Ch7 - Statistical Intervals Based on a Single Sample

Thursday, June 22, 2023 11:53 AM

1. [–/4 Points] DETAILS **DEVORESTAT9 7.1.001.MI.S.**

Consider a normal population distribution with the value of σ known.

USE SALT

(a) What is the confidence level for the interval $\bar{x} \pm 2.88\sigma/\sqrt{n}$? (Round your answer to one decimal place.) (two tailed)

[ ] % $z_{\alpha/2} = 2.88$, $\frac{\alpha}{2} = 1 - \Phi(2.88) = 0.0020$, $\frac{\alpha}{2} = 0.0020 \Rightarrow \alpha = 0.0020(2) = 0.0040$, CI $= 1 - \alpha = 1 - 0.0040 = 0.996$ or 99.6%

(b) What is the confidence level for the interval $\bar{x} \pm 1.43\sigma/\sqrt{n}$? (Round your answer to one decimal place.)

[ ] % $z_{\alpha/2} = 1.43$, $\frac{\alpha}{2} = 1 - \Phi(1.43) = 0.0764$, $\frac{\alpha}{2} = 0.0764 \Rightarrow \alpha = 0.0764(2) = 0.1527$, CI $= 1 - \alpha = 1 - 0.1527 = 0.847$ or 84.7%

(c) What value of $z_{\alpha/2}$ in the CI formula below results in a confidence level of 99.7%? (Round your answer to two decimal places.)

$$\left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

$CI = \alpha - 1 \Rightarrow \alpha = 1 - CI = 1 - 0.997 = 0.003$, $\frac{\alpha}{2} = \frac{0.003}{2} = 0.0015$, $z_{\alpha/2} = \text{invNorm}(0.0015) = \pm 2.97$

$z_{\alpha/2}$ = [ ]

(d) Answer the question posed in part (c) for a confidence level of 70%. (Round your answer to two decimal places.)

$z_{\alpha/2}$ = [ ] $\alpha = 1 - CI = 1 - 0.70 = 0.30$, $\frac{\alpha}{2} = \frac{0.30}{2} = 0.15$, $z_{\alpha/2} = \text{invNorm}(0.15) = \pm 1.04$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It Watch It Master It

Topics:

7.1 Basic Properties of Confidence Intervals
- Interpreting a Confidence Level
- Other Levels of Confidence
- Confidence Level, Precision, and Sample Size
- Deriving a Confidence Interval
- Bootstrap Confidence Intervals

Exercises Section 7.1 (1–11)

7.2 Large-Sample Confidence Intervals for a Population Mean and Proportion
- A Large-Sample Interval for μ
- A General Large-Sample Confidence Interval
- A Confidence Interval for a Population Proportion
- One-Sided Confidence Intervals (Confidence Bounds)

7.3 Intervals Based on a Normal Population Distribution
- Properties of t Distributions
- The One-Sample t Confidence Interval
- A Prediction Interval for a Single Future Value
- Tolerance Intervals
- Intervals Based on Nonnormal Population Distributions

Exercises Section 7.3 (28–41)

7.4 Confidence Intervals for the Variance and Standard Deviation of a Normal Population

2. [–/2 Points] DETAILS **DEVORESTAT9 7.1.002.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Each of the following is a confidence interval for μ = true average (i.e., population mean) resonance frequency (Hz) for all tennis rackets of a certain type:

(113.4, 114.6) (113.1, 114.9)

(a) What is the value of the sample mean resonance frequency?

[114] Hz Lower & upper limits of confidence interval tend to 114

(b) Both intervals were calculated from the same sample data. The confidence level for one of these intervals is 90% and for the other is 99%. Which of the intervals has the 90% confidence level, and why?

- ○ The first interval has the 90% confidence level because it is a wider interval.
- ○ The second interval has the 90% confidence level because it is a narrower interval.
- ○ The second interval has the 90% confidence level because it is a wider interval.
- ● The first interval has the 90% confidence level because it is a narrower interval.

Need Help? Read It Narrower CI, smaller confidence level %

3. [–/9 Points] DETAILS **DEVORESTAT9 7.1.004.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A CI is desired for the true average stray-load loss μ (watts) for a certain type of induction motor when the line current is held at 10 amps for a speed of 1500 rpm. Assume that stray-load loss is normally distributed with $\sigma = 3.1$. (Round your answers to two decimal places.)

(a) Compute a 95% CI for μ when $n = 25$ and $\bar{x} = 53.6$.

([52.385] , [54.815]) watts TI-84: STAT → TESTS → ZInterval (7)

(b) Compute a 95% CI for μ when $n = 100$ and $\bar{x} = 53.6$.

([52.992] , [54.208]) watts

(c) Compute a 99% CI for μ when $n = 100$ and $\bar{x} = 53.6$.

([52.801] , [54.399]) watts

(d) Compute an 82% CI for μ when $n = 100$ and $\bar{x} = 53.6$.

([53.184] , [54.016]) watts

(e) How large must n be if the width of the 99% interval for μ is to be 1.0? (Round your answer up to the nearest whole number.) $w = 1.0$

n = [ ] $\alpha = 1 - CI = 1 - 0.99 = 0.01$, $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$, $z_{\alpha/2} = \text{invNorm}(0.005) = \pm 2.58$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It $n = \left(2 z_{\alpha/2} \cdot \frac{\sigma}{w}\right)^2 = \left(2 \cdot 2.58 \cdot \frac{3.1}{1}\right)^2 = 256$

A general formula for the sample size n necessary to ensure an interval width w is obtained from equating w to $2 \cdot z_{\alpha/2} \cdot \sigma/\sqrt{n}$ and solving for n.

The sample size necessary for the CI (7.5) to have a width w is

$$n = \left(2z_{\alpha/2} \cdot \frac{\sigma}{w}\right)^2$$

4. [0/6 Points] DETAILS | PREVIOUS ANSWERS | DEVORESTAT9 7.1.005. | MY NOTES | ASK YOUR TEACHER | P

Assume that the helium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with true standard deviation 0.76. $\sigma = 0.76$

(a) Compute a 95% CI for the true average porosity of a certain seam if the average porosity for 21 specimens from the seam was 4.85. (Round your answers to two decimal places.)

(4.52 , 5.18) $CI = 0.95,\ n = 21,\ \bar{x} = 4.85$

(b) Compute a 98% CI for true average porosity of another seam based on 13 specimens with a sample average porosity of 4.56. (Round your answers to two decimal places.)

(4.07 , 5.05) $CI = 0.98,\ n = 13,\ \bar{x} = 4.56$

(c) How large a sample size is necessary if the width of the 95% interval is to be 0.5? (Round your answer up to the nearest whole number.) $w = 0.5$

[ ] specimens $\alpha = 1 - CI = 1 - 0.95 = 0.05,\ \frac{\alpha}{2} = \frac{0.05}{2} = 0.025,\ z_{\alpha/2} = \text{invNorm}(0.025) = \pm 1.96,\ n = \left(2 z_{\alpha/2} \cdot \frac{\sigma}{w}\right)^2 = \left(2 \cdot 1.96 \cdot \frac{0.76}{0.5}\right)^2 = 36$

(d) What sample size is necessary to estimate true average porosity to within 0.23 with 99% confidence? (Round your answer up to the nearest whole number.)

[ ] ✖ specimens $z_{\alpha/2} = \pm 2.56$, $w = 2 \cdot 0.23 = 0.46,\ n = \left(2 z_{\alpha/2} \cdot \frac{\sigma}{w}\right)^2 = \left(2 \cdot 2.56 \cdot \frac{0.76}{0.46}\right)^2 = 73$

given $CI = 99\%$

5. [–/4 Points] DETAILS | DEVORESTAT9 7.2.013. | MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

$n = 40$ $\bar{x} = 654.16$ $\sigma = 165.4$

An article reported that for a sample of 40 kitchens with gas cooking appliances monitored during a one-week period, the sample mean CO_2 level (ppm) was 654.16, and the sample standard deviation was 165.4.

(a) Calculate and interpret a 95% (two-sided) confidence interval for true average CO_2 level in the population of all homes from which the sample was selected. (Round your answers to two decimal places.)

(602.90 , 705.42) ppm $CI = 0.95$

Interpret the resulting interval.

- ○ We are 95% confident that this interval does not contain the true population mean.
- ○ We are 95% confident that the true population mean lies below this interval.
- ○ We are 95% confident that the true population mean lies above this interval.
- ● We are 95% confident that this interval contains the true population mean.

$s = 171$ find n $w = 44$

(b) Suppose the investigators had made a rough guess of 171 for the value of s before collecting data. What sample size would be necessary to obtain an interval width of 44 ppm for a confidence level of 95%? (Round your answer up to the nearest whole number.)

[ ] kitchens

$CI = 0.95 \Rightarrow z_{\alpha/2} = \pm 1.96,\ n = \left(2 z_{\alpha/2} \cdot \frac{\sigma}{w}\right)^2 = \left(2 \cdot 1.96 \cdot \frac{171}{44}\right)^2 = 233$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It

6. [–/3 Points] DETAILS | DEVORESTAT9 7.2.015.MI.S. | MY NOTES

Determine the confidence level for each of the following large-sample one-sided confidence bounds. (Round your answers to the nearest whole number.)

USE SALT

(a) Upper bound: $\bar{x} + 1.04s/\sqrt{n}$

[ ] % $z_\alpha = 1.04 \Rightarrow \Phi(1.04) = 0.85 = 85\%$

(b) Lower bound: $\bar{x} - 2.33s/\sqrt{n}$ (double negative is pos.)

[ ] % B/c LB is $\bar{x} - z_\alpha \cdot \frac{s}{\sqrt{n}}$ then $z_\alpha = 2.33 \Rightarrow \Phi(2.33) = 0.99 = 99\%$

(c) Upper bound: $\bar{x} + 0.64s/\sqrt{n}$

[ ] % $z_\alpha = 0.64 \Rightarrow \Phi(0.64) = 0.74 = 74\%$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It | Watch It | Master It

7. [–/2 Points] DETAILS | DEVORESTAT9 7.2.019. | MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

$n = 356$ $x = 195$

An article reported that, in a study of a particular wafer inspection process, 356 dies were examined by an inspection probe and 195 of these passed the probe. Assuming a stable process, calculate a 95% (two-sided) confidence interval for the proportion of all dies that pass the probe. (Round your answers to three decimal places.) $CI = 0.95$

(0.496 , 0.599) STAT → TESTS → 1-PropZInt (A) $\hat{p} = \frac{x}{n}$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It

8. [–/5 Points] DETAILS DEVORESTAT9 7.3.028.MI.S.

Determine the values of the following quantities. (Round your answers to three decimal places.)

USE SALT

(a) $t_{0.10,\ 12}$
1.356

$\alpha = 0.10$, $df = 12$ Use TI-84: distr (2nd vars) → invT (4)

(b) $t_{0.05,\ 12}$
1.782

$\alpha = 0.05$, $df = 12$

(c) $t_{0.05,\ 23}$
1.714

$\alpha = 0.05$, $df = 23$

(d) $t_{0.05,\ 60}$
1.671

$\alpha = 0.05$, $df = 60$

(e) $t_{0.005,\ 60}$
2.660

$\alpha = 0.005$, $df = 60$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It Watch It Master It

9. [–/6 Points] DETAILS DEVORESTAT9 7.3.029.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Determine the t critical value(s) that will capture the desired t-curve area in each of the following cases. (Assume that central areas are centered at $t = 0$. Round your answers to three decimal places.)

USE SALT

Use invT dist.

(a) Central area = 0.95, df = 10
± 2.228

$\alpha = \frac{1-CI}{2} = \frac{1-0.95}{2} = 0.025$, $df = 10$

(b) Central area = 0.95, df = 30
± 2.042

$\alpha = 0.025$, $df = 30$

(c) Central area = 0.99, df = 30
± 2.750

$\alpha = \frac{1-CI}{2} = \frac{1-0.99}{2} = 0.005$, $df = 30$

(d) Central area = 0.99, df = 50
± 2.678

$\alpha = 0.005$, $df = 50$

(e) Upper-tail area = 0.01, df = 25
2.485

$\alpha = 1-0.01 = 0.99$, $df = 25$

(f) Lower-tail area = 0.025, df = 5
-2.571

$\alpha = 0.025$, $df = 5$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It

10. [–/6 Points] DETAILS DEVORESTAT9 7.3.030.S. MY N

Determine the t critical value for a two-sided confidence interval in each of the following situations. (Round your answers to three decimal places.)

USE SALT

Use invT dist.

(a) Confidence level = 95%, df = 5
2.571

$\alpha = \frac{1+CI}{2} = \frac{1+0.95}{2} = 0.975$, $df = 5$

(b) Confidence level = 95%, df = 20
2.086

$\alpha = 0.975$, $df = 20$

(c) Confidence level = 99%, df = 20
2.845

$\alpha = \frac{CI+1}{2} = \frac{0.99+1}{2} = 0.995$, $df = 20$

(d) Confidence level = 99%, n = 10
3.250

$\alpha = 0.995$, $df = n-1 = 10-1 = 9$

(e) Confidence level = 98%, df = 23
2.500

$\alpha = \frac{CI+1}{2} = \frac{0.98+1}{2} = 0.99$, $df = 23$

(f) Confidence level = 99%, n = 36
2.724

$\alpha = 0.995$, $df = n-1 = 36-1 = 35$

11. [–/6 Points] DETAILS **DEVORESTAT9 7.4.042.S.**

Determine the values of the following quantities. (Round your answers to two decimal places.)

USE SALT

Use chisq.inv(prob, df) in Excel

(a) $\chi^2_{0.05,\,5}$ 11.07 $\alpha = 1-0.05 = 0.95$, $df = 5$

(b) $\chi^2_{0.05,\,20}$ 31.41 $\alpha = 0.95$, $df = 20$

(c) $\chi^2_{0.01,\,20}$ 37.57 $\alpha = 1-0.01 = 0.99$, $df = 20$

(d) $\chi^2_{0.005,\,20}$ 40.00 $\alpha = 1-0.005 = 0.995$, $df = 20$

(e) $\chi^2_{0.99,\,20}$ 8.26 $\alpha = 1-0.99 = 0.01$, $df = 20$

(f) $\chi^2_{0.975,\,20}$ 9.59 $\alpha = 1-0.975 = 0.025$, $df = 20$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It

12. [–/4 Points] DETAILS **DEVORESTAT9 7.4.043.MI.S.**

Determine the following:

USE SALT

(a) The 95th percentile of the chi-squared distribution with $\nu = 13$ (Round your answer to three decimal places.)

22.362 $\alpha = 0.95$, $df = 13$, chisq.inv()

(b) The 5th percentile of the chi-squared distribution with $\nu = 13$ (Round your answer to three decimal places.)

5.892 $\alpha = 0.05$, $df = 13$

(c) $P(11.689 \le \chi^2 \le 38.076)$, where χ^2 is a chi-squared rv with $\nu = 23$ (Round your answer to two decimal places.)

0.95 dist $\rightarrow \chi^2$ cdf (8)

(d) $P(\chi^2 < 18.493$ or $\chi^2 > 43.773)$, where χ^2 is a chi-squared rv with $\nu = 30$ (Round your answer to two decimal places.)

$P(\chi^2 < 18.493) + P(\chi^2 > 43.773) = 0.05 + 0.05 = 0.10$

add χ^2 cdf's

You may need to use the appropriate table in the Appendix of Tables to answer this question.

13. [–/4 Points] DETAILS **DEVORESTAT9 7.4.044.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHI

The amount of lateral expansion (mils) was determined for a sample of $n = 7$ pulsed-power gas metal arc welds used in LNG ship containment tanks. The resulting sample standard deviation was $s = 2.82$ mils. Assuming normality, derive a 95% CI for σ^2 and for σ. (Round your answers to two decimal places.)

CI for σ^2 (3.30, 38.56) mils^2

CI for σ (1.82, 6.21) mils

$\chi^2_{\frac{\alpha}{2},n-1} = \chi^2_{\frac{0.05}{2},7-1} = \chi^2_{0.025,6} = \text{chisq.inv}(0.025, 6) = 1.237$

$\chi^2_{1-\frac{\alpha}{2},n-1} = \chi^2_{1-\frac{0.05}{2},7-1} = \chi^2_{0.975,6} = \text{chisq.inv}(0.975, 6) = 14.449$

$\alpha = 1 - CI = 1 - 0.95 = 0.05$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It Watch It

$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}} = \frac{(7-1)(2.82)^2}{1.237} = 38.56$ $\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}} = \frac{(7-1)(2.82)^2}{14.449} = 3.30$

$\sqrt{38.56} = 6.21$ $\sqrt{3.30} = 1.82$

The inequalities in (7.17) are equivalent to

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

Substituting the computed value s^2 into the limits gives a CI for σ^2, and taking square roots gives an interval for σ.