1. [-/4 Points] DETAILS DEVORESTAT9 3.4.046.MI.

Compute the following binomial probabilities directly from the formula for b(x; n, p). (Round your answers to three decimal places.)

$$b(x; n, 7) = {n \choose x} 2^{x} (1-2)^{n-x} = {n \choose y} (0.3)^{y} (1-0.3)^{y-y} = 0.136$$

$$b(6; 8, 0.65) = (8)(0.65)^{6}(1-0.65)^{8-6} = 0.259$$

(c)
$$P(3 \le X \le 5)$$
 when $n = 7$ and $p = 0.55$

$$P(3 \le X \le 5) = b(3;7,0.55) + b(4;7,0.55) + b(5;7,0.55) = 0.745$$
(d) $P(1 \le X)$ when $n = 9$ and $p = 0.15$

Need Help? Read It Watch It Master It

Notation

Because the pmf of a binomial rv X depends on the two parameters n and p, we denote the pmf by b(x; n, p).

$$b(x;n,p) = \left\{ \begin{pmatrix} n \\ x \end{pmatrix} p^x (1-p)^{n-x} & x=0,1,2,\ldots,n \\ 0 & \text{otherwise} \end{cases}$$

For $X \sim \operatorname{Bin}(n,p)$, the cdf will be denoted by

$$B(x;n,p)=P(X\leq x)=\sum_{y=0}^{x}b(y;n,p) \qquad x=0,1,\ldots,n$$

Topics:

3.4 The Binomial Probability Distribution

The Binomial Random Variable and Distribution

Using Binomial Tables

The Mean and Variance of X

Exercises Section 3.4 (46-67)

3.5 Hypergeometric and Negative Binomial Distributions

The Hypergeometric Distribution

The Negative Binomial Distribution

Exercises Section 3.5 (68-78)

3.6 The Poisson Probability Distribution

The Poisson Distribution as a Limit

The Mean and Variance of X

The Poisson Process

2. [-/4 Points] DETAILS DEVORESTAT9 3.4.050.S MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER A particular telephone number is used to receive both voice calls and fax messages. Suppose that 25% of the incoming calls involve fax messages, and consider a sample of 25 incoming calls. (Round your answers to three decimal places.) (a) What is the probability that at most 8 of the calls involve a fax message? $?(X \le 8) = \underbrace{\$}_{3} b(y; 25, 0.25) = B(8; 25, 0.25) = 0.85!$ (b) What is the probability that exactly 8 of the calls involve a fax message? ?(X = 8) = b(8; 25, 0.25) = 0.124(c) What is the probability that at least 8 of the calls involve a fax message? $?(X \ge 8) = [-?(X \le 8) =$ $P(X>8)=1-3(x \le 8)=1-B(8:325,0.35)=1-0.851=0.144$ You may need to use the appropriate table in the Appendix of Tables to answer this question

Twenty percent of all telephones of a certain type are submitted for service while under warranty. Of these, 60% can be repaired, whereas the other 40% must be replaced with new units. If a company purchases

7= % nee) servicy . Monee) replaced = (0.20) (0.40) = 0.08 X=2

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

DETAILS

DEVORESTAT9 3.4.055.S.

?(x=2)= b(2; 10, 0.08)= 0.148

n=10

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3. [-/1 Points]

An instructor who taught two sections of engineering statistics last term, the first with 25 students and the second with 40, decided to assign a term project. After all projects had been turned in, the instructor randomly ordered them before grading. Consider the first 15 graded projects. N = 25 + 40 = 65 n = 15(a) What is the probability that exactly 10 of these are from the second section? (Round your answer to four decimal places.) M = 40 $P(X = | 0) = (\frac{1}{10}) \cdot (\frac{1}{10} + \frac{1}{10}) = \frac{1}{10} \cdot (\frac{1}{10} + \frac{1}{10}) = 0.2172$ (b) What is the probability that at least 10 of these are from the second section? (Round your answer to four decimal places.) M = 40 $P(X \ge | 0) = -\frac{1}{10} \times (X \le 0) = 1 - \frac{1}{10} \times (X \le 0) = \frac{1}{10} \times (X \le 0) = 1 - \frac{1}{10} \times (X \le 0) = 1$

Proposition

If X is the number of S's in a completely random sample of size n drawn from a population consisting of M S's and (N-M) F's, then the probability distribution of X, called the

If $X \sim \text{Bin}(n, p)$, then E(X) = np, V(X) = np(1-p) = npq, and $\sigma_X = \sqrt{npq}$ (where q = 1-p).

hypergeometric distribution, is given by

$$P(X=x) = h(x;n,M,N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$
 (3.15)

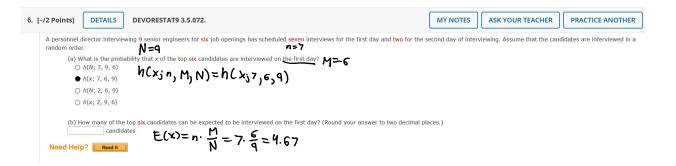
let H(x; n, M, N) be the cot

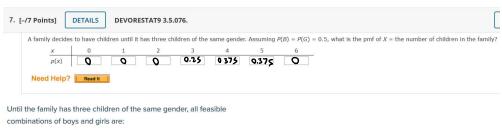
for x an integer satisfying $\max (0, n-N+M) \le x \le \min (n, M)$.

Proposition

The mean and variance of the hypergeometric rv X having pmf h(x; n, M, N) are

$$E(X) = n \cdot \frac{M}{N} \qquad V(X) = \left(\frac{N-n}{N-1}\right) \cdot n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right)$$





{BBB, GGG, BBGB, BGBB, GBBB, GGBG, GBGG, BGGG, BGGGB, BGGGB, BGGBB, BGGBB, GBBGB, GBBBB, BBGGG, BGBGG, BGBGG, BGBGG, BGBGG, BBGGG, BGBGG, BBGGG, BGBGG, BBGGG, BGBGG, BBGGG, BGBGG, BBGGG, BBGGG, BGBGG, BBGGG, BGBGG, BGBGG

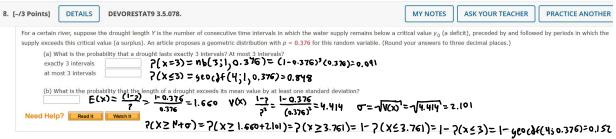
Then we see that there are two combinations that result in three children, six combinations that result in four children, and twelve combinations that result in five children.

Let X be the family's total number of children. Add up the probability for

each conceivable combination:
$$P(X=3) = P(BBB) + P(GGG) \\ = 0.125 + 0.125 \\ = 0.25 \\ = \frac{1}{4}$$

$$P(X=4) = P(BBGB) + \dots + P(BGGG) \\ = 6P(BBGB) \\ = 6(0.0625) \\ = 0.375 \\ = \frac{3}{8}$$

$$P(X=5) = P(BBGB) + \dots + P(BBBGG) \\ = 12P(BBGGB) \\ = 12(0.03125) \\ = 0.375$$



In the special case r=1, the pmf is

$$nb(x;1,p) = (1-p)^x p$$
 $x = 0, 1, 2, ...$ (3.17)

In Example 3.12, we derived the pmf for the number of trials necessary to obtain the first S, and the pmf there is similar to Expression (3.17). Both $X = \text{number of } F^* \text{s}$ and Y = number of trials (= 1 + X) are referred to in the literature as **geometric** random variables, and the pmf in Expression (3.12) is called the **geometric** distribution.

Proposition

If X is a negative binomial rv with pmf nb(x;r,p), then

$$E(X) = \frac{r(1-p)}{p}$$
 $V(X) = \frac{r(1-p)}{p^2}$

9. [-/5 Points] DETAILS DEVORESTAT9 3.6.080.MI.

Let X be the number of material anomalies occurring in a particular region of an aircraft gas-turbine disk. The article "Methodology for Probabilistic Life Prediction of Multiple-Anomaly Materials" + proposes a Poisson distribution for X. Suppose that $\mu = 4$. (Round your answers to three decimal places.)

(a) Compute both $P(X \le 4)$ and P(X < 4). $P(X \le 4) = P(X \le 4)$

Definition

A discrete random variable X is said to have a **Poisson distribution** with parameter μ (μ > 0) if the pmf of X is

$$p(x; \mu) = \frac{e^{-\mu} \cdot \mu^x}{x!}$$
 $x = 0, 1, 2, 3, \dots$

Proposition

If X has a Poisson distribution with parameter μ , then $E(X)=V(X)=\mu$.

10. [-/3 Points] DETAILS DEVORESTAT9 3.6.082.

Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter $\mu = 0.5$. (Round your answers to three decimal places.)

(a) What is the probability that a disk has a seastly one missing pulse? $P(X = 1) = \frac{1}{2}(1; 0.5) = 0.303$ (b) What is the probability add isk has at least two missing pulses? $P(X = 1) = \frac{1}{2}(1; 0.5) = 0.303$ (c) If two disks are independently selected, what is the probability that neither contains a missing pulse? $P(X = 0) = P(X \le 1) = \frac{1}{2}(1; 0.5) = 0.368$ You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help?

11. [-/3 Points] DEVORESTAT9 3.6.089. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

An article suggests that a Poisson process can be used to represent the occurrence of structural loads over time. Suppose the mean time between occurrences of loads is 0.4 year.

(a) How many loads can be expected to occur during a 4-year period?

Proposition

 $P_k(t) = e^{-at} \cdot (\alpha t)^k/k!$, so that the number of events during a time interval of length t is a Poisson rv with parameter $\mu = \alpha t$. The expected number of events during any such time interval is then αt , so the expected number during a unit interval of time is α .

The occurrence of events over time as described is called a *Poisson process*; the parameter α specifies the *rate* for the process.

Automobiles arrive at a vehicle equipment inspection station according to a Poisson process with rate $\alpha=8$ per hour. Suppose that with probability 0.5 an arriving vehicle will have no equipment violations.

(a) What is the probability that exactly eight arrive during the hour and all eight have no violations? (Round your answer to four decimal places.)

(b) For any fixed $y \ge 8$, what is the probability that y arrive during the hour, of which eight have no violations?

(c) What is the probability that eight 'no-violation' cars arrive during the next hour? (Hint: Sum the probabilities in part (b) from y = 8 to ∞ .] (Round your answer to three decimal places.)

(c) What is the probability that eight "no-violation" cars arrive during the next hour? (Hint: Sum the probabilities in part (b) from y = 8 to ∞ .] (Round your answer to three decimal places.)

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read II Weight III White the probability which is the probability that the probability of the probability of the probability of Tables to answer this question.