



# ALGEBRA

## Chapter 17

**5th**  
SECONDARY

**RADICACIÓN–RADICALES**  
**DOBLES**

**SESIÓN 1**

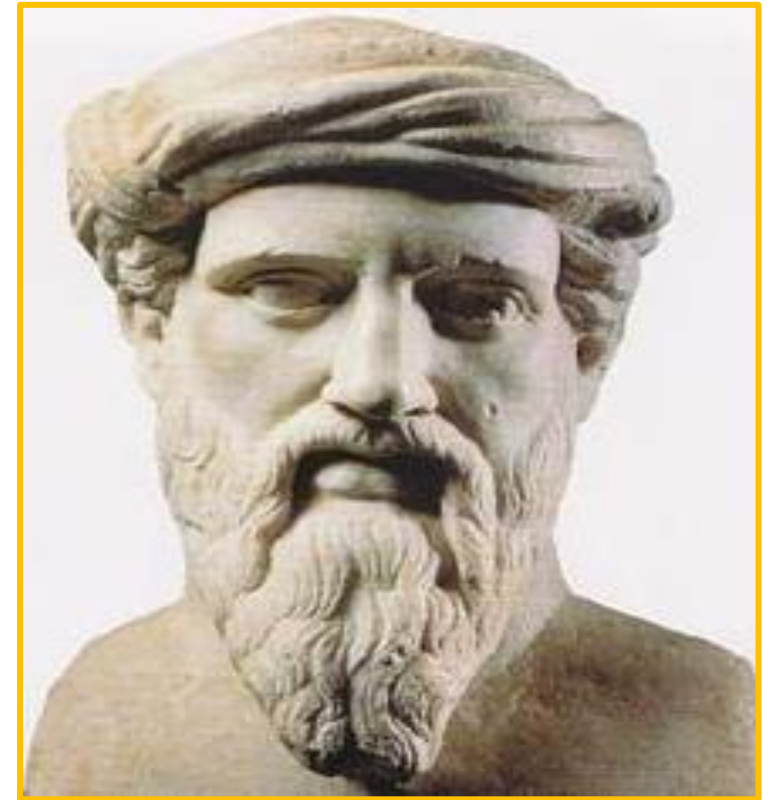


 **SACO OLIVEROS**

# *MOTIVATING STRATEGY*

## HISTORIA DE LA RADICACION

- Todo comenzó con el llamado Teorema de Pitágoras, el cual establece la relación entre los lados de un triángulo rectángulo, que para el cálculo se emplea las raíces cuadradas.
- La raíz cúbica fue estimulada para hallar el lado de un cubo conociendo su volumen. O bien para hallar el radio de una esfera también conociendo su volumen.





## PROPIEDADES

$$\sqrt[n]{a \times b} = \sqrt[n]{a} \times \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Ejemplo:

Hallar el equivalente de:

$$\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$$

$$\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

## RADICALES SEMEJANTES

Ejemplos

$$1) 5\sqrt[3]{2} + 7\sqrt[3]{2} - 9\sqrt[3]{2} = (5 + 7 - 9)\sqrt[3]{2} = 3\sqrt[3]{2}$$

$$2) 8\sqrt{3} - 4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$$



## DEFINICIÓN

Son radicales en los cuales dentro de un radical (radical mayor) se encuentra otros radicales ligados con operaciones de adición y sustracción.

## *Ejemplos*

$$\sqrt{8 + \sqrt{28}}$$

$$\sqrt{6 - 2\sqrt{10}}$$

# TRANSFORMACIÓN DE RADICALES DOBLES A RADICALES SIMPLES



Radicales de la forma

$$\sqrt{A \pm \sqrt{B}}$$

## 1.- Forma general

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A+C}{2}} \pm \sqrt{\frac{A-C}{2}}$$

Donde:  $A > \sqrt{B}$

$$C = \sqrt{A^2 - B}$$

**Ejemplo:** Transforme a radicales simples:

$$\sqrt{8 + \sqrt{28}} \quad \left\{ \begin{array}{l} A=8 \\ B=28 \\ C = \sqrt{8^2 - 28} = \sqrt{36} = 6 \end{array} \right.$$

$$\sqrt{8 + \sqrt{28}} = \sqrt{\frac{8+6}{2}} + \sqrt{\frac{8-6}{2}}$$

$$\sqrt{7} + 1$$

## 2.- Forma práctica



$$\sqrt{A \pm 2\sqrt{B}} = \sqrt{x} \pm \sqrt{y}$$

$\swarrow \quad \searrow$        $\swarrow \quad \searrow$   
 $x+y$        $x \times y$

Donde  $x > y$

### Ejemplo 1:

*Transforme a radicales simples:*

$$\sqrt{10 + 2\sqrt{21}} = \sqrt{7} + \sqrt{3}$$

$\swarrow \quad \searrow$        $\swarrow \quad \searrow$   
 $7 + 3$        $7 \times 3$

### Ejemplo 2:

*Transforme a radicales simples:*

$$\sqrt{13 - 2\sqrt{22}} = \sqrt{11} - \sqrt{2}$$

$\swarrow \quad \searrow$        $\swarrow \quad \searrow$   
 $11 + 2$        $11 \times 2$



# HELICO PRACTICE





*Efectúe*

$$E = \sqrt{2} + 3\sqrt{2} + 10\sqrt{2} - 5\sqrt{2}$$

*Resolución*

$$E = \underbrace{1}_{\text{green dashed circle}} \underbrace{\sqrt{2}}_{\text{yellow underline}} + \underbrace{3}_{\text{green dashed circle}} \underbrace{\sqrt{2}}_{\text{yellow underline}} + \underbrace{10}_{\text{green dashed circle}} \underbrace{\sqrt{2}}_{\text{yellow underline}} - \underbrace{5}_{\text{green dashed circle}} \underbrace{\sqrt{2}}_{\text{yellow underline}}$$

$$E = (1 + 3 + 10 - 5) \sqrt{2}$$

$$E = 9\sqrt{2}$$

$$\therefore E = 9\sqrt{2}$$



*Efectúe*

$$T = 12\sqrt[3]{2} - 7\sqrt[3]{2} + 3\sqrt[3]{2} - 8\sqrt[3]{2}$$

*Resolución*

$$T = \underbrace{12}_{\text{green dashed circle}} \underbrace{\sqrt[3]{2}}_{\text{yellow underline}} - \underbrace{7}_{\text{green dashed circle}} \underbrace{\sqrt[3]{2}}_{\text{yellow underline}} + \underbrace{3}_{\text{green dashed circle}} \underbrace{\sqrt[3]{2}}_{\text{yellow underline}} - \underbrace{8}_{\text{green dashed circle}} \underbrace{\sqrt[3]{2}}_{\text{yellow underline}}$$

$$T = (12 - 7 + 3 - 8) \sqrt[3]{2}$$

$$T = 0 \sqrt[3]{2}$$

$$\therefore T = 0$$



La edad de la psicóloga del colegio Saco Oliveros es  $\frac{4A^4}{9}$ , luego de simplificar  $A = \frac{5\sqrt{2}+3\sqrt{2}+7\sqrt{2}}{3\sqrt{2}+2\sqrt{2}}$  ¿Cuál será su edad dentro de 4 años?

### Resolución

$$A = \frac{5\sqrt{2} + 3\sqrt{2} + 7\sqrt{2}}{3\sqrt{2} + 2\sqrt{2}}$$

$$A = \frac{15\sqrt{2}}{5\sqrt{2}}$$

$$A = 3$$

$$\begin{aligned} \text{Edad} &= \frac{4A^4}{9} = \frac{4(3^4)}{9} \\ &= \frac{4(81)}{9} = 36 \end{aligned}$$

∴ Dentro de 4 años la edad será = 40



*Indique el resultado de*

$$H = \frac{\sqrt{32} + \sqrt{50} + \sqrt{18}}{\sqrt{2}}$$

*Resolución*

$$H = \frac{\sqrt{32} + \sqrt{50} + \sqrt{18}}{\sqrt{2}}$$

$$H = \sqrt{\frac{32}{2}} + \sqrt{\frac{50}{2}} + \sqrt{\frac{18}{2}}$$

$$H = \sqrt{16} + \sqrt{25} + \sqrt{9}$$

$$H = 4 + 5 + 3$$

$$\therefore H = 12$$

*Reduzca*

$$E = \sqrt{10 + 2\sqrt{24}} + \sqrt{15 - 2\sqrt{54}}$$

$$E = \sqrt{10 + 2\sqrt{24}} + \sqrt{15 - 2\sqrt{54}}$$

Diagram showing the breakdown of the terms under the square roots:

- $10 = 6 + 4$  (indicated by yellow arrows from 10 to 6 and 4)
- $2\sqrt{24} = 2 \times \sqrt{6 \times 4}$  (indicated by yellow arrows from 24 to 6 and 4, and a purple arrow from 2 to the coefficient 2)
- $15 = 9 + 6$  (indicated by yellow arrows from 15 to 9 and 6)
- $2\sqrt{54} = 2 \times \sqrt{9 \times 6}$  (indicated by yellow arrows from 54 to 9 and 6, and a purple arrow from 2 to the coefficient 2)

$$E = \sqrt{6} + \sqrt{4} + \sqrt{9} - \sqrt{6}$$

(Note: The  $\sqrt{6}$  terms are crossed out with a purple arrow, indicating they cancel out.)

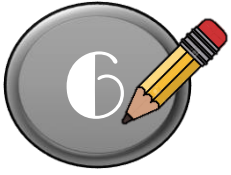
$$E = 2 + 3$$

$$E = 5$$

$$\therefore E = 5$$

**RECUERDA***FORMA PRÁCTICA*

$$\sqrt{A \pm \sqrt{B}} = \sqrt{(x + y) \pm 2\sqrt{xy}}$$



*Indica el equivalente de*

$$F = \sqrt{5 + 2\sqrt{6}} - \sqrt{5 - 2\sqrt{6}}$$

*Resolución*

$$F = \sqrt{5 + 2\sqrt{6}} - \sqrt{5 - 2\sqrt{6}}$$

Diagram showing the decomposition of the radicands into products of two numbers:

- For  $\sqrt{5 + 2\sqrt{6}}$ , the radicand is decomposed into  $3 + 2$  and  $3 \times 2$ .
- For  $\sqrt{5 - 2\sqrt{6}}$ , the radicand is decomposed into  $3 + 2$  and  $3 \times 2$ .

$$F = \sqrt{3} + \sqrt{2} - (\sqrt{3} - \sqrt{2})$$

$$F = \cancel{\sqrt{3}} + \sqrt{2} - \cancel{\sqrt{3}} + \sqrt{2}$$

$$F = 2\sqrt{2}$$

$$\therefore F = 2\sqrt{2}$$



*Simplifique*

$$B = \sqrt{8 + 2\sqrt{15}} + \sqrt{12 - 2\sqrt{35}}$$

Resolución

$$B = \sqrt{8 + 2\sqrt{15}} + \sqrt{12 - 2\sqrt{35}}$$

$\swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow \quad \searrow$   
 $5 + 3 \quad 5 \times 3 \quad 7 + 5 \quad 7 \times 5$

$$B = \cancel{\sqrt{5}} + \sqrt{3} + \sqrt{7} - \cancel{\sqrt{5}}$$

$$B = \sqrt{3} + \sqrt{7}$$

$$\therefore B = \sqrt{3} + \sqrt{7}$$



Obtenga el resultado de

$$M = \sqrt{7 + \sqrt{40}} - \sqrt{2}$$

## Resolución

$$M = \sqrt{7 + \sqrt{40}} - \sqrt{2}$$

$$M = \sqrt{7 + \sqrt{4 \times 10}} - \sqrt{2}$$

$$M = \sqrt{7 + 2\sqrt{10}} - \sqrt{2}$$

$\begin{array}{cc} \swarrow & \searrow & \swarrow & \searrow \\ 5 + 2 & & 5 \times 2 & \end{array}$

$$M = \sqrt{5} + \cancel{\sqrt{2}} - \cancel{\sqrt{2}}$$

### RECUERDA

FORMA PRÁCTICA

$$\sqrt{A \pm \sqrt{B}} = \sqrt{(x + y) \pm 2\sqrt{xy}}$$

$$\therefore M = \sqrt{5}$$