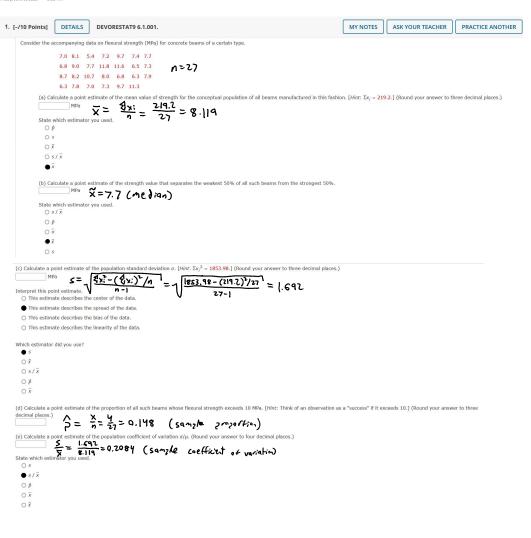
## Ch6 - Point Estimation

Eriday June 9, 2023 8:10 PM



## 2. [-/8 Points] DETAILS DEVORESTAT9 6.1.003.S. MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER Consider the following sample of observations on coating thickness for low-viscosity paint. 0.84 0.88 0.88 1.01 1.09 1.17 1.29 1.31 1.36 1.49 1.59 1.62 1.65 1.71 1.76 1.83 L USE SALT Assume that the distribution of coating thickness is normal (a normal probability plot strongly supports this assumption). (a) Calculate a point estimate of the mean value of coating thickness. (Round your answer to four decimal places.) $\overline{X} = \underbrace{\frac{1}{3}}_{1} \frac{X_1}{1} = \underbrace{\frac{21.98}{16}}_{16} = \underbrace{1.3405}_{1.3405}$ State which estimator you used. O s 0 s / x Op (b) Calculate a point estimate of the median of the coating thickness distribution. (Round $\hat{X}=1.3350$ Cn $\hat{T}$ evan $S^{\circ}$ we any) of the coating thickness distribution. (Round your answer to four decimal places.) State which estimator you used and which estimator you might have used instead. (Select all that apply.) ✓ x̃ □ p̂ □ s $\Box s/\bar{x}$ ☑ x

90th 20/Contin=4+5000 a= (3122+1.612.0.3321=1.8841)

ning 90%. [Hint: Express what you are trying to estimate in terms of  $\mu$  and

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(c) Calculate a point estimate of the value and operation of (Round your answer to four decimal places.)

2ap = inumary (0.90)=1.645

State which estimator you used.  $\bigcirc \overline{x}$   $\bigcirc$  10th percentile

(c) Calculate a point estimate of the value that separates the largest 10% of all values in the thickness distribution from the

## Topics

6.1 Some General Concepts of Point Estimation

Unbiased Estimators

Estimators with Minimum Variance

Some Complications

Reporting a Point Estimate: The Standard Error

## Exercises Section 6.1 (1-19)

6.2 Methods of Point Estimation

The Method of Moments

Maximum Likelihood Estimation

Estimating Functions of Parameters

Large Sample Behavior of the MLE

Large Sample Bellavi

Some Complications

□ p̂ □ s

□ s/x

☑ x

90th zercentin= M+Z.,000= 1.3425+1.645.0.3357=1.8947

○ 10th percentile

05

90th percentile

(d) Estimate P(X < 1.1), i.e., the proportion of all thickness values less than 1.1. [Hint: If you knew the values of  $\mu$  and  $\sigma$ , you could calculate this probability. These values are not available, but they can be estimated.] (Round your answer to four decimal places.) P(X < 1.1) = P(3 < 1.1 - 1.3425) = P(3 < -0.72) = 0.2350(e) What is the estimated standard error of the estimator that you used in part (b)? (Round your answer to four decimal places.)  $\frac{\sigma_X}{\sqrt{n}} = \frac{\sigma.3357}{\sqrt{n}} = 0.0839$ 

3. [-/7 Points] DEVORESTAT9 6.1.004.MI.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

5.7	7.2	7.3	6.3	8.1	6.8	7.0	7.4	6.8	6.5	7.0	6.3	7.9	9.0
8.4	8.7	7.8	9.7	7.4	7.7	9.7	8.0	7.7	11.6	11.3	11.8	10.7	

The data be

pelow gi	ve aco	ompan	ying s	trengt	n obse	rvation	s for cy	linders.		
6.5	5.8	7.8	7.1	7.2	9.2	6.6	8.3	7.0	8.1	n = 20
8.0	8.1	7.4	8.5	8.9	9.8	9.7	14.1	12.6	12.0	y=8.635

Prior to obtaining data, denote the beam strengths by  $X_1, ..., X_m$  and the cylinder strengths by  $Y_1, ..., Y_n$ . Suppose that the  $X_i$ 's constitute a random sample from a distribution with mean  $\mu_1$  and standard deviation  $\sigma_1$  and that the  $Y_i$ 's form a random sample (independent of the  $X_i$ 's) from another distribution with mean  $\mu_2$  and standard deviation  $\sigma_2$ .

1=27  $\bar{x} = 8.141$ 

(a) Use rules of expected value to show that  $\overline{X}-\overline{Y}$  is an unbiased estimator of  $\mu_1-\mu_2$ .

$$\bigcirc \ E(\overline{X}-\overline{Y}) = \left(E(\overline{X})-E(\overline{Y})\right)^2 = \mu_1 - \mu_2$$

$$\bigcirc \ E(\overline{X}-\overline{Y}) = \frac{E(\overline{X})-E(\overline{Y})}{nm} = \mu_1 - \mu_2$$

$$\bigcirc E(\overline{X} - \overline{Y}) = \sqrt{E(\overline{X}) - E(\overline{Y})} = \mu_1 - \mu_2$$

$$E(\overline{X} - \overline{Y}) = E(X) - E(Y) = \mu_1 - \mu_2$$

$$O(E(\overline{X} - \overline{Y})) = nm(E(\overline{X}) - E(\overline{Y})) = \mu_1 - \mu_2$$

Calculate the estimate (in MPa) for the given data. (Round your answer to three decimal places.)  $E(\vec{x}) - E(\vec{y}) = 8.141 - 8.635 = -0.499$ 

(b) Use rules of variance to obtain an expression for the variance and standard deviation (standard error) of the estimator in part (a)

$$V(X - Y) = V(X) + V(Y)$$

Identify the next step in this rule from the options below

$$\bigcirc V(\overline{X} - \overline{Y}) = \frac{{\sigma_1}^2}{n_1} - \frac{{\sigma_2}^2}{n_2}$$

$$\bigcirc V(\overline{X} - \overline{Y}) = \frac{\sigma_1}{n_1} - \frac{\sigma_2}{n_2}$$

$$\bigcirc V(\overline{X} - \overline{Y}) = \frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}$$

$$\sigma_{\overline{X}-\overline{Y}} = \sqrt{v(\overline{X}-\overline{Y})} \qquad \sigma_{\overline{I}} = 1.673, \quad \sigma_{\overline{z}} = 2.13q$$

$$\bullet \sigma_{\overline{X}-\overline{Y}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{(1.673)^3}{27} + \frac{(2.13q)^3}{20}} = 0.577$$

$$\circ \sigma_{\overline{X}-\overline{Y}} = \sqrt{\frac{\sigma_1}{n_1} - \frac{\sigma_2}{n_2}}$$

$$\circ \sigma_{\overline{X}-\overline{Y}} = \sqrt{\frac{\sigma_1^2}{n_1} - \frac{\sigma_2^2}{n_2}}$$

Compute the estimated standard error (in MPa). (Round your answer to three decimal places.)

(c) Calculate a point estimate of the ratio  $\sigma_1/\sigma_2$  of the two standard deviations. (Round your answer to three decimal places.)  $\frac{l.673}{2.134} = 0.782$ 

$$\frac{1.673}{2.139} = 0.782$$

(d) Suppose a <u>single beam and single cylinder</u> are randomly selected. Calculate a point estimate (in MPa<sup>2</sup>) of the variance of the difference X – Y between beam strength and cylinder strength. (Round your answer to two decimal places.)

MPa<sup>2</sup> V (X - Y) =  $\frac{\sigma_1^{Y}}{n_1} + \frac{\sigma_3^{Y}}{n_2} = \frac{1.673}{1} + \frac{2.139}{1} = 7.379$ 

MPa<sup>2</sup> 
$$V(\bar{x} - \bar{y}) = \frac{\sigma_1^2}{\sigma_1} + \frac{\sigma_3^2}{2\sigma_2} = \frac{1.673}{1} + \frac{2.139}{1} = 7.379$$

4. [-/6 Points] DETAILS DEVORESTAT9 6.1.005.

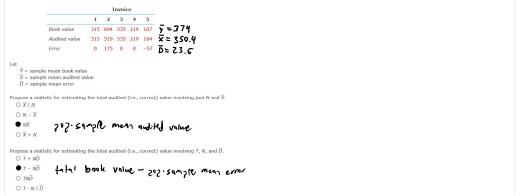
MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

As an example of a situation in which several different statistics could reasonably be used to calculate a point estimate, consider a population of N invoices. Associated with each invoice is its "book value," the recorded amount of that invoice. Let T denote the total book value, a known amount. Some of these book values are erroneous. An audit will be carried out by randomly selecting n invoices and determining audited (correct) value for each one. Suppose that the sample gives the following results (in dollars).

1 2 3 4 5

Book value 315 694 535 219 107 7 = 374

Audited value 315 519 535 219 164 X = 350.4



Propose a statistic for estimating the total audited (i.e., correct) value involving T and  $\overline{X}/\overline{Y}$ . O  $T - \overline{X}/\overline{Y}$ O T - T  $\overline{X}/\overline{Y}$ O T  $\overline{X}/\overline{Y$ 

