3.5 - Set identities

Wednesday, June 7, 2023 6:49 PM

Table 3.5.1: Set identities.

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Feedback?

EXERCISE 3.5.1: Name the set identity.

Name the set identity that is used to justify each of the identities given below.

(a) $(B \cap C) \cup \overline{B \cap C} = U$ Complement law

(b) $\overline{A \cup (A \cap B)} = \overline{A}$ Absorption law

(c) $A \cup (\overline{B \cap C}) = A \cup (\overline{B} \cup \overline{C})$ De Morgan's law

EXERCISE 3.5.2: Proving set identities.

Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

(a) $(\overline{A} \cap C) \cup (A \cap C) = C$

(b) $(B \cup A) \cap (\overline{B} \cup A) = A$

(a) $(\overline{A} \cap C) \cup (A \cap C)$

(a) $(A \cap C) \cup (A \cap C)$

$(C \cap \bar{A}) \cup (C \cap A)$ Commutative law

$C \cap (\bar{A} \cup A)$ Distributive law

$C \cap (A \cup \bar{A})$ Commutative law

$C \cap U$ complement law

C Identity law

$\therefore (\bar{A} \cap C) \cup (A \cap C) = C$

(b) $(B \cup A) \cap (\bar{B} \cup A)$

$(A \cup B) \cap (A \cup \bar{B})$ commutative law

$A \cup (B \cap \bar{B})$ Distributive law

$A \cup \emptyset$ Complement law

A Identity law

$\therefore (B \cup A) \cap (\bar{B} \cup A) = A$

3.6 - Cartesian products

Wednesday, June 7, 2023 7:03 PM

EXERCISE 3.6.2: Cartesian product of two small sets.

Define the sets X and Y as: X = {*, +, $} and Y = {52, 67}. Use the definitions for X and Y to answer the questions.

(a) Write the set X × Y using roster notation. $X \times Y = \{(*, 52), (*, 67), (+, 52), (+, 67), (\$, 52), (\$, 67)\}$

(b) Give an element of X^4. Express your answer as a 4-tuple, not as a string. $X^4 = X \times X \times X \times X = (*, +, \$, *), (*, *, \$, +), \ldots$

EXERCISE 3.6.6: Roster notation for sets defined using set builder notation and the Cartesian product.

Express the following sets using the roster method. Express the elements as strings, not n-tuples.

(a) $\{0x : x \in \{0, 1\}^2\}$ $\{0x : x \in \{0,1\}^2\} = \{000, 001, 010, 011\}$

(b) $\{0, 1\}^0 \cup \{0, 1\}^1 \cup \{0, 1\}^2$ $\{0,1\}^0 \cup \{0,1\}^1 \cup \{0,1\}^2 = \{\lambda\} \cup \{0,1\} \cup \{00, 01, 10, 11\} = \{\lambda, 0, 1, 00, 01, 10, 11\}$

$\{0,1\}^2 = \{0,1\} \cup \{0,1\} = \{00, 01, 10, 11\}$

3.7 - Partitions

Wednesday, June 7, 2023 7:31 PM

Two sets, A and B, are said to be ***disjoint*** if their intersection is empty ($A \cap B = \emptyset$). A sequence of sets, $A_1, A_2, \ldots, A_n$, is ***pairwise disjoint*** if every pair of distinct sets in the sequence is disjoint (i.e., $A_i \cap A_j = \emptyset$ for any i and j in the range from 1 through n where $i \neq j$).

A ***partition*** of a non-empty set A is a collection of non-empty subsets of A such that each element of A is in exactly one of the subsets. $A_1, A_2, \ldots, A_n$ is a partition for a non-empty set A if all of the following conditions hold:

- For all i, $A_i \subseteq A$.
- For all i, $A_i \neq \emptyset$
- $A_1, A_2, \ldots, A_n$ are pairwise disjoint.
- $A = A_1 \cup A_2 \cup \ldots \cup A_n$

$\subseteq$ (subset)

EXERCISE 3.7.1: Recognizing partitions - small finite sets.

Define the sets A, B, C, D, and E as follows:

- $A = \{1, 2, 6\}$
- $B = \{2, 3, 4\}$
- $C = \{5\}$
- $D = \{x \in \mathbf{Z}: 1 \leq x \leq 6\}$
- $E = \{x \in \mathbf{Z}: 1 < x < 6\}$

Use the definitions for A, B, C, D, and E to answer the questions.

(a) Do the sets A, B, and C form a partition of the set D? If not, which condition of a partition is not satisfied?

(b) Do the sets B and C form a partition of the set D? If not, which condition of a partition is not satisfied?

(c) Do the sets B and C form a partition of the set E? If not, which condition of a partition is not satisfied?

(a) The sets A, B, & C do not form a partition of the set D. $A \cap B = \{5\} \neq \emptyset$, the sets are not disjoint, $\therefore$ they do not form a partition.

(b) The sets B & C do not form a partition of the set D. $B \cup C = \{2, 3, 4, 5\} \neq \{x \in \mathbf{Z} : 1 \leq x \leq 6\} = D$, $\therefore$ the sets do not form a partition.

(c) $B \subseteq E$, $C \subseteq E$, $B \neq \{\emptyset\}$, $C \neq \{\emptyset\}$, $B \cap C = \{\emptyset\}$, $B \cup C = \{2, 3, 4, 5\} = \{x \in \mathbf{Z} : 1 < x < 6\} = E$ $\therefore$ the

$B \cup C = \{2,3,4,5\} = \{x \in \mathbf{Z} : 1 < x < 6\} = E$ $\therefore$ the sets form a partition.

EXERCISE 3.7.3: Recognizing partitions - the real numbers.

Define the sets A, B, C, D, and E as follows:

- $A = \{x \in \mathbf{R}: x < -2\}$
- $B = \{x \in \mathbf{R}: x > 2\}$
- $C = \{x \in \mathbf{R}: |x| < 2\}$ $= \{x \in \mathbf{R} : -2 < x < 2\}$
- $D = \{x \in \mathbf{R}: |x| \leq 2\}$ $= \{x \in \mathbf{R} : -2 \leq x \leq 2\}$
- $E = \{x \in \mathbf{R}: x \leq -2\}$

Use the definitions for A, B, C, D, and E to answer the questions.

(a) Do the sets A, B, and C form a partition of **R**? If not, which condition of a partition is not satisfied?

(b) Do the sets A, B, and D form a partition of **R**? If not, which condition of a partition is not satisfied?

(c) Do the sets B, D, and E form a partition of **R**? If not, which condition of a partition is not satisfied?

(a) The sets A, B, C do not form a partition of **R**. $A \cap B \cap C = \{-2, 2\} \neq \{\emptyset\}$, the sets are not disjoint, $\therefore$ they do not form a partition.

(b) $A \subseteq \mathbf{R}$, $B \subseteq \mathbf{R}$, $D \subseteq \mathbf{R}$, $A \neq \{\emptyset\}$, $B \neq \{\emptyset\}$, $D \neq \{\emptyset\}$, $A \cap B \cap D = \{\emptyset\}$, $A \cup B \cup D = \{x \in \mathbf{R} : -\infty < x < \infty\}$ $\therefore$ the sets form a partition.

(c) The sets B, D, & E do not form a partition of **R**. $D \cap E = \{-2\} \neq \emptyset$, the sets are not disjoint,

$D \cap E = \{-2\} \neq \emptyset$, the sets are not disjoint,
$\therefore$ they do not form a partition.