Module 2 Test

Friday, May 19, 2023 11:11 PM

1. [–/6 Points] DETAILS **DEVORESTAT9 3.1.006.** MY NOTES ASK YOUR TEACHER

Starting at a fixed time, each car entering an intersection is observed to see whether it turns left (L), right (R), or goes straight ahead (A). The experiment terminates as soon as a car is observed to turn left. Let X = the number of cars observed. What are possible X values?

- ○ 0, 1, 2, 3, ...
- ● 1, 2, 3, 4, ...
- ○ 0, 1, 2, 3, 4
- ○ 1, 2, 3

Same as problem 3 of Ch.3

Given the outcomes below, find their associated X values.

Outcome:	AAL	RRAL	ARRAL	RRL	ARL
X:	3	4	5	3	3

2. [–/4 Points] DETAILS **DEVORESTAT9 3.2.019.** MY NOTES ASK YOUR TEACHER

A library subscribes to two different weekly news magazines, each of which is supposed to arrive in Wednesday's mail. In actuality, each one may arrive on Wednesday, Thursday, Friday, or Saturday. Suppose the two arrive independently of one another, and for each one P(Wed.) = 0.3, P(Thurs.) = 0.35, P(Fri.) = 0.2, and P(Sat.) = 0.15. Let Y = the number of days beyond Wednesday that it takes for both magazines to arrive (so possible Y values are 0, 1, 2, or 3). Compute the pmf of Y. [*Hint:* There are 16 possible outcomes; $Y(W,W) = 0$, $Y(F,Th) = 2$, and so on.] (Enter your answers to four decimal places.)

Y	0	1	2	3
$P(Y)$	0.0900	0.3325	0.3000	0.2775

y = number of days after W until both magazines are delivered

Similar to problem 6 of Ch.3

Sample space of probability $= \{ \underset{y=0}{\overset{1}{P(W,W)}}, \underset{y=1}{\overset{2}{P(W,Th)}}, \underset{y=2}{\overset{3}{P(W,F)}}, \underset{y=3}{\overset{4}{P(W,Sat)}}, \underset{y=1}{\overset{5}{P(Th,W)}}, \underset{y=1}{\overset{6}{P(Th,Th)}}, \underset{y=2}{\overset{7}{P(Th,F)}}, \underset{y=3}{\overset{8}{P(Th,Sat)}}, \underset{y=2}{\overset{9}{P(F,W)}}, \underset{y=2}{\overset{10}{P(F,Th)}}, \underset{y=2}{\overset{11}{P(F,F)}}, \underset{y=}{\overset{12}{P(F,S}}$

$P(Y=0) = P(W,W) = (0.30)(0.30) = 0.0900$

$P(Y=1) = P(W,Th) + P(Th,W) + P(Th,Th) = (0.30)(0.35) + (0.35)(0.30) + (0.35)(0.35) = 0.3325$

$P(Y=2) = P(W,F) + P(Th,F) + P(F,W) + P(F,Th) + P(F,F) = (0.30)(0.20) + (0.35)(0.20) + (0.20)(0.30) + (0.20)(0.35) + (0.20)(0.20) = 0.3000$

$P(Y=3) = P(W,Sat) + P(Th,Sat) + P(F,Sat) + P(Sat,W) + P(Sat,Th) + P(Sat,F) + P(Sat,Sat) = (0.30)(0.15) + (0.35)(0.15) + (0.20)(0.15) + (0.15)(0.30) + (0.15)(0.35) + (0.15)(0.20) + (0.15)(0.15) = 0.2775$

3. [–/4 Points] DETAILS **DEVORESTAT9 3.3.039.** MY NOTES ASK YOUR TEACHER

A chemical supply company currently has in stock 100 lb of a certain chemical, which it sells to customers in 3-lb batches. Let X = the number of batches ordered by a randomly chosen customer, and suppose that X has the following pmf.

x	1	2	3	4
$p(x)$	0.3	0.5	0.1	0.1

Compute $E(X)$ and $V(X)$.

$E(X)$ = ______ batches

$V(X)$ = ______ batches^2

$E(x) = \sum x \cdot p(x) = (1)(0.3) + (2)(0.5) + (3)(0.1) + (4)(0.1) = 2$

$E(x^2) = \sum x^2 \cdot p(x) = (1)^2(0.3) + (2)^2(0.5) + (3)^2(0.1) + (4)^2(0.1) = 4.8$

$V(x) = E(x^2) - [E(x)]^2 = 4.8 - (2)^2 = 0.8$

Compute the expected number of pounds left after the next customer's order is shipped and the variance of the number of pounds left. [*Hint:* The number of pounds left is a linear function of X.]

expected weight left ______ lb

variance of weight left ______ lb^2

$E(100-3x) = \sum (100-3x) \cdot p(x) = (100-3\cdot1)(0.3) + (100-3\cdot2)(0.5) + (100-3\cdot3)(0.1) + (100-3\cdot4)(0.1) = 94$

$V(100-3x) = (3)^2 \cdot V(x) = (3)^2 \cdot 0.8 = 7.2$

Using: $V(aX+b) = a^2 \cdot V(x)$

4. [–/5 Points] DETAILS **DEVORESTAT9 4.1.006.**

$$\begin{array}{c|cc} & x & -3 \\ \hline x & x^2 & -3x \\ -3 & -3x & 9 \end{array} \quad x^2 - 6x + 9$$

The actual tracking weight of a stereo cartridge that is set to track at 3 g on a particular changer can be regarded as a continuous rv X with the following pdf.

$$f(x) = \begin{cases} k[1-(x-3)^2] & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

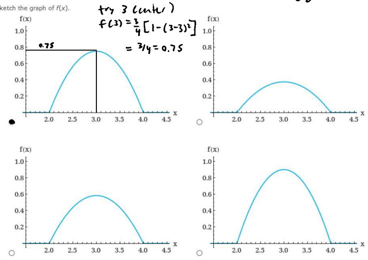
$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_2^4 k \cdot [1-(x-3)^2]dx = 1 \Rightarrow k \cdot \int_2^4 1-(x^2-6x+9)dx = 1 \Rightarrow k \cdot \int_2^4 1 - x^2 + 6x - 9\,dx = 1 \Rightarrow k \cdot \frac{4}{3} = 1 \Rightarrow k = \frac{3}{4}$

(a) Sketch the graph of $f(x)$.

try 3 (center)

$f(3) = \frac{3}{4}[1-(3-3)^2]$

$= 3/4 = 0.75$



(b) Find the value of k.

0.75 ✔

(c) What is the probability that the actual tracking weight is greater than the prescribed weight?

$P(x \ge 3) = P(3 \le x \le 4) = \int_a^b f(x)dx = \int_3^4 \frac{3}{4}[1-(x-3)^2]dx = 0.50$

(d) What is the probability that the actual weight is within 0.55 g of the prescribed weight? (Round your answer to four decimal places.)

$P(3 - \text{deviation} \le x \le 3 + \text{deviation}) = P(3 - 0.55 \le x \le 3 + 0.55) = P(2.45 \le x \le 3.55) = \int_a^b f(x)dx = \int_{2.45}^{3.55} \frac{3}{4}[1-(x-3)^2]dx = 0.7418$

(e) What is the probability that the actual weight differs from the prescribed weight by more than 0.6 g? (Round your answer to four decimal places.)

$P(X < 3 - 0.6 \cup x > 3 + 0.6) = P(x < 2.4 \cup x > 3.6) = 1 - P(2.4 \le x \le 3.6) = 1 - \int_{2.4}^{3.6} \frac{3}{4}[1-(x-3)^2]dx = 0.208$

$\underset{3}{\text{Sat})}, \underset{y=3}{\overset{13}{?(\text{Sat}, \text{W})}}, \underset{y=3}{\overset{14}{?(\text{Sat}, \text{Th})}}, \underset{y=3}{\overset{15}{?(\text{Sat}, \text{F})}}, \underset{y=3}{\overset{16}{?(\text{Sat}, \text{Sat})}} \}$

5. [–/10 Points] DETAILS DEVORESTAT9 4.2.015.

Let X denote the amount of space occupied by an article placed in a 1-ft[3] packing container. The pdf of X is below.

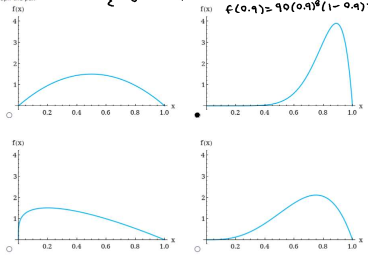
$$f(x) = \begin{cases} 90x^8(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \Rightarrow \begin{cases} 90x^8 - 90x^9 & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

$f(0.1) = 90(0.1)^8(1-0.1) = 8.1 \times 10^{-7}$

$f(0.5) = 90(0.5)^8(1-0.5) = 0.18$

$f(0.9) = 90(0.9)^8(1-0.9) = 3.87$

(a) Graph the pdf.



Obtain the cdf of X.

$$F(x) = \begin{cases} 0 & x < 0 \\ \square & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

$$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_0^x 90y^8(1-y)\,dy = \int_0^x 90y^8 - 90y^9\,dy = \frac{90y^9}{9} - \frac{90y^{10}}{10}\Big|_0^x = \left[\frac{90x^9}{9} - \frac{90x^{10}}{10}\right] - [0] = 10x^9 - 9x^{10}$$

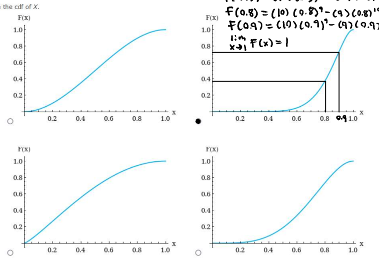
$F(0.5) = (10)(0.5)^9 - (9)(0.5)^{10} = 0.01$

$F(0.8) = (10)(0.8)^9 - (9)(0.8)^{10} = 0.38$

$F(0.9) = (10)(0.9)^9 - (9)(0.9)^{10} = 0.74$

$\lim_{x \to 1} F(x) = 1$

Graph the cdf of X.



(b) What is $P(X \le 0.7)$ [i.e., $F(0.7)$]? (Round your answer to four decimal places.)

$P(X \le 0.7) = F(0.7) = (10)(0.7)^9 - (9)(0.7)^{10} = 0.1493$

(c) Using the cdf from (a), what is $P(0.45 < X \le 0.7)$? (Round your answer to four decimal places.)

$P(0.45 \le X \le 0.7) = F(0.7) - F(0.45) = [(10)(0.7)^9 - (9)(0.7)^{10}] - [(10)(0.45)^9 - (10)(0.45)^{10}] = 0.1448$

What is $P(0.45 \le X \le 0.7)$? (Round your answer to four decimal places.)

$P(0.45 \le X \le 0.7) = 0.1448$ (same b/c continuous, not discrete, see image below)

(d) What is the 75th percentile of the distribution? (Round your answer to four decimal places.)

$F(k) = 0.75 \Rightarrow 10k^9 - 9k^{10} = 0.75 \Rightarrow -9k^{10} + 10k^9 - 0.75 = 0 \Rightarrow k = 0.9036$ (use eq'n solver on calculator)

(e) Compute $E(X)$ and σ_X. (Round your answers to four decimal places.)

$E(X) =$

$\sigma_X =$

$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^1 x[90x^8 - 90x^9]\,dx = 0.8182$

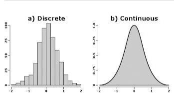
$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx = \int_0^1 x^2[90x^8 - 90x^9]\,dx = 0.6818$

$V(X) = E(X^2) - [E(X)]^2 = 0.6818 - (0.8182)^2 = 0.0124$

$\sigma_X = \sqrt{V(X)} = \sqrt{(0.0124)} = 0.1113$

(f) What is the probability that X is more than 1 standard deviation from its mean value? (Round your answer to four decimal places.)

$P(\mu_X - \sigma_X > X \cup X < \mu_X + \sigma_X) = 1 - P(\mu_X - \sigma_X \le X \le \mu_X + \sigma_X) = 1 - [F(\mu_X + \sigma_X) - F(\mu_X - \sigma_X)] = 1 - [F(0.8182 + 0.1113) - F(0.8182 - 0.1113)] = 1 - [F(0.9295) - F(0.7069)] = 1 - [(10(0.9295)^9 - 9(0.9295)^{10}) - (10(0.7069)^9 - 9(0.7069)^{10})]$



Definition

Let p be a number between 0 and 1. The **(100p)th percentile** of the distribution of a continuous rv X, denoted by $\eta(p)$, is defined by

$$p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\,dy \qquad (4.2)$$

6. [–/10 Points] DETAILS DEVORESTAT9 5.1.018. MY NOTES ASK YOUR TEACHER

A service station has both self-service and full-service islands. On each island, there is a single regular unleaded pump with two hoses. Let X denote the number of hoses being used on the self-service island at a particular time, and let Y denote the number of hoses on the full-service island in use at that time. The joint pmf of X and Y appears in the accompanying tabulation.

$p(x, y)$		y = 0	1	2
x	0	0.10	0.03	0.02
	1	0.07	0.20	0.07
	2	0.05	0.14	0.32

(a) Given that $X = 1$, determine the conditional pmf of Y—i.e., $p_{Y|X}(0|1)$, $p_{Y|X}(1|1)$, $p_{Y|X}(2|1)$. (Round your answers to four decimal places.)

y	0	1	2
$p_{Y\|X}(y\|1)$			

(b) Given that two hoses are in use at the self-service island, what is the conditional pmf of the number of hoses in use on the full-service island? (Round your answers to four decimal places.)

y	0	1	2
$p_{Y\|X}(y\|2)$			

(c) Use the result of part (b) to calculate the conditional probability $P(Y \le 1 \mid X = 2)$. (Round your answer to four decimal places.)

$P(Y \le 1 \mid X = 2) =$

(d) Given that two hoses are in use at the full-service island, what is the conditional pmf of the number in use at the self-service island? (Round your answers to four decimal places.)

x	0	1	2
$p_{X\|Y}(x\|2)$			

Definition

Let X and Y be two continuous rv's with joint pdf $f(x, y)$ and marginal X pdf $f_X(x)$. Then for any X value x for which $f_X(x) > 0$, the **conditional probability density function of Y given that $X = x$** is

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} \qquad -\infty < y < \infty$$

If X and Y are discrete, replacing pdf's by pmf's in this definition gives the **conditional probability mass function of Y when $X = x$.**

(a)

$$P_{Y|X}(Y|X) = \frac{f(x,y)}{f_X(x)}$$

$$P_{Y|X}(y=0|x=1) = \frac{P(y=0 \cap x=1)}{P(x=1)} = \frac{0.07}{0.07+0.20+0.07} = 0.2059$$

$$P_{Y|X}(y=1|x=1) = \frac{P(y=1 \cap x=1)}{P(x=1)} = \frac{0.20}{0.07+0.20+0.07} = 0.5882$$

$$P_{Y|X}(y=2|x=1) = \frac{P(y=2 \cap x=1)}{P(x=1)} = \frac{0.07}{0.07+0.20+0.07} = 0.2059$$

$= 0.3137$

(b)

$$P_{Y|X}(y=0|x=2)=\frac{P(y=0\cap x=2)}{P(x=2)}=\frac{0.05}{0.05+0.14+0.32}=0.0980$$

$$P_{Y|X}(y=1|x=2)=\frac{P(y=1\cap x=2)}{P(x=2)}=\frac{0.14}{0.05+0.14+0.32}=0.2745$$

$$P_{Y|X}(y=2|x=2)=\frac{P(y=2\cap x=2)}{P(x=2)}=\frac{0.32}{0.05+0.14+0.32}=0.6275$$

(c)

$$P_{Y|X}(Y\le 1\,|\,x=2)=P_{Y|X}(y=0\,|\,x=2)+P_{Y|X}(y=1\,|\,x=2)$$
$$=0.0980+0.2745$$
$$=0.3725$$

(d)

$$P_{X|Y}(x=0|y=2)=\frac{P(x=0\cap y=2)}{P(y=2)}=\frac{0.02}{0.02+0.07+0.32}=0.0488$$

$$P(x=1\,|\,y=2)=\frac{P(x=1\cap y=2)}{P(y=2)}=\frac{0.07}{0.02+0.07+0.32}=0.1707$$

$$P(x=2\,|\,y=2)=\frac{P(x=2\cap y=2)}{P(y=2)}=\frac{0.32}{0.02+0.07+0.32}=0.7805$$

7. [–/2 Points] DETAILS DEVORESTAT9 5.2.031. MY NOTES ASK YOUR TEACHER

Each front tire on a particular type of vehicle is supposed to be filled to a pressure of 26 psi. Suppose the actual air pressure in each tire is a random variable—X for the right tire and Y for the left tire, with joint pdf given below.

$$f(x,y)=\begin{cases}K(x^2+y^2) & 22\le x\le 31,\ 22\le y\le 31\\ 0 & \text{otherwise}\end{cases}$$

(a) Compute the covariance between X and Y. (Round your answer to four decimal places.)
Cov(X, Y) =

(b) Compute the correlation coefficient ρ for this X and Y. (Round your answer to four decimal places.)
ρ =

similar to ch5 problem 8

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy=1 \Rightarrow \int_{22}^{31}\int_{22}^{31} K(x^2+y^2)\,dx\,dy=1 \Rightarrow K=\frac{1}{114{,}858}$$

$$f_Y(y)=f_X(x)=\int_{-\infty}^{\infty} f(x,y)\,dy=\int_{22}^{31} K(x^2+y^2)\,dy=\frac{x^2+709}{12{,}762}$$

$$E(Y)=E(X)=\int_{-\infty}^{\infty} x\cdot f(x)\,dx=\int_{22}^{31} x\cdot\frac{x^2+709}{12{,}762}\,dx=26.7523$$

$$E(XY)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\cdot f(x,y)\,dx\,dy=\int_{22}^{31}\int_{22}^{31} xy\cdot\left[K(x^2+y^2)\,dx\,dy\right]=715.6215$$

$$\mathrm{Cov}(X,Y)=E(XY)-E(X)\cdot E(Y)=(715.6215)-(26.7523)(26.7523)=-0.0637$$

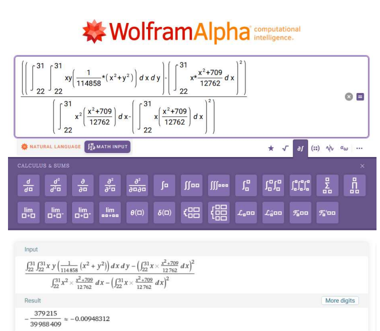
$$E(Y^2)=E(X^2)=\int_{-\infty}^{\infty} x^2\cdot f(x)\,dx=\int_{22}^{31} x^2\cdot\frac{x^2+709}{12{,}762}\,dx=722.3971$$

$$V(Y)=V(X)=E(X^2)-[E(X)]^2=6.7121$$

$$\sigma_Y=\sigma_X=\sqrt{V(X)}=\sqrt{6.7121}=2.5908$$

$$\rho=\frac{\mathrm{Cov}(X,Y)}{\sigma_X\cdot\sigma_Y}=\frac{-0.0637}{(2.5908)(2.5908)}=-0.0095$$

correlation coefficient



Covariance: