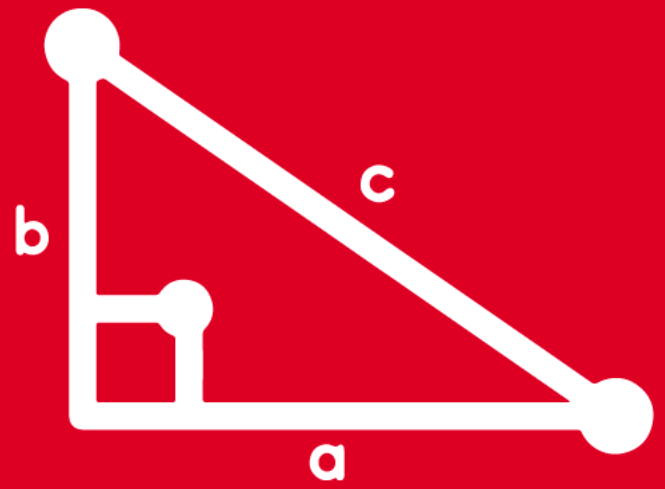
# TRIGONOMETRY

## Chapter 22




**3rd**
**SECONDARY**

## IDENTIDADES TRIGONOMÉTRICAS DE ÁNGULOS COMPUESTOS

SACO OLIVEROS

# HELICO-MOTIVACIÓN

## ¿ EL TODO ES IGUAL A LA SUMA DE LAS PARTES ?

# IDENTIDADES TRIGONOMÉTRICAS DE ÁNGULOS COMPUESTOS

## I) PARA LA SUMA DE DOS ÁNGULOS:

$$\operatorname{sen}(\alpha + \beta) = \operatorname{sen}\alpha \cos\beta + \cos\alpha \operatorname{sen}\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

# II ) PARA LA RESTA DE DOS ÁNGULOS:

$$\operatorname{sen}(\alpha - \beta) = \operatorname{sen}\alpha \cos\beta - \cos\alpha \operatorname{sen}\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}$$

# HELICO-PRACTICA

1 ) Calcule sen15°

**Recordar**

$$\text{sen}(\alpha - \beta) = \text{sen}\alpha\cos\beta - \cos\alpha\,\text{sen}\beta$$

**Resolución:**

$$\text{sen}(45° - 30°) = \text{sen}45°\cos30° - \cos45°\,\text{sen}30°$$

$$\text{sen}15° = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$\therefore \text{sen}15° = \frac{\sqrt{6}-\sqrt{2}}{4}$$

# HELICO-PRACTICE

2 ) Calcule $\cos 8°$

**Recordar**

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \operatorname{sen}\alpha \operatorname{sen}\beta$$

**Resolución:**

$$\cos(45° - 37°) = \cos 45° \cos 37° + \operatorname{sen} 37° \operatorname{sen} 45°$$

$$\cos 8° = \frac{\sqrt{2}}{2} \cdot \frac{4}{5} - \frac{3}{5} \cdot \frac{\sqrt{2}}{2}$$

$$\therefore \cos 8° = \frac{\sqrt{2}}{10}$$

# HELICO-PRACTICE

3 ) Efectúe $E = 2\cos(60° - x) - \sqrt{3}\,\text{sen}x$

**Resolución:**

$$E = 2(\cos 60°.\cos x + \text{sen}60°.\text{sen}x) - \sqrt{3}\,\text{sen}x$$

$$E = 2\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\text{sen}x\right) - \sqrt{3}\,\text{sen}x$$

$$E = \cos x + \cancel{\sqrt{3}\,\text{sen}x} - \cancel{\sqrt{3}\,\text{sen}x}$$

$\therefore E = \cos x$

# HELICO-PRACTICE

4 ) Determine el valor de:

$$P = \frac{\text{sen}80° \,.\, \cos 10° - \cos 80° \,.\, \text{sen}10°}{\text{sen}55° \,.\, \cos 15° + \cos 55° \,.\, \text{sen}15°}$$

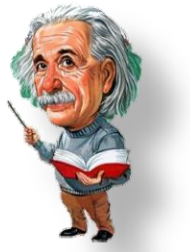
**Recordar**

$$\text{sen}\alpha\cos\beta \pm \cos\alpha\,\text{sen}\beta = \text{sen}(\alpha \pm \beta)$$

**Resolución:**

$$\Rightarrow P = \frac{\text{sen}(80° - 10°)}{\text{sen}(55° + 15°)} = \frac{\text{sen}70°}{\text{sen}70°}$$

$$\therefore P = 1$$



# HELICO-PRACTICE

5 ) Ana ha realizado una encuesta en su aula sobre qué residuos reciclan en su casa, obteniendo los siguientes resultados:

| Residuos | Cantidad de alumnos |
|---|---|
| Papel y cartón | 24 A |
| Vidrio | 20 B |
| Envases y plásticos | $5\sqrt{3}$ C |

# HELICO-PRACTICE

Donde :

$A = \text{sen}18°.\cos12° + \cos18°.\text{sen}12° = \text{sen}(30°) = \frac{1}{2}$

$B = \cos23°.\cos14° - \text{sen}23°.\text{sen}14° = \cos(37°) = \frac{4}{5}$

$$C = \frac{\tan32° + \tan28°}{1 - \tan32°.\tan28°} = \tan(60°) = \sqrt{3}$$

Halle la cantidad de alumnos que reciclan cada residuo.

**Rptas:** **Papel y carton = 24** $\left(\frac{1}{2}\right)$ **= 12**

**Vidrio = 20** $\left(\frac{4}{5}\right)$ **= 16**

**Envases y plásticos =** $5\sqrt{3}$ $\left(\sqrt{3}\right)$ **= 15**

# HELICO-PRACTICE

6 ) Si $\tan x = \frac{1}{2}$ y $\tan y = 4$ ; calcule $\tan(x + y)$

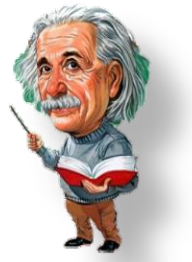
**Recordar**

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$$

**Resolución:**

$$\tan(x + y) = \frac{\frac{1}{2} + 4}{1 - (\frac{1}{2})(4)} = \frac{\frac{9}{2}}{1 - 2} = \frac{\frac{9}{2}}{-1}$$

$$\therefore \tan(x + y) = -\frac{9}{2}$$



# HELICO-PRACTICE

**7 )** Si $\tan(37° + \alpha) = \frac{5}{3}$, calcule $\tan\alpha$

**Resolución:**

$$\tan(37° + \alpha) = \frac{5}{3}$$

$$\frac{\tan 37° + \tan\alpha}{1 - \tan 37° \tan\alpha} = \frac{5}{3}$$

$$\frac{\frac{3}{4} + \tan\alpha}{1 - \frac{3}{4}\tan\alpha} = \frac{5}{3}$$

$$\frac{\frac{3 + 4\tan\alpha}{4}}{\frac{4 - 3\tan\alpha}{4}} = \frac{5}{3}$$

$$\frac{3 + 4\tan\alpha}{4 - 3\tan\alpha} = \frac{5}{3}$$

$$9 + 12\tan\alpha = 20 - 15\tan\alpha$$

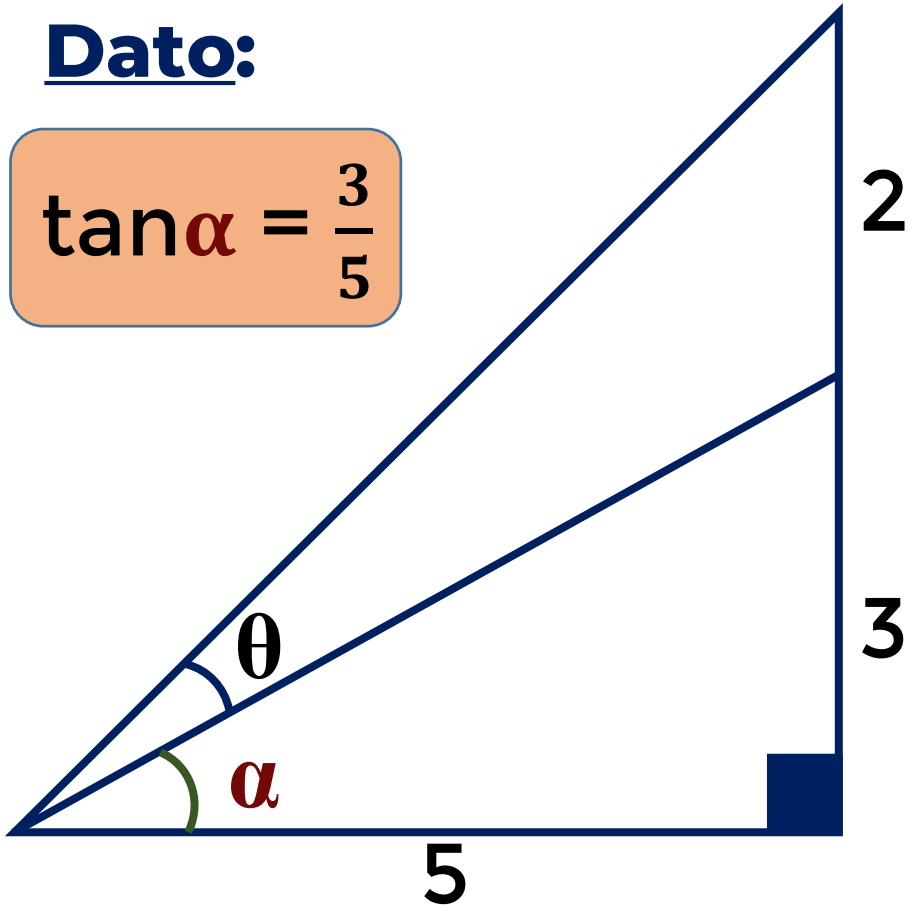
$$\therefore \tan\alpha = \frac{11}{27}$$

# HELICO-PRACTICE

8 ) Del gráfico, calcule tanθ

**Dato:**

$\tan\alpha = \frac{3}{5}$




**Resolución:**

$$\underbrace{\tan(\theta+\alpha)}_{\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}} = \frac{5}{5}$$

$$\frac{\tan\theta+\frac{3}{5}}{1-\tan\theta(\frac{3}{5})} = 1$$

$$\Rightarrow \tan\theta+\frac{3}{5} = 1-\tan\theta(\frac{3}{5})$$

$$\Rightarrow \left(\frac{8}{5}\right)\tan\theta = \frac{2}{5}$$

$$\therefore \tan\theta = \frac{1}{4}$$

COLEGIOS
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# MUCHAS GRACIAS POR TU ATENCIÓN

Tu curso amigo
TRIGONOMETRÍA