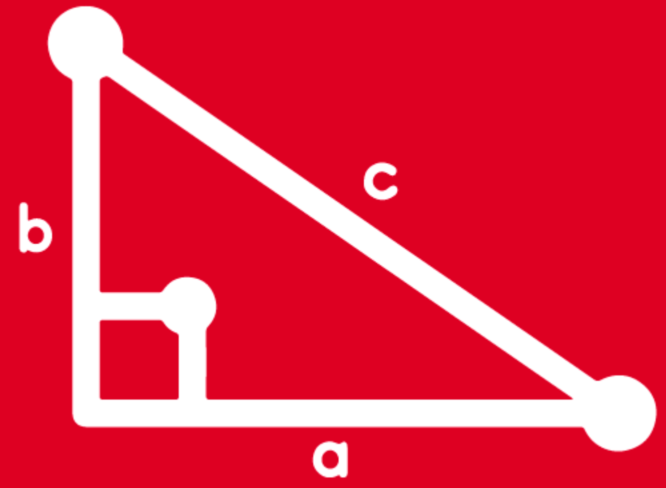
TRIGONOMETRY

Chapter 5

IDENTIDADES TRIGONOMETRICAS DE LOS ANGULOS COMPUESTOS



TOMO 2

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HELICO-MOTIVACIÓN

Las fórmulas de suma y diferencia para el seno y el coseno tienen una historia larga y antigua. Originalmente se desarrollaron para ayudar a estudiar el movimiento de los cuerpos celestes, siglos después se emplearon para desarrollar conceptos como las funciones trigonométricas, la teoría de los números complejos y el movimiento de ondas.

Estas Identidades también se utilizan para encontrar resultados exactos para muchos ángulos de gran importancia para los antiguos astrónomos y aún importante hoy.

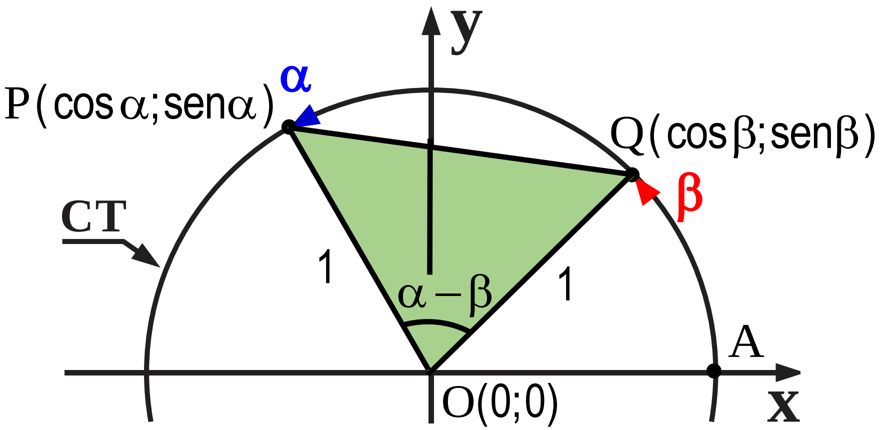
Usando el teorema de Ptolomeo (siglo II) , se demuestra :

$$\text{sen}(\alpha-\beta)=\text{sen}\alpha.\cos\beta-\cos\alpha.\text{sen}\beta$$

HELICOTEORÍA

IDENTIDADES TRIGONOMÉTRICAS DEL ÁNGULO COMPUESTO

En la CT, ubicamos los arcos α y β



Del gráfico : $\text{Área } \Delta POQ = \dfrac{(1)(1)}{2}\operatorname{sen}(\alpha-\beta)$

Luego : $\text{Área } \Delta POQ = \dfrac{\operatorname{sen}(\alpha-\beta)}{2}$ **… (I)**

También :

$$(+)\left\{\begin{array}{c|cc|c} 0 & 0 & 0 & \\ 0 & \cos\beta & \operatorname{sen}\beta & 0 \\ \cos\alpha.\operatorname{sen}\beta & \cos\alpha & \operatorname{sen}\alpha & \operatorname{sen}\alpha.\cos\beta \\ 0 & 0 & 0 & 0 \\ \hline \cos\alpha.\operatorname{sen}\beta & & & \operatorname{sen}\alpha.\cos\beta \end{array}\right\}(+)$$

$\Rightarrow \text{Área } \Delta POQ = \dfrac{\operatorname{sen}\alpha.\cos\beta - \cos\alpha.\operatorname{sen}\beta}{2}$ **… (II)**

(I) = (II) : $\dfrac{\operatorname{sen}(\alpha-\beta)}{2} = \dfrac{\operatorname{sen}\alpha.\cos\beta - \cos\alpha.\operatorname{sen}\beta}{2}$

$$\Rightarrow \boxed{\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha.\cos\beta - \cos\alpha.\operatorname{sen}\beta}$$

En la identidad anterior , cambiar β por $(-\beta)$:

$$\operatorname{sen}(\alpha-(-\beta))=\underbrace{\operatorname{sen}\alpha.\cos(-\beta)}_{\cos\beta}-\underbrace{\cos\alpha.\operatorname{sen}(-\beta)}_{-\operatorname{sen}\beta}$$

$$\Rightarrow \boxed{\operatorname{sen}(\alpha+\beta)=\operatorname{sen}\alpha.\cos\beta+\cos\alpha.\operatorname{sen}\beta}\ (*)$$

Usando propiedad del ángulos complementarios :

$$\cos(\alpha+\beta)=\operatorname{sen}(90^\circ-(\alpha+\beta))$$

$$\cos(\alpha+\beta)=\operatorname{sen}((90^\circ-\alpha)-\beta)$$

Desarrollando, tenemos :

$$\cos(\alpha+\beta)=\underbrace{\operatorname{sen}(90^\circ-\alpha)}_{\cos\alpha}.\cos\beta-\underbrace{\cos(90^\circ-\alpha)}_{\operatorname{sen}\alpha}.\operatorname{sen}\beta$$

$$\Rightarrow \boxed{\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\operatorname{sen}\alpha.\operatorname{sen}\beta}\ (**)$$

Usando la identidad por cociente :

$$\tan(\alpha+\beta)=\frac{\operatorname{sen}(\alpha+\beta)}{\cos(\alpha+\beta)}$$

Usando **(*)** y **(**)** :

$$\tan(\alpha+\beta)=\frac{\operatorname{sen}\alpha.\cos\beta+\cos\alpha.\operatorname{sen}\beta}{\cos\alpha.\cos\beta-\operatorname{sen}\alpha.\operatorname{sen}\beta}$$

En el 2º miembro , dividiendo el numerador y denominador por $\cos\alpha.\cos\beta$

$$\tan(\alpha+\beta)=\frac{\dfrac{\operatorname{sen}\alpha.\cancel{\cos\beta}}{\cos\alpha.\cancel{\cos\beta}}+\dfrac{\cancel{\cos\alpha}.\operatorname{sen}\beta}{\cancel{\cos\alpha}.\cos\beta}}{\dfrac{\cancel{\cos\alpha}.\cancel{\cos\beta}}{\cancel{\cos\alpha}.\cancel{\cos\beta}}-\dfrac{\operatorname{sen}\alpha.\operatorname{sen}\beta}{\cos\alpha.\cos\beta}}$$

$$\Rightarrow \boxed{\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}}$$

IDENTIDADES FUNDAMENTALES

Para la suma de dos ángulos :

$$\text{sen}(x+y)=\text{sen}x.\cos y+\cos x.\text{sen}y$$

$$\cos(x+y)=\cos x.\cos y-\text{sen}x.\text{sen}y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$$

Para la diferencia de dos ángulos :

$$\text{sen}(x-y)=\text{sen}x.\cos y-\cos x.\text{sen}y$$

$$\cos(x-y)=\cos x.\cos y+\text{sen}x.\text{sen}y$$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x.\tan y}$$

Ejemplo : Calcular sen75º

Resolución :

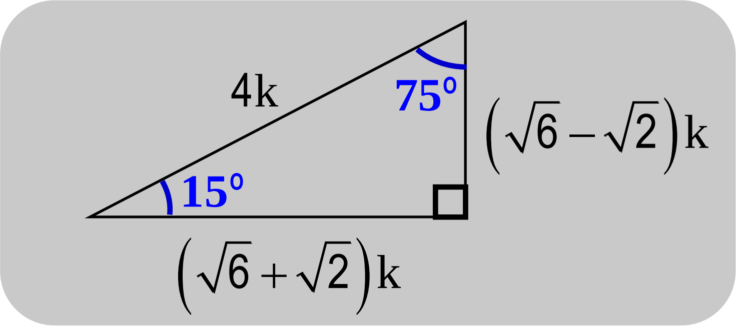
$$\text{sen}75^\circ=\text{sen}(45^\circ+30^\circ)$$

$$\text{sen}75^\circ=\text{sen}45^\circ.\cos30^\circ+\cos45^\circ.\text{sen}30^\circ$$

$$\text{sen}75^\circ=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}.\frac{1}{2}$$

$$\Rightarrow \text{sen}75^\circ=\frac{\sqrt{6}+\sqrt{2}}{4}$$

15º y **75º**



IDENTIDADES TRIGONOMÉTRICAS AUXILIARES

1. $\boxed{\operatorname{sen}(x+y).\operatorname{sen}(x-y)=\operatorname{sen}^2 x-\operatorname{sen}^2 y}$

2. $\boxed{\cos(x+y).\cos(x-y)=\cos^2 x-\operatorname{sen}^2 y}$

Ejemplo : Si $\sqrt{2}=1{,}41$

Calcule : $E=\operatorname{sen}^2 41^\circ-\operatorname{sen}^2 4^\circ$

Resolución :

Para x = 41° , y = 4° ; usamos la identidad **1.**

$$E=\operatorname{sen}(41^\circ+4^\circ).\operatorname{sen}(41^\circ-4^\circ)$$

$$E=\operatorname{sen}45^\circ.\operatorname{sen}37^\circ$$

$$E=\frac{\sqrt{2}}{2}.\frac{3}{5}=\frac{3\sqrt{2}}{10}=\frac{3(1{,}41)}{10} \qquad \therefore E=0{,}423$$

3. $\boxed{\dfrac{\operatorname{sen}(x+y)}{\cos x.\cos y}=\tan x+\tan y}$

4. $\boxed{\dfrac{\operatorname{sen}(x-y)}{\cos x.\cos y}=\tan x-\tan y}$

Ejemplo :

Calcule : $E=(\tan 50^\circ+\tan 20^\circ)\operatorname{sen}40^\circ$

Resolución :

Para x = 50° , y = 20° ; usamos la identidad **3.**

$$E=\left(\frac{\operatorname{sen}(50^\circ+20^\circ)}{\cos 50^\circ.\cos 20^\circ}\right)\operatorname{sen}40^\circ$$

$$E=\left(\frac{\cancel{\operatorname{sen}70^\circ}}{\cancel{\cos 50^\circ}.\cancel{\cos 20^\circ}}\right)\cancel{\operatorname{sen}40^\circ} \qquad \therefore E=1$$

5. $\boxed{\tan x+\tan y+\tan(x+y).\tan x.\tan y=\tan(x+y)}$

6. $\boxed{\tan x-\tan y-\tan(x-y).\tan x.\tan y=\tan(x-y)}$

Ejemplo :

Calcule : $E=\tan 40°+\tan 20°+\sqrt{3}.\tan 40°.\tan 20°$

Resolución :

$$E=\tan 40°+\tan 20°+\sqrt{3}.\tan 40°.\tan 20°$$

$$E=\tan 40°+\tan 20°+\tan 60°.\tan 40°.\tan 20°$$

Para $x = 40°$, $y = 20°$; usamos la identidad **5.**

$E=\tan 60°$ $\therefore E=\sqrt{3}$

7. $$\boxed{\underbrace{-\sqrt{a^2+b^2}}_{\text{mín}}\leq a.\text{sen}x+b.\cos x\leq\underbrace{\sqrt{a^2+b^2}}_{\text{máx}}}$$

Ejemplos :

- $-5\leq 4.\text{sen}x+3.\cos x\leq 5$
- $-2\leq\sqrt{3}.\text{sen}x-\cos x\leq 2$
- $-\sqrt{2}\leq\text{sen}x+\cos x\leq\sqrt{2}$

8. Si : $\alpha+\beta+\theta=n\pi$; $n\in Z$

$\Rightarrow \tan\alpha+\tan\beta+\tan\theta=\tan\alpha.\tan\beta.\tan\theta$

$\Rightarrow \cot\alpha.\cot\beta+\cot\beta.\cot\theta+\cot\alpha.\cot\theta=1$

9. Si : $x+y+z=(2n+1)\frac{\pi}{2}$; $n\in Z$

$\Rightarrow \cot x+\cot y+\cot z=\cot x.\cot y.\cot z$

$\Rightarrow \tan x.\tan y+\tan y.\tan z+\tan x.\tan z=1$

HELICO-PRACTICA 1

1. Calcule el valor aproximado de la siguiente expresión.

$$\text{sen}67° - \frac{3\sqrt{3}}{10}$$

Recuerda:

$$\text{sen}(\alpha - \beta) = \text{sen}\alpha . \cos\beta - \cos\alpha . \text{sen}\beta$$

Resolución:

$$\text{Sen}67° = \text{Sen}(30° + 37°)$$

$$\text{Sen}(30° + 37°) = \text{Sen}30°\,\text{Cos}37° + \text{Sen}37°\,\text{Cos}30°$$

$$\text{Sen}(30° + 37°) = \left(\frac{1}{2}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$$

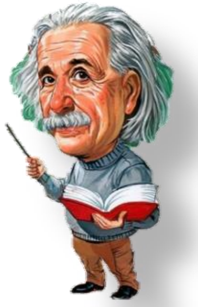
$$\text{Sen}67° = \left(\frac{4}{10}\right) + \left(\frac{3\sqrt{3}}{10}\right)$$

Piden:

$$\text{Sen}67° - \left(\frac{3\sqrt{3}}{10}\right)$$

$$\overbrace{\left(\frac{4}{10}\right) + \cancel{\left(\frac{3\sqrt{3}}{10}\right)}} - \cancel{\left(\frac{3\sqrt{3}}{10}\right)} = \left(\frac{4}{10}\right) \Rightarrow \therefore \frac{2}{5}$$

¡Muy bien!



HELICO-PRACTICE 2

2. Si sen(37°+x)=2cos(60°+x), calcule cotx.

Resolución:

Recuerda:

$$\text{sen}(\alpha-\beta)=\text{sen}\alpha.\cos\beta-\cos\alpha\,.\,\text{sen}\beta$$
$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\text{sen}\alpha.\text{sen}\beta$$

$$5\text{Sen}(37°+x)=2\cos(60°+x)$$

$$5(\text{Sen}37°\text{Cosx}+\text{SenxCos}37°)=2(\text{Cos}60°\text{Cosx}-\text{Sen}60°\text{senx})$$

$$\cancel{5}\left(\left(\frac{3}{\cancel{5}}\right)\text{Cosx}+\text{Senx}\left(\frac{4}{\cancel{5}}\right)\right)\cancel{=2}\left(\left(\frac{1}{\cancel{2}}\right)\text{Cosx}-\left(\frac{\sqrt{3}}{\cancel{2}}\right)\text{senx}\right)$$

$$3\text{Cosx}+4\text{Senx}=\text{Cosx}-\sqrt{3}\text{senx}$$

$$2\text{Cosx}=-(4+\sqrt{3})\text{senx}$$

$$\text{Cotx}=-\left(\frac{4+\sqrt{3}}{2}\right)$$

¡Muy bien!

HELICO-PRACTICE 3

3. Simplifique la expresión

$$E = \frac{\text{Sen}37° - \text{Sen}22°\text{Cos}15°}{\text{Cos}22°\text{Cos}15°}$$

Recuerda:

$$\text{sen}(\alpha - \beta) = \text{sen}\alpha.\cos\beta - \cos\alpha.\text{sen}\beta$$
$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \text{sen}\alpha.\text{sen}\beta$$

Resolución:

$$\text{Sen}37° = \text{Sen}(22° + 15°)$$

$$\text{Sen}37° = \text{Sen}22°\text{Cos}15° + \text{Sen}15°\text{Cos}22° \quad \ldots \text{I}$$

I en E

$$E = \frac{\cancel{\text{Sen}22°\text{Cos}15°} + \text{Sen}15°\text{Cos}22° - \cancel{\text{Sen}22°\text{Cos}15°}}{\text{Cos}22°\text{Cos}15°}$$

$$E = \frac{\text{Sen}15°\cancel{\text{Cos}22°}}{\cancel{\text{Cos}22°}\text{Cos}15°}$$

$$E = \text{Tan}15°$$

¡Muy bien!

$$\mathbf{E = 2 - \sqrt{3}}$$

HELICO-PRACTICE 4

4. Simplifique

$$E = \text{Cot}3x\left[\frac{\text{Tan}^2 2x - \text{Tan}^2 x}{1 - \text{Tan}^2 2x.\text{Tan}^2 x}\right]$$

Recuerda:

$$a^2 - b^2 = (a+b)(a-b)$$

Resolución:

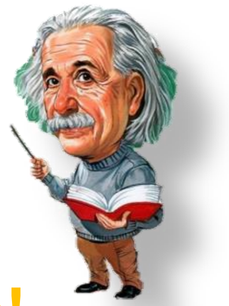
$$E = \text{Cot}3x\left[\left(\frac{\text{Tan}2x - \text{Tan}x}{1 - \text{Tan}2x\text{Tan}x}\right)\left(\frac{\text{Tan}2x + \text{Tan}x}{1 + \text{Tan}2x\text{Tan}x}\right)\right]$$

$$E = \text{Cot}3x.\ \mathbf{Tan(2x - x).Tan(2x + x)}$$

$$E = \text{Cot}3x.\text{Tan}(x).\text{Tan}(3x)$$

$$E = \underbrace{\text{Cot}3x.\text{Tan}(3x)}.\text{Tan}(x)$$

$$E = 1.\text{Tan}(x)$$



¡Muy bien!

$$\mathbf{E = Tan(x)}$$

HELICO-PRACTICE 5

5. Si $\operatorname{Tan}(x+y)=2$

$$\operatorname{Tan}(2x+y-z)=\frac{3}{5}$$

Calcule $\tan(x-z)$

Notamos que:

$$\underbrace{(2x+y-z)}_{\mathbf{A}}-\underbrace{(x+y)}_{\mathbf{B}}=\underbrace{x-z}_{\mathbf{A-B}}$$

Recuerda:

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}$$

Resolución:

$$\operatorname{Tan}(A)=\frac{3}{5}\qquad \operatorname{Tan}(B)=2$$

$$\operatorname{Tan}(x-z)=\operatorname{Tan}(A-B)\ \ldots.*$$

$$\operatorname{Tan}(A-B)=\frac{\operatorname{Tan}A-\operatorname{Tan}B}{1+\operatorname{Tan}A.\operatorname{Tan}B}$$

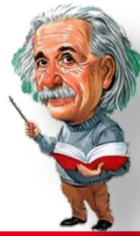
$$\operatorname{Tan}(A-B)=\frac{\frac{3}{5}-2}{1+\left(\frac{3}{5}\right)(2)}$$

$$\operatorname{Tan}(A-B)=\frac{-\frac{7}{\cancel{5}}}{\frac{11}{\cancel{5}}}$$

$$\operatorname{Tan}(A-B)=-\frac{7}{11}$$

De $(*)$

$$\mathbf{Tan(x-z)=-\frac{7}{11}}$$



HELICO-PRACTICE 6

6. El valor de la expresión

$$K = \frac{\text{Sen}7° - \sqrt{3}\text{Cos}7°}{\sqrt{3}\text{Sen}14° - \text{Cos}14°}$$

Recuerda:

$$\text{sen}(\alpha - \beta) = \text{sen}\alpha.\cos\beta - \cos\alpha.\text{sen}\beta$$
$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \text{sen}\alpha.\text{sen}\beta$$

Resolución:

Multiplicamos por ½ en el numerador
Y denominador

$$K = \frac{\left(\frac{1}{2}\right)\text{Sen}7° - \left(\frac{\sqrt{3}}{2}\right)\text{Cos}7°}{\left(\frac{\sqrt{3}}{2}\right)\text{Sen}14° - \left(\frac{1}{2}\right)\text{Cos}14°}$$

$$K = \frac{\text{Cos}60°\text{Sen}7° - \text{Sen}60°\text{Cos}7°}{\text{Cos}30°\text{Sen}14° - \text{Sen}30°\text{Cos}14°}$$

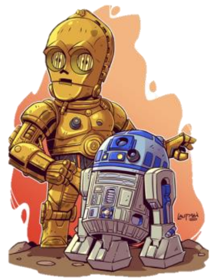
$$K = \frac{\text{Sen}(7° - 60°)}{\text{Sen}(14° - 30°)}$$

$$K = \frac{\text{Sen}(-53°)}{\text{Sen}(-16°)}$$

$$K = \frac{-\text{Sen}(53°)}{-\text{Sen}(16°)}$$

$$K = \frac{\text{Sen}(53°)}{\text{Sen}(16°)}$$

$$K = \frac{\frac{4}{\cancel{5}}}{\frac{7}{\cancel{25}}}$$



$$\mathbf{K = \frac{20}{7}}$$

HELICO-PRACTICE 7

7. Si $\text{Sen}5° = \frac{a}{2}$, obtenga;

$$\text{Cos}35°.\text{Cos}25°$$

Recuerda:

$$\cos(x+y).\cos(x-y) = \cos^2 x - \text{sen}^2 y$$

Resolución:

Sea:

$$E = \text{Cos}35°.\text{Cos}25°$$

$$E = \text{Cos}(30° + 5°).\text{Cos}(30° - 5°)$$

$$E = \text{Cos}^2 30° - \text{Sen}^2 5°$$

$$E = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{a}{2}\right)^2$$

$$E = \frac{3}{4} - \frac{a^2}{4}$$

$$E = \frac{3-a^2}{4}$$

HELICO-PRACTICE 8

8. Halle el valor del cociente

$$R = \frac{\text{Sen}5°}{\text{Sen}^2 25° - \text{Sen}^2 20°}$$

Recuerda:

$$\text{sen}(x+y).\text{sen}(x-y) = \text{sen}^2 x - \text{sen}^2 y$$

Resolución:

$$R = \frac{\text{Sen}5°}{\text{Sen}^2 25° - \text{Sen}^2 20°}$$

$$R = \frac{\text{Sen}5°}{\text{Sen}(25° + 20°)\text{Sen}(25° - 20°)}$$

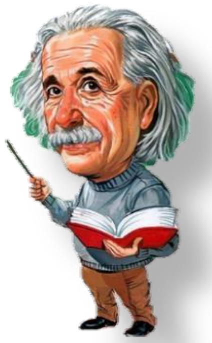
$$R = \frac{\cancel{\text{Sen}5°}}{\text{Sen}45°\cancel{\text{Sen}5°}}$$

$$R = \frac{1}{\text{Sen}45°}$$

$$R = \frac{1}{\frac{1}{\sqrt{2}}}$$

$$R = \sqrt{2}$$

¡Muy bien!



HELICO-PRACTICE 9

9. Calcule

$$\frac{\text{Sen}50° - \text{Cos}50°}{\text{Sen}5°} + \frac{\text{Tan}65° - \text{Tan}25°}{\sqrt{2}\text{Tan}40°}$$

Recuerda:

$$\text{sen}(\alpha - \beta) = \text{sen}\alpha.\cos\beta - \cos\alpha.\text{sen}\beta$$
$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \text{sen}\alpha.\text{sen}\beta$$

Resolución:

$$\frac{\text{Sen}(45° + 5°) - \text{Cos}(45° + 5°)}{\text{Sen}5°} + \frac{\text{Tan}65° - \text{Tan}25°}{\sqrt{2}\text{Tan}40°}$$

$$\frac{(\text{Sen}45°\text{Cos}5° + \text{Cos}45°\text{Sen}5°) - (\text{Cos}45°\text{Cos}5° - \text{Sen}45°\text{Sen}5°)}{\text{Sen}5°} + \frac{\text{Tan}65° - \text{Tan}25°}{\sqrt{2}\text{Tan}(65° - 25°)}$$

$$\frac{\cancel{\frac{\sqrt{2}}{2}\text{Cos}5°} + \frac{\sqrt{2}}{2}\text{Sen}5° - \cancel{\frac{\sqrt{2}}{2}\text{Cos}5°} + \frac{\sqrt{2}}{2}\text{Sen}5°}{\text{Sen}5°} + \frac{\text{Tan}65° - \text{Tan}25°}{\sqrt{2}\left(\frac{\text{Tan}65° - \text{Tan}25°}{1 + \text{Tan}65°\text{Cot}25°}\right)}$$

$$\frac{\sqrt{2}\cancel{\text{Sen}5°}}{\cancel{\text{Sen}5°}} + \frac{1 + 1}{\sqrt{2}}$$

$$\frac{\sqrt{2}}{1} + \frac{2}{\sqrt{2}}$$

$$\sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

¡Muy bien!

HELICO-PRACTICE 10

10. La expresión

$$\mathrm{Sen}^2(a+b)+\mathrm{Sen}^2 b-2\mathrm{Sen}(a+b).\mathrm{Senb}.\mathrm{Cosa}$$

Es identica a:

Recuerda:

$$\mathrm{sen}(\alpha-\beta)=\mathrm{sen}\alpha.\cos\beta-\cos\alpha.\mathrm{sen}\beta$$
$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\mathrm{sen}\alpha.\mathrm{sen}\beta$$
$$\mathrm{sen}(x+y).\mathrm{sen}(x-y)=\mathrm{sen}^2x-\mathrm{sen}^2y$$

Resolución:

$$\mathrm{Sen}^2(a+b)+\mathrm{Sen}^2 b-2\mathrm{Sen}(a+b).\mathrm{Senb}.\mathrm{Cosa}$$

$$\mathrm{Sen}(a+b)[\mathrm{Sen}(a+b)-2\mathrm{SenbCosa}]+\mathrm{Sen}^2 b$$

$$\mathrm{Sen}(a+b)[\mathrm{SenaCosb}+\cancel{\mathrm{SenbCosa}}-\cancel{2\mathrm{SenbCosa}}]+\mathrm{Sen}^2 b$$

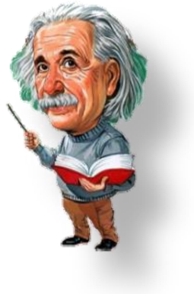
$$\mathrm{Sen}(a+b)[\mathrm{SenaCosb}-\mathrm{SenbCosa}]+\mathrm{Sen}^2 b$$

$$\mathrm{Sen}(a+b)\mathrm{Sen}(a-b)+\mathrm{Sen}^2 b$$

$$\mathrm{sen}^2(a)-\mathrm{sen}^2(b)+\mathrm{Sen}^2 b$$

Rpta: $\mathrm{sen}^2(a)$

¡Muy bien!



HELICO-PRACTICE 11

11. Si $(m-n)Sen(\alpha-\beta)=(m+n)Sen(\alpha+\beta)$

Determine $\dfrac{Tan(\beta)}{Tan(\alpha)}$

Recuerda:

$$\frac{a}{b}=\frac{c}{d}\rightarrow\frac{a-b}{a+b}=\frac{c-d}{c+d}$$

Resolución:

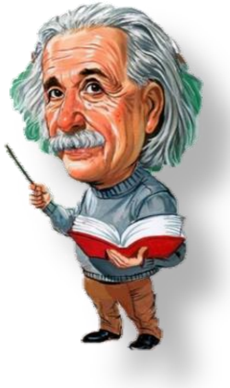
$$\frac{Sen(\alpha-\beta)}{Sen(\alpha+\beta)}=\frac{m+n}{m-n}$$

$$\frac{Sen(\alpha-\beta)-Sen(\alpha+\beta)}{Sen(\alpha-\beta)+Sen(\alpha+\beta)}=\frac{(m+n)-(m-n)}{(m+n)+(m-n)}$$

$$\frac{-2Sen(\beta)Cos(\alpha)}{2Sen(\alpha)Cos(\beta)}=\frac{2n}{2m}$$

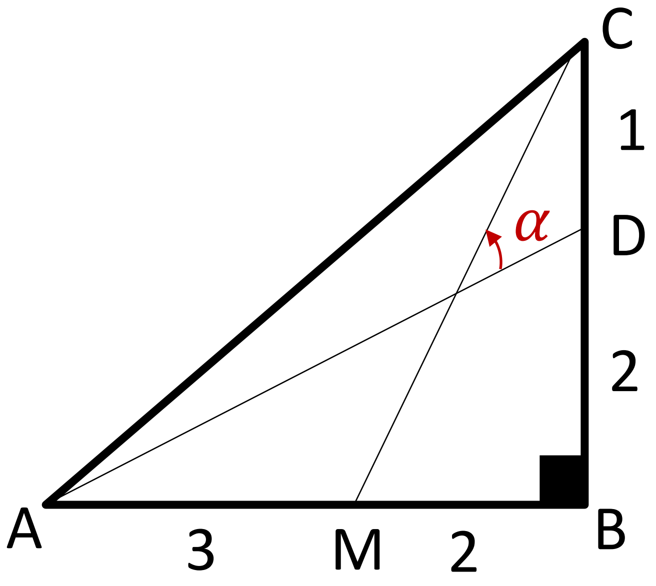
$$\frac{\frac{Sen\beta}{Cos\beta}}{\frac{Sen\alpha}{Cos\alpha}}=-\frac{n}{m}$$

$$\frac{Tan\beta}{Tan\alpha}=-\frac{\mathbf{n}}{\mathbf{m}}$$

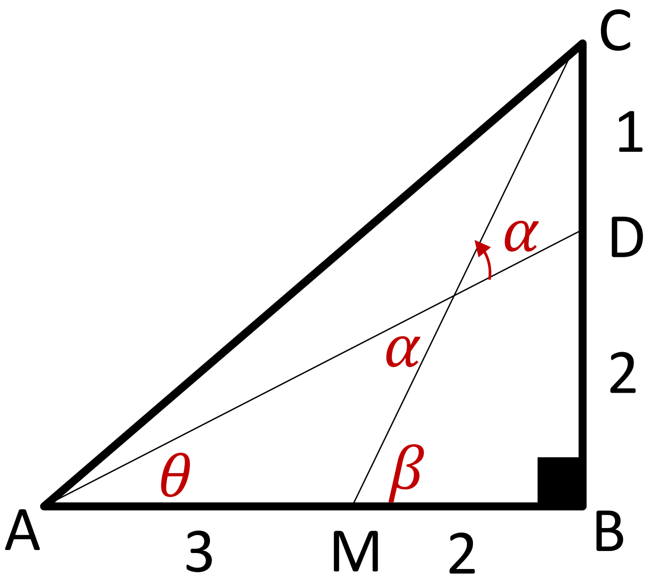


HELICO-PRACTICE 12

12. Del gráfico calcule Tanα



Resolución:



Del gráfico

$$\alpha = \beta - \theta$$

$$\Rightarrow \text{Tan}\alpha = \text{Tan}(\beta - \theta)$$

$$\text{Tan}\alpha = \frac{\text{Tan}\beta - \text{Tan}\theta}{1 + \text{Tan}\beta.\text{Tan}\theta}$$

$$\text{Tan}\alpha = \frac{\frac{3}{2} - \frac{2}{5}}{1 + \left(\frac{3}{2}\right)\left(\frac{2}{5}\right)}$$

$$\text{Tan}\alpha = \frac{\frac{11}{\cancel{10}\ 2}}{\frac{8}{\cancel{5}}}$$

$$\text{Tan}\alpha = \frac{11}{16}$$

Recuerda:

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$$

HELICO-PRACTICE 13

13. Al reducir

$$\frac{\operatorname{Sen}(x+3y)\operatorname{Sen}(3x-y)}{\operatorname{Cos}^2(2y-x)-\operatorname{Cos}^2(2x+y)}$$

Recuerda:

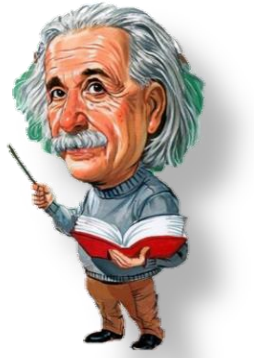
$$\operatorname{sen}(x+y).\operatorname{sen}(x-y)=\operatorname{sen}^2x-\operatorname{sen}^2y$$

Resolución:

$$\frac{\operatorname{Sen}(x+3y)\operatorname{Sen}(3x-y)}{[1-\operatorname{Sen}^2(2y-x)]-[1-\operatorname{Sen}^2(2x+y)]}$$

$$\frac{\operatorname{Sen}(x+3y)\operatorname{Sen}(3x-y)}{[\operatorname{Sen}^2(2x+y)]-[\operatorname{Sen}^2(2y-x)]}$$

$$\frac{\operatorname{Sen}(x+3y)\operatorname{Sen}(3x-y)}{\operatorname{Sen}(x+3y)\operatorname{Sen}(3x-y)}=\mathbf{1}$$



HELICO-PRACTICE 14

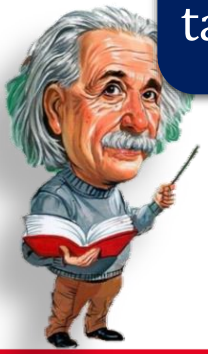
14. Reduzca

$$C = \frac{\text{Tan}10° + \text{Tan}12° + \text{Tan}10°.\text{Tan}12°.\text{Tan}22°}{\tan 15° + \text{Tan}7° + \text{Tan}15°\text{Tan}7°\text{Tan}22°}$$

Recuerda:

$$\tan x + \tan y + \tan(x+y).\tan x.\tan y = \tan(x+y)$$

$$\tan x - \tan y - \tan(x-y).\tan x.\tan y = \tan(x-y)$$



¡Muy bien!

Resolución:

$$C = \frac{\text{Tan}(10° + 12°)}{\text{Tan}(15° + 7°)}$$

$$C = \frac{\text{Tan}(22°)}{\text{Tan}(22°)}$$

$$C = 1$$

HELICO-PRACTICE 15

15. Señale un valor agudo de x, si

$$\mathrm{Tan2x} + \mathrm{Tan3x} + \mathrm{Tan2x.\,Tan3x.\,Tan5x} = \frac{1}{2}\mathrm{Sec5x}$$

Recuerda:

$$\tan x + \tan y + \tan(x+y).\tan x.\tan y = \tan(x+y)$$

$$\tan x - \tan y - \tan(x-y).\tan x.\tan y = \tan(x-y)$$

Resolución:

$$\mathrm{Tan}(2x + 3x) = \frac{1}{2}\mathrm{Sec5x}$$

$$\mathrm{Tan}(5x) = \frac{\mathrm{Sec5x}}{2}$$

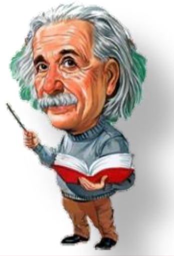
$$\frac{\mathrm{Sen5x}}{\mathrm{Cos5x}} = \frac{\mathrm{Sec5x}}{2}$$

$$\mathrm{Sen5x} = \frac{\mathrm{Cos5x.\,Sec5x}}{2}$$

$$\mathrm{Sen5x} = \frac{1}{2}$$

$$5x = 30° \qquad x = 6°$$

¡Muy bien!



HELICO-PRACTICE 16

16. Calcule el valor de

$$C = \text{Tanx} + \text{Tan2x} + \text{Tanx}.\text{Tan2x}.\text{Tan3x}$$

Si se sabe que

$\text{Tanx} + \text{Tan3x} + \text{Tan4x}.\text{Tan3x}.\text{Tan4x} = \sqrt{3}$; x Es agudo.

Recuerda:

$$\tan x + \tan y + \tan(x+y).\tan x.\tan y = \tan(x+y)$$

$$\tan x - \tan y - \tan(x-y).\tan x.\tan y = \tan(x-y)$$

Resolución:

Piden:

$$C = \text{Tanx} + \text{Tan2x} + \text{Tanx}.\text{Tan2x}.\text{Tan3x}$$

$$C = \text{Tan}(x + 2x)$$

$$C = \text{Tan}(3x) \quad ...(I)$$

Del dato:

$$\text{Tan}(x + 3x) = \sqrt{3}$$

$$\text{Tan}(4x) = \sqrt{3}$$

$$4x = 60°$$

$$x = 15° \; ...(II)$$

(II) En (I)

$$C = \text{Tan}(3x)$$

$$C = \text{Tan}(3(15°))$$

$$C = \text{Tan}(45°)$$

$$C = 1$$

¡Muy bien!

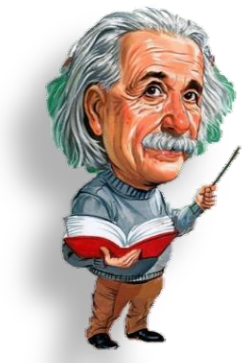
HELICO-PRACTICE 17

17. En un triángulo ABC; se cumple

$$\text{TanA} + \text{TanB} + \text{TanC} = 7\text{TanC}$$

Calcule TanA. TanB

Recuerda:



Si : $\alpha + \beta + \theta = n\pi$; $n \in Z$

$\Rightarrow \tan\alpha + \tan\beta + \tan\theta = \tan\alpha.\tan\beta.\tan\theta$

$\Rightarrow \cot\alpha.\cot\beta + \cot\beta.\cot\theta + \cot\alpha.\cot\theta = 1$

Resolución:

Dato

$A + B + C = 180°$

$$\text{TanA. TanB.}\cancel{\text{TanC}} = 7\cancel{\text{TanC}}$$

$$\text{TanA. TanB} = 7$$

¡Muy bien!

HELICO-PRACTICE 18

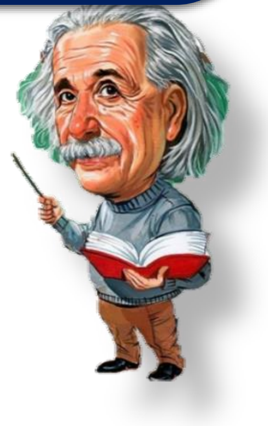
18. Simplifique

$$\frac{\mathrm{Senx}}{\mathrm{Cos3xCos2x}} + \frac{\mathrm{Senx}}{\mathrm{Cos4x.\,Cos3x}} - \frac{\mathrm{Sen2x}}{\mathrm{Cos4x.\,Cos2x}}$$

Recuerda:

$$\frac{\mathrm{sen}(x+y)}{\cos x\,.\cos y} = \tan x + \tan y$$

$$\frac{\mathrm{sen}(x-y)}{\cos x\,.\cos y} = \tan x - \tan y$$



Resolución:

$$\cancel{\mathrm{Tan3x}} - \cancel{\mathrm{Tan2x}} + \cancel{\mathrm{Tan4x}} - \cancel{\mathrm{Tan3x}} - (\cancel{\mathrm{Tan4x}} - \cancel{\mathrm{Tan2x}})$$

$Rpta: 0$

¡Muy bien!

HELICO-PRACTICE 19

19. $Si\ x + y + z = 90°$; $\text{Tanx} + \text{Tany} + \text{Tanz} = 4$;

Calcule $U = \text{Sec}^2x + \text{Sec}^2y + \text{Sec}^2z$

Recuerda:

$\text{Si}:\ x + y + z = (2n+1)\dfrac{\pi}{2}\ ;\ n \in Z$

$\Rightarrow \cot x + \cot y + \cot z = \cot x.\cot y.\cot z$

$\Rightarrow \tan x.\tan y + \tan y.\tan z + \tan x.\tan z = 1$

Resolución:

$$(\text{Tanx} + \text{Tany} + \text{Tanz})^2 = (4)^2$$

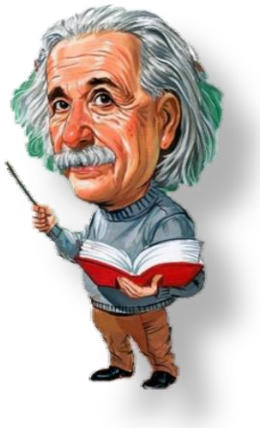
$$\text{Tan}^2x + \text{Tan}^2y + \text{Tan}^2z + 2(\underbrace{\text{TanxTany} + \text{TanxTanz} + \text{TanyTanz}}_{1}) = 16$$

$$\text{Tan}^2x + \text{Tan}^2y + \text{Tan}^2z = 14$$

$$(\text{Sec}^2x - 1) + (\text{Sec}^2y - 1) + (\text{Sec}^2z - 1) = 14$$

$$\text{Sec}^2x + \text{Sec}^2y + \text{Sec}^2z = 17$$

$$\mathbf{U = 17}$$



¡Muy bien!

HELICO-PRACTICE 20

20. Si $\text{Cos}(45° - x) = \dfrac{\sqrt{2}}{3}$, halle el valor de

$$E = \text{Sen}^3x + \text{Cos}^3x$$

Resolución:

Del dato:

$$\text{Cos}(45° - x) = \frac{\sqrt{2}}{3}$$

$$\text{Cos}45°\text{Cos}x + \text{Sen}45°\text{Sen}x = \frac{\sqrt{2}}{3}$$

$$\frac{\cancel{\sqrt{2}}}{2}\text{Cos}x + \frac{\cancel{\sqrt{2}}}{2}\text{Sen}x = \frac{\cancel{\sqrt{2}}}{3}$$

$$\text{Cos}x + \text{Sen}x = \frac{2}{3}$$

$$(E)^2 \quad (\text{Cos}x + \text{Sen}x)^2 = \left(\frac{2}{3}\right)^2$$

$$\underbrace{\text{Cos}^2x + \text{Sen}^2x}_{1} + 2\text{Sen}x\text{Cos}x = \frac{4}{9}$$

$$2\text{Sen}x\text{Cos}x = -\frac{5}{9}$$

$$\text{Sen}x\text{Cos}x = -\frac{5}{18}$$

Continuara...

HELICO-PRACTICE 20

Aquí

$$\mathrm{SenxCosx} = -\frac{5}{18}$$

Piden

$$E = \mathrm{Sen}^3x + \mathrm{Cos}^3x$$

Recuerda:

$$a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$E = \underbrace{(\mathrm{Senx} + \mathrm{Cosx})}_{\frac{2}{3}}\underbrace{(\mathrm{Sen}^2x + \mathrm{Cos}^2x}_{1} - \underbrace{\mathrm{SenxCosx})}_{-\frac{5}{18}}$$

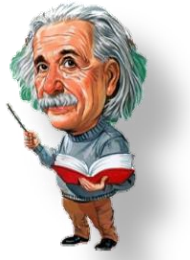
$$E = \left(\frac{2}{3}\right)\left(1 + \frac{5}{18}\right)$$

$$E = \left(\frac{\cancelto{1}{2}}{3}\right)\left(\frac{23}{\cancelto{9}{18}}\right)$$

$$E = \left(\frac{1}{3}\right)\left(\frac{23}{9}\right)$$

$$\mathbf{E = \frac{23}{27}}$$

¡Muy bien!



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