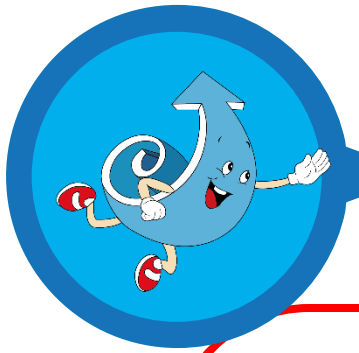
ALGEBRA

Chapter 13

3th
SECONDARY

FACTORIZACION II

SACO OLIVEROS



¡ RECORDEMOS !

- $P(x) = ax^2 - a - bx^2 + b$

  $\Rightarrow \quad P(x) = (a - b)(x - 1)(x + 1)$

- $P(a) = ma^4 - m$

  $\Rightarrow \quad P(a) = m(a - 1)(a + 1)(a^2 + 1)$

FACTORIZACIÓN

Es el proceso transformar un polinomio en una multiplicación indicada de dos o más factores primos o irreductibles.

Ejemplo:

$$P(x) = \underbrace{x^2 - 81}_{} = \underbrace{(x-9)(x+9)}_{}$$

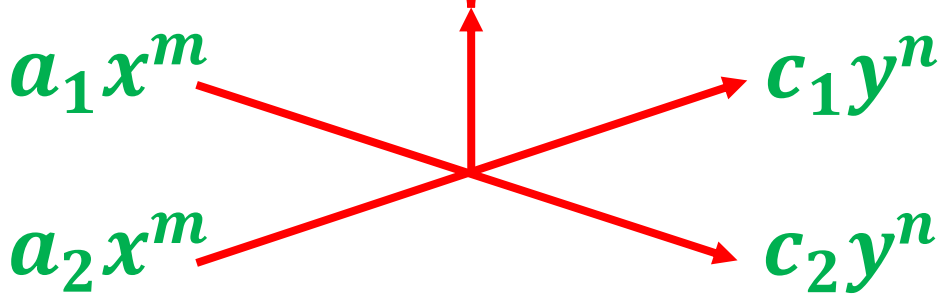
factorización

Factores primos: $x-9$ y $x+9$

I. CRITERIO DEL ASPA SIMPLE:

$$P(x,y) = Ax^{2m} + Bx^m y^n + Cy^{2m}$$

$$\{A, B, C\} \subset \mathbb{Z}$$



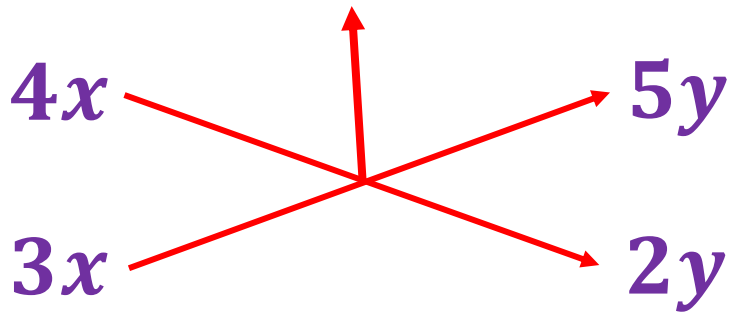
$$P(x,y) = (a_1x^m + c_1y^n)(a_2x^m + c_2y^n)$$

Ejemplo:

Factorice $P(x,y) = 10x^2 + 23xy + 10y^2$

Resolución:

$$P(x,y) = 12x^2 + 23xy + 10y^2$$

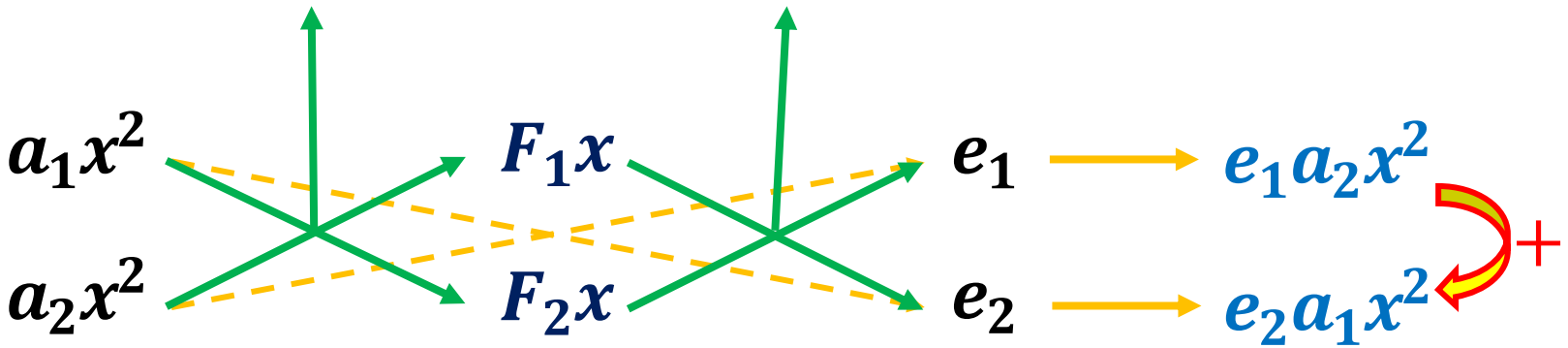


$$P(x,y) = (4x + 5y)(3x + 2y)$$

II. CRITERIO DEL ASPA DOBLE ESPECIAL:

TC

$$P(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$$



$$S.T = Ex^2$$

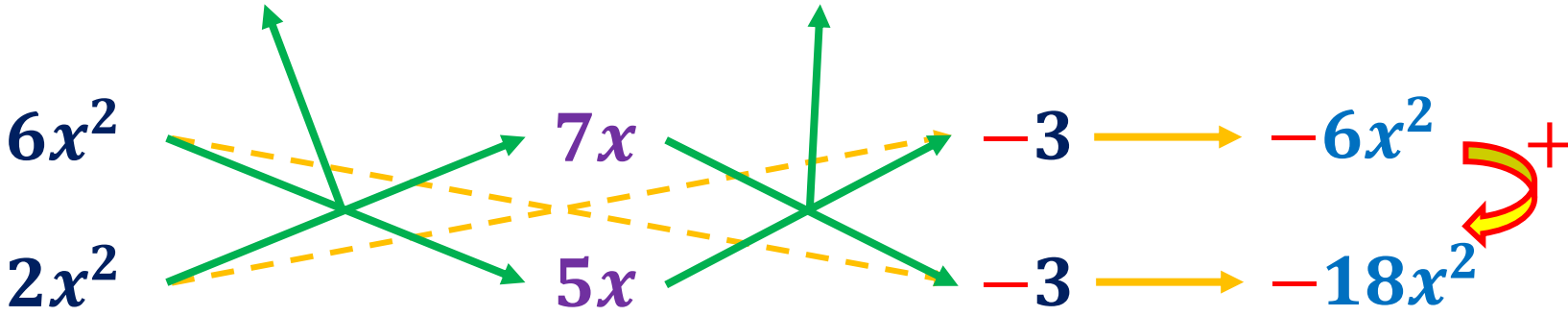
$$FALTA = T.C - S.T = Fx^2$$

$$P(x) = (a_1x^2 + F_1x + e_1)(a_2x^2 + F_2x + e_2)$$

Ejemplo: Factorice $P(x) = 12x^4 + 44x^3 + 11x^2 - 36x + 9$

Resolución:

$$P(x) = 12x^4 + 44x^3 + \underbrace{11x^2}_{TC} - 36x + 9$$



$$FALTA = 11x^2 - (-24x^2) = \boxed{35x^2}$$

$$P(x) = (6x^2 + 7x - 3)(2x^2 + 5x - 3)$$

$$\boxed{P(x) = (3x - 1)(2x + 3)(2x - 1)(x + 3)}$$

III. CRITERIO DE LOS DIVISORES BINÓMICOS:

$$P(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots\ldots + a_{n-1}x + a_n \quad , a_0 \neq 0$$

Si $x = \alpha$ **es una raíz de** $P(x) \Rightarrow P(\alpha) = 0$

Luego, $(x - \alpha)$ **es un factor de** $P(x)$

Calculamos los otros factores de $P(x)$ *usando la regla de RUFFINI*

$$\frac{P(x)}{x - \alpha}$$

Posibles ceros o raíces:

$$PC = \pm\left\{\frac{div(a_n)}{div(a_0)}\right\}$$

Ejemplo: *Factorice* $P(x) = x^3 - x^2 - 2x - 12$

Resolución:

Posibles ceros o raíces:

$$PC = \pm\left\{\frac{div(12)}{div(1)}\right\}$$

$$PC = \pm\left\{\frac{\{1; 2; 3; 4; 6; 12\}}{1}\right\}$$

$$PC = \pm\{1; 2; 3; 4; 6; 12\}$$

	1	-1	-2	-12
$x = 3$		3	6	12
	1	2	4	0

$$P(x) = (x - 3)(x^2 + 2x + 4)$$

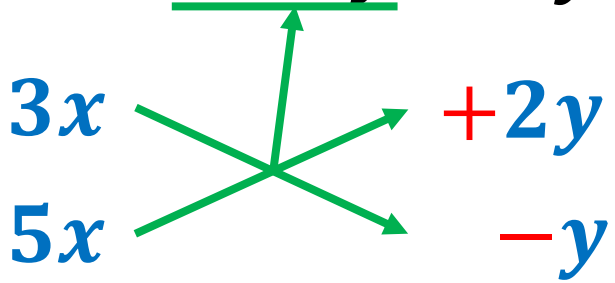
Problema 1

Indique un factor primo, luego de factorizar

$$P(x,y) = 15x^2 + 7xy - 2y^2$$

Resolución:

$$P(x,y) = 15x^2 + 7xy - 2y^2$$



$$P(x,y) = (3x + 2y)(5x - y)$$

Factores primos:

$$(3x + 2y) \quad y \quad (5x - y)$$

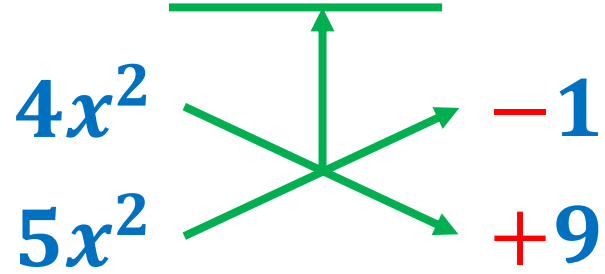
Problema 2

Determine el número de factores primos en

$$M(x) = 20x^4 + 31x^2 - 9$$

Resolución:

$$M(x) = 20x^4 + 31x^2 - 9$$



$$M(x) = (4x^2 - 1)(5x^2 + 9)$$

$$M(x) = (2x + 1)(2x - 1)(5x^2 + 9)$$

$\therefore$ *M(x) tiene 3 factores primos*

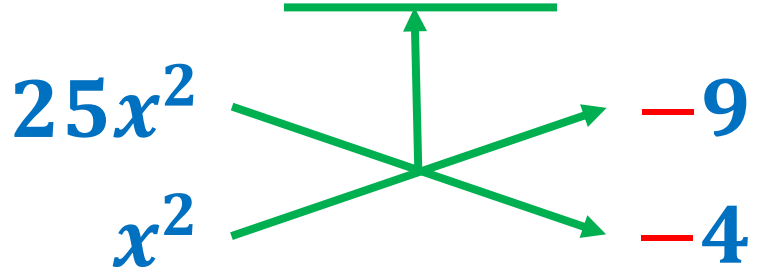
Problema 3

Calcule la suma de los términos independientes de los factores primos de

$$P(x) = 25x^4 - 109x^2 + 36$$

Resolución:

$$P(x) = 25x^4 - 109x^2 + 36$$



$$P(x) = (25x^2 - 9)(x^2 - 4)$$

$$P(x) = (5x + 3)(5x - 3)(x + 2)(x - 2)$$

$$\sum TI = 3 - 3 + 2 - 2$$

$$\therefore \quad \sum TI = 0$$

Problema 4

Un factor primo de

$P(x) = x^3 + 3x^2 + 5x + 3$

luego de factorizarlo es

Resolución:

$$P(x) = x^3 + 3x^2 + 5x + 3$$

$$PC = \pm\left\{\frac{div(3)}{div(1)}\right\}$$

$$PC = \pm\left\{\frac{\{1;3\}}{1}\right\}$$

$$PC = \pm\{1;3\}$$

	1	3	5	3
$x=-1$		-1	-2	-3
×	1	2	3	0

$$P(x) = (x+1)(x^2 + 2x + 3)$$

Factores primos: $(x+1)$ y $(x^2 + 2x + 3)$

Problema 5

Luego de factorizar

$$P(x) = x^3 - x^2 - x - 2$$

el quíntuplo del valor de la mayor suma de coeficientes de sus factores primos representa la temperatura en $°C$ del medio ambiente de la ciudad de Lima. ¿Cuál es la temperatura?

Resolución:

$$P(x) = x^3 - x^2 - x - 2$$

$$PC = \pm\left\{\frac{div(2)}{div(1)}\right\}$$

$$PC = \pm\left\{\frac{\{1;2\}}{1}\right\}$$

$$PC = \pm\{1;2\}$$

	1	−1	−1	−2
$x=2$		2	2	2
×	1	1	1	0

$$P(x) = (x-2)(x^2+x+1)$$

Factores Primos:

- $(x-2)$ ⇨ $\sum Coef = 1 - 2 = \boxed{-1}$
- (x^2+x+1) ⇨ $\sum Coef = 1 + 1 + 1 = \boxed{3}$ *Mayor*

$\therefore$ *La temperatura de Lima es* $15°C$

Problema 6

Un factor primo de

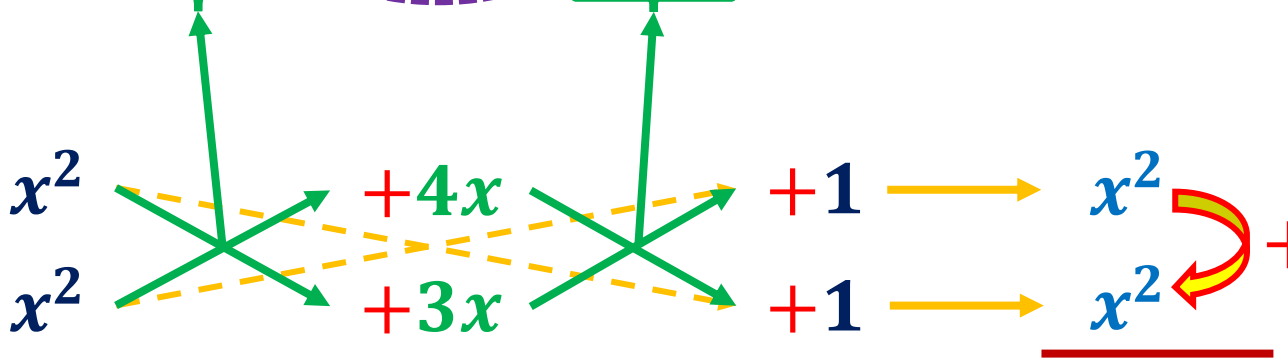
$P(x) = x^4 + 7x^3 + 14x^2 + 7x + 1$

es

Resolución:

$$P(x) = x^4 + 7x^3 + 14x^2 + 7x + 1$$

TC



$$FALTA = 14x^2 - 2x^2 = 12x^2$$

$$S.T = +2x^2$$

$$P(x) = (x^2 + 4x + 1)(x^2 + 3x + 1)$$

Factores primos: $(x^2 + 4x + 1)$ *y* $(x^2 + 3x + 1)$

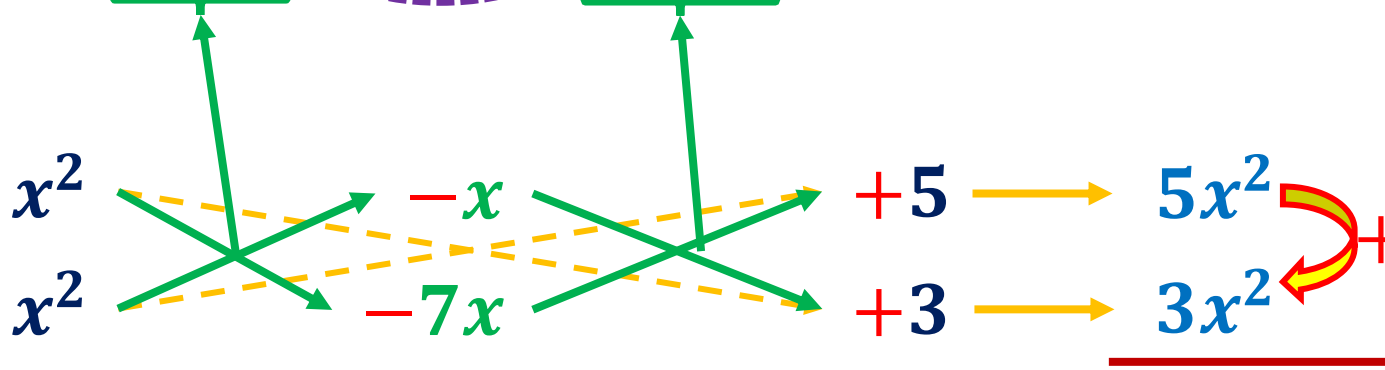
Problema 7

Factorice e indique el término independiente de un factor primo.

$$P(x) = x^4 - 8x^3 + 15x^2 - 38x + 15$$

Resolución:

$$P(x) = x^4 - 8x^3 + \underbrace{15x^2}_{TC} - 38x + 15$$



$$S.T = +8x^2$$

$$FALTA = 15x^2 - 8x^2 = 7x^2$$

$$P(x) = (x^2 - x + 5)(x^2 - 7x + 3)$$

Términos independientes de los Factores primos:

$$TI_1 = 5 \quad y \quad TI_2 = 3$$

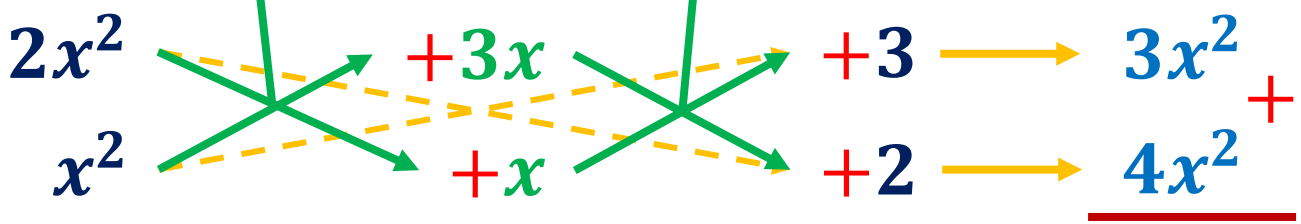
Problema 8

Calcule el número de factores primos luego de factorizar

$$P(x) = 2x^4 + 5x^3 + 10x^2 + 9x + 6$$

Resolución:

$$P(x) = 2x^4 + 5x^3 + \underbrace{10x^2}_{TC} + 9x + 6$$



$$S.T = +7x^2$$

$$FALTA = 10x^2 - 7x^2 = 3x^2$$

$$P(x) = (2x^2 + 3x + 3)(x^2 + x + 2)$$

$\therefore$ *$P(x)$ tiene 2 factores primos.*