



# TRIGONOMETRY

TOMO VII

**5th**  
SECONDARY

**Feedback**

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 **SACO OLIVEROS**

## HELICO-PRACTICE 1



Reduzca:  $E = \frac{\text{sen}65^\circ + \text{sen}55^\circ}{\text{cos}65^\circ + \text{cos}55^\circ}$

**Resolución:**

**Recordar:**

$$\text{sen}A + \text{sen}B = 2\text{sen}\left(\frac{A+B}{2}\right) \cdot \text{cos}\left(\frac{A-B}{2}\right)$$

$$\text{cos}A + \text{cos}B = 2\text{cos}\left(\frac{A+B}{2}\right) \cdot \text{cos}\left(\frac{A-B}{2}\right)$$



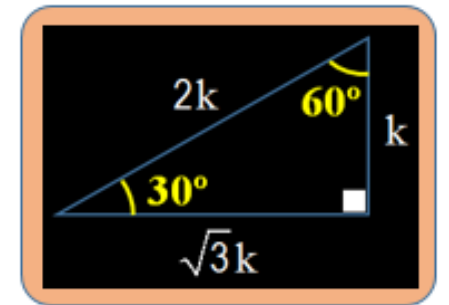
$$\cancel{2\text{sen}60^\circ \text{cos}5^\circ}$$

$$E = \frac{\text{sen}65^\circ + \text{sen}55^\circ}{\text{cos}65^\circ + \text{cos}55^\circ}$$

$$\cancel{2\text{cos}60^\circ \text{cos}5^\circ}$$

$$\Rightarrow E = \frac{\text{sen}60^\circ}{\text{cos}60^\circ}$$

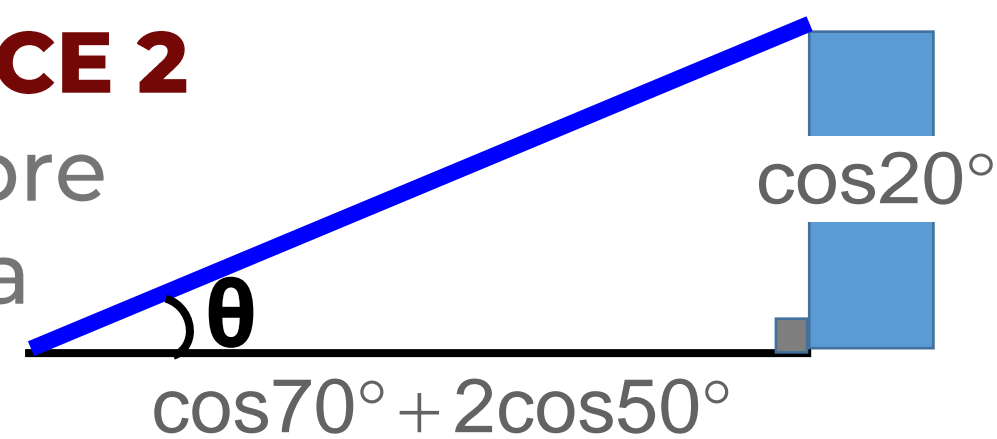
$$\Rightarrow E = \text{tan}60^\circ$$



$$\therefore E = \sqrt{3}$$

# HELICO-PRACTICE 2

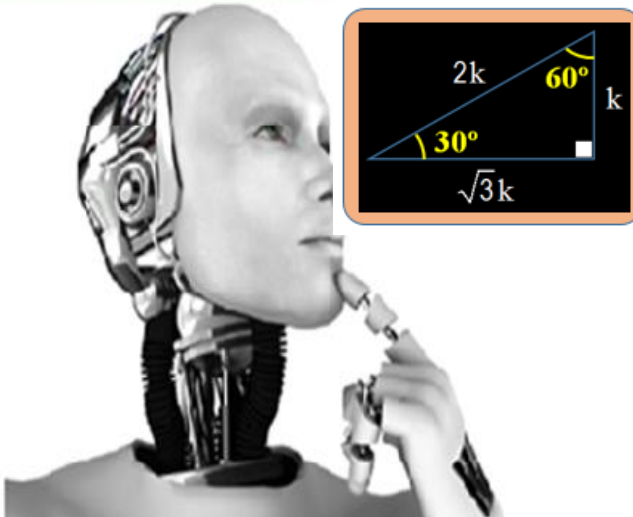
Una barra metálica descansa sobre una pared lisa, tal como se muestra en la figura. Calcule el valor de  $\theta$ .



**Resolución:**

**Recordar:**

$$\cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cdot \cos \left( \frac{A-B}{2} \right)$$



$$\cot \theta = \frac{\cos 70^\circ + 2 \cos 50^\circ}{\cos 20^\circ} = \frac{2 \cos 60^\circ \cos 10^\circ + \cos 70^\circ + \cos 50^\circ}{\cos 20^\circ}$$

$$\cot \theta = \frac{2 \left( \frac{1}{2} \right) \cos 10^\circ + \cos 50^\circ}{\cos 20^\circ} = \frac{2 \cos 30^\circ \cos 20^\circ + \cos 50^\circ + \cos 10^\circ}{\cos 20^\circ}$$

$$\cot \theta = 2 \left( \frac{\sqrt{3}}{2} \right) \Rightarrow \cot \theta = \sqrt{3}$$

$$\therefore \theta = 30^\circ$$

## HELICO-PRACTICE 3

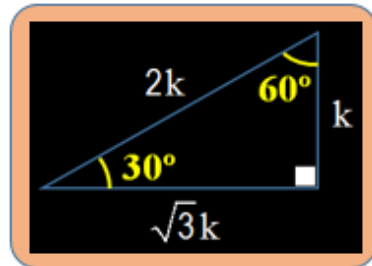


Halle el valor de  $m$ , si:  $\frac{\cos 20^\circ \cdot \cos 10^\circ}{\sin 30^\circ} = m + \sin 80^\circ$

Resolución:

Recordar

$$2\cos x \cos y = \cos(x + y) + \cos(x - y)$$



$$\frac{2\cos 20^\circ \cos 10^\circ}{2\sin 30^\circ} = m + \sin 80^\circ$$

$$\frac{\cos 30^\circ + \cos 10^\circ}{2\left(\frac{1}{2}\right)} = m + \sin 80^\circ$$

$$\frac{\sqrt{3}}{2} + \cancel{\sin 80^\circ} = m + \cancel{\sin 80^\circ}$$

$$\therefore m = \frac{\sqrt{3}}{2}$$



Determine el rango de la función:  $f(x) = 4\text{sen}x + 5$

**Resolución:**

Se sabe que:  $-1 \leq \text{sen}x \leq 1$

Ahora le damos la forma de la función  $f$ :

$$-1 \leq \text{sen}x \leq 1 \quad \dots\dots\dots (x4)$$

$$-4 \leq 4 \text{sen}x \leq 4 \quad \dots\dots\dots (+5)$$

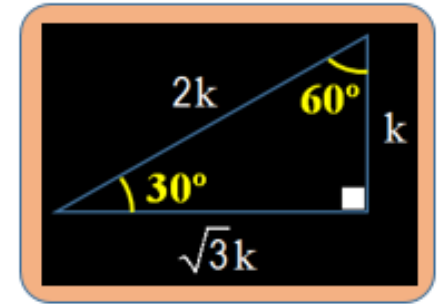
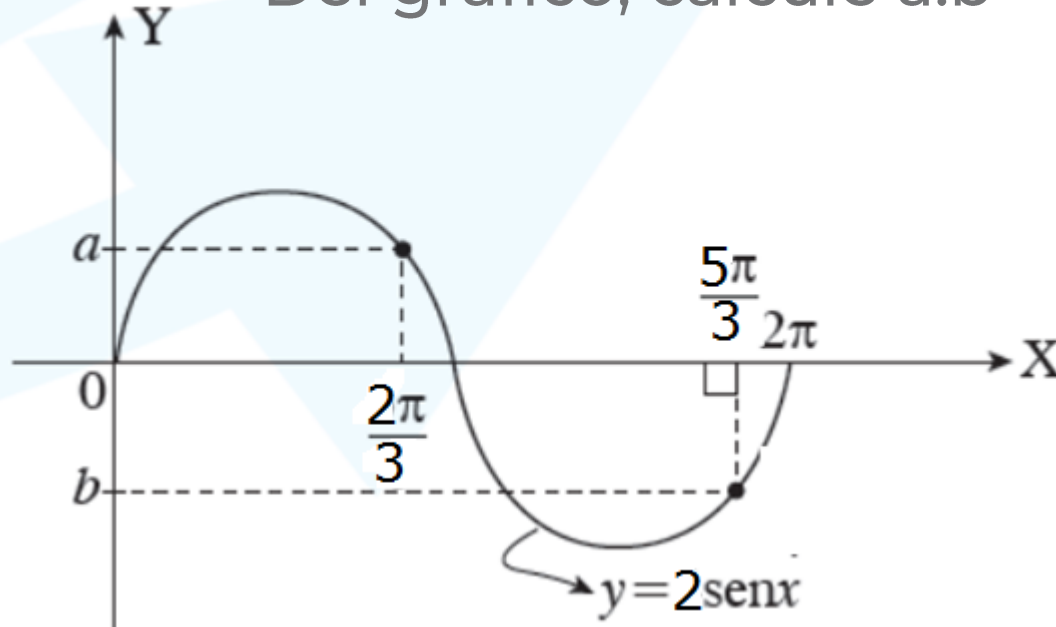
$$1 \leq \underbrace{4 \text{sen}x + 5}_{f(x)} \leq 9$$

$$\therefore \text{Ran}f = [1; 9]$$

# HELICO-PRACTICE 5



Del gráfico, calcule a.b



**Resolución:**

Sea:  
 $f(x) = y = 2\text{sen}x$

$$P\left(\frac{2\pi}{3}; a\right) \in f$$

$$a = 2\text{sen}\left(\frac{2\pi}{3}\right)$$

$$a = 2\left(\frac{\sqrt{3}}{2}\right) \Rightarrow a = \sqrt{3}$$

$$Q\left(\frac{5\pi}{3}; b\right) \in f$$

$$b = 2\text{sen}\left(\frac{5\pi}{3}\right)$$

$$b = 2\left(-\frac{\sqrt{3}}{2}\right) \Rightarrow b = -\sqrt{3}$$

Piden:

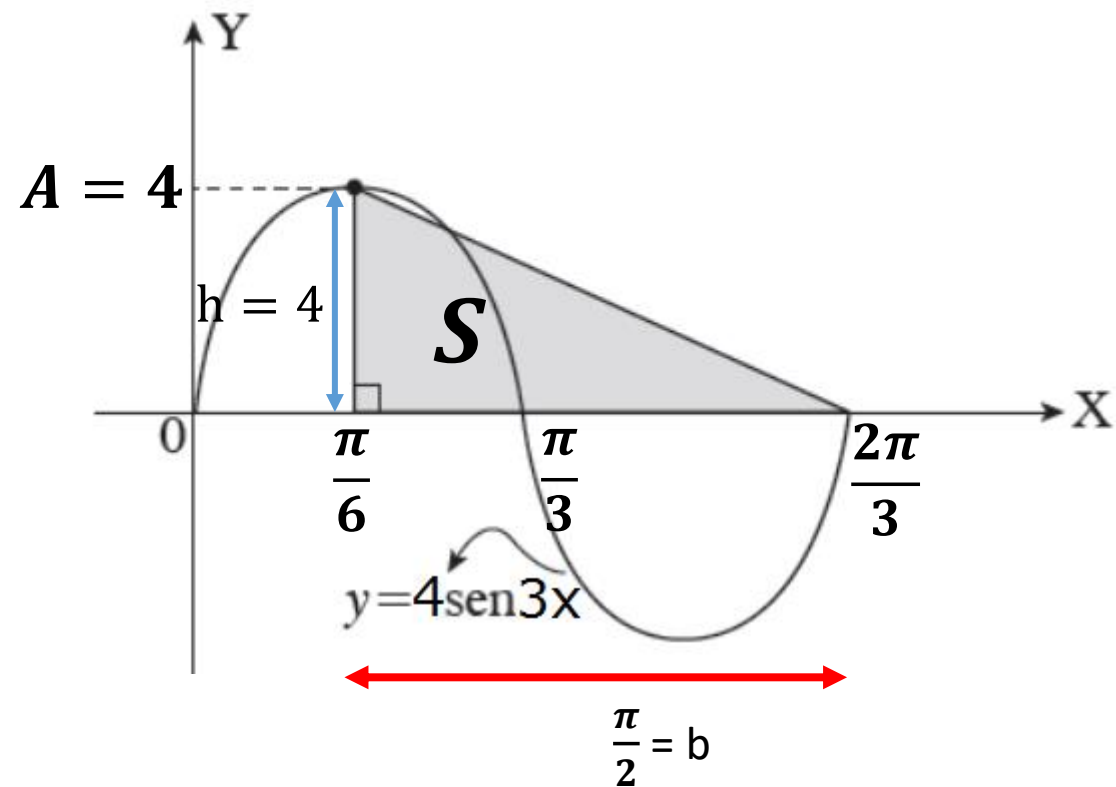
$$a.b = (\sqrt{3}) \cdot (-\sqrt{3})$$



$$a.b = -3$$



Del gráfico, determine el área de la región sombreada.



### Resolución:

Sea la función:  $f(x) = y = 4\text{sen}3x$

Periodo de la función:

$$\Rightarrow T = \frac{2\pi}{B} \Rightarrow T = \frac{2\pi}{3}$$

Amplitud:  $A = 4$

Calculando el área:

$$S = \frac{b \cdot h}{2} \Rightarrow S = \frac{\left(\frac{\pi}{2}\right) \cdot (4)}{2}$$



$$S = \pi u^2$$



Determine el rango de la función:  $f(x) = 3\cos x - 2$

### Resolución:

Se sabe que:  $-1 \leq \cos x \leq 1$

Ahora le damos la forma de la función  $f$ :

$$-1 \leq \cos x \leq 1 \dots \dots (x^3)$$

$$-3 \leq 3\cos x \leq 3 \dots \dots (-2)$$

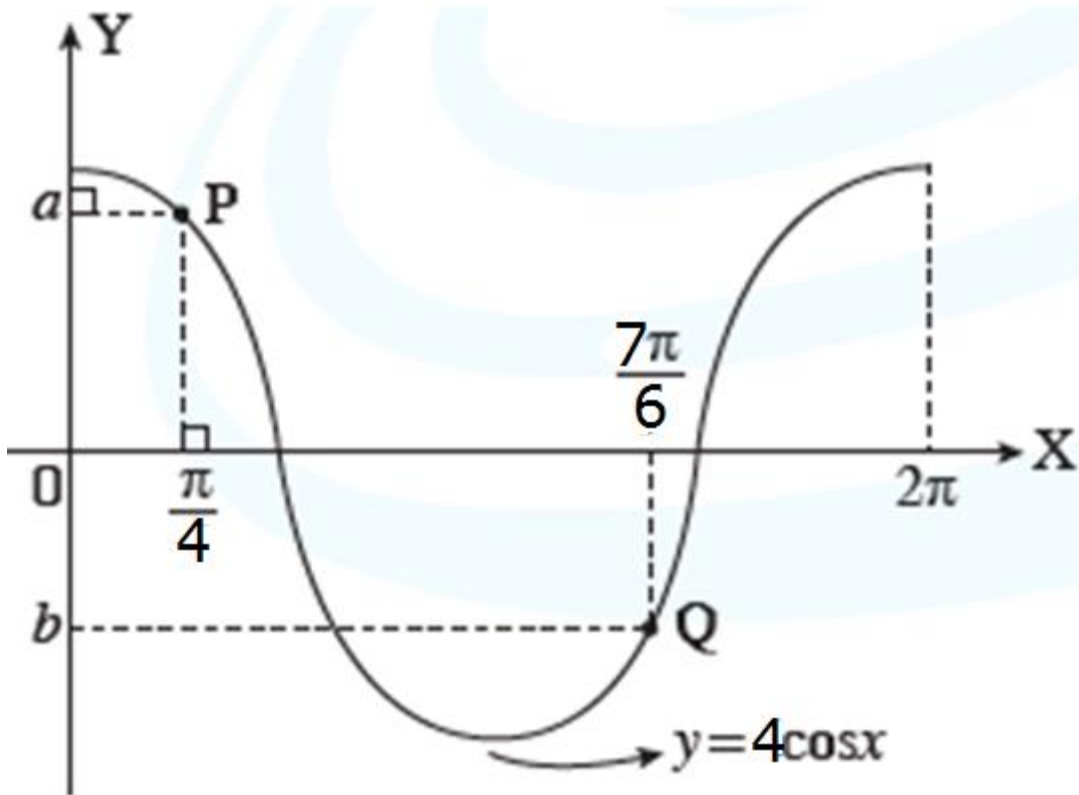
$$-5 \leq \underbrace{3\cos x - 2}_{f(x)} \leq 1$$

$$\therefore \text{Ran } f = [-5; 1]$$



# HELICO-PRACTICE 8

Del gráfico, calcule  $a \cdot b$



## Resolución:

Sea :  $f(x) = y = 4\cos x$

$$Q\left(\frac{7\pi}{6}; b\right) \in f$$

$$\Rightarrow b = 4\cos\left(\frac{7\pi}{6}\right)$$

$$\Rightarrow b = -4\cos\left(\frac{\pi}{6}\right)$$

$$\Rightarrow b = -4\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow b = -2\sqrt{3}$$

$$P\left(\frac{\pi}{4}; a\right) \in f$$

$$\Rightarrow a = 4\cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow a = 4\left(\frac{\sqrt{2}}{2}\right)$$

$$\Rightarrow a = 2\sqrt{2}$$

NOS PIDEN:

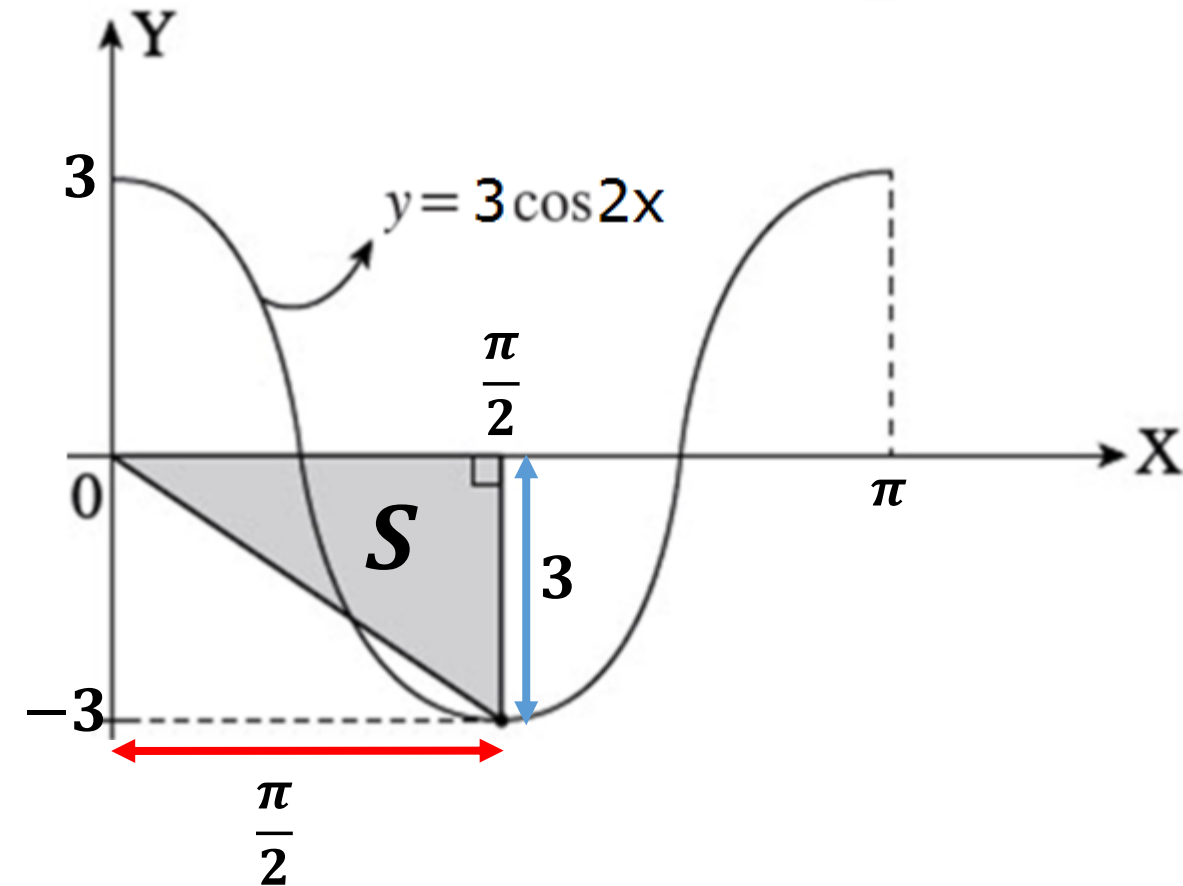
$$a \cdot b = (2\sqrt{2})(-2\sqrt{3})$$

$$\therefore a \cdot b = -4\sqrt{6}$$

# HELICO-PRACTICE 9



Del gráfico, determine el área de la región sombreada.



## Resolución:

Sea la función:  $f(x) = y = 3\cos 2x$

Periodo de la función:

$$\Rightarrow T = \frac{2\pi}{B} \rightarrow T = \frac{2\pi}{2} = \pi$$

Amplitud:  $A = 3$

Calculando el área:  $S = \frac{\left(\frac{\pi}{2}\right) \cdot (3)}{2}$

$$\therefore S = \frac{3\pi}{4} u^2$$

# HELICO-PRACTICE 10

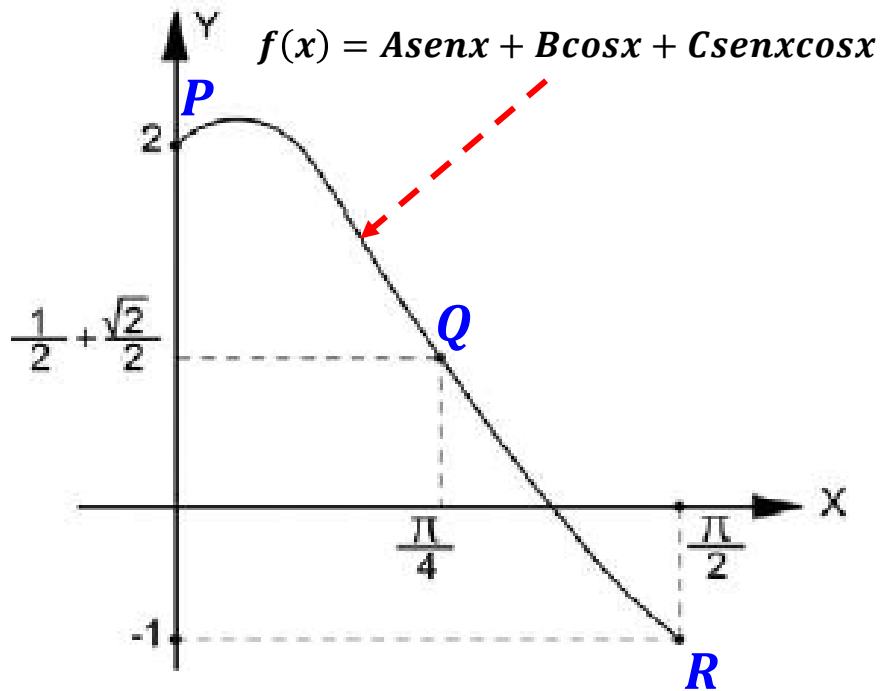
## HELICO | PRACTICE



Sean  $A, B, C$  constantes y  $f: \mathbb{R} \rightarrow \mathbb{R}$  dada por

$$f(x) = A \sen(x) + B \cos(x) + C \sen(x)\cos(x)$$

cuya gráfica parcial se muestra a continuación:



Calcule  $A + B + C$

## Resolución:

**El punto  $P \in f(x)$**

$$f(0) = A \sen(0) + B \cos(0) + C \sen(0)\cos(0)$$

$$2 = A(0) + B(1) + C(0)(1) \rightarrow \boxed{2 = B}$$

**El punto  $R \in f(x)$**

$$f\left(\frac{\pi}{2}\right) = A \sen\left(\frac{\pi}{2}\right) + B \cos\left(\frac{\pi}{2}\right) + C \sen\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right)$$

$$-1 = A(1) + 2(0) + C(1)(0) \rightarrow \boxed{-1 = A}$$

**El punto  $Q \in f(x)$**

$$f\left(\frac{\pi}{4}\right) = A \sen\left(\frac{\pi}{4}\right) + B \cos\left(\frac{\pi}{4}\right) + C \sen\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$$

$$\frac{1}{2} + \frac{\sqrt{2}}{2} = -1\left(\frac{\sqrt{2}}{2}\right) + 2\left(\frac{\sqrt{2}}{2}\right) + C\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} + C\left(\frac{1}{2}\right) \rightarrow \boxed{1 = C}$$



$$\boxed{A + B + C = 2}$$