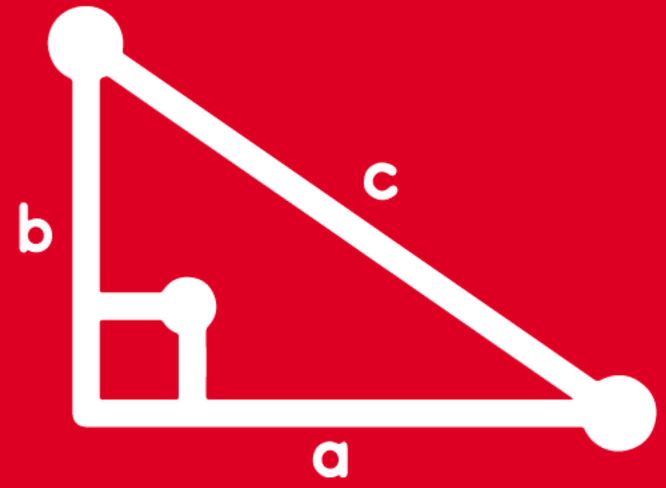
TRIGONOMETRY

Chapter 6

Identidades trigonométricas del ángulo doble, mitad y triple



TOMO 2

SACO OLIVEROS

HELICO-MOTIVACIÓN

Los **polinomios de Chebyshev** son una familia de polinomios, definidas por la relación de recurrencia :

$$\begin{cases} T_0(x)=1 \\ T_1(x)=x \\ T_n(x)=2x.T_{n-1}(x)-T_{n-2}(x)\ ;\ n>2 \end{cases}$$

Sea : $T_n(\cos\alpha)=\cos(n\alpha)$

Para : $n=0 \Rightarrow T_0(\cos\alpha)=\cos(0\alpha)=1$

$n=1 \Rightarrow T_1(\cos\alpha)=\cos(1\alpha)=\cos\alpha$

En general :

$$\cos(n\alpha)=2\cos\alpha.\cos(n-1)\alpha-\cos(n-2)\alpha$$

Para : $n=2$

$\cos(2\alpha)=2\cos\alpha.\cos(2-1)\alpha-\cos(2-2)\alpha$

$\cos(2\alpha)=2\cos\alpha.\cos\alpha-\cos0\alpha$

$\therefore \cos(2\alpha)=2\cos^2\alpha-1$

Para : $n=3$

$\cos(3\alpha)=2\cos\alpha.\cos(3-1)\alpha-\cos(3-2)\alpha$

$\cos(3\alpha)=2\cos\alpha.\cos2\alpha-\cos1\alpha$

$\cos(3\alpha)=2\cos\alpha(2\cos^2\alpha-1)-\cos\alpha$

$\therefore \cos(3\alpha)=4\cos^3\alpha-3\cos\alpha$

¿Podrías hallar formulas para $\cos(4\alpha)$, $\cos(5\alpha)$, $\cos(6\alpha)$, etc?

HELICOTEORÍA

IDENTIDADES TRIGONOMÉTRICAS DE ∢ MÚLTIPLES

IDENTIDADES DEL ÁNGULO DOBLE

IDENTIDADES FUNDAMENTALES

$$\operatorname{sen}2x = 2\operatorname{sen}x.\cos x$$

$$\cos 2x = \cos^2 x - \operatorname{sen}^2 x$$

$$\cos 2x = 1 - 2\operatorname{sen}^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} \quad \ldots (**)$$

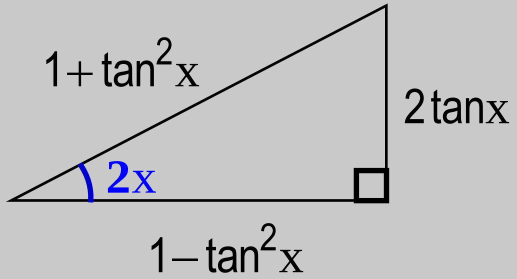
Identidades de degradación :

$$2\operatorname{sen}^2 x = 1 - \cos 2x$$

$$2\cos^2 x = 1 + \cos 2x$$

Triángulo Rectángulo del Ángulo Doble

A partir de **(**)** , tenemos :



Del gráfico, podemos obtener cualquier Razón Trigonométrica del ángulo **2x** en función de la **tanx** .

Ejemplos :

$$\operatorname{sen}2x = \frac{2\tan x}{1+\tan^2 x}$$

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

IDENTIDADES AUXILIARES

1. $\boxed{(\operatorname{sen}x+\cos x)^2=1+\operatorname{sen}2x}$

2. $\boxed{(\operatorname{sen}x-\cos x)^2=1-\operatorname{sen}2x}$

3. $\boxed{\cot x+\tan x=2\csc 2x}$

4. $\boxed{\cot x-\tan x=2\cot 2x}$

Demostración 3.

Sea : $E=\cot x+\tan x$

$$\Rightarrow E=\frac{\cos x}{\operatorname{sen}x}+\frac{\operatorname{sen}x}{\cos x}=\frac{\overbrace{\cos^2x+\operatorname{sen}^2x}}{\cos x\operatorname{sen}x}$$

$$\Rightarrow E=\frac{2.1}{2\operatorname{sen}x\cos x}=\frac{2}{\operatorname{sen}2x}=2\csc 2x$$

$\therefore \cot x+\tan x=2\csc 2x$

5. $\boxed{\sec 2x-1=\tan 2x.\tan x}$

6. $\boxed{\sec 2x+1=\tan 2x.\cot x}$

Demostración 5.

Sea : $E=\sec 2x-1$

Usando el ⊿ **Dobles** para la $\sec 2x$:

$$\Rightarrow E=\frac{1+\tan^2x}{1-\tan^2x}-1=\frac{2\tan^2x}{1-\tan^2x}$$

$$\Rightarrow E=\boxed{\frac{2\tan x}{1-\tan^2x}}.\tan x=\tan 2x.\tan x$$

$\therefore \sec 2x-1=\tan 2x.\tan x$

7. $\boxed{\operatorname{sen}^4x+\cos^4x=\frac{3}{4}+\frac{1}{4}\cos 4x}$

8. $\boxed{\operatorname{sen}^6x+\cos^6x=\frac{5}{8}+\frac{3}{8}\cos 4x}$

IDENTIDADES DEL ÁNGULO MITAD

IDENTIDADES FUNDAMENTALES

$$\operatorname{sen}\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

NOTA :

El signo $\pm$ (positivo o negativo) depende del cuadrante al que pertenece el ángulo $\frac{x}{2}$ y su Razón Trigonométrica.

Ejemplo : Si $\cos\alpha = \frac{4}{5}$; $270^\circ < \alpha < 360^\circ$

Calcular : $\tan\left(\frac{\alpha}{2}\right)$

Resolución :

Del **DATO :** $135^\circ < \frac{\alpha}{2} < 180^\circ$

IIC

$$\tan\left(\frac{x}{2}\right) = -\sqrt{\frac{1-\cos x}{1+\cos x}} = -\sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}} = -\sqrt{\frac{\frac{1}{\cancel{5}}}{\frac{9}{\cancel{5}}}}$$

$$\therefore \tan\left(\frac{x}{2}\right) = -\frac{1}{3}$$

IDENTIDADES AUXILIARES

$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x \qquad \cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

También se les llama **Fórmulas Racionalizadas**.

IDENTIDADES DEL ÁNGULO TRÍPLE

IDENTIDADES FUNDAMENTALES

$$\boxed{\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3x} \quad \ldots (\text{ I })$$

$$\boxed{\cos 3x = 4\cos^3x - 3\cos x} \quad \ldots (\text{ II })$$

$$\boxed{\tan 3x = \frac{3\tan x - \tan^3x}{1 - 3\tan^2x}}$$

Ejemplos :

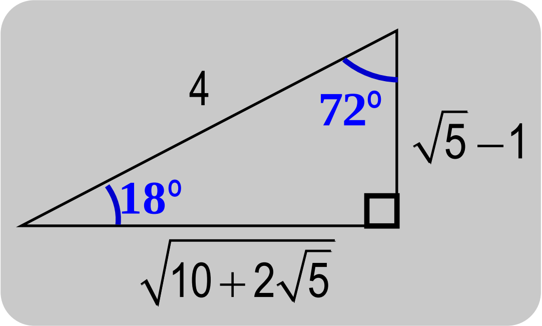
- $3\operatorname{sen}10^\circ - 4\operatorname{sen}^3 10^\circ = \operatorname{sen}30^\circ = 1/2$
- $4\cos^3 15^\circ - 3\cos 15^\circ = \cos 45^\circ = \sqrt{2}/2$

Identidades de degradación :

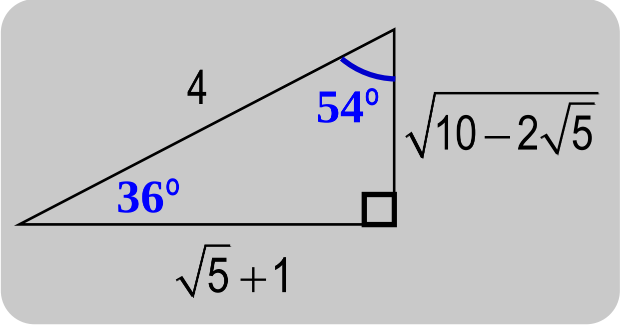
De (I) : $\boxed{4\operatorname{sen}^3x = 3\operatorname{sen}x - \operatorname{sen}3x}$

De (II) : $\boxed{4\cos^3x = 3\cos x + \cos 3x}$

$$\boxed{\operatorname{sen}18^\circ = \frac{\sqrt{5}-1}{4}}$$



$$\boxed{\cos 36^\circ = \frac{\sqrt{5}+1}{4}}$$



IDENTIDADES AUXILIARES

1. $\boxed{\operatorname{sen}3x = \operatorname{sen}x(2\cos 2x + 1)}$

2. $\boxed{\cos 3x = \cos x(2\cos 2x - 1)}$

3. $\boxed{\dfrac{\tan 3x}{\tan x} = \dfrac{2\cos 2x + 1}{2\cos 2x - 1}}$

Demostración 1.

Sea : $\operatorname{sen}3x = 3\operatorname{sen}x - 4\operatorname{sen}^3 x$

$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x(3 - 4\operatorname{sen}^2 x)$

$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2(2\operatorname{sen}^2 x)\right)$

$\Rightarrow \operatorname{sen}3x = \operatorname{sen}x\left(3 - 2(1 - \cos 2x)\right)$

$\therefore \operatorname{sen}3x = \operatorname{sen}x(2\cos 2x + 1)$

4. $\boxed{\operatorname{sen}3x = 4\operatorname{sen}x\operatorname{sen}(60^\circ - x)\operatorname{sen}(60^\circ + x)}$

5. $\boxed{\cos 3x = 4\cos x\cos(60^\circ - x)\cos(60^\circ + x)}$

6. $\boxed{\tan 3x = \tan x\tan(60^\circ - x)\tan(60^\circ + x)}$

Ejemplo :

Calcule : $E = \cos 10^\circ . \cos 50^\circ . \cos 70^\circ$

Resolución :

$\times 4 : 4E = 4\cos 10^\circ . \cos 50^\circ . \cos 70^\circ$

$\Rightarrow 4E = \underbrace{4\cos 10^\circ \cos(60^\circ - 10^\circ)\cos(60^\circ + 10^\circ)}_{\cos(3x10^\circ)}$

$\Rightarrow 4E = \cos 30^\circ$

$\Rightarrow 4E = \dfrac{\sqrt{3}}{2}$ $\qquad \therefore E = \dfrac{\sqrt{3}}{8}$

HELICO-PRACTICE 1

1. Siendo θ un ángulo agudo, tal que: $\cot\theta = 4$, calcule $\operatorname{sen}2\theta$.

A) $\frac{4}{15}$ B) $\frac{4}{17}$ C) $\frac{8}{15}$

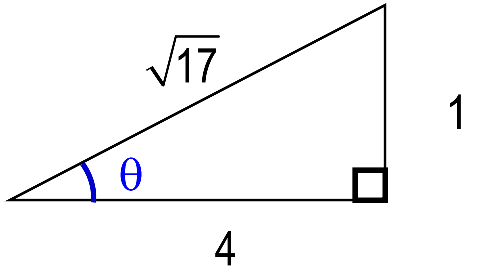
~~D)~~ $\frac{8}{17}$ E) $\frac{15}{17}$

Recordar

$\operatorname{sen}2x = 2\operatorname{sen}x.\cos x$

Resolución:

siendo: $\cot\theta = \frac{4}{1} \Rightarrow$



Nos piden : $\operatorname{sen}2\theta$

$$\operatorname{sen}2\theta = 2\operatorname{sen}\theta.\cos\theta$$

$$\operatorname{sen}2\theta = 2\left(\frac{1}{\sqrt{17}}\right).\left(\frac{4}{\sqrt{17}}\right) = \frac{8}{17}$$

$$\therefore \boxed{E = \frac{8}{17}}$$

HELICO-PRACTICE 2

2. Reduzca $D = \operatorname{sen}2x \cdot \sec x - \tan x \cdot \cos x.$

A) senx B) 2senx C) cosx
D) 2cosx E) 0

Resolución:

Tenemos :

$$D = \operatorname{sen}2x.\sec x - \tan x.\cos x$$

$$D = 2\operatorname{sen}x.\underbrace{\cos x.\sec x}_{1} - \frac{\operatorname{sen}x}{\cancel{\cos x}}.\cancel{\cos x}$$

$$D = 2\operatorname{sen}x - \operatorname{sen}x$$

$$\therefore \boxed{D = \operatorname{sen}x}$$

Recordar:

$\operatorname{sen}2x = 2\operatorname{sen}x.\cos x$

$\cos x.\sec x = 1$

$\tan x = \dfrac{\operatorname{sen}x}{\cos x}$

HELICO-PRACTICE 3

3. Si $\operatorname{sen}x + \cos x = \sqrt{\frac{1}{5}}$; calcule sen2x.

A) 0,6 B) 0,8 ~~C)~~ –0,8
D) –0,6 E) –0,4

Recordar

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\operatorname{sen}2x = 2\operatorname{sen}x.\cos x$

Resolución:

dato : $\operatorname{sen}x + \cos x = \sqrt{\frac{1}{5}}$

Elevando al cuadrado:

$$(\operatorname{sen}x + \cos x)^2 = \left(\sqrt{\frac{1}{5}}\right)^2$$

$$\underbrace{\operatorname{sen}^2 x + \cos^2 x}_{1} + \underbrace{2\operatorname{sen}x.\cos x}_{\operatorname{sen}2x} = \frac{1}{5}$$

$$\operatorname{sen}2x = \frac{1}{5} - 1 = -\frac{4}{5}$$

$$\therefore \boxed{\operatorname{sen}2x = -0.8}$$

HELICO-PRACTICE 4

4. Reduzca $Q=\sqrt{\dfrac{1-\cos 4x}{1+\cos 2x}}\left(0<x<\dfrac{\pi}{4}\right)$.

A) senx ~~B)~~ 2senx C) 3senx
D) 4senx E) sen^2x

Recordar la identidad

$$\boxed{2\text{sen}^2x=1-\cos 2x}$$

$$\boxed{2\cos^2x=1+\cos 2x}$$

$$\boxed{\text{sen}2x=2\text{sen}x.\cos x}$$

Resolución:

dato : $Q=\sqrt{\dfrac{1-\cos 4x}{1+\cos 2x}}$

Aplicando las identidades de degradación:

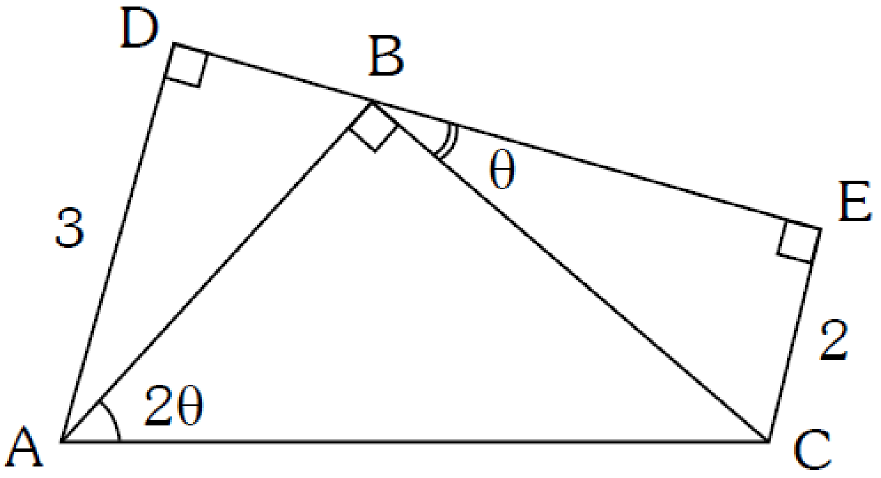
$$Q=\sqrt{\dfrac{\cancel{2}\text{sen}^2 2x}{\cancel{2}\cos^2 x}}=\dfrac{\sqrt{\text{sen}^2 2x}}{\sqrt{\cos^2 x}}=\dfrac{|\text{sen}2x|}{|\cos x|}$$

$$Q=\dfrac{\text{sen}2x}{\cos x}=\dfrac{2\text{sen}x.\cancel{\cos x}}{\cancel{\cos x}}$$

$$\therefore\ \boxed{Q=2\text{sen}x}$$

HELICO-PRACTICE 5

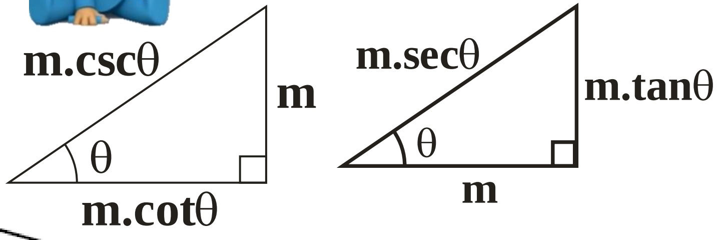
5. Del gráfico, calcule $\tan\theta$.



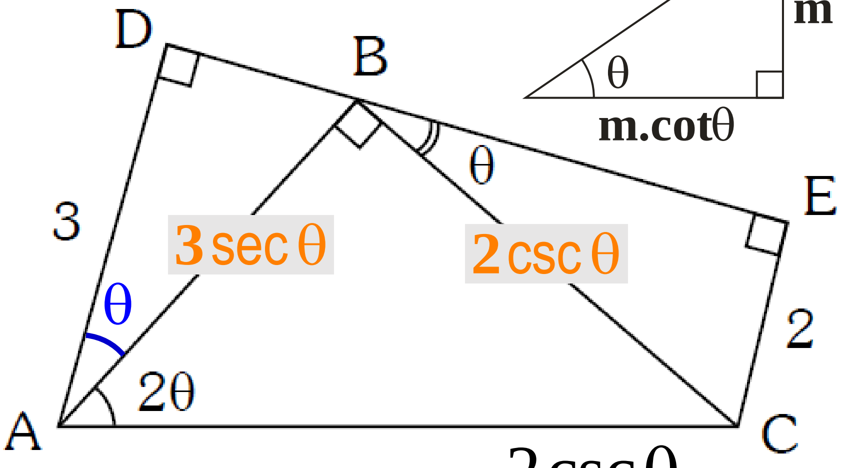
A) $\frac{1}{2}$ B) 2 C) $\frac{2}{3}$

D) $\frac{\sqrt{3}}{4}$ E) $\frac{\sqrt{2}}{4}$

Recordar



Resolución:



Del gráfico: $\tan 2\theta = \dfrac{2\csc\theta}{3\sec\theta} \Rightarrow \tan 2\theta = \dfrac{2\dfrac{1}{\operatorname{sen}\theta}}{3\dfrac{1}{\cos\theta}}$

$$\tan 2\theta = \frac{2}{3}\cot\theta \Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cot\theta}{3}$$

operando: $\therefore$ $\boxed{\tan\theta = \frac{1}{2}}$

HELICO-PRACTICE 6

6. Si $\tan x + \tan^2 x + \tan^3 x = 1$, calcule

$$C = \cos 2x + \cos^2 2x + \cos^3 2x$$

~~A)~~ 1 B) 2 C) 0
D) –1 E) –2

Resolución:

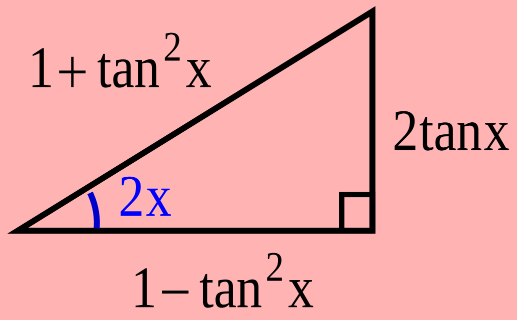
Dándole forma al dato:

$$\tan x + \tan^3 x = 1 - \tan^2 x$$

$$\tan x(1 + \tan^2 x) = 1 - \tan^2 x$$

$$\Rightarrow \tan x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

Recordar



$$\Rightarrow \tan x = \cos 2x \quad \dots(*)$$

piden : $C = \cos 2x + \cos^2 2x + \cos^3 2x$

En **(*)**: $C = \tan x + \tan^2 x + \tan^3 x$

$$\therefore \boxed{C = 1}$$

HELICO-PRACTICE 7

7. Calcule el valor de

$$K = \cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}$$

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$

~~D)~~ $-\frac{1}{8}$ E) $\frac{1}{7}$

Resolución:

piden : $P = \cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}$

$$2\operatorname{sen}\frac{\pi}{7}.P = 2\operatorname{sen}\frac{\pi}{7}.\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}$$

$$2\operatorname{sen}\frac{\pi}{7}.P = \underbrace{2\operatorname{sen}\frac{\pi}{7}.\cos\frac{\pi}{7}}_{\operatorname{sen}\frac{2\pi}{7}}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}$$

$$2.2\operatorname{sen}\frac{\pi}{7}.P = \underbrace{2.\operatorname{sen}\frac{2\pi}{7}.\cos\frac{2\pi}{7}}_{\operatorname{sen}\frac{4\pi}{7}}.\cos\frac{4\pi}{7}$$

$$2.2.2\operatorname{sen}\frac{\pi}{7}.P = \underbrace{2.\operatorname{sen}\frac{4\pi}{7}.\cos\frac{4\pi}{7}}_{\operatorname{sen}\frac{8\pi}{7}}$$

$$\operatorname{sen}\frac{8\pi}{7} = \operatorname{sen}(\pi+\frac{\pi}{7}) = -\operatorname{sen}\frac{\pi}{7}$$

$$8\cancel{\operatorname{sen}\frac{\pi}{7}}.P = -\cancel{\operatorname{sen}\frac{\pi}{7}} \Rightarrow \therefore \boxed{P = -\frac{1}{8}}$$

HELICO-PRACTICE 8

8. Si $\tan\theta = \dfrac{\sqrt{5}}{2}$ $(180° < \theta < 270°)$, calcule $\cos\dfrac{\theta}{2}$.

$$90° < \frac{\theta}{2} < 135° \Rightarrow \theta \in IIC$$

A) $-\sqrt{\dfrac{1}{3}}$ B) $-\sqrt{\dfrac{1}{4}}$ C) $-\sqrt{\dfrac{1}{5}}$

D) $-\sqrt{\dfrac{1}{6}}$ E) $\sqrt{\dfrac{1}{5}}$

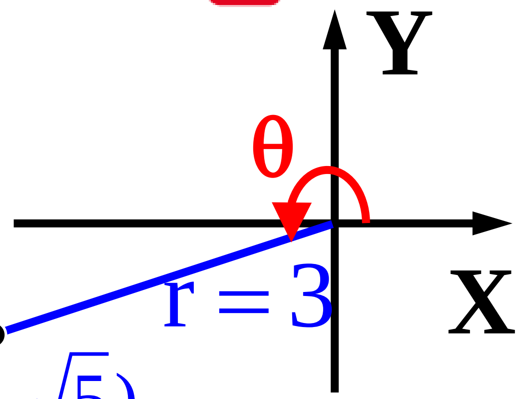
Resolución:

Recuerda:

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

Dato :

$$\tan\theta = \frac{\sqrt{5}}{2} = \frac{y}{x} \Rightarrow P(\underbrace{-2}_{x};\underbrace{-\sqrt{5}}_{y})$$



$$\cos\theta = \frac{x}{r} = \frac{-2}{3}$$

Piden :

$$\cos\left(\frac{\theta}{2}\right) = \overset{IIC}{-}\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{-2}{3}}{2}}$$

$$\therefore \cos\frac{\theta}{2} = -\sqrt{\frac{1}{6}}$$

HELICO-PRACTICE 9

9. Si $\operatorname{sen}\theta=\frac{1}{3}$, calcule $\tan\left(45^\circ-\frac{\theta}{2}\right)$.

A) $\pm\frac{1}{\sqrt{3}}$ ~~B)~~ $\pm\frac{1}{\sqrt{2}}$ C) $\pm\frac{1}{2}$

D) $\pm\frac{1}{\sqrt{5}}$ E) $\pm\frac{1}{\sqrt{6}}$

Resolución:

Recuerda:

$$\tan\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

Dato : $\operatorname{sen}\theta=\frac{1}{3}$ $\Rightarrow$ $\cos(90^\circ-\theta)=\frac{1}{3}$

Piden : $\tan(45^\circ-\frac{\theta}{2})$

$$\tan\left(\frac{90-\theta}{2}\right)=\pm\sqrt{\frac{1-\cos(90^\circ-\theta)}{1+\cos(90^\circ-\theta)}}=\pm\sqrt{\frac{1-\frac{1}{3}}{1+\frac{1}{3}}}$$

$$\therefore\ \tan\left(45^\circ-\frac{\theta}{2}\right)=\pm\frac{1}{\sqrt{2}}$$

HELICO-PRACTICE 10

10. Si $\operatorname{sen}\theta = \dfrac{a-b}{a+b}$, halle $\tan\left(45^\circ + \dfrac{\theta}{2}\right)$.

A) $\pm\sqrt{ab}$ ~~B)~~ $\pm\sqrt{\dfrac{a}{b}}$ C) $\pm\sqrt{\dfrac{b}{a}}$

D) $\pm\sqrt{\dfrac{a}{2b}}$ E) $\pm\sqrt{\dfrac{b}{2a}}$

Resolución:

Recuerda:

$$\cot\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{1-\cos x}}$$

Dato : $\operatorname{sen}\theta = \dfrac{a-b}{a+b} \Rightarrow \cos(90^\circ - \theta) = \dfrac{a-b}{a+b}$

Piden : $\tan(45^\circ + \dfrac{\theta}{2}) = \cot(45^\circ - \dfrac{\theta}{2})$

$$\cot\left(\frac{90-\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(90^\circ-\theta)}{1-\cos(90^\circ-\theta)}} = \pm\sqrt{\frac{1+\dfrac{a-b}{a+b}}{1-\dfrac{a-b}{a+b}}}$$

$$\therefore\quad \tan\left(45^\circ + \frac{\theta}{2}\right) = \pm\sqrt{\frac{a}{b}}$$

HELICO-PRACTICE 11

11. Si $\csc 2\alpha + \csc 2\beta + \csc 2\theta = \cot 2\alpha + \cot 2\beta + \cot 2\theta$

Halle $H = \dfrac{\tan^3\alpha + \tan^3\beta + \tan^3\theta}{\tan\alpha \cdot \tan\beta \cdot \tan\theta}$.

A) 1 B) 2 ~~C)~~ 3
D) –3 E) 9

Resolución:

Recordar la identidad

$$\tan\left(\frac{x}{2}\right) = \csc x - \cot x \qquad \cot\left(\frac{x}{2}\right) = \csc x + \cot x$$

si: $a + b + c = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

Dato :

$$\csc 2\alpha + \csc 2\beta + \csc 2\theta = \cot 2\alpha + \cot 2\beta + \cot 2\theta$$

$$\underbrace{\csc 2\alpha - \cot 2\alpha}_{\tan\alpha} + \underbrace{\csc 2\beta - \cot 2\beta}_{\tan\beta} + \underbrace{\csc 2\theta - \cot 2\theta}_{\tan\theta} = 0$$

Entonces :

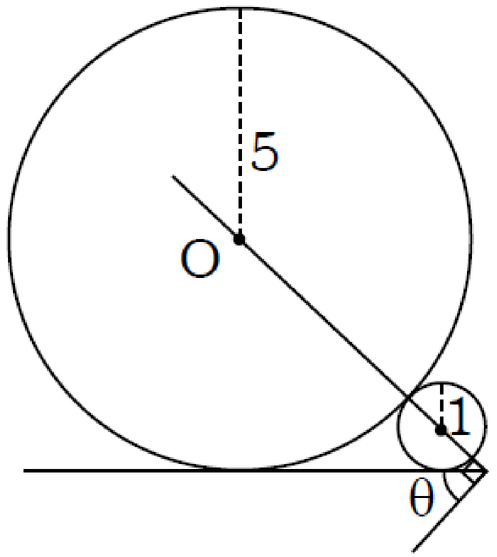
$$\tan^3\alpha + \tan^3\beta + \tan^3\theta = 3\tan\alpha\tan\beta\tan\theta$$

$$\underbrace{\frac{\tan^3\alpha + \tan^3\beta + \tan^3\theta}{\tan\alpha\tan\beta\tan\theta}}_{H} = 3$$

$\therefore \boxed{H = 3}$

HELICO-PRACTICE 12

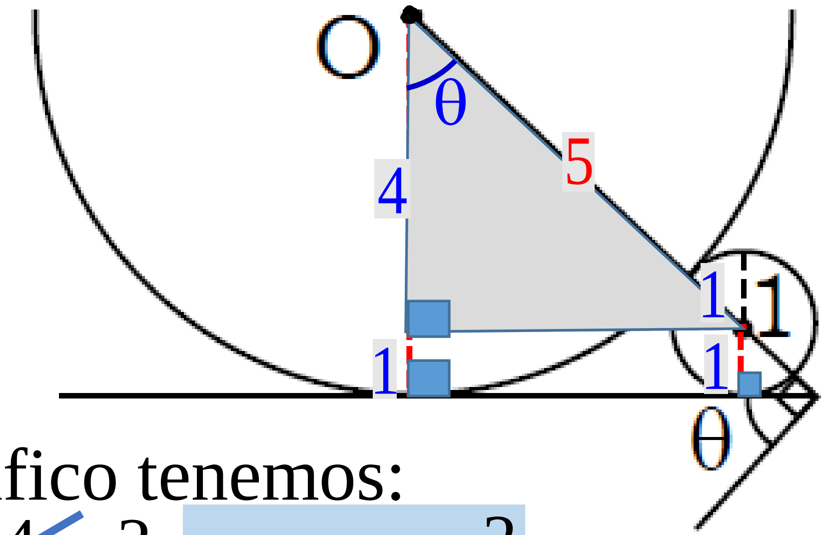
12. Del gráfico, calcule $\tan\dfrac{\theta}{2}$.



A) $\sqrt{5}$ B) $\sqrt{6}$ C) $\dfrac{\sqrt{5}}{5}$

D) $\dfrac{\sqrt{6}}{6}$ E) $\dfrac{\sqrt{6}}{5}$

Resolución:



Del gráfico tenemos:

$$\cos\theta = \cancel{\frac{4}{6}} = \frac{2}{3} \Rightarrow \boxed{\cos\theta = \frac{2}{3}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\frac{2}{3}}{1+\frac{2}{3}}} \qquad \therefore\ \boxed{\tan\frac{\theta}{2} = \frac{\sqrt{5}}{5}}$$

HELICO-PRACTICE 13

13. Reduzca

$$P=\csc 10^\circ+\csc 20^\circ+\csc 40^\circ+\csc 80^\circ+\csc 160^\circ+\cot 160^\circ$$

~~A)~~ $\cot 5^\circ$ B) $\tan 5^\circ$ C) $\sec 10^\circ$

D) $\cot 10^\circ$ E) 1

Recordar la identidad

$$\cot\left(\frac{x}{2}\right)=\csc x+\cot x$$

Resolución:

Dato :

$$P=\underbrace{\csc 10^\circ+\underbrace{\csc 20^\circ+\underbrace{\csc 40^\circ+\underbrace{\csc 80^\circ+\underbrace{\csc 160^\circ+\cot 160^\circ}_{\cot 80^\circ}}_{\cot 40^\circ}}_{\cot 20^\circ}}_{\cot 10^\circ}}_{\cot 5^\circ}$$

$$\therefore\ \boxed{P=\cot 5^\circ}$$

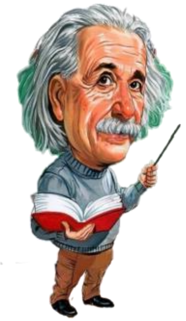
HELICO-PRACTICE 14

14. Si $\operatorname{sen}3x = n \cdot \operatorname{sen}x$, halle $S = \dfrac{\cos 3x}{\cos x}$.

A) $n-1$ B) $n+1$ ~~C)~~ $n-2$
D) $n+2$ E) $2n-1$

Resolución:

IDENTIDADES AUXILIARES



$$\operatorname{sen}3x = \operatorname{sen}x(2\cos 2x + 1)$$

$$\cos 3x = \cos x(2\cos 2x - 1)$$

Dato : $\operatorname{sen}3x = n\operatorname{sen}x$

$$\underbrace{\frac{\operatorname{sen}3x}{\operatorname{sen}x}}_{2\cos 2x+1} = n \qquad \Rightarrow 2\cos 2x = n-1$$

Piden : $S = \dfrac{\cos 3x}{\cos x} = \underbrace{2\cos 2x}_{n-1} - 1$

$$\therefore \boxed{S = n-2}$$

HELICO-PRACTICE 15

15. Reduzca $C=\dfrac{\operatorname{sen}^3 20^\circ+\cos^3 10^\circ}{\operatorname{sen}20^\circ+\cos 10^\circ}$.

A) $\frac{2}{3}$ B) $\frac{1}{4}$ C) $\frac{3}{2}$

~~D)~~ $\frac{3}{4}$ E) $\frac{4}{3}$

Resolución:

Recordar

$$4\operatorname{sen}^3 x = 3\operatorname{sen}x-\operatorname{sen}3x$$

$$4\cos^3 x = 3\cos x+\cos 3x$$

Piden reducir :

$$4.C=\frac{4.\operatorname{sen}^3 20^\circ+4.\cos^3 10^\circ}{\operatorname{sen}20^\circ+\cos 10^\circ}$$

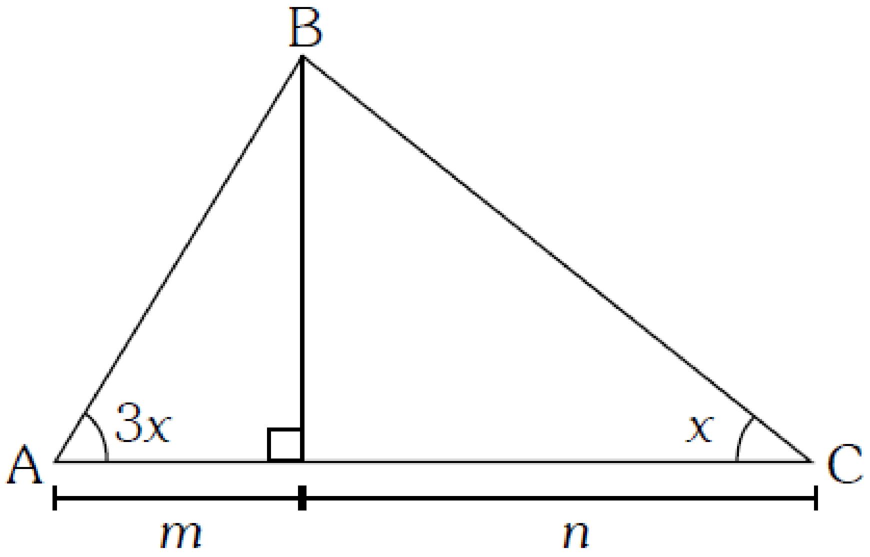
$$4C=\frac{3\operatorname{sen}20^\circ-\cancel{\operatorname{sen}60^\circ}+3\cos 10^\circ+\cancel{\cos 30^\circ}}{\operatorname{sen}20^\circ+\cos 10^\circ}$$

$$4C=\frac{3\cancel{(\operatorname{sen}20^\circ+\cos 10^\circ)}}{\cancel{\operatorname{sen}20^\circ+\cos 10^\circ}}$$

$$\therefore\ \boxed{C=\frac{3}{4}}$$

HELICO-PRACTICE 16

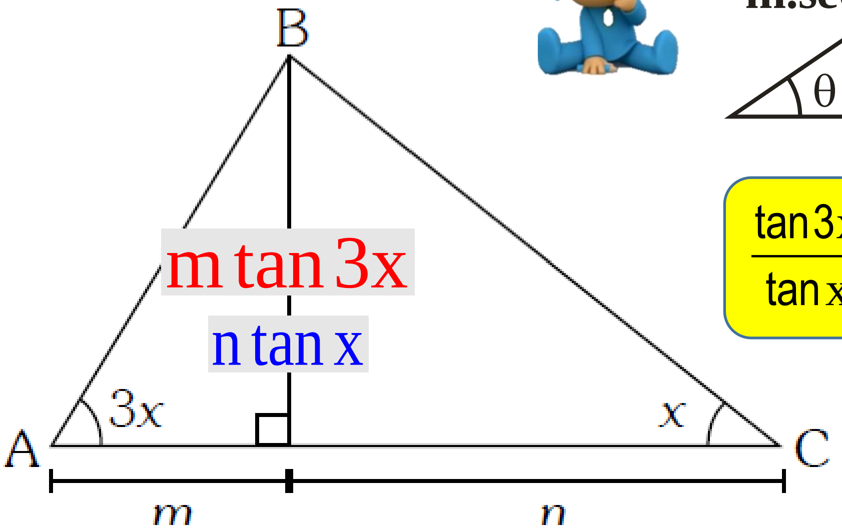
16. Del gráfico, calcule $2\cos 2x$.



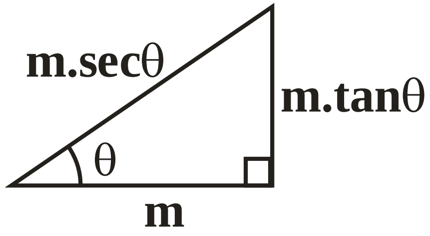
A) $\dfrac{n-m}{n+m}$ ~~B)~~ $\dfrac{n+m}{n-m}$ C) $\dfrac{n-m}{m}$

D) $\dfrac{n-m}{n}$ E) $\dfrac{n-m}{2m}$

Resolución:



Recordar



$$\frac{\tan 3x}{\tan x}=\frac{2\cos 2x+1}{2\cos 2x-1}$$

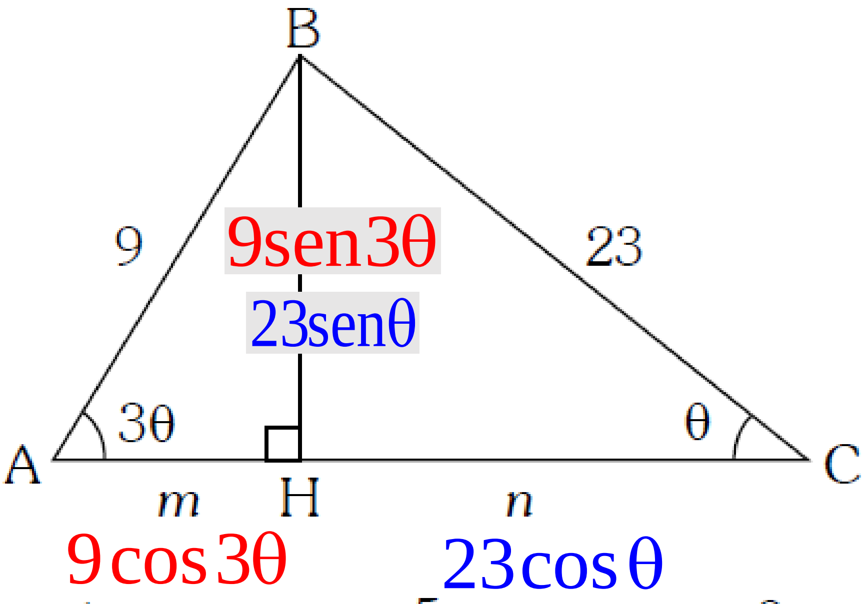
Igualando: $m\tan 3x = n\tan x$

$$\frac{\tan 3x}{\tan x}=\frac{n}{m} \Rightarrow \frac{2\cos 2x+1}{2\cos 2x-1}=\frac{n}{m}$$

$$\therefore \boxed{2\cos 2x=\frac{n+m}{n-m}}$$

HELICO-PRACTICE 17

17. En el gráfico, determine $\dfrac{m}{n}$.

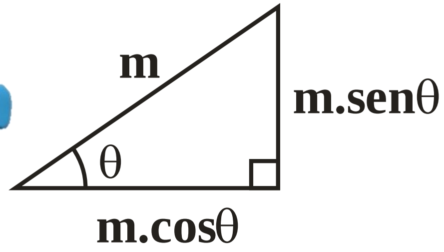


A) $\dfrac{1}{23}$ ~~B)~~ $\dfrac{5}{23}$ C) $\dfrac{9}{23}$

D) $\dfrac{3}{23}$ E) $\dfrac{7}{23}$

Resolución:

Recordar



$$\frac{\text{sen}3x}{\text{sen}x} = 2\cos 2x + 1 \qquad \frac{\cos 3x}{\cos x} = 2\cos 2x - 1$$

Igualando: $9\text{sen}3\theta = 23\text{sen}\theta$

$$\frac{\text{sen}3\theta}{\text{sen}\theta} = \frac{23}{9} \Rightarrow 2\cos 2\theta + 1 = \frac{23}{9} \Rightarrow \boxed{\cos 2\theta = \frac{7}{9}}$$

Piden : $\dfrac{m}{n} = \dfrac{9\cos 3\theta}{23\cos\theta} = \dfrac{9}{23}(2\cos 2\theta - 1)$

Reemplazando: $\therefore \dfrac{m}{n} = \dfrac{5}{23}$

HELICO-PRACTICE 18

18. Calcule el valor numérico de

$$F = \frac{1}{6\text{sen}18° \cdot \cos 36°}$$

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$

~~D)~~ $\frac{2}{3}$ E) $\frac{4}{3}$

Resolución:

Recuerda:

$$\text{sen}18° = \frac{\sqrt{5}-1}{4}$$

$$\cos 36° = \frac{\sqrt{5}+1}{4}$$

Piden :

$$F = \frac{1}{6\text{sen}18°.\cos 36°}$$

Reemplazando:

$$F = \frac{1}{6\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)} = \frac{1}{6\left(\frac{4}{16}\right)}$$

$$\therefore \boxed{F = \frac{2}{3}}$$

HELICO-PRACTICE 19

19. Reduzca

$$Q = \cos^3 x + \cos^3(60° - x) + \cos^3(60° + x) + \frac{1}{4}\cos 3x$$

Recordar la identidad

$$4\text{sen}^3 x = 3\text{sen}x - \text{sen}3x$$

$$4\cos^3 x = 3\cos x + \cos 3x$$

Resolución:

Piden : $4.Q = 4.\cos^3 x + 4.\cos^3(60° - x) + 4.\cos^3(60° + x) + \cos 3x$

$$4Q = 3\cos x + \cancel{\cos 3x} + 3\cos(60° - x) + \underbrace{\cos(180° - 3x)}_{-\cancel{\cos 3x}} + 3\cos(60° + x) + \underbrace{\cos(180° + 3x)}_{-\cancel{\cos 3x}} + \cancel{\cos 3x}$$

$$4Q = 3\cos x + 3(\underbrace{\cos(60° - x) + \cos(60° + x)}_{\cos 60°.\cos x + \cancel{\text{sen}60°.\text{sen}x} + \cos 60°.\cos x - \cancel{\text{sen}60°.\text{sen}x}})$$

$$4Q = 3\cos x + 3.2\cos 60°.\cos x \Rightarrow 4Q = 6\cos x$$

$$\therefore \boxed{Q = \frac{3}{2}\cos x}$$

HELICO-PRACTICE 20

20. Si $x=\sqrt{2-\sqrt{2+\sqrt{2+x}}}$

Calcule el valor numérico de $E=\dfrac{x^3+1}{x}$.

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $3\sqrt{3}$

D) 3 E) 6

Resolución:

Haciendo un cambio de variable:

$$x=2\cos 8\theta$$

Reemplazando:

$$2\cos 8\theta=\sqrt{2-\sqrt{2+\sqrt{2+2\cos 8\theta}}}$$

$$2\cos 8\theta=\sqrt{2-\sqrt{2+\sqrt{2\underbrace{(1+\cos 8\theta)}_{2\cos^2 4\theta}}}}$$

$$2\cos 8\theta=\sqrt{2-\sqrt{2+2\cos 4\theta}}$$

$$2\cos 8\theta=\sqrt{2-\sqrt{2\underbrace{(1+\cos 4\theta)}_{2\cos^2 2\theta}}}$$

$$2\cos 8\theta=\sqrt{2-2\cos 2\theta}$$

$$2\cos 8\theta=\sqrt{2\underbrace{(1-\cos 2\theta)}_{2\text{sen}^2\theta}} \Rightarrow \cancel{2}\cos 8\theta=\cancel{2}\text{sen}\,\theta$$

$$\cos 8\theta=\text{sen}\,\theta \quad \Rightarrow x=2\cos 80^\circ$$

$$\theta=10^\circ$$

$$\boxed{x=2\text{sen}\,10^\circ}\quad \ldots(*)$$

HELICO-PRACTICE 20

Piden : $E = \dfrac{x^3+1}{x}$

Recordar

$$4\text{sen}^3x = 3\text{sen}x - \text{sen}3x$$

En **(*)**: $E = \dfrac{(2\text{sen}10°)^3+1}{2\text{sen}10°}$

$$E = \frac{2(4\text{sen}^3 10°)+1}{2\text{sen}10°} \Rightarrow E = \frac{2(\overbrace{4\text{sen}^3 10°}^{3\text{sen}10° - \text{sen}30°})+1}{2\text{sen}10°}$$

$$E = \frac{6\text{sen}10° - \overbrace{\cancel{2\text{sen}30°}}^{1} \cancel{+1}}{2\text{sen}10°}$$

$$E = \frac{6\cancel{\text{sen}10°}}{2\cancel{\text{sen}10°}}$$

$$\therefore \quad \boxed{E = 3}$$

COLEGIOS
SACO OLIVEROS APEIRON
SISTEMA HELICOIDAL

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Tu curso amigo

TRIGONOMETRÍA