# Module 3 Test

Tuesday, June 6, 2023 4:29 PM

**1. [–/6 Points]** DETAILS DEVORESTAT9 3.SE.509.XP.S. MY NOTES ASK YOUR TEACHER

When circuit boards used in the manufacture of compact disc players are tested, the long-run percentage of defectives is 5%. Let $X$ = the number of defective boards in a random sample of size $n = 25$, so $X \sim \text{Bin}(25, 0.05)$. (Round your probabilities to three decimal places.)

$p = 0.05 \quad n = 25$

USE SALT

(a) Determine $P(X \leq 2)$.

$P(X \leq 2) = B(x; n, p) = B(2; 25, 0.05) =$

(b) Determine $P(X \geq 5)$.

$P(X \geq 5) = 1 - P(X \leq 4) = 1 - B(4; 25, 0.05) = 0.007$

(c) Determine $P(1 \leq X \leq 4)$.

$P(1 \leq X \leq 4) = B(4; 25, 0.05) - B(0; 25, 0.05) = 0.715$

(d) What is the probability that none of the 25 boards is defective?

$P(X = 0) = B(0; 25, 0.05) = 0.277$

(e) Calculate the expected value and standard deviation of $X$. (Round your standard deviation to two decimal places.)

expected value ______ boards $E(X) = np = (25)(0.05) = 1.250$

standard deviation ______ boards $\sigma = \sqrt{npq} = \sqrt{(25)(0.05)(1-0.05)} = 1.090$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

**2. [–/6 Points]** DETAILS DEVORESTAT9 3.SE.510.XP. MY NOTES ASK YOUR TEACHER

An electronics store has received a shipment of 30 table radios that have connections for an iPod or iPhone. Ten of these have two slots (so they can accommodate both devices), and the other twenty have a single slot. Suppose that six of the 30 radios are randomly selected to be stored under a shelf where the radios are displayed, and the remaining ones are placed in a storeroom. Let $X$ = the number among the radios stored under the display shelf that have two slots.

(a) What kind of distribution does $X$ have (name and values of all parameters)?

- ○ hypergeometric with $N = 10$, $M = 6$, and $n = 10$
- ○ binomial with $n = 30$, $x = 10$, and $p = 6/30$
- ○ binomial with $n = 10$, $x = 6$, and $p = 6/10$
- ● hypergeometric with $N = 30$, $M = 10$, and $n = 6$

$N = 30$ total table radios (iPod or iPhone, 10 both + 20 single)

$M = 10$ radios with both

$n = 6$ randomly selected

(b) Compute $P(X = 2)$, $P(X \leq 2)$, and $P(X \geq 2)$. (Round your answers to four decimal places.)

$P(X = 2) =$ ______

$P(X \leq 2) =$ ______

$P(X \geq 2) =$ ______

$P(X=2) = \dfrac{\binom{M}{x}\cdot\binom{N-M}{n-x}}{\binom{N}{n}} = \dfrac{\binom{10}{2}\cdot\binom{30-10}{6-2}}{\binom{30}{6}} = 0.3672$

$P(X \leq 2) = H(x; n, M, N) = H(2; 6, 10, 30) = 0.6936$

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - H(1, 6, 10, 30) = 0.6736$

(c) Calculate the mean value and standard deviation of $X$. (Round your standard deviation to two decimal places.)

mean value ______ radios

standard deviation ______ radios

$E(X) = n \cdot \frac{M}{N} = 6 \cdot \frac{10}{30} = 2.00$

$V(X) = \left(\frac{N-n}{N-1}\right) \cdot n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right) = \left(\frac{30-6}{30-1}\right) \cdot 6 \cdot \frac{10}{30} \cdot \left(1 - \frac{10}{30}\right) = 1.10$

$\sigma = \sqrt{V(X)} = \sqrt{1.10} = 1.05$

**3. [–/5 Points]** DETAILS DEVORESTAT9 3.SE.511.XP. MY NOTES ASK YOUR TEACHER

Let $X$, the number of flaws on the surface of a randomly selected boiler of a certain type, have a Poisson distribution with parameter $\mu = 5$. Use the cumulative Poisson probabilities from the Appendix Tables to compute the following probabilities. (Round your answers to three decimal places.)

(a) $P(X \leq 7)$

$P(X \leq 7) = p_{cdf}(x; \mu) = p_{cdf}(7; 5) = 0.867$

(b) $P(X = 7)$

$P(X = 7) = p_{pdf}(7; 5) = 0.104$

(c) $P(8 \leq X)$

$P(8 \leq X) = P(X \geq 8) = 1 - P(X \leq 7) = 1 - p_{cdf}(7; 5) = 0.133$

(d) $P(5 \leq X \leq 7)$

$P(5 \leq X \leq 7) = p_{cdf}(7; 5) - p_{cdf}(4; 5) = 0.426$

(e) $P(5 < X < 7)$

$P(5 < X < 7) = P(X = 6) = p_{pdf}(6; 5) = 0.146$

**4. [–/4 Points]** DETAILS **DEVORESTAT9 4.SE.503.XP.** MY NOTES ASK YOUR TEACHER

Suppose the time spent by a randomly selected student who uses a terminal connected to a local time-sharing computer facility has a gamma distribution with mean 20 min and variance 80 $\text{min}^2$.

(a) What are the values of $\alpha$ and $\beta$?

$\alpha$ = 

$\beta$ = 

$\beta = \frac{\alpha\beta^2}{\alpha\beta} = \frac{V(X)}{E(X)} = \frac{80}{20} = 4 \qquad E(x) = \alpha\beta \Rightarrow \alpha = \frac{E(x)}{\beta} = \frac{20}{4} = 5$

(b) What is the probability that a student uses the terminal for at most 24 min? (Round your answer to three decimal places.)

$P(X \le 24) = F(x;\alpha,\beta) = F(24;5,4) = \underset{\text{Excel}}{\text{gamma.dist}}(x,\alpha,\beta,\text{cum.}) = \text{gamma.dist}(24,5,4) = 0.715$

(c) What is the probability that a student spends between 20 and 40 min using the terminal? (Round your answer to three decimal places.)

$P(20 \le x \le 40) = \text{gamma.dist}(40;5,4) - \text{gamma.dist}(20;5,4) = 0.411$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

The mean and variance of a random variable $X$ having the gamma distribution $f(x;\alpha,\beta)$ are

$$E(X) = \mu = \alpha\beta \qquad V(X) = \sigma^2 = \alpha\beta^2$$

**Definition**

A continuous random variable $X$ is said to have a **gamma distribution** if the pdf of $X$ is

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} & x \ge 0 \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(4.8)}$$

where the parameters $\alpha$ and $\beta$ satisfy $\alpha > 0$, $\beta > 0$. The **standard gamma distribution** has $\beta = 1$, so the pdf of a standard gamma rv is given by (4.7).

When $X$ is a standard gamma rv, the cdf of $X$,

$$F(x;\alpha) = \int_0^x \frac{y^{\alpha-1}e^{-y}}{\Gamma(\alpha)}dy \qquad x > 0 \qquad \textbf{(4.9)}$$

is called the **incomplete gamma function** [sometimes the incomplete gamma function refers to Expression (4.9) without the denominator $\Gamma(\alpha)$ in the integrand]. There are extensive tables of $F(x;\alpha)$ available; in Appendix Table A.4, we present a small tabulation for $\alpha = 1, 2, \ldots, 10$ and $x = 1, 2, \ldots, 15$.

**Proposition**

Let $X$ have a gamma distribution with parameters $\alpha$ and $\beta$. Then for any $x > 0$, the cdf of $X$ is given by

$$P(X \le x) = F(x;\alpha,\beta) = F\left(\frac{x}{\beta};\alpha\right)$$

where $F(\,\cdot\,;\alpha)$ is the incomplete gamma function.

**5. [–/5 Points]** DETAILS **DEVORESTAT9 4.SE.504.XP.** MY NOTES ASK YOUR TEACHER

Let $X$ = the time (in $10^{-1}$ weeks) from shipment of a defective product until the customer returns the product. Suppose that the minimum return time is $\gamma = 4.5$ and that the excess $X - 4.5$ over the minimum has a Weibull distribution with parameters $\alpha = 2$ and $\beta = 2.5$.

(a) What is the cdf of $X$?

$$F(x) = \begin{cases} 0 & x < 4.5 \\ \square & x \ge 4.5 \end{cases}$$

$1 - e^{-(x/\beta)^\alpha} = 1 - e^{-\left(\frac{x-4.5}{2.5}\right)^2}$

(b) What are the expected return time and variance of return time? [*Hint*: First obtain $E(X - 4.5)$ and $V(X - 4.5)$.] (Round your answers to three decimal places.)

$E(X)$ = $10^{-1}$ weeks

$V(X)$ = $(10^{-1}\text{ weeks})^2$

$E(X) = \beta \cdot \Gamma(1 + 1/\alpha) = \beta \cdot \int_0^\infty x^{\alpha-1}e^{-x}dx = 2.5 \cdot \int_0^\infty x^{(3/2)-1}e^{-x}dx = 2.216 \qquad E(X-4.5) = E(x) + 4.5 = 2.216 + 4.5 = 6.716$

$V(x) = \beta^2\left[\Gamma(1+2/\alpha) - (\Gamma(1+1/\alpha))^2\right] = (2.5)^2\left[\underset{\text{Excel:}}{\text{gamma}}(1+2/2) - (\text{gamma}(1+1/2))^2\right] = 1.341 = V(X-4.5)$

(c) Compute $P(X > 7)$. (Round your answer to four decimal places.)

$P(X>7) = 1 - P(X \le 7) = 1 - \text{weibull.dist}(x-4.5,\alpha,\beta,\text{cum.}) = 1 - \text{weibull.dist}(7-4.5,2,2.5,\text{TRUE}) = 0.3679$

(d) Compute $P(7 \le X \le 9.5)$. (Round your answer to four decimal places.)

$P(7 \le x \le 9.5) = P(x \le 9.5) - P(x \le 7) = \text{weibull.dist}(9.5-4.5,2,2.5,\text{TRUE}) - \text{weibull.dist}(7-4.5,2,2.5) = 0.3496$

**6. [–/7 Points]** DETAILS **DEVORESTAT9 4.SE.505.XP.S.** MY NOTES ASK YOUR TEACHER

A theoretical justification based on a certain material failure mechanism underlies the assumption that ductile strength *X* of a material has a lognormal distribution. Suppose the parameters are $\mu = 5.0$ and $\sigma = 0.11$.

USE SALT

(a) Compute $E(X)$ and $V(X)$. (Round your answers to three decimal places.)

$E(X) =$ ____

$V(X) =$ ____

$E(x) = e^{\mu+\sigma^2/2} = e^{5.0+(0.11)^2/2} = 149.314$

$V(x) = e^{2\mu+\sigma^2} \cdot (e^{\sigma^2}-1) = e^{2(5.0)+(0.11)^2} \cdot (e^{(0.11)^2}-1) = 271.403$

(b) Compute $P(X > 125)$. (Round your answer to four decimal places.)

$P(x>125) = 1-P(x\le 125) = 1-P(\ln(x)\le \ln(125)) = 1-P(z\le \frac{\ln(x)-\mu}{\sigma}) = 1-P(z\le \frac{\ln(125)-5.0}{0.11}) = 1-P(z\le -1.56) = \Phi(-1.56) = 0.9407$

(c) Compute $P(110 \le X \le 125)$. (Round your answer to four decimal places.)

$P(110\le x\le 125) = P(\frac{\ln(110)-5.0}{0.11} \le z \le \frac{\ln(125)-5.0}{0.11}) = P(-2.72\le z\le -1.56) = \Phi(-1.56)-\Phi(-2.72) = 0.0561$

(d) What is the value of median ductile strength? (Round your answer to three decimal places.) $\Phi(z)=0.5 \Rightarrow \text{invnorm}(0.5)=0$

$P(X\le x)=0.5 \Rightarrow P(z\le \frac{\ln(x)-5.0}{0.11})=0.5 \Rightarrow \Phi(\frac{\ln(x)-5.0}{0.11})=0.5 \Rightarrow \frac{\ln(x)-5.0}{0.11}=0 \Rightarrow \ln(x)-5.0=0 \Rightarrow \ln(x)=5 \Rightarrow x=e^5=148.413$

(e) If ten different samples of an alloy steel of this type were subjected to a strength test, how many would you expect to have strength of at least 125? (Round your answer to three decimal places.)

____ samples $p=0.9407$ (from (b)) $n=10$

$\underset{\text{binomial}}{E(Y)} = n\cdot p = (0.9407)(10) = 9.407$

(f) If the smallest 5% of strength values were unacceptable, what would the minimum acceptable strength be? (Round your answer to three decimal places.) $\Phi(z)=0.05 \Rightarrow \text{invnorm}(0.05)=-1.64$

$P(X\le x)=0.05 \Rightarrow P(z\le \frac{\ln(x)-5.0}{0.11})=0.05 \Rightarrow \Phi(\frac{\ln(x)-5.0}{0.11})=0.05 \Rightarrow \frac{\ln(x)-5.0}{0.11}=-1.64 \Rightarrow \ln(x)-5.0=-0.181 \Rightarrow \ln(x)=4.819 \Rightarrow x=e^{4.819}=123.849$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

**7. [–/3 Points]** DETAILS **DEVORESTAT9 4.3.040.S.** MY NOTES ASK YOUR TEACHER

An article suggested that yield strength (ksi) for A36 grade steel is normally distributed with $\mu = 45$ and $\sigma = 5.5$.

USE SALT

(a) What is the probability that yield strength is at most 39? Greater than 67? (Round your answers to four decimal places.)

at most 39 ____

greater than 67 ____

$P(X\le 39) = P(z\le \frac{x-\mu}{\sigma}) = P(z\le \frac{39-45}{5.5}) = P(z\le -1.09) = 0.1379$

$P(x>67) = 1-P(x\le 67) = 1-P(z\le \frac{67-45}{5.5}) = 1-P(z\le 4) = 0.0000$

(b) What yield strength value separates the strongest 75% from the others? (Round your answer to three decimal places.) $\Phi(z)=0.25 \Rightarrow \text{invnorm}(0.25)=-0.67$

____ ksi $P(X>x)=0.75 \Rightarrow 1-P(X\le x)=0.75 \Rightarrow -P(X\le x)=0.75-1 \Rightarrow P(X\le x)=1-0.75 \Rightarrow P(z\le \frac{x-45}{5.5})=0.25 \Rightarrow \Phi(\frac{x-45}{5.5})=0.25 \Rightarrow \frac{x-45}{5.5}=-0.67 \Rightarrow x-45=-3.710$

$\Rightarrow x=41.290$

You may need to use the appropriate table in the Appendix of Tables to answer this question.