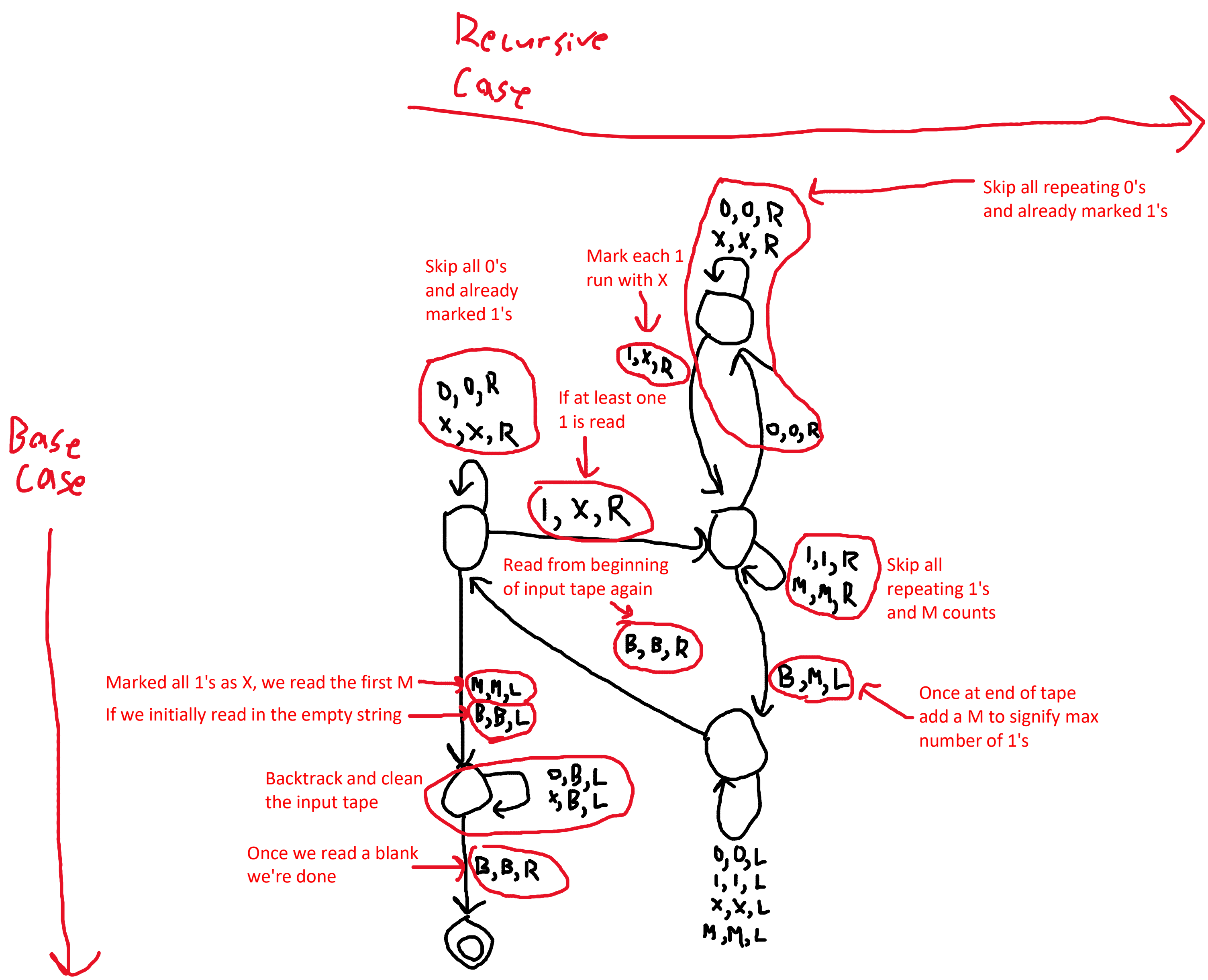
Question 1

Tuesday, December 3, 2024 4:29 PM

Q1 Devise a TM that takes in a string and returns the length of the longest run of ones in the string. For example, if the input is 101101110001111, then the output must be 4 in some representation of your choice. **[6 points]**

M is the max count, which will be the output of the Turing Machine

Example:
Input: 10110<u>111</u>
Output: MMM
Our final result was 3, the correct output



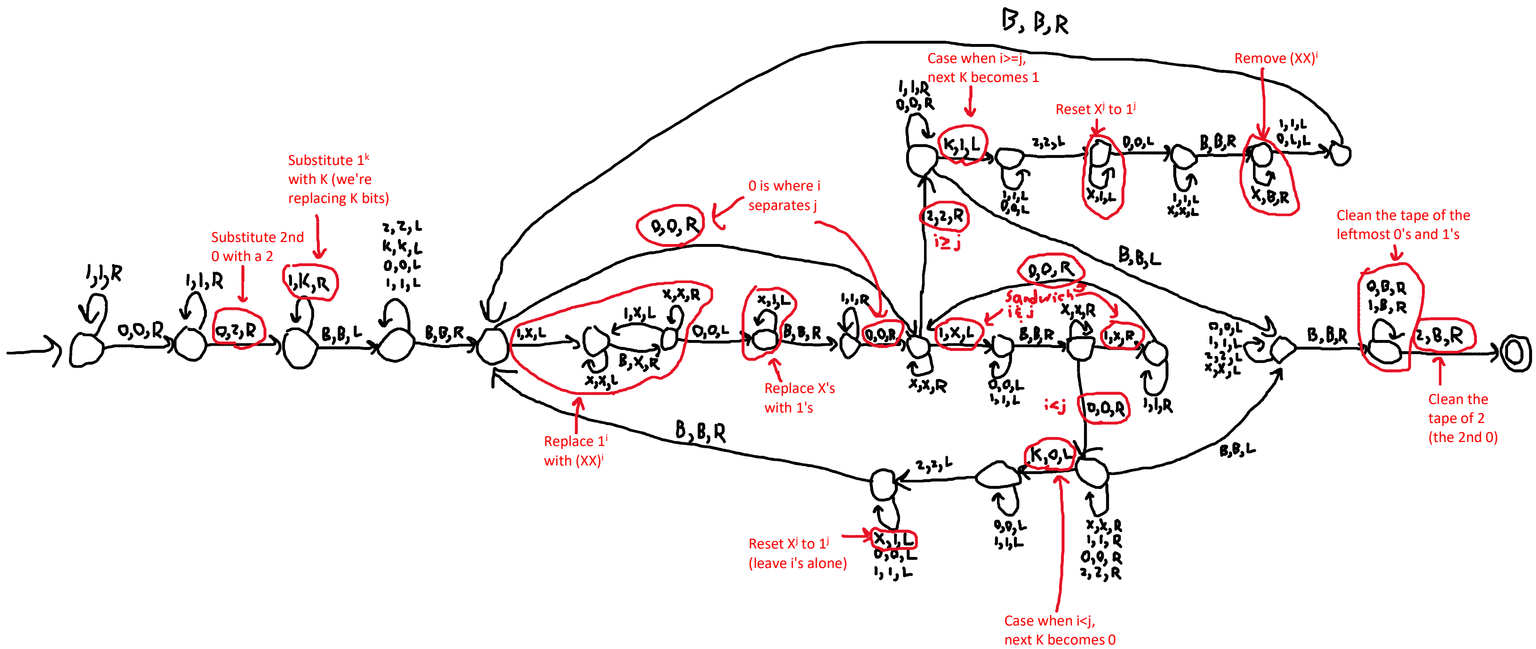
Question 2

Wednesday, December 4, 2024 1:13 PM

Q2 Assume one represents three positive numbers i, j, k using the string $1^i01^j01^k$. Suppose that $i < j$. Devise a TM whose input is three positive numbers i, j, k in the form of the string $1^i01^j01^k$ and returns k bits in the binary representation of $\frac{i}{j}$. For example, if $i = 3$, $j = 5$ and $k = 6$, the output must be 6 bits of the fraction $0.6 = \frac{3}{5}$ and therefore must be 100110, since $(0.6)_{10} = (0.\overline{1001})_2$. Assume that the TM is always presented a valid string of the correct format and with $i < j$, i.e., the machine can behave erratically/unexpectedly when the input is not valid. **[6 points]**

No empty string allowed

Output of the TM will be the final string in binary as K becomes a binary string (assuming i < j in the input)



Question 3

Wednesday, December 4, 2024 1:13 PM

Q3 Devise a TM that **decides**

$$L = \{0^i 1^j 2^k 3^\ell : i, j, k, \ell \geq 0 \text{ and } i - 2j = 2k - 3\ell\}.$$

[5 points]

Deriving the count:
$i - 2j = 2k - 3l$
$i - 2j - 2k + 3l = 0$

Count:
1. Increment i (0) by 1
2. Decrement j (1) by 2
3. Decrement k (2) by 2
4. Increment l (3) by 3

We use Y and Z to track the count of the TM

Same idea as question 7 from Assignment 4 on PDAs (increment/decrement)

1. Case for 0:

Add 1 Y (no Z's available yet)

2. Cases for 1:

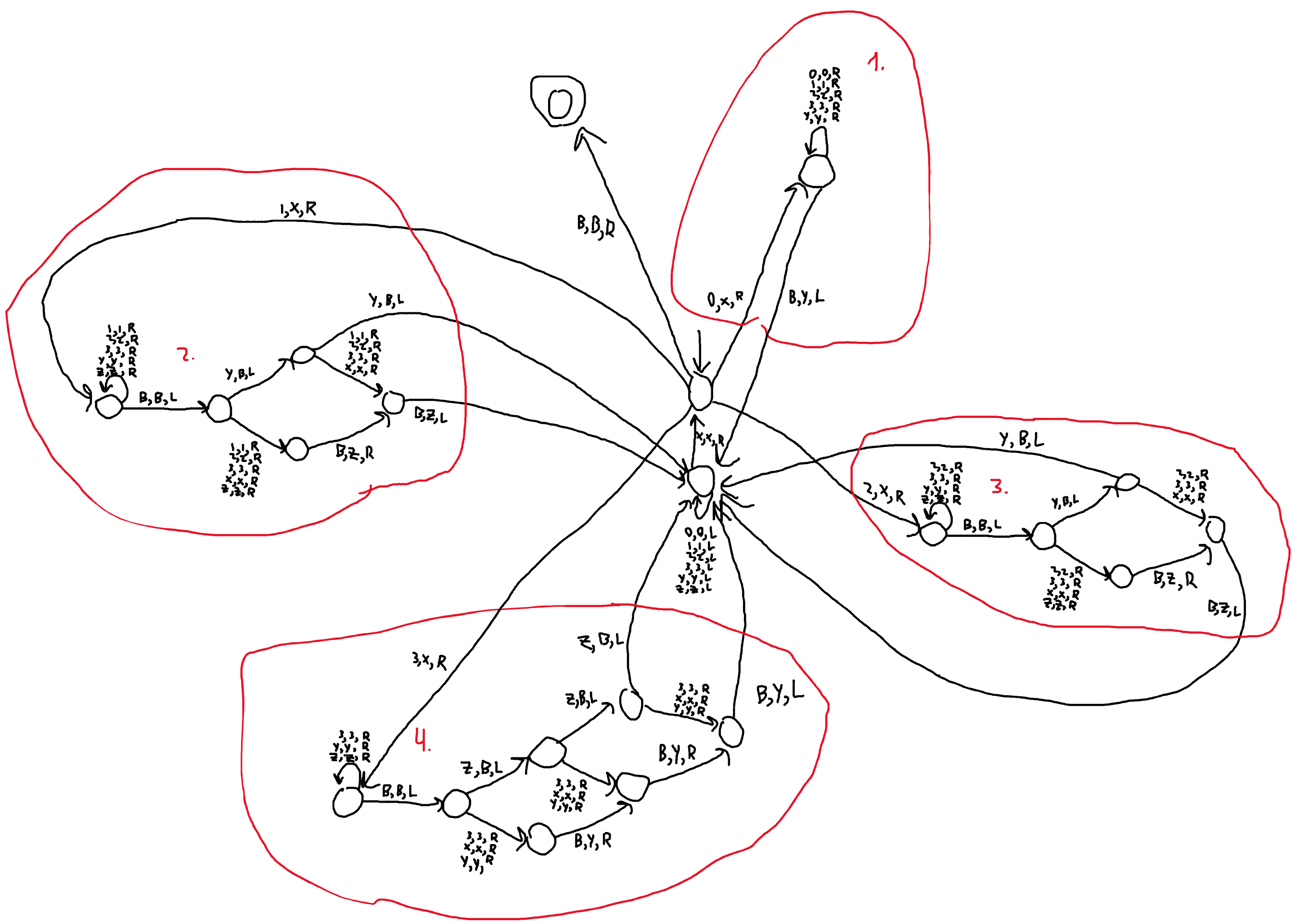
Pop 2 Y's (001)
Pop 1 Y, add 1 Z (01)
Add 2 Z's (1)

3. Cases for 2:

Pop 2 Y's (00…2)
Pop 1 Y, add 1 Z (0…2)
Add 2 Z's (…2)

4. Cases for 3:

Pop 3 Z's (0…11 or 12 or 22…3)
Pop 2 Z's, add 1 Y
Pop 1 Z, add 2 Y's
Add 3 Y's



Question 4

Wednesday, December 4, 2024 6:50 PM

Q4 Devise a TM whose input is a non-empty list of positive numbers $(i_1, i_2, \ldots, i_k)$ represented as the string $1^{i_1}01^{i_2}01^{i_3}\cdots01^{i_k}$ (note: k can be any positive number), and calculates the integer part (floor) of the average of the entries in the list. For example, if the list is (1,3,7), the TM must return $\lfloor\frac{1+3+7}{3}\rfloor = 3$, and if the list is (1,5,2,9), the TM must return $\lfloor\frac{1+5+2+9}{4}\rfloor = 4$. **[6 points]**

Non-empty list, empty string not allowed
Min case: 1
Other cases: 11, 111, 101, 1011

Example:
$(1, 3, 7) = 1^1 01^3 01^7$
$$\left\lfloor\frac{1+3+7}{3}\right\rfloor = \left\lfloor\frac{11}{3}\right\rfloor = 3$$

NOTE: r will be the remainder, every X will become an r (if floor(p/q) >= 1)