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Analysis and compensation of control valve stiction-induced limit cycles

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Abstract— **In this paper time-domain analysis is performed for limit cycles in control loops that are induced by control valve stiction, and an analytical relationship between the proportional-integral controller parameters and the oscillation amplitude and period of process output is established. Based on this relationship, a novel controller design approach is proposed to achieve the desired oscillation amplitude. Compared with the qualitative controller tuning methods in the literature, the proposed approach is quantitative, and it avoids tuning the controller parameters in a trial-and-error manner for compensation of control valve stiction. The validity of the limit cycle analysis and the effectiveness of the proposed compensation approach are illustrated with numerical examples.**

I. Introduction

Industrial surveys [1][2][3] reported that about $20-30\%$ of control loops in process industries suffer from control valve nonlinearities. Among these nonlinearities, the control valve stiction usually leads to oscillations in closed-loop systems, which has received increasing attention recently [4]. The first step to deal with control valve stiction is to detect the existence and quantify the severity of stiction. Once the stiction has been detected and quantified, a typical method for eliminating or mitigating it is to perform valve maintenance or replacement; however, this is feasible only during the plant shutdowns, which are usually scheduled every 6 months to 3 years. Consequently, it is desirable to continue the operation of the sticky control valves and to apply some compensation techniques to reduce or remove the detrimental effects of control valve stiction.

Existing compensation methods can be classified mainly into four categories, namely, the knocker method [5][6][7], the constant reinforcement method [8], the two-movement method [9][10][11], and the controller tuning method [12][13]. In particular, the controller tuning method does not change the control loop configuration, and it is relatively easy to implement in practice. However, the controller tuning method has two drawbacks: first, this method is based on the describing function approach for the limit cycle analysis, which involves a rough approximation of the control valve stiction and possibly has significant errors in calculating the oscillation amplitude and period; second, only qualitative tuning suggestions are provided on increasing or decreasing the controller parameters, and thus the controller parameters are determined in a trial-and-error manner.

The work in this paper is inspired by the time-domain limit cycle analysis of feedback systems with backlash [14] and relay [15]. However, the authors in the latter papers only consider symmetric limit cycles in the state-space description based on switched linear subsystems. In this paper we calculate the time-domain expression of the asymmetric signals in the proportional-integral-controlled feedback loop, where the sticky control valve is described by He's stiction model [16], and formulate an exact relationship between the proportional-integral (PI) controller parameters and the oscillation amplitude and period of the process output. Based on this relationship, a novel controller design approach is proposed to obtain the PI controller parameters for reducing the oscillation amplitude to a desired value. To our best knowledge, these are the first results in the literature on exact analysis of limit cycles caused by the control valve stiction, and on quantitative controller tuning methods for compensating the negative effects of control valve stiction.

The rest of this paper is organized as follows. In Section II the system under consideration is described. The limit cycle analysis is presented in Section III, followed by the discussion on the controller design in Section IV. The proposed approach is illustrated through two numerical examples in Section V. Concluding remarks are given in Section VI.

II. Problem Formulation

Consider a closed-loop system, where a plant $G(s)$ is actuated by a sticky control valve f as shown in Fig. 1. Here, $r(t)$, $e(t)$, $u(t)$, $m(t)$, $y(t)$, $w(t)$ and $y_m(t)$ denote the reference, control error, controller output, valve position, process output, process noise, and measured process output, respectively. The parameter c is a real-valued constant to take care of the static offset of $y_m(t)$ so that the noise-free process output $y(t)$ is a signal derivative from zero. The process $G(s)$ is assumed to be a first-order plus dead time (FOPDT) model,

$$G(s)=\frac{K_p}{T_p s+1}e^{-\theta s}. \quad (1)$$

It is known that this model can well approximate a higher-order linear time-invariant system [17]. A PI controller $C(s)$ is used

$$C(s)=K_c+\frac{K_i}{s}. \quad (2)$$

There are several data-driven stiction models in the literature [4], among which He's stiction model [16] is adopted

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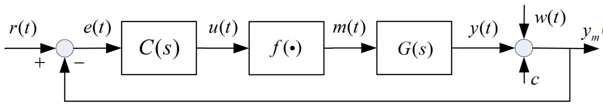

Fig. 1. A closed-loop system where the plant $G(s)$ is actuated by a sticky valve f.

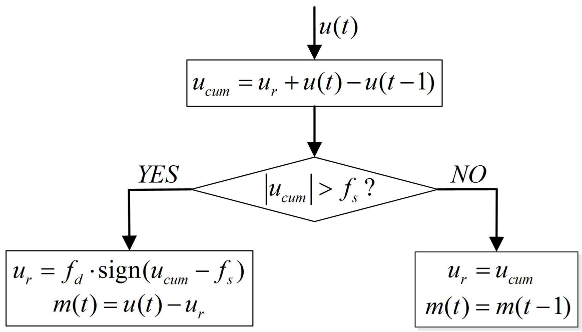

Fig. 2. Flowchart of He's stiction model (adapted from Fig. 2 of [16]).

here to describe the behaviors of the sticky control valve in oscillations due to its structural simplicity and effectiveness in modeling valve stiction [11]. The flowchart of He's stiction model is presented in Fig. 2. The two parameters f_s and f_d in Fig. 2 stand for the static and kinetic friction bands, respectively. The variable u_r is the residual force acting on the valve which has not resulted in a valve movement, and the variable u_{cum} is a current cumulative force acting on the valve.

The following assumptions are made throughout the paper:

A1. The process and sticky control valve can be described by $G(s)$ in (1) and He's stiction model in Fig. 2, respectively. The parameters of $G(s)$, namely, K_p, T_p and θ, the parameters f_s and f_d of He's stiction model, and the static offset c are known *a priori*, and the sign of the process gain K_p is positive.

A2. The reference $r(t)$ stays at a known constant value r_0. Under some non-zero initial condition, the valve stiction leads to oscillations in the control loop, in which the valve position $m(t)$ moves back and forth at two known positions.

Assumption A1 is not restrictive. In industrial processes many controlled plants can be well modelled by a FOPDT model [17], and He's stiction model is widely used to capture the sticky control valves [16][11]. With the available oscillatory measurements $\{u(t), y_m(t)\}$, all the unknown system parameters (K_p, T_p, θ, c, f_s and f_d) and the initial valve position $m_0 \triangleq m(0)$ can be well estimated using the Hammerstein model identification techniques [11][18][19][20]. The phenomenon stated in Assumption A2, namely, the oscillation appears under a constant reference, and the valve moves between two positions, has been observed from many industrial control loops [4]. These two valve positions, denoted as m_1 and m_2 (with $m_1 > m_2$), are determined only by the reference r_0 and the initial valve position m_0 from He's stiction model shown in Fig. 2 in relatively broad ranges for

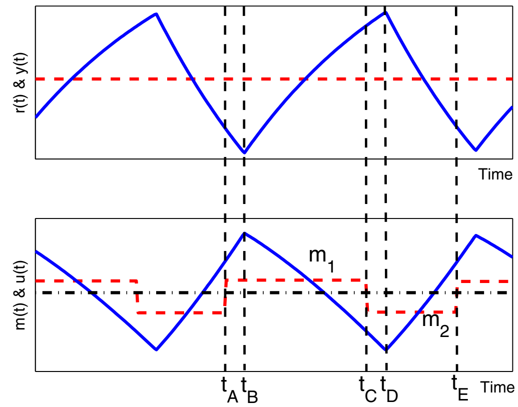

Fig. 3. Illustration of oscillatory signals in the closed-loop system: the process output $y(t)$ (top, solid), reference $r(t)$ (top, dash), controller output $u(t)$ (bottom, solid), valve position $m(t)$ (bottom, dash) and desired valve position m_{ss} (bottom, dashdot).

the PI controller parameters. That is,

$$m_1 = m_0 + iJ,\ m_2 = m_1 - J,\ i \triangleq \lfloor \frac{\frac{r_0 - c}{K_p} - m_0}{J} \rfloor + 1,\ J \triangleq f_s - f_d,$$

where the symbol $\lfloor \cdot \rfloor$ is the largest integer less than or equal to the operand. Hence, with the definition of He's stiction model in Fig. 2 and good estimation of m_0 from Hammerstein model identification, it is reliable to say that the valve position $m(t)$ moves back and forth at two known positions, even after the variation of the controller parameters for the compensation purpose.

In the next two sections we perform the time-domain analysis of limit cycle and propose a controller design method to reduce the oscillation amplitude to a desired value based on Assumptions A1-A2.

III. Limit Cycle Analysis

In this section time-domain limit cycle analysis is conducted for the closed-loop system in Fig. 1. In order to ease the presentation, the process noise $w(t)$ is assumed to be absent for the time being.

A. Main Idea

If the control valve has no stiction, the valve position $m(t)$, under a constant reference r_0, can reach its steady value m_{ss} defined as

$$m_{ss} \triangleq (r_0 - c)/K_p. \tag{3}$$

However, $m(t)$ cannot accurately arrive at m_{ss} when the stiction is present; instead, $m(t)$ jumps around m_{ss} in a periodic manner, leading to oscillations in the control loop. Since the static offset c always appears together with r_0, we denote $r_{ss} \triangleq r_0 - c$ as the nominal reference for the control loop with zero offset, and use $y(t)$ to discuss the oscillation amplitude instead of $y_m(t)$.

When the oscillation appears, the system signals evolve as illustrated in Fig. 3. During one time period $[t_A, t_E]$, the valve jumps up at the time instant t_A from the lower position m_2 to the upper position m_1, and holds until t_C when it moves back to m_2. At t_E, $m(t)$ experiences an upward jump again. Define two time instants $t_B = t_A + \theta$ and $t_D = t_C + \theta$, and two (unknown) time intervals $T_1 = t_C - t_A$ and $T_2 = t_E - t_C$, where θ is the time delay of the FOPDT model (1). Assume that $T_1 > \theta$ and $T_2 > \theta$. When the control valve moves, the relationship between $u(t)$ and $m(t)$ is

$$m(t) = u(t) \pm f_d, \tag{4}$$

where the summation (subtraction) sign is applied if $u(t)$ is decreasing (increasing).

The key objective of limit cycle analysis is to compute T_1 and T_2, which characterize the limit cycle. The main idea of limit cycle analysis is stated as follows. The first step is to utilize the step response of the FOPDT model (1) to establish the relationship between the oscillation amplitude and period of the process output. Then, the character of the valve motion (4) is exploited to formulate two equations for the unknown period parameters T_1 and T_2. Furthermore, the Newton-Raphson method [21] is utilized to solve for T_1 and T_2. Finally, the special case of a symmetric limit cycle is discussed to have a deeper insight into the relationship between the oscillation amplitude and period.

B. Analysis of Process Output $y(t)$

The variation of the process output $y(t)$ during one oscillation period is analyzed in this section. Separate $G(s)$ as a time delay $e^{-\theta s}$ followed by $G_0(s) = K_p/(T_p s + 1)$. Then, the subsystem $G_0(s)$ of $G(s)$ takes the constant m_2 as the input during $[t_A, t_B]$. The process output $y(t)$, which is the same as the output of $G_0(s)$, is

$$y(t) = K_p m_2 + [y(t_A) - K_p m_2]\, e^{-(t-t_A)/T_p}. \tag{5}$$

Thus, $y(t)$ at $t_B = t_A + \theta$ is,

$$y(t_B) = K_p m_2 + [y(t_A) - K_p m_2]\, e^{-\theta/T_p}. \tag{6}$$

Because the input to $G_0(s)$ is switched from the minimum value m_2 to the maximum value m_1 at t_B, the process output $y(t)$ is minimized at this moment, namely, $y_{\min} = y(t_B)$. Similarly, $y_{\max} = y(t_D)$. Rewrite (6) to let $y(t_B)$ be the independent variable, i.e.,

$$y(t_A) = K_p m_2 + [y(t_B) - K_p m_2]\, e^{\theta/T_p}. \tag{7}$$

Similarly, the subsystem $G_0(s)$ encounters a constant input m_1 in $[t_B, t_C]$, then

$$y(t_C) = K_p m_1 + [y(t_B) - K_p m_1]\, e^{-(T_1-\theta)/T_p}. \tag{8}$$

Moreover, the process output $y(t)$ at the time instants t_D and t_E is respectively

$$y(t_D) = K_p m_1 + [y(t_C) - K_p m_1]\, e^{-\theta/T_p}, \tag{9}$$

$$y(t_E) = K_p m_2 + [y(t_D) - K_p m_2]\, e^{-(T_2-\theta)/T_p}. \tag{10}$$

The existence of a limit cycle requires $y(t_A) = y(t_E)$. This condition, together with (7) and (10), results in

$$[y(t_B) - K_p m_2]\, e^{\theta/T_p} = [y(t_D) - K_p m_2]\, e^{-(T_2-\theta)/T_p}. \tag{11}$$

Substituting (8) and (9) into (11) to eliminate $y(t_D)$, we can obtain

$$y(t_B) = K_p \frac{m_2 + J e^{-T_2/T_p} - m_1 e^{-(T_1+T_2)/T_p}}{1 - e^{-(T_1+T_2)/T_p}}, \tag{12}$$

where $J \triangleq m_1 - m_2 = f_s - f_d$ is the jump amplitude when the valve moves. The oscillation amplitude of the process output $y(t)$ can be defined as

$$\begin{aligned} Y(T_1, T_2) &\triangleq y_{\max} - y_{\min} \\ &= y(t_D) - y(t_B) \\ &= K_p m_1 + [y(t_B) - K_p m_1]\, e^{-T_1/T_p} - y(t_B) \\ &= [K_p m_1 - y(t_B)] \left(1 - e^{-T_1/T_p}\right). \end{aligned} \tag{13}$$

With $y(t_B)$ in (12), we can rewrite (13) as

$$Y(T_1, T_2) = K_p J \frac{\left(1 - e^{-T_1/T_p}\right)\left(1 - e^{-T_2/T_p}\right)}{1 - e^{-(T_1+T_2)/T_p}}. \tag{14}$$

C. Analysis of Controller Output $u(t)$

Since two unknown period parameters T_1 and T_2 exist in (14), we consider the characteristics of the controller output $u(t)$ and formulate two equalities on T_1 and T_2 here.

At the time instants t_A and t_C, the controller output $u(t)$ and valve position $m(t)$ have the relationship (4), namely,

$$u(t_A) = m_1 + f_d, \tag{15}$$

$$u(t_C) = m_2 - f_d. \tag{16}$$

Because the contribution of the integrator in the PI controller (2) at an arbitrary time t_X is $K_i \int_{-\infty}^{t_X} e(\tau)\mathrm{d}\tau = u(t_X) - K_c e(t_X)$, and the control error is $e(t) = r_{ss} - y(t)$, the controller output $u(t)$ is, for $t \in [t_A, t_B]$,

$$\begin{aligned} u(t) &= K_c e(t) + u(t_A) - K_c e(t_A) + K_i \int_{t_A}^{t} e(\tau)\mathrm{d}\tau \\ &= u(t_A) + K_c [y(t_A) - y(t)] + K_i r_{ss}(t - t_A) - K_i \int_{t_A}^{t} y(\tau)\mathrm{d}\tau. \end{aligned} \tag{17}$$

Substituting (5) into (17) and calculating the integral term until $t = t_B$, one obtains

$$u(t_B) = u(t_A) + (K_c - K_i T_p)[y(t_A) - y(t_B)] + K_i (r_{ss} - K_p m_2)\theta. \tag{18}$$

Similarly, the controller output $u(t)$ at the time instants t_C, t_D and t_E is respectively

$$\begin{aligned} u(t_C) = u(t_B) &+ (K_c - K_i T_p)[y(t_B) - y(t_C)] \\ &+ K_i (r_{ss} - K_p m_1)(T_1 - \theta), \end{aligned} \tag{19}$$

$$\begin{aligned} u(t_D) = u(t_C) &+ (K_c - K_i T_p)[y(t_C) - y(t_D)] \\ &+ K_i (r_{ss} - K_p m_1)\theta, \end{aligned} \tag{20}$$

$$\begin{aligned} u(t_E) = u(t_D) &+ (K_c - K_i T_p)[y(t_D) - y(t_E)] \\ &+ K_i (r_{ss} - K_p m_2)(T_2 - \theta). \end{aligned} \tag{21}$$

With the conditions of the limit cycle, $u(t_A) = u(t_E)$ and $y(t_A) = y(t_E)$, and the definition $m_{ss} = r_{ss}/K_p$ in (3), the summation of both sides of (18) – (21) yields

$$H_1(T_1, T_2) \triangleq m_1 T_1 + m_2 T_2 - m_{ss}(T_1 + T_2) = 0, \tag{22}$$

which indicates that the desired valve position m_{ss} is the time average of two actual valve positions m_1 and m_2.

The value $u(t_C)$ in (16) can be represented using (18) and (19) as

$$\begin{aligned} u(t_C) &= m_2 - f_d \\ &= u(t_A) + (K_c - K_i T_p)\left[y(t_A) - y(t_C)\right] + K_i r_{ss} T_1 \\ &\quad - K_i K_p \left[m_2 \theta + m_1 (T_1 - \theta)\right]. \end{aligned}$$

Replacing $y(t_A)$ and $y(t_C)$ in the above equation with (7) and (8) in terms of $y(t_B)$, respectively, and considering (12), (15) and $J = m_1 - m_2$ ultimately give (23) (see the top of next page).

D. Solving for the Period Parameters of the Limit Cycle

Given He's stiction model parameters, f_s, f_d, m_1 and m_2, the process model parameters, K_p, T_p, θ, the controller parameters, K_c and K_i, the reference r_0, and the static offset c, Eqns. (22) and (23) yield two equations for two unknowns T_1 and T_2. Since these are nonlinear equations, the Newton-Raphson method [21] is utilized to solve for (T_1, T_2). Define $\Phi \triangleq [T_1, T_2]^T$ and

$$H(\Phi) \triangleq [H_1(\Phi), H_2(\Phi)]^T = [0, 0]^T.$$

A solution Φ^* can be obtained using the iterative formula

$$\Phi^{i+1} = \Phi^i - J_a^{-1}(\Phi^i) H(\Phi^i), \tag{24}$$

where $J_a(\Phi)$ is Jacobian matrix of $H(\Phi)$. Note that the solution Φ^* completely characterizes the behavior of the limit cycle.

E. Special Case of Symmetric Limit Cycle

In this section we discuss the special case of a symmetric limit cycle, where $T_1 = T_2 \triangleq h$, to gain a deeper insight into (14). The symmetric limit cycles have been observed in some industrial control loops [4]. In this case, the oscillation amplitude (14) can be rewritten as

$$Y(h) = K_p J \frac{1 - e^{-h/T_p}}{1 + e^{-h/T_p}} = K_p J \tanh\left(\frac{h}{2T_p}\right), \tag{25}$$

which is a monotonically increasing function of h. This intuitively reveals that an increment of the oscillation frequency implies the reduction of the oscillation amplitude.

Other interesting conclusions can also be drawn from (25). First, the oscillation amplitude Y is proportional to J, which has the potential in estimating the stiction severity. That is, with the amplitude Y and the period parameters T_1 and T_2 easily read from the measurement of $y_m(t)$, the stiction parameter $J = f_s - f_d$ can be estimated from (25) if the gain K_p of the process model (1) is known *a priori*. Second, there exists a maximum amplitude for $y(t)$ if the oscillation period is sufficiently long, namely, $Y_{\max} \approx K_p J$ when $h \gg 2T_p$. This can be understood as follows. Because h is sufficiently large, $G_0(s)$ has reached its steady states for the two constant inputs m_1 and m_2 before the next jump; then, the difference between the two steady-state values is $K_p(m_1 - m_2) = K_p J$. Third, the relationship between the oscillation amplitude and period can be well approximated by a straight line $Y(h) = K_p J h/(2T_p)$ if $h/(2T_p) < 2$. This quantitatively illustrates the price of the compensation in practice: the frequency of oscillation is doubled if the amplitude is expected to be reduced by half.

IV. Controller Design

The objective of controller design is to reduce the amplitude of oscillatory $y(t)$ to a desired value Y_d to meet the predetermined control loop performance by only adjusting the PI controller parameters.

Recall that Assumption A2 states that the valve moves back and forth at only two positions m_1 and m_2 in the oscillation. This requires $u(t) \in (m_2 - J, m_1 + J),\ t \in [t_A, t_E]$ even after the tuning of the controller parameters. Note also that the process output $y(t)$ is minimized and maximized at the time instants t_B and t_D, respectively. Due to the existence of the integrator in the PI controller, the maximum value of $u(t)$ may not be accessed at t_B; instead, it could be reached in (t_B, t_D). The controller output $u(t)$ in the time interval $[t_B, t_D]$ can be expressed as

$$\begin{aligned} u(t) = \ & u(t_B) + (K_c - K_i T_p)\left[y(t_B) - y(t)\right] \\ & + K_i (r_{ss} - K_p m_1)(t - t_B). \end{aligned}$$

Therefore, the derivative of $u(t)$ with respect to t is

$$\dot{u}(t) = -(K_c - K_i T_p)\dot{y}(t) + K_i (r_{ss} - K_p m_1). \tag{26}$$

Note that in the time interval $[t_B, t_D]$

$$y(t) = K_p m_1 + \left[y(t_B) - K_p m_1\right] e^{-(t - t_B)/T_p}.$$

Substituting the derivative of $y(t)$ into (26) and letting $\dot{u}(t_1) = 0$ yield

$$t_1 = t_B + T_p \ln \frac{(K_c - K_i T_p)\left[y(t_B) - K_p m_1\right]}{K_i T_p (K_p m_1 - r_{ss})},$$

where t_1 corresponds to the time instant when $u(t)$ reaches its maximum value. Correspondingly, the process output $y(t)$ at t_1 becomes

$$y(t_1) = \frac{K_c K_p m_1 - K_i T_p r_{ss}}{K_c - K_i T_p}.$$

Thus, the upper threshold of $u(t)$ is limited to

$$\begin{aligned} u(t_1) = \ & u(t_B) + (K_c - K_i T_p) y(t_B) - K_c K_p m_1 + K_i T_p r_{ss} \\ & + K_i T_p (r_{ss} - K_p m_1) \ln \frac{(K_c - K_i T_p)\left[y(t_B) - K_p m_1\right]}{K_i T_p (K_p m_1 - r_{ss})} \\ < \ & m_1 + J. \end{aligned} \tag{27}$$

Note that in (27), it is clear that $y(t_B) - K_p m_1 < 0$, $K_p m_1 - r_{ss} > 0$; hence, the definition of the natural logarithm requires

$$K_c - K_i T_p < 0, \tag{28}$$

which is natural since the integral time constant $T_i \triangleq K_c/K_i$ is usually expected to be less than the time constant T_p of the process to accelerate the systematic dynamics. Analogous to

$$H_2(T_1,T_2) \triangleq f_s + f_d - K_iK_p(m_1T_1 - \theta J) + K_i r_{ss} T_1 + (K_c - K_iT_p)K_pJ\left[\frac{e^{\frac{\theta}{T_p}}}{1-e^{-\frac{T_1+T_2}{T_p}}}\left(e^{-\frac{T_1}{T_p}} + e^{-\frac{T_2}{T_p}} - 2e^{-\frac{T_1+T_2}{T_p}}\right) - 1\right] = 0. \tag{23}$$

the analysis of $u(t_1)$, the lower threshold of $u(t)$ is obtained as

$$\begin{aligned} u(t_2) &= u(t_D) + (K_c - K_iT_p)y(t_D) - K_cK_pm_2 + K_iT_pr_{ss} \\ &\quad + K_iT_p(r_{ss} - K_pm_2)\ln\frac{(K_c - K_iT_p)\left[y(t_D) - K_pm_2\right]}{K_iT_p(K_pm_2 - r_{ss})} \\ &> m_2 - J. \end{aligned} \tag{29}$$

Based on the limit cycle analysis in the last section and the inequality conditions established here, the objective of controller design is to find the controller parameters K_c and K_i, such that $Y(T_1,T_2)$ is equal to a desired oscillation amplitude Y_d, while meeting the following constraints:

$$\begin{aligned} \text{Equalities:}\ & (14),\ (22),\ (23), \\ \text{Inequalities:}\ & (27),\ (28),\ (29),\ K_c > 0,\ K_i > 0. \end{aligned}$$

With He's stiction model parameters, f_s, f_d, m_1 and m_2, the process model parameters, K_p, T_p and θ, the reference r_0 and the static offset c, the steps of our proposed controller design to achieve $Y(T_1,T_2) = Y_d$ are summarized as follows.

1) Calculate the expected period parameters T_1 and T_2 corresponding to the desired amplitude Y_d from (14) and (22).
2) Formulate the curve (23) that the controller parameters K_c and K_i need to satisfy.
3) Check the inequality conditions (27), (28) and (29) for the validity of Assumption A2, and obtain the feasible region for (K_c,K_i).

V. Examples

Two simulated examples are provided to illustrate the effectiveness of the limit cycle analysis and proposed controller design method.

Example 1. A closed-loop simulation experiment is performed in the same configuration as an experimental study for the closed-loop control valve stiction compensation in [11]. That is, the PI controller and the process model are respectively

$$C(s) = 0.25\left(1 + \frac{1}{50}\right), \quad G(s) = \frac{3.8163}{156.46s+1}e^{-2.5s}.$$

The static offset is $c = -192.3411$, the parameters of He's stiction model are $f_s = 8.4$ and $f_d = 3.5243$, and the sampling period is 0.5 sec. These parameters are reported to be capable of capturing a water tank level control system actuated by a sticky control valve. The process noise $w(t)$ is set as a Gaussian white noise with the variance 0.5 passing a discrete low-pass filter $F(z) = 1/(z-0.5)$. The reference $r(t)$ is kept at a constant value $r_0 = 35$.

First, the oscillatory signals are collected from the simulation and plotted in Fig. 4. From this figure, the parameters

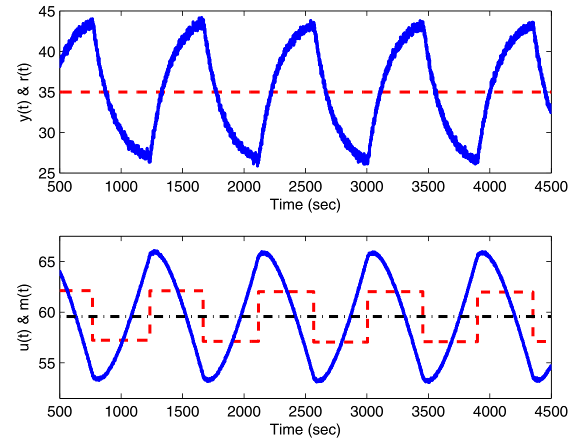

Fig. 4. Signals in Example 1 before the compensation: the measured process output $y_m(t)$ (top, solid), reference $r(t)$ (top, dash), controller output $u(t)$ (bottom, solid), valve position $m(t)$ (bottom, dash) and desired valve position m_{ss} (bottom, dashdot).

of oscillation are determined as $\hat{T}_1 = 449.5$ sec, $\hat{T}_2 = 435.5$ sec, and $\hat{Y} = 16.5865$. Moreover, the two valve positions are $\hat{m}_1 = 62.0190$ and $\hat{m}_2 = 57.0491$. Note that the limit cycle is slightly asymmetric.

Second, the Newton-Raphson method (24) is used to solve for the period parameters T_1 and T_2 of the limit cycle. In this case, the two valve positions m_1 and m_2 are assumed to be known *a priori*, namely, $m_1 = 62.0190$ and $m_2 = 57.0491$. The Newton-Raphson method gives the numerical solution as $\check{T}_1 = 448.0877$ sec and $\check{T}_2 = 434.9368$ sec. Correspondingly, the amplitude of oscillation in (14) is calculated as $\check{Y} = 16.5155$. The slight discrepancies between the simulation results and the numerical solutions for the oscillation period parameters are primarily attributed to the error in reading off $\hat{Y}$ from the noisy simulation trajectories. These results support the limit cycle analysis and prove the effectiveness of Newton-Raphson method in solving for the period parameters T_1 and T_2.

Example 2. The compensation objective is to reduce the oscillation amplitude by half through tuning the controller parameters, that is, $Y_d = \hat{Y}/2 = 8.2933$. First, the expected period parameters T_1 and T_2 are obtained from solving (14) and (22) with the Newton-Raphson method, $\bar{T}_1 = 152.2604$ sec and $\bar{T}_2 = 147.7918$ sec. Then, the desired controller parameters (K_c,K_i) are determined by (23), with the constraints (27), (28) and (29). The final feasible region for (K_c,K_i) is shown in Fig. 5. Here, the middle point (marked as $*$ in Fig. 5) of the feasible region, $\bar{K}_c = 1.1137$ and $\bar{K}_i = 0.0249$, is chosen for the compensation without consideration of other factors such as robustness and process noise.

The compensation is implemented at $t = 4500$ sec, and

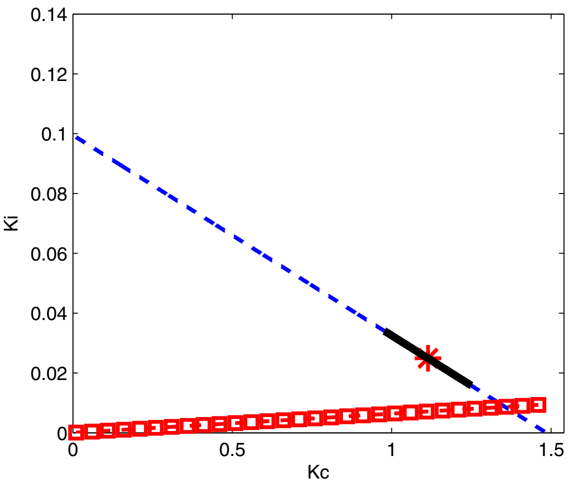

Fig. 5. Controller design in Example 2: the function in (23) (dash), the inequality in (28) (square-dash), the final feasible region satisfying (27) and (29) (thick solid), and the selected controller parameter pair (K_c, K_i) $(*)$.

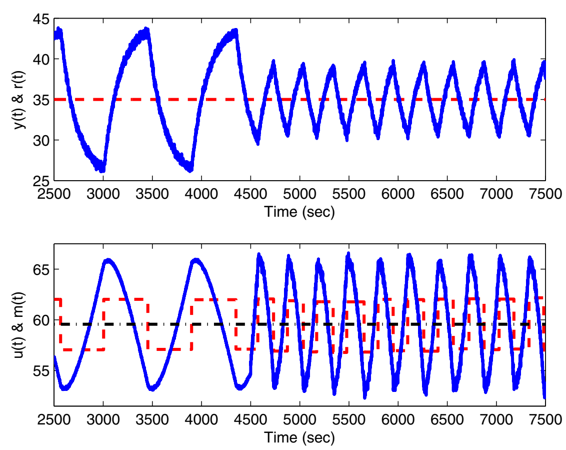

Fig. 6. Signals in Example 2 after the compensation: $y_m(t)$ (top, solid), $r(t)$ (top, dash), $u(t)$ (bottom, solid), $m(t)$ (bottom, dash) and m_{ss} (bottom, dashdot).

the signals after compensation are exhibited in Fig. 6. The amplitude and the period after the compensation become 8.3009 and 305.5 sec, respectively. Note that the amplitude reduction from 16.5865 to 8.3009 is achieved at the cost of period decrement from 895 sec to 305.5 sec. This example demonstrates the effectiveness of our proposed controller design method.

Finally, we compare the proposed compensation method against the controller tuning method in [13]. In the present circumstance, the latter only suggests qualitatively to reduce the integral effect K_c in the controller $C(s)$. As a result, the parameter pair (K_c, K_i) has to be determined in a trial-and-error manner. By contrast, the proposed method directly gives the desired controller parameters.

VI. Conclusions

In this paper time-domain limit cycle analysis was provided for control loops with sticky control valves described by He's stiction model. Based on this analysis, a novel PI controller tuning approach was proposed to reduce the oscillation amplitude to a desired value. Compared with the qualitative-type controller tuning methods in the literature, the proposed approach is quantitative, and directly gives the desired control parameters, which can significantly improve the efficiency in compensating control valve stiction.

For future work, robustness to model uncertainty and noises will be incorporated into the controller design in determining the optimal controller parameters (K_c, K_i). Experimental validation of the proposed analysis and controller design method will also be pursued.

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