Module 5 Test

Saturday, June 24, 2023 4:26 PM

1. [–/2 Points] DETAILS **DEVORESTAT9 7.SE.050.S.** MY NOTES ASK YOUR TEACHER

A journal article reports that a sample of size 5 was used as a basis for calculating a 95% CI for the true average natural frequency (Hz) of delaminated beams of a certain type. The resulting interval was (229.265, 233.003). You decide that a confidence level of 99% is more appropriate than the 95% level used. What are the limits of the 99% interval? [*Hint*: Use the center of the interval and its width to determine $\bar{x}$ and s.] (Round your answers to three decimal places.)

$n = 5$ $\alpha = 1 - CI = 1 - 0.95 = 0.05$, $\frac{\alpha}{2} = 0.025$

USE SALT

$\bar{x} = \frac{LL + UL}{2} = \frac{229.265 + 233.003}{2} = 231.134$

(228.034 , 234.234) Hz

$t_{\frac{\alpha}{2}, n-1} = t_{0.025, 4} = \text{invT}(0.025, 4) = 2.776$

You may need to use the appropriate table in the Appendix of Tables to answer this question.

$w = UL - LL = 233.003 - 229.265 = 3.738$

$w = 2 t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} \Rightarrow 3.738 = 2(2.776) \cdot \frac{s}{\sqrt{5}} \Rightarrow s = 1.5055$

$\left(\bar{x} - t_{\frac{\alpha}{2}, n-1} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\frac{\alpha}{2}, n-1} \cdot \frac{s}{\sqrt{n}}\right) = \left(231.134 - 4.604 \cdot \frac{1.5055}{\sqrt{5}}, 231.134 + 4.604 \cdot \frac{1.5055}{\sqrt{5}}\right)$

for $\alpha = 1 - 0.99 = 0.01$, $\frac{\alpha}{2} = 0.005$, $t_{\frac{\alpha}{2}, n-1} = t_{0.005, 4} = \text{invT}(0.005, 4) = 4.604$

Proposition

Let $\bar{x}$ and s be the sample mean and sample standard deviation computed from the results of a random sample from a normal population with mean μ. Then a **$100(1 - \alpha)\%$ confidence interval for μ** is

$$\left(\bar{x} - t_{\alpha/2,\, n-1} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,\, n-1} \cdot \frac{s}{\sqrt{n}}\right) \qquad \textbf{(7.15)}$$

or, more compactly, $\bar{x} \pm t_{\alpha/2,\, n-1} \cdot s/\sqrt{n}$.

An **upper confidence bound for μ** is

$$\bar{x} + t_{\alpha,\, n-1} \cdot \frac{s}{\sqrt{n}}$$

and replacing + by – in this latter expression gives a **lower confidence bound for μ**, both with confidence level $100(1 - \alpha)\%$.

A general formula for the sample size n necessary to ensure an interval width w is obtained from equating w to $2 \cdot z_{\alpha/2} \cdot \sigma/\sqrt{n}$ and solving for n.

2. [–/2 Points] DETAILS **DEVORESTAT9 7.SE.054.** MY NOTES ASK YOUR TEACHER

It is important that face masks used by firefighters be able to withstand high temperatures because firefighters commonly work in temperatures of 200-500°F. In a test of one type of mask, 11 of 55 masks had lenses pop out at 250°. Construct a 90% upper confidence limit for the true proportion of masks of this type whose lenses would pop out at 250°. (Round your answers to four decimal places.)

(0.1309 ,)

X

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Using Z_{α}, df

3. [–/3 Points] DETAILS **DEVORESTAT9 7.SE.501.XP.** MY NOTES ASK YOUR TEACHER

A random sample of 120 lightning flashes in a certain region resulted in a sample average radar echo duration of 0.90 sec and a sample standard deviation of 0.31 sec. Calculate a 99% (two-sided) confidence interval for the true average echo duration μ. (Round your answers to two decimal places.) $n = 120$, $\bar{x} = 0.90$, $s = 0.31$

(0.83 , 0.97) sec Z Interval

Interpret the resulting interval.

- ○ We are 99% confident that this interval does not contain the true population mean.
- ○ We are 99% confident that the true population mean lies below this interval.
- ○ We are 99% confident that the true population mean lies above this interval.
- ● We are 99% confident that this interval contains the true population mean.

You may need to use the appropriate table in the Appendix of Tables to answer this question.

4. [–/3 Points] DETAILS **DEVORESTAT9 7.SE.506.XP.**

The following observations were made on fracture toughness of a base plate of 18% nickel maraging steel (in ksi $\sqrt{\text{in.}}$, given in increasing order)]:

65.4	71.9	72.6	73.1	73.1	73.5	75.5	75.7	75.8	76.1	76.2
76.2	77.0	77.9	78.1	79.6	79.7	79.9	80.1	82.2	83.5	93.8

$s = 5.41$
$n = 22$

Calculate a 99% CI for the standard deviation of the fracture toughness distribution. (Round your answers to one decimal place.)

(3.9 , 8.8) ksi $\sqrt{\text{in}}$ $\chi^2_{\alpha/2,n-1} = \chi^2_{0.005,21} = \text{chisq.inv}(0.005, 21) = 8.034$, $\chi^2_{1-\frac{\alpha}{2},n-1} = \chi^2_{0.995,21} = \text{chisq.inv}(0.995, 21) = 41.401$

$\alpha = 1 - CI = 1 - 0.99 = 0.01$, $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$, $1 - \frac{\alpha}{2} = 1 - \frac{0.01}{2} = 0.995$

$$\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}} = \frac{(22-1)(5.41)^2}{41.401} = 14.862, \ \sqrt{14.862} = 3.9$$

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}} = \frac{(22-1)(5.41)^2}{8.034} = 76.591, \ \sqrt{76.591} = 8.8$$

Is this interval valid whatever the nature of the distribution? Explain.

- ○ The distribution needs to be skewed to the right.
- ● The distribution needs to be approximately normal.
- ○ The interval is always valid.
- ○ The distribution needs to be approximately uniform.

You may need to use the appropriate table in the Appendix of Tables to answer this question.

The inequalities in (7.17) are equivalent to

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$$

Substituting the computed value s^2 into the limits gives a CI for σ^2, and taking square roots gives an interval for σ.

5. [–/3 Points] DETAILS **DEVORESTAT9 9.1.011.** MY NOTES ASK YOUR TEACHER

The level of lead in the blood was determined for a sample of 152 male hazardous-waste workers age 20-30 and also for a sample of 86 female workers, resulting in a mean ± standard error of 5.2 ± 0.3 for the men and 3.5 ± 0.2 for the women. Calculate an estimate of the difference between true average blood lead levels for male and female workers in a way that provides information about reliability and precision. (Use a 95% confidence interval. Round your answers to two decimal places.)

$m = 152$, $\bar{x} = 5.2$, $SE_1 = 0.3$ $n = 86$, $\bar{y} = 3.5$, $SE_2 = 0.2$ $SE = \frac{s}{\sqrt{n}}$

(,) $\alpha = 1 - CI = 1 - 0.95 = 0.05$

$\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$, $z_{\frac{\alpha}{2}} = \text{invNorm}(0.025) = 1.96$

$(\bar{x} - \bar{y}) \pm z_{\alpha/2}\sqrt{(SE_1)^2 + (SE_2)^2} = (5.2 - 3.5) \pm 1.96\sqrt{(0.3)^2 + (0.2)^2} = 1.7 \pm 0.71 = (0.99, 2.41)$

Interpret the interval.

- ○ We are 95% confident that the true average blood lead level for male workers is less than that of female workers by an amount within the confidence interval.
- ○ We are 95% confident that the true average blood lead level for male workers is greater than that of female workers by an amount outside the confidence interval.
- ● We are 95% confident that the true average blood lead level for male workers is greater than that of female workers by an amount within the confidence interval.
- ○ We cannot draw a conclusion from the given information.

You may need to use the appropriate table in the Appendix of Tables to answer this question.

Provided that m and n are both large, a CI for $\mu_1 - \mu_2$ with a confidence level of approximately $100(1 - \alpha)\%$ is

$$\bar{x} - \bar{y} \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$

$(\bar{x} - \bar{y}) \pm z_{\alpha/2}\sqrt{(SE_1)^2 + (SE_2)^2}$

$$\overline{x} - \overline{y} \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$

$$(\bar{x}-\bar{y}) \pm z_{\alpha/2}\sqrt{(SE_1)^2+(SE_2)^2}$$

where – gives the lower limit and + the upper limit of the interval. An upper or a lower confidence bound can also be calculated by retaining the appropriate sign (+ or –) and replacing $z_{\alpha/2}$ by z_{α}.

6. [–/3 Points] DETAILS **DEVORESTAT9 9.2.020.S.** MY NOTES ASK YOUR TEACHER

Suppose μ_1 and μ_2 are true mean stopping distances at 50 mph for cars of a certain type equipped with two different types of braking systems. The data follows: $m = 8$, $\overline{x} = 113.5$, $s_1 = 5.07$, $n = 8$, $\overline{y} = 129.5$, and $s_2 = 5.36$. Calculate a 95% CI for the difference between true average stopping distances for cars equipped with system 1 and cars equipped with system 2. (Round your answers to two decimal places.)

$\alpha = 1 - CI = 1 - 0.95 = 0.05$, $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$, $t_{\alpha/2, m+n-2} = t_{0.025, 14} = 2.14$

USE SALT

$$(\bar{x}-\bar{y}) \pm t_{\frac{\alpha}{2}, m+n-2}\sqrt{\frac{s_1^2}{m}+\frac{s_2^2}{n}} = (113.5 - 129.5) \pm 2.14\sqrt{\frac{(5.07)^2}{8} + \frac{(5.36)^2}{8}} = -16 \pm 5.59 = (-21.60, -10.40)$$

(______ , ______)

Does the interval suggest that precise information about the value of this difference is available?

- ○ Because the interval is so narrow, it appears that precise information is available.
- ○ Because the interval is so narrow, it appears that precise information is not available.
- ○ Because the interval is so wide, it appears that precise information is available.
- ● Because the interval is so wide, it appears that precise information is not available.

You may need to use the appropriate table in the Appendix of Tables to answer this question.

The **two-sample t confidence interval for $\mu_1 - \mu_2$** with confidence level $100(1 - \alpha)\%$ is then

$$\overline{x} - \overline{y} \pm t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$