Ch2 - Probability

Thursday, May 11, 2023 9:51 AM

1. [1/6 Points] DETAILS PREVIOUS ANSWERS **DEVORESTAT9 2.1.001.** MY NOTES ASK YOUR TEACHER

Four universities—1, 2, 3, and 4—are participating in a holiday basketball tournament. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in first-round games, and then 1 beats 3 and 2 beats 4). (Enter your answers in set notation. Enter EMPTY or ∅ for the empty set.)

(a) List all outcomes in $\mathcal{S}$.

$\mathcal{S}$ =

(b) Let A denote the event that 1 wins the tournament. List outcomes in A.

A =

(c) Let B denote the event that 2 gets into the championship game. List outcomes in B.

B =

(d) What are the outcomes in $A \cup B$?

$A \cup B$ =

What are the outcomes in $A \cap B$?

$A \cap B$ =

What are the outcomes in A'?

A' =

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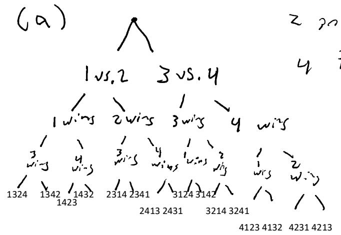
Topics:

(a)

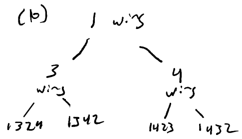


2 possibilities: win, lose

4 teams

$2^4 = 16$ total possibilities

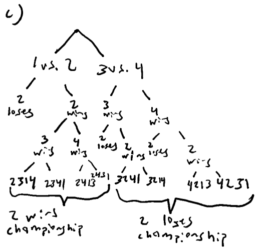
(b)



win/lose, 3 teams

$2^3 = 8$

(c)



(d)

Definition

1. The **complement** of an event A, denoted by A', is the set of all outcomes in $\mathcal{S}$ that are not contained in A.
2. The **union** of two events A and B, denoted by $A \cup B$ and read "A *or* B," is the event consisting of all outcomes that are *either in A or in B or in both events* (so that the union includes outcomes for which both A and B occur as well as outcomes for which exactly one occurs)—that is, all outcomes in at least one of the events.
3. The **intersection** of two events A and B, denoted by $A \cap B$ and read "A *and* B," is the event consisting of all outcomes that are in *both A and B*.

(b) Let A denote the event that 1 wins the tournament. List outcomes in A.

$A = \{1324, 1342, 1423, 1432\}$

(c) Let B denote the event that 2 gets into the championship game. List outcomes in B.

$B = \{2314, 2341, 2413, 2431, 3241, 3214, 4213, 4231\}$

$A \cup B = A$ or $B = \{1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3241, 3214, 4213, 4231\}$

Every event in A & every event in B is in $A \cup B$

$A \cap B = A$ & $B = \emptyset$ (null event, A & B are mutually exclusive or disjoint events)

A' = complement of $A = \{2314, 2341, 2413, 2431, 3124, 3142, 3241, 3214, 4123, 4132, 4231, 4213\}$

Every event not in A

2. [–/7 Points] DETAILS DEVORESTAT9 2.1.002. MY NOTES ASK YOUR TEACHER

Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles. (Enter your answers in set notation. Enter EMPTY or ∅ for the empty set.)

(a) List all outcomes in the event A that all three vehicles go in the same direction.

A =

(b) List all outcomes in the event B that all three vehicles take different directions.

B =

(c) List all outcomes in the event C that exactly two of the three vehicles turn right.

C =

(d) List all outcomes in the event D that exactly two vehicles go in the same direction.

D =

(e) List outcomes in D'.

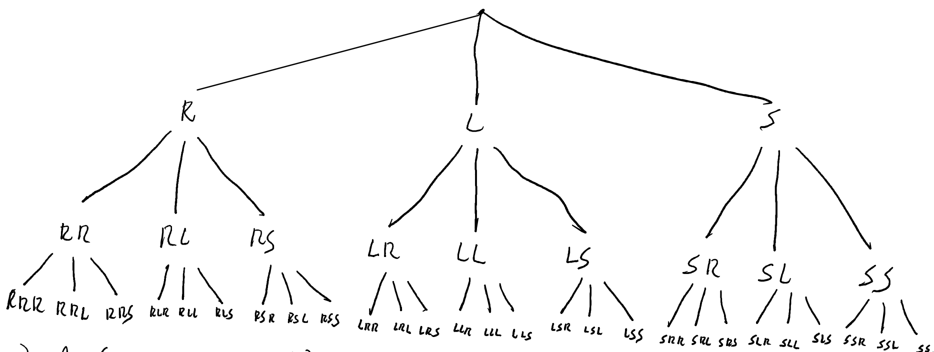
D' =

List outcomes in $C \cup D$.

$C \cup D$ =

List outcomes in $C \cap D$.

$C \cap D$ =



(a) A = {RRR, SSS, LLL}
same direction

(b) B = {RLS, RSL, LRS, LSR, SRL, SLR}
different direction

(c) C = {RRL, RRS, RLR, RSR, LRR, SRR}
two vehicles go right

(d) D = {RRL, RRS, RLR, RLL, RSR, RSS, LRR, LRL, LLR, LLS, LSL, LSS, SRR, SRS, SLL SLS SSR SSL}

(e) D' = {RRR, RLS, RSL, LRS, LLL, LSR, SRL, SLR, SSS}

C∪D = C or D = {RRL, RRS, RLR, RLL, RSR, RSS, LRR, LRL, LLR, LLS, LSL, LSS, SRR, SRS, SLL SLS SSR SSL}

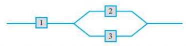
C∪D is the same as D b/c all event in C also occur in D

C∩D = {RRL, RRS RLR, RSR, LRR, SRR}

C∩D is the same as C b/c all events in C also occur in D (except D has additional events that do not occur in C)

3. [–/8 Points] DETAILS DEVORESTAT9 2.1.003. MY NOTES ASK YOUR TEACHER

Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2–3 subsystem.



The experiment consists of determining the condition of each component [*S* (success) for a functioning component and *F* (failure) for a nonfunctioning component]. (Enter your answers in set notation. Enter EMPTY or ∅ for the empty set.)

(a) Which outcomes are contained in the event *A* that exactly two of the three components function?

$A =$

(b) Which outcomes are contained in the event *B* that at least two of the components function?

$B =$

(c) Which outcomes are contained in the event *C* that the system functions?

$C =$

(d) List outcomes in C'.

$C' =$

List outcomes in $A \cup C$.

$A \cup C =$

List outcomes in $A \cap C$.

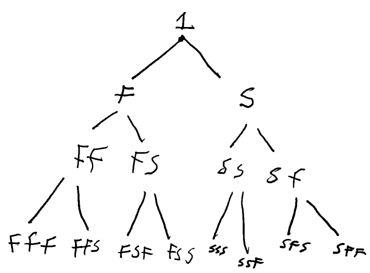
$A \cap C =$

List outcomes in $B \cup C$.

$B \cup C =$

List outcomes in $B \cap C$.

$B \cap C =$



(a)

$A = \{FSS, SSF, SFS\}$

outcomes where 2 out of 3 succeed

(b) $B = \{FSS, SSS, SSF, SFS\}$

outcomes where at least 2 out of 3 succeed

(c) B/c either one or both of component 2 & 3 must work for the system to function then,

$C = \{SSS, SSF, SFS\}$

are the outcomes where the system works

(d) $C' = \{FFF, FFS, FSF, FSS, SFF\}$

The complement of C is when the system does not work

$A \cup C = A$ or $C = \{FSS, SSF, SSS, SFS\}$

$A \cap C = A$ & $C = \{SSF, SFS\}$ (both events occur in A & C)

$B \cup C = B$ or $C = \{FSS, SSS, SSF, SFS\}$

$B \cap C = B$ & $C = \{SSF, SFS, SSS\}$

4. [–/3 Points] DETAILS **DEVORESTAT9 2.2.011.MI.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A mutual fund company offers its customers a variety of funds: a money-market fund, three different bond funds (short, intermediate, and long-term), two stock funds (moderate and high-risk), and a balanced fund. Among customers who own shares in just one fund, the percentages of customers in the different funds are as follows.

Money-market	23%	**High-risk stock**	16%
Short bond	14%	**Moderate-risk stock**	25%
Intermediate bond	7%	**Balanced**	10%
Long bond	5%		

A customer who owns shares in just one fund is randomly selected.

(a) What is the probability that the selected individual owns shares in the balanced fund?

0.10

(b) What is the probability that the individual owns shares in a bond fund?

$0.14+0.07+0.05=0.26$

(c) What is the probability that the selected individual does not own shares in a stock fund?

$1-(0.16+0.25) = 1-0.41 = 0.59$

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5. [–/2 Points] DETAILS **DEVORESTAT9 2.2.014.MI.** MY NOTES ASK

Suppose that 65% of all adults regularly consume coffee, 50% regularly consume carbonated soda, and 75% regularly consume at least one of these two products.

(a) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

$P(\text{coffee} \cap \text{soda}) = P(\text{coffee}) + P(\text{soda}) - P(\text{coffee} \cup \text{soda}) = 0.65 + 0.50 - 0.75 = 0.40$

(b) What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

$P'(\text{coffee} \cap \text{soda}) = 1 - P(\text{coffee} \cup \text{soda}) = 1 - 0.75 = 0.25$

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Proposition

For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A) + P(B) - P(A \cap B) = P(A \cup B)$

$-P(A \cap B) = P(A \cup B) - P(A) - P(B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

6. [–/4 Points] DETAILS **DEVORESTAT9 2.2.016.** MY NOTES ASK YOUR TEACHER

An individual is presented with three different glasses of cola, labeled C, D, and P. He is asked to taste all three and then list them in order of preference. Suppose the same cola has actually been put into all three glasses.

(a) What are the simple events in this ranking experiment? (Enter your answer in set notation.)

What probability would you assign to each one?

- The probability of an individual event where D is ranked first is $\frac{1}{12}$. The probability of another individual event is $\frac{1}{4}$.
- It is impossible to determine the probability of the simple events with the given information.
- The probability of an individual event where D is ranked first is $\frac{1}{5}$. The probability of another individual event is $\frac{1}{15}$.
- All of the simple events have the same probability, $\frac{1}{6}$.
- All of the simple events have the same probability, $\frac{1}{3}$.

(b) What is the probability that C is ranked first? (Round your answer to three decimal places.)

(c) What is the probability that C is ranked first and D is ranked last? (Round your answer to three decimal places.)

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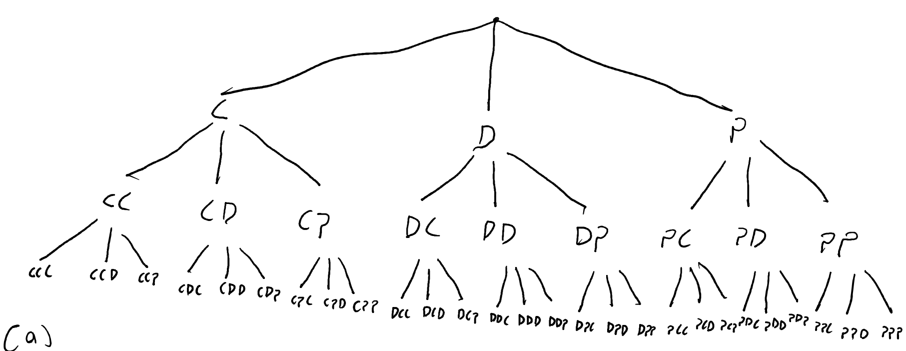
Definition

An **event** is any collection (subset) of outcomes contained in the sample space $\mathscr{S}$. An event is **simple** if it consists of exactly one outcome and **compound** if it consists of more than one outcome.



(a)

Simple (unique) events: { CDP CPD DC…

CC? C?D C?? DCC DCD DC? DDC DDD DD? D?C D?D D?? ?CC ?CD ?C? ?DC ?DD ?D? ??C ??D ???

(a)

Simple (unique) events: {CD?, C?D, DC?, D?C, ?CD, ?DC}

probability of simple events: 1/6

(b) probability that C is ranked first: $2/6 = 1/3 = 0.333$

(c) probability that C is ranked first & D is ranked last: $1/6 = 0.167$

7. [–/2 Points] DETAILS DEVORESTAT9 2.2.019. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Human visual inspection of solder joints on printed circuit boards can be very subjective. Part of the problem stems from the numerous types of solder defects (e.g., pad non-wetting, knee visibility, voids) and even the degree to which a joint possesses one or more of these defects. Consequently, even highly trained inspectors can disagree on the disposition of a particular joint. In one batch of 10,000 joints, inspector A found 742 that were judged defective, inspector B found 747 such joints, and 884 of the joints were judged defective by at least one of the inspectors. Suppose that one of the 10,000 joints is randomly selected.

(a) What is the probability that the selected joint was judged to be defective by neither of the two inspectors? (Enter your answer to four decimal places.)

$P'(A \cap B) = 1 - P(A \cup B) = 1 - 0.0884 = 0.9116$

(b) What is the probability that the selected joint was judged to be defective by inspector B but not by inspector A? (Enter your answer to four decimal places.)

$P(A' \cap B) = P(A \underset{or}{\cup} B) - P(A) = 0.0884 - 0.0742 = 0.0142$

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we use $P(A \cup B)$ b/c we want to exclude $P(A)$

$P(A) = \frac{742}{10,000} = 0.0742$ &

$A \cup B$ removes duplicate sets

$P(B) = \frac{747}{10,000} = 0.0747$

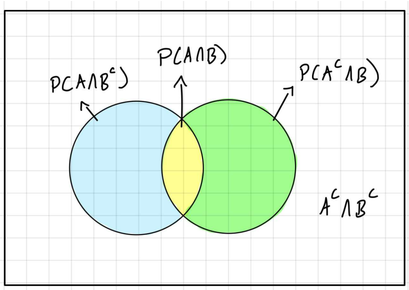
$P(A \cup B) = \frac{884}{10,000} = 0.0884$

Proof Note first that $A \cup B$ can be decomposed into two *disjoint* events, A and $B \cap A'$; the latter is the part of B that lies outside A (see Figure 2.4). Furthermore, B itself is the union of the two disjoint events $A \cap B$ and $A' \cap B$, so $P(B) = P(A \cap B) + P(A' \cap B)$. Thus

$$P(A \cup B) = P(A) + P(B \cap A') = P(A) + [P(B) - P(A \cap B)]$$
$$= P(A) + P(B) - P(A \cap B)$$

$P(A \overset{or}{\cup} B) = P(A) + P(\overset{not}{A'} \overset{\&}{\cap} B)$

$P(A \underset{or}{\cup} B) - P(A) = P(A' \underset{\&}{\cap} B)$



8. [–/5 Points] DETAILS DEVORESTAT9 2.3.030.MI. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A friend of mine is giving a dinner party. His current wine supply includes 7 bottles of zinfandel, 11 of merlot, and 13 of cabernet (he only drinks red wine), all from different wineries.

(a) If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?

$P_{k,n} = P_{3,7} = 210$

(b) If 6 bottles of wine are to be randomly selected from the 31 for serving, how many ways are there to do this?

$\binom{n}{k} = \binom{31}{6} = 736,281$

(c) If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?

combo of zinfandel · combo of merlot · combo of cabernet $= \binom{7}{2} \cdot \binom{11}{2} \cdot \binom{13}{2} = 90,090$

(d) If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen? (Round your answer to three decimal places.)

$\frac{\text{num. of outcomes for two bottles of each variety}}{\text{num. of ways to select 6 bottles}} = \frac{90,090}{736,281} = 0.122$

(e) If 6 bottles are randomly selected, what is the probability that all of them are the same variety? (Round your answer to three decimal places.)

$\frac{\text{combo of zinfandel}}{\text{combo of 6 bottles}} + \frac{\text{combo of merlot}}{\text{combo of 6 bottles}} + \frac{\text{combo of cabernet}}{\text{combo of 6 bottles}} = \frac{\binom{7}{6}}{\binom{31}{6}} + \frac{\binom{11}{6}}{\binom{31}{6}} + \frac{\binom{13}{6}}{\binom{31}{6}} = 0.003$

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Definition

An ordered subset is called a **permutation**. The number of permutations of size k that can be formed from the n individuals or objects in a group will be denoted by $P_{k,n}$. An unordered subset is called a **combination**. One way to denote the number of combinations is $C_{k,n}$, but we shall instead use notation that is quite common in probability books: $\binom{n}{k}$, read "n choose k."

Proposition

$$\binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$$

Notice that $\binom{n}{n} = 1$ and $\binom{n}{0} = 1$ since there is only one way to choose a set of (all) n elements or of no elements, and $\binom{n}{1} = n$ since there are n subsets of size 1.

Fundamental Counting Principle

The Fundamental Counting Principle is a method to determine the number of ways multiple independent events can occur.

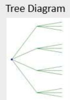
Event 1: a_1 ways
Event 2: a_2 ways
.
.
Event n: a_n ways

Total number of ways is $a_1 \bullet a_2 \dots \bullet a_n$

Example:

If event 1 can occur in 4 ways and event 2 can occur in 3 ways then the total number of ways that events 1 and 2 can occur is 3 x 4 = 12.

Tree Diagram



9. [–/5 Points] DETAILS DEVORESTAT9 2.3.035. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A production facility employs 10 workers on the day shift, 8 workers on the swing shift, and 6 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 24).

(a) How many selections result in all 6 workers coming from the day shift?

selections $\binom{n}{k} = \binom{10}{6} = 210$

What is the probability that all 6 selected workers will be from the day shift? (Round your answer to four decimal places.)

$$\frac{\text{ways to get 6 workers from day shift}}{\text{ways to get 6 workers from 24 total}} = \frac{\binom{10}{6}}{\binom{24}{6}} = 0.0016$$

(b) What is the probability that all 6 selected workers will be from the same shift? (Round your answer to four decimal places.)

$$\frac{\text{ways to get dayshift workers}}{\text{ways to get total workers}} + \frac{\text{ways to get swing shift workers}}{\text{ways to get total workers}} + \frac{\text{ways to get graveyard shift workers}}{\text{ways to get total workers}} = \frac{\binom{10}{6}}{\binom{24}{6}} + \frac{\binom{8}{6}}{\binom{24}{6}} + \frac{\binom{6}{6}}{\binom{24}{6}} = 0.0018$$

(c) What is the probability that at least two different shifts will be represented among the selected workers? (Round your answer to four decimal places.)

P(2 shifts of 6 workers) = 1 − P(1 shift of 6 workers) = 1 − 0.0018 = 0.9982

(d) What is the probability that at least one of the shifts will be unrepresented in the sample of workers? (Round your answer to four decimal places.)

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Proposition

For any three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

(d)
A = day shift is unrepresented   d = 10
B = swing shift is unrepresented   s = 8
C = graveyard shift is unrepresented   g = 6

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - \underbrace{P(A \cap B)}_{\text{graveyard}} - \underbrace{P(A \cap C)}_{\text{swing}} - \underbrace{P(B \cap C)}_{\text{day}} + \underbrace{P(A \cap B \cap C)}_{0}$$

$$= \frac{\binom{8+6}{6}}{\binom{24}{6}} + \frac{\binom{10+6}{6}}{\binom{24}{6}} + \frac{\binom{10+8}{6}}{\binom{24}{6}} - \frac{\binom{6}{6}}{\binom{24}{6}} - \frac{\binom{8}{6}}{\binom{24}{6}} - \frac{\binom{10}{6}}{\binom{24}{6}} + 0 = 0.2180$$

10. [–/4 Points] DETAILS DEVORESTAT9 2.3.039. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A box in a supply room contains 21 compact fluorescent lightbulbs, of which 7 are rated 13-watt, 9 are rated 18-watt, and 5 are rated 23-watt. Suppose that three of these bulbs are randomly selected. (Round your answers to three decimal places.)

2 same, 1 different

(a) What is the probability that exactly two of the selected bulbs are rated 23-watt?

$$\frac{\text{ways to select two 23-watt lightbulbs}}{\text{total num. of outcomes}} = \frac{\binom{5}{2} \cdot \binom{7+9}{1}}{\binom{21}{3}} = 0.120$$

(b) What is the probability that all three of the bulbs have the same rating?

$$\frac{\text{3 13-watt lights combo}}{\text{total num. of outcomes}} + \frac{\text{3 18-watt lights combo}}{\text{total num. of outcomes}} + \frac{\text{3 23-watt lights}}{\text{total num. of outcomes}} = \frac{\binom{7}{3}}{\binom{21}{3}} + \frac{\binom{9}{3}}{\binom{21}{3}} + \frac{\binom{5}{3}}{\binom{21}{3}} = 0.097$$

(c) What is the probability that one bulb of each type is selected?

$$\frac{\text{each type combo}}{\text{total num. of outcomes}} = \frac{\binom{7}{1} \cdot \binom{9}{1} \cdot \binom{5}{1}}{\binom{21}{3}} = 0.237$$

(d) If bulbs are selected one by one until a 23-watt bulb is obtained, what is the probability that it is necessary to examine at least 6 bulbs?

$$\frac{\text{combo of bulbs not 23-watts}}{\text{6-1 sample combo with no 23-watt lights}} = \frac{\binom{7+9}{6-1}}{\binom{21}{6-1}} = 0.215$$

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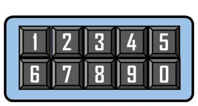
How many different 4-digit combinations can be made on this keypad?



Does order matter?

$$\textit{Number of Unique Choices} = \textit{Product Rule For Counting}$$
$$= \textit{No. of } 1^{st} \textit{ Choices} \times \textit{No. of } 2^{nd} \textit{ Choices} \times \dots$$
$$= 10 \times 10 \times 10 \times 10$$
$$= 10000$$

11. [–/4 Points] DETAILS **DEVORESTAT9 2.4.048.** MY NOTES ASK YOUR TEACHER P

A certain system can experience three different types of defects. Let A_i (i = 1,2,3) denote the event that the system has a defect of type i. Suppose that the following probabilities are true.

$P(A_1) = 0.10$ $P(A_2) = 0.08$ $P(A_3) = 0.05$
$P(A_1 \cup A_2) = 0.12$ $P(A_1 \cup A_3) = 0.12$
$P(A_2 \cup A_3) = 0.11$ $P(A_1 \cap A_2 \cap A_3) = 0.01$

(a) Given that the system has a type 1 defect, what is the probability that it has a type 2 defect? (Round your answer to four decimal places.)

$$\underbrace{P(A_2|A_1)}_{\text{prob. of } A_2 \text{ given } A_1} = \frac{P(A_2 \cap A_1)}{P(A_1)} = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{P(A_1)+P(A_2)-P(A_1 \cup A_2)}{P(A_1)} = \frac{0.10+0.08-0.12}{0.10} = 0.60$$

(b) Given that the system has a type 1 defect, what is the probability that it has all three types of defects? (Round your answer to four decimal places.)

$$P(A_1 \cap A_2 \cap A_3 | A_1) = \frac{P[(A_1 \cap A_2 \cap A_3) \cap A_1]}{P(A_1)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{0.01}{0.10} = 0.10$$

(c) Given that the system has at least one type of defect, what is the probability that it has exactly one type of defect? (Round your answer to four decimal places.)

See below

(d) Given that the system has both of the first two types of defects, what is the probability that it does not have the third type of defect? (Round your answer to four decimal places.)

$$P(A_3' | A_1 \cap A_2) = 1 - P(A_3 | A_1 \cap A_2) = 1 - \frac{P(A_3 \cap (A_1 \cap A_2))}{P(A_1 \cap A_2)} = 1 - \frac{0.01}{0.10+0.08-0.12} = 0.8333$$

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Definition

For any two events A and B with $P(B) > 0$, the **conditional probability of A given that B has occurred** is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{2.3}$$

$$P(A \cap B) = P(B) \cdot P(A|B)$$

(c)

$P(\text{exactly one defect} \mid \text{at least one defect})$

$$= \frac{P(A_1 \cap A_2' \cap A_3')}{P(A_1 \cup A_2 \cup A_3)} + \frac{P(A_1' \cap A_2 \cap A_3')}{P(A_1 \cup A_2 \cup A_3)} + \frac{P(A_1' \cap A_2' \cap A_3)}{P(A_1 \cup A_2 \cup A_3)}$$

$$= \frac{P(A_1)+P(A_2)+P(A_3) - 2P(A_1 \cap A_2) - 2P(A_1 \cap A_3) - 2P(A_2 \cap A_3) + 3P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)}$$

$$= \frac{0.10+0.08+0.05-2(0.10+0.08-0.12)-2(0.10+0.05-0.12)-2(0.08+0.05-0.11)+3(0.01)}{0.10+0.08+0.05-(0.10+0.08-0.12)-(0.10+0.05-0.12)-(0.08+0.05-0.11)+0.01}$$

$= 0.3077$

12. [–/9 Points] DETAILS **DEVORESTAT9 2.4.050.** MY NOTES

A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.

Short-sleeved

	Pattern		
Size	Pl	Pr	St
S	0.04	0.02	0.05
M	0.05	0.06	0.12
L	0.03	0.07	0.08

Long-sleeved

	Pattern		
Size	Pl	Pr	St
S	0.03	0.02	0.03
M	0.07	0.12	0.07
L	0.04	0.02	0.08

(a) What is the probability that the next shirt sold is a medium, long-sleeved, print shirt?

0.12

(b) What is the probability that the next shirt sold is a medium print shirt?

$P(\text{med, print shirt}) = P(\text{med, print, long shirt}) + P(\text{med, print, short}) = 0.12 + 0.06 = 0.18$

(c) What is the probability that the next shirt sold is a short-sleeved shirt? A long-sleeved shirt?

short-sleeved shirt $0.04+0.02+0.05+0.05+0.06+0.12+0.03+0.07+0.08 = 0.52$

long-sleeved shirt $0.03+0.02+0.03+0.07+0.12+0.07+0.04+0.02+0.08 = 0.48$

(d) What is the probability that the size of the next shirt sold is medium?

$P(\text{med shirt}) = P(\text{med, short}) + P(\text{med, long}) = (0.05+0.06+0.12)+(0.07+0.12+0.07) = 0.49$

What is the probability that the pattern of the next shirt sold is a print?

$P(\text{print shirt}) = P(\text{print, short shirt}) + P(\text{print, long}) = (0.02+0.06+0.07)+(0.02+0.12+0.02) = 0.31$

(e) Given that the shirt just sold was a short-sleeved plaid, what is the probability that its size was medium? (Round your answer to three decimal places.)

$$P(\text{med} | \text{short} \cap \text{plaid}) = \frac{P(\text{med} \cap (\text{short} \cap \text{plaid}))}{P(\text{short} \cap \text{plaid})} = \frac{0.05}{0.04+0.05+0.03} = 0.4167$$

(f) Given that the shirt just sold was a medium plaid, what is the probability that it was short-sleeved? Long-sleeved? (Round your answer to three decimal places.)

short-sleeved

long-sleeved

$$P(\text{short} | \text{medium} \cap \text{plaid}) = \frac{P(\text{short} \cap (\text{med} \cap \text{plaid}))}{P(\text{med} \cap \text{plaid})} = \frac{0.05}{0.05+0.07} = 0.4167$$

$$P(\text{long} | \text{med} \cap \text{plaid}) = \frac{P(\text{long} \cap (\text{med} \cap \text{plaid}))}{P(\text{med} \cap \text{plaid})} = \frac{0.07}{0.05+0.07} = 0.5833$$

13. [–/4 Points] DETAILS **DEVORESTAT9 2.5.071.MI.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

An oil exploration company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.9$.

(a) If the Asian project is not successful, what is the probability that the European project is also not successful?

$P(B'|A') = P(B') = 1 - P(B) = 1 - 0.9 = 0.1$

Explain your reasoning.

- ● Since the events are independent, then A' and B' are independent.
- ○ Since the events are not independent, then A' and B' are mutually exclusive.
- ○ Since the events are independent, then A' and B' are mutually exclusive.
- ○ Since the events are independent, then A' and B' are not independent.

$P(A \cap B) = P(A) \cdot P(B)$ b/c they're ind.

(b) What is the probability that at least one of the two projects will be successful?

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A) \cdot P(B) = 0.5 + 0.9 - (0.5 \cdot 0.9)$
$= 0.95$

(c) Given that at least one of the two projects is successful, what is the probability that only the Asian project is successful? (Round your answer to three decimal places.)

$$P(AB' | A \cup B) = \frac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \frac{P(AB')}{P(A \cup B)} = \frac{P(A) \cdot P(B')}{P(A \cup B)} = \frac{0.5 \cdot (1-0.9)}{0.95} = 0.053$$

Definition

Two events A and B are **independent** if $P(A|B) = P(A)$ and are **dependent** otherwise.

Proposition

A and B are independent if and only if (iff)

$$P(A \cap B) = P(A) \cdot P(B) \quad \textbf{(2.8)}$$

The verification of this multiplication rule is as follows:

$$P(A \cap B) = P(A|B) \cdot P(B) = P(A) \cdot P(B) \quad \textbf{(2.9)}$$

14. [–/2 Points] DETAILS **DEVORESTAT9 2.5.077.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

An aircraft seam requires 21 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability. (Round your answers to four decimal places.)

(a) If 22% of all seams need reworking, what is the probability that a rivet is defective?

(b) How small should the probability of a defective rivet be to ensure that only 11% of all seams need reworking?

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(a) $0.22 = P(A) = 1 - P(A')$ A: event seam needs reworking

$0.22 = 1 - (\text{1st not working} \cap \text{2nd not working} \cap \ldots \text{21st not working})$ A': event seam does not need fixed

$0.22 = 1 - (1-p)^{21}$

$1 - (1-p)^{21} = 0.22$

$-(1-p)^{21} = -0.78$

$(1-p)^{21} = 0.78$

$1 - p = 0.9882$

$-p = -0.0118$

$p = 0.0118$

(b) $P(A) = 1 - (1-p)^{21}$

$0.11 = 1 - (1-p)^{21}$

$1 - (1-p)^{21} = 0.11$

$-(1-p)^{21} = -0.89$

$(1-p)^{21} = 0.89$

$1 - p = 0.9945$

$-p = -0.0055$

$p = 0.0055$

Definition

Events $A_1, \ldots, A_n$ are **mutually independent** if for every k ($k = 2, 3, \ldots, n$) and every subset of indices $i_1, i_2, \ldots, i_k$,

$$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdot \cdots \cdot P(A_{i_k})$$

15. [–/10 Points] DETAILS **DEVORESTAT9 2.5.086.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A lumber company has just taken delivery on a shipment of 10,000 2 × 4 boards. Suppose that 30% of these boards (3000) are actually too green to be used in first-quality construction. Two boards are selected at random, one after the other. Let A = {the first board is green} and B = {the second board is green}.

(a) Compute $P(A)$, $P(B)$, and $P(A \cap B)$ (a tree diagram might help). (Round your answer for $P(A \cap B)$ to five decimal places.)

$P(A)$ = $P(A) = \frac{3{,}000}{10{,}000} = 0.30$

$P(B)$ = $P(B) = [\text{first board was green}] + [\text{first board wasn't green}] = [P(A) \cdot P(B|A)] + [P(A') \cdot P(B|A')] = \left[0.30 \cdot \left(\frac{3{,}000-1}{10{,}000-1}\right)\right] + \left[(1-0.30) \cdot \left(\frac{3{,}000}{10{,}000-1}\right)\right] = 0.30$

$P(A \cap B)$ = $P(A \cap B) = P(A) \cdot P(B|A) = 0.30 \cdot \left(\frac{3{,}000-1}{10{,}000-1}\right) = 0.08998$

Are A and B independent?

- ○ Yes, the two events are independent.
- ● No, the two events are not independent.

ind if $P(B) = P(B|A)$

$0.30 \neq 0.08998$ ∴ not ind.

(b) With A and B independent and $P(A) = P(B) = 0.3$, what is $P(A \cap B)$?

$P(A \cap B) = P(A) \cdot P(B) = (0.3)(0.3) = 0.09$

How much difference is there between this answer and $P(A \cap B)$ in part (a)?

- ○ There is no difference.
- ● There is very little difference.
- ○ There is a very large difference.

For purposes of calculating $P(A \cap B)$, can we assume that A and B of part (a) are independent to obtain essentially the correct probability?

- ● Yes
- ○ No

(c) Suppose the lot consists of ten boards, of which three are green. Does the assumption of independence now yield approximately the correct answer for $P(A \cap B)$?

- ○ Yes
- ● No

$P(A) = \frac{3}{10} = 0.30$

$P(B) = P(A) \cdot P(B|A) + P(A') \cdot P(B|A') = \frac{3}{10} \cdot \frac{(3-1)}{(10-1)} + \left(1 - \frac{3}{10}\right) \cdot \frac{3}{(10-1)} = 0.30$

$0.0667 \neq 0.09$ $\quad P(A \cap B) = P(A) \cdot P(B|A) = \frac{3}{10} \cdot \frac{(3-1)}{(10-1)} = 0.0667$, if ind. would have been $P(A \cap B) = P(A) \cdot P(B) = 0.30 \cdot 0.30 = 0.09$

What is the critical difference between the situation here and that of part (a)?

- ○ The critical difference is that the percentage of green boards is smaller in part (a).
- ● The critical difference is that the population size in part (a) is huge compared to the random sample of two boards.
- ○ The critical difference is that the percentage of green boards is larger in part (a).
- ○ The critical difference is that the population size in part (a) is small compared to the random sample of two boards.

When do you think that an independence assumption would be valid in obtaining an approximately correct answer to $P(A \cap B)$?

- ○ This assumption would be valid when there are fewer green boards in the sample.
- ○ This assumption would be valid when there are more green boards in the sample.
- ● This assumption would be valid when the population is much larger than the sample size.
- ○ This assumption would be valid when the sample size is very large.