# Module 6 Test

Saturday, July 15, 2023 2:46 PM

**1. [11/12 Points]** DETAILS PREVIOUS ANSWERS **DEVORESTAT9 1.2.010.** MY NOTES ASK YOUR TEACHER

Flexural strength is a measure of a material's ability to resist failure in bending. The accompanying data are on flexural strength of concrete (in MegaPascal, MPa, where 1 Pa (Pascal) = $1.45 \times 10^{-4}$ psi):

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5.5 | 7.2 | 7.3 | 6.1 | 8.1 | 6.8 | 7.0 | 7.6 | 6.8 | 6.5 | 7.0 | 6.3 | 7.9 | 9.0 |
| 8.2 | 8.7 | 7.8 | 9.7 | 7.4 | 7.7 | 9.7 | 7.8 | 7.7 | 11.6 | 11.2 | 11.8 | 10.9 | |

(a) Construct a stem-and-leaf display of the data. (Enter numbers from smallest to largest separated by spaces. Enter NONE for stems with no values.)

| Stems | Leaves |
|---|---|
| 5 | 5 |
| 6 | 1 3 5 8 8 |
| 7 | 0 0 2 3 4 6 7 7 8 8 9 |
| 8 | 1 2 7 |
| 9 | 0 7 7 |
| 10 | 9 |
| 11 | 2 6 8 |

What appears to be a representative strength value?

7 MPa

Do the observations appear to be highly concentrated about the representative value or rather spread out?

- highly concentrated around the representative value
- spread out with a large range

(c) Do there appear to be any outlying strength values?

- Yes
- No

(d) What proportion of strength observations in this sample exceed 10 MPa? (Round your answer to two decimal places.)

0.15

(d)
4 strength values greater than 10 MPa: 10.9, 11.2, 11.6, 11.8
(4 strength values greater than 10 MPa) / (27 total strength values) = 4 / 27 = 0.15

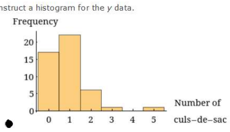
**2. [–/6 Points]** DETAILS **DEVORESTAT9 1.2.021.S.** MY NOTES ASK YOUR TEACHER

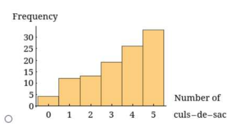
The article "Determination of Most Representative Subdivision"† gave data on various characteristics of subdivisions that could be used in deciding whether to provide electrical power using overhead lines or underground lines. Here are the values of the variables $y$ = number of culs-de-sac and $z$ = number of intersections:

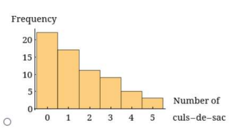
| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 1 | 0 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 1 | 2 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| $z$ | 1 | 8 | 6 | 1 | 1 | 5 | 3 | 0 | 0 | 4 | 4 | 0 | 0 | 1 | 2 | 1 | 4 | 0 | 4 | 0 | 3 | 0 | 1 | 1 |
| $y$ | 1 | 1 | 2 | 0 | 1 | 2 | 2 | 1 | 1 | 0 | 2 | 1 | 1 | 0 | 1 | 5 | 0 | 3 | 0 | 1 | 1 | 0 | 0 | |
| $z$ | 0 | 1 | 3 | 2 | 4 | 6 | 6 | 0 | 1 | 1 | 8 | 3 | 3 | 5 | 0 | 5 | 2 | 3 | 1 | 0 | 0 | 0 | 3 | |

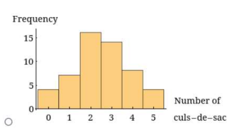
USE SALT

(a) Construct a histogram for the $y$ data.



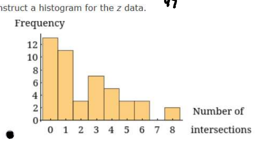



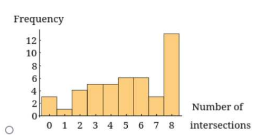

What proportion of these subdivisions had no culs-de-sac? At least one cul-de-sac? (Round your answers to three decimal places.)

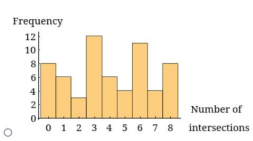
no culs-de-sac $\frac{17}{47} = 0.362$

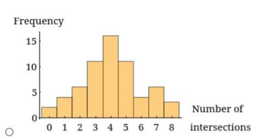
at least one cul-de-sac $\frac{30}{47} = 1 - 0.362 = 0.638$

(b) Construct a histogram for the $z$ data.




What proportion of these subdivisions had at most five intersections? Fewer than five intersections? (Round your answers to three decimal places.)

at most fiver intersections $\frac{42}{47} = 0.894$

fewer than five intersections $\frac{39}{47} = 0.830$

**3. [–/3 Points]** DETAILS DEVORESTAT9 1.3.039.S. MY NOTES ASK YOUR TEACHER

The propagation of fatigue cracks in various aircraft parts has been the subject of extensive study in recent years. The accompanying data consists of propagation lives (flight hours/$10^4$) to reach a given crack size in fastener holes intended for use in military aircraft.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.744 | 0.847 | 0.866 | 0.915 | 0.929 | 0.942 | 0.966 | 1.010 |
| 1.039 | 1.041 | 1.091 | 1.111 | 1.132 | 1.160 | 1.232 | 1.366 |

USE SALT

(a) Compute and compare the values of the sample mean $\bar{x}$ and median $\tilde{x}$. (Round your mean to four decimal places.)

$\bar{x}$ = 1.0244 flight hours/$10^4$ from Excel

$\tilde{x}$ = 1.0245 flight hours/$10^4$ $\frac{x_8+x_9}{2} = \tilde{x}$

(b) By how much could the largest sample observation be decreased without affecting the value of the median? (Enter your answer to three decimal places.)

0.327 flight hours/$10^4$ largest sample observation $- x_9$

**4. [–/4 Points]** DETAILS DEVORESTAT9 1.SE.505.XP.S. MY NOTES ASK YOUR TEACHER

An article reported the following data on oxygen consumption (mL/kg/min) for a sample of ten firefighters performing a fire-suppression simulation:

29.7 49.3 30.4 28.4 28.4 25.7 33.1 29.5 23.0 31.1

USE SALT

Compute the following. (Round your answers to four decimal places.)

(a) The sample range

26.3000 mL/kg/min largest - smallest

(b) The sample variance $s^2$ from the definition (i.e., by first computing deviations, then squaring them, etc.)

49.8916 mL$^2$/kg$^2$/min$^2$

(c) The sample standard deviation

7.0634 mL/kg/min

Using Excel

(d) $s^2$ using the shortcut method

mL$^2$/kg$^2$/min$^2$

**5. [–/8 Points]** DETAILS DEVORESTAT9 2.1.004. MY NOTES ASK YOUR TEACHER

Each of a sample of four home mortgages is classified as fixed rate ($F$) or variable rate ($V$). (Enter your answers in set notation. Enter EMPTY or ∅ for the empty set.)

(a) What are the 16 outcomes in $\mathcal{S}$?

$\mathcal{S}$ = {FFFF,FFFV,FFVF,FFVV,FVFF,FVFV,FVVF,FVVV,VFFF,VFFV,VFVF,VFVV,VVFF,VVFV,VVVF,VVVV}

(b) Which outcomes are in the event that exactly three of the selected mortgages are fixed rate?

{FFFV, FFVF, FVFF, VFFF}

(c) Which outcomes are in the event that all four mortgages are of the same type?

{FFFF, VVVV}

(d) Which outcomes are in the event that at most one of the four is a variable-rate mortgage? (same as (b) but include FFFF)

{FFFV, FFVF, FVFF, VFFF, FFFF}

(e) What is the union of the events in parts (c) and (d)?

{FFFV, FFVF, FVFF, VFFF, FFFF, VVVV}

What is the intersection of these two events?

{FFFF}

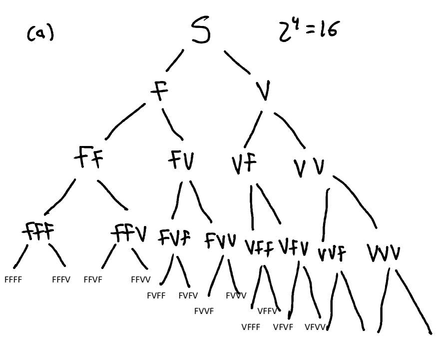
(f) What is the union of the two events in parts (b) and (c)?

{FFFV, FFVF, FVFF, VFFF, FFFF, VVVV}

What is the intersection of the two events in parts (b) and (c)?

∅

(a) $2^4 = 16$




**6. [–/2 Points]** DETAILS DEVORESTAT9 2.2.014.MI. MY NOTES ASK YOUR TEACHER

Suppose that 65% of all adults regularly consume coffee, 40% regularly consume carbonated soda, and 75% regularly consume at least one of these two products.

(a) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

$P(\text{coffee} \cap \text{soda}) = P(\text{coffee}) + P(\text{soda}) - P(\text{coffee} \cup \text{soda}) = 0.65 + 0.40 - 0.75 = 0.30$

(b) What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

$P'(\text{coffee} \cap \text{soda}) = 1 - P(\text{coffee} \cup \text{soda}) = 1 - 0.75 = 0.25$

**Proposition**

For any two events $A$ and $B$,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

$-P(A \cap B) = P(A \cup B) - P(A) - P(B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

**7. [–/2 Points]** DETAILS DEVORESTAT9 2.3.042. MY NOTES ASK YOUR TEACHER

A starting lineup in basketball consists of two guards, two forwards, and a center.

(a) A certain college team has on its roster four centers, four guards, five forwards, and one individual (X) who can play either guard or forward. How many different starting lineups can be created? [*Hint*: Consider lineups without X, then lineups with X as guard, then lineups with X as forward.]

____ lineups

$P(\text{lineups w/o } X) + P(\text{lineups w/X as guard}) + P(\text{lineups w/X as forward}) = \underbrace{\binom{4}{1}\binom{4}{2}\binom{5}{2}}_{\text{center}\cdot\text{guards}\cdot\text{forwards}} + \underbrace{\binom{4}{1}\binom{4}{1}\binom{5}{2}}_{\text{w/o guard}} + \underbrace{\binom{4}{1}\binom{4}{2}\binom{5}{1}}_{\text{w/o forward}} = 520$

(b) Now suppose the roster has 3 guards, 4 forwards, 4 centers, and 2 "swing players" (X and Y) who can play either guard or forward. If 5 of the 13 players are randomly selected, what is the probability that they constitute a legitimate starting lineup? (Round your answer to three decimal places.)

____

$P(\text{w/o } X \,\&\, Y) = \underbrace{\binom{3}{1}}_{\text{center}}\underbrace{\binom{4}{2}}_{\text{guard}}\underbrace{\binom{4}{2}}_{\text{forward}} = 108$

$P(X \text{ as guard}) = \binom{3}{1}\binom{4}{1}\binom{4}{2} = 72$

$P(Y \text{ as guard})$ is same

$P(X \text{ as forward}) = \binom{3}{1}\binom{4}{2}\binom{4}{1} = 72$

$P(Y \text{ as forward})$ is same

$P(X \text{ guard}; Y \text{ forward}) = \binom{3}{1}\binom{4}{1}\binom{4}{1} = 48$

$P(Y \text{ guard}; X \text{ forward})$ is same

$P(X \,\&\, Y \text{ guard}) = \binom{3}{1}\binom{4}{0}\binom{4}{2} = 18$

$P(X \,\&\, Y \text{ forward}) = \binom{3}{1}\binom{4}{2}\binom{4}{0} = 18$

$P(\text{arrangement of 5-out-of-13}) = \binom{13}{5} = 1{,}287$

$$P(\text{lineup}) = \frac{P(\text{w/o } X \,\&\, Y) + P(X \text{ as guard}) + P(Y \text{ as guard}) + P(X \text{ as forward}) + P(Y \text{ as forward}) + P(X \text{ guard}; Y \text{ forward}) + P(Y \text{ guard}; X \text{ forward}) + P(X \,\&\, Y \text{ guard}) + P(X \,\&\, Y \text{ forward})}{P(\text{arrangement of 5-out-of-13})}$$

$$= \frac{108 + 72(4) + 48(2) + 18(2)}{1{,}287}$$

$= 0.410$ X

**8. [–/8 Points]** DETAILS DEVORESTAT9 2.4.054. MY NOTES ASK YOUR TEACHER

A computer consulting firm presently has bids out on three projects. Let $A_i$ = {awarded project $i$}, for $i$ = 1, 2, 3, and suppose that $P(A_1) = 0.22$, $P(A_2) = 0.26$, $P(A_3) = 0.28$, $P(A_1 \cap A_2) = 0.11$, $P(A_1 \cap A_3) = 0.08$, $P(A_2 \cap A_3) = 0.09$, $P(A_1 \cap A_2 \cap A_3) = 0.01$. Use the probabilities given above to compute the following probabilities, and explain in words the meaning of each one. (Round your answers to four decimal places.)

(a) $P(A_2 \mid A_1) =$ ____ $\quad P(A_2 \mid A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.11}{0.22} = 0.5000$

Explain this probability in words.

- ○ If the firm is awarded project 2, this is the chance they will also be awarded project 1.
- ● If the firm is awarded project 1, this is the chance they will also be awarded project 2.
- ○ This is the probability that the firm is awarded either project 1 or project 2.
- ○ This is the probability that the firm is awarded both project 1 and project 2.

(b) $P(A_2 \cap A_3 \mid A_1) =$ ____ $\quad P(A_2 \cap A_3 \mid A_1) = \frac{P(A_1 \cap (A_2 \cap A_3))}{P(A_1)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{0.01}{0.22} = 0.0455$

Explain this probability in words.

- ○ If the firm is awarded projects 2 and 3, this is the chance they will also be awarded project 1.
- ● If the firm is awarded project 1, this is the chance they will also be awarded projects 2 and 3.
- ○ This is the probability that the firm is awarded at least one of the projects.
- ○ This is the probability that the firm is awarded projects 1, 2, and 3.

(c) $P(A_2 \cup A_3 \mid A_1) =$ ____ $\quad P(A_2 \cup A_3 \mid A_1) = \frac{P(A_1 \cap (A_2 \cup A_3))}{P(A_1)} = \frac{P((A_1 \cap A_2) \cup (A_1 \cap A_3))}{P(A_1)} = \frac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{0.11 + 0.08 - 0.01}{0.22} = 0.8182$

Explain this probability in words.

- ○ If the firm is awarded at least one of projects 2 and 3, this is the chance they will also be awarded project 1.
- ○ This is the probability that the firm is awarded projects 1, 2, and 3.
- ● If the firm is awarded project 1, this is the chance they will also be awarded at least one of the other two projects.
- ○ This is the probability that the firm is awarded at least one of the projects.

Explain this probability in words.

- ○ If the firm is awarded at least one of projects 2 and 3, this is the chance they will also be awarded project 1.
- ○ This is the probability that the firm is awarded projects 1, 2, and 3.
- ● If the firm is awarded project 1, this is the chance they will also be awarded at least one of the other two projects.
- ○ This is the probability that the firm is awarded at least one of the projects.

(d) $P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) =$ [ ]

$$P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)+P(A_2)+P(A_3)-P(A_1 \cap A_2)-P(A_1 \cap A_3)-P(A_2 \cap A_3)+P(A_1 \cap A_2 \cap A_3)} = \frac{0.01}{0.22+0.26+0.28-0.11-0.08-0.09+0.01} = 0.0204$$

Explain this probability in words.

- ○ This is the probability that the firm is awarded projects 1, 2, and 3.
- ○ This is the probability that the firm is awarded at least one of the projects.
- ● If the firm is awarded at least one of the projects, this is the chance that they will be awarded all three projects.
- ○ If the firm is awarded at least two of the projects, this is the chance that they will be awarded all three projects.

**9. [5/5 Points]** DETAILS | PREVIOUS ANSWERS | DEVORESTAT9 3.AE.012. | MY NOTES | ASK YOUR TEACHER

**Example 3.12** Starting at a fixed time, we observe the gender of each newborn child at a certain hospital until a boy ($B$) is born. Let $p = P(B)$, assume that successive births are independent, and define the rv $X$ by $X$ = number of births observed. Then

$p(1) = P(X = 1) = P(B) =$ $p$ ✓

$p(2) = P(X = 2) = P(GB) = P(G) \cdot P(B) =$ $(1-p)p$ ✓

and

$p(3) = P(X = 3) = P(GGB) = P(G) \cdot P(G) \cdot P(B) =$ $(1-p)^2 p$ ✓

Continuing in this way, a general formula emerges:

$$p(x) = \begin{cases} (1-p)^{x-1}p & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases}$$

The parameter $p$ can assume any value between 0 ✓ and 1 ✓. The expression above describes the family of *geometric* distributions. In the gender example, $p = 0.51$ might be appropriate, but if we were looking for the first child with Rh-positive blood, then we might have $p = 0.85$.

**10. [19/19 Points]** DETAILS | PREVIOUS ANSWERS | DEVORESTAT9 3.2.020. | MY NOTES | ASK YOUR TEACHER

Three couples and two single individuals have been invited to an investment seminar and have agreed to attend. Suppose the probability that any particular couple or individual arrives late is 0.39 (a couple will travel together in the same vehicle, so either both people will be on time or else both will arrive late). Assume that different couples and individuals are on time or late independently of one another. Let $X$ = the number of people who arrive late for the seminar.

(a) Determine the probability mass function of $X$. [*Hint:* label the three couples #1, #2, and #3 and the two individuals #4 and #5.] (Round your answers to four decimal places.)

| $x$ | $P(X = x)$ |
|---|---|
| 0 | 0.0845 ✓ |
| 1 | 0.1080 ✓ |
| 2 | 0.1965 ✓ |
| 3 | 0.2071 ✓ |
| 4 | 0.1698 ✓ |
| 5 | 0.1324 ✓ |
| 6 | 0.0644 ✓ |
| 7 | 0.0282 ✓ |
| 8 | 0.0090 ✓ |

(b) Obtain the cumulative distribution function of $X$. (Round your answers to four decimal places.)

| $x$ | $F(x)$ |
|---|---|
| 0 | 0.0845 ✓ |
| 1 | 0.1925 ✓ |
| 2 | 0.3890 ✓ |
| 3 | 0.5961 ✓ |
| 4 | 0.7659 ✓ |
| 5 | 0.8983 ✓ |
| 6 | 0.9627 ✓ |
| 7 | 0.9909 ✓ |
| 8 | 1.0000 ✓ |

Use the cumulative distribution function of $X$ to calculate $P(2 \le X \le 6)$. (Round your answer to four decimal places.)

$P(2 \le X \le 6) =$ 0.7702 ✓

Following table shows the pdf:

| Possible ways of X | X | P(X=x) | Formula for calcaulating P(X=x) |
|---|---|---|---|
| No late | 0 | 0.08446 | (1-0.39)^5 |
| I | 1 | 0.107998 | C(2,1)*(1-0.39)^4*0.39 |
| C+II | 2 | 0.19652 | (C(3,1)*(1-0.39)^4*0.39)+ ((1-0.39)^3*0.39^2) |
| CI | 3 | 0.207143 | C(3,1)*C(2,1)*(1-0.39)^3*0.39^2 |
| CC+CII | 4 | 0.169789 | C(3,2)*0.39^2*(1-0.39)^3+C(3,1)*0.39^3*(1-0.39)^2 |
| CCI | 5 | 0.132436 | C(3,1)*C(2,1)*0.39^3*(1-0.39)^2 |
| CCC+CCII | 6 | 0.064409 | 0.39^3*(1-0.39)^2+C(3,2)*0.39^4*(1-0.39) |
| CCCI | 7 | 0.028224 | C(2,1)*0.39^4*(1-0.39) |
| All late | 8 | 0.009022 | 0.39^5 |
| Total | | 1 | |

Following is the CDF:

| X | P(X=x) | F(X) |
|---|---|---|
| 0 | 0.0845 | 0.0845 |
| 1 | 0.108 | 0.1925 |
| 2 | 0.1965 | 0.389 |
| 3 | 0.2071 | 0.5961 |
| 4 | 0.1698 | 0.7659 |
| 5 | 0.1324 | 0.8983 |
| 6 | 0.0644 | 0.9627 |
| 7 | 0.0282 | 0.9909 |
| 8 | 0.009 | 0.9999 |
| Total | 1 | |

The required probability is

$$P(2 \le X \le 6) = P(X \le 6) - P(X \le 1) = 0.9627 - 0.1925 = 0.7702$$

**11. [–/7 Points]** DETAILS DEVORESTAT9 3.2.024. MY NOTES ASK YOUR TEACHER

An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let $X$ = the number of months between successive payments. The cdf of $X$ is as follows:

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.38 & 1 \le x < 3 \\ 0.49 & 3 \le x < 4 \\ 0.53 & 4 \le x < 6 \\ 0.89 & 6 \le x < 12 \\ 1 & 12 \le x \end{cases}$$

(a) What is the pmf of $X$?

$0.49 - 0.38 = 0.11$  $0.53 - 0.49 = 0.04$  $0.89 - 0.53 = 0.36$  $1 - 0.89 = 0.11$

| $x$ | 1 | 3 | 4 | 6 | 12 |
|---|---|---|---|---|---|
| $p(x)$ | 0.38 | | | | |

(b) Using just the cdf, compute $P(3 \le X \le 6)$ and $P(4 \le X)$.

$P(3 \le X \le 6)$ = $P(3 \le x \le 6) = P(x \le 6) - P(x \le 3) + P(x = 3) = 0.89 - 0.49 + 0.11 = 0.51$

$P(4 \le X)$ = $P(4 \le x) = 1 - P(x < 4) = 1 - 0.49 = 0.51$

**12. [4/8 Points]** DETAILS PREVIOUS ANSWERS DEVORESTAT9 4.1.004. MY NOTES ASK YOUR TEACHER

Let $X$ denote the vibratory stress (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. Use the Rayleigh distribution, with pdf

$$f(x;\theta) = \begin{cases} \dfrac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

as a model for the $X$ distribution.

(a) Verify that $f(x; \theta)$ is a legitimate pdf.

$u = \frac{x^2}{2\theta^2}$; $du = 2x \cdot \frac{1}{2\theta^2} dx = \frac{x}{\theta^2} dx \Rightarrow dx = \frac{\theta^2}{x} du$

$$\int_0^\infty \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)}\, dx = \left[ -e^{-\left(\frac{x^2}{2\theta^2}\right)} \right]_0^\infty$$

$= 0 - (-1)$

$= 1$

$\int_0^\infty \frac{x}{\theta^2} \cdot e^{-\frac{x^2}{2\theta^2}} dx = \int_0^\infty \frac{x}{\theta^2} \cdot e^{-u} \cdot \frac{\theta^2}{x} du = \int_0^\infty e^{-u} = -e^{-u}\Big|_0^\infty = -e^{-\frac{x^2}{2\theta^2}}$

$= -\frac{1}{e^\infty} - \left(-\frac{1}{e^0}\right) = 0 - (-1) = 1$

(b) Suppose $\theta$ = 133. What is the probability that $X$ is at most 200? Less than 200? At least 200? (Round your answer to four decimal places.)

$f(x; \theta = 133) = \frac{x}{133^2} \cdot e^{-\frac{x^2}{2(133)^2}}$

at most 200 $P(X \le 200) = P(0 \le x \le 200) = \int_a^b f(x)dx = \int_0^{200} \frac{x}{133^2} \cdot e^{-\frac{x^2}{2(133)^2}} dx = 0.6772$

less than 200 $P(X < 200) = P(0 \le x \le 200) = 0.6772$

at least 200 $P(x \ge 200) = 1 - P(x < 200) = 1 - 0.6772 = 0.3228$

(c) What is the probability that $X$ is between 100 and 200 (again assuming $\theta$ = 133)? (Round your answer to four decimal places.)

$P(100 \le x \le 200) = \int_a^b f(x)dx = \int_{100}^{200} \frac{x}{133^2} \cdot e^{-\frac{x^2}{2(133)^2}} dx = 0.4310$

(d) Give an expression for $P(X \le x)$. or $P(x \ge X)$

$$P(X \le x) = 1 - e^{-\left(\frac{x^2}{2\theta^2}\right)}$$

$1 - P(x < X) = 1 - e^{-\frac{x^2}{2\theta^2}}$

when $e^{-\frac{x^2}{2\theta^2}}$ is the integral simplified

**13. [–/4 Points]** DETAILS DEVORESTAT9 4.SE.502.XP. MY NOTES ASK YOUR TEACHER

"Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let $X$ = the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The pdf of $X$ is the following.

$$f(x) = \begin{cases} 0.13e^{-0.13(x - 0.5)} & x \ge 0.5 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the probability that time headway is at most 5 sec? (Round your answer to three decimal places.)

$P(\text{at most } 5 \text{ sec}) = P(x \le 5) = \int_{0.5}^{5} (0.13e^{-0.13(x-0.5)})dx = 0.443$

(b) What is the probability that time headway is more than 5 sec? At least 5 sec? (Round your answers to three decimal places.)

more than 5 sec $P(x > 5) = 1 - P(x \le 5) = 1 - 0.443 = 0.557$

at least 5 sec Same as above, 0.557

(c) What is the probability that time headway is between 4 and 5 sec? (Round your answer to three decimal places.)

$P(4 \le x \le 5) = \int_4^5 (0.13e^{-0.13(x-0.5)})dx = 0.077$

**14. [–/6 Points]** DETAILS DEVORESTAT9 5.1.003. MY NOTES ASK YOUR TEACHER

A certain market has both an express checkout line and a superexpress checkout line. Let $X_1$ denote the number of customers in line at the express checkout at a particular time of day, and let $X_2$ denote the number of customers in line at the superexpress checkout at the same time. Suppose the joint pmf of $X_1$ and $X_2$ is as given in the accompanying table.

| | | $x_2$ | | | |
|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 |
| $x_1$ | 0 | 0.09 | 0.06 | 0.04 | 0.00 |
| | 1 | 0.06 | 0.15 | 0.05 | 0.04 |
| | 2 | 0.05 | 0.03 | 0.10 | 0.06 |
| | 3 | 0.00 | 0.02 | 0.04 | 0.07 |
| | 4 | 0.00 | 0.01 | 0.05 | 0.08 |

(a) What is $P(X_1 = 1, X_2 = 1)$, that is, the probability that there is exactly one customer in each line?

$P(X_1 = 1, X_2 = 1)$ = $P(1,1) = 0.15$

(b) What is $P(X_1 = X_2)$, that is, the probability that the numbers of customers in the two lines are identical?

$P(X_1 = X_2)$ = $P(0,0) + P(1,1) + P(2,2) + P(3,3) = 0.09 + 0.15 + 0.10 + 0.07 = 0.41$

(c) Let $A$ denote the event that there are at least two more customers in one line than in the other line. Express $A$ in terms of $X_1$ and $X_2$.

- ○ $A = \{X_1 \le 2 + X_2 \cup X_2 \le 2 + X_1\}$
- ● $A = \{X_1 \ge 2 + X_2 \cup X_2 \ge 2 + X_1\}$
- ○ $A = \{X_1 \le 2 + X_2 \cup X_2 \ge 2 + X_1\}$
- ○ $A = \{X_1 \ge 2 + X_2 \cup X_2 \le 2 + X_1\}$

Calculate the probability of this event. $P(2,0) + P(3,1) + P(4,2) + P(3,0) + P(4,1) + P(4,0) + P(0,2) + P(1,3) + P(0,3)$

$P(A)$ = $= 0.05 + 0.02 + 0.05 + 0.00 + 0.01 + 0.00 + 0.04 + 0.04 + 0.00 = 0.21$

(d) What is the probability that the total number of customers in the two lines is exactly four? At least four?

P(exactly four) = $P(x_1 + x_2 = 4) = P(1,3) + P(2,2) + P(3,1) + P(4,0) = 0.04 + 0.10 + 0.02 + 0.00 = 0.16$

P(at least four) =

$P(x_1 + x_2 = 4) + P(x_1 + x_2 = 5) + P(x_1 + x_2 = 6) + P(x_1 + x_2 = 7)$

$P(A) =$ ☐ $= 0.05 + 0.02 + 0.05 + 0.00 + 0.01 + 0.00 + 0.04 + 0.04 + 0.00 = 0.21$

(d) What is the probability that the total number of customers in the two lines is exactly four? At least four?

$P$(exactly four) = ☐ $P(x_1 + x_2 = 4) = P(1,3) + P(2,2) + P(3,1) + P(4,0) = 0.04 + 0.10 + 0.02 + 0.00 = 0.16$

$P$(at least four) = ☐

$P(x_1 + x_2 = 4) + P(x_1 + x_2 = 5) + P(x_1 + x_2 = 6) + P(x_1 + x_2 = 7)$

$= P(1,3) + P(2,2) + P(3,1) + P(4,0) + P(2,3) + P(3,2) + P(3,3) + P(4,1) + P(4,2) + P(4,3)$

$= 0.04 + 0.10 + 0.02 + 0.00 + 0.06 + 0.04 + 0.07 + 0.01 + 0.05 + 0.08$

$= 0.47$

**15. [–/2 Points]** DETAILS **DEVORESTAT9 5.2.031.** MY NOTES ASK YOUR TEACHER

Each front tire on a particular type of vehicle is supposed to be filled to a pressure of 26 psi. Suppose the actual air pressure in each tire is a random variable—$X$ for the right tire and $Y$ for the left tire, with joint pdf given below.

$$f(x, y) = \begin{cases} K(x^2 + y^2) & 22 \le x \le 31,\ 22 \le y \le 31 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the covariance between $X$ and $Y$. (Round your answer to four decimal places.)

Cov($X$, $Y$) = ☐

-0.0637

(b) Compute the correlation coefficient $\rho$ for this $X$ and $Y$. (Round your answer to four decimal places.)

$\rho$ = ☐

-0.0095

**16. [–/10 Points]** DETAILS **DEVORESTAT9 6.1.001.** MY NOTES ASK YOUR TEACHER

Consider the accompanying data on flexural strength (MPa) for concrete beams of a certain type.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7.0 | 7.0 | 8.2 | 6.8 | 9.0 | 7.2 | 6.3 | |
| 6.3 | 8.1 | 11.6 | 5.6 | 7.8 | 8.7 | 7.3 | $n = 27$ |
| 7.7 | 7.1 | 6.5 | 7.9 | 7.4 | 6.8 | 8.1 | |
| 11.8 | 9.7 | 9.7 | 11.3 | 7.7 | 10.7 | | |

(a) Calculate a point estimate of the mean value of strength for the conceptual population of all beams manufactured in this fashion. [*Hint*: $\Sigma x_i = 219.3$.] (Round your answer to three decimal places.)

☐ MPa $\bar{x} = \frac{\sum x_i}{n} = \frac{219.3}{27} = 8.122$

State which estimator you used.

- ○ $\hat{p}$
- ○ $s$
- ● $\bar{x}$
- ○ $\tilde{x}$
- ○ $s / \bar{x}$

(b) Calculate a point estimate of the strength value that separates the weakest 50% of all such beams from the strongest 50%.

☐ MPa $\tilde{x} = \text{median} = 7.7$

State which estimator you used.

- ○ $\bar{x}$
- ● $\tilde{x}$
- ○ $s / \bar{x}$
- ○ $s$
- ○ $\hat{p}$

(c) Calculate a point estimate of the population standard deviation $\sigma$. [*Hint*: $\Sigma x_i^2 = 1854.91$.] (Round your answer to three decimal places.)

☐ MPa $s = \sqrt{\frac{\sum x_i^2 - (\sum x_i)^2/n}{n-1}} = \sqrt{\frac{1{,}854.91 - (219.3)^2/27}{27-1}} = 1.684$

Interpret this point estimate.

- ○ This estimate describes the bias of the data.
- ● This estimate describes the spread of the data.
- ○ This estimate describes the linearity of the data.
- ○ This estimate describes the center of the data.

Which estimator did you use?

- ○ $\bar{x}$
- ○ $\hat{p}$
- ● $s$
- ○ $s / \bar{x}$
- ○ $\tilde{x}$

(d) Calculate a point estimate of the proportion of all such beams whose flexural strength exceeds 10 MPa. [*Hint*: Think of an observation as a "success" if it exceeds 10.] (Round your answer to three decimal places.)

☐ $\hat{p} = \frac{x}{n} = \frac{4}{27} = 0.148$

(e) Calculate a point estimate of the population coefficient of variation $\sigma/\mu$. (Round your answer to four decimal places.)

☐ $\frac{s}{\bar{x}} = \frac{1.684}{8.122} = 0.2073$

State which estimator you used.

- ○ $\hat{p}$
- ○ $\bar{x}$
- ○ $s$
- ○ $\tilde{x}$
- ● $s / \bar{x}$

**17. [–/6 Points]** DETAILS **DEVORESTAT9 6.1.007.** MY NOTES ASK YOUR TEACHER

(a) A random sample of 10 houses in a particular area, each of which is heated with natural gas, is selected and the amount of gas (therms) used during the month of January is determined for each house. The resulting observations are 82, 118, 125, 143, 155, 122, 99, 138, 109, 103. Let $\mu$ denote the average gas usage during January by all houses in this area. Compute a point estimate of $\mu$.

[ ] therms $\bar{x} = 119.4$

(b) Suppose there are 24,000 houses in this area that use natural gas for heating. Let $\tau$ denote the total amount of gas used by all of these houses during January. Estimate $\tau$ using the data of part (a).

[ ] therms $n\bar{x} = (24{,}000)(119.4) = 2{,}865{,}600$

What estimator did you use in computing your estimate?

- ○ $s$
- ○ $\hat{p}$
- ○ $\tilde{x}$
- ○ $s/\bar{x}$
- ● $n\bar{x}$

(c) Use the data in part (a) to estimate $p$, the proportion of all houses that used at least 100 therms.

[ ] $\hat{p} = \frac{x}{n} = \frac{8}{10} = 0.8$

(d) Give a point estimate of the population median usage (the middle value in the population of all houses) based on the sample of part (a).

[ ] therms $\tilde{x} = 120$

What estimator did you use?

- ○ $s$
- ○ $\bar{x}$
- ○ $s/\bar{x}$
- ● $\tilde{x}$
- ○ $\hat{p}$

**18. [–/4 Points]** DETAILS **DEVORESTAT9 7.1.001.MI.S.** MY NOTES ASK YOUR TEACHER

Consider a normal population distribution with the value of $\sigma$ known.

USE SALT

(a) What is the confidence level for the interval $\bar{x} \pm 2.88\sigma/\sqrt{n}$? (Round your answer to one decimal place.)

99.6 % $\Phi(2.88) = 99.8$ or $0.002$, $\frac{\alpha}{2} = \frac{1-CI}{2} = 0.002 \Rightarrow CI = 99.6\%$

(b) What is the confidence level for the interval $\bar{x} \pm 1.49\sigma/\sqrt{n}$? (Round your answer to one decimal place.)

86.4 % $\Phi(1.49) = 0.068$, $\frac{1-CI}{2} = 0.068 \Rightarrow CI = 86.4\%$

(c) What value of $z_{\alpha/2}$ in the CI formula below results in a confidence level of 99.7%? (Round your answer to two decimal places.)

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

$CL = 99.7\% = 0.997$

$z_{\alpha/2} =$ 2.97 $\frac{\alpha}{2} = \frac{1-CL}{2} = \frac{1-0.997}{2} = 0.0015$

invNorm(0.0015) = 2.97

(d) Answer the question posed in part (c) for a confidence level of 78%. (Round your answer to two decimal places.)

$z_{\alpha/2} =$ 1.23 $\frac{\alpha}{2} = \frac{1-0.78}{2} = 0.11$, invNorm(0.11) = 1.23

**19. [–/5 Points]** DETAILS **DEVORESTAT9 7.3.035.** MY NOTES ASK YOUR TEACHER

Silicone implant augmentation rhinoplasty is used to correct congenital nose deformities. The success of the procedure depends on various biomechanical properties of the human nasal periosteum and fascia. An article reported that for a sample of 16 (newly deceased) adults, the mean failure strain (%) was 26.0, and the standard deviation was 3.3.

(a) Assuming a normal distribution for failure strain, estimate true average strain in a way that conveys information about precision and reliability. (Use a 95% confidence interval. Round your answers to two decimal places.)

( 24.24 %, 27.76 %) $\bar{x} \pm$ t.inv.2t(1 − CI, n−1) $\cdot \frac{s}{\sqrt{n}}$

(b) Predict the strain for a single adult in a way that conveys information about precision and reliability. (Use a 95% prediction interval. Round your answers to two decimal places.)

( 18.75 %, 33.25 %) $\bar{x} \pm$ t.inv.2t(1 − CI, n−1) $\cdot s \cdot \sqrt{1+\frac{1}{n}}$

x

prediction interval

How does the prediction compare to the estimate calculated in part (a)?

- ○ The prediction interval is the same as the confidence interval in part (a).
- ○ The prediction interval is much narrower than the confidence interval in part (a).
- ● The prediction interval is much wider than the confidence interval in part (a).

You may need to use the appropriate table in the Appendix of Tables to answer this question.

**20. [–/4 Points]** DETAILS **DEVORESTAT9 8.2.022.** MY NOTES ASK YOUR TEACHER

To obtain information on the corrosion-resistance properties of a certain type of steel conduit, 45 specimens are buried in soil for a 2-year period. The maximum penetration (in mils) for each specimen is then measured, yielding a sample average penetration of $\bar{x} = 53.1$ and a sample standard deviation of $s = 4.7$. The conduits were manufactured with the specification that true average penetration be at most 50 mils. They will be used unless it can be demonstrated conclusively that the specification has not been met. What would you conclude? (Use $\alpha = 0.05$.)

State the appropriate null and alternative hypotheses.

- ○ $H_0$: $\mu \neq 50$
  $H_a$: $\mu > 50$
- ● $H_0$: $\mu = 50$
  $H_a$: $\mu > 50$
- ○ $H_0$: $\mu = 50$
  $H_a$: $\mu \neq 50$
- ○ $H_0$: $\mu > 50$
  $H_a$: $\mu = 50$

$n = 45$

$\bar{x} = 53.1$

$s = 4.7$

Calculate the test statistic and determine the $P$-value. (Round your test statistic to two decimal places and your $P$-value to four decimal places.)

$z =$ 4.42

$P$-value = 0

STAT → TESTS → Z-TEST

State the conclusion in the problem context.

- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that the true average penetration is more than 50 mils.
- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the true average penetration is more than 50 mils.
- ○ Do not reject the null hypothesis. There is not sufficient evidence to conclude that the true average penetration is more than 50 mils.
- ● Reject the null hypothesis. There is sufficient evidence to conclude that the true average penetration is more than 50 mils.

p-value & $\alpha$: $0 < 0.05$ ∴ reject $H_0$

**21. [–/4 Points]** DETAILS **DEVORESTAT9 8.SE.062.S.** MY NOTES ASK YOUR TEACHER

An article reports the following values for soil heat flux of eight plots covered with coal dust.

36.1 34.0 36.1 36.4 35.9 28.5 18.3 24.1

$n = 8$
$\bar{x} = 31.175$
$s = 6.857$

USE SALT

The mean soil heat flux for plots covered only with grass is 29.0. Assuming that the heat-flux distribution is approximately normal, does the data suggest that the coal dust is effective in increasing the mean heat flux over that for grass? Test the appropriate hypotheses using $\alpha = 0.05$.

State the appropriate hypotheses.

- ○ $H_0: \mu = 29$
  $H_a: \mu < 29$
- ○ $H_0: \mu = 29$
  $H_a: \mu \neq 29$
- ● $H_0: \mu = 29$
  $H_a: \mu > 29$
- ○ $H_0: \mu \neq 29$
  $H_a: \mu = 29$

Calculate the test statistic and determine the $P$-value. (Round your test statistic to two decimal places and your $P$-value to three decimal places.)

$t$ = 0.90 STAT→TESTS→T-TEST
$P$-value = 0.200

State the conclusion in the problem context.

- ● Do not reject the null hypothesis. There is not sufficient evidence to conclude that there was an increase in mean heat flux. P-value & $\alpha$: $0.200 > 0.05$ ∴ do not reject Ho
- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that there was an increase in mean heat flux.
- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that there was an increase in mean heat flux.
- ○ Reject the null hypothesis. There is sufficient evidence to conclude that there was an increase in mean heat flux.

**22. [–/5 Points]** DETAILS **DEVORESTAT9 9.1.010.** MY NOTES ASK YOUR TEACHER

An experiment was performed to compare the fracture toughness of high-purity 18 Ni maraging steel with commercial-purity steel of the same type. For $m = 32$ specimens, the sample average toughness was $\bar{x} = 65.1$ for the high-purity steel, whereas for $n = 37$ specimens of commercial steel $\bar{y} = 59.9$. Because the high-purity steel is more expensive, its use for a certain application can be justified only if its fracture toughness exceeds that of commercial-purity steel by more than 5. Suppose that both toughness distributions are normal.

(a) Assuming that $\sigma_1 = 1.4$ and $\sigma_2 = 1.1$, test the relevant hypotheses using $\alpha = 0.001$. (Use $\mu_1 - \mu_2$, where $\mu_1$ is the average toughness for high-purity steel and $\mu_2$ is the average toughness for commercial steel.)

State the relevant hypotheses.

- ○ $H_0: \mu_1 - \mu_2 = 5$
  $H_a: \mu_1 - \mu_2 < 5$
- ○ $H_0: \mu_1 - \mu_2 = 5$
  $H_a: \mu_1 - \mu_2 \neq 5$
- ○ $H_0: \mu_1 - \mu_2 = 5$
  $H_a: \mu_1 - \mu_2 \leq 5$
- ● $H_0: \mu_1 - \mu_2 = 5$
  $H_a: \mu_1 - \mu_2 > 5$

$m = 32$ $n = 37$
$\bar{x} = 65.1$ $\bar{y} = 59.9$
$\sigma_1 = 1.4$ $\sigma_2 = 1.1$

Calculate the test statistic and determine the $P$-value. (Round your test statistic to two decimal places and your $P$-value to four decimal places.)

$z$ = 0.65 STAT→TESTS→2-SampZTest
$P$-value = 0.2570 Note: subtract 5 from $\bar{x}_1$

State the conclusion in the problem context.

- ● Fail to reject $H_0$. The data does not suggest that the fracture toughness of high-purity steel exceeds that of commercial-purity steel by more than 5. P-value & $\alpha$: $0.2570 > 0.001$ ∴ fail to reject Ho
- ○ Reject $H_0$. The data does not suggest that the fracture toughness of high-purity steel exceeds that of commercial-purity steel by more than 5.
- ○ Fail to reject $H_0$. The data suggests that the fracture toughness of high-purity steel exceeds that of commercial-purity steel by more than 5.
- ○ Reject $H_0$. The data suggests that the fracture toughness of high-purity steel exceeds that of commercial-purity steel by more than 5.

(b) Compute $\beta$ for the test conducted in part (a) when $\mu_1 - \mu_2 = 6$. (Round your answer to four decimal places.)

$\beta$ =

$\Delta' = 6$, $\Delta_0 = 5$, $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{1.4^2}{32} + \frac{1.1^2}{37}} = 0.3065$, $z_\alpha = z_{0.001} = 3.09$, $\beta(\Delta') = \beta(6) = \Phi\left(z_\alpha - \frac{\Delta' - \Delta_0}{\sigma_{\bar{x}_1 - \bar{x}_2}}\right) = \Phi\left(3.09 - \frac{6-5}{0.3065}\right) = 0.4316$

**23. [–/4 Points]** DETAILS **DEVORESTAT9 9.5.063.** MY NOTES ASK YOUR TEACHER

Toxaphene is an insecticide that has been identified as a pollutant in the Great Lakes ecosystem. To investigate the effect of toxaphene exposure on animals, groups of rats were given toxaphene in their diet. A study reports weight gains (in grams) for rats given a low dose (4 ppm) and for control rats whose diet did not include the insecticide. The sample standard deviation for 21 female control rats was 30 g and for 20 female low-dose rats was 55 g. Does this data suggest that there is more variability in low-dose weight gains than in control weight gains? Assuming normality, carry out a test of hypotheses at significance level 0.05.

$\alpha = 0.05$

State the relevant hypotheses. (Use $\sigma_1$ for low-dose treatment and $\sigma_2$ for control condition.)

- ○ $H_0: \sigma_1^2 = \sigma_2^2$
  $H_a: \sigma_1^2 \geq \sigma_2^2$
- ● $H_0: \sigma_1^2 = \sigma_2^2$
  $H_a: \sigma_1^2 > \sigma_2^2$
- ○ $H_0: \sigma_1^2 = \sigma_2^2$
  $H_a: \sigma_1^2 < \sigma_2^2$
- ○ $H_0: \sigma_1^2 = \sigma_2^2$
  $H_a: \sigma_1^2 \neq \sigma_2^2$

$n_2 = 21$ $n_1 = 20$
$s_2 = 30$ $s_1 = 55$

Calculate the test statistic. (Round your answer to two decimal places.)

$f$ =

$f = \frac{s_1^2}{s_2^2} = \frac{(55)^2}{(30)^2} = 3.36$

What can be said about the $P$-value for the test?

- ○ $P$-value > 0.100
- ○ 0.050 < $P$-value < 0.100
- ○ 0.010 < $P$-value < 0.050
- ● 0.001 < $P$-value < 0.010
- ○ $P$-value < 0.001

STAT→TESTS→2-SampFTest
p-value = 0.0049

State the conclusion in the problem context.

- ○ Fail to reject $H_0$. The data suggests that there is more variability in the low-dose weight gains than in control weight gains.
- ● Reject $H_0$. The data suggests that there is more variability in the low-dose weight gains than in control weight gains. P-value & $\alpha$: $0.0049 < 0.05$ ∴ reject Ho
- ○ Reject $H_0$. The data does not suggest that there is more variability in the low-dose weight gains than in control weight gains.
- ○ Fail to reject $H_0$. The data does not suggest that there is more variability in the low-dose weight gains than in control weight gains.