Module 1 Test

Friday, May 12, 2023   5:16 PM

Temperature transducers of a certain type are shipped in batches of 50. A sample of 60 batches was selected, and the number of transducers in each batch not conforming to design specifications was determined, resulting in the following data:

1	1	2	3	1	1	3	2	0	5	3	3	1	3	2	4	7	0	2	3
0	4	2	1	3	1	1	3	4	1	2	3	2	2	8	4	5	1	3	1
5	0	2	3	2	1	0	6	4	2	1	6	0	3	3	3	7	1	2	2

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(a) Determine frequencies and relative frequencies for the observed values of x = number of nonconforming transducers in a batch. (Round your relative frequencies to four decimal places.)

x	Nonconforming Frequency	Relative Frequency
0	6	0.1000
1	14	0.2333
2	13	0.2167
3	14	0.2333
4	5	0.0833
5	3	0.0500
6	2	0.0333
7	2	0.0333
8	1	0.0167

(b) What proportion of batches in the sample have at most *six* nonconforming transducers? (Round your answer to four decimal places.)

$P(x \leq 6) = 0.1000 + 0.2333 + 0.2167 + 0.2333 + 0.0833 + 0.0500 + 0.0333 = 0.9499$
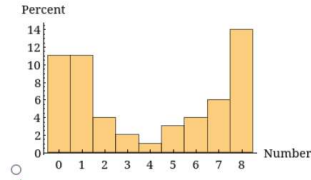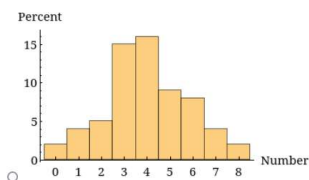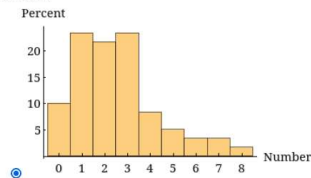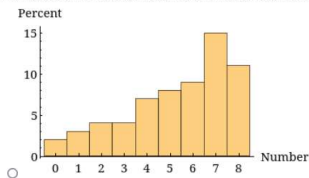
What proportion have fewer than *six*? (Round your answer to four decimal places.)

$P(x < 6) = 0.1000 + 0.2333 + 0.2167 + 0.2333 + 0.0833 + 0.0500 = 0.9166$

What proportion have at least *six* nonconforming units? (Round your answer to four decimal places.)

$P(x \geq 6) = 0.0333 + 0.0333 + 0.0167 = 0.0833$

(c) Draw a histogram of the data using relative frequency on the vertical scale.

Comment on its features. (Select all that apply.)

- ☑ There is some positive skewness in the data.
- ☐ The center of the histogram is around 1.
- ☐ The center of the histogram is around 7.
- ☐ The distribution is fairly symmetric.
- ☐ There is some negative skewness in the data.
- ☑ The center of the histogram is around 2 or 3. ✔

relative frequency of a value $= \dfrac{\text{number of times the value occurs}}{\text{number of observations in the data set}}$

In a study of author productivity, a large number of authors were classified according to the number of articles they had published during a certain period. The results were presented in the accompanying frequency distribution:

Number of papers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Frequency	796	204	127	50	33	28	19	19	6	7	6	7	4	4	5	3	3

USE SALT

(a) Construct a histogram/bar chart corresponding to this frequency distribution.

What is the most interesting feature of the shape of the distribution?
- ○ slight positive skewness
- ○ heavy negative skewness
- ○ bimodal
- ○ normally distributed
- ⦿ heavy positive skewness
- ○ slight negative skewness

(b) What proportion of these authors published at least five papers? At least ten papers? More than ten papers? (Round your answers to four decimal places.)

at least five papers [＿＿＿]

at least ten papers [＿＿＿]   *Answer below*

more than ten papers [＿＿＿]

(c) Suppose the five 15s, three 16s, and three 17s had been lumped into a single category displayed as "≥15." Would you be able to draw a histogram? Explain.
- ○ Yes; this class does have a finite width of 3, so the relative frequency of category "≥15" can be plotted as a rectangle of height 0.831.
- ○ No; the classes cannot be combined in such a manner because the increments would not be equivalent.
- ⦿ No; this class has no upper boundary, so it is impossible to draw a rectangle above it having finite area (i.e., frequency).

(d) Suppose that instead of the values 15, 16, and 17 being listed separately, they had been combined into 15–17 category with frequency 11. Would you be able to draw a histogram? Explain.
- ⦿ Yes; this class does have a finite width of 3, so the relative frequency of category "15–17" can be plotted as a rectangle of height 0.831.
- ○ No; this class has no upper boundary, so it is impossible to draw a rectangle above it having finite area (i.e., frequency).
- ○ No; the classes cannot be combined in such a manner because the increments would not be equivalent.

	Number of papers	Frequency	Relative Frequency						
1	Number of papers	Frequency	Relative Frequency						
2	1	796	0.6026						
3	2	204	0.1544						
4	3	127	0.0961						
5	4	50	0.0379	P(X>=5)					
6	5	33	0.0250	SUM	0.1090				
7	6	28	0.0212						
8	7	19	0.0144						
9	8	19	0.0144						
10	9	6	0.0045		P(X>=10)				
11	10	7	0.0053		SUM	0.0295	P(X>10)		
12	11	6	0.0045				SUM	0.0242	
13	12	7	0.0053						
14	13	4	0.0030						
15	14	4	0.0030						
16	15	5	0.0038						
17	16	3	0.0023						
18	17	3	0.0023						

The minimum injection pressure (psi) for injection molding specimens of high amylose corn was determined for eight different specimens (higher pressure corresponds to greater processing difficulty), resulting in the following observations.

15.0 12.9 17.7 14.5 12.5 10.7 9.5 8.0

(a) Determine the values of the sample mean $\bar{x}$, sample median $\tilde{x}$, and 12.5% trimmed mean $\bar{x}_{tr}$. (Round your answers to two decimal places.)

$\bar{x} =$ [＿＿＿] psi

$\tilde{x} =$ [＿＿＿] psi

$\bar{x}_{tr} =$ [＿＿＿] psi

Compare these values.
- ○ The mean is much larger than the median and trimmed mean, indicating positive skewness.
- ○ All three measures of center are similar, indicating little skewness to the data set.
- ○ The median is much larger than the mean and trimmed mean, indicating negative skewness.
- ○ The mean is much larger than the median and trimmed mean, indicating negative skewness.
- ○ The median is much larger than the mean and trimmed mean, indicating positive skewness.

(b) By how much could the smallest sample observation, currently 8.0, be increased without affecting the value of the sample median?
[＿＿＿] psi

(c) Suppose we want the values of the sample mean and median when the observations are expressed in kilograms per square inch (ksi) rather than psi. Is it necessary to reexpress each observation in ksi, or can the values calculated in part (a) be used directly? [Hint: 1 kg = 2.2 lb.]
- ○ Yes, it is necessary to reexpress each observation.
- ○ No, the values obtained in part (a) can be used directly.

(a)   8.0,   9.5,   10.7,   12.5,   12.9,   14.5,   15.0,   17.7
       1   2   3   4   5   6   7   8

$$\bar{x} = \frac{(8.0 + 9.5 + 10.7 + 12.5 + 12.9 + 14.5 + 15.0 + 17.7)}{8} = 12.6$$

8.0, ~~9.5~~ ~~10.7~~ 12.5, 12.9, ~~14.5~~ ~~15.0~~ ~~17.7~~

$$\tilde{x} = \frac{12.5 + 12.9}{2} = 12.7$$

12.5% (⅛) trimmed data set:

9.5, 10.7, 12.5, 12.9, 14.5, 15.0

$$\bar{x}_{tr} = \frac{(9.5 + 10.7 + 12.5 + 12.9 + 14.5 + 15.0)}{6} = 12.52$$

(b) $\tilde{x}$ — smallest value to change sample median

12.5 − 8.0 = 4.5

(c) No, the conversion factor will not affect the statistics

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4. [-/4 Points] DETAILS DEVORESTAT9 1.SE.505.XP.S. MY NOTES ASK YOUR TEACHER

An article reported the following data on oxygen consumption (mL/kg/min) for a sample of ten firefighters performing a fire-suppression simulation:

29.1 49.0 30.2 28.3 28.8 25.8 33.0 29.2 23.0 30.0

🔲 **USE SALT**

Compute the following. (Round your answers to four decimal places.)

(a) The sample range
[____] mL/kg/min   range = largest − smallest

(b) The sample variance s^2 from the definition (i.e., by first computing deviations, then squaring them, etc.)
[____] mL²/kg²/min² $s^2 = \frac{\sum(x - \bar{x})^2}{n-1}$

(c) The sample standard deviation
[____] mL/kg/min $s = \sqrt{s^2}$

(d) s^2 using the shortcut method
[____] mL²/kg²/min² $s^2 = \frac{S_{xx}}{n-1}$ where $S_{xx} = \sum[x]^2 - \frac{(\sum[x])^2}{n}$

23		
25.8	RANGE:	26.0000
28.3	SAMPLE VARIANCE:	48.7071
28.8	SAMPLE STANDARD DEVIATION:	6.9790
29.1		
29.2		
30		
30.2		
33		
49		

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5. [-/4 Points] DETAILS DEVORESTAT9 2.SE.501.XP. MY NOTES ASK YOUR TEACHER

Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cap B) = 0.15$.

(a) Compute the probability that the selected individual has at least one of the two types of cards (i.e., the probability of the event $A \cup B$).
[____] $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.5 - 0.15 = 0.65$

(b) What is the probability that the selected individual has neither type of card?
[____] $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.65 = 0.35$

(c) Describe, in terms of A and B, the event that the selected student has a Visa card but not a MasterCard.
🔘 $A \cap B'$ A   B'
○ $A' \cup B'$
○ $A' \cap B'$
○ $A \cup B'$
○ $A' \cap B$

Calculate the probability of this event.
[____] $P(A \cap B') =$ have A − having A & B = $P(A) - P(A \cap B) = 0.3 - 0.15 = 0.15$

A box contains six 40-W bulbs, seven 60-W bulbs, and nine 75-W bulbs. If bulbs are selected one by one in random order, what is the probability that at least two bulbs must be selected to obtain one that is rated 75 W? (Round your answer to three decimal places.)

[   ]

$P(75W$ in at least 2 trials$) = 1 - P(75W$ on first try$) = 1 - \dfrac{9}{6+7+9} = 0.591$

if we don't get it on the first try, then we get it on the second try (the complement of the first try)

A box in a certain supply room contains four 40-W lightbulbs, three 60-W bulbs, and five 75-W bulbs. Suppose that three bulbs are randomly selected. (Round your answers to four decimal places.)

(a) What is the probability that exactly two of the selected bulbs are rated 75-W?

[   ] $P(\text{exactly } 2 \, 75\text{-W}) = \dfrac{\text{combo of 2 75-W} \cdot \text{combo of other lights}}{\text{total ways to select any lights}} = \dfrac{\binom{5}{2} \cdot \binom{4+3}{1}}{\binom{4+3+5}{3}} = 0.3182$

(b) What is the probability that all three of the selected bulbs have the same rating?

[   ] $P(\text{all the same rating}) = \dfrac{\text{combo of 40-W + combo of 60-W + combo of 75-W}}{\text{total ways to select any light}} = \dfrac{\binom{4}{3} + \binom{3}{3} + \binom{5}{3}}{\binom{4+3+5}{3}} = 0.0682$

(c) What is the probability that one bulb of each type is selected?

[   ] $P(\text{one of each}) = \dfrac{\text{combo of 1 40-W} \cdot \text{combo of 1 60-W} \cdot \text{combo of 75-W}}{\text{total ways to select any light}} = \dfrac{\binom{4}{1} \cdot \binom{3}{1} \cdot \binom{5}{1}}{\binom{4+3+5}{3}} = \dfrac{4 \cdot 3 \cdot 5}{\binom{4+3+5}{3}} = 0.2727$

(d) Suppose now that bulbs are to be selected one by one until a 75-W bulb is found. What is the probability that it is necessary to examine at least six bulbs?

[   ] $P(\text{at least 6 until 75-W}) = P(\text{first 5 bulbs not 75-W}) = \dfrac{\text{combo of 5 bulbs either 40-W or 60-W}}{\text{total combo of bulbs out of 5}} = \dfrac{\binom{4+3}{5}}{\binom{4+3+5}{5}} = 0.0265$

Fifteen telephones have just been received at an authorized service center. Five of these telephones are cellular, five are cordless, and the other five are corded phones. Suppose that these components are randomly allocated the numbers 1, 2, . . . , 15 to establish the order in which they will be serviced. (Round your answers to four decimal places.)

(a) What is the probability that all the cordless phones are among the first ten to be serviced?

[   ] $P(\text{first 10 are cordless}) = \dfrac{\text{combo of cordless phones} \cdot \text{combo of remaining phones}}{\text{total ways of 10 phones}} = \dfrac{\binom{5}{10-5} \cdot \binom{5+5}{10-5}}{\binom{5+5+5}{10}} = 0.0839$

(b) What is the probability that after servicing ten of these phones, phones of only two of the three types remain to be serviced?

[   ] $P(\text{only 2 of 3 types remaining after 10}) = \dfrac{\text{combo of 2 out of 3 phones} \, (\text{combo of remaining phones} - 2 \text{ phones of same type})}{\text{total ways of 10 phones}} = \dfrac{\binom{3}{2}\left[\binom{5+5}{5}-2\right]}{\binom{5+5+5}{10}} = 0.2498$

(c) What is the probability that two phones of each type are among the first six serviced?

[   ] $P(\text{2 of each type in 6}) = \dfrac{\text{combo of cellular in 2} \cdot \text{combo of cordless in 2} \cdot \text{combo of corded in 2}}{\text{total ways of 6 phones}} = \dfrac{\binom{5}{2} \cdot \binom{5}{2} \cdot \binom{5}{2}}{\binom{5+5+5}{6}} = 0.1998$

Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard where P(A) = 0.50, P(B) = 0.45, and P(A ∩ B) = 0.30. Calculate and interpret each of the following probabilities (a Venn diagram might help). (Round your answers to four decimal places.)

(a) $P(B \mid A)$ [   ] $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.30}{0.50} = 0.6000$

(b) $P(B' \mid A)$ [   ] $P(B' \mid A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{P(A) - P(B \cap A)}{P(A)} = \dfrac{0.50 - 0.30}{0.50} = 0.4000$

(c) $P(A \mid B)$ [   ] $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.30}{0.45} = 0.6667$

(d) $P(A' \mid B)$ [   ] $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)} = \dfrac{0.45 - 0.30}{0.45} = 0.3333$

(e) Given that the selected individual has at least one card, what is the probability that he or she has a Visa Card?

[   ] $P(A \mid A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A) + P(B) - P(A \cap B)} = \dfrac{0.50}{0.50 + 0.45 - 0.30} = 0.7692$

A company that manufactures video cameras produces a basic model and a deluxe model. Over the past year, 45% of the cameras sold have been of the basic model. Of those buying the basic model, 32% purchase an extended warranty, whereas 37% of all deluxe purchasers do so. If you learn that a randomly selected purchaser has an extended warranty, how likely is it that he or she has a basic model? (Round your answer to four decimal places.)

$$P(\text{basil} \mid \text{warranty}) = \frac{P(\text{basil} \cap \text{warranty})}{P(\text{warranty})} = \frac{(0.45)(0.32)}{(0.45)(0.32)+(1-0.45)(0.37)} = 0.4144$$