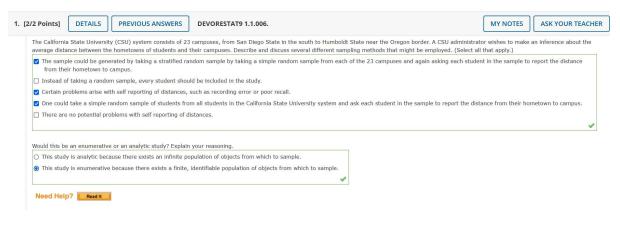
## Ch1 - Descriptive Statistics

Tuesday, May 9, 2023 4:42 PM



Analytical study: infinite population of objects to sample Enumerative study: finite, identifiable population of objects to sample

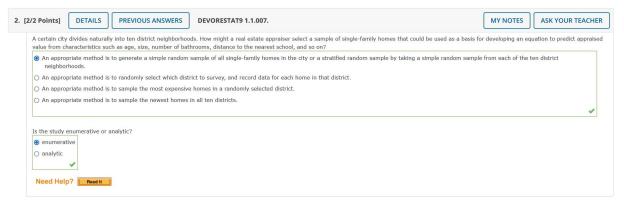
Sampling frame: a list of individuals or objects to be sampled Simple random sample: data collection that entails selecting individuals or objects from a

Stratified random sample: separating the population units into nonoverlapping groups and taking samples from each one

We could either take a stratified random sample (option 1) or a simple random sample (option 4), we cannot include every student in a sample (option 2).

Certain problems do arise when reporting distance (option 3), excluding option 5.

Because the population is finite and identifiable, this is an enumerative study.

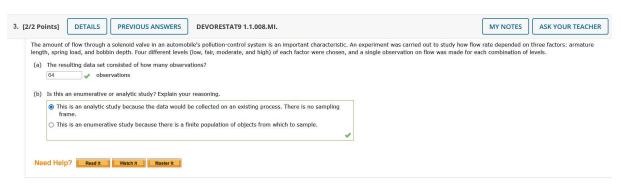


Option 4 is bad because it only includes new homes.

Option 3 is bad because it only includes the most expensive homes.

Option 2 is bad because it only includes one district (a single subpopulation).

Option 1 is best because we either take a simple random sample or stratified random sample with no constraints.



(a)

Three factors, four different level possibilities for each

Armature Length	Spring Load	Bobbin Depth
Low	Low	Low
Low	Fair	Low
Low	Low	Fair

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1.4 Measures of Variability

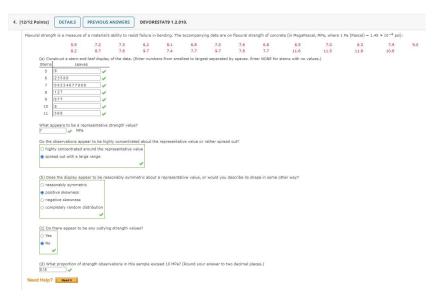
Measures of Variability for S

Motivation for s<sup>2</sup>
A Comparing Formula for s<sup>2</sup>
Bosplots
Bosplots That Show Outliers

Fair	Low	Low
Fair	Fair	Low
High	High	High

4 levels for Armature Length x 4 levels for Spring Load x 4 levels for Bobbin Depth =  $4^3$  = 64

Because we are trying to improve the flow rate of the solenoid valve in this study instead of on an unchanging population, we have an analytical study with no sampling frame.



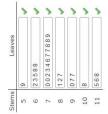
Our leaf plot is simply organizing our data from smallest to largest and seeing where it clusters based upon the leading digit (essentially the following below except just the leading digit)

```
11 11.5 11.6 11.8 🗶
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Our "representative value" is 7 because it occurs the most frequently (protruding out the most)

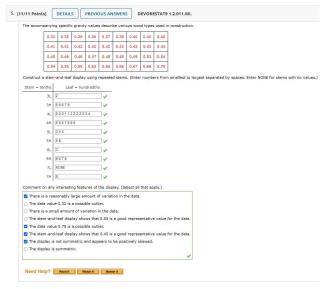
But our data is not concentrated largely around the center, it's spread out (8 has much fewer than the nearby representative value)

Our leaf plot has positive skewness (AKA right skewness), consider inverting the leaf plot to see this:



There are no values (leaves) completely disconnected completely such that their stems do not border other leaves (meaning we don't skip any spaces in between values).

4 strength values greater than 10 MPa: 10.8, 11.5, 11.6, 11.8 (4 strength values greater than 10 MPa) / (27 total strength values) = 4 / 27 = 0.15



We're only concerned with the rightmost digit in the stem-and-leaf plot.

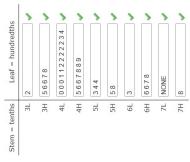
3L means 3-lower and 3H means 3-higher (0.32 is 3L, 0.35 is 3H, 0.38 is 3H, 4.0 is 4L, etc.)

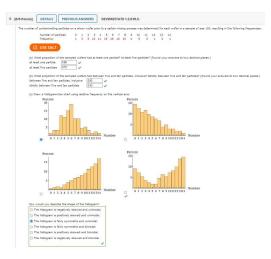
Our data is very spread out so it has a large amount of variation.

0.32 is not a possible outlier because it's not disconnected or 'stranded' from other nearby leaves. However 0.78 is a possible outlier because it is not near any other leaves and is 'stranded'.

 $0.45 \ (around\ the\ 4L\ stem)\ is\ a\ good\ representative\ value\ because\ the\ most\ frequent\ values\ are\ here\ and\ the\ leaf\ plot\ bulges\ out\ the\ furthest\ here.\ This\ is\ not\ true\ for\ 0.55.$ 

The display is positively skewed (right skewed) and therefore cannot be symmetric, this is visible by tilting the leaf plot:





(a)

All but 1 out of 100 wafers has at least one particle (one wafer has 0 particles). Therefore 99% or 0.99 of wafers has at least one particle.

Number of wafers with at least five particles = (15 wafers w/five particles) + (19 wafers w/six particles) + (10 wafers w/seven particles) + (10 wafers w/eight particles) + (4 wafers w/nine particles) + (5 wafers w/ten particles) + (3 wafers w/eleven particles) + (1 wafer w/twelve particles) + (2 wafers w/thirteen particles) + (1 wafer/fourteen particles). There are 70 out of 30 wafers that have at least five particles. Therefore 70% or 0.70 of wafers have at least five particles.

(b)

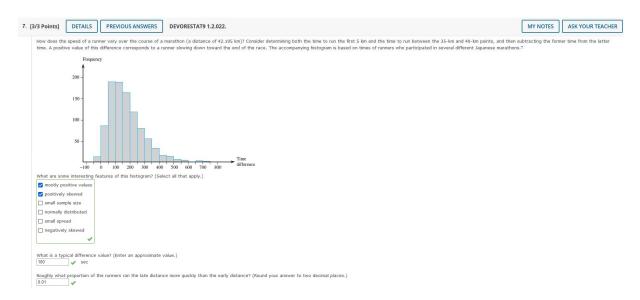
Number of wafers with between five and ten particles (inclusive) = (15 wafers w/five particles) + (19 wafers w/six particles) + (10 wafers w/seven particles) + (10 wafers w/eight particles) + (4 wafers w/nine particles) + (5 wafers w/ten particles). 15 + 19 + 10 + 10 + 4 + 5 = 63

Number of wafers with between five and ten particles (restricted/exclusive) = (19 wafers w/six particles) + (10 wafers w/seven particles) + (10 wafers w/eight particles) + (4 wafers w/nine particles)

19 + 10 + 10 + 4 = 43

(c)

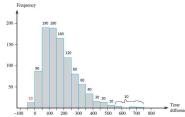
The frequency of the data is greatest at the center and only there as it spreads out further. Therefore the data is symmetric and unimodal.



The histogram is positively skewed (skewed rightwards) with mostly positive values.

The "typical" difference value is 100 sec because it's where most of the data is centered.

Only 1% or 0.01 of runners ran their last 35-km and 40-km faster than their first 5-km. This can be proven by the graph below.



Number of runners that ran their late distance slower than their early distance = 90 + 190 + 180 + 160 + 120 + 80 + 60 + 40 + 30 + 20 + 10 + 10 = 990

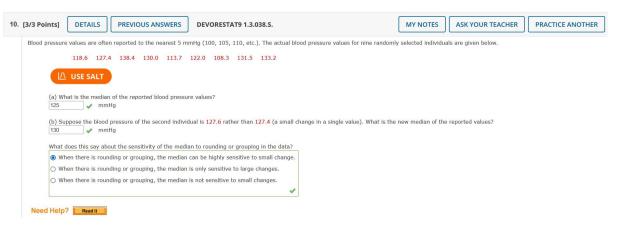
(Number of runners that ran their late distance more quickly than their early distance) / (number of runners that ran their late distance slower than their early distance) = 10/990 = 0.01





The median and mean are not always the best central measures, it depends on the circumstances (excluding options 1 and 4).

The median, not the mean, takes the centermost value and ignores potential outliers (whereas mean calculates as an average of all values, including outliers unless trimmed), excluding option 2, leaving option 3 as the only correct choice.



Sample size = 9

Median position = (sample size) / 2 = 9 / 2 = 4.5 = 4 (truncated)

ported (grouped) values of blood pressures (rounded to nearest 5mmHg) and sorted: 110 115 120 120 125 130 130 135 140

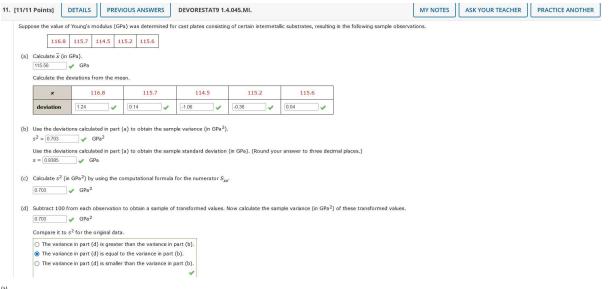
2 3 4 (position)

The median of the reported blood pressure values = 125

If we round 127.6 down to 127.4, the new reported (grouped) values of blood pressures, sorted, are: 110 115 120 120 130 130 130 135 140 0 1 2 3 4 (position)

The new median reported blood pressure value = 130

This proves that when rounding or grouping data, the median can be highly sensitive to small changes.



(a)

Sample mean =  $\bar{x}$  = (116.8 + 115.7 + 114.5 + 115.2 + 115.6) / 5 = 115.56

Sample deviation =  $(x - \bar{x})$ 

х	116.8	115.7	114.5	115.2	115.6
(x - x̄)	116.8 - 115.56 = 1.24	115.7 - 115.56 = 0.14	114.5 - 115.56 = -1.06	115.2 - 115.56 = -0.36	115.6 - 115.56 = 0.04

Sample variance =  $(x - \bar{x})^2$ 

x	116.8	115.7	114.5	115.2	115.6
(x - x̄)	116.8 - 115.56 = 1.24	115.7 - 115.56 = 0.14	114.5 - 115.56 = -1.06	115.2 - 115.56 = -0.36	115.6 - 115.56 = 0.04
(x - x)^2	(1.24)^2 = 1.5376	(0.14)^2 = 0.0196	(-1.06)^2 = 1.1236	(-0.36)^2 = 0.1296	(0.04)^2 = 0.0016

Sample variance =  $s^2 = (sum \ of the sample variances) / (sample size - 1) = <math>\sum (x - \bar{x})^2 / (n - 1) = (1.5376 + 0.0196 + 1.1236 + 0.1296 + 0.0016) / (5-1) = 0.703$ 

Sample standard deviation =  $s = Sqrt(s^2) = Sqrt(sample variance) = Sqrt(0.703) = 0.8385$ 

Alternative method to calculate sample variance and sample standard deviation using the sum of squared deviations from the sample mean (or  $S_{xx}$ )

116.8 115.7 114.5 115.2 115.6

 $\sum [x] = 116.8 + 115.7 + 114.5 + 115.2 + 115.6 = 577.8$   $\sum [x]^2 = 13,642.24 + 13,386.49 + 13,110.25 + 13,271.04 + 13,363.36 = 66,773.38$ 

 $S_{xx}$  = sum of squared deviations from the sample mean =  $\sum [x - \bar{x}]^2 = \sum [x]^2 - (\sum [x])^2 / n = (66,773.38) - (577.8)^2 / 5 = 2.812$ 

Sample variance = s^2 = (sum of squared deviations from the sample mean) / (sample size - 1) = ( $S_{xx}$ ) / (n - 1) = (2.812) / (5 - 1) = 0.703

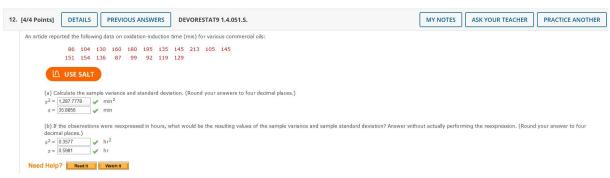
x	116.8	115.7	114.5	115.2	115.6
(x - 100)	116.8 - 100 = 16.8	115.7 - 100 = 15.7	114.5 - 100 = 14.5	115.2 - 100 = 15.2	115.6 - 100 = 15.6
(x - 100)^2	(16.8)^2 = 282.24	(15.7)^2 = 246.49	(14.5)^2 = 210.25	(15.2)^2 = 231.04	(15.6)^2 = 243.36

$$\begin{split} &\sum[x\cdot 100] = 16.8 + 15.7 + 14.5 + 15.2 + 15.6 = 77.8 \\ &\sum[x\cdot 100]^2 = 282.24 + 246.49 + 210.25 + 231.04 + 243.36 = 1,213.38 \end{split}$$

 $S_{xx}$  = sum of squared deviations from the sample mean =  $\sum [x - \bar{x}]^2 = \sum [x]^2 - (\sum [x])^2 / n = (1,213.38) - (77.8)^2 / 5 = 2.812$ 

Sample variance =  $s^2$  = (sum of squared deviations from the sample mean) / (sample size - 1) = (S<sub>nt</sub>) / (n - 1) = (2.812) / (5 - 1) = 0.703

 $Changing each observation consistently \ (\underline{when adding or subtracting}) \ will yield \ the same \ variance \ and \ thus \ the same \ deviations for \ the \ transformed \ values.$ 



(a)

1	oxidation-induction time (r	nin)	Squared Values
2		86	7396
3		104	10816
4		130	16900
5		160	25600
6		180	32400
7		195	38025
8		135	18225
9		145	21025
10		213	45369
11		105	11025
12		145	21025
13		151	22801
14		154	23716
15		136	18496
16		87	7569
17		99	9801
18		92	8464
19		119	14161
20		129	16641
21			
22	Sum		2565
23	Sum of Squares		369455

Sample size = 19

 $S_{xx}$  = sum of squared deviations from the sample mean =  $\sum [x - \bar{x}]^2 = \sum [x]^2 - (\sum [x])^2 / n = (369,455) - (2,565)^2 / 19 = 23,180$ 

Sample variance =  $s^2 = (sum \ of \ squared \ deviations \ from \ the \ sample \ mean) / (sample \ size - 1) = (S_{xx}) / (n - 1)$ = 23,180 / (19 - 1) = 1,287.7778

 $Sample\ standard\ deviation = s = Sqrt(s^2) = Sqrt(sample\ variance) = Sqrt(1,287.7778) = 35.8856$ 

(b	)	

	A	В	C
1	oxidation-induction time (min)	oxidation-induction time (hr)	Squared Values
2	86	1.433333333	2.05444444
3	104	1.733333333	3.00444444
4	130	2.166666667	4.69444444
5	160	2.666666667	7.111111111
6	180	3	9
7	195	3.25	10.5625
8	135	2.25	5.0625
9	145	2.416666667	5.840277778
10	213	3.55	12.602
11	105	1.75	3.062
12	145	2.416666667	5.840277778
13	151	2.516666667	6.33361111
14	154	2.566666667	6.587777778
15	136	2.266666667	5.137777778
16	87	1.45	2.102
17	99	1.65	2.722
18	92	1.533333333	2.35111111
19	119	1.983333333	3.93361111
20	129	2.15	4.622
21			
22	Sum	42.75	
23	Sum of Squares	102.6263889	
24	•	6.438888889	

 $S_{xx}$  = sum of squared deviations from the sample mean =  $\sum [x - x]^2 = \sum [x]^2 - (\sum [x])^2 / n (102.6264) - (42.75)^2 / 19 = 6.4389$ 

Sample variance = s^2 = (sum of squared deviations from the sample mean) / (sample size - 1) = ( $S_{xx}$ ) / (n - 1) = 6.4389 / (19 - 1) = 0.3577

Sample standard deviation = s = Sqrt(s^2) = Sqrt(sample variance) = Sqrt(0.3577) = 0.5981

Unlike the previous problem, changing each observation consistently when multiplying or dividing will yield different variances and deviations from the transformed values.