Ch9 - Inferences Based on Two Samples (Part I)

Friday, June 23, 2023 8:30 PM

Topics:

1. [–/6 Points] DETAILS **DEVORESTAT9 9.1.005.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Persons having Raynaud's syndrome are apt to suffer a sudden impairment of blood circulation in fingers and toes. In an experiment to study the extent of this impairment, each subject immersed a forefinger in water and the resulting heat output (cal/cm^2/min) was measured. For $m = 9$ subjects with the syndrome, the average heat output was $\bar{x} = 0.62$, and for $n = 9$ nonsufferers, the average output was 2.04. Let μ_1 and μ_2 denote the true average heat outputs for the sufferers and nonsufferers, respectively. Assume that the two distributions of heat output are normal with $\sigma_1 = 0.1$ and $\sigma_2 = 0.5$.

$\alpha = 0.01$

(a) Consider testing H_0: $\mu_1 - \mu_2 = -1.0$ versus H_a: $\mu_1 - \mu_2 < -1.0$ at level 0.01. Describe in words what H_a says, and then carry out the test.

- ● H_a says that the average heat output for sufferers is more than 1 cal/cm^2/min below that of non-sufferers.
- ○ H_a says that the average heat output for sufferers is the same as that of non-sufferers.
- ○ H_a says that the average heat output for sufferers is less than 1 cal/cm^2/min below that of non-sufferers.

for $Ha: \mu_1 - \mu_2 < -1.0 \Rightarrow \mu_1 + 1 < \mu_2$ where μ_1 is sufferers & μ_2 is non-sufferers

Calculate the test statistic and P-value. (Round your test statistic to two decimal places and your P-value to four decimal places.)

z = -2.47

P-value = 0.0067

TI-84: STAT→TESTS→2-SampZTest

1: w/ Syndrome: $n_1 = 9$, $\bar{x}_1 = 0.62$, $\sigma_1 = 0.1$

2: w/o Syndrome: $m = n_2 = 9$, $\bar{x}_2 = 2.04$, $\sigma_2 = 0.5$

To test the hypothesis, when $\mu_1 < \mu_2$ add $1 + \bar{x}_1 = 1 + 0.62 = 1.62$

State the conclusion in the problem context.

- ● Reject H_0. The data suggests that the average heat output for sufferers is more than 1 cal/cm^2/min below that of non-sufferers.
- ○ Fail to reject H_0. The data suggests that the average heat output for sufferers is the same as that of non-sufferers.
- ○ Reject H_0. The data suggests that the average heat output for sufferers is the same as that of non-sufferers.
- ○ Fail to reject H_0. The data suggests that the average heat output for sufferers is less than 1 cal/cm^2/min below that of non-sufferers.

p-value & α: $0.0067 < 0.01$ ∴ reject H_0, Ha is true

(b) What is the probability of a type II error when the actual difference between μ_1 and μ_2 is $\mu_1 - \mu_2 = -1.4$? (Round your answer to four decimal places.)

$$\Delta' = -1.4,\ \Delta_0 = -1.0,\ \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{(0.1)^2}{9} + \frac{(0.5)^2}{9}} = 0.1700,\ z_\alpha = z_{0.01} = 2.33 \quad \beta(\Delta') = \beta(-1.4) = 1 - \Phi\left(-z_\alpha - \frac{\Delta' - \Delta_0}{\sigma_{\bar{x}_1 - \bar{x}_2}}\right) = 1 - \Phi\left(-2.33 - \frac{-1.4-(-1.0)}{0.1700}\right) = 0.4892$$

(c) Assuming that $m = n$, what sample sizes are required to ensure that $\beta = 0.1$ when $\mu_1 - \mu_2 = -1.4$? (Round your answer up to the nearest whole number.)

subjects

$$z_\alpha = z_{0.01} = 2.33,\ z_\beta = z_{0.1} = 1.28,\ m = n = \frac{(\sigma_1^2 + \sigma_2^2)(z_\alpha + z_\beta)^2}{(\Delta' - \Delta_0)^2} = \frac{(0.1^2 + 0.5^2)(2.33 + 1.28)^2}{(-1.4-(-1.0))^2} = 22$$

$$Z = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}$$

Alternative Hypothesis	$\beta(\Delta') = P(\text{type II error when} \mu_1 - \mu_2 = \Delta')$
H_a: $\mu_1 - \mu_2 > \Delta_0$	$\Phi\left(z_\alpha - \frac{\Delta' - \Delta_0}{\sigma}\right)$
H_a: $\mu_1 - \mu_2 < \Delta_0$	$1 - \Phi\left(-z_\alpha - \frac{\Delta' - \Delta_0}{\sigma}\right)$
H_a: $\mu_1 - \mu_2 \neq \Delta_0$	$\Phi\left(z_{\alpha/2} - \frac{\Delta' - \Delta_0}{\sigma}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta' - \Delta_0}{\sigma}\right)$

where $\sigma = \sigma_{\overline{X} - \overline{Y}} = \sqrt{(\sigma_1^2/m) + (\sigma_2^2/n)}$

As in Chapter 8, sample sizes m and n can be determined that will satisfy both $P(\text{type I error}) = \text{a specified } \alpha$ and $P(\text{type II error when } \mu_1 - \mu_2 = \Delta') = \text{a specified } \beta$. For an upper-tailed test, equating the previous expression for $\beta(\Delta')$ to the specified value of β gives

$$\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n} = \frac{(\Delta' - \Delta_0)^2}{(z_\alpha + z_\beta)^2}$$

When the two sample sizes are equal, this equation yields

$$m = n = \frac{(\sigma_1^2 + \sigma_2^2)(z_\alpha + z_\beta)^2}{(\Delta' - \Delta_0)^2}$$

2. [–/6 Points] DETAILS **DEVORESTAT9 9.1.006.** MY NOTES ASK YOUR TEACHER PRACTICE ANOT

An experiment to compare the tension bond strength of polymer latex modified mortar (Portland cement mortar to which polymer latex emulsions have been added during mixing) to that of unmodified mortar resulted in $\bar{x} = 18.13$ kgf/cm^2 for the modified mortar ($m = 42$) and $\bar{y} = 16.89$ kgf/cm^2 for the unmodified mortar ($n = 32$). Let μ_1 and μ_2 be the true average tension bond strengths for the modified and unmodified mortars, respectively. Assume that the bond strength distributions are both normal.

(a) Assuming that $\sigma_1 = 1.6$ and $\sigma_2 = 1.3$, test H_0: $\mu_1 - \mu_2 = 0$ versus H_a: $\mu_1 - \mu_2 > 0$ at level 0.01.

Calculate the test statistic and determine the P-value. (Round your test statistic to two decimal places and your P-value to four decimal places.)

z = 3.68

P-value = 0.0001

$m = n_1 = 42$, $\bar{x}_1 = 18.13$, $\sigma_1 = 1.6$

$n = n_2 = 32$, $\bar{y} = \bar{x}_2 = 16.89$, $\sigma_2 = 1.3$

State the conclusion in the problem context.

- ○ Fail to reject H_0. The data does not suggest that the difference in average tension bond strengths exceeds from 0.
- ○ Fail to reject H_0. The data suggests that the difference in average tension bond strengths exceeds 0.
- ● Reject H_0. The data suggests that the difference in average tension bond strengths exceeds 0.
- ○ Reject H_0. The data does not suggest that the difference in average tension bond strengths exceeds 0.

Compare p-value & α: $0.0001 < 0.01$ ∴ reject H_0, Ha true $(\mu_1 - \mu_2 > 0 \Rightarrow \mu_1 > \mu_2)$

(b) Compute the probability of a type II error for the test of part (a) when $\mu_1 - \mu_2 = 1$. (Round your answer to four decimal places.)

$$\Delta' = 1,\ \Delta_0 = 0,\ \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{1.6^2}{42} + \frac{1.3^2}{32}} = 0.3373,\ z_\alpha = z_{0.01} = 2.33,\ \beta(\Delta') = \beta(1) = \Phi\left(z_\alpha - \frac{\Delta' - \Delta_0}{\sigma_{\bar{x}_1 - \bar{x}_2}}\right) = \Phi\left(2.33 - \frac{1-0}{0.3373}\right) = 0.2628$$

(c) Suppose the investigator decided to use a level 0.05 test and wished $\beta = 0.10$ when $\mu_1 - \mu_2 = 1$. If $m = 42$, what value of n is necessary? (Round your answer up to the nearest whole number.)

n =

$$z_\alpha = z_{0.05} = 1.64,\ z_\beta = z_{0.1} = 1.28,\ \frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n} = \frac{(\Delta' - \Delta_0)^2}{(z_\alpha + z_\beta)^2} \Rightarrow \frac{1.6^2}{42} + \frac{1.3^2}{n} = \frac{(1-0)^2}{(1.64+1.28)^2} \Rightarrow n = 31$$

(d) How would the analysis and conclusion of part (a) change if σ_1 and σ_2 were unknown but $s_1 = 1.6$ and $s_2 = 1.3$?

- ● Since $n = 32$ is not a large sample, it would no longer be appropriate to use the large sample test. A small sample t procedure should be used, and the appropriate conclusion would follow.
- ○ Since $n = 32$ is a large sample, it would no longer be appropriate to use the large sample test. Any other test can be used, and the conclusions would stay the same.
- ○ Since $n = 32$ is not a large sample, it still be appropriate to use the large sample test. The analysis and conclusions would stay the same.
- ○ Since $n = 32$ is a large sample, it would be more appropriate to use the t procedure. The appropriate conclusion would follow.

3. [–/6 Points] DETAILS DEVORESTAT9 9.1.008. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Tensile strength tests were carried out on two different grades of wire rod, resulting in the accompanying data.

Grade	Sample Size	Sample Mean (kg/mm^2)	Sample SD
AISI 1064	m = 126	$\bar{x}$ = 106.5	s_1 = 1.2
AISI 1078	n = 126	$\bar{y}$ = 129.3	s_2 = 2.2

(a) Does the data provide compelling evidence for concluding that true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 kg/mm^2? Test the appropriate hypotheses using a significance level of 0.01. State the relevant hypotheses.

Ha:

$\mu_{1078} > \mu_{1064} + 10$

$\Rightarrow \mu_{1078} - \mu_{1064} > 10$

$\Rightarrow \mu_{1078} - \mu_{1064} < -10$

○ H_0: $\mu_{1064} - \mu_{1078} = -10$
H_a: $\mu_{1064} - \mu_{1078} \neq -10$

○ H_0: $\mu_{1064} - \mu_{1078} = -10$
H_a: $\mu_{1064} - \mu_{1078} \geq -10$

○ H_0: $\mu_{1064} - \mu_{1078} = -10$
H_a: $\mu_{1064} - \mu_{1078} > -10$

● H_0: $\mu_{1064} - \mu_{1078} = -10$
H_a: $\mu_{1064} - \mu_{1078} < -10$

○ H_0: $\mu_{1064} - \mu_{1078} = -10$
H_a: $\mu_{1064} - \mu_{1078} \leq -10$

Calculate the test statistic and P-value. (Round your test statistic to two decimal places and your P-value to four decimal places.)

z = -57.33

P-value = 0

Add $\bar{x}_1 + 10$

State the conclusion in the problem context.

● Reject H_0. The data suggests that the true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 kg/mm^2.

○ Fail to reject H_0. The data suggests that the true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 kg/mm^2.

○ Reject H_0. The data does not suggest that the true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 kg/mm^2.

○ Fail to reject H_0. The data does not suggest that the true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 kg/mm^2.

Compare P-value & α: 0<0.01, reject Ho, accept Ha

(b) Estimate the difference between true average strengths for the two grades in a way that provides information about precision and reliability. (Use a 95% confidence interval. Round your answers to two decimal places.)

(-23.24 , -22.36) kg/mm^2

TI-84:
STAT → TESTS → 2-SampZInt (9)

You may need to use the appropriate table in the Appendix of Tables to answer this question.

4. [–/4 Points] DETAILS DEVORESTAT9 9.2.017. MY NOTES ASK YOU

Determine the number of degrees of freedom for the two-sample t test or CI in each of the following situations. (Round your answers down to the nearest whole number.)

(a) m = 12, n = 10, s_1 = 3.0, s_2 = 5.0

19

$$df = \frac{\left(\frac{3.0^2}{12} + \frac{5.0^2}{10}\right)^2}{\frac{(3.0^2/12)^2}{12-1} + \frac{(5.0^2/10)^2}{10-1}} = 14$$

(b) m = 12, n = 21, s_1 = 3.0, s_2 = 5.0

30

(c) m = 12, n = 21, s_1 = 2.0, s_2 = 5.0

28

$$df = \frac{\left(\frac{S_1^2}{m} + \frac{S_2^2}{n}\right)^2}{\frac{(S_1^2/m)^2}{(m-1)} + \frac{(S_2^2/n)^2}{(n-1)}}$$

(d) m = 10, n = 24, s_1 = 3.0, s_2 = 5.0

27

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It

has approximately a t distribution with df ν estimated from the data by

$$\nu = \frac{\left(\frac{s_1^2}{m} + \frac{s_2^2}{n}\right)^2}{\frac{(s_1^2/m)^2}{m-1} + \frac{(s_2^2/n)^2}{n-1}} = \frac{[(se_1)^2 + (se_2)^2]^2}{\frac{(se_1)^4}{m-1} + \frac{(se_2)^4}{n-1}}$$

5. [–/3 Points] DETAILS DEVORESTAT9 9.2.019.MI.S. MY NOTES ASK YOUR TEACHER PRACTICE AN

Suppose μ_1 and μ_2 are true mean stopping distances at 50 mph for cars of a certain type equipped with two different types of braking systems. Use the two-sample t test at significance level 0.01 to test H_0: $\mu_1 - \mu_2 = -10$ versus H_a: $\mu_1 - \mu_2 < -10$ for the following data: m = 5, $\bar{x}$ = 114.8, s_1 = 5.09, n = 5, $\bar{y}$ = 129.5, and s_2 = 5.33.

USE SALT

Calculate the test statistic and determine the P-value. (Round your test statistic to one decimal place and your P-value to three decimal places.)

t = -1.4

P-value = 0.096

TI-84:
STAT → TESTS → 2-SampTTest (4)

Add 10 to $\bar{x}$ for hypothesis

State the conclusion in the problem context. Compare P-value & α: 0.096 > 0.01 ∴ fail to reject Ho

○ Reject H_0. The data suggests that the difference between mean stopping distances is less than −10.

○ Reject H_0. The data does not suggest that the difference between mean stopping distances is less than −10.

○ Fail to reject H_0. The data suggests that the difference between mean stopping distances is less than −10.

● Fail to reject H_0. The data does not suggest that the difference between mean stopping distances is less than −10.

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It Watch It Master It

6. [–/4 Points] DETAILS DEVORESTAT9 9.2.021.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Quantitative noninvasive techniques are needed for routinely assessing symptoms of peripheral neuropathies, such as carpal tunnel syndrome (CTS). An article reported on a test that involved sensing a tiny gap in an otherwise smooth surface by probing with a finger; this functionally resembles many work-related tactile activities, such as detecting scratches or surface defects. When finger probing was not allowed, the sample average gap detection threshold for m = 7 normal subjects was 1.68 mm, and the sample standard deviation was 0.54; for n = 12 CTS subjects, the sample mean and sample standard deviation were 2.39 and 0.84, respectively. Does this data suggest that the true average gap detection threshold for CTS subjects exceeds that for normal subjects?

$\mu_2 > \mu_1$ (CTS exceeds normal)

USE SALT

Quantitative noninvasive techniques are needed for routinely assessing symptoms of peripheral neuropathies, such as carpal tunnel syndrome (CTS). An article reported on a test that involved sensing a tiny gap in an otherwise smooth surface by probing with a finger; this functionally resembles many work-related tactile activities, such as detecting scratches or surface defects. When finger probing was not allowed, the sample average gap detection threshold for $m = 7$ normal subjects was 1.68 mm, and the sample standard deviation was 0.54; for $n = 12$ CTS subjects, the sample mean and sample standard deviation were 2.39 and 0.84, respectively. Does this data suggest that the true average gap detection threshold for CTS subjects exceeds that for normal subjects?

USE SALT

$\mu_2 > \mu_1$ (CTS exceeds normal)

$\Rightarrow 0 > \mu_1 - \mu_2 \Rightarrow \mu_1 - \mu_2 < 0$

State and test the relevant hypotheses using a significance level of 0.01. (Use μ_1 for normal subjects and μ_2 for CTS subjects.)

$\alpha = 0.01$

- ● $H_0: \mu_1 - \mu_2 = 0$
  $H_a: \mu_1 - \mu_2 < 0$
- ○ $H_0: \mu_1 - \mu_2 = 0$
  $H_a: \mu_1 - \mu_2 > 0$
- ○ $H_0: \mu_1 - \mu_2 = 0$
  $H_a: \mu_1 - \mu_2 \geq 0$
- ○ $H_0: \mu_1 - \mu_2 = 0$
  $H_a: \mu_1 - \mu_2 \neq 0$

$m=7$ $\quad n=12$

$\bar{x}=1.68$ $\quad \bar{y}=2.39$

$\sigma_1=0.54$ $\quad \sigma_2=0.84$

Calculate the test statistic and determine the P-value. (Round your test statistic to one decimal place and your P-value to three decimal places.)

$t =$ -2.2

P-value = 0.019

State the conclusion in the problem context. Compare P-value & α: $0.019 > 0.01$ ∴ fail to reject H_0

- ● Fail to reject H_0. The data suggests that the true average gap detection threshold for CTS subjects is the same as that for normal subjects.
- ○ Reject H_0. The data suggests that the true average gap detection threshold for CTS subjects exceeds that for normal subjects.
- ○ Fail to reject H_0. The data suggests that the true average gap detection threshold for CTS subjects exceeds that for normal subjects.
- ○ Reject H_0. The data suggests that the true average gap detection threshold for CTS subjects is the same as that for normal subjects.