Ch3 - Discrete Random Variables and Probability Distributions (Part I)

Wednesday, May 17, 2023 8:12 PM

Topics:

1. [–/8 Points] DETAILS **DEVORESTAT9 3.1.001.** MY NOTES ASK YOUR TEACHER

A concrete beam may fail either by shear (*S*) or flexure (*F*). Suppose that three failed beams are randomly selected and the type of failure is determined for each one. Let *X* = the number of beams among the three selected that failed by shear. List each outcome in the sample space along with the associated value of *X*.

S:	FFF	SFF	FSF	FFS	FSS	SFS	SSF	SSS
X:	0S	1S	1S	1S	2S	2S	2S	3S

Need Help? Read It

2. [–/4 Points] DETAILS **DEVORESTAT9 3.1.004.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Let *X* = the number of nonzero digits in a randomly selected 4-digit PIN that has no restriction on the digits. What are the possible values of *X*?

- ● 0, 1, 2, 3, 4 — 0 if no nonzero digits, 4 if all 4 digits are 0's
- ○ 0, 1, 2, 3
- ○ 1, 2, 3, 4
- ○ 1, 2, 3, 4, ...
- ○ 0, 1, 2, 3, 4, ...

For the following possible outcomes, give their associated *X* values.

PIN	associated value
1533	4
2020	2
7270	3

Need Help? Read It Watch It

3. [–/6 Points] DETAILS **DEVORESTAT9 3.1.006.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Starting at a fixed time, each car entering an intersection is observed to see whether it turns left (*L*), right (*R*), or goes straight ahead (*A*). The experiment terminates as soon as a car is observed to turn left. Let *X* = the number of cars observed. What are possible *X* values?

- ○ 1, 2, 3
- ○ 0, 1, 2, 3, ...
- ● 1, 2, 3, 4, ... — infinite possibilities of X until a car turns left (L)
- ○ 0, 1, 2, 3, 4

Given the outcomes below, find their associated *X* values.

Outcome:	RRL	RL	RARRL	RRAL	RRRRL
X:	3	2	5	4	5

we include L as an observation

Need Help? Read It

4. [–/4 Points] DETAILS **DEVORESTAT9 3.2.012.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Airlines sometimes overbook flights. Suppose that for a plane with 50 seats, 55 passengers have tickets. Define the random variable *Y* as the number of ticketed passengers who actually show up for the flight. The probability mass function of *Y* appears in the accompanying table.

y	45	46	47	48	49	50	51	52	53	54	55
p(y)	0.06	0.10	0.12	0.14	0.25	0.16	0.06	0.05	0.03	0.02	0.01

(a) What is the probability that the flight will accommodate all ticketed passengers who show up?

$P(y \le 50) = 0.06+0.10+0.12+0.14+0.25+0.16 = 0.83$

(b) What is the probability that not all ticketed passengers who show up can be accommodated?

$P(y > 50) = 0.06+0.05+0.03+0.02+0.01 = 0.17 = 1 - P(y \le 50)$

(c) If you are the first person on the standby list (which means you will be the first one to get on the plane if there are any seats available after all ticketed passengers have been accommodated), what is the probability that you will be able to take the flight?

$P(y \le 50-1) = 0.06+0.10+0.12+0.14+0.25 = 0.67$

What is this probability if you are the third person on the standby list?

$P(y \le 50-3) = 0.06+0.10+0.12 = 0.28$

Need Help? Read It

5. [–/6 Points] DETAILS **DEVORESTAT9 3.2.013.MI.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A mail-order company business has six telephone lines. Let *X* denote the number of lines in use at a specified time. Suppose the pmf of *X* is as given in the accompanying table.

x	0	1	2	3	4	5	6
p(x)	0.11	0.15	0.20	0.25	0.19	0.09	0.01

Calculate the probability of each of the following events.

(a) {at most three lines are in use}

$P(x \le 3) = 0.11+0.15+0.20+0.25 = 0.71$

(b) {fewer than three lines are in use}

$P(X < 3) = 0.11+0.15+0.20 = 0.46$

(c) {at least three lines are in use}

$P(x \ge 3) = 1 - P(x<3) = 1 - 0.46 = 0.54$

(d) {between two and five lines, inclusive, are in use}

$P(2 \le x \le 5) = 0.20+0.25+0.19+0.09 = 0.73$

(e) {between two and four lines, inclusive, are not in use} Not in use: 6−x (6 phones & x number of them aren't used)

$P(2 \le 6-x \le 4) = P(2 \le x \le 4) = 0.20+0.25+0.19 = 0.64$

(f) {at least four lines are not in use}

$P(4 \ge 6-x) = P(2 \le x) = P(x<3) = 0.46$

$4 \ge 6-x$
$-2 \ge -x$
$2 \le x$

$2 \le 6-x \le 4$
$-4 \le -x \le -2$
$4 \ge x \ge 2$
$\Rightarrow 2 \le x \le 4$

Need Help? Read It Watch It Master It

Definition

The **probability distribution** or **probability mass function** (pmf) of a discrete rv is defined for every number x by $p(x) = P(X = x) = P(\text{all } \omega \in \mathscr{S}\text{: } X(\omega) = x)$.

rv = random variable

6. [–/8 Points] DETAILS DEVORESTAT9 3.2.016. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Some parts of California are particularly earthquake-prone. Suppose that in one metropolitan area, 34% of all homeowners are insured against earthquake damage. Four homeowners are to be selected at random. Let X denote the number among the four who have earthquake insurance.

(a) Find the probability distribution of X. [*Hint:* Let S denote a homeowner that has insurance and F one who does not. Then one possible outcome is $SFSS$, with probability (0.34)(0.66)(0.34)(0.34) and associated X value 3. There are 15 other outcomes.] (Round your answers to four decimal places.)

x	0	1	2	3	4
$p(x)$					

$S = 0.34$

$F = 0.66$

Sample space = {FFFF, SFFF, FSFF, FFSF, FFFS, SSFF, SFSF, SFFS, FSSF, FSFS, FFSS, SSSF, FSSS, SFSS, SSFS, SSSS}

16 total outcomes

$P(X=0) = P(FFFF) = (0.66)^4 = 0.1897$

$P(x=1) = P(SFFF) + P(FSFF) + P(FFSF) + P(FFFS)$

$= (0.34)(0.66)^3 + (0.34)(0.66)^3 + (0.34)(0.66)^3 + (0.34)(0.66)^3$

$= 4(0.34)(0.66)^3 = 0.3910$

$P(x=2) = P(SSFF) + P(SFSF) + P(SFFS) + P(FSSF) + P(FSFS) + P(FFSS)$

$= 6(0.34)^2(0.66)^2 = 0.3021$

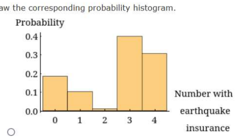
$P(x=3) = P(SSSF) + P(FSSS) + P(SFSS) + P(SSFS)$

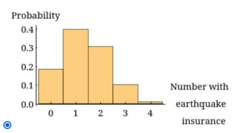
$= 4(0.34)^3(0.66) = 0.1038$

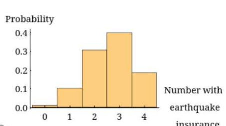
$P(x=4) = P(SSSS) = (0.34)^4 = 0.0134$

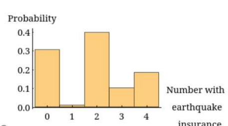
x	0	1	2	3	4
$p(x)$	0.1897 ✔	0.3910 ✔	0.3021 ✔	0.1038 ✔	0.0134 ✔

(b) Draw the corresponding probability histogram.









(c) What is the most likely value for X?

1 ✔

(d) What is the probability that at least two of the four selected have earthquake insurance? (Round your answer to four decimal places.)

$P(x \geq 2) = P(x=2) + P(x=3) + P(x=4) = 0.3021 + 0.1038 + 0.0134 = 0.4193$

7. [–/4 Points] DETAILS DEVORESTAT9 3.3.029.MI.

The pmf of the amount of memory X (GB) in a purchased flash drive is given as the following.

x	1	2	4	8	16
$p(x)$	0.05	0.15	0.25	0.30	0.25

(a) Compute $E(X)$ (in GB). (Enter your answer to two decimal places.)

GB $E(X) = \sum[x \cdot p(x)] = (1)(0.05) + (2)(0.15) + (4)(0.25) + (8)(0.30) + (16)(0.25) = 7.75$

(b) Compute $V(X)$ (in GB^2) directly from the definition. (Enter your answer to four decimal places.) where $\mu = E(x)$

GB^2 $V(x) = \sum[(x-\mu)^2 \cdot p(x)] = [(1-7.75)^2 \cdot 0.05] + [(2-7.75)^2 \cdot 0.15] + [(4-7.75)^2 \cdot 0.25] + [(8-7.75)^2 \cdot 0.30] + [(16-7.75)^2 \cdot 0.25] = 27.7875$

(c) Compute the standard deviation of X (in GB). (Round your answer to three decimal places.)

GB $\sigma_X = \sqrt{V(x)} = \sqrt{27.7875} = 5.271$

(d) Compute $V(X)$ (in GB^2) using the shortcut formula. (Enter your answer to four decimal places.)

GB^2 $E(X^2) = \sum[x^2 \cdot p(x)] = [(1)^2(0.05)] + [(2)^2(0.15)] + [(4)^2(0.25)] + [(8)^2(0.30)] + [(16)^2(0.25)] = 87.85$

Need Help? Read It Watch It Master It

$V(x) = E(X^2) - [E(x)]^2 = 87.85 - (7.75)^2 = 27.7875$

Definition

Let X be a discrete rv with set of possible values D and pmf $p(x)$. The **expected value** or **mean value** of X, denoted by $E(X)$ or μ_X or just μ, is

$$E(X) = \mu_X = \sum_{x \in D} x \cdot p(x)$$

Definition

Let X have pmf $p(x)$ and expected value μ. Then the **variance** of X, denoted by $V(X)$ or σ_X^2, or just σ^2, is

$$V(X) = \sum_D (x-\mu)^2 \cdot p(x) = E[(X-\mu)^2]$$

The **standard deviation** (SD) of X is

$$\sigma_X = \sqrt{\sigma_X^2}$$

A Shortcut Formula for σ^2

The number of arithmetic operations necessary to compute σ^2 can be reduced by using an alternative formula.

Proposition

$$V(X) = \sigma^2 = \left[\sum_D x^2 \cdot p(x)\right] - \mu^2 = E(X^2) - [E(X)]^2$$

8. [–/2 Points] DETAILS DEVORESTAT9 3.3.030. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

An individual who has automobile insurance from a certain company is randomly selected. Let Y be the number of moving violations for which the individual was cited during the last 3 years. The pmf of Y is the following.

y	0	1	2	3
$p(y)$	0.50	0.25	0.20	0.05

(a) Compute $E(Y)$.
$E(Y) =$

$E(Y) = \sum[y \cdot p(y)] = [(0)(0.50)] + [(1)(0.25)] + [(2)(0.20)] + [(3)(0.05)] = 0.80$

(b) Suppose an individual with Y violations incurs a surcharge of \$$90Y^2$. Calculate the expected amount of the surcharge.
\$

$E(90Y^2) = \sum[90 \cdot y^2 \cdot p(y)] = [(90)(0)^2(0.50)] + [(90)(1)^2(0.25)] + [(90)(2)^2(0.20)] + [(90)(3)^2(0.05)] = 135$

Need Help? Read It

Expected Value of a Linear Function

The $h(X)$ function of interest is quite frequently a linear function $aX + b$. In this case, $E[h(X)]$ is easily computed from $E(X)$ without the need for additional summation.

Proposition

$$E(aX+b) = a \cdot E(X) + b$$

9. [–/6 Points] DETAILS DEVORESTAT9 3.3.032. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

An certain brand of upright freezer is available in three different rated capacities: 16 ft^3, 18 ft^3, and 20 ft^3. Let X = the rated capacity of a freezer of this brand sold at a certain store. Suppose that X has the following pmf.

x	16	18	20
$p(x)$	0.3	0.2	0.5

$E(x) = \sum[x \cdot p(x)] = [(16)(0.3)] + [(18)(0.2)] + [(20)(0.5)] = 18.4$

$E(x^2) = \sum[x^2 \cdot p(x)] = [(16)^2(0.3)] + [(18)^2(0.2)] + [(20)^2(0.5)] = 341.6$

$V(x) = \sum[(x-\mu)^2 \cdot p(x)] = [(16-18.4)^2 \cdot (0.3)] + [(18-18.4)^2 \cdot (0.2)] + [(20-18.4)^2 \cdot (0.5)] = 3.04$

(a) Compute $E(X)$, $E(X^2)$, and $V(X)$.
$E(X) =$ ft^3
$E(X^2) =$
$V(X) =$

(b) If the price of a freezer having capacity X is $69X - 650$, what is the expected price paid by the next customer to buy a freezer?
\$

$E(69X-650) = 69 \cdot E(x) - 650 = (69)(18.4) - (650) = 619.6$

(c) What is the variance of the price paid by the next customer?

$V(69X-650) = (69)^2 \cdot V(X) = (69)^2(3.04) = 14,473.44$

Use: $V(aX+b) = a^2 \cdot V(X)$

(d) Suppose that although the rated capacity of a freezer is X, the actual capacity is $h(X) = X - 0.009X^2$. What is the expected actual capacity of the freezer purchased by the next customer? (Enter your answer to four decimal places.)
ft^3

$E(X - 0.009X^2) = E(X) - (0.009) \cdot E(X^2) = (18.4) - (0.009)(341.6) = 15.3256$

Need Help? Read It Watch It

Variance of a Linear Function

The variance of $h(X)$ is the expected value of the squared difference between $h(X)$ and its expected value:

$$V[h(X)] = \sigma_{h(X)}^2 = \sum_D \{h(x) - E[h(X)]\}^2 \cdot p(x) \quad \textbf{(3.13)}$$

Proposition

$$V(aX+b) = \sigma_{aX+b}^2 = a^2 \cdot \sigma_X^2$$

$= a^2 \cdot V(X)$

and

$$\sigma_{aX+b} = |a| \cdot \sigma_X$$

In particular,

$$\sigma_{aX} = |a| \cdot \sigma_X, \quad \sigma_{X+b} = \sigma_X \quad \textbf{(3.14)}$$