

$$\operatorname{sen}(x + y) = \operatorname{sen}x \cdot \operatorname{cos}y + \operatorname{cos}x \cdot \operatorname{sen}y$$

$$\operatorname{cos}(x + y) = \operatorname{cos}x \cdot \operatorname{cos}y - \operatorname{sen}x \cdot \operatorname{sen}y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

Para la resta de dos ángulos

$$\operatorname{sen}(x - y) = \operatorname{sen}x \cdot \operatorname{cos}y - \operatorname{cos}x \cdot \operatorname{sen}y$$

$$\operatorname{cos}(x - y) = \operatorname{cos}x \cdot \operatorname{cos}y + \operatorname{sen}x \cdot \operatorname{sen}y$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$



1. Halle el valor de $\text{sen}82^\circ$ y $\text{cos}15^\circ$

RESOLUCIÓN

Como: $82^\circ = 45^\circ + 37^\circ \rightarrow \text{sen}(82^\circ) = \text{sen}(45^\circ + 37^\circ)$

$$\rightarrow \text{sen}82^\circ = \underbrace{\text{sen}45^\circ}_{\left(\frac{\sqrt{2}}{2}\right)} \underbrace{\text{cos}37^\circ}_{\left(\frac{4}{5}\right)} + \underbrace{\text{cos}45^\circ}_{\left(\frac{\sqrt{2}}{2}\right)} \underbrace{\text{sen}37^\circ}_{\left(\frac{3}{5}\right)}$$

$$\therefore \text{sen}82^\circ = \frac{7\sqrt{2}}{10}$$

Como: $15^\circ = 45^\circ - 30^\circ \rightarrow \text{cos}(15^\circ) = \text{cos}(45^\circ - 30^\circ)$

$$\rightarrow \text{cos}15^\circ = \underbrace{\text{cos}45^\circ}_{\left(\frac{\sqrt{2}}{2}\right)} \underbrace{\text{cos}30^\circ}_{\left(\frac{\sqrt{3}}{2}\right)} + \underbrace{\text{sen}45^\circ}_{\left(\frac{\sqrt{2}}{2}\right)} \underbrace{\text{sen}30^\circ}_{\left(\frac{1}{2}\right)}$$

$$\therefore \text{cos}15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$$



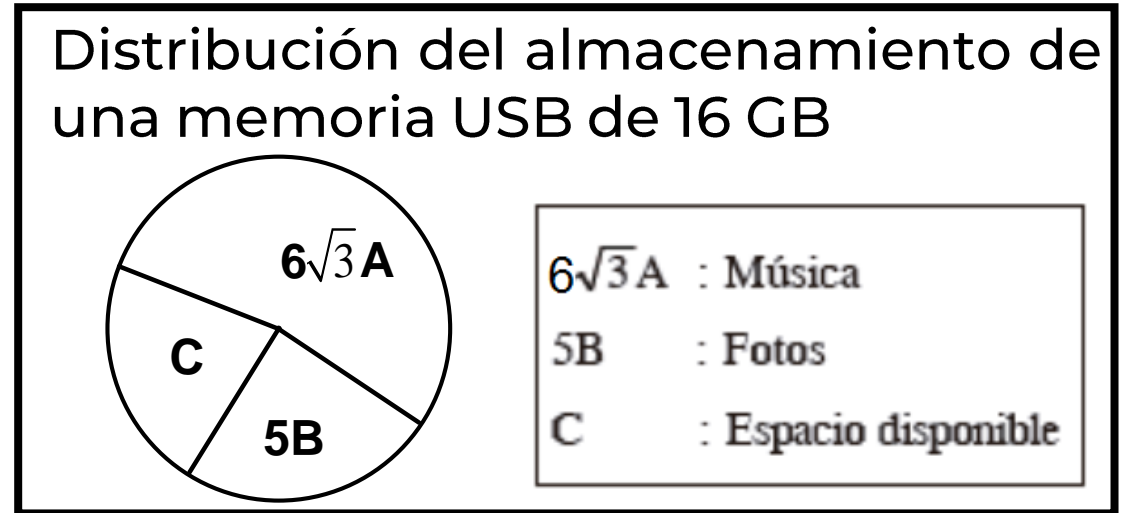
2. Observe el siguiente diagrama que indica el espacio utilizado de la memoria USB (GB):

Donde

$$A = \sin 40^\circ \cdot \cos 20^\circ + \cos 40^\circ \cdot \sin 20^\circ$$

$$B = \cos 63^\circ \cdot \cos 10^\circ + \sin 63^\circ \cdot \sin 10^\circ$$

Indique el espacio disponible de la memoria USB.



RESOLUCIÓN

$$A = \sin 40^\circ \cdot \cos 20^\circ + \cos 40^\circ \cdot \sin 20^\circ$$

$$\sin(40^\circ + 20^\circ) = \sin 60^\circ$$

$$B = \cos 63^\circ \cdot \cos 10^\circ + \sin 63^\circ \cdot \sin 10^\circ$$

$$\cos(63^\circ - 10^\circ) = \cos 53^\circ$$

$$A = \frac{\sqrt{3}}{2}$$

$$B = \frac{3}{5}$$

MÚSICA: $6\sqrt{3} \cdot \left(\frac{\sqrt{3}}{2}\right) = 9 \text{ GB}$

FOTOS: $5 \cdot \left(\frac{3}{5}\right) = 3 \text{ GB}$

∴ **ESPACIO
DISPONIBLE: 4 GB**



3. Halle el valor de $\tan x$ si:

$$\tan 37^\circ + \tan x = \frac{2}{3}$$

RESOLUCIÓN

$$\frac{\tan 37^\circ + \tan x}{1 - \tan 37^\circ \cdot \tan x} = \frac{2}{3}$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\Rightarrow \frac{\left(\frac{3}{4}\right) + \tan x}{1 - \left(\frac{3}{4}\right) \tan x} = \frac{2}{3}$$

$$\Rightarrow \frac{3 + 4 \tan x}{4 - 3 \tan x} = \frac{2}{3}$$

$$\Rightarrow \frac{3 + 4 \tan x}{4 - 3 \tan x} = \frac{2}{3}$$

$$\Rightarrow 9 + 12 \tan x = 8 - 6 \tan x$$

$$\Rightarrow 18 \tan x = -1$$

$$\therefore \tan x = -\frac{1}{18}$$



4. Si se cumple que $\left(-\frac{\pi}{4}\right) = \sqrt{\quad}$, calcule $\text{sen}x\cos x$.

RESOLUCIÓN

$$4 \left[\cancel{\cos x \cos \frac{\pi}{4}} + \cancel{\text{sen} x \text{sen} \frac{\pi}{4}} \right] = \sqrt{2} \quad \Rightarrow \quad 2 \cancel{\sqrt{2}} [\cancel{\cos x} + \cancel{\text{sen} x}] = \cancel{\sqrt{2}}$$

$\frac{\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2}$

$$\left\{ \cos x + \text{sen} x = \frac{1}{2} \right\}^2 \quad \Rightarrow \quad 1 + 2\text{sen}x\cos x = \frac{1}{4} \quad \Rightarrow \quad 2\text{sen}x\cos x = -\frac{3}{4}$$

Recordar: $(a+b)^2 = a^2 + 2ab + b^2$

$$\cos(x-y) = \cos x \cdot \cos y + \text{sen} x \cdot \text{sen} y$$

\therefore

$$\text{sen}x\cos x = -\frac{3}{8}$$

5. Simplifique la expresión

$$= \frac{(\quad - \quad)}{\cos x \sen y} -$$

RESOLUCIÓN

$$M = \frac{\cos(x - y)}{\cos x \sen y} - \tan x$$

$$M = \frac{\cos x \cos y + \sen x \sen y}{\cos x \sen y} - \tan x$$

$$M = \frac{\cancel{\cos x} \cos y}{\cancel{\cos x} \sen y} + \frac{\sen x \cancel{\sen y}}{\cancel{\cos x} \cancel{\sen y}} - \tan x$$

Recordar las identidades



$$\cos(x - y) = \cos x \cos y + \sen x \sen y$$

$$\cot \theta = \frac{\cos \theta}{\sen \theta}$$

$$\tan \theta = \frac{\sen \theta}{\cos \theta}$$

$$M = \frac{\cos y}{\sen y} + \frac{\sen x}{\cos x} - \tan x$$

$$M = \cot y + \cancel{\tan x} - \cancel{\tan x}$$

$$\therefore M = \cot y$$



6. Siendo $\alpha - \beta = 30^\circ$, halle el valor de

$$E = (\sin \alpha + \cos \beta)^2 + (\cos \alpha - \sin \beta)^2$$

RESOLUCIÓN

Recordar: $(a \pm b)^2 = a^2 \pm 2ab + b^2$

$$\sin(x - y) = \sin x \cdot \cos y - \cos x \cdot \sin y$$

$$E = (\sin \alpha + \cos \beta)^2 + (\cos \alpha - \sin \beta)^2$$

$$E = \sin^2 \alpha + 2\sin \alpha \cos \beta + \cos^2 \beta + \cos^2 \alpha - 2\cos \alpha \sin \beta + \sin^2 \beta$$

$$E = \underbrace{\sin^2 \alpha + \cos^2 \alpha}_1 + 2(\underbrace{\sin \alpha \cos \beta - \cos \alpha \sin \beta}_{\sin(\alpha - \beta)}) + \underbrace{\cos^2 \beta + \sin^2 \beta}_1$$

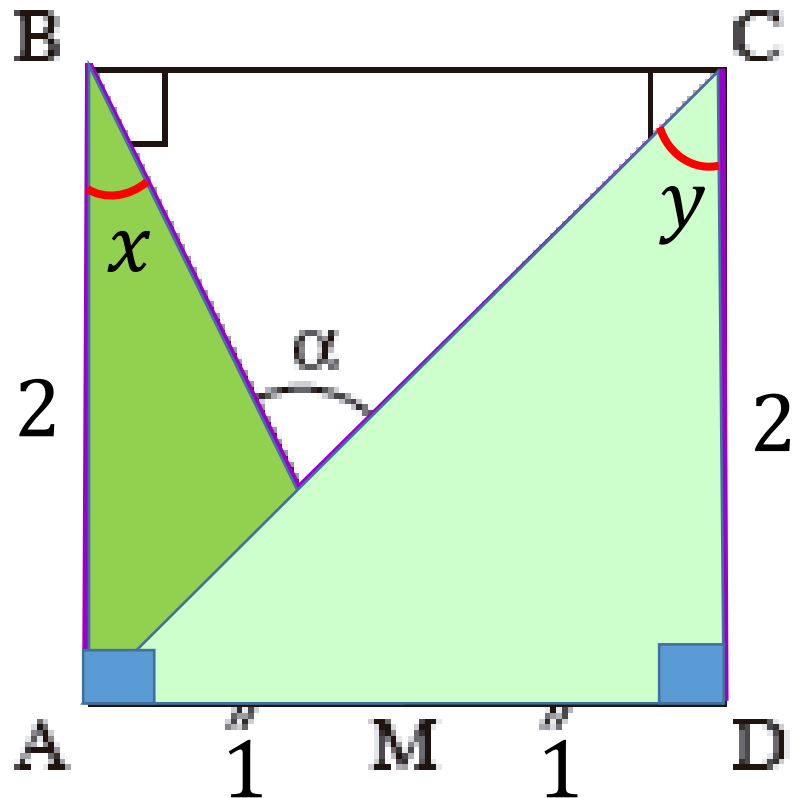
$$E = 2 + 2\sin 30^\circ = 2 + \cancel{2} \left(\cancel{\frac{1}{2}} \right)$$

\therefore

$$E = 3$$

7. En la figura adjunta, ABCD es un cuadrado. Calcule $\tan \alpha$

RESOLUCIÓN



Del grafico: $\tan x = \frac{1}{2}$; $\tan y = 1$

Como $\overline{AB} \parallel \overline{CD}$

$$\rightarrow \alpha = x + y \Rightarrow \tan \alpha = \tan(x + y)$$

$$\tan \alpha = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan \alpha = \frac{\frac{1}{2} + 1}{1 - \frac{1}{2}(1)} \Rightarrow \tan \alpha = \frac{3}{\cancel{\frac{1}{2}}}$$

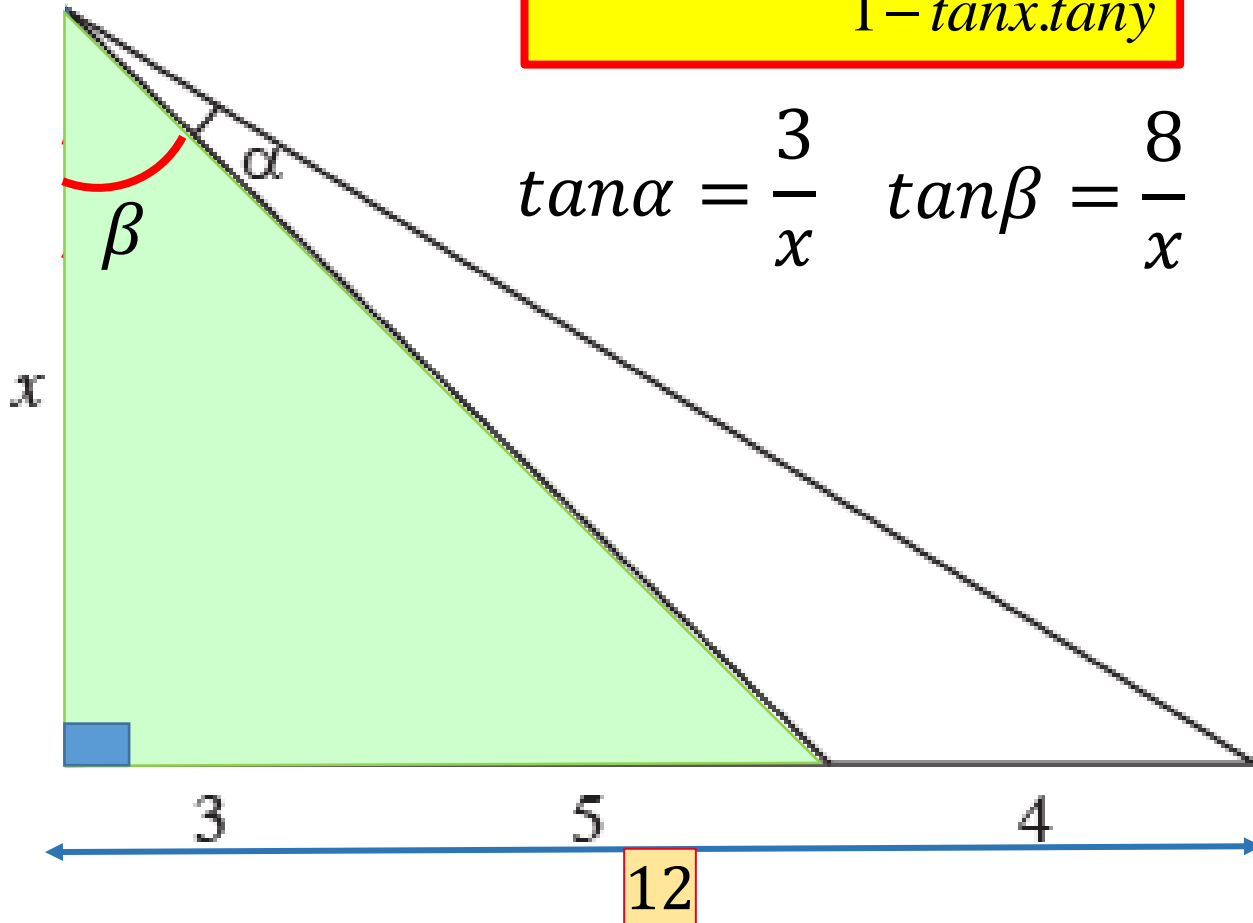
$$\therefore \tan \alpha = 3$$

8. Del gráfico, halle el valor de x .

RESOLUCIÓN

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\tan \alpha = \frac{3}{x} \quad \tan \beta = \frac{8}{x}$$



Se observa que: $\tan(\alpha + \beta) = \frac{12}{x}$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{12}{x}$$

$$\frac{\frac{3}{x} + \frac{8}{x}}{1 - \left(\frac{3}{x}\right)\left(\frac{8}{x}\right)} = \frac{12}{x} \quad \Rightarrow \quad \frac{\frac{11}{x}}{1 - \frac{24}{x^2}} = \frac{12}{x}$$

$$11 = 12 \left(1 - \frac{24}{x^2}\right) \quad \Rightarrow \quad 11 = 12 - \frac{12(24)}{x^2}$$

$$\frac{12(24)}{x^2} = 1 \quad \Rightarrow \quad x^2 = 12^2 \cdot 2$$

$$\therefore x = 12\sqrt{2}$$