Ch9 - Inferences Based on Two Samples (Part II)

Saturday, July 8, 2023 6:24 PM

1. [–/3 Points] DETAILS DEVORESTAT9 9.3.036.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Consider the accompanying data on breaking load (kg/25 mm width) for various fabrics in both an unabraded condition and an abraded condition. Use the paired t test to test H_0: $\mu_D = 0$ versus H_a: $\mu_D > 0$ at significance level 0.01. (Use $\mu_D = \mu_U - \mu_A$.)

	Fabric							
	1	2	3	4	5	6	7	8
U	36.5	55.0	51.4	38.8	43.2	48.8	25.6	49.7
A	28.5	20.0	46.0	34.0	36.0	52.5	26.5	46.5

USE SALT

Calculate the test statistic and determine the P-value. (Round your test statistic to one decimal place and your P-value to three decimal places.)

t = 1.8

P-value = 0.061 See Excel below

State the conclusion in the problem context.

- ○ Reject H_0. The data suggests a significant mean difference in breaking load for the two fabric load conditions.
- ○ Reject H_0. The data does not suggest a significant mean difference in breaking load for the two fabric load conditions.
- ● Fail to reject H_0. The data does not suggest a significant mean difference in breaking load for the two fabric load conditions. 0.061 > 0.01
- ○ Fail to reject H_0. The data suggests a significant mean difference in breaking load for the two fabric load conditions.

Topics:

9.3 Analysis of Paired Data
- The Paired t Test
- The Paired t Confidence Interval
- Paired Data and Two-Sample t Procedures
- Paired versus Unpaired Experiments

Exercises Section 9.3 (36–48)

9.4 Inferences Concerning a Difference Between Population Proportions
- A Large-Sample Test Procedure
- Type II Error Probabilities and Sample Sizes
- A Large-Sample Confidence Interval
- Small-Sample Inferences

Exercises Section 9.4 (49–58)

9.5 Inferences Concerning Two Population Variances
- The F Distribution
- The F Test for Equality of Variances
- A Confidence Interval for σ_1/σ_2

The Paired t Test

Null Hypothesis:

$H_0: \mu_D = \Delta_0$ (where $D = X - Y$ is the difference between the first and second observations within a pair, and $\mu_D = \mu_1 - \mu_2$)

Test Statistic Value:

$t = \frac{\bar{d} - \Delta_0}{s_D/\sqrt{n}}$ (where $\bar{d}$ and s_D are the sample mean and standard deviation, respectively, of the d_i's)

Alternative Hypothesis	P-Value Determination
$H_a: \mu_D > \Delta_0$	Area under the t_{n-1} curve to the right of t
$H_a: \mu_D < \Delta_0$	Area under the t_{n-1} curve to the left of t
$H_a: \mu_D \neq \Delta_0$	$2 \cdot$ (Area under the t_{n-1} curve to the right of $\lvert t \rvert$)

Assumptions: The D_is constitute a random sample from a normal "difference" population.

Difference (Di)		
8		
35	D-bar (Di mean)	7.375
5.4	STD (Di STD)	11.84818
4.8	Null difference	0
7.2	t=(d-bar-null)/(std/sqrt(n))	1.8
-3.7	P(t>1.8)=1-P(t<1.8)	0.061
-0.9		
3.2		

2. [–/6 Points] DETAILS DEVORESTAT9 9.3.040. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Lactation promotes a temporary loss of bone mass to provide adequate amounts of calcium for milk production. A paper gave the following data on total body bone mineral content (TBBMC) (g) for a sample both during lactation (L) and in the postweaning period (P).

	Subject									
	1	2	3	4	5	6	7	8	9	10
L	1927	2546	2825	1922	1628	2175	2113	2621	1843	2542
P	2127	2885	2895	1944	1750	2183	2164	2626	2006	2626

(a) Does the data suggest that true average total body bone mineral content during postweaning exceeds that during lactation by more than 25 g? State and test the appropriate hypotheses using a significance level of 0.05. [*Note:* The appropriate normal probability plot shows some curvature but not enough to cast substantial doubt on a normality assumption.] (Use $\mu_D = \mu_P - \mu_L$.)

- ● H_0: $\mu_D = 25$ / H_a: $\mu_D > 25$
- ○ H_0: $\mu_D = 25$ / H_a: $\mu_D < 25$
- ○ H_0: $\mu_D = 25$ / H_a: $\mu_D \le 25$
- ○ H_0: $\mu_D = 25$ / H_a: $\mu_D \neq 25$

Calculate the test statistic and determine the P-value. (Round your test statistic to two decimal places and your P-value to three decimal places.)

t = 2.46

P-value = 0.018

State the conclusion in the problem context.

- ○ Fail to reject H_0. The data suggests that the true average total body bone mineral content during postweaning does not exceed that during lactation by more than 25 g.
- ○ Reject H_0. The data suggests that the true average total body bone mineral content during postweaning does not exceed that during lactation by more than 25 g.
- ● Reject H_0. The data suggests that the true average total body bone mineral content during postweaning exceeds that during lactation by more than 25 g. 0.018 < 0.05
- ○ Fail to reject H_0. The data suggests that the true average total body bone mineral content during postweaning exceeds that during lactation by more than 25 g.

(b) Calculate an upper confidence bound using a 95% confidence level for the true average difference between TBBMC during postweaning and during lactation. (Round your answer to two decimal places.)

166.98 g

(c) Does the (incorrect) use of the two-sample t test to test the hypotheses suggested in (a) lead to the same conclusion that you obtained there? Explain.

- ○ Yes, if the two samples were independent, the result would be the same.
- ● No, if the two samples were independent, the result would not be the same.

Di		
200	D-bar (Di mean)	106.4
339	STD (Di STD)	104.5065
70	Null difference	25
22	t=(d-bar-null)/(std/sqrt(n))	2.46
122	P(t>2.46)=1-P(t<2.46)	0.018
8	95% CI, invT dist:	1.833113
51	UB: dbar+invT*std/sqrt(n)	166.98
5		
163		
84		

3. [–/6 Points] DETAILS DEVORESTAT9 9.3.042. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Many freeways have service (or logo) signs that give information on attractions, camping, lodging, food, and gas services prior to off-ramps. These signs typically do not provide information on distances. An article reported that in one investigation, six sites along interstate highways where service signs are posted were selected. For each site, crash data was obtained for a three-year period before distance information was added to the service signs and for a one-year period afterward. The number of crashes per year before and after the sign changes were as follows.

Before:	16	22	65	119	68	62
After:	17	21	43	82	79	71

(a) The article included the statement "A paired t test was performed to determine whether there was any change in the mean number of crashes before and after the addition of distance information on the signs." Carry out such a test. [*Note:* The relevant normal probability plot shows a substantial linear pattern.]

State and test the appropriate hypotheses. (Use $\alpha = 0.05$.)

- ○ H_0: $\mu_D = 0$
 H_a: $\mu_D > 0$
- ○ H_0: $\mu_D = 0$
 H_a: $\mu_D \geq 0$
- ○ H_0: $\mu_D = 0$
 H_a: $\mu_D \leq 0$
- ○ H_0: $\mu_D = 0$
 H_a: $\mu_D < 0$
- ● H_0: $\mu_D = 0$
 H_a: $\mu_D \neq 0$

Calculate the test statistic and P-value. (Round your test statistic to two decimal places and your P-value to three decimal places.)

t = 0.84

P-value = 0.440

State the conclusion in the problem context.

- ● Fail to reject H_0. The data does not suggest a significant mean difference in the average number of accidents after information was added to road signs. 0.440 > 0.05
- ○ Reject H_0. The data suggests a significant mean difference in the average number of accidents after information was added to road signs.
- ○ Reject H_0. The data does not suggest a significant mean difference in the average number of accidents after information was added to road signs.
- ○ Fail to reject H_0. The data suggests a significant mean difference in the average number of accidents after information was added to road signs.

(b) If a seventh site were to be randomly selected among locations bearing service signs, between what values would you predict the difference in number of crashes to lie? (Use a 95% prediction interval. Round your answers to two decimal places.)

(-46.23 , 59.23)

Di		
-1	D-bar (Di mean)	6.5
1	STD (Di STD)	18.9921
22	Null difference	0
37	t=(d-bar-null)/(std/sqrt(n))	0.84
-11	P-value = T.DIST.2T	0.440
-9	95% CI, invT dist (2-tail):	2.570582
	UB: dbar+invT*std*sqrt(1+1/n)	59.23
	LB: dbar-invT*std*sqrt(1+1/n)	-46.23

4. [–/3 Points] DETAILS DEVORESTAT9 9.4.052.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Do teachers find their work rewarding and satisfying? An article reports the results of a survey of 392 elementary school teachers and 261 high school teachers. Of the elementary school teachers, 222 said they were very satisfied with their jobs, whereas 127 of the high school teachers were very satisfied with their work. Estimate the difference between the proportion of all elementary school teachers who are satisfied and all high school teachers who are satisfied by calculating a 95% CI. (Use $p_{\text{elementary}} - p_{\text{high school}}$. Round your answers to four decimal places.)

USE SALT

$n_1 = 392$ $n_2 = 261$

$x_1 = 222$ $x_2 = 127$

(0.0017 , 0.1577)

Interpret your 95% confidence interval.

- ● We are 95% confident that the difference between the proportions of elementary school teachers who are satisfied and high school teachers who are satisfied falls between these values.
- ○ We are 95% confident that the difference between the proportions of elementary school teachers who are satisfied and high school teachers who are satisfied falls above the upper bound.
- ○ We are 95% confident that the difference between the proportions of elementary school teachers who are satisfied and high school teachers who are satisfied falls outside these values.
- ○ We are 95% confident that the difference between the proportions of elementary school teachers who are satisfied and high school teachers who are satisfied falls below the lower bound.

5. [–/6 Points] DETAILS DEVORESTAT9 9.4.056. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Sometimes experiments involving success or failure responses are run in a paired or before/after manner. Suppose that before a major policy speech by a political candidate, n individuals are selected and asked whether (S) or not (F) they favor the candidate. Then after the speech the same n people are asked the same question. The responses can be entered in a table as follows:

		After	
		S	F
Before	S	X_1	X_2
	F	X_3	X_4

where $X_1 + X_2 + X_3 + X_4 = n$. Let p_1, p_2, p_3, and p_4 denote the four cell probabilities, so that $p_1 = P(S$ before and S after), and so on. We wish to test the hypothesis that the true proportion of supporters (S) after the speech has not increased against the alternative that it has increased.

(a) State the two hypotheses of interest in terms of p_1, p_2, p_3, and p_4.

- ○ H_0: $p_1 = p_3$
  H_a: $p_1 < p_3$
- ● H_0: $p_3 = p_2$
  H_a: $p_3 > p_2$
- ○ H_0: $p_3 = p_2$
  H_a: $p_3 < p_2$
- ○ H_0: $p_1 = p_3$
  H_a: $p_1 > p_3$

(b) Construct an estimator for the after/before difference in success probabilities.

- ○ $(X_2 - X_1)\sqrt{n}$
- ○ $(X_3 - X_2)\sqrt{n}$
- ● $\dfrac{X_3 - X_2}{n}$
- ○ $\dfrac{X_2 - X_1}{n}$

(c) When n is large, it can be shown that the rv $(X_i - X_j)/n$ has approximately a normal distribution with variance given by $[p_i + p_j - (p_i - p_j)^2]/n$. Use this to construct a test statistic with approximately a standard normal distribution when H_0 is true (the result is called McNemar's test).

- ○ $(X_3 - X_2)\sqrt{X_2 + X_3}$
- ○ $\dfrac{X_2 - X_1}{\sqrt{X_1 + X_2}}$
- ● $\dfrac{X_3 - X_2}{\sqrt{X_2 + X_3}}$
- ○ $(X_2 - X_1)\sqrt{X_1 + X_2}$

(d) If $x_1 = 350$, $x_2 = 150$, $x_3 = 200$, and $x_4 = 300$, what do you conclude? (Round your test statistic to two decimal places and your P-value to four decimal places.)

z =

P-value =

$z = \dfrac{x_3 - x_2}{\sqrt{x_2 + x_3}} = \dfrac{200 - 150}{\sqrt{150 + 200}} = 2.67$, p-value $= P(Z > 2.67) = 1 - P(Z \le 2.67) = 0.0037$

What do you conclude?

- ○ Reject H_0 at level 0.001.
- ● Reject H_0 at level 0.01 but not at level 0.001.
- ○ Reject H_0 at level 0.05 but not at level 0.01.
- ○ Reject H_0 at level 0.10 but not at level 0.05.

6. [–/1 Points] DETAILS DEVORESTAT9 9.4.058. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Using the traditional formula, a 95% CI for $p_1 - p_2$ is to be constructed based on equal sample sizes from the two populations. For what value n (= m) will the resulting interval have width at most 0.2 irrespective of the results of the sampling? (Round your answer up to the nearest whole number.)

n = 193

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It Watch It

$z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} = 1.96$

$2 \cdot z_{\frac{\alpha}{2}} \sqrt{\dfrac{\hat{p}_1 \hat{q}_1}{m} + \dfrac{\hat{p}_2 \hat{q}_2}{n}} = 0.2$

$2 \cdot 1.96 \sqrt{\dfrac{(0.5)(0.5)}{m} + \dfrac{(0.5)(0.5)}{n}} = 0.2$

$\Rightarrow m = n = 192.08 = 193$

A CI for $p_1 - p_2$ with confidence level approximately $100(1 - \alpha)\%$ is

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{m} + \frac{\hat{p}_2\hat{q}_2}{n}}$$

This interval can safely be used as long as $m\hat{p}_1$, $m\hat{q}_1$, $n\hat{p}_2$, and $n\hat{q}_2$ are all at least 10.

7. **[–/5 Points]** DETAILS DEVORESTAT9 9.5.060.MI.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Give as much information as you can about the *P*-value of the *F* test in each of the following situations:

USE SALT

(a) $v_1 = 9$, $v_2 = 10$, upper-tailed test, $f = 3.50$

○ *P*-value < 0.001
○ 0.001 < *P*-value < 0.01
● 0.01 < *P*-value < 0.05 1- f.dist(3.5, 9, 10, true) = 0.03
○ 0.05 < *P*-value < 0.10
○ *P*-value > 0.10

(b) $v_1 = 9$, $v_2 = 10$, upper-tailed test, $f = 1.25$

○ *P*-value < 0.001
○ 0.001 < *P*-value < 0.01
○ 0.01 < *P*-value < 0.05
○ 0.05 < *P*-value < 0.10
● *P*-value > 0.10 1- f.dist(1.5, 9, 10, true) = 0.36

(c) $v_1 = 9$, $v_2 = 10$, two-tailed test, $f = 4.94$

○ *P*-value < 0.001
○ 0.001 < *P*-value < 0.01
● 0.01 < *P*-value < 0.05 1- f.dist(4.94, 9, 10, true) = 0.01
○ 0.05 < *P*-value < 0.10
○ *P*-value > 0.10

(d) $v_1 = 9$, $v_2 = 10$, lower-tailed test, $f = 0.15$

○ *P*-value < 0.001
● 0.001 < *P*-value < 0.01 f.dist(0.15, 9, 10, true) = 0.004
○ 0.01 < *P*-value < 0.05
○ 0.05 < *P*-value < 0.10
○ *P*-value > 0.10

(e) $v_1 = 35$, $v_2 = 20$, upper-tailed test, $f = 3.27$

○ *P*-value < 0.001
● 0.001 < *P*-value < 0.01 1- f.dist(3.27, 35, 20, true) = 0.003
○ 0.01 < *P*-value < 0.05
○ 0.05 < *P*-value < 0.10
○ *P*-value > 0.10

8. **[–/4 Points]** DETAILS DEVORESTAT9 9.5.061. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

As the population ages, there is increasing concern about accident-related injuries to the elderly. An article reported on an experiment in which the maximum lean angle—the furthest a subject is able to lean and still recover in one step—was determined for both a sample of younger females (21–29 years) and a sample of older females (67–81 years). The following observations are consistent with summary data given in the article:

YF: 29, 36, 32, 27, 28, 32, 31, 36, 32, 25
OF: 19, 16, 21, 13, 12

Carry out a test at significance level 0.10 to see whether the population standard deviations for the two age groups are different (normal probability plots support the necessary normality assumption).
State the relevant hypotheses. (Use σ_1 for YF and σ_2 for OF.)

○ $H_0: \sigma_1 = \sigma_2$
$H_a: \sigma_1 \le \sigma_2$

● $H_0: \sigma_1 = \sigma_2$
$H_a: \sigma_1 \ne \sigma_2$

○ $H_0: \sigma_1 = \sigma_2$
$H_a: \sigma_1 < \sigma_2$

○ $H_0: \sigma_1 = \sigma_2$
$H_a: \sigma_1 > \sigma_2$

Calculate the test statistic. (Round your answer to two decimal places.)
$f =$ 0.89

What can be said about the *P*-value for the test?

● *P*-value > 0.100
○ 0.050 < *P*-value < 0.100
○ 0.010 < *P*-value < 0.050
○ 0.001 < *P*-value < 0.010
○ *P*-value < 0.001

State the conclusion in the problem context.

○ Reject H_0. The data does not suggest that there is a difference between the two standard deviations.
○ Fail to reject H_0. The data suggests that there is a difference between the two standard deviations.
● Fail to reject H_0. The data does not suggest that there is a difference between the two standard deviations.
○ Reject H_0. The data suggests that there is a difference between the two standard deviations.

S1	3.6148
S2	3.8341
f=S1^2/S2^2	0.89
V1 (n1 - 1)	9
V2 (n2 - 1)	4
p-value:	
F.DIST(f,v1,v2,true)	0.4032
	0.4032>0.1000