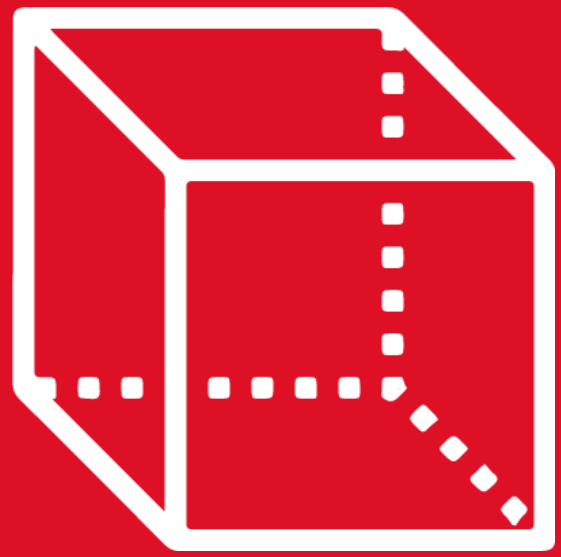
GEOMETRÍA

Capítulo 9



5th
SECONDARY

RELACIONES METRICAS EN EL TRIÁNGULO RECTÁNGULO Y LA CIRCUNFERENCIA

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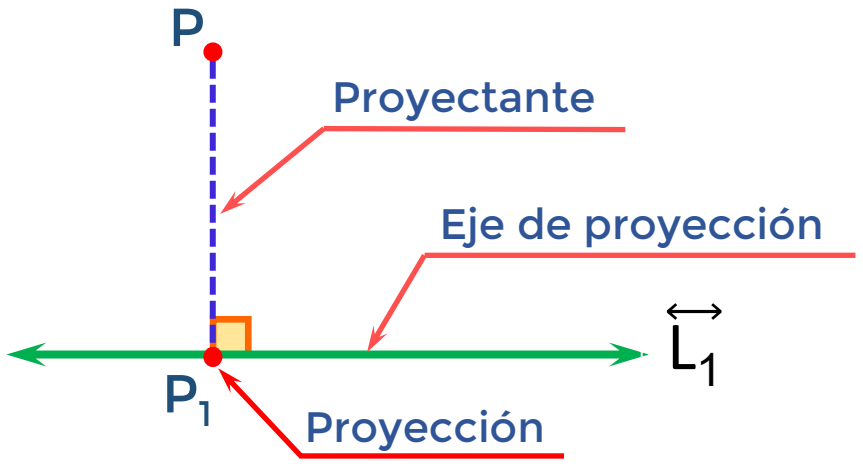
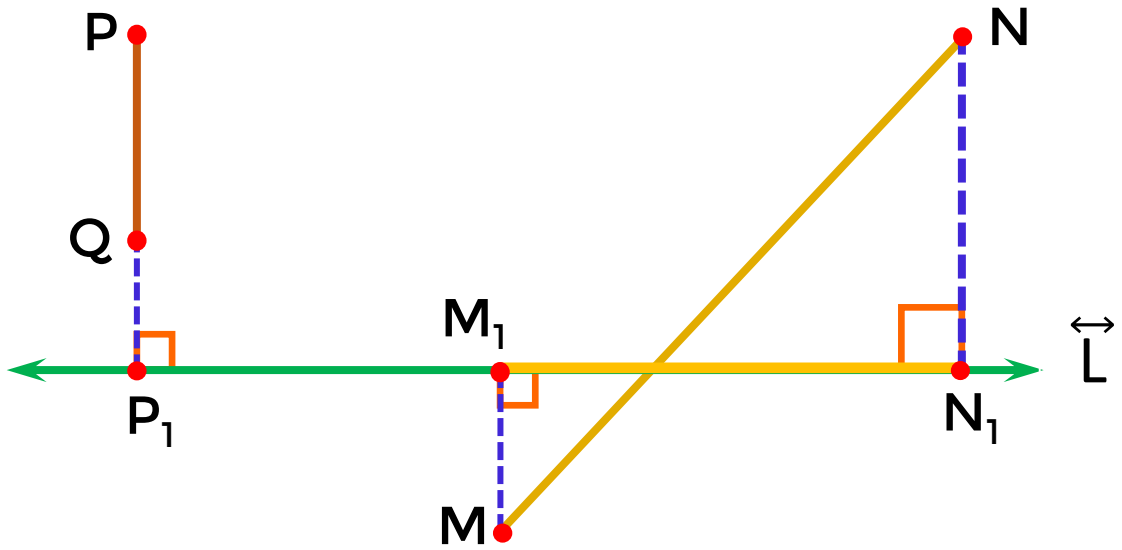
PROYECCIÓN ORTOGONAL

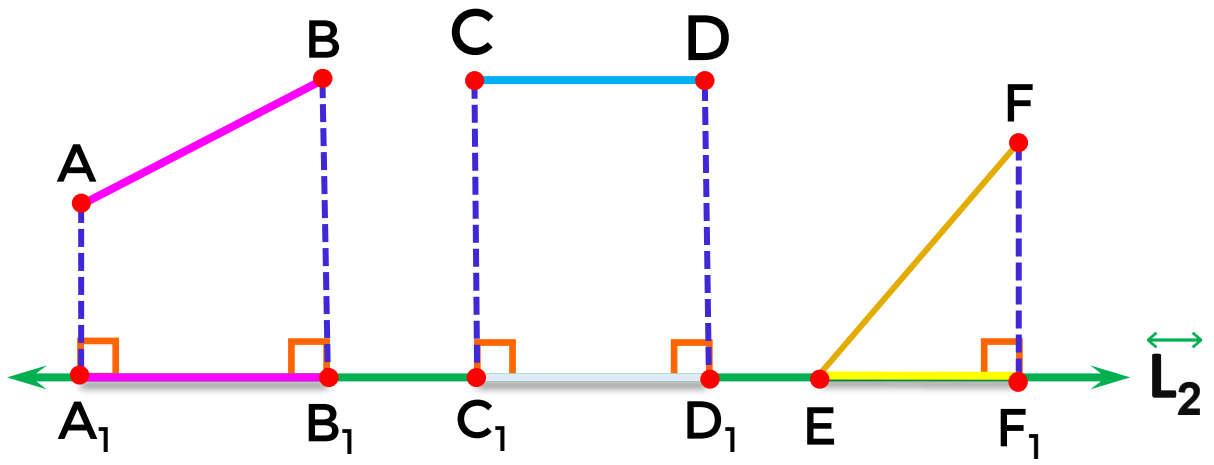
I. De un punto a una recta



NOTA :



II. De un segmento a una recta

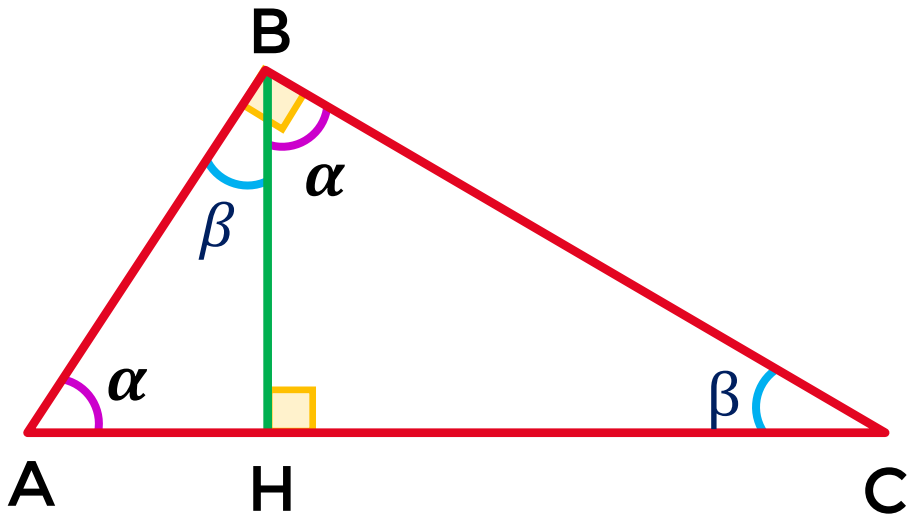


$\overline{A_1B_1}$: Proyección de $\overline{AB}$ sobre $\overleftrightarrow{L_2}$

$\overline{C_1D_1}$: Proyección de $\overline{C_1D_1}$ sobre $\overleftrightarrow{L_2}$

$\overline{EF_1}$: Proyección de $\overline{EF}$ sobre $\overleftrightarrow{L_2}$

RELACIONES MÉTRICAS EN EL TRIÁNGULO RECTÁNGULO



* $\overline{AB}$ y $\overline{BC}$ son catetos
* $\overline{AC}$: hipotenusa

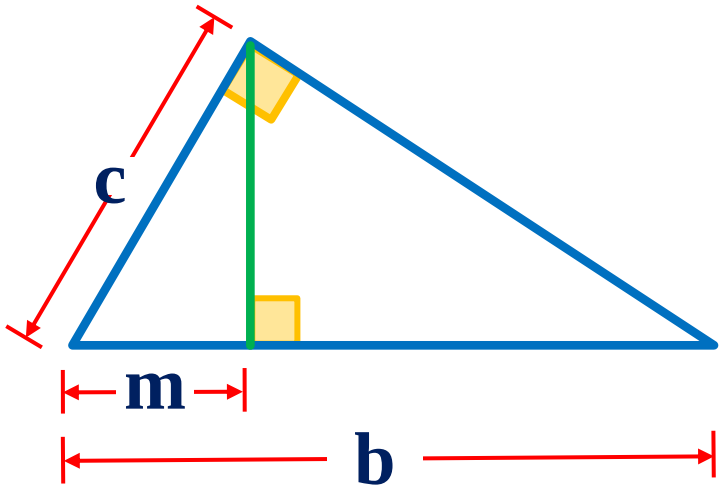
$\overline{AH}$: proyección ortogonal $\overline{AB}$ sobre $\overline{AC}$

$\overline{HC}$: proyección ortogonal $\overline{BC}$ sobre $\overline{AC}$

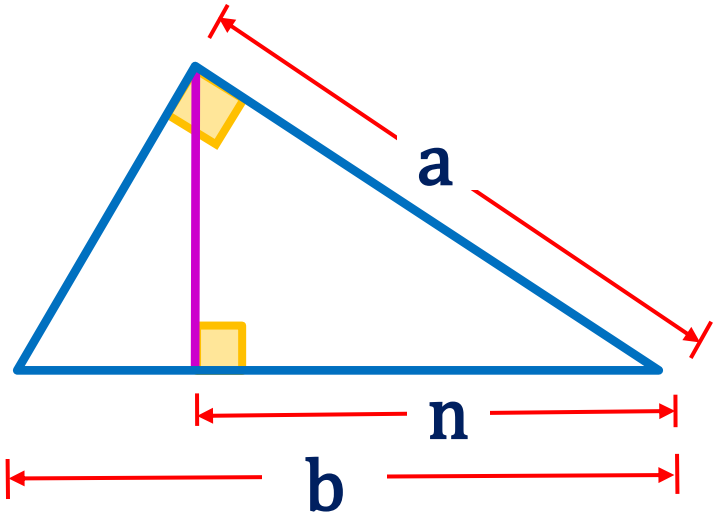
$\triangle ABC \sim \triangle AHB \sim \triangle BHC$

$(AC)^2 = (AB)^2 + (BC)^2$

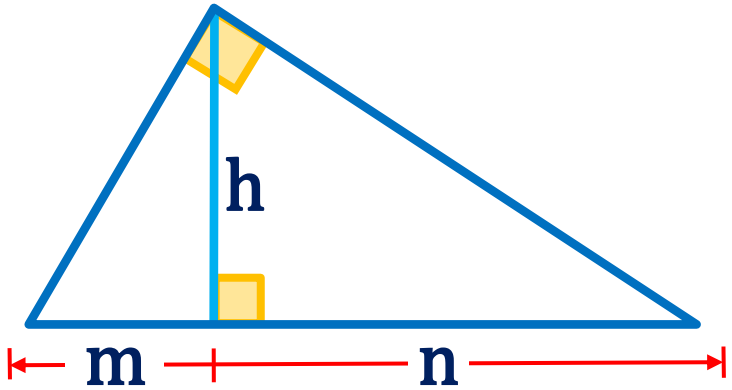
Longitud de cateto al cuadrado



$c^2 = bm$

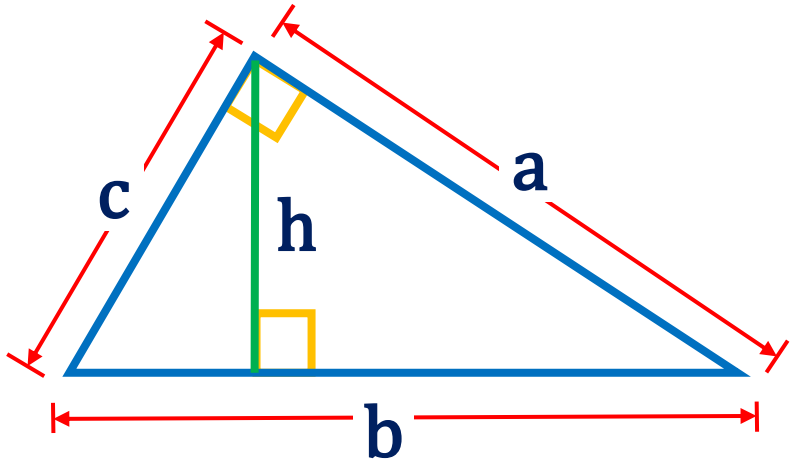


$a^2 = bn$

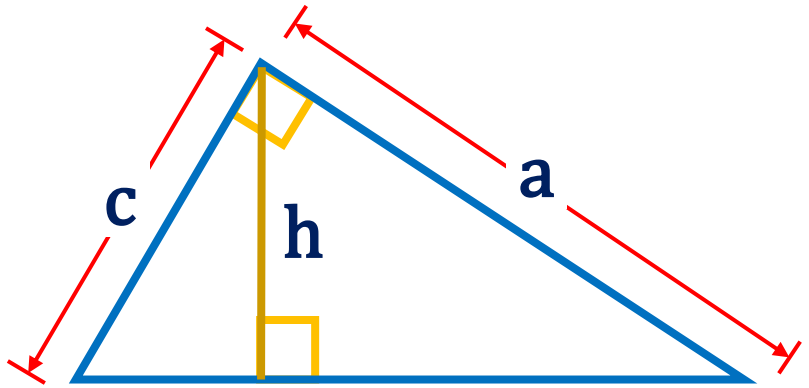


Longitud de la altura elevada al cuadrado

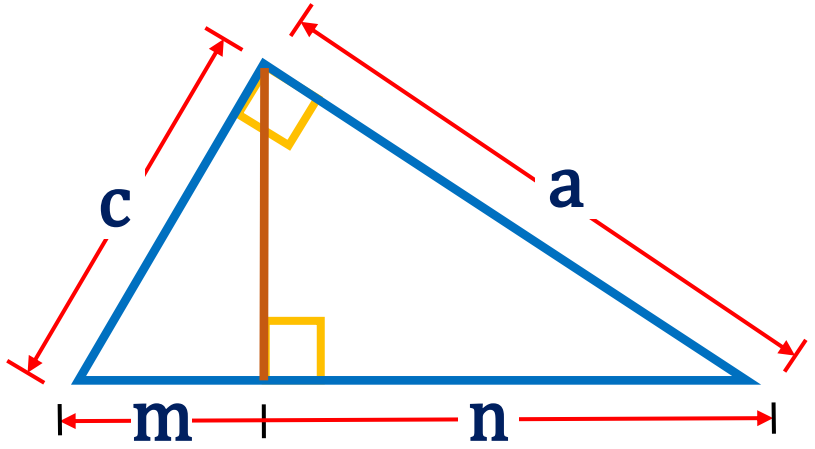
$h^2 = mn$



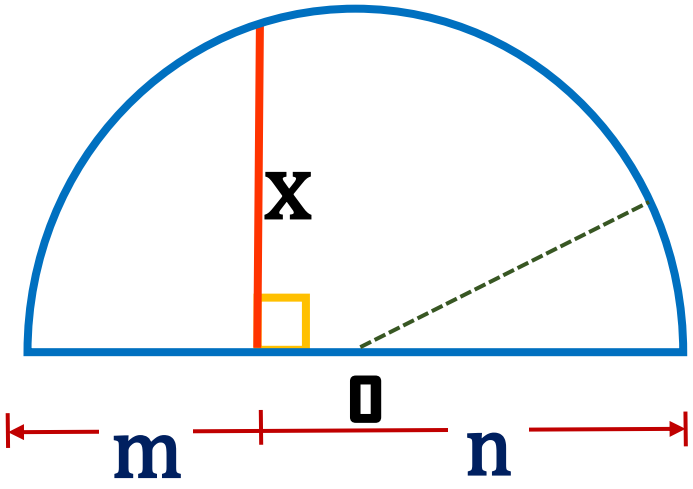
$$c \cdot a = h \cdot b$$



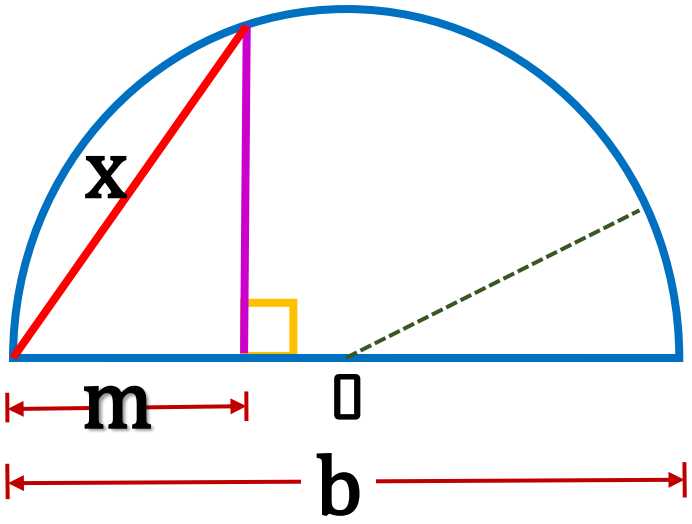
$$\frac{1}{c^2} + \frac{1}{a^2} = \frac{1}{h^2}$$



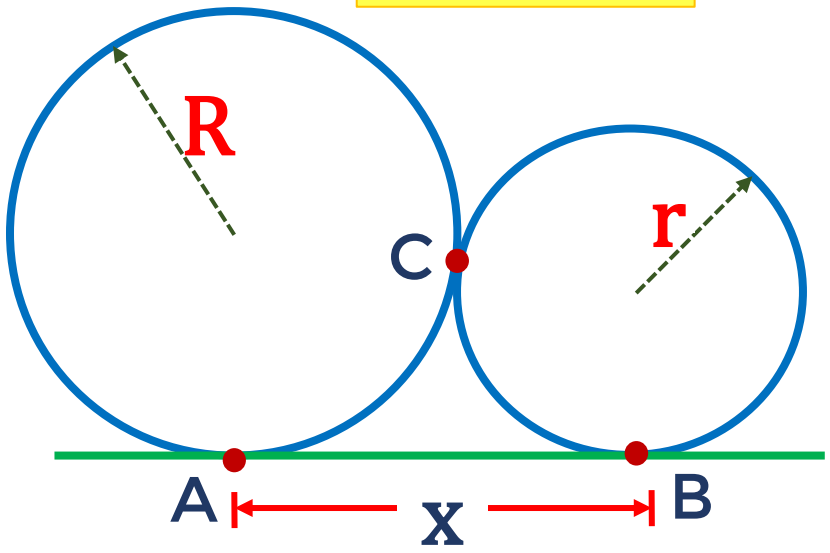
$$\frac{m}{n} = \frac{c^2}{a^2}$$



$$x^2 = m \cdot n$$



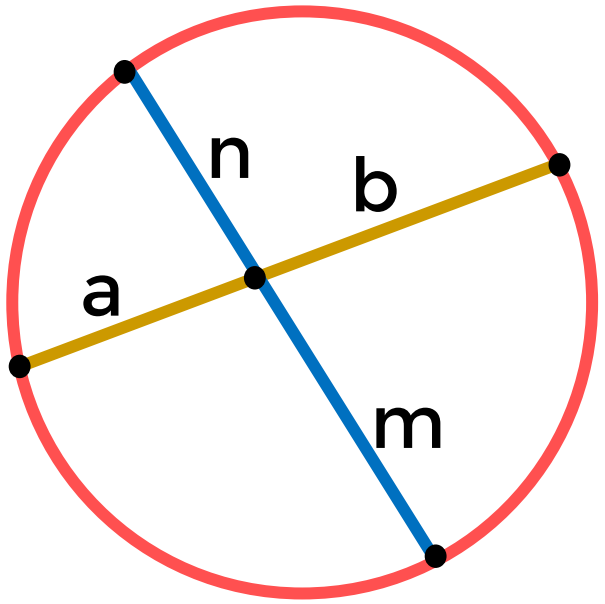
$$x^2 = b \cdot m$$



$$x = 2\sqrt{R.r}$$

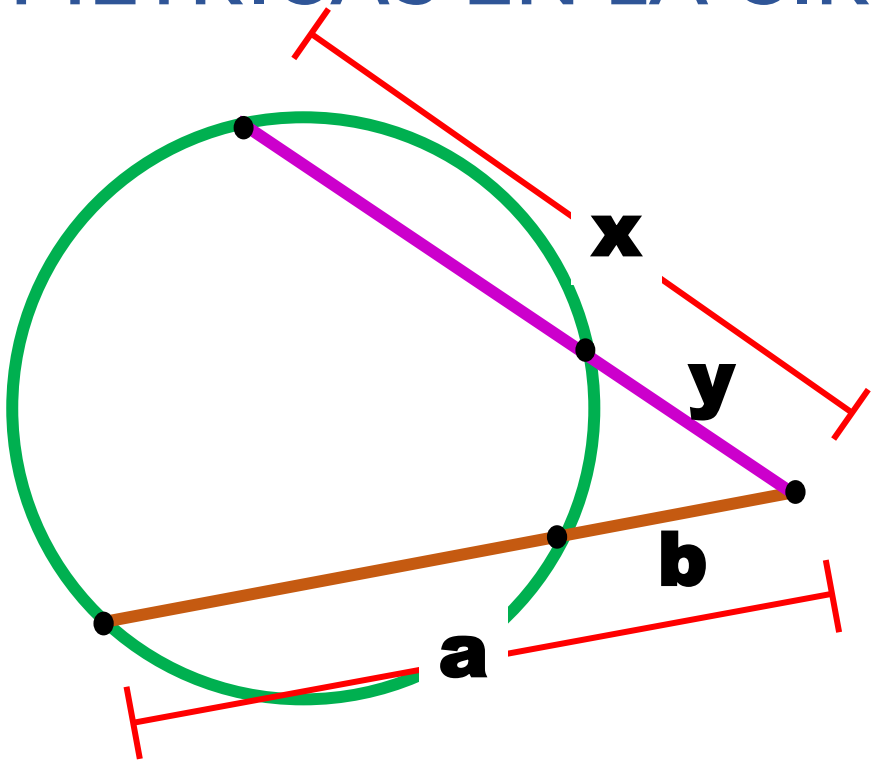
A, B y C son puntos de tangencia

RELACIONES MÉTRICAS EN LA CIRCUNFERENCIA



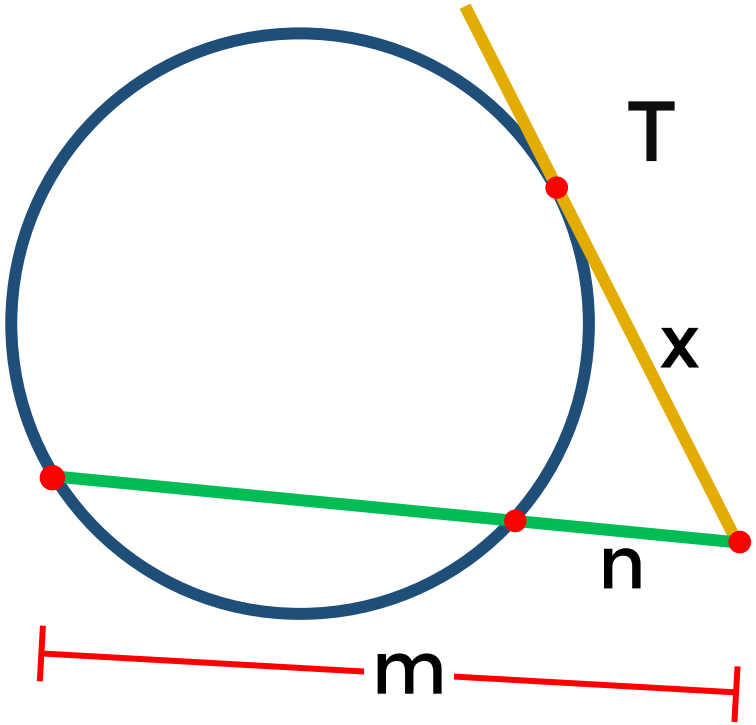
T. de Cuerdas

$$a \cdot b = m \cdot n$$



T. de las Secantes

$$x \cdot y = a \cdot b$$

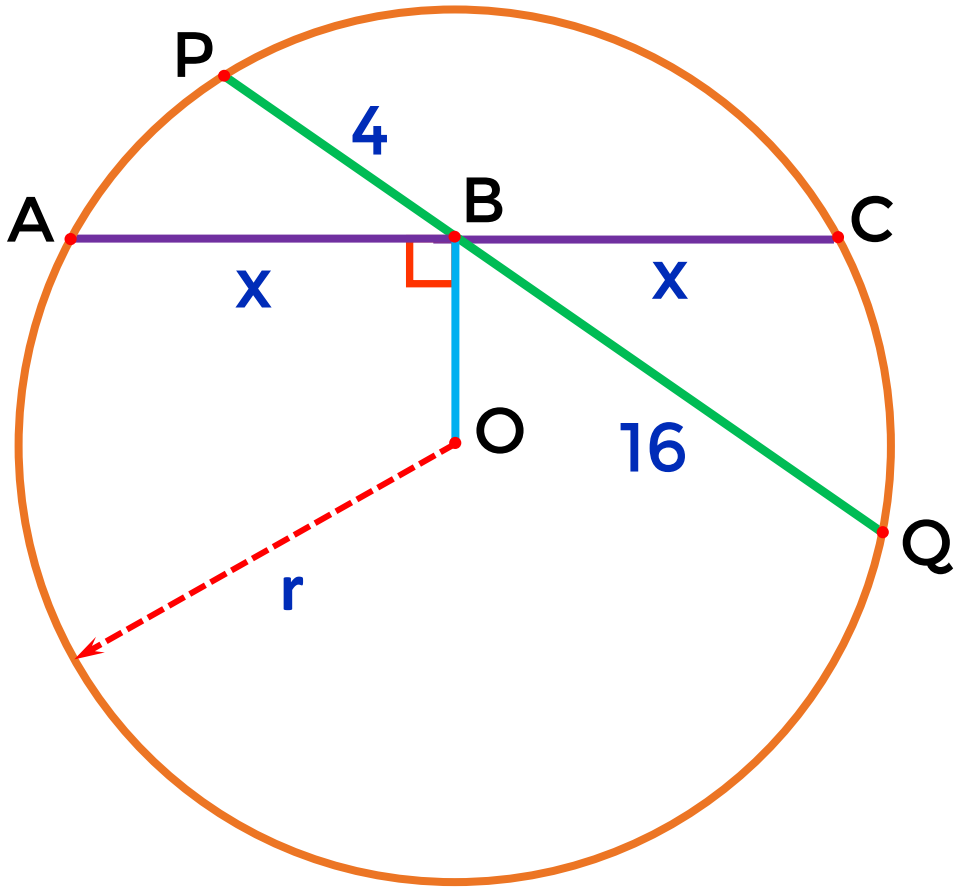


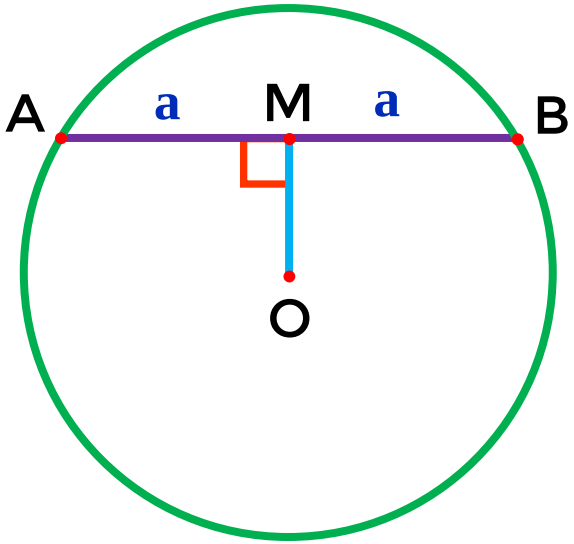
T. de la Tangente

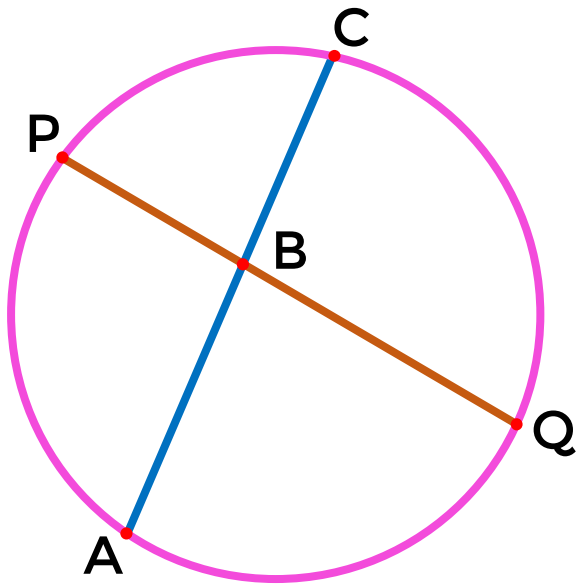
$$x^2 = n \cdot m$$

T : punto de tangencia

1. Halle el valor de x, si O es centro.







Teorema de cuerdas

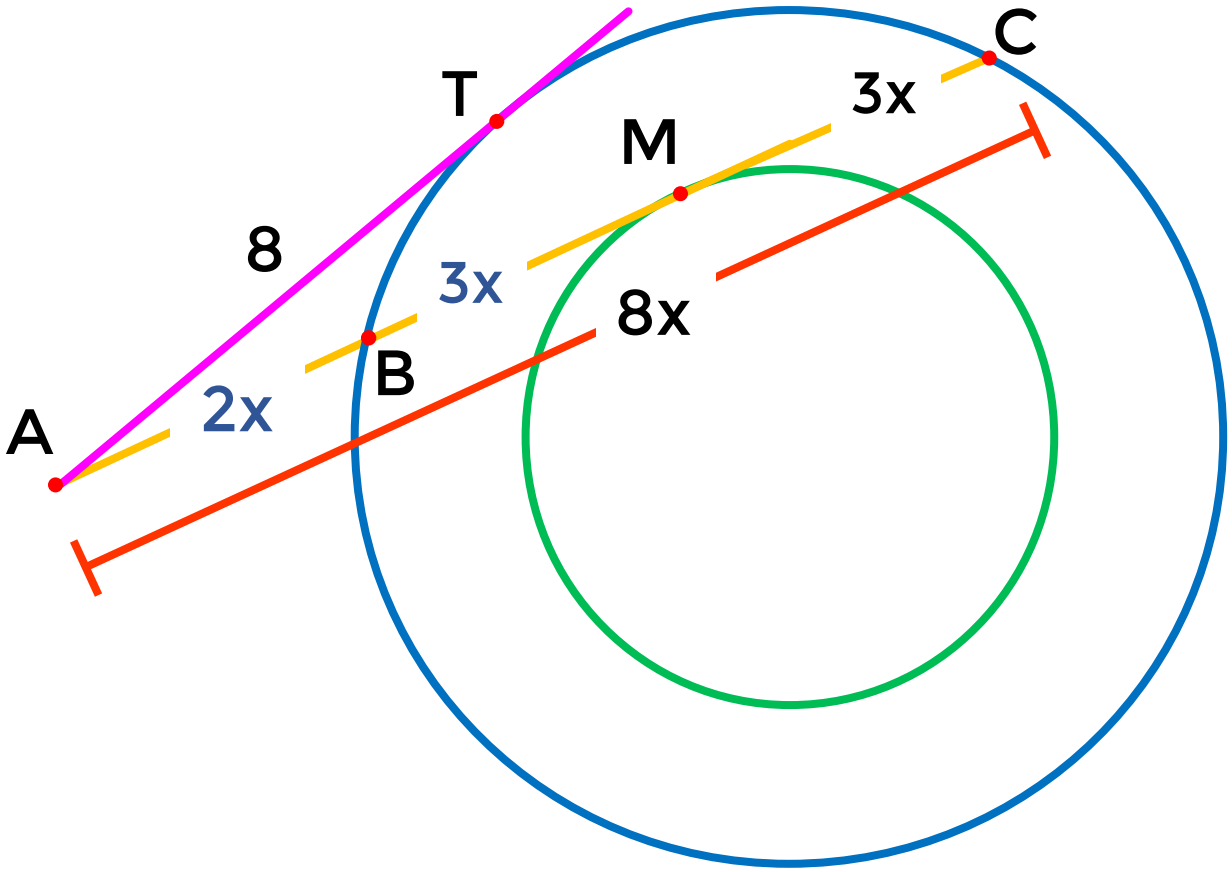
(PB)(BQ) = (AB)(BC)

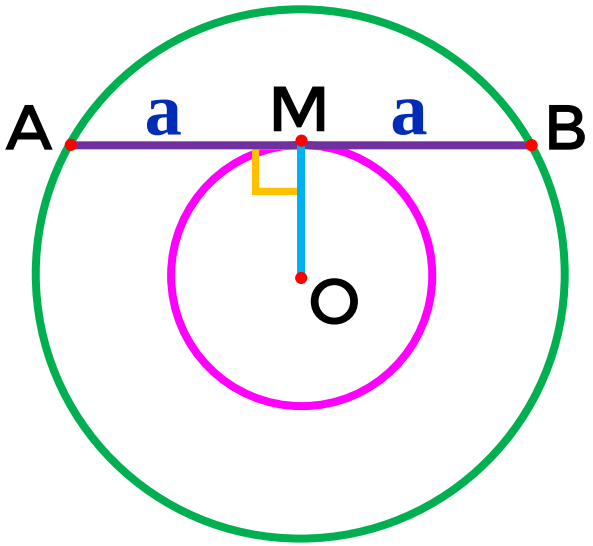
Resolución:

Del gráfico: $(x)(x) = (4)(16) \Rightarrow x^2 = 64$

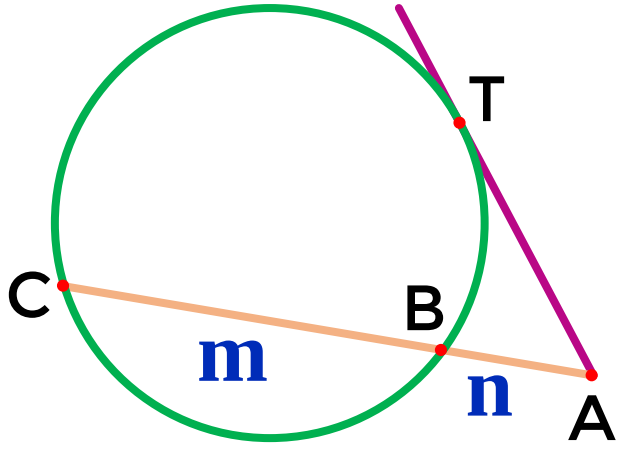
$\therefore x = 8$

2. Hallar el valor de x, si las circunferencias son concéntricas.





T: punto de tangencia



Teorema de la tangente

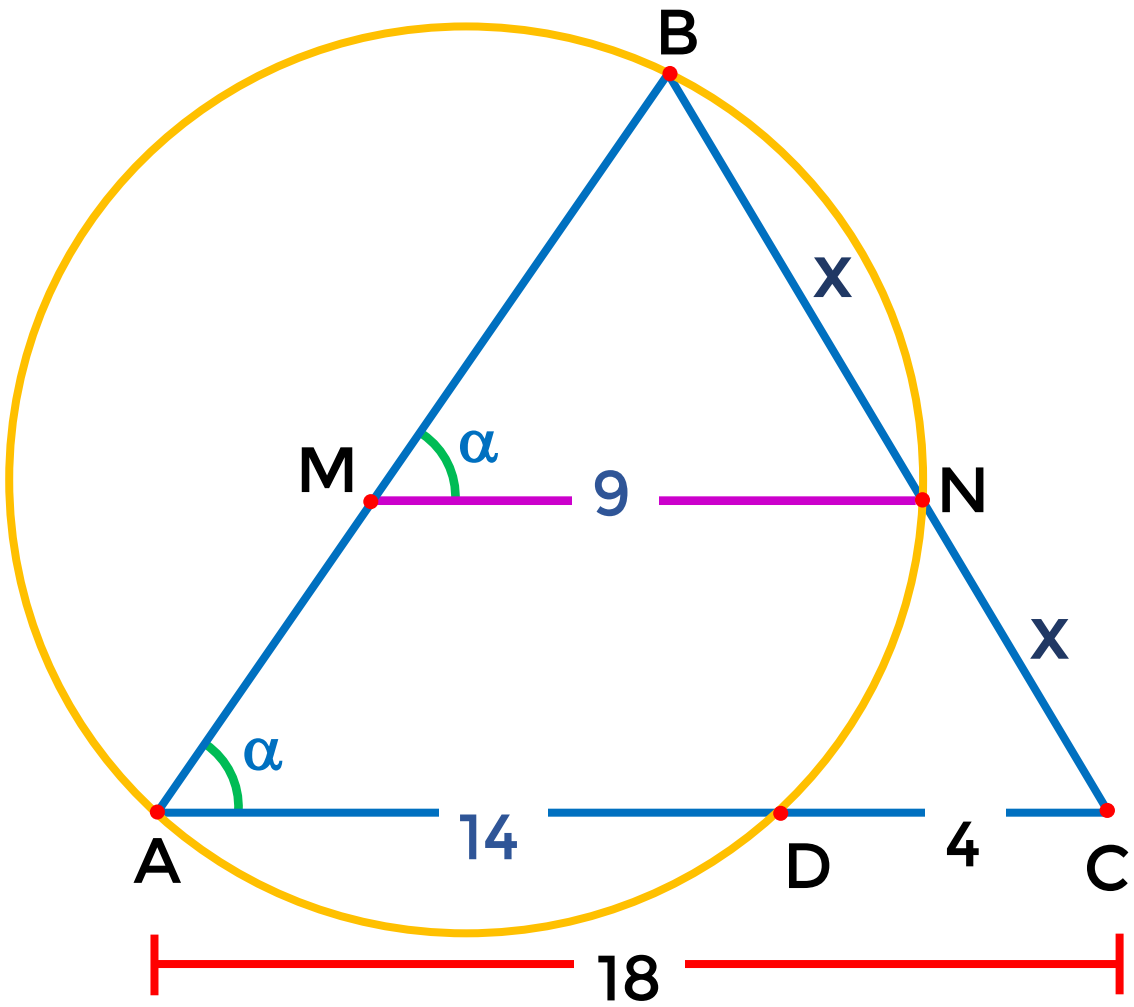
$$(AT)^2 = (AB)(AC)$$

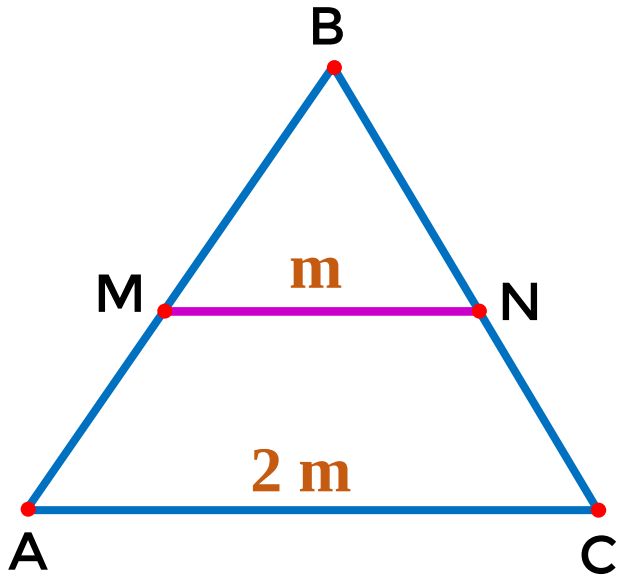
Resolución:

$$8^2 = 8x \cdot 2x \Rightarrow \cancel{8} = \cancel{2}x^2 \Rightarrow 4 = x^2$$

$$\therefore x = 2$$

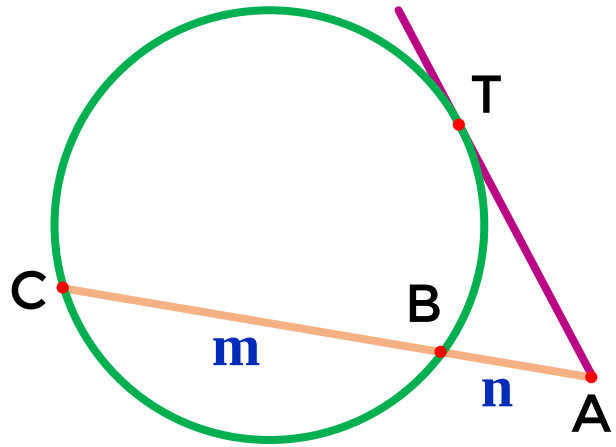
3. Hallar el valor de x.





$\overline{MN}$: Base media

$AC = 2(MN)$



Teorema de la tangente

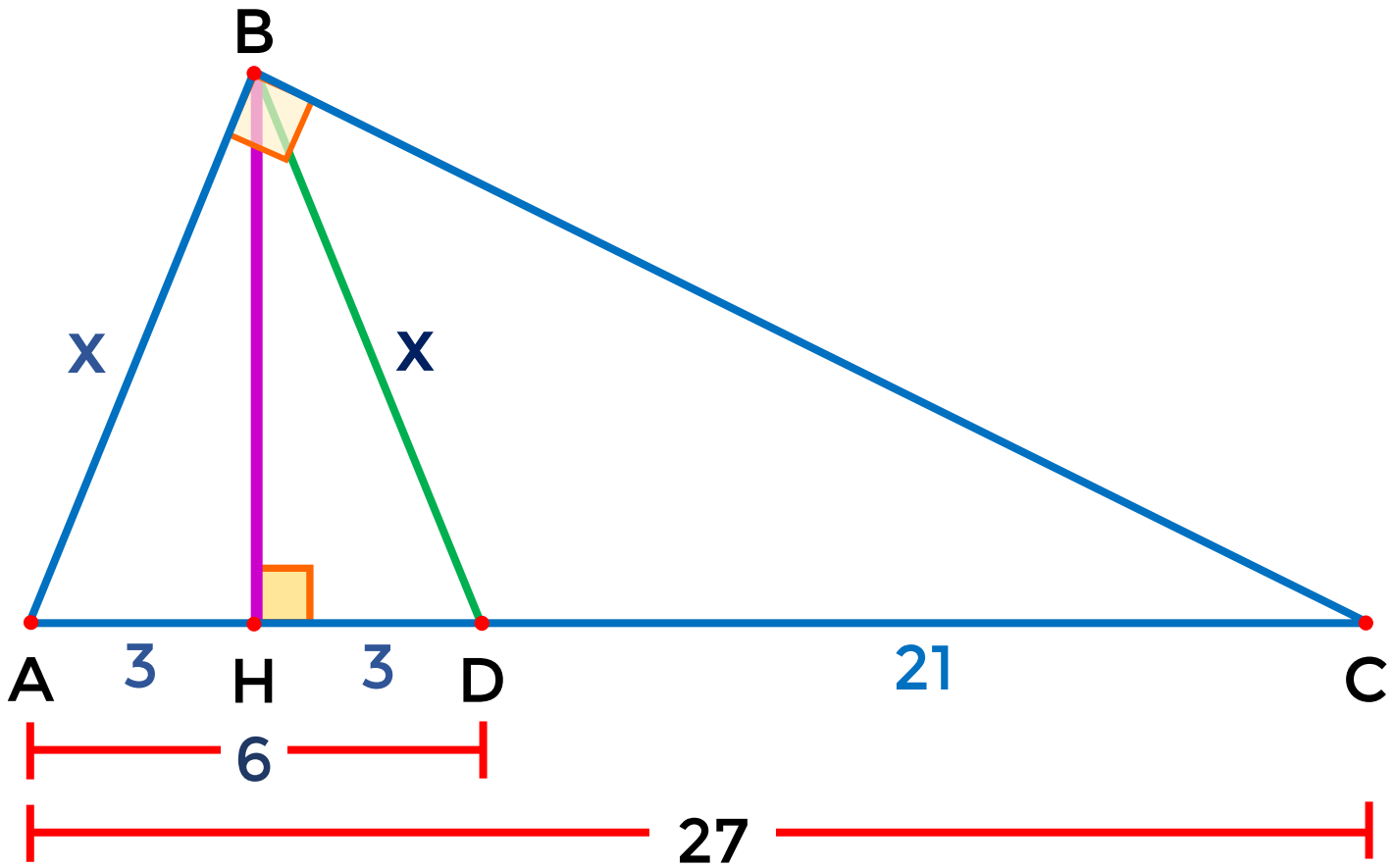
$(AT)^2 = (AB)(AC)$

Resolución:

$2x(x) = 18(4) \Rightarrow x^2 = 36$

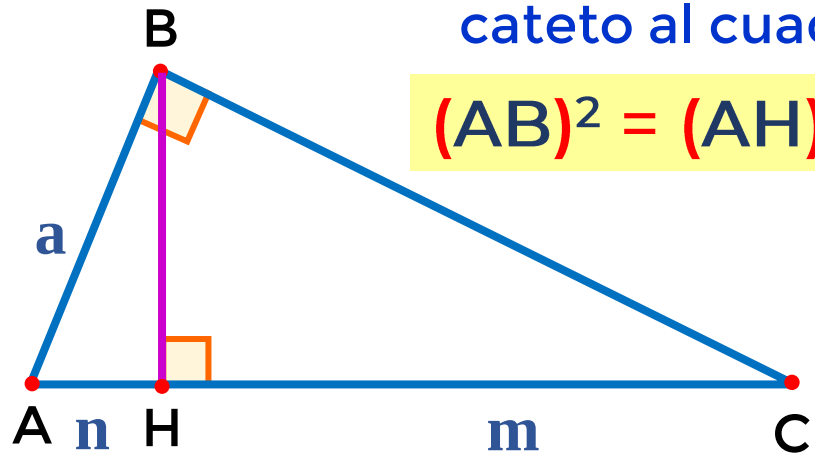
$\therefore x = 6$

4. En un triángulo rectángulo ABC, recto en B, se traza la ceviana interior $\overline{BD}$, tal que AD = 6, DC = 21 y AB = BD. Hallar AB.



Teorema de calculo de cateto al cuadrado

$(AB)^2 = (AH)(AC)$

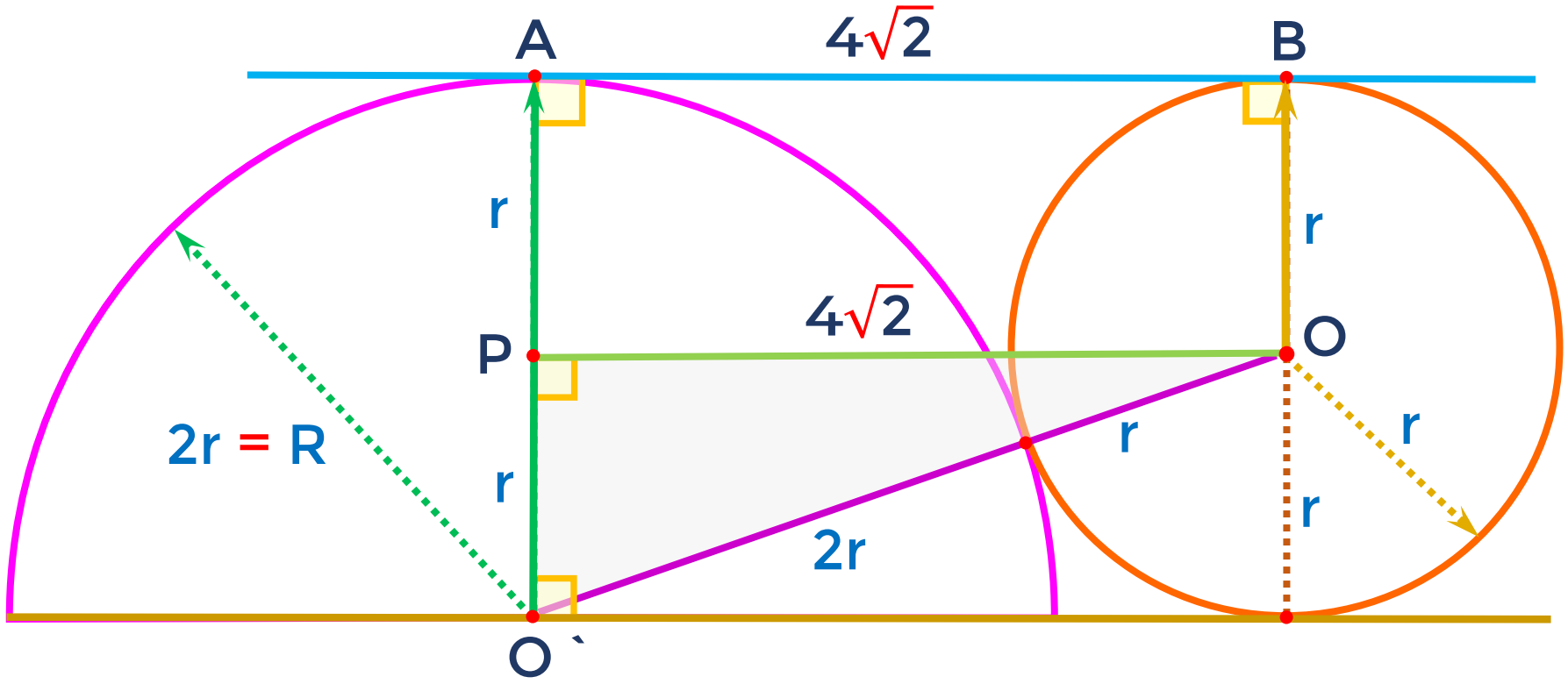


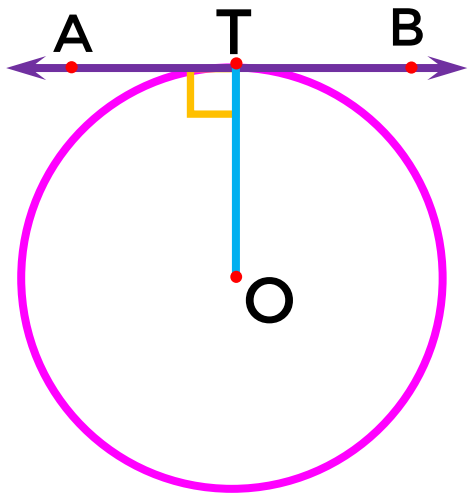
Resolución:

$x^2 = 3(27) \Rightarrow x^2 = 81$

$\therefore x = 9$

5. En la figura, si AB = $4\sqrt{2}$. Halle R.





T: punto de tangencia

En ⊿ O´PO:

$(3r)^2 = (r)^2 + (4\sqrt{2})^2 \Rightarrow 9r^2 = r^2 + 32$

$8r^2 = 32 \Rightarrow r^2 = 4 \Rightarrow r = 2$

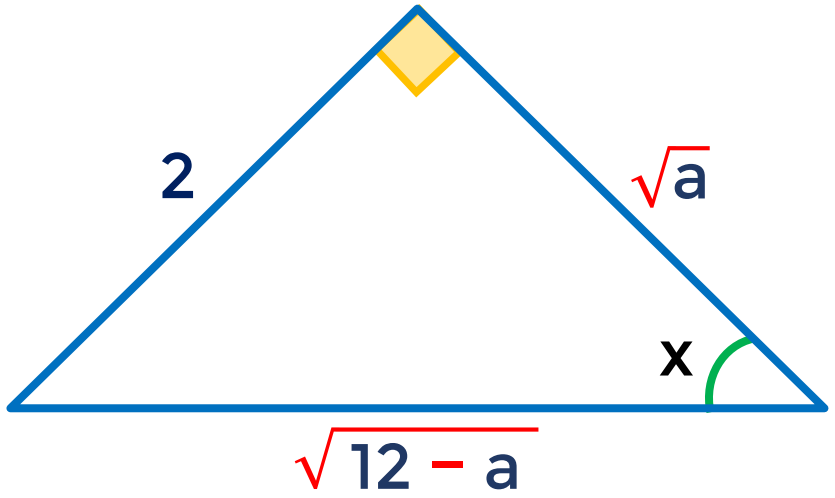
Nos piden:

$R = 2r \Rightarrow R = 2(2)$

∴ R = 4

6. Halle la medida de uno de los ángulos agudos de un triángulo rectángulo si la hipotenusa tiene una longitud igual a $\sqrt{12 - a}$ y los otros lados sus longitudes son 2 y $\sqrt{a}$.

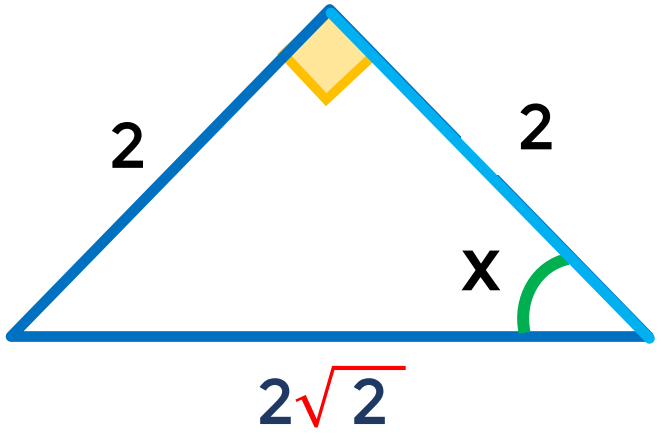
Resolución:



Por teorema de Pitágoras

$$(\sqrt{12 - a})^2 = (\sqrt{a})^2 + 2^2$$

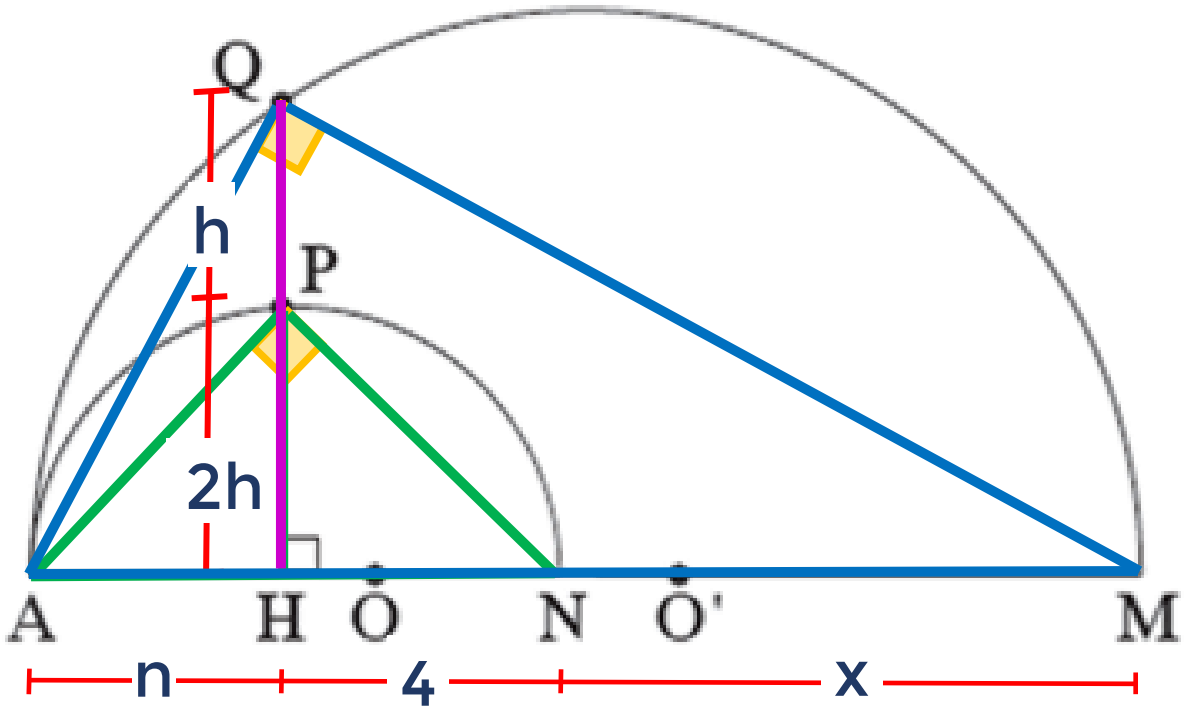
$$\Rightarrow 12 - a = a + 4 \quad \Rightarrow 8 = 2a \quad \Rightarrow 4 = a$$

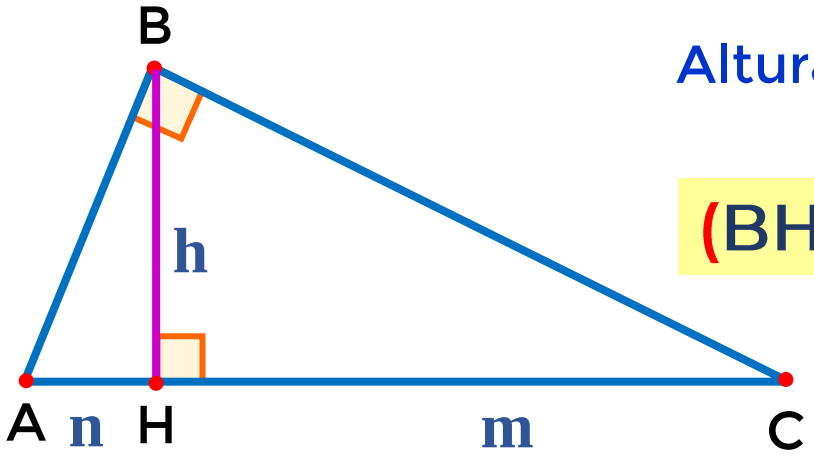


Por Notable
(45°- 45°)

$\therefore x = 4$

7. En la siguiente figura, PH = 2(PQ). Si HN = 4. Calcule MN. (O y O' centros de las semicircunferencias).





Altura relativa sobre la hipotenusa

$(BH)^2 = (AH)(HC)$

Resolución:

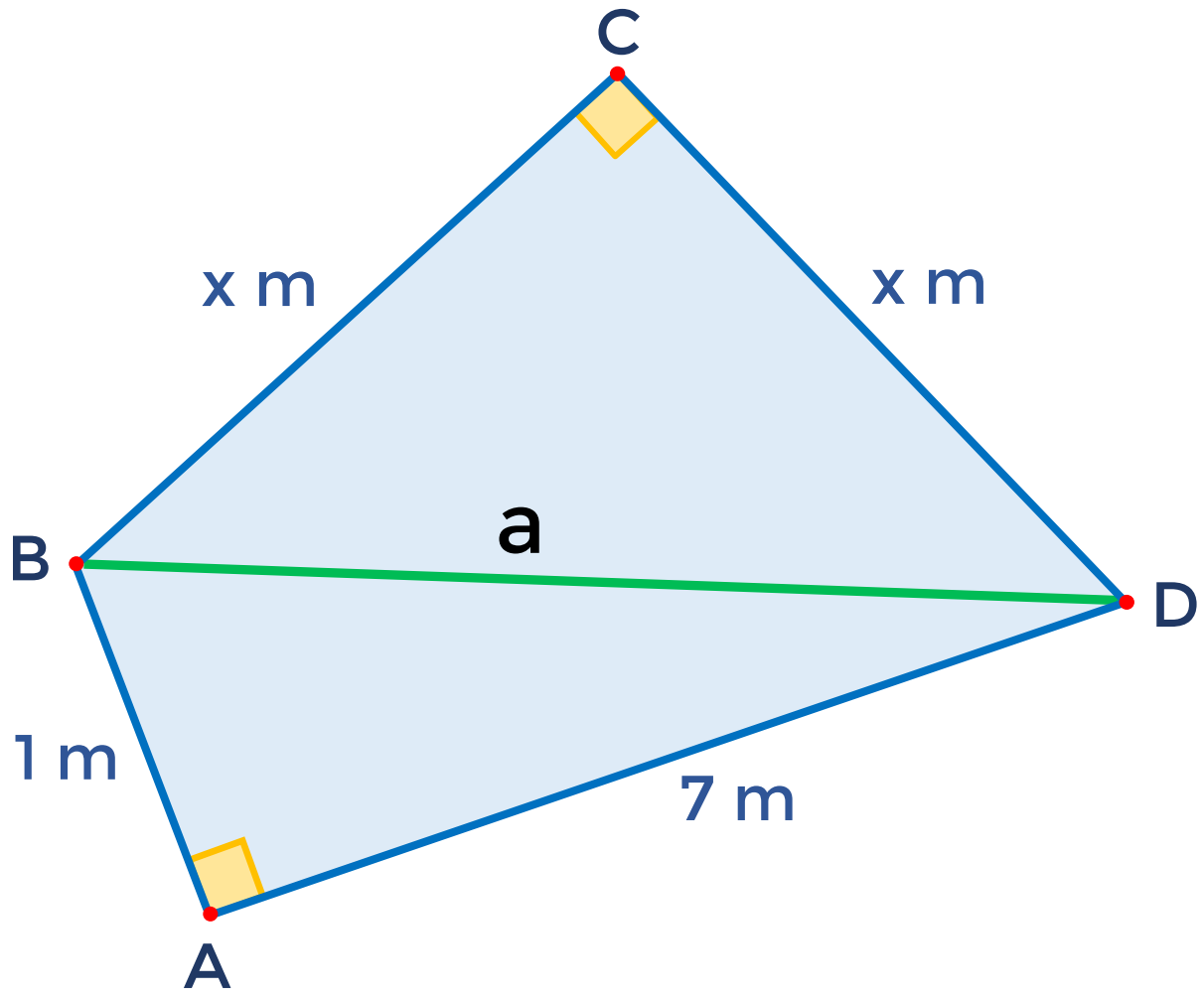
△ ANP: $(2h)^2 = n.4 \Rightarrow 4h^2 = n.4 \Rightarrow h^2 = n$

△ AMQ: $(3h)^2 = n(x + 4)$

$9h^2 = n(x + 4) \Rightarrow 9 = x + 4$

$\therefore x = 5$

8. En la figura se muestra un patio cuyo contorno tiene forma de cuadrilátero. Halle el valor de x.



Resolución:

* Trazamos la diagonal $\overline{BD}$

Por teorema de Pitágoras

△ ABD:

$$a^2 = 7^2 + 1^2$$

$$a^2 = 50$$

△ BCD:

$$\underbrace{a^2}_{50} = x^2 + x^2$$

$$50 = 2x^2 \quad \Rightarrow \quad 25 = x^2$$

$$\therefore x = 5$$