

1. [-/14 Points] DETAILS DEVORESTAT9 5.3.037.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

A particular brand of dishwasher soap is sold in three sizes: 25 oz, 40 oz, and 65 oz. Twenty percent of all purchasers select a 25-oz box, 50% select a 40-oz box, and the remaining 30% choose a 65-oz box. Let  $X_1$  and  $X_2$  denote the package sizes selected by two independently selected purchasers.

(a) Determine the sampling distribution of  $\bar{X}$ .  $E(\bar{X}) = 25 \cdot 0.2 + 40 \cdot 0.5 + 65 \cdot 0.3 = 44.5$

$\bar{x}$	25	32.5	40	45	52.5	65
$p(\bar{x})$	0.04	0.10	0.25	0.20	0.30	0.09

$x_1 \backslash x_2$	25	40	65
25	0.2*0.2 =0.04	0.2*0.5 =0.10	0.2*0.3 =0.06
40	0.5*0.2 =0.10	0.5*0.5 =0.25	0.5*0.3 =0.15
65	0.3*0.2 =0.06	0.3*0.5 =0.15	0.3*0.3 =0.09

Calculate  $E(\bar{X})$ . $E(\bar{X}) =$   ozCompare  $E(\bar{X})$  to  $\mu$ .☐  $E(\bar{X}) < \mu$ ☐  $E(\bar{X}) > \mu$ ☒  $E(\bar{X}) = \mu$ (b) Determine the sampling distribution of the sample variance  $S^2$ .

$s^2$	0	112.5	312.5	800
$p(s^2)$	0.38	0.20	0.30	0.12

Calculate  $E(S^2)$ . $E(S^2) =$  Compare  $E(S^2)$  to  $\sigma^2$ .☐  $E(S^2) > \sigma^2$ ☐  $E(S^2) < \sigma^2$ ☐  $E(S^2) = \sigma^2$ 

$$E(S^2) = 0 \cdot 0.38 + 112.5 \cdot 0.20 + 312.5 \cdot 0.30 + 800 \cdot 0.12 = 212.25$$

2. [-/13 Points] DETAILS DEVORESTAT9 5.3.038.MI.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

There are two traffic lights on a commuter's route to and from work. Let  $X_1$  be the number of lights at which the commuter must stop on his way to work, and  $X_2$  be the number of lights at which he must stop when returning from work. Suppose that these two variables are independent, each with the pmf given in the accompanying table (so  $X_1, X_2$  is a random sample of size  $n = 2$ ).

$x_1$	0	1	2
$p(x_1)$	0.4	0.2	0.4

 $\mu = 1, \sigma^2 = 0.8$ (a) Determine the pmf of  $T_0 = X_1 + X_2$ .

$t_0$	0	1	2	3	4
$p(t_0)$	0.16	0.16	0.32	0.16	0.16

(b) Calculate  $\mu_{T_0}$ .

$$\mu_{T_0} = 0 \cdot 0.16 + 1 \cdot 0.16 + 2 \cdot 0.32 + 3 \cdot 0.16 + 4 \cdot 0.16 = 2$$

How does it relate to  $\mu$ , the population mean? $\mu_{T_0} =$    $\cdot \mu$  $\mu_{T_0} = 2, \mu = 1$ (c) Calculate  $\sigma_{T_0}^2$ . $\sigma_{T_0}^2 =$  

$$E(T_0^2) = 0^2 \cdot 0.16 + 1^2 \cdot 0.16 + 2^2 \cdot 0.32 + 3^2 \cdot 0.16 + 4^2 \cdot 0.16 = 5.6$$

 $\sigma_{T_0}^2 =$  

$$\sigma_{T_0}^2 = E(T_0^2) - [E(T_0)]^2 = 5.6 - [2]^2 = 5.6 - 4 = 1.6$$

How does it relate to  $\sigma^2$ , the population variance? $\sigma_{T_0}^2 =$    $\cdot \sigma^2$ 

$$\sigma_{T_0}^2 = 0.8, \sigma^2 = 1.6$$

(d) Let  $X_3$  and  $X_4$  be the number of lights at which a stop is required when driving to and from work on a second day assumed independent of the first day. With  $T_0 =$  the sum of all four  $X_i$ 's, what now are the values of  $E(T_0)$  and  $V(T_0)$ ?

$$T_0 = X_1 + X_2 + X_3 + X_4 \quad \mu = 1, \sigma^2 = 0.8$$

 $E(T_0) =$  

$$E(T_0) = 4 \mu = 4(1) = 4$$

 $V(T_0) =$  

$$V(T_0) = 4 \sigma^2 = 4(0.8) = 3.2$$

(e) Referring back to (d), what are the values of  $P(T_0 = 8)$  and  $P(T_0 \geq 7)$  [Hint: Don't even think of listing all possible outcomes!]

 $P(T_0 = 8) =$  

$$P(T_0 = 8) = P(X_1=2) \cdot P(X_2=2) \cdot P(X_3=2) \cdot P(X_4=2) = (0.4)(0.4)(0.4)(0.4) = 0.0256$$

 $P(T_0 \geq 7) =$  

$$P(T_0 \geq 7) = P(T_0 = 7) + P(T_0 = 8) = 0.0512 + 0.0256 = 0.0768$$

$$P(T_0 = 7) = 7 \cdot 0.4 \cdot 0.3 \cdot 0.3 \cdot 0.4 = 0.0512$$

Need Help?

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Topics:

5.3 [Statistics and Their Distributions](#)[Random Samples](#)[Deriving a Sampling Distribution](#)[Simulation Experiments](#)Exercises Section 5.3 [\(37–45\)](#)5.4 [The Distribution of the Sample Mean](#)[The Case of a Normal Population Distribution](#)[The Central Limit Theorem](#)[Other Applications of the Central Limit Theorem](#)

3. [32/32 Points] DETAILS PREVIOUS ANSWERS DEVORESTAT9 5.3.039.S.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

It is known that 85% of all brand A external hard drives work in a satisfactory manner throughout the warranty period (are "successes"). Suppose that  $n = 15$  drives are randomly selected. Let  $X$  = the number of successes in the sample. The statistic  $X/n$  is the sample proportion (fraction) of successes. Obtain the sampling distribution of this statistic. [Hint: One possible value  $X/n$  is 0.2, corresponding to  $X = 3$ . What is the probability of this value (what kind of random variable is  $X$ )?] (Round your answers to three decimal places.)

USE SALT

X	X/n	p(X/n)
0	<input type="text"/>	<input type="text"/>
1	<input type="text"/>	<input type="text"/>
2	<input type="text"/>	<input type="text"/>
3	<input type="text"/>	<input type="text"/>
4	<input type="text"/>	<input type="text"/>
5	<input type="text"/>	<input type="text"/>
6	<input type="text"/>	<input type="text"/>
7	<input type="text"/>	<input type="text"/>
8	<input type="text"/>	<input type="text"/>
9	<input type="text"/>	<input type="text"/>
10	<input type="text"/>	<input type="text"/>
11	<input type="text"/>	<input type="text"/>
12	<input type="text"/>	<input type="text"/>
13	<input type="text"/>	<input type="text"/>
14	<input type="text"/>	<input type="text"/>
15	<input type="text"/>	<input type="text"/>

$X/n$  with  $n=15$  so  $\frac{X}{15}$   
 Binomial:  
 $p(X/n) = \binom{n}{x} p^x q^{n-x} = \binom{15}{x} (0.85)^x (0.15)^{15-x}$

4. [12/12 Points] DETAILS PREVIOUS ANSWERS DEVORESTAT9 5.3.042.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

A company maintains three offices in a certain region, each staffed by two employees. Information concerning yearly salaries (1000s of dollars) is as follows:

Office	1	1	2	2	3	3
Employee	1	2	3	4	5	6
Salary	28.7	32.6	29.2	32.6	24.8	28.7

(a) Suppose two of these employees are randomly selected from among the six (without replacement). Determine the sampling distribution of the sample mean salary  $\bar{X}$ . (Enter your answers for  $p(\bar{x})$  as fractions.)

$\bar{x}$	26.75	27	28.70	28.95	30.65	30.90	32.60
$p(\bar{x})$	2/15	1/15	1/5	2/15	4/15	2/15	1/15

Make Excel table of  $\binom{6}{2}=15$  possibilities of employee salaries of two different employees. Their frequency is  $p(\bar{x})$ .

(b) Suppose one of the three offices is randomly selected. Let  $X_1$  and  $X_2$  denote the salaries of the two employees. Determine the sampling distribution of  $\bar{X}$ . (Enter your answers as fractions.)

$\bar{x}$	26.75	30.65	30.90
$p(\bar{x})$	1/3	1/3	1/3

2 employees at each of 3 offices

(c) How does  $E(\bar{X})$  from parts (a) and (b) compare to the population mean salary  $\mu$ ?  
 $E(\bar{X})$  from part (a) is equal to  $\mu$ , and  $E(\bar{X})$  from part (b) is equal to  $\mu$ .

Need Help? Read It

5. [-/5 Points] DETAILS DEVORESTAT9 5.4.046.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

Young's modulus is a quantitative measure of stiffness of an elastic material. Suppose that for aluminum alloy sheets of a particular type, its mean value and standard deviation are 70 GPa and 1.6 GPa, respectively (values given in the article "Influence of Material Properties Variability on Springback and Thinning in Sheet Stamping Processes: A Stochastic Analysis" (Int'l. J. of Advanced Manuf. Tech., 2010: 117–134)).

(a) If  $\bar{X}$  is the sample mean Young's modulus for a random sample of  $n = 16$  sheets, where is the sampling distribution of  $\bar{X}$  centered, and what is the standard deviation of the  $\bar{X}$  distribution?

$E(\bar{X}) = \mu = 70$  GPa  
 $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{16}} = 0.4$  GPa

(b) Answer the questions posed in part (a) for a sample size of  $n = 256$  sheets.

$E(\bar{X}) = \mu = 70$  GPa  
 $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{256}} = 0.1$  GPa

(c) For which of the two random samples, the one of part (a) or the one of part (b), is  $\bar{X}$  more likely to be within 1 GPa of 70 GPa? Explain your reasoning.

- ☐  $\bar{X}$  is more likely to be within 1 GPa of the mean in part (a). This is due to the decreased variability of  $\bar{X}$  that comes with a smaller sample size.  
☒  $\bar{X}$  is more likely to be within 1 GPa of the mean in part (b). This is due to the decreased variability of  $\bar{X}$  that comes with a larger sample size.  
☐  $\bar{X}$  is more likely to be within 1 GPa of the mean in part (b). This is due to the increased variability of  $\bar{X}$  that comes with a larger sample size.  
☐  $\bar{X}$  is more likely to be within 1 GPa of the mean in part (a). This is due to the increased variability of  $\bar{X}$  that comes with a smaller sample size.

Need Help? Read It

### Proposition

Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution with mean value  $\mu$  and standard deviation  $\sigma$ . Then

- $E(\bar{X}) = \mu_{\bar{X}} = \mu$
- $V(\bar{X}) = \sigma_{\bar{X}}^2 = \sigma^2/n$  and  $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

In addition, with  $T_n = X_1 + \dots + X_n$  (the sample total),  $E(T_n) = n\mu$ ,  $V(T_n) = n\sigma^2$ , and  $\sigma_{T_n} = \sqrt{n}\sigma$ .

6. [-/2 Points] DETAILS DEVORESTAT9 5.4.053.S.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

Rockwell hardness of pins of a certain type is known to have a mean value of 50 and a standard deviation of 1.6. (Round your answers to four decimal places.)

USE SALT

(a) If the distribution is normal, what is the probability that the sample mean hardness for a random sample of 18 pins is at least 51?

$\mu_{\bar{x}} = \mu = 50$   $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{18}} = 0.377$   $P(\bar{x} \geq 51) = P\left(Z \geq \frac{51-50}{0.377}\right) = P(Z \geq 2.65) = 1 - P(Z \leq 2.65) = 1 - \Phi(2.65) = 0.004$

(b) What is the (approximate) probability that the sample mean hardness for a random sample of 38 pins is at least 51?

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{38}} = 0.26$   $P(\bar{x} \geq 51) = P\left(Z \geq \frac{51-50}{0.26}\right) = P(Z \geq 3.85) = 1 - P(Z \leq 3.85) = 1 - \Phi(3.85) = 0$  (approximate)

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Need Help? Read It

7. [-/3 Points] DETAILS DEVORESTAT9 5.4.054.5.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

Suppose the sediment density (g/cm) of a randomly selected specimen from a certain region is normally distributed with mean 2.66 and standard deviation 0.86.

USE SALT

- (a) If a random sample of 23 specimens is selected, what is the probability that the sample average sediment density is at most 3.00? Between 2.66 and 3.00? (Round your answers to four decimal places.)
- at most 3.00   $P(\bar{X} \leq 3.00) = P\left(Z \leq \frac{3.00 - 2.66}{\frac{0.86}{\sqrt{23}}}\right) = P(Z \leq 1.98) = \Phi(1.98) = 0.9760$
- between 2.66 and 3.00   $P(2.66 \leq \bar{X} \leq 3.00) = P\left(\frac{2.66 - 2.66}{\frac{0.86}{\sqrt{23}}} \leq Z \leq \frac{3.00 - 2.66}{\frac{0.86}{\sqrt{23}}}\right) = P(0 \leq Z \leq 1.98) = \Phi(1.98) - \Phi(0) = 0.476$

- (b) How large a sample size would be required to ensure that the first probability in part (a) is at least 0.99? (Round your answer up to the nearest whole number.)

$$P(\bar{X} \leq 3.00) = 0.99$$

$$P\left(Z \leq \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\right) = 0.99$$

$$P\left(Z \leq \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = 0.99; \text{invnorm}(0.99) = 2.33$$

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = 2.33$$

$$\bar{X} - \mu = 2.33 \cdot \frac{\sigma}{\sqrt{n}}$$

$$3.00 - 2.66 = 2.33 \cdot \frac{0.86}{\sqrt{n}}$$

$$\Rightarrow n = 34.7 \approx 35$$

8. [-/3 Points] DETAILS DEVORESTAT9 5.5.058.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

A shipping company handles containers in three different sizes: (1) 27 ft<sup>3</sup> (3 × 3 × 3), (2) 125 ft<sup>3</sup>, and (3) 512 ft<sup>3</sup>. Let  $X_i$  ( $i = 1, 2, 3$ ) denote the number of type  $i$  containers shipped during a given week. With  $\mu_i = E(X_i)$  and  $\sigma_i^2 = V(X_i)$ , suppose that the mean values and standard deviations are as follows:

$$\begin{array}{lll} \mu_1 = 200 & \mu_2 = 240 & \mu_3 = 140 \\ \sigma_1 = 9 & \sigma_2 = 13 & \sigma_3 = 6 \end{array}$$

$$V_0 = 27X_1 + 125X_2 + 512X_3$$

- (a) Assuming that  $X_1, X_2, X_3$  are independent, calculate the expected value and variance of the total volume shipped. [Hint: Volume =  $27X_1 + 125X_2 + 512X_3$ .]

expected value  ft<sup>3</sup>  $E(V_0) = E(27X_1 + 125X_2 + 512X_3) = 27E(X_1) + 125E(X_2) + 512E(X_3) = 27(200) + 125(240) + 512(140) = 107,980$

variance  ft<sup>6</sup>  $V(V_0) = V(27X_1 + 125X_2 + 512X_3) = (27)^2 V(X_1) + (125)^2 V(X_2) + (512)^2 V(X_3) = (27)^2(9) + (125)^2(13) + (512)^2(6) = 12,136,858$

- (b) Would your calculations necessarily be correct if the  $X_i$ 's were not independent? Explain.

- ☐ Both the expected value and the variance would be correct.
- ☒ The expected value would be correct, but the variance would not be correct. They would be covariances
- ☐ Neither the expected value nor the variance would be correct.
- ☐ The expected value would not be correct, but the variance would be correct.

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9. [-/1 Points] DETAILS DEVORESTAT9 5.5.062.5.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

Manufacture of a certain component requires three different machining operations. Machining time for each operation has a normal distribution, and the three times are independent of one another. The mean values are 15, 30, and 20 min, respectively, and the standard deviations are 2, 1, and 1.8 min, respectively. What is the probability that it takes at most 1 hour of machining time to produce a randomly selected component? (Round your answer to four decimal places.)

USE SALT

$$E(T_0) = E(X_1 + X_2 + X_3) = 15 + 30 + 20 = 65$$

$$V(T_0) = V(X_1 + X_2 + X_3) = (2)^2 + (1)^2 + (1.8)^2 = 8.24$$

$$\sigma_{T_0} = \sqrt{V(T_0)} = \sqrt{8.24} = 2.87$$

$$P(T_0 \leq 60) = P\left(Z \leq \frac{T_0 - E(T_0)}{\sigma_{T_0}}\right) = P\left(Z \leq \frac{60 - 65}{2.87}\right) = P(Z \leq -1.74) = 0.0409$$

You may need to use the appropriate table in the [Appendix of Tables](#) to answer this question.

Need Help? [Read It](#)

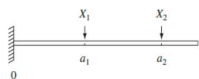
10. [-/6 Points] DETAILS DEVORESTAT9 5.5.066.5.

MY NOTES

ASK YOUR TEACHER

PRACTICE ANOTHER

If two loads are applied to a cantilever beam as shown in the accompanying drawing, the bending moment at 0 due to the loads is  $a_1X_1 + a_2X_2$ .



USE SALT

- (a) Suppose that  $X_1$  and  $X_2$  are independent  $nv$ 's with means 4 and 8 kips, respectively, and standard deviations 0.7 and 1.4 kip, respectively. If  $a_1 = 6$  ft and  $a_2 = 12$  ft, what is the expected bending moment and what is the standard deviation of the bending moment? (Round your standard deviation to three decimal places.)
- expected bending moment  kip-ft  $E(T_0) = E(a_1X_1 + a_2X_2) = a_1E(X_1) + a_2E(X_2) = 6(4) + 12(8) = 120$
- standard deviation  kip-ft  $V(T_0) = V(a_1X_1 + a_2X_2) = a_1^2V(X_1) + a_2^2V(X_2) = (6)^2(0.7)^2 + (12)^2(1.4)^2 = 299.88$

- (b) If  $X_1$  and  $X_2$  are normally distributed, what is the probability that the bending moment will exceed 75 kip-ft? (Round your answer to four decimal places.)
- $$P(T_0 \geq 75) = P\left(Z \geq \frac{T_0 - \mu_{T_0}}{\sigma_{T_0}}\right) = P\left(Z \geq \frac{75 - 120}{\sqrt{299.88}}\right) = P(Z \geq -2.60) = 1 - P(Z \leq -2.60) = 0.9953$$

moment and what is the standard deviation of the bending moment? (Round your standard deviation to three decimal places.)

expected bending moment  kip-ft  $E(T_0) = E(a_1 X_1 + a_2 X_2) = a_1 E(X_1) + a_2 E(X_2) = 6(4) + 12(8) = 120$

standard deviation  kip-ft  $V(T_0) = V(a_1 X_1 + a_2 X_2) = a_1^2 V(X_1) + a_2^2 V(X_2) = (6)^2 (0.7)^2 + (12)^2 (1.4)^2 = 299.88$

(b) If  $X_1$  and  $X_2$  are normally distributed, what is the probability that the bending moment will exceed 75 kip-ft? (Round your answer to four decimal places.)

$P(T_0 \geq 75) = P\left(Z \geq \frac{75 - 120}{\sqrt{299.88}}\right) = P\left(Z \geq \frac{75 - 120}{17.317}\right) = P(Z \geq -2.60) = 1 - P(Z \leq -2.60) = 0.9953$

(c) Suppose the positions of the two loads are random variables. Denoting them by  $A_1$  and  $A_2$ , assume that these variables have means of 6 and 12 ft, respectively, that each has a standard deviation of 0.5, and that all  $A_i$ 's and  $X_i$ 's are independent of one another. What is the expected moment now?

kip-ft  $E(M_0) = E(A_1 X_1 + A_2 X_2) = A_1 E(X_1) + A_2 E(X_2) = 6(4) + 12(8) = 120$

(d) For the situation of part (c), what is the variance of the bending moment? (Round your answer to two decimal places.)

kip-ft<sup>2</sup>  $V(M_0) = E(M_0^2) - [E(M_0)]^2 = 19,720.4925 - [120]^2 = 320.49$

(e) If the situation is as described in part (a) except that  $\text{Corr}(X_1, X_2) = 0.5$  (so that the two loads are not independent), what is the variance of the bending moment?

kip-ft<sup>2</sup>  $V(a_1 X_1 + a_2 X_2) = a_1^2 V(X_1) + a_2^2 V(X_2) + 2 a_1 a_2 \text{Cov}(X_1, X_2) = (6)^2 (0.7)^2 + (12)^2 (1.4)^2 + 2(6)(12)(0.49) = 370.44$

$$\rho_{X_1, X_2} = \frac{\text{Cov}(X_1, X_2)}{\sigma_{X_1} \cdot \sigma_{X_2}} = 0.5 \Rightarrow \frac{\text{Cov}(X_1, X_2)}{(0.7)(1.4)} = 0.5 \Rightarrow \text{Cov}(X_1, X_2) = 0.49$$