11.3 - The pigeonhole principle

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EXERCISE | 11.3.2: Generalized pigeonhole principle.

(a) There are 121.4 million people in the United States who earn an annual income that is at least $10,000$ and less than $1,000,000$ dollars. Annual income is rounded to the nearest dollar. Show that there are 123 people who earn the same annual income in dollars.

(b) Show that among a group of 621 people, there are at least 21 who are born on the same day of the month (e.g., the 21st or the 12th, etc.). Is the same fact true if there are only 620 people?

Feedback?

(a) $n = 121.4E6$ elements in domain (people in price range)

$k = 1{,}000{,}000 - 10{,}000 = 990{,}000 = 9.9E5$ elements in target (price range)

$$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{121.4E6}{9.9E5} \right\rceil = \lceil 122.63 \rceil = 123$$

∴ there are 123 people who earn the same annual income

(b)

$n = 621$ people

$k = 31$ days in month (max)

$$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{621}{31} \right\rceil = \lceil 20.003 \rceil = 21$$

∴ there are at least 21 people born on the same day of the month (for 621 people)

$n = 620$

$k = 31$

$$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{620}{31} \right\rceil = \lceil 20 \rceil = 20$$

∴ there are NOT at least 21 people

∴ there are NOT at least 21 people born on the same day of the month (for 620 people)