Module 4 Test

Saturday, June 10, 2023 12:23 PM

1. [–/5 Points] DETAILS DEVORESTAT9 5.SE.502.XP. MY NOTES ASK YOUR TEACHER

The inside diameter of a randomly selected piston ring is a random variable with mean value 9 cm and standard deviation 0.07 cm.

(a) If $\overline{X}$ is the sample mean diameter for a random sample of n = 16 rings, where is the sampling distribution of $\overline{X}$ centered and what is the standard deviation of the $\overline{X}$ distribution? (Enter your standard deviation to five decimal places.)

center [ ] cm $\bar{x} = \mu_{\bar{x}} = \mu = 9$

standard deviation [ ] cm $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.07}{\sqrt{16}} = 0.0175$

(b) Answer the questions posed in part (a) for a sample size of n = 64 rings. (Enter your standard deviation to five decimal places.)

center [ ] cm $\bar{x} = \mu_{\bar{x}} = \mu = 9$

standard deviation [ ] cm $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.07}{\sqrt{64}} = 0.00875$

(c) For which of the two random samples, the one of part (a) or the one of part (b), is $\overline{X}$ more likely to be within 0.01 cm of 9 cm? Explain your reasoning.

- ○ $\overline{X}$ is more likely to be within 0.01 cm of 9 cm in sample (a) because of the decreased variability with a smaller sample size.
- ● $\overline{X}$ is more likely to be within 0.01 cm of 9 cm in sample (b) because of the decreased variability with a larger sample size.
- ○ $\overline{X}$ is more likely to be within 0.01 cm of 9 cm in sample (a) because of the increased variability with a smaller sample size.
- ○ $\overline{X}$ is more likely to be within 0.01 cm of 9 cm in sample (b) because of the increased variability with a larger sample size.

2. [–/2 Points] DETAILS DEVORESTAT9 5.SE.503.XP.S. MY NOTES ASK YOUR TEACHER

The inside diameter of a randomly selected piston ring is a random variable with mean value 16 cm and standard deviation 0.03 cm. Suppose the distribution of the diameter is normal. (Round your answers to four decimal places.)

USE SALT

$\mu_{\bar{x}} = \mu = 16$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.03}{\sqrt{16}} = 0.0075$

(a) Calculate $P(15.99 \le \overline{X} \le 16.01)$ when n = 16.

$P(15.99 \le \overline{X} \le 16.01)$ = [ ]

$P(15.99 \le \bar{x} \le 16.01) = P\left(\frac{15.99-16}{0.0075} \le z \le \frac{16.01-16}{0.0075}\right) = P(-1.33 \le z \le 1.33) = \Phi(1.33) - \Phi(-1.33) = 0.8165$

(b) How likely is it that the sample mean diameter exceeds 16.01 when n = 25? $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.03}{\sqrt{25}} = 0.006$

$P(\overline{X} \ge 16.01)$ = [ ] $P(\bar{x} > 16.01) = P\left(z > \frac{16.01-16}{0.006}\right) = P(z > 1.67) = 1 - P(z < 1.67) = 0.0475$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

3. [–/2 Points] DETAILS DEVORESTAT9 5.SE.504.XP.S. MY NOTES

The breaking strength of a rivet has a mean value of 10,050 psi and a standard deviation of 504 psi.

USE SALT

(a) What is the probability that the sample mean breaking strength for a random sample of 40 rivets is between 9,950 and 10,250? (Round your answer to four decimal places.)

[ ] $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{504}{\sqrt{40}} = 79.69$ $P(9,950 \le \bar{x} \le 10,250) = P\left(\frac{9,950-10,050}{79.69} \le z \le \frac{10,250-10,050}{79.69}\right) = P(-1.25 \le z \le 2.51) = \Phi(2.51) - \Phi(-1.25) = 0.8892$

(b) If the sample size had been 15 rather than 40, could the probability requested in part (a) be calculated from the given information? Explain your reasoning.

- ○ Yes, the probability in part (a) can still be calculated from the given information.
- ● No, n should be greater than 30 in order to apply the Central Limit Theorem.
- ○ No, n should be greater than 20 in order to apply the Central Limit Theorem.
- ○ No, n should be greater than 50 in order to apply the Central Limit Theorem.

You may need to use the appropriate table in the Appendix of Tables to answer this question.

4. [–/2 Points] DETAILS DEVORESTAT9 6.SE.036.MI. MY NOTES ASK YOUR TEACHER

When the population distribution is normal, the statistic median $\{|X_1 - \tilde{x}|, \ldots, |X_n - \tilde{x}|\}/0.6745$ can be used to estimate σ. This estimator is more resistant to the effects of outliers (observations far from the bulk of the data) than is the sample standard deviation. Compute both the corresponding point estimate and s for the data below. (Round your answers to three decimal places.)

24.83	25.92	26.40	26.57	26.81	27.30	27.46	27.69	27.89	27.97
28.12	28.19	28.43	28.64	28.65	29.02	29.26	29.28	29.55	30.81

point estimate 1.275

sample standard deviation s = 1.393

24.83	25.92	26.4	26.57	26.81	27.3	27.46	27.69	27.89	27.97
28.12	28.19	28.43	28.64	28.65	29.02	29.26	29.28	29.55	30.81
Median	28.045								
Mean	27.9395								
ABS(data-median):									
3.215	2.125	1.645	1.475	1.235	0.745	0.585	0.355	0.155	0.075
0.075	0.145	0.385	0.595	0.605	0.975	1.215	1.235	1.505	2.765
ABS(data-mean):									
3.1095	2.0195	1.5395	1.3695	1.1295	0.6395	0.4795	0.2495	0.0495	0.0305
0.1805	0.2505	0.4905	0.7005	0.7105	1.0805	1.3205	1.3405	1.6105	2.8705
Median	0.86								
Median/0.6745		1.275	(point estimate)						
(ABS(data-mean))^2									
10.33623	4.515625	2.706025	2.175625	1.525225	0.555025	0.342225	0.126025	0.024025	0.005625
0.005625	0.021025	0.148225	0.354025	0.366025	0.950625	1.476225	1.525225	2.265025	7.645225
Sum	37.0689								
Divide n	1.950995								
Sqrt	1.397		1.393						

5. [–/2 Points] DETAILS **DEVORESTAT9 6.SE.502.XP.** MY NOTES ASK YOUR TEACHER

A sample of 20 students who had recently taken elementary statistics yielded the following information on brand of calculator owned. (T = Texas Instruments, H = Hewlett Packard, C = Casio, S = Sharp):

$n=20$

C T S S C S H T S T

S H H C S T H C T H

$X = \#\text{ of } T = 5$

(a) Estimate the true proportion of all such students who own a Texas Instruments calculator.

$\hat{p} = \frac{X}{n} = \frac{5}{20} = \frac{1}{4} = 0.25$

(b) Of the 5 students who owned a TI calculator, 3 had graphing calculators. Estimate the proportion of students who do not own a TI graphing calculator.

$\hat{p} = 1 - \frac{3}{20} = 0.85$

6. [–/3 Points] DETAILS **DEVORESTAT9 6.SE.503.XP.** MY NOTES ASK YOUR TEACHER

Consider the accompanying observations on stream flow (1000s of acre-feet) recorded at a station in Colorado for the period April 1–August 31 over a 31-year span.

185.36	299.87	114.79	285.37	200.19
123.30	125.86	100.85	550.56	150.58
311.13	183.00	302.74	203.24	330.33
117.64	210.07	280.55	204.91	89.59
109.11	94.33	108.91	66.24	127.96
145.11	103.18	247.11	79.98	262.09
133.56				

$n=31$

An appropriate probability plot supports the use of the lognormal distribution as a reasonable model for stream flow.

(a) Estimate the parameters of the distribution. [*Hint*: Remember that X has a lognormal distribution with parameters μ and σ^2 if $\ln(X)$ is normally distributed with mean μ and variance σ^2.] (Round your estimate for the mean to three decimal places, and round your estimate for the variance to four decimal places.)

$\hat{\mu} =$ $\hat{\mu} = \frac{\sum \ln(x_i)}{n} = 5.113$

$\hat{\sigma}^2 =$ $\hat{\sigma}^2 = \frac{\sum [(\ln(x_i) - \ln(\bar{x}))^2]}{n-1} = 0.2539$

(b) Use the estimates of part (a) to calculate an estimate of the expected value of stream flow. [*Hint*: What is $E(X)$?] (Round your answer to two decimal places.)

thousand acre-feet

$E(x) = e^{\hat{\mu} + \hat{\sigma}^2/2} = e^{5.113 + 0.2539/2} = 188.65$