ALGEBRA

3rd
SECONDARY

Academic advising

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Problema 1

Calcule la suma de los factores primos de

$$P(x) = 36x^4 - 25x^2 + 4$$

Resolución:

$$P(x) = 36x^4 \underline{-25x^2} + 4$$

$$\begin{array}{ll} 9x^2 & -4 \\ 4x^2 & -1 \end{array}$$

$$P(x) = \underline{(9x^2 - 4)}\,\underline{(4x^2 - 1)}$$

$$\boxed{P(x) = (3x+2)(3x-2)(2x+1)(2x-1)}$$

$$\sum FP = 3x \cancel{+2} + 3x \cancel{-2} + 2x \cancel{+1} + 2x \cancel{-1}$$

$$\boxed{\therefore \quad \sum FP = 10x}$$

Problema 2

Indique un factor primo luego de factorizar

$$Q(x) = 2x^3 + 5x^2 - 2x + 3$$

Resolución:

$$Q(x) = 2x^3 + 5x^2 - 2x + 3$$

$\boxed{a_0 = 2}$ $\boxed{a_n = 3}$

$div(a_0) = \{1; 2\}$

$div(a_n) = \{1; 3\}$

$$\boxed{PC = \pm\left\{1; 3; \frac{1}{2}; \frac{3}{2}\right\}}$$

	2	5	−2	3
$x = -3$		−6	3	−3
	2	−1	1	0

$$\boxed{Q(x) = (x+3)(2x^2 - x + 1)}$$

Factores primos: $\boxed{(x+3)\ y\ (2x^2 - x + 1)}$

Problema 3

Factorice e indique el número de factores primos

$$R(x) = x^4 - 2x^2 + 5x - 6$$

Resolución:

$$R(x) = x^4 - 2x^2 + 5x - 6$$

Completando el polinomio:

$$R(x) = x^4 + \underline{0x^3} \overset{TC}{- 2x^2} \underline{+ 5x} - 6$$

$$\boxed{-x^2}$$

$$\begin{array}{cccccc} x^2 & +x & -3 & \rightarrow & -3x^2 & \\ x^2 & -x & +2 & \rightarrow & +2x^2 & + \\ \hline & & & & -x^2 & \end{array}$$

$TC = -2x^2$

$ST = -x^2$

$$\begin{array}{rl} -2x^2 & \\ -x^2 & - \\ \hline \boxed{-x^2} & \end{array}$$

$$\boxed{R(x) = (x^2 + x - 3)(x^2 - x + 2)}$$

$\therefore$ $R(x)$ *tiene 2 factores primos.*

Problema 4

Simplifique:

$$M = \sqrt[n]{\sqrt{3}-2}\,.\,\sqrt[2n]{7-4\sqrt{3}}$$

Recordemos:

TRINOMIO CUADRADO PERFECTO (Binomio al cuadrado):

$$\boxed{(a \pm b)^2 = a^2 + b^2 \pm 2ab}$$

DIFERENCIA DE CUADRADOS:

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

Resolución:

$$M = \sqrt[2n]{\left(\sqrt{3}-2\right)^2}\,.\,\sqrt[2n]{7+4\sqrt{3}}$$

$$M = \sqrt[2n]{7-4\sqrt{3}}\,.\,\sqrt[2n]{7+4\sqrt{3}}$$

$$M = \sqrt[2n]{\left(7-4\sqrt{3}\right)\left(7+4\sqrt{3}\right)}$$

$$M = \sqrt[2n]{7^2-\left(4\sqrt{3}\right)^2}$$

$$M = \sqrt[2n]{49-16\,.\,3}$$

$$M = \sqrt[2n]{1}$$

$$\boxed{\therefore\ M = 1}$$

Problema 5

Racionalice

$$E = \frac{10}{\sqrt[7]{16}}$$

Resolución:

$$E = \frac{10}{\sqrt[7]{16}}$$

$$E = \frac{10}{\sqrt[7]{2^4}} \times \frac{\sqrt[7]{2^{7-4}}}{\sqrt[7]{2^{7-4}}}$$

$$E = \frac{10\ \sqrt[7]{2^3}}{\sqrt[7]{2^4}\,.\,\sqrt[7]{2^3}}$$

$$E = \frac{10\ \sqrt[7]{8}}{\sqrt[7]{2^7}}$$

$$E = \frac{10\sqrt[7]{8}}{2}$$

$$\boxed{\therefore\quad E = 5\sqrt[7]{8}}$$

Problema 6

Reduzca

$$R=\sqrt{8+\sqrt{28}}-\sqrt{10-\sqrt{84}}+\sqrt{7-\sqrt{48}}$$

Recordemos:

$$\boxed{\sqrt{A\pm\sqrt{B}}=\sqrt{(x+y)\pm 2\sqrt{x.y}}=\sqrt{x}\pm\sqrt{y}}$$

Resolución:

$$R=\sqrt{8+\sqrt{28}}-\sqrt{10-\sqrt{84}}+\sqrt{7-\sqrt{48}}$$

$$R=\sqrt{8+\sqrt{4}\,.\sqrt{7}}-\sqrt{10-\sqrt{4}\,.\sqrt{21}}+\sqrt{7-\sqrt{4}\,.\sqrt{12}}$$

$$R=\sqrt{\underbrace{8}_{7+1}+2\sqrt{\underbrace{7}_{7\times 1}}}-\sqrt{\underbrace{10}_{7+3}-2\sqrt{\underbrace{21}_{7\times 3}}}+\sqrt{\underbrace{7}_{4+3}-2\sqrt{\underbrace{12}_{4\times 3}}}$$

$$R=\sqrt{7}+\sqrt{1}-(\sqrt{7}-\sqrt{3})+\sqrt{4}-\sqrt{3}$$

$$R=\cancel{\sqrt{7}}+\sqrt{1}-\cancel{\sqrt{7}}+\cancel{\sqrt{3}}+\sqrt{4}-\cancel{\sqrt{3}}$$

$$R=1+2$$

$$\boxed{\therefore\ R=3}$$

Problema 7

Si $z_1 = 7 + 3i$
$z_2 = 1 - 5i$

al efectuar

$$z = \bar{z}_1 . z_2^*$$

la diferencia entre la parte real y la parte imaginaria de z representa la edad de la profesora Verónica hace dos años. ¿Cuántos años tiene la profesora actualmente?

Recordemos:

Sea: $z = a + bi$

Conjugado de z: $\boxed{\bar{z} = a - bi}$

Opuesto de z: $\boxed{z^* = -a - bi}$

Resolución:

$$z = \bar{z}_1 . z_2^*$$

$$z = (7 - 3i)(-1 + 5i)$$

$$z = -7 + 35i + 3i - 15i^2$$

$$z = -7 + 35i + 3i + 15$$

$$\boxed{z = 38i + 8}$$

Edad de la profesora Verónica hace 2 años: $38 - 8 = 30$ *años*

$\therefore$ *La profesora Verónica tiene 32 años.*

Problema 8

Luego de efectuar:

$$z = \frac{15i^{72}(3-i)}{2+i^{83}}; \quad \left(i = \sqrt{-1}\right)$$

Calcule $Im(z)$

Recordemos:

POTENCIAS DE i:

$\boxed{i^{4k} = 1}$ $\boxed{i^{4k+1} = i}$

$\boxed{i^{4k+2} = -1}$ $\boxed{i^{4k+3} = -i}$

Resolución:

$$z = \frac{15i^{72}(3-i)}{2+i^{83}}$$

$$z = \frac{15(1)(3-i)}{(2-i)} \times \frac{(2+i)}{(2+i)}$$

$$z = \frac{15\left(6+3i-2i-i^2\right)}{4-i^2}$$

$$z = \frac{15(6+3i-2i+1)}{4+1}$$

$$z = \frac{\overset{3}{\cancel{15}}(7+i)}{\underset{1}{\cancel{5}}}$$

$$\boxed{z = 21+3i}$$

$$\boxed{\therefore\ Im(z) = 3}$$

Problema 9

Sea $z_1 = -3 - 5i$
$z_2 = 6 + 4i$
$z_3 = -1 + 7i$

Si $z = \bar{z}_1 - z_2 + z_3^*$

calcule $|Z|$

Recordemos:

Sea: $z = a + bi$

Conjugado de z: $\boxed{\bar{z} = a - bi}$

Opuesto de z: $\boxed{z^* = -a - bi}$

Módulo de z: $\boxed{|z| = \sqrt{a^2 + b^2}}$

Resolución:

$$z = \bar{z}_1 - z_2 + z_3^*$$

$$z = (-3 + 5i) - (6 + 4i) + (1 - 7i)$$

$$z = -3 + 5i - 6 - 4i + 1 - 7i$$

$$\boxed{z = -8 - 6i}$$

Nos piden: $|z|$

$$|z| = \sqrt{(-8)^2 + (-6)^2}$$

$$|z| = \sqrt{100}$$

$$\boxed{\therefore \ |z| = 10}$$

Problema 10

Efectúe

$$P = 20\left[\frac{3+i}{3-i} - \frac{3-i}{3+i}\right] \quad ;\left(i = \sqrt{-1}\right)$$

Recordemos:

IDENTIDAD DE LEGENDRE:

$$\boxed{(a+b)^2 - (a-b)^2 = 4ab}$$

DIFERENCIA DE CUADRADOS:

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

Resolución:

$$P = 20\left[\frac{3+i}{3-i} - \frac{3-i}{3+i}\right]$$

$$P = 20\left[\frac{(3+i)^2 - (3-i)^2}{(3-i)\ (3+i)}\right]$$

$$P = 20\left[\frac{4\ (3)\ (i)}{9 - i^2}\right] \qquad i^2 \to -1$$

$$P = \overset{2}{\cancel{20}}\left[\frac{12i}{\underset{1}{\cancel{10}}}\right]$$

$$\boxed{\therefore \quad P = 24i}$$