Ch8 - Tests of Hypotheses Based on a Single Sample

Tuesday, June 27, 2023 1:41 PM

1. [8/8 Points] DETAILS | PREVIOUS ANSWERS | **DEVORESTAT9 8.1.002.** | MY NOTES | ASK YOUR TEACHER

For the following pairs of assertions, indicate which do not comply with our rules for setting up hypotheses and why (the subscripts 1 and 2 differentiate between quantities for two different populations or samples):

(a) H_0: $\mu = 100$, H_a: $\mu > 100$

- ◉ These hypotheses comply with our rules.
- ○ H_0 cannot include equality, so these hypotheses are not in compliance.
- ○ Each μ is a statistic, so these hypotheses do not comply with our rules.
- ○ The asserted value in H_0 should not appear in H_a, so these hypotheses are not in compliance.

✓

(b) H_0: $\sigma = 20$, H_a: $\sigma \le 20$

- ○ These hypotheses comply with our rules.
- ◉ H_a cannot include equality, so these hypotheses are not in compliance.
- ○ Each σ is a statistic, so these hypotheses do not comply with our rules.
- ○ The asserted value in H_0 should not appear in H_a, so these hypotheses are not in compliance.

✓

(c) H_0: $p \ne 0.25$, H_a: $p = 0.25$

- ○ These hypotheses comply with our rules.
- ◉ H_a cannot include equality, so these hypotheses are not in compliance.
- ○ Each p is a statistic, so these hypotheses do not comply with our rules.
- ○ The asserted value in H_0 should not appear in H_a, so these hypotheses are not in compliance.

✓

(d) H_0: $\mu_1 - \mu_2 = 25$, H_a: $\mu_1 - \mu_2 > 100$

- ○ These hypotheses comply with our rules.
- ○ H_0 cannot include equality, so these hypotheses are not in compliance.
- ○ Each μ is a statistic, so these hypotheses do not comply with our rules.
- ◉ The asserted value in H_0 should also appear in H_a, so these hypotheses are not in compliance.

✓

(e) H_0: $S_1{}^2 = S_2{}^2$, H_a: $S_1{}^2 \ne S_2{}^2$

- ○ These hypotheses comply with our rules.
- ○ H_0 cannot include equality, so these hypotheses are not in compliance.
- ◉ Each S is a statistic, so these hypotheses do not comply with our rules.
- ○ The asserted value in H_0 should not appear in H_a, so these hypotheses are not in compliance.

✓

(f) H_0: $\mu = 120$, H_a: $\mu = 150$

- ○ These hypotheses comply with our rules.
- ◉ H_a cannot include equality, so these hypotheses are not in compliance.
- ○ Each μ is a statistic, so these hypotheses do not comply with our rules.
- ○ If μ appears in H_0, then it should not appear in H_a, so these hypotheses are not in compliance.

✓

(g) H_0: $\sigma_1/\sigma_2 = 1$, H_a: $\sigma_1/\sigma_2 \ne 1$

- ◉ These hypotheses comply with our rules.
- ○ H_0 cannot include equality, so these hypotheses are not in compliance.
- ○ Each σ is a statistic, so these hypotheses do not comply with our rules.
- ○ The asserted value in H_0 should not appear in H_a, so these hypotheses are not in compliance.

✓

(h) H_0: $p_1 - p_2 = -0.1$, H_a: $p_1 - p_2 < -0.1$

- ◉ These hypotheses comply with our rules.
- ○ H_0 cannot include equality, so these hypotheses are not in compliance.
- ○ Each p is a statistic, so these hypotheses do not comply with our rules.
- ○ The asserted value in H_0 should not appear in H_a, so these hypotheses are not in compliance.

✓

Topics:

2. [6/6 Points] DETAILS | PREVIOUS ANSWERS | **DEVORESTAT9 8.1.004.MI.** | MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

Pairs of *P*-values and significance levels, α, are given. For each pair, state whether the observed *P*-value would lead to rejection of H_0 at the given significance level.

(a) *P*-value = 0.076, $\alpha = 0.05$

- ○ reject H_0
- ◉ do not reject H_0

✓

$0.076 > 0.005$
$\therefore$ do not reject

(b) *P*-value = 0.009, $\alpha = 0.001$

- ○ reject H_0
- ◉ do not reject H_0

✓

$0.009 > 0.001$
$\therefore$ do not reject

(c) *P*-value = 0.492, $\alpha = 0.05$

- ○ reject H_0
- ◉ do not reject H_0

✓

$0.492 > 0.05$
$\therefore$ do not reject

(d) *P*-value = 0.076, $\alpha = 0.10$

- ◉ reject H_0
- ○ do not reject H_0

✓

$0.076 < 0.10$
$\therefore$ reject

(e) *P*-value = 0.045, $\alpha = 0.01$

- ○ reject H_0
- ◉ do not reject H_0

✓

$0.045 > 0.01$
$\therefore$ do not reject

(f) *P*-value = 0.256, $\alpha = 0.10$

- ○ reject H_0
- ◉ do not reject H_0

✓

$0.256 > 0.10$
$\therefore$ do not reject

3. [1/1 Points] DETAILS PREVIOUS ANSWERS DEVORESTAT9 8.1.005. MY NOTES ASK YOUR TEACHER

To determine whether the pipe welds in a nuclear power plant meet specifications, a random sample of welds is selected, and tests are conducted on each weld in the sample. Weld strength is measured as the force required to break the weld. Suppose the specifications state that mean strength of welds should exceed 100 lb/in^2; the inspection team decides to test H_0: $\mu = 100$ versus H_a: $\mu > 100$. Explain why it might be preferable to use this H_a rather than $\mu < 100$.

- ◉ We want to determine if there is significant evidence that the mean strength of welds exceeds 100 lb/in^2. The current hypotheses correctly place the burden of proof on those who wish to assert that the specification is satisfied.
- ○ We want to determine if there is significant evidence that the mean strength of welds differs from 100 lb/in^2. The current hypotheses correctly place the burden of proof on those who wish to assert that the specification is not satisfied.
- ○ We want to determine if there is significant evidence that the mean strength of welds is less than 100 lb/in^2. The current hypotheses correctly place the burden of proof on those who wish to assert that the specification is not satisfied.
- ○ We want to determine if there is significant evidence that the mean strength of welds equals 100 lb/in^2. The current hypotheses correctly place the burden of proof on those who wish to assert that the specification is satisfied.

Need Help? Read It

4. [–/7 Points] DETAILS DEVORESTAT9 8.2.018. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Consider a paint-drying situation in which drying time for a test specimen is normally distributed with $\sigma = 8$. The hypotheses H_0: $\mu = 74$ and H_a: $\mu < 74$ are to be tested using a random sample of $n = 25$ observations.

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{25}} = 1.6$

(a) How many standard deviations (of $\overline{X}$) below the null value is $\bar{x} = 72.3$? (Round your answer to two decimal places.)

1.06 standard deviations $\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{72.3 - 74}{1.6} = -1.06$

(b) If $\bar{x} = 72.3$, what is the conclusion using $\alpha = 0.006$?

Calculate the test statistic and determine the P-value. (Round your test statistic to two decimal places and your P-value to four decimal places.)

z = −1.06

P-value = 0.1440

TI-84:
STAT→TESTS→Z-Test

State the conclusion in the problem context. compare p-value & α: 0.1440 > 0.006 ∴ do not reject H_0, H_a true

- ○ Reject the null hypothesis. There is sufficient evidence to conclude that the mean drying time is less than 74.
- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the mean drying time is less than 74.
- ● Do not reject the null hypothesis. There is not sufficient evidence to conclude that the mean drying time is less than 74.
- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that the mean drying time is less than 74.

(c) For the test procedure with $\alpha = 0.006$, what is $\beta(70)$? (Round your answer to four decimal places.)

$\beta(70)$ = $\mu_0 = 74$, $\mu' = 70$, $z_\alpha = z_{0.006}$ = invNorm(0.005) = 2.51, b/c $\mu < \mu_0$ then $\beta(70) = 1 - \Phi\left(-z_\alpha + \frac{\mu_0 - \mu'}{\sigma_{\bar{x}}}\right) = 1 - \Phi\left(-2.51 + \frac{74-70}{1.6}\right) = 0.5039$

(d) If the test procedure with $\alpha = 0.006$ is used, what n is necessary to ensure that $\beta(70) = 0.01$? (Round your answer up to the next whole number.)

n = specimens $z_\alpha = z_{0.006} = 2.51$, $z_\beta = z_{0.01} = 2.33$, $n = \left[\frac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}\right]^2 = \left[\frac{8(2.51 + 2.33)}{74 - 70}\right]^2 = 94$

(e) If a level 0.01 test is used with $n = 100$, what is the probability of a type I error when $\mu = 76$? (Round your answer to four decimal places.)

$\mu = 76$ → H_0 false → Type I error impossible, P(Type I) = 0

Type I error: when H_0 is true

Alternative Hypothesis	Type II Error Probability $\beta(\mu')$ for a Level α Test
H_a: $\mu > \mu_0$	$\Phi\left(z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
H_a: $\mu < \mu_0$	$1 - \Phi\left(-z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$
H_a: $\mu \neq \mu_0$	$\Phi\left(z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) - \Phi\left(-z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$

where $\Phi(z)$ = the standard normal cdf.

The sample size n for which a level α test also has $\beta(\mu') = \beta$ at the alternative value μ' is

$$n = \begin{cases} \left[\frac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}\right]^2 & \text{for a one-tailed(upper or lower)test} \\ \left[\frac{\sigma(z_{\alpha/2} + z_\beta)}{\mu_0 - \mu'}\right]^2 & \text{for a two-tailed test(an approximate solution)} \end{cases}$$

5. [–/3 Points] DETAILS DEVORESTAT9 8.2.020. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Lightbulbs of a certain type are advertised as having an average lifetime of 750 hours. The price of these bulbs is very favorable, so a potential customer has decided to go ahead with a purchase arrangement unless it can be conclusively demonstrated that the true average lifetime is smaller than what is advertised. A random sample of 51 bulbs was selected, the lifetime of each bulb determined, and the appropriate hypotheses were tested using MINITAB, resulting in the accompanying output.

Variable	N	Mean	StDev	SEMean	Z	P-Value
lifetime	51	738.44	38.69	5.42	-2.13	0.016

What conclusion would be appropriate for a significance level of 0.05? compare p-value & α: 0.016 < 0.05 ∴ reject H_0

- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.
- ● Reject the null hypothesis. There is sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.
- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.
- ○ Do not reject the null hypothesis. There is not sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.

What conclusion would be appropriate for a significance level of 0.01? Compare p-value & α: 0.016 > 0.01 ∴ do not reject H_0

- ○ Reject the null hypothesis. There is sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.
- ● Do not reject the null hypothesis. There is not sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.
- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.
- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the lifetime of a bulb is less than 750 hours.

What significance level and conclusion would you recommend and why?

A common significance level used for hypothesis testing is alpha = 0.05, which I recommend for this reason. This would give us the conclusion that the lifetime of a bulb is less than 750 hours, rejecting the null hypothesis.

6. [–/4 Points] DETAILS DEVORESTAT9 8.3.029.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

The true average diameter of ball bearings of a certain type is supposed to be 0.5 in. A one-sample *t* test will be carried out to see whether this is the case. What conclusion is appropriate in each of the following situations?

USE SALT

$H_0: \mu = 0.5$
$H_a: \mu \neq 0.5$

(a) $n = 20$, $t = 1.53$, $\alpha = 0.05$ $t_{\frac{\alpha}{2}, n-1} = t_{0.025, 19} = \text{invT}(0.025, 19) = 2.09$

- ○ Reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in.
- ○ Reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in.
- ○ Do not reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in.
- ● Do not reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in. compare t & $t_{\frac{\alpha}{2}, n-1}$: $1.53 < 2.09$

(b) $n = 20$, $t = -1.53$, $\alpha = 0.05$ $t_{0.025, 19} = 2.09$

- ○ Reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in.
- ○ Reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in.
- ○ Do not reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in.
- ● Do not reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in. $|t| < t_{\frac{\alpha}{2}, n-1}$: $1.53 < 2.09$

(c) $n = 26$, $t = -2.5$, $\alpha = 0.01$ $t_{0.005, 25} = 3.45$

- ○ Reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in.
- ○ Reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in.
- ○ Do not reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in.
- ○ Do not reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in. $|t| < t_{\frac{\alpha}{2}, n-1}$; $2.5 < 3.45$

(d) $n = 26$, $t = -3.91$, $\alpha = 0.05$ $t_{0.025, 25} = 2.06$

- ● Reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in. $|t| > t_{\frac{\alpha}{2}, n-1}$: $3.91 > 2.06$
- ○ Reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in.
- ○ Do not reject the null hypothesis. There is not sufficient evidence that the true diameter differs from 0.5 in.
- ○ Do not reject the null hypothesis. There is sufficient evidence that the true diameter differs from 0.5 in.

7. [–/6 Points] DETAILS DEVORESTAT9 8.3.031.MI.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

The paint used to make lines on roads must reflect enough light to be clearly visible at night. Let μ denote the true average reflectometer reading for a new type of paint under consideration. A test of H_0: $\mu = 20$ versus H_a: $\mu > 20$ will be based on a random sample of size *n* from a normal population distribution. What conclusion is appropriate in each of the following situations? (Round your *P*-values to three decimal places.)

USE SALT

(a) $n = 11$, $t = 3.3$, $\alpha = 0.05$

P-value = [ ] $P\text{-value} = 1 - t_{cdf}(3.3, 10) = 0.004$

State the conclusion in the problem context.

- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ○ Do not reject the null hypothesis. There is not sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ● Reject the null hypothesis. There is sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20. P-value & α: $0.004 < 0.05$
- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.

(b) $n = 7$, $t = 1.7$, $\alpha = 0.01$

P-value = [ ] $P\text{-value} = 1 - t_{cdf}(1.7, 6) = 0.070$

State the conclusion in the problem context.

- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ○ Reject the null hypothesis. There is sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ● Do not reject the null hypothesis. There is not sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20. P-value & α: $0.070 > 0.01$

(c) $n = 26$, $t = -0.4$

P-value = [ ] $P\text{-value} = 1 - t_{cdf}(-0.4, 25) = 0.6537$

State the conclusion in the problem context.

- ○ Reject the null hypothesis. There is not sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ○ Reject the null hypothesis. There is sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20.
- ● Do not reject the null hypothesis. There is not sufficient evidence to conclude that the new paint has a reflectometer reading higher than 20. no α larger than P-value 0.6537

8. [–/8 Points] DETAILS DEVORESTAT9 8.4.044.MI.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A manufacturer of nickel-hydrogen batteries randomly selects 100 nickel plates for test cells, cycles them a specified number of times, and determines that 13 of the plates have blistered.

USE SALT

(a) Does this provide compelling evidence for concluding that more than 10% of all plates blister under such circumstances? State and test the appropriate hypotheses using a significance level of 0.05.

- ○ H_0: $p = 0.10$
  H_a: $p \neq 0.10$
- ○ H_0: $p = 0.10$
  H_a: $p < 0.10$
- ○ H_0: $p > 0.10$
  H_a: $p = 0.10$
- ● H_0: $p = 0.10$
  H_a: $p > 0.10$

Calculate the test statistic and determine the *P*-value. (Round your test statistic to two decimal places and your *P*-value to four decimal places.)

z = 1.00

P-value = 0.1586 STAT → TESTS → 1-Prop Z Test (5)

State the conclusion in the problem context.

- ○ Reject the null hypothesis. There is not sufficient evidence that more than 10% of plates blister under the experimental conditions.
- ● Do not reject the null hypothesis. There is not sufficient evidence that more than 10% of plates blister under the experimental conditions. P-value & α: $0.1586 > 0.05$
- ○ Reject the null hypothesis. There is sufficient evidence that more than 10% of plates blister under the experimental conditions.
- ○ Do not reject the null hypothesis. There is sufficient evidence that more than 10% of plates blister under the experimental conditions.

In reaching your conclusion, what type of error might you have committed?

- ○ type I
- ● type II Not rejecting Ho when it was false

(b) If it is really the case that 16% of all plates blister under these circumstances and a sample size 100 is used, how likely is it that the null hypothesis of part (a) will not be rejected by the 0.05 test? (Round your answer to four decimal places.)

[ ] $z_\alpha = z_{0.05} = 1.64$, $p_0 = 0.10$, $p' = 0.16$, $\Phi\left(\frac{p_0 - p' + z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right) = \Phi\left(\frac{0.10 - 0.16 + 1.64\sqrt{0.10(1-0.10)/100}}{\sqrt{0.16(1-0.16)/100}}\right) = \Phi(-0.29) = 0.3842$

If it is really the case that 16% of all plates blister under these circumstances and a sample size 200 is used, how likely is it that the null hypothesis of part (a) will not be rejected by the 0.05 test? (Round your answer to four decimal places.)

0.1654 same as above except $n = 200$ instead of $n = 100$

(c) How many plates would have to be tested to have $\beta(0.16) = 0.10$ for the test of part (a)? (Round your answer up to the next whole number.)

[ ] plates $z_\beta = z_{0.10} = 1.28$, $n = \left[\frac{z_\alpha\sqrt{p_0(1-p_0)} + z_\beta\sqrt{p'(1-p')}}{p' - p_0}\right]^2 = \left[\frac{1.64\sqrt{0.10(1-0.10)} + 1.28\sqrt{0.16(1-0.16)}}{0.16 - 0.10}\right]^2 = 257$

Alternative Hypothesis	$\beta(p')$
$H_a\colon p > p_0$	$\Phi\left[\dfrac{p_0 - p' + z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right]$
$H_a\colon p < p_0$	$1 - \Phi\left[\dfrac{p_0 - p' - z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right]$
$H_a\colon p \neq p_0$	$\Phi\left[\dfrac{p_0 - p' + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right] - \Phi\left[\dfrac{p_0 - p' - z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right]$

The sample size n for which the level α test also satisfies $\beta(p') = \beta$ is

$$n = \begin{cases} \left[\dfrac{z_\alpha\sqrt{p_0(1-p_0)}+z_\beta\sqrt{p'(1-p')}}{p'-p_0}\right]^2 & \text{one-tailed test} \\ \left[\dfrac{z_{\alpha/2}\sqrt{p_0(1-p_0)}+z_\beta\sqrt{p'(1-p')}}{p'-p_0}\right]^2 & \text{two-tailed test(an approximate solution)} \end{cases}$$

9. [–/5 Points] DETAILS DEVORESTAT9 8.4.045.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

A random sample of 151 recent donations at a certain blood bank reveals that 85 were type A blood. Does this suggest that the actual percentage of type A donations differs from 40%, the percentage of the population having type A blood? Carry out a test of the appropriate hypotheses using a significance level of 0.01.

USE SALT

State the appropriate null and alternative hypotheses.

○ H_0: $p = 0.40$
  H_a: $p > 0.40$

○ H_0: $p \neq 0.40$
  H_a: $p = 0.40$

○ H_0: $p = 0.40$
  H_a: $p < 0.40$

● H_0: $p = 0.40$
  H_a: $p \neq 0.40$

Calculate the test statistic and determine the P-value. (Round your test statistic to two decimal places and your P-value to four decimal places.)

z = 4.99 STAT → TESTS → 1-Prop Z Test
P-value = 0

State the conclusion in the problem context.

○ Reject the null hypothesis. There is not sufficient evidence to conclude that the percentage of type A donations differs from 40%.
● Reject the null hypothesis. There is sufficient evidence to conclude that the percentage of type A donations differs from 40%. P-value < α : 0 < 0.01
○ Do not reject the null hypothesis. There is sufficient evidence to conclude that the percentage of type A donations differs from 40%.
○ Do not reject the null hypothesis. There is not sufficient evidence to conclude that the percentage of type A donations differs from 40%.

Would your conclusion have been different if a significance level of 0.05 had been used?

○ Yes
● No 0 < 0.05

10. [–/7 Points] DETAILS DEVORESTAT9 8.5.053.MI. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

The drying time of a certain type of paint under specified test conditions is known to be normally distributed with mean value 75 min and standard deviation 9 min. Chemists have proposed a new additive designed to decrease average drying time. It is believed that drying times with this additive will remain normally distributed with $\sigma = 9$. Because of the expense associated with the additive, evidence should strongly suggest an improvement in average drying time before such a conclusion is adopted. Let μ denote the true average drying time when the additive is used. The appropriate hypotheses are H_0: $\mu = 75$ versus H_a: $\mu < 75$. Consider the alternative value $\mu = 74$, which in the context of the problem would presumably not be a practically significant departure from H_0. $\mu' = 74$, $\mu_0 = 75$, $\sigma = 9$, $z_\alpha = z_{0.01} = 2.33$

(a) For a level 0.01 test, compute β at this alternative for sample sizes n = 64, 784, and 2,500. (Round your answers to four decimal places.)

n	β
64	0.9252
784	0.2174
2,500	0.0006

$\beta(\mu') = \beta(74) = 1 - \Phi\left(-z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) = 1 - \Phi\left(-2.33 + \frac{75-74}{9/\sqrt{n}}\right)$

plug in n

(b) If the observed value of $\overline{X}$ is $\overline{x} = 74$, what can you say about the resulting P-value when n = 2,500? Is the data statistically significant at any of the standard values of α? (Round your z to two decimal places. Round your P-value to four decimal places.)

z = −5.56 STAT → TESTS → Z-Test
P-value = 0

State the conclusion in the problem context.

○ Do not reject the null hypothesis. The data are not statistically significant at any standard value of α.
○ Reject the null hypothesis. The data are not statistically significant at any standard value of α.
○ Do not reject the null hypothesis. The data are statistically significant at any standard value of α.
● Reject the null hypothesis. The data are statistically significant at any standard value of α. α > 0

(c) Would you really want to use a sample size of 2,500 along with a level 0.01 test (disregarding the cost of such an experiment)? Explain.

○ Yes, even when the departure from H_0 is significant from a practical point of view, a statistically significant result is not likely to appear; it is difficult for the test to detect departures from H_0.
○ Yes, it is always advantageous to have a very large sample size, because it will detect very small departures from H_0.
● No, even when the departure from H_0 is insignificant from a practical point of view, a statistically significant result is highly likely to appear; the test is too likely to detect small departures from H_0.
○ No, it is never advantageous to have a very large sample size, because it cannot detect very small departures from H_0.

11. [–/10 Points] DETAILS DEVORESTAT9 8.5.054.S. MY NOTES ASK YOUR TEACHER PRACTICE ANOTHER

Consider a large-sample level 0.01 test for testing H_0: $p = 0.2$ against H_a: $p > 0.2$.

USE SALT

$p' = 0.21, \; p_0 = 0.2, \; z_\alpha = z_{0.01} = 2.33$

(a) For the alternative value $p = 0.21$, compute $\beta(0.21)$ for sample sizes n = 400, 1600, 16,900, 40,000, and 90,000. (Round your answers to four decimal places.)

n	β
400	0.9636
1600	0.9036
16,900	0.1822
40,000	0.0043
90,000	0

$$\beta(p') = \beta(0.21) = \Phi\left(\frac{p_0 - p' + z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right) = \Phi\left(\frac{0.2 - 0.21 + 2.33\sqrt{0.2(1-0.2)/n}}{\sqrt{0.21(1-0.21)/n}}\right)$$

(b) For $\hat{p} = x/n = 0.21$, compute the P-value when n = 400, 1600, 16,900, and 40,000. (Round your answers to four decimal places.)

n	P-value
400	0.3085
1600	0.1587
16,900	0.0005
40,000	0

$$P(\hat{p} \geq 0.21) = 1 - P(\hat{p} \leq 0.21) = 1 - P\left(z \leq \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}\right) = 1 - P\left(z \leq \frac{0.21 - 0.2}{\sqrt{0.2(1-0.2)/n}}\right)$$

(c) In most situations, would it be reasonable to use a level 0.01 test in conjunction with a sample size of 40,000? Why or why not?

- ○ Yes, even when the departure from H_0 is significant from a practical point of view, a statistically significant result is not likely to appear; it is difficult for the test to detect departures from H_0.
- ○ Yes, it is always advantageous to have a very large sample size, because it will detect very small departures from H_0.
- ● No, even when the departure from H_0 is insignificant from a practical point of view, a statistically significant result is highly likely to appear; the test is too likely to detect small departures from H_0.
- ○ No, it is never advantageous to have a very large sample size, because it cannot detect very small departures from H_0.

Null Hypothesis:

$$H_0 : p = p_0$$

Test Statistic Value:

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

Alternative Hypothesis	P-Value Determination
H_a: $p > p_0$	Area under the standard normal curve to the right of z
H_a: $p < p_0$	Area under the standard normal curve to the left of z
H_a: $p \neq p_0$	2·(Area under the standard normal curve to the right of $\lvert z \rvert$)

These test procedures are valid provided that $np_0 \geq 10$ and $n(1-p_0) \geq 10$.

They are referred to as *upper-tailed*, *lower-tailed*, and *two-tailed*, respectively, for the three different alternative hypotheses.