Ch6 - Point Estimation

Friday, June 9, 2023 8:10 PM

1. [–/10 Points] DETAILS **DEVORESTAT9 6.1.001.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Consider the accompanying data on flexural strength (MPa) for concrete beams of a certain type.

7.0	8.1	5.4	7.2	9.7	7.4	7.7
6.8	9.0	7.7	11.8	11.6	6.5	7.3
8.7	8.2	10.7	8.0	6.8	6.3	7.9
6.3	7.8	7.0	7.3	9.7	11.3	

$n = 27$

(a) Calculate a point estimate of the mean value of strength for the conceptual population of all beams manufactured in this fashion. [*Hint*: Σx_i = 219.2.] (Round your answer to three decimal places.)

[ ] MPa $\bar{x} = \frac{\sum x_i}{n} = \frac{219.2}{27} = 8.119$

State which estimator you used.

- ○ $\hat{p}$
- ○ s
- ○ $\tilde{x}$
- ○ $s/\bar{x}$
- ● $\bar{x}$

(b) Calculate a point estimate of the strength value that separates the weakest 50% of all such beams from the strongest 50%.

[ ] MPa $\tilde{x} = 7.7$ (median)

State which estimator you used.

- ○ $s/\bar{x}$
- ○ $\hat{p}$
- ○ $\bar{x}$
- ● $\tilde{x}$
- ○ s

(c) Calculate a point estimate of the population standard deviation σ. [*Hint*: Σx_i^2 = 1853.98.] (Round your answer to three decimal places.)

[ ] MPa $s = \sqrt{\frac{\sum x_i^2 - (\sum x_i)^2/n}{n-1}} = \sqrt{\frac{1853.98 - (219.2)^2/27}{27-1}} = 1.692$

Interpret this point estimate.

- ○ This estimate describes the center of the data.
- ● This estimate describes the spread of the data.
- ○ This estimate describes the bias of the data.
- ○ This estimate describes the linearity of the data.

Which estimator did you use?

- ● s
- ○ $\tilde{x}$
- ○ $s/\bar{x}$
- ○ $\hat{p}$
- ○ $\bar{x}$

(d) Calculate a point estimate of the proportion of all such beams whose flexural strength exceeds 10 MPa. [*Hint*: Think of an observation as a "success" if it exceeds 10.] (Round your answer to three decimal places.)

[ ] $\hat{p} = \frac{x}{n} = \frac{4}{27} = 0.148$ (sample proportion)

(e) Calculate a point estimate of the population coefficient of variation σ/μ. (Round your answer to four decimal places.)

[ ] $\frac{s}{\bar{x}} = \frac{1.692}{8.119} = 0.2084$ (sample coefficient of variation)

State which estimator you used.

- ○ s
- ● $s/\bar{x}$
- ○ $\hat{p}$
- ○ $\bar{x}$
- ○ $\tilde{x}$

2. [–/8 Points] DETAILS **DEVORESTAT9 6.1.003.S.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Consider the following sample of observations on coating thickness for low-viscosity paint.

0.84	0.88	0.88	1.01	1.09	1.17	1.29	1.31
1.36	1.49	1.59	1.62	1.65	1.71	1.76	1.83

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Assume that the distribution of coating thickness is normal (a normal probability plot strongly supports this assumption).

(a) Calculate a point estimate of the mean value of coating thickness. (Round your answer to four decimal places.)

[ ] $\bar{x} = \frac{\sum x_i}{n} = \frac{21.48}{16} = 1.3425$

State which estimator you used.

- ○ $\tilde{x}$
- ○ s
- ○ $s/\bar{x}$
- ○ $\hat{p}$
- ● $\bar{x}$

(b) Calculate a point estimate of the median of the coating thickness distribution. (Round your answer to four decimal places.)

[ ] $\tilde{x} = 1.3350$ (n is even so we avg)

State which estimator you used and which estimator you might have used instead. (Select all that apply.)

- ☑ $\tilde{x}$
- ☐ $\hat{p}$
- ☐ s
- ☐ $s/\bar{x}$
- ☑ $\bar{x}$

✔

(c) Calculate a point estimate of the value that separates the largest 10% of all values in the thickness distribution from the remaining 90%. [*Hint*: Express what you are trying to estimate in terms of μ and σ.] (Round your answer to four decimal places.)

$\sigma = 0.3357$

[ ] $z_{0.90} = \text{invnorm}(0.90) = 1.645$ 90^{th} percentile $= \mu + z_{0.90}\sigma = 1.3425 + 1.645 \cdot 0.3357 = 1.8947$

State which estimator you used.

- ○ $\bar{x}$
- ○ 10th percentile

State which estimator you used and which estimator you might have used instead. (Select all that apply.)

- ☑ $\tilde{x}$
- ☐ $\hat{p}$
- ☐ s
- ☐ $s / \bar{x}$
- ☑ $\bar{x}$

✓

(c) Calculate a point estimate of the value that separates the largest 10% of all values in the thickness distribution from the remaining 90%. [*Hint*: Express what you are trying to estimate in terms of μ and σ.] (Round your answer to four decimal places.)

$\sigma = 0.3357$

$z_{0.90} = \text{invnorm}(0.90) = 1.645$ $\quad$ 90^{th} percentile $= \mu + z_{0.90}\sigma = 1.3425 + 1.645 \cdot 0.3357 = 1.8947$

State which estimator you used.

- ○ $\bar{x}$
- ○ 10th percentile
- ○ s
- ○ $\tilde{x}$
- ● 90th percentile

(d) Estimate $P(X < 1.1)$, i.e., the proportion of all thickness values less than 1.1. [*Hint*: If you knew the values of μ and σ, you could calculate this probability. These values are not available, but they can be estimated.] (Round your answer to four decimal places.)

Assume $\bar{x} = \mu$ & $s = \sigma$

$P(X < 1.1) = P\left(Z < \frac{1.1 - 1.3425}{0.3357}\right) = P(Z < -0.72) = 0.2350$

(e) What is the estimated standard error of the estimator that you used in part (b)? (Round your answer to four decimal places.)

$\hat{\sigma}_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.3357}{\sqrt{16}} = 0.0839$

3. [–/7 Points] DETAILS **DEVORESTAT9 6.1.004.MI.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Consider the accompanying data on flexural strength (MPa) for concrete beams of a certain type.

5.7	7.2	7.3	6.3	8.1	6.8	7.0	7.4	6.8	6.5	7.0	6.3	7.9	9.0
8.4	8.7	7.8	9.7	7.4	7.7	9.7	8.0	7.7	11.6	11.3	11.8	10.7	

$n = 27$

$\bar{x} = 8.141$

The data below give accompanying strength observations for cylinders.

6.5	5.8	7.8	7.1	7.2	9.2	6.6	8.3	7.0	8.1
8.0	8.1	7.4	8.5	8.9	9.8	9.7	14.1	12.6	12.0

$n = 20$

$\bar{y} = 8.635$

Prior to obtaining data, denote the beam strengths by $X_1, \ldots, X_m$ and the cylinder strengths by $Y_1, \ldots, Y_n$. Suppose that the X_i's constitute a random sample from a distribution with mean μ_1 and standard deviation σ_1 and that the Y_i's form a random sample (independent of the X_i's) from another distribution with mean μ_2 and standard deviation σ_2.

(a) Use rules of expected value to show that $\bar{X} - \bar{Y}$ is an unbiased estimator of $\mu_1 - \mu_2$.

- ○ $E(\bar{X} - \bar{Y}) = \left(E(\bar{X}) - E(\bar{Y})\right)^2 = \mu_1 - \mu_2$
- ○ $E(\bar{X} - \bar{Y}) = \frac{E(\bar{X}) - E(\bar{Y})}{nm} = \mu_1 - \mu_2$
- ○ $E(\bar{X} - \bar{Y}) = \sqrt{E(\bar{X}) - E(\bar{Y})} = \mu_1 - \mu_2$
- ● $E(\bar{X} - \bar{Y}) = E(\bar{X}) - E(\bar{Y}) = \mu_1 - \mu_2$
- ○ $E(\bar{X} - \bar{Y}) = nm\left(E(\bar{X}) - E(\bar{Y})\right) = \mu_1 - \mu_2$

Calculate the estimate (in MPa) for the given data. (Round your answer to three decimal places.)

MPa $E(\bar{x}) - E(\bar{y}) = 8.141 - 8.635 = -0.494$

(b) Use rules of variance to obtain an expression for the variance and standard deviation (standard error) of the estimator in part (a).

$$V(\bar{X} - \bar{Y}) = V(\bar{X}) + V(\bar{Y}) = \sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2$$

Identify the next step in this rule from the options below.

- ○ $V(\bar{X} - \bar{Y}) = \frac{\sigma_1^2}{n_1} - \frac{\sigma_2^2}{n_2}$
- ○ $V(\bar{X} - \bar{Y}) = \frac{\sigma_1}{n_1} - \frac{\sigma_2}{n_2}$
- ○ $V(\bar{X} - \bar{Y}) = \frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}$
- ● $V(\bar{X} - \bar{Y}) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

Since standard deviation is the square root of variance, it follows that

$\sigma_{\bar{X} - \bar{Y}} = \sqrt{V(\bar{X} - \bar{Y})}$ $\quad$ $\sigma_1 = 1.673, \ \sigma_2 = 2.139$

- ● $\sigma_{\bar{X} - \bar{Y}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{(1.673)^2}{27} + \frac{(2.139)^2}{20}} = 0.577$
- ○ $\sigma_{\bar{X} - \bar{Y}} = \sqrt{\frac{\sigma_1}{n_1} - \frac{\sigma_2}{n_2}}$
- ○ $\sigma_{\bar{X} - \bar{Y}} = \sqrt{\frac{\sigma_1^2}{n_1} - \frac{\sigma_2^2}{n_2}}$
- ○ $\sigma_{\bar{X} - \bar{Y}} = \sqrt{\frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}}$

Compute the estimated standard error (in MPa). (Round your answer to three decimal places.)

MPa

(c) Calculate a point estimate of the ratio σ_1/σ_2 of the two standard deviations. (Round your answer to three decimal places.)

$\frac{1.673}{2.139} = 0.782$

(d) Suppose a single beam and a single cylinder are randomly selected. Calculate a point estimate (in MPa^2) of the variance of the difference $X - Y$ between beam strength and cylinder strength. (Round your answer to two decimal places.)

MPa^2 $V(\bar{x} - \bar{y}) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{1.673}{1} + \frac{2.139}{1} = 7.379$

4. [–/6 Points] DETAILS **DEVORESTAT9 6.1.005.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

As an example of a situation in which several different statistics could reasonably be used to calculate a point estimate, consider a population of N invoices. Associated with each invoice is its "book value," the recorded amount of that invoice. Let T denote the total book value, a known amount. Some of these book values are erroneous. An audit will be carried out by randomly selecting n invoices and determining the audited (correct) value for each one. Suppose that the sample gives the following results (in dollars).

Invoice

	1	2	3	4	5
Book value	315	694	535	219	107
Audited value	315	519	535	219	164
Error	0	175	0	0	−57

$\bar{y} = 374$

$\bar{x} = 350.4$

	Invoice				
	1	2	3	4	5
Book value	315	694	535	219	107
Audited value	315	519	535	219	164
Error	0	175	0	0	−57

$\bar{y} = 374$
$\bar{x} = 350.4$
$\bar{D} = 23.6$

Let
$\bar{Y}$ = sample mean book value
$\bar{X}$ = sample mean audited value
$\bar{D}$ = sample mean error

Propose a statistic for estimating the total audited (i.e., correct) value involving just N and $\bar{X}$.

- ○ $\bar{X} / N$
- ○ $N - \bar{X}$
- ● $N\bar{X}$ pop. sample mean audited value
- ○ $\bar{X} + N$

Propose a statistic for estimating the total audited (i.e., correct) value involving T, N, and $\bar{D}$.

- ○ $T + N\bar{D}$
- ● $T - N\bar{D}$ total book value − pop. sample mean error
- ○ $TN\bar{D}$
- ○ $T \cdot N / \bar{D}$

Propose a statistic for estimating the total audited (i.e., correct) value involving T and $\bar{X} / \bar{Y}$.

- ○ $T - \bar{X} / \bar{Y}$
- ○ $1 - T \cdot \bar{X} / \bar{Y}$
- ○ $T + \bar{X} / \bar{Y}$
- ● $T \cdot \bar{X} / \bar{Y}$ total book value · sample mean audited value / sample mean book value

If N = 5000 and T = 1,761,100, calculate the corresponding point estimates. (Round your answers to the nearest whole number.)

based on just N and $\bar{X}$ $ [ ] $N \cdot \bar{x} = (5{,}000)(350.4) = 1{,}752{,}000$
based on T, N, and $\bar{D}$ $ [ ] $T - N \cdot \bar{D} = (1{,}761{,}100) - (5{,}000)(23.6) = 1{,}643{,}100$
based on T and $\bar{X} / \bar{Y}$ $ [ ] $T \cdot \bar{x}/\bar{y} = (1{,}761{,}100) \cdot 350.4/374 = 1{,}649{,}972$

5. [–/4 Points] DETAILS DEVORESTAT9 6.2.020. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A diagnostic test for a certain disease is applied to n individuals known to not have the disease. Let X = the number among the n test results that are positive (indicating presence of the disease, so X is the number of false positives) and p = the probability that a disease-free individual's test result is positive (i.e., p is the true proportion of test results from disease-free individuals that are positive). Assume that only X is available rather than the actual sequence of test results.

(a) Derive the maximum likelihood estimator of p.

$\hat{p}$ = [ ] $\frac{x}{n} \Rightarrow \frac{d}{dp}\left[\ln(f(x;p))\right] = \frac{d}{dp}\left[\ln\left(\binom{n}{x} p^x (1-p)^{n-x}\right)\right] = 0$

If n = 25 and x = 5, what is the estimate?

$\hat{p}$ = [ ] $\frac{5}{25} = \frac{1}{5} = 0.2$

(b) Is the estimator of part (a) unbiased?

- ● Yes
- ○ No

maximum likelihood estimator

(c) If n = 25 and x = 5, what is the mle of the probability $(1 - p)^5$ that none of the next five tests done on disease-free individuals are positive? (Round your answer to four decimal places.)

[ ] $\left(1 - \frac{5}{25}\right)^5 = 0.3277$

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6. [–/4 Points] DETAILS DEVORESTAT9 6.2.022.MI. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Let X denote the proportion of allotted time that a randomly selected student spends working on a certain aptitude test. Suppose the pdf of X is

$$f(x; \theta) = \begin{cases} (\theta + 1)x^\theta & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

where $-1 < \theta$. A random sample of ten students yields data x_1 = 0.65, x_2 = 0.86, x_3 = 0.45, x_4 = 0.73, x_5 = 0.99, x_6 = 0.79, x_7 = 0.92, x_8 = 0.90, x_9 = 0.94, x_{10} = 0.79.

(a) Use the method of moments to obtain an estimator of θ

- ● $\frac{1}{(1 - \bar{X})} - 2$
- ○ $\frac{1}{(1 + \bar{X})}$
- ○ $\frac{1}{(1 + \bar{X})} - 1$
- ○ $\frac{1}{(\bar{X} - 1)} - 2$
- ○ $\frac{1}{(\bar{X} - 1)} - 1$

$$E(x) = \int_0^1 x(\theta+1)x^\theta \, dx = 1 - \frac{1}{\theta+2}$$

$$\bar{x} = 1 - \frac{1}{\theta+2} \Rightarrow \hat{\theta} = \frac{1}{1-\bar{x}} - 2$$

Compute the estimate for this data. (Round your answer to two decimal places.) $\bar{x} = 0.802$

[ ] $\hat{\theta} = \frac{1}{1-\bar{x}} - 2 = \frac{1}{1-0.802} - 2 = 3.05$

MLE

(b) Obtain the maximum likelihood estimator of θ.

- ○ $\frac{n}{\Sigma \ln(X_i)}$
- ○ $\frac{\Sigma \ln(X_i)}{n} - 1$
- ○ $\frac{\Sigma \ln(X_i)}{-n}$
- ● $\frac{-n}{\Sigma \ln(X_i)} - 1$ $\Rightarrow \frac{d}{d\theta}\left[n \cdot \ln(\theta+1) + \theta \sum_{i=1}^{n} \ln(x_i)\right] = 0$
- ○ $\frac{\Sigma \ln(X_i)}{n}$

Compute the estimate for the given data. (Round your answer to two decimal places.) $\sum \ln(x_i) = -2.427$

[ ] $\theta = \frac{-n}{\sum \ln(x_i)} - 1 = \frac{-10}{-2.427} - 1 = 3.12$

7. **[–/4 Points]** DETAILS **DEVORESTAT9 6.2.025.** MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

The shear strength of each of ten test spot welds is determined, yielding the following data (psi).

389 367 362 389 374 409 358 375 397 415

(a) Assuming that shear strength is normally distributed, estimate the true average shear strength and standard deviation of shear strength using the method of maximum likelihood. (Round your answers to two decimal places.)

average [ ] psi

standard deviation [ ] psi

$\hat{\mu} = \bar{x} = \frac{\sum x_i}{n} = 383.50$

S.D. estimator

$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = 381.389 \quad \hat{\sigma}^2 = \frac{n-1}{n} \cdot s^2 = \frac{10-1}{10} \cdot 381.389 = 343.25 \quad \hat{\sigma} = \sqrt{\hat{\sigma}^2} = \sqrt{343.25} = 18.53$

(b) Again assuming a normal distribution, estimate the strength value below which 95% of all welds will have their strengths. [*Hint*: What is the 95th percentile in terms of μ and σ? Now use the invariance principle.] (Round your answer to two decimal places.)

$\Phi(z) = 0.95 \Rightarrow \text{invnorm}(0.95) = 1.64$

[ ] psi $P(X < x) = 0.95 \Rightarrow P(Z < x) = 0.95 \Rightarrow P\left(Z < \frac{x - \hat{\mu}}{\hat{\sigma}}\right) = 0.95 \Rightarrow \frac{x - \hat{\mu}}{\hat{\sigma}} = 1.64 \Rightarrow \frac{x - 383.50}{18.53} = 1.64 \Rightarrow x = 413.89$

(c) Suppose we decide to examine another test spot weld. Let X = shear strength of the weld. Use the given data to obtain the mle of $P(X \le 400)$. [*Hint*: $P(X \le 400) = \Phi((400 - \mu)/\sigma)$.] (Round your answer to four decimal places.)

[ ] $P(X \le 400) = P\left(Z \le \frac{400 - 383.50}{18.53}\right) = P(Z \le 0.89) = 0.8134$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

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