Bits, Bytes and Integers – Part 1

CS2011 Introduction to Computer Systems (Fall 2022)
Lecture 3

Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**

An Amazing & Successful Abstraction.

(which we won't dig into in 2011)

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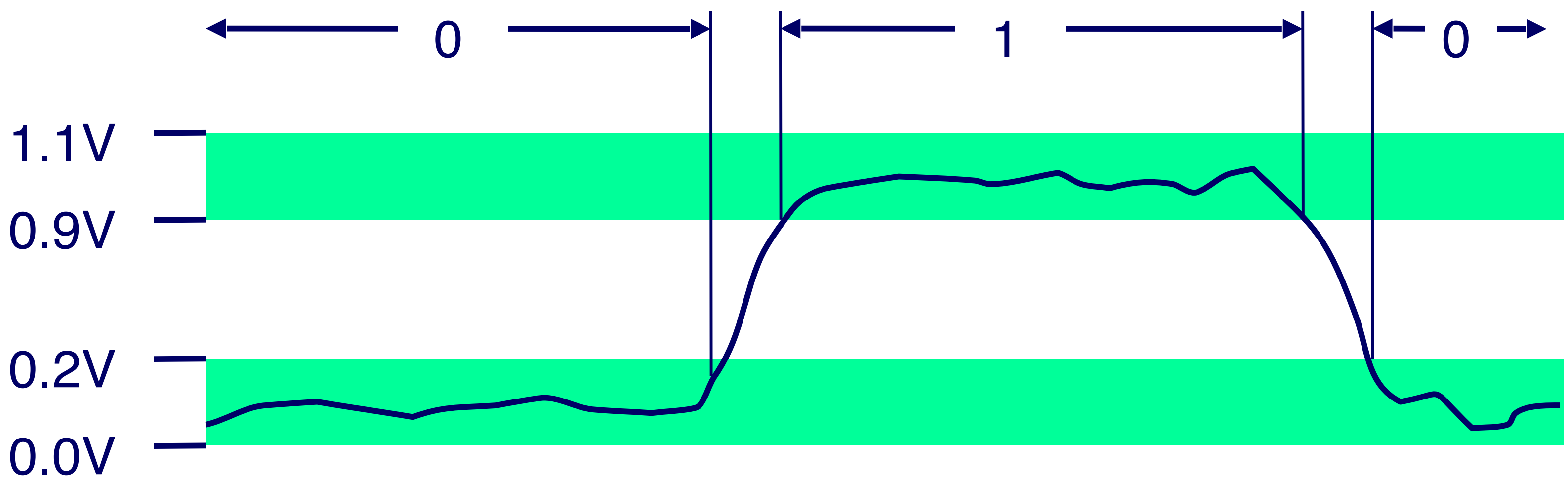
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



For example, can count in binary

Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Numbering Systems

- **Decimal** numbering system
  - RA decimal digit can have 10 possible values: {0,1,2,3,4,5,6,7,8,9}
  - E.g., 180
  - Base-10
- **A digit in a base-n numbering system can have n possible values:**
  - **Binary**: **Base-2**
    - A digit can have 2 possible values: {0,1}
    - E.g., 1011
  - **Hexadecimal**: **Base-16**
    - A digit can have how many possible values?
    - 16 possible values: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, **A, B, C, D, E, F**}

Numbering Systems (Conversions)

Dec	Hex	Binary 8421	Dec	Hex	Binary 8421
0	0	0000	8	8	1000
1	1	0001	9	9	1001
2	2	00***10***	10	A	1010
3	3	00***11***	11	B	1011
4	4	0***100***	12	C	1100
5	**5**	**0101**	13	D	1101
6	6	0110	14	E	1110
7	7	0111	15	F	1111

- 1 hex = 4 bits (16 possible numbers)
- 1 byte = 2 hex = 8 bits (256 possible numbers)

Binary to Decimal Conversion

	Place Value								
	2^7 or 128	2^6 or 64	2^5 or 32	2^4 or 16	2^3 or 8	2^2 or 4	2^1 or 2	2^0 or 1	
10011001 shown as	1	0	0	1	1	0	0	1	= 153
00101011 shown as	0	0	1	0	1	0	1	1	= 43

Figure 1-34. Binary system.

- Examples:

  10011001 (binary) = 153 (decimal)
  00101011 (binary) = 43 (decimal)

Encoding Byte Values

- **Byte = 8 bits**
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters '0' to '9' and 'A' to 'F'
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

15213: $\underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

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- **Bit-level manipulations**
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Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode “True” as 1 and “False” as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- **All of the Properties of Boolean Algebra Apply**

Common Uses: Representing & Manipulating Sets

Representation

- Width w bit vector represents subsets of {0, …, w–1}
- $a_j = 1$ if $j \in A$
    - 01101001 { 0, 3, 5, 6 } —> 1s here means 0, 3, 5, and 6 are in the set
    - *76543210*
    - 01010101 { 0, 2, 4, 6 } —> 1s here means 0, 2, 4, and 6 are in the set
    - *76543210*

Operations

- & Intersection 01000001 { 0, 6 }
- | Union 01111101 { 0, 2, 3, 4, 5, 6 }
- ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
- ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any "integral" data type
    - `long`, `int`, `short`, `char`, `unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - `~0x41` →
  - `~0x00` →
  - `0x69 & 0x55` →
  - `0x69 | 0x55` →

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
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Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - `~0x41 → 0xBE`
    - $\sim 0100\ 0001_2 \rightarrow 1011\ 1110_2$
  - `~0x00 → 0xFF`
    - $\sim 0000\ 0000_2 \rightarrow 1111\ 1111_2$
  - `0x69 & 0x55 → 0x41`
    - $0110\ 1001_2\ \&\ 0101\ 0101_2 \rightarrow 0100\ 0001_2$
  - `0x69 | 0x55 → 0x7D`
    - $0110\ 1001_2\ |\ 0101\ 0101_2 \rightarrow 0111\ 1101_2$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
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C	12	1100
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Contrast: Logic Operations in C

- **Contrast to Bit-Level Operators**
  - **Logic Operations: &&, ||, !**
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - `!0x41` → `0x00`
  - `!0x00` → `0x01`
  - `!!0x41` → `0x01`
  - `0x69 && 0x55` → `0x01`
  - `0x69 || 0x55` → `0x01`
  - `p && *p` (avoids null pointer access)

Watch out for && vs. & (and || vs. |)...
Super common C programming pitfall!

Shift Operations

- **Left Shift: `x << y`**
  - Shift bit-vector **x** left **y** positions
    - – Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift: `x >> y`**
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 (e.g., -1), or ≥ word size (e.g., 32, 64, or even 100)
  - Result is not same across different C compilers

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

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Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- **C does not mandate using two's complement**
  - But, most machines do, and we will assume so
- **C `short` 2 bytes long**

	Decimal	Hex	Binary
`x`	15213	`3B 6D`	`00111011 01101101`
`y`	-15213	`C4 93`	`11000100 10010011`

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Two's complement (Origin)

Sign bit only	Decimal	One's Complement	Decimal	Two's Complement	Decimal
1111	-7	1000	-7	**1000**	**-8**
1110	-6	1001	-6	1001	-7
1101	-5	1010	-5	1010	-6
1100	-4	1011	-4	1011	-5
1011	-3	1100	-3	1100	-4
1010	-2	1101	-2	1101	-3
1001	-1	1110	-1	1110	-2
1000	**-0**	**1111**	**-0**	**1111**	**-1**
0000	**0**	**0000**	**0**	**0000**	**0**
0001	1	0001	1	0001	1
0010	2	0010	2	0010	2
0011	3	0011	3	0011	3
0100	4	0100	4	0100	4
0101	5	0101	5	0101	5
0110	6	0110	6	0110	6
0111	7	0111	7	0111	7

Sign bit only:
- MSB is the sign bit
- 0 → positive
- 1 → negative

One's Complement:
Flipping bits is a simple CPU operation

Two's Complement:
- Eliminates the **-0**
- An additional number **-8** can now be represented
- The **MSB**'s position now presents a **-8**
e.g., 1101 = -8 + 4 + 0 + 1 = -3

Two's complement (Origin)

- **One easy way to find the negative number equivalent is to start with positive number, take its ones complement then add a one.**
- **e.g., -64 = 11000000 = 0xC0**

Decimal	Binary	Hex	
64	01000000	0x40	
	10111111	0xBF	**1's** Complement
-64	11000000	0xC0	**2's** Complement

Two-complement: Simple Example

	-16	8	4	2	1	
10 =	0	1	0	1	0	8+2 = 10

	-16	8	4	2	1	
-10 =	1	0	1	1	0	-16+4+2 = -10

Two-complement Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1

- **Two's Complement Values**
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
  - Minus 1
    111…1

Values for *W* = 16

	Decimal	Hex	Binary
`UMax`	65535	`FF FF`	`11111111 11111111`
`TMax`	32767	`7F FF`	`01111111 11111111`
`TMin`	-32768	`80 00`	`10000000 00000000`
`-1`	-1	`FF FF`	`11111111 11111111`
`0`	0	`00 00`	`00000000 00000000`

Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

Observations

- $|TMin| = TMax + 1$
  - Asymmetric range
- $UMax = 2 * TMax + 1$
- Question: abs(TMin)?

C Programming

- #include <limits.h>
- Declares constants, e.g.,
  - ULONG_MAX
  - LONG_MAX
  - LONG_MIN
- Values platform specific

Unsigned & Signed Numeric Values

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- **$\Rightarrow$ Can Invert Mappings**
  - U2B(x) = B2U^{-1}(x)
    - Bit pattern for unsigned integer
  - T2B(x) = B2T^{-1}(x)
    - Bit pattern for two's comp integer

Practice!

1) 11100110_b = 0x?
2) 0x5d | 0xd5 = ?
3) (0x5d && 0x00) || 0x5d = ?
4) -2 = ? (4-bit signed integer)

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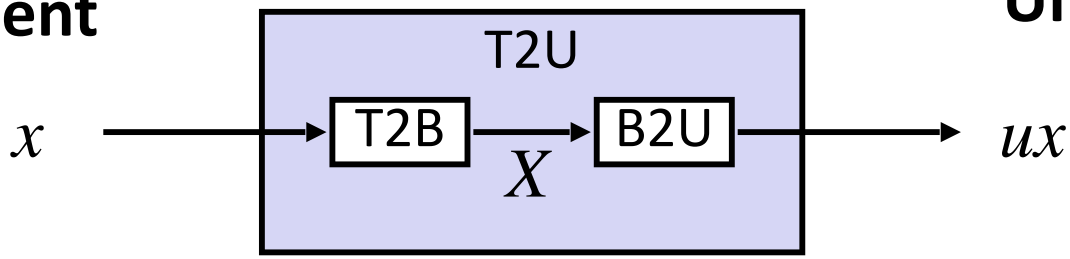
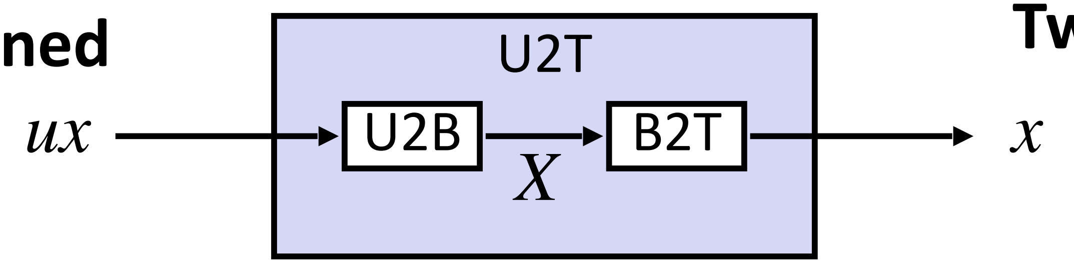
Mapping Between Signed & Unsigned (Casting)

Two's Complement → **Unsigned**



Maintain Same Bit Pattern

Unsigned → **Two's Complement**



Maintain Same Bit Pattern

- **Mappings between unsigned and two's complement numbers:**
  Keep bit representations and reinterpret

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

→ T2U →

← U2T ←

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
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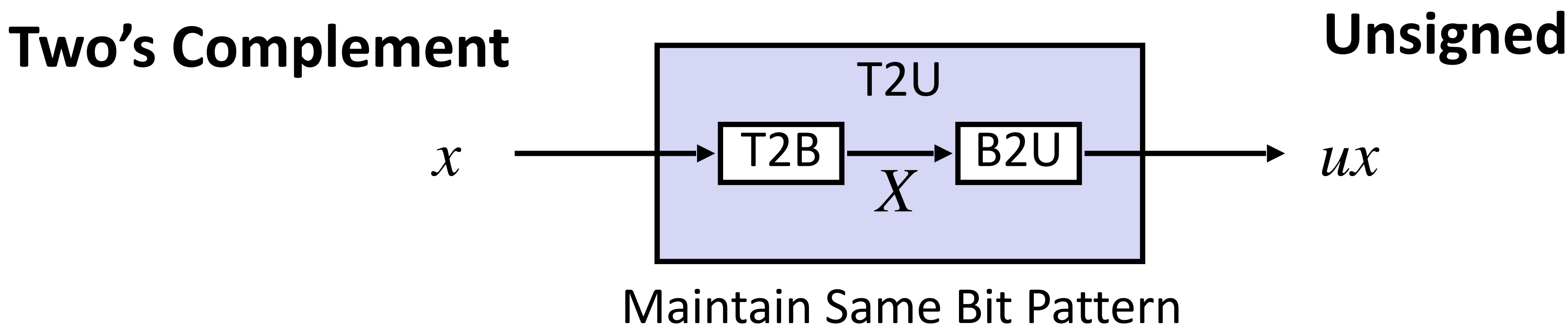
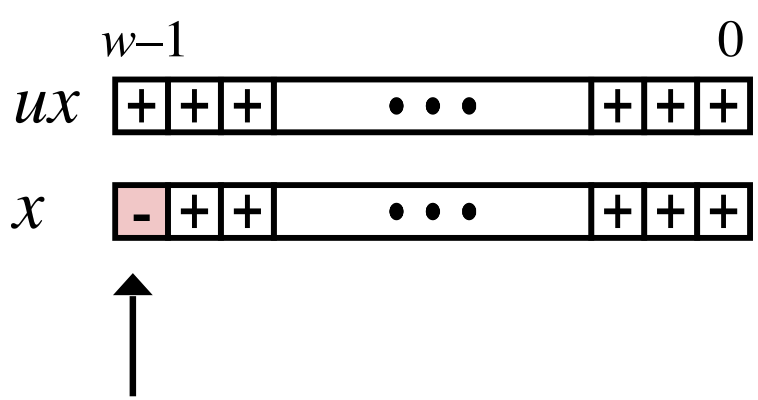
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

=

+/- 16

Relation between Signed & Unsigned





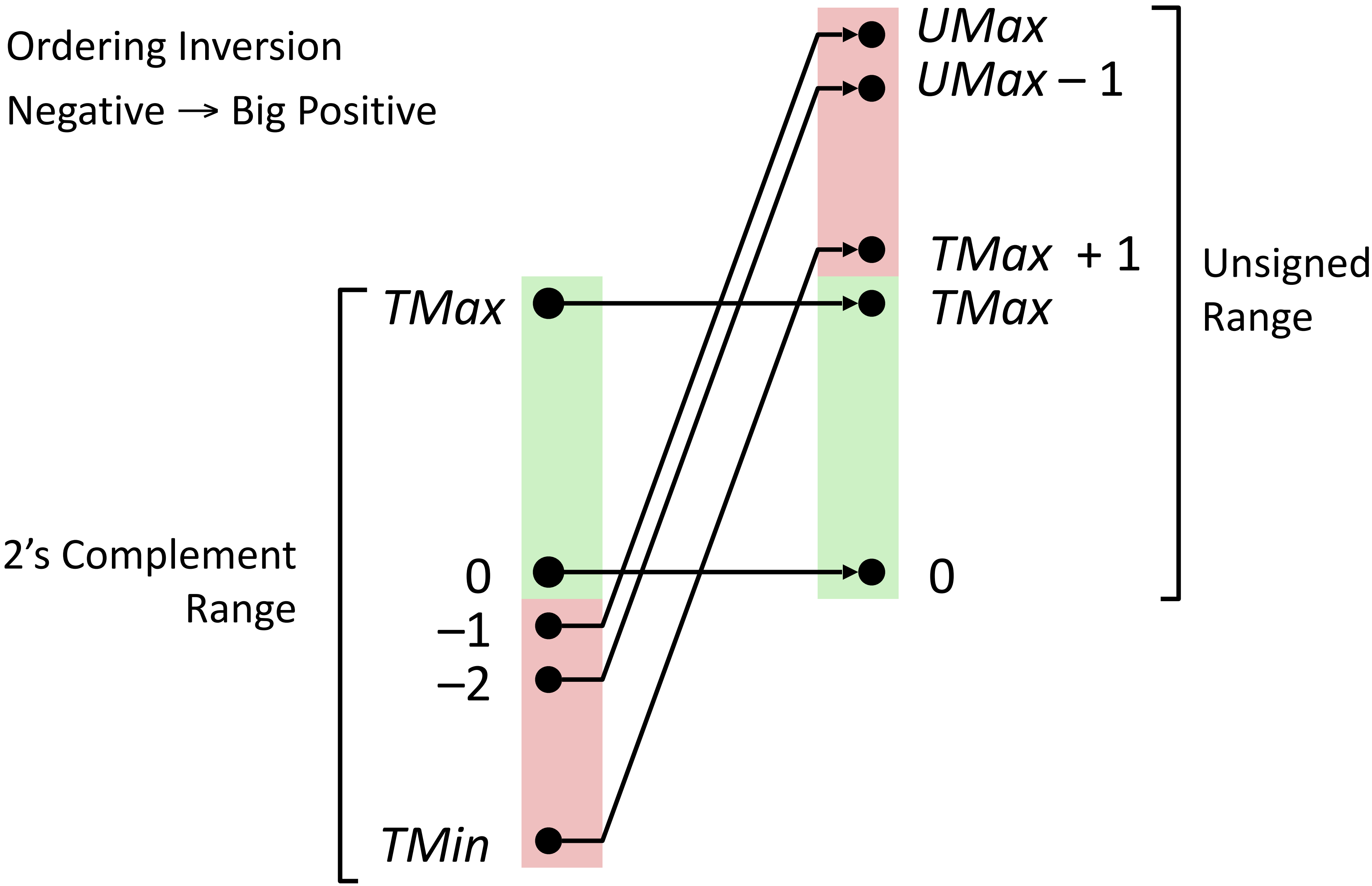
Large negative weight

becomes

Large positive weight

Conversion Visualized

- **2's Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

Constants

- By **default** are considered to be **signed integers**
- Unsigned if have "**U**" as suffix

```
0U, 4294967259U
```

Casting

- ***Explicit*** casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

- ***Implicit*** casting also occurs via *assignments and procedure calls*

```
tx = ux;
uy = ty;
```

```
int fun(unsigned u);
uy = fun(tx);
```

Casting Surprises

- **Expression Evaluation**
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 , TMAX = 2,147,483,647**

Constant$_1$	**Constant$_2$**	**Relation**	**Evaluation**
0	0U	**==**	**unsigned**
-1	0	**<**	**signed**
-1	0U	**>**	**unsigned**
2147483647	-2147483647-1	**>**	**signed**
2147483647U	-2147483647-1	**<**	**unsigned**
-1	-2	**>**	**signed**
(unsigned)-1	-2	**>**	**unsigned**
2147483647	2147483648U	**<**	**unsigned**
2147483647	(int) 2147483648U	**>**	**signed**

Unsigned vs. Signed: Easy to Make Mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
  a[i] += a[i+1];
```

>>> could lead to an infinite loop / or a segmentation fault
(i will never be less than zero / decrementing i may result in huge number)

- Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
  . . .
```

>>> could lead to an infinite loop (sizeof returns size_t which is *unsigned*)
(I will never be less than zero)

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

- **Expression containing signed and unsigned int**
  - `int` is cast to `unsigned`!!

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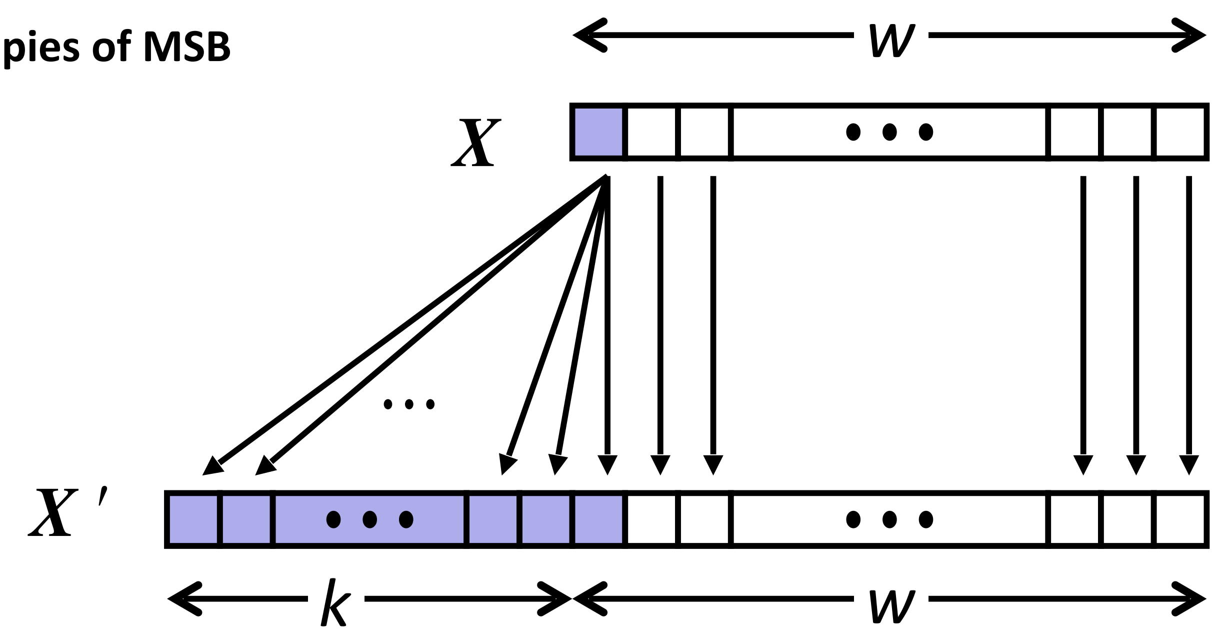
Sign Extension

- **Task:**
  - Given *w*-bit signed integer *x*
  - Convert it to *w*+*k*-bit integer with same value
- **Rule:**
  - Make *k* copies of sign bit:
  - $X' = \underbrace{x_{w-1},\ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2},\ldots, x_0$



Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Larger Sign Extension Example

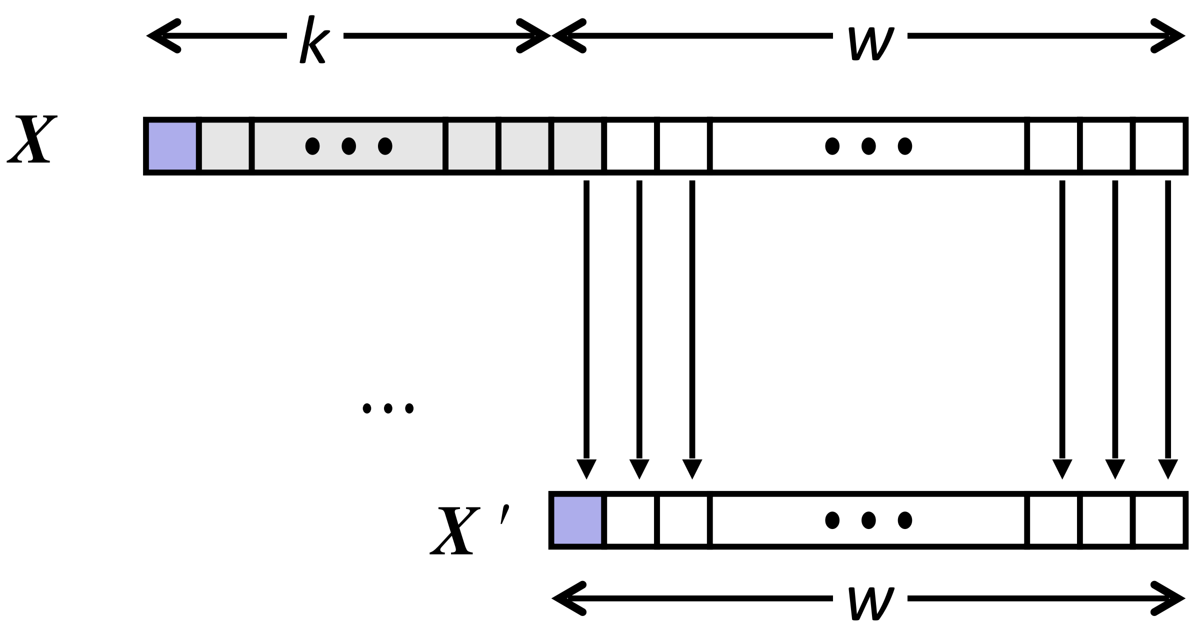
```
short int x =  15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Truncation

- **Task:**
  - Given k+*w*-bit signed or unsigned integer *X*
  - Convert it to *w*-bit integer X’ with same value for “small enough” X
- **Rule:**
  - Drop top *k* bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



Truncation: Simple Example

No sign change

2 =

-16	8	4	2	1
0	0	0	1	0

2 =

-8	4	2	1
0	0	1	0

`2 mod 16 = 2`

-6 =

-16	8	4	2	1
1	1	0	1	0

-6 =

-8	4	2	1
1	0	1	0

`-6 mod 16 = 26U mod 16 = 10U = -6`

Sign change

10 =

-16	8	4	2	1
0	1	0	1	0

-6 =

-8	4	2	1
1	0	1	0

`10 mod 16 = 10U mod 16 = 10U = -6`

-10 =

-16	8	4	2	1
1	0	1	1	0

6 =

-8	4	2	1
0	1	1	0

`-10 mod 16 = 22U mod 16 = 6U = 6`

Summary: Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small (in magnitude) numbers yields expected behavior

Misunderstanding integers can lead to disastrous consequences!