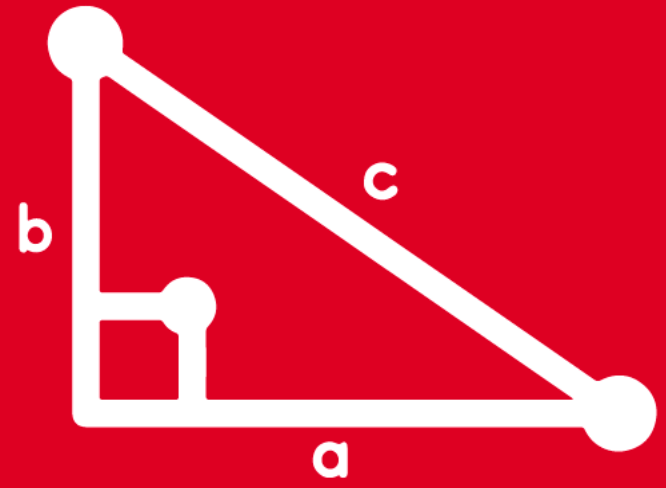
TRIGONOMETRY

Chapter 22
Session I



4th
SECONDARY

Funciones trigonométricas

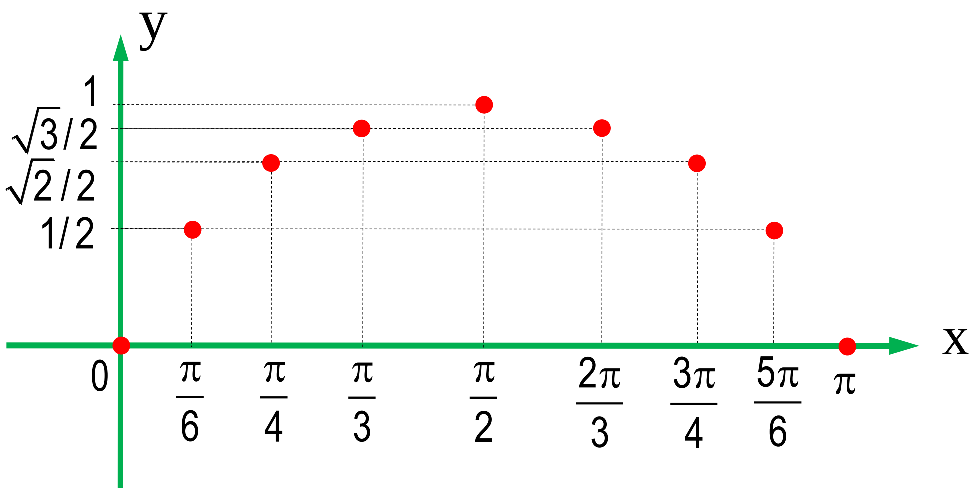
SACO OLIVEROS

FUNCION SENO:

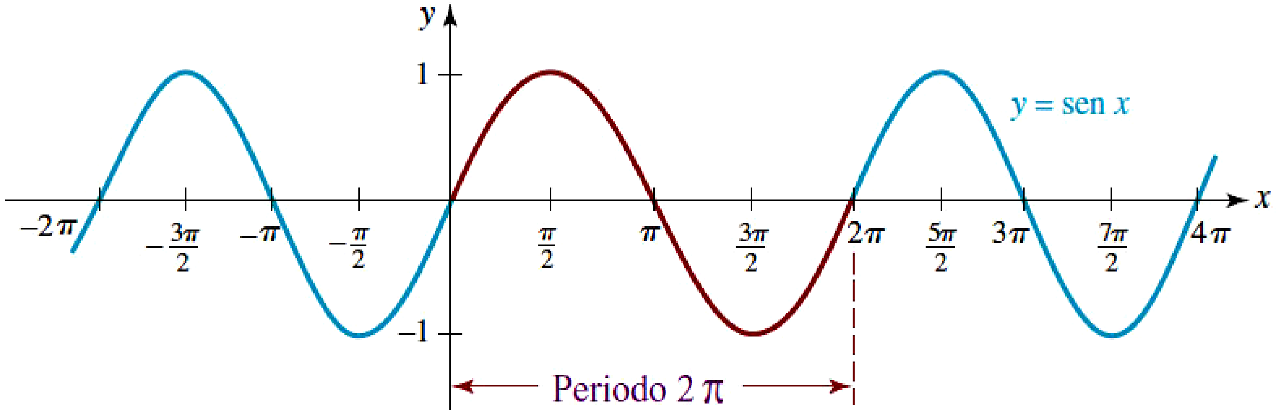
$$F = \{(x;y)/y = \text{sen}x\ ;\ x \in R\}$$

Tabulando algunos valores para x e y :

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y = senx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0



Tabulando mas valores y uniendo con una curva dichos puntos , tenemos :



Dominio : $\text{Dom}\,F \in R\ ;\ x \in R$

Rango : $\text{Ran}\,F \in [-1;1] \Rightarrow -1 \leq \text{sen}x \leq 1$

Periodo : $T = 2\pi$

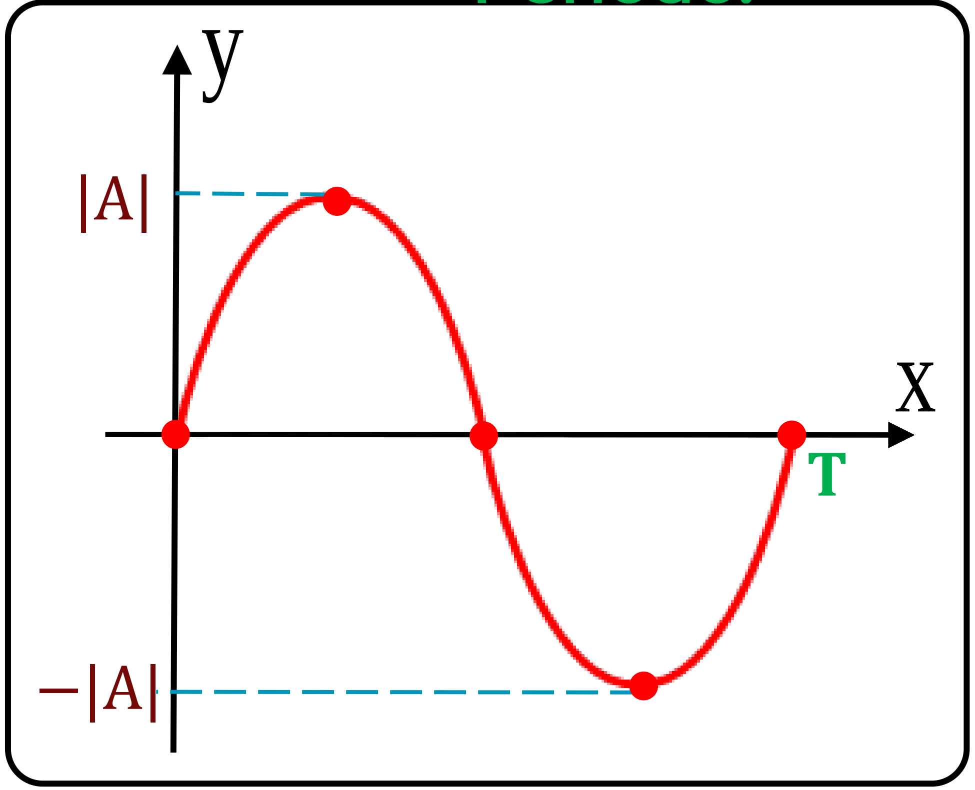
Es una función impar : $\text{sen}(-x) = -\text{sen}x$

OBSERVACION:

Sea la función: $y = A.\text{sen}Bx$

- Amplitud: $|A|$;

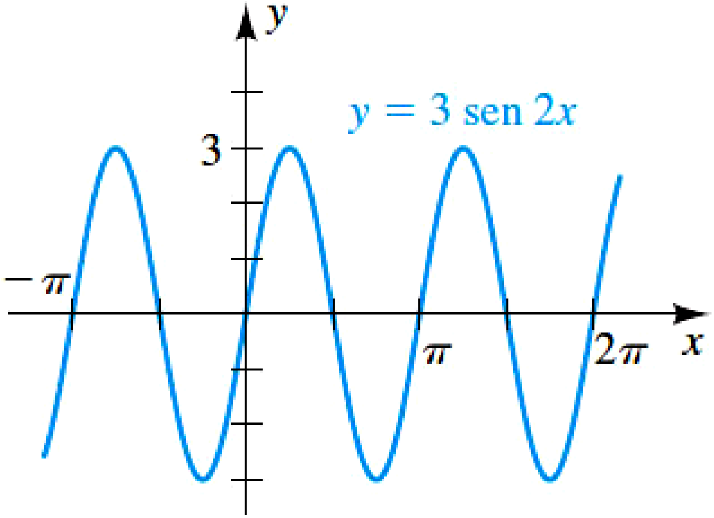
Período: $T = \dfrac{2\pi}{|B|}$



Ejemplos:

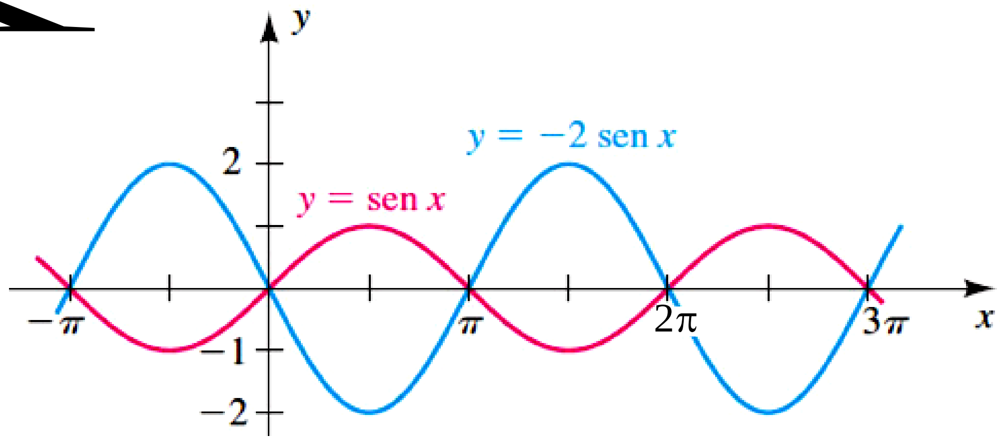
- y

$$\begin{cases} |A| = 3 \\ T = \pi \end{cases}$$



- x

$$\begin{cases} |A| = 2 \\ T = 2\pi \end{cases}$$



PROBLEMA 1

Determine el rango de la función: $\mathbf{f(x) = 5senx - 3}$

Resolución

Recordemos que:

$$\boxed{\forall x \in \mathbb{R}: -1 \leq senx \leq 1} \quad \ldots (*)$$

De (*):

$$-1 \leq senx \leq 1$$

x 5

$$-5 \leq 5senx \leq 5$$

– 3

$$-8 \leq \underbrace{5senx - 3}_{} \leq 2$$

$$-8 \leq f(x) \leq 2$$

$$\boxed{\therefore \text{Ran } f = [-8; 2]}$$

PROBLEMA 2

Determine el rango de la función: $\mathbf{g(x)} = \dfrac{2sen3x - 1}{3}$

Resolución

Recordemos que:

$\boxed{\forall x \in \mathbb{R}: -1 \leq sen3x \leq 1}$ ... (*)

De (*):

$$-1 \leq sen3x \leq 1$$

x 2

$$-2 \leq 2sen3x \leq 2$$

− 1

$$-3 \leq 2sen3x - 1 \leq 1$$

÷3

$$-1 \leq \underbrace{\frac{2sen3x - 1}{3}}_{g(x)} \leq \frac{1}{3}$$

$$\boxed{\therefore \text{Ran } g = \left[-1; \frac{1}{3}\right]}$$

PROBLEMA 3

Determine el rango de la función: $\mathbf{f(x) = 6.senx.cosx + 2}$

Resolución

Recordemos que:

$$\forall x \in \mathbb{R}: \quad -1 \leq \overbrace{sen2x}^{\mathbf{2senxcosx}} \leq 1$$

$$\Rightarrow \boxed{-\frac{1}{2} \leq \mathbf{senxcosx} \leq \frac{1}{2}} \quad \ldots (*)$$

De (*):

$$-\frac{1}{2} \leq \mathbf{senxcosx} \leq \frac{1}{2}$$

x 6

$$-3 \leq \mathbf{6senxcosx} \leq 3$$

+ 2

$$-1 \leq \underbrace{\mathbf{6senxcosx + 2}}_{} \leq 5$$

$$-1 \leq \mathbf{f(x)} \leq 5$$

$$\boxed{\therefore \mathbf{Ran\ f} = [-1; 5]}$$

PROBLEMA 4

Calcule $T_1 + T_2$, siendo T_1 y T_2 periodos de las funciones $f(x)$ y $g(x)$ respectivamente, donde: $\boldsymbol{f(x) = 3sen(5x)}$ y $\boldsymbol{g(x) = 2sen\left(\frac{x}{3}\right)}$

Resolución

Recordemos que:

$$y = Asen(Bx) \wedge y = Acos(Bx)$$

$$\boxed{T = \frac{2\pi}{|B|}} ; B \neq 0$$

Para : $f(x) = 3sen(5x)$ ➡ $T_1 = \frac{2\pi}{5}$

Para : $g(x) = 2sen\left(\frac{x}{3}\right)$ ➡ $T_2 = \frac{2\pi}{\frac{1}{3}}$ ➡ $T_2 = 6\pi$

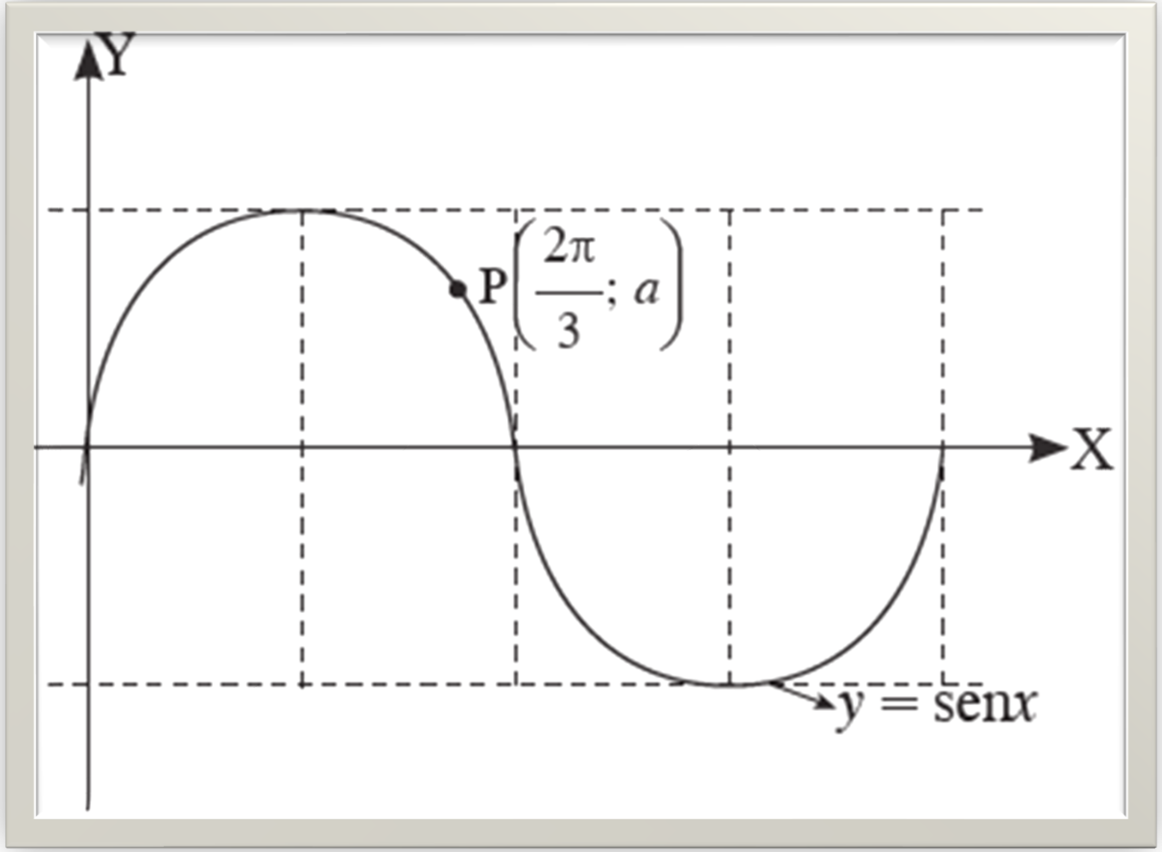
Piden:

$$T_1 + T_2 = \frac{2\pi}{5} + 6\pi$$

$$\boxed{\therefore T_1 + T_2 = \frac{32\pi}{5}}$$

PROBLEMA 5

Del gráfico, halle el valor de a.



Resolución

Sea f(x) = y = senx

Se cumple:

$$P\left(\frac{2\pi}{3}; a\right) \in f \Rightarrow a = \operatorname{sen}\left(\frac{2\pi}{3}\right)$$

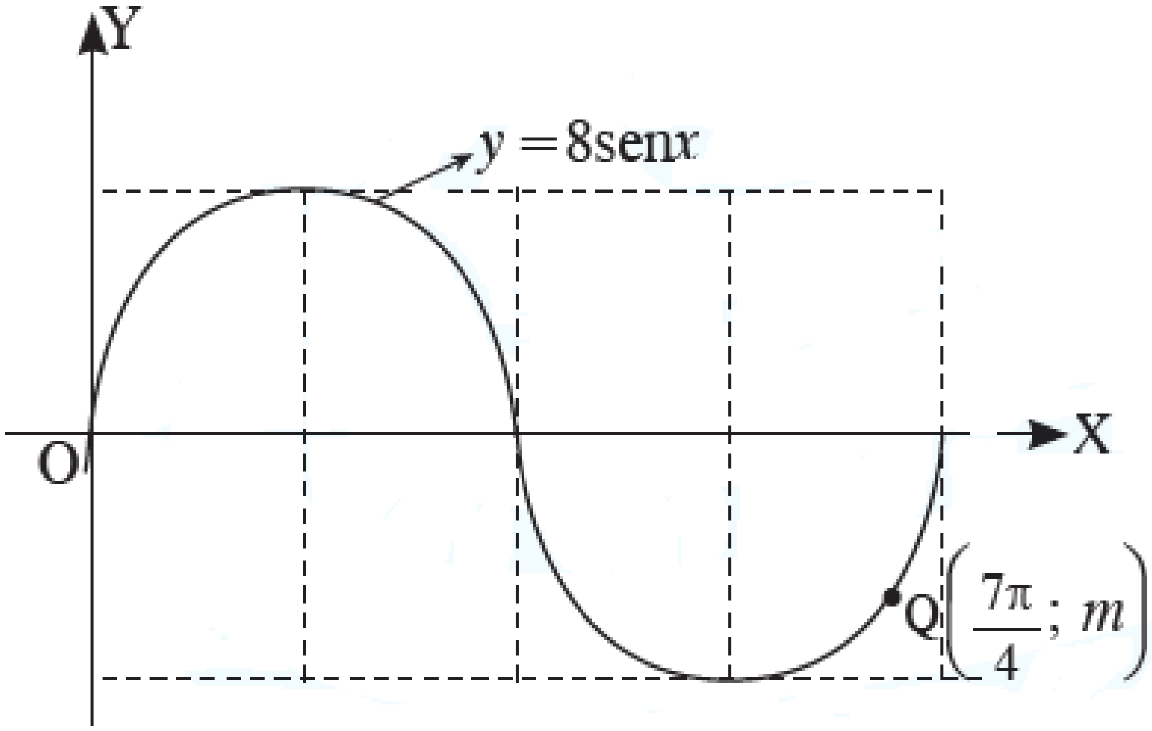
Entonces:

$$a = \operatorname{sen}\underbrace{\left(\pi - \frac{\pi}{3}\right)}_{IIC} = +\operatorname{sen}\left(\frac{\pi}{3}\right)$$

$$\boxed{\therefore a = \frac{\sqrt{3}}{2}}$$

PROBLEMA 6

Del gráfico, halle el valor de m.



Resolución

Sea: $f(x) = y = 8\text{sen}x$

Se cumple:

$$Q\left(\frac{7\pi}{4}; m\right) \in f \quad \Rightarrow \quad m = 8\text{sen}\left(\frac{7\pi}{4}\right)$$

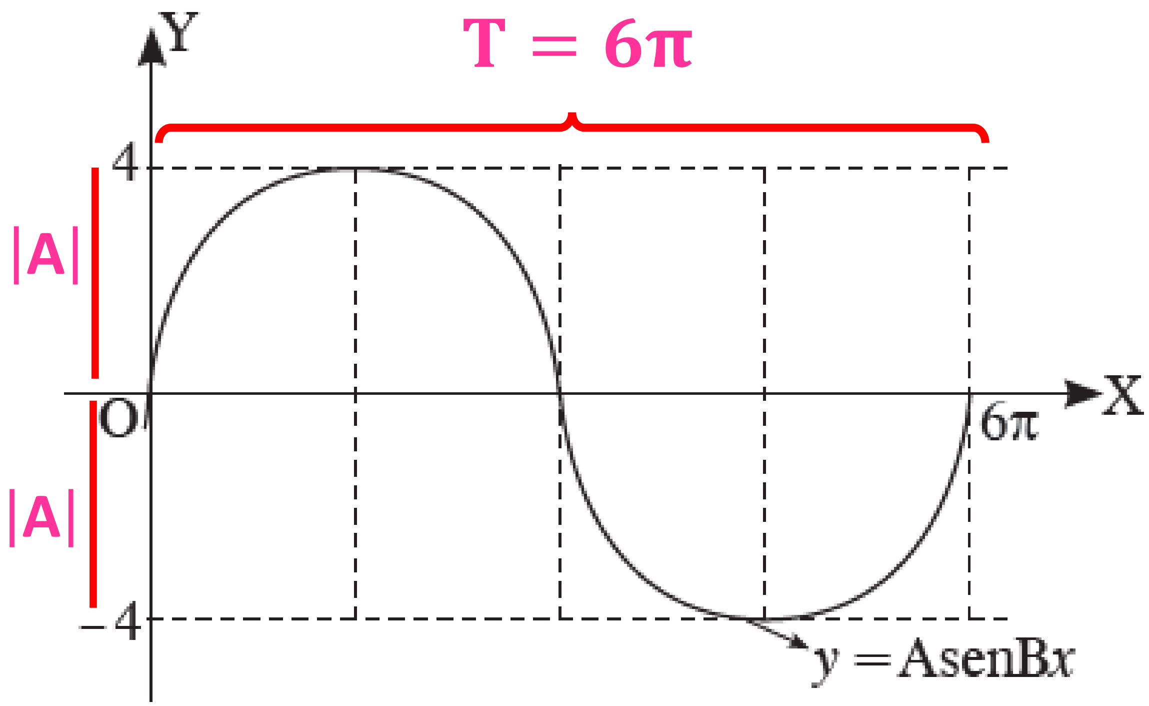
Entonces:

$$m = 8\text{sen}\underbrace{\left(2\pi - \frac{\pi}{4}\right)}_{\text{IVC}} = -8\,\underbrace{sen\left(\frac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}}$$

$$\boxed{\therefore\ m = -4\sqrt{2}}$$

PROBLEMA 7

Del gráfico, calcule A + B



Resolución

De la figura: $A = \boxed{4}$

Sea la función: $f(x) = y = A\text{sen}Bx$

El periodo (T): $T = \dfrac{2\pi}{|B|}$

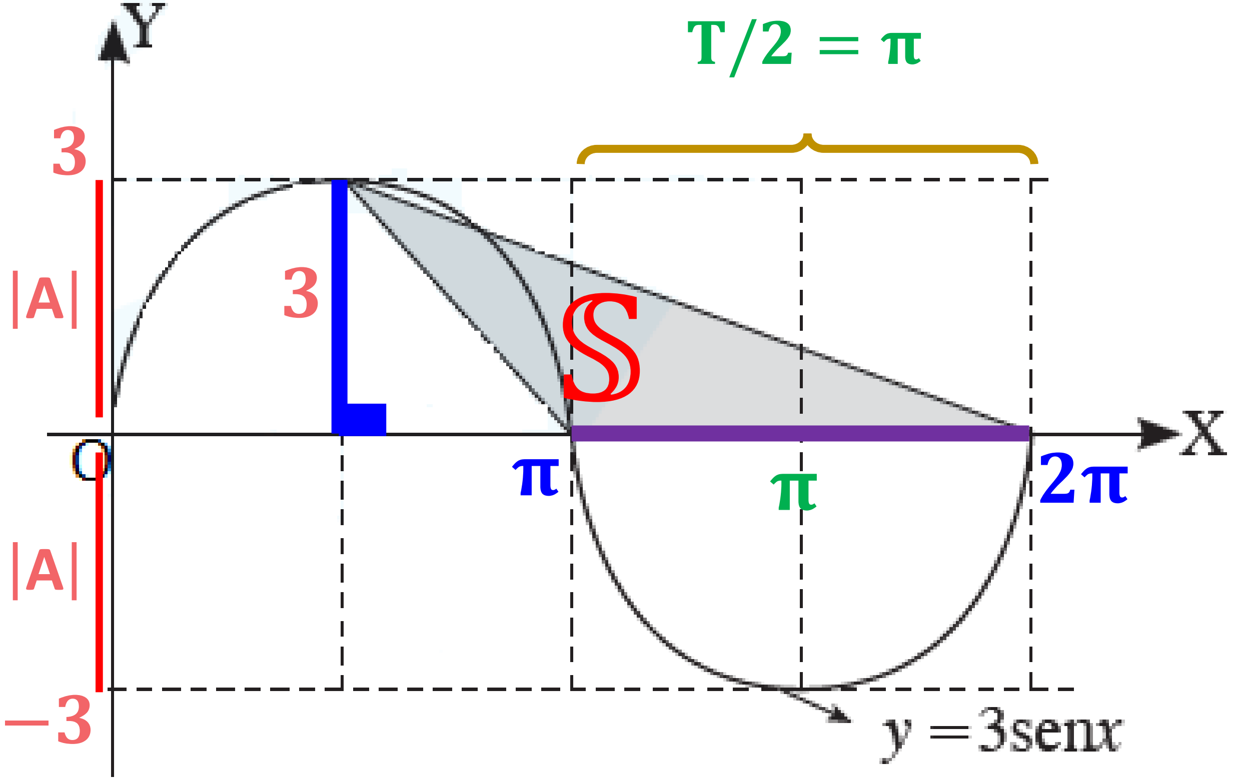
$\Rightarrow\ 6\pi = \dfrac{2\pi}{B}$ $\boxed{B = \dfrac{1}{3}}$

Piden: A + B $= 4 + \dfrac{1}{3}$

$\boxed{\therefore\ A + B = \dfrac{13}{3}}$

PROBLEMA 8

El siguiente grafico muestra las ondas emitidas por un teléfono móvil. Calcule el área de la región triangular sombreada.



Resolución

De la figura: $\boxed{A = 3}$

Sea la función: $f(x) = y = 3\text{senx}$

El periodo (T): $T = \dfrac{2\pi}{1}$ ➡ $\boxed{T = 2\pi}$

Área sombreada:

$$\Rightarrow \quad \mathbb{S} = \frac{(\pi)(3)}{2}$$

$$\boxed{\therefore \mathbb{S} = \frac{3\pi}{2}u^2}$$