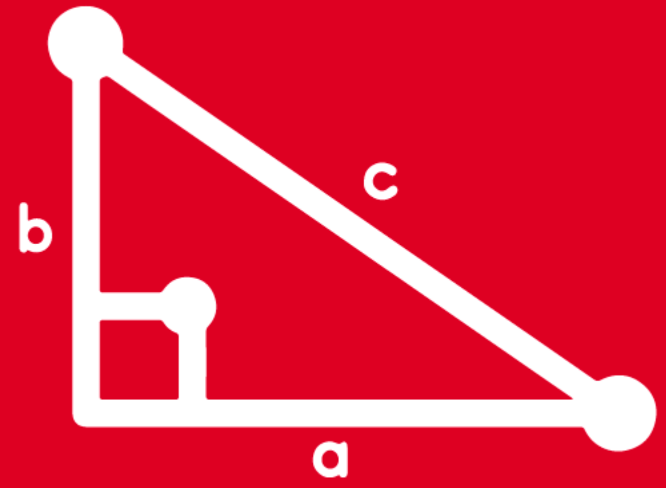
TRIGONOMETRY

5th
SECONDARY



ADVISORY

SACO OLIVEROS

1. Si senθ = – 9/41 , además $\theta \in \langle 270°;360° \rangle$, halle el valor de M = cscθ + cotθ

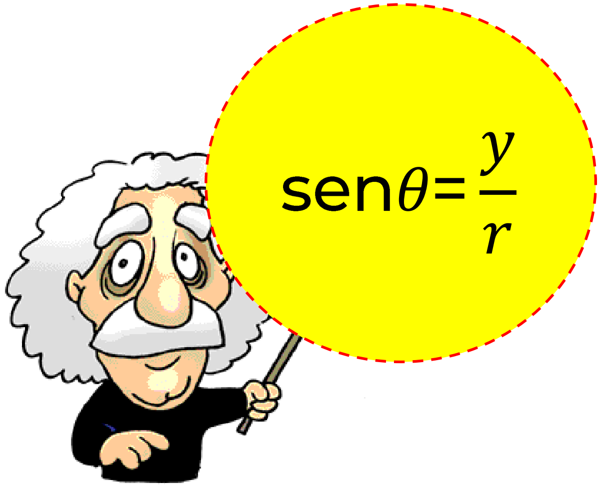
RESOLUCIÓN

- Como $\theta \in \langle 270°;360° \rangle$

$\theta \in$ IVC ➡ x(+), y(–) , r(+)

- Además:

$$\text{sen}\theta = \frac{-9}{41} = \frac{y}{r}$$



Luego: y = – 9 y r = 41

Sabemos: $r = \sqrt{x^2 + y^2}$

$$41 = \sqrt{x^2 + (-9)^2} \Rightarrow x = 40$$

Piden: M = cscθ + cotθ

$$M = \frac{41}{-9} + \frac{40}{-9}$$

$\therefore$ M = – 9

2. Si $\beta \in$ IIIC, además $\tan(270 - \beta) = 0{,}75$
Reduzca: $E = \csc(270° - \beta) + \tan(180° + \beta)$

RESOLUCIÓN

$$E = \underbrace{\csc(\overbrace{270° - \beta}^{\text{IIIC}})}_{-\sec\beta} + \underbrace{\tan(\overbrace{180° + \beta}^{\text{IIIC}})}_{\tan\beta}$$

$$T = -\sec\beta + \tan\beta \quad \ldots(*)$$

Del dato:

$$\tan(\overbrace{270° - \beta}^{\text{IIIC}}) = 0{,}75$$

$$\cot\beta = \frac{3}{4}$$

$$\cot\beta = \frac{-3}{-4} = \frac{x}{y}$$

Por radio vector:

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-3)^2 + (-4)^2} \qquad r = 5$$

Reemplazando en $(*)$:

$$T = -\frac{r}{x} + \frac{y}{x}$$

$$T = -\frac{5}{-3} + \frac{-4}{-3} = \frac{-5-4}{-3}$$

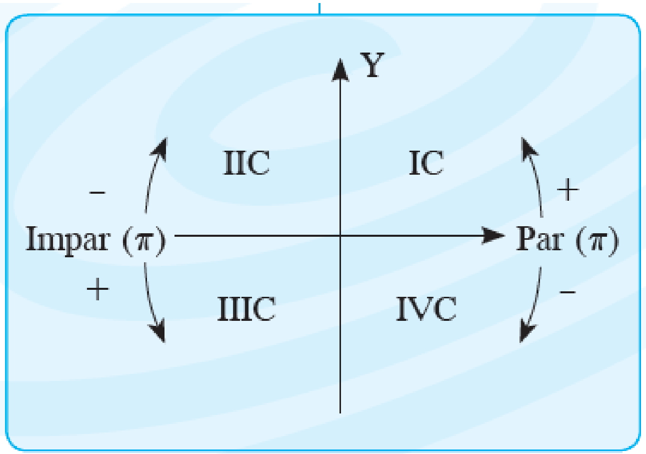
$$T = \frac{\cancel{-}9}{\cancel{-}3}$$

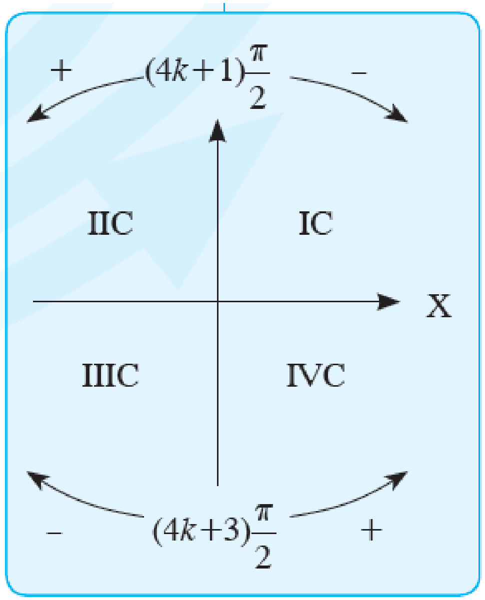
$$\boxed{\therefore T = 3}$$

3. Se cumple que tan(11π + x) = 3

Efectúe: $K = \cos\left(\frac{15\pi}{2} - x\right).\sec(26\pi - x)$

si x es un ángulo agudo.





RESOLUCIÓN

$$K = \cos\left(\overset{4k+3}{15}\frac{\pi}{2} - x\right).\sec(\overset{PAR}{26}\,\pi - x)$$

$$K = \underbrace{\cos\left(\overbrace{3\frac{\pi}{2} - x}^{IIIC}\right)}_{-senx}.\underbrace{\sec\overbrace{(2\pi - x)}^{IVC}}_{\sec x}$$

$K = -senx.secx \;...(*)$

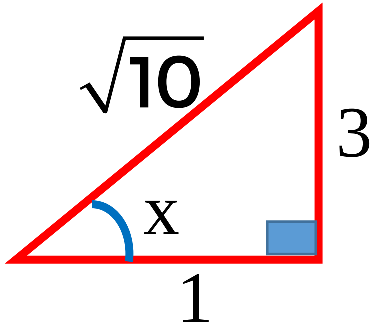
Del dato:

$$\tan(\overset{IMPAR}{11}\;\pi + x) = 3$$

$$\underbrace{\tan(\overbrace{\pi + x}^{IIIC})}_{\tan x} = 3$$

$\tan x = 3$

$\tan x = \frac{3}{1}$



Reemplazando en (*):

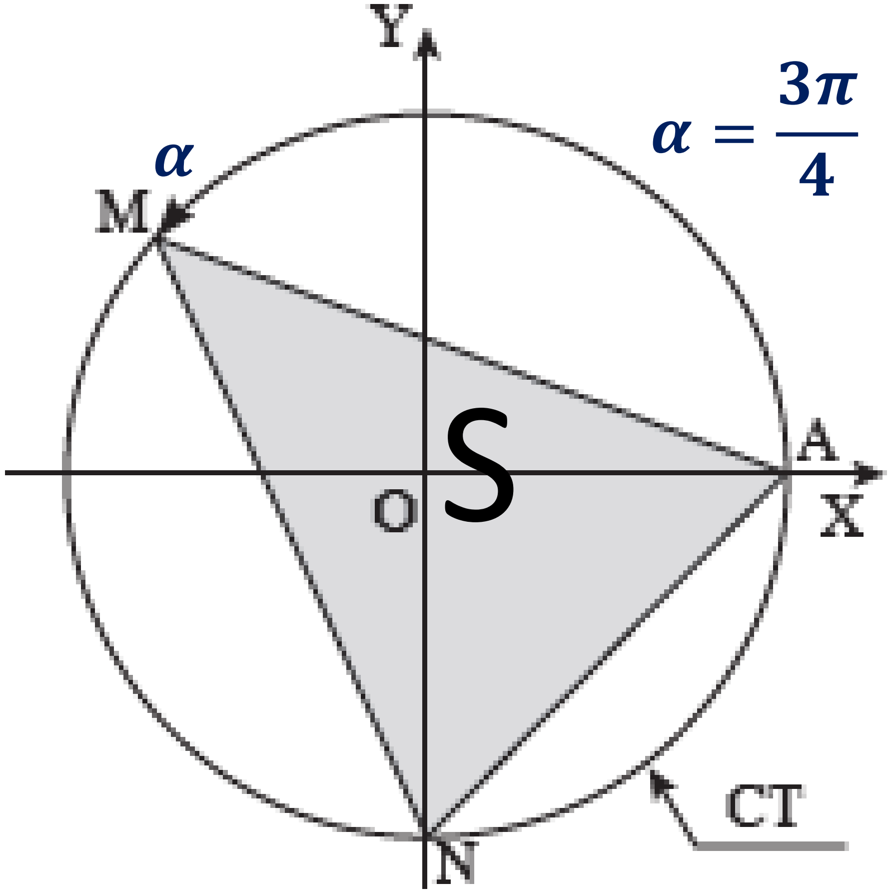
$K = -senx.secx$

$$K = -\frac{3}{\cancel{\sqrt{10}}}\cdot\frac{\cancel{\sqrt{10}}}{1}$$

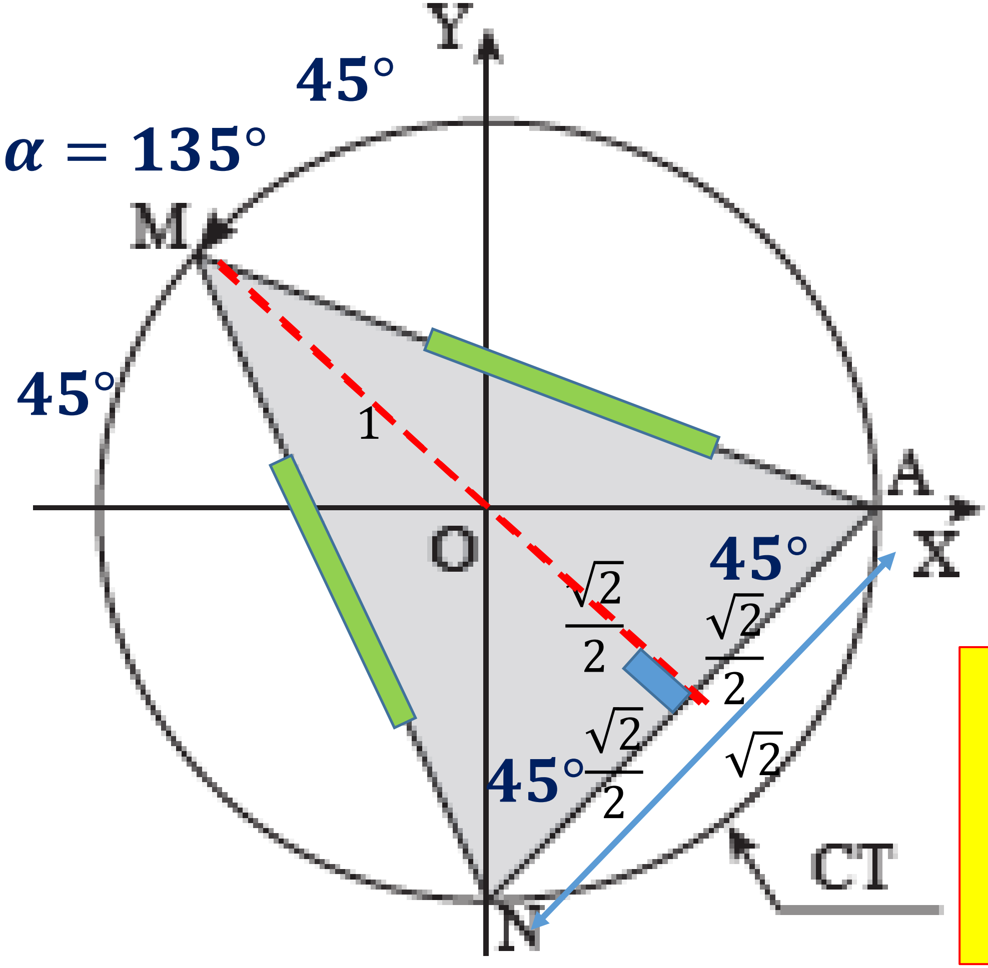
$$\boxed{\therefore K = -3}$$

4. En la circunferencia trigonométrica mostrada. Halle el área de la región triangular AMN .

RESOLUCIÓN



Calculamos el área S:



Entonces:

$$S = \frac{\sqrt{2}\left(1 + \frac{\sqrt{2}}{2}\right)}{2}$$

5. Si se cumple que: $\mathrm{senx}-\mathrm{cosx}=\mathrm{a}$ y $\mathrm{senx}.\mathrm{cosx}=\mathrm{b}$
determine una relación entre m y n
independiente de x.

RESOLUCIÓN

$$\mathrm{senx}-\mathrm{cosx}=\mathrm{a}$$

$$(\mathrm{senx}-\mathrm{cosx})^2=\mathrm{a}^2$$

$$\mathrm{sen}^2\mathrm{x}-2\mathrm{senxcosx}+\mathrm{cos}^2\mathrm{x}=\mathrm{a}^2$$

$$\mathrm{sen}^2\mathrm{x}+\mathrm{cos}^2\mathrm{x}-2\mathrm{senxcosx}=\mathrm{a}^2$$

$$1-\underbrace{2\mathrm{senxcosx}}_{\mathrm{b}}=\mathrm{a}^2$$

$$\therefore\ \boxed{1-2\mathrm{b}=\mathrm{a}^2}$$

6. Si se cumple que $\operatorname{senx} + \cos x = \frac{2}{3}$, calcule

$$E = (1 - \operatorname{senx})(1 - \cos x)$$

RESOLUCIÓN

$$E = (1 - \operatorname{senx})(1 - \cos x)$$

$$2E = \underbrace{2(1 - \operatorname{senx})(1 - \cos x)}$$

$$2E = (1 - senx - \cos x)^2$$

$$2E = (1 - \underbrace{(\operatorname{senx} + \cos x)}_{\frac{2}{3}})^2$$

$$2E = \left(1 - \frac{2}{3}\right)^2$$

$$2E = \left(\frac{1}{3}\right)^2$$

$$\boxed{\therefore E = \frac{1}{18}}$$

7. Calcule el valor de:

$$E = \frac{\csc 2730°}{\text{sen}4005°}$$

RESOLUCIÓN

$$\begin{array}{r|l} 2730° & 360° \\ \cline{2-2} 2520° & 7 \\ 210° & \end{array}$$

$$\begin{array}{r|l} 4005° & 360° \\ \cline{2-2} 3960° & 11 \\ \hline 45° & \end{array}$$

$$E = \frac{\csc 210°}{\text{sen}45°}$$

$$E = \frac{\csc(\overbrace{180° + 30°}^{\text{IIIC}})}{\text{sen}45°}$$

$$E = \frac{-\csc 30°}{sen45°}$$

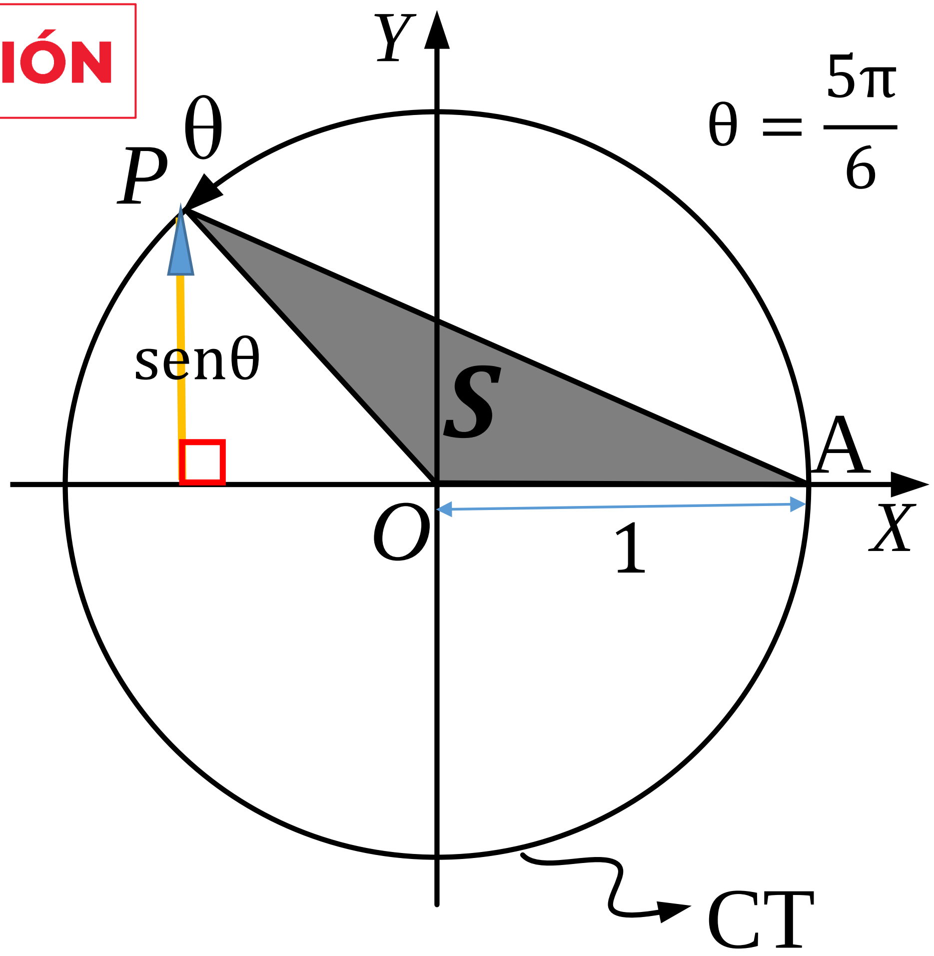
$$E = \frac{-2}{\frac{1}{\sqrt{2}}}$$

$$E = \frac{-2}{\frac{1}{\frac{1}{\sqrt{2}}}}$$

$$\boxed{\therefore E = -2\sqrt{2}}$$

8. De la circunferencia trigonométrica mostrada, determine el área de la región triangular sombreada AOP.

RESOLUCIÓN



Calculamos el área S:

$$S = \frac{1 * \operatorname{sen}\theta}{2} \qquad \theta = \frac{5(180°)}{6} = 150°$$

$$S = \frac{1 * \operatorname{sen}150°}{2}$$

$$S = \frac{1 * \operatorname{sen}\overbrace{(180° - 30°)}^{\text{IIC}}}{2}$$

$$S = \frac{1 * \operatorname{sen}30°}{2}$$

$$S = \frac{1\left(\frac{1}{2}\right)}{2}$$

$$\boxed{\therefore S = \frac{1}{4}}$$

9. Si: $\tan^2\alpha = 2\tan^2 x + 1$, halle el valor de

$y = \cos^2\alpha + \operatorname{sen}^2 x$, en términos de α

RESOLUCIÓN

$$\tan^2\alpha = 2\tan^2 x + 1$$

$$\underbrace{1 + \tan^2\alpha} = 2\tan^2 x + 1 + 1$$

$$\sec^2\alpha = 2\tan^2 x + 2$$

$$\sec^2\alpha = 2\underbrace{(\tan^2 x + 1)}_{\sec^2 x}$$

$$\sec^2\alpha = 2\sec^2 x$$

$$\frac{1}{\cos^2\alpha} = \frac{2}{\cos^2 x}$$

$$\cos^2 x = 2\cos^2\alpha$$

$$1 - \operatorname{sen}^2 x = \cos^2\alpha + \cos^2\alpha$$

$$\underbrace{1 - \cos^2\alpha}_{\operatorname{sen}^2\alpha} = \cos^2\alpha + \operatorname{sen}^2 x$$

$$\boxed{\therefore \cos^2\alpha + \operatorname{sen}^2 x = \operatorname{sen}^2\alpha}$$

10. Si se cumple que $\mathrm{secxcscx} = 3$, calcule: $F = \tan^3 x + \cot^3 x$

RESOLUCIÓN

$$\mathrm{secxcscx} = 3$$

$$\mathrm{tanx} + \mathrm{cotx} = 3$$

$$(\mathrm{tanx} + \mathrm{cotx})^2 = 3^2$$

$$\underbrace{\tan^2 x + \cot^2 x}_{7} + \underbrace{2\mathrm{tanxcotx}}_{1} = 9$$

$$\boxed{a^3 + b^3 = (a+b)(a^2 + b^2 - ab)}$$

$$F = \tan^3 x + \cot^3 x$$

$$F = \underbrace{(\mathrm{tanx} + \mathrm{cotx})}_{3}\underbrace{(\tan^2 x + \cot^2 x}_{7} - \underbrace{\mathrm{tanxcotx})}_{1}$$

$$\boxed{\therefore F = 18}$$