# Mitigation of the Detection Efficiency Loophole through Quantum Computer Simulations

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## Abstract

In the realm of quantum mechanics, Bell's Inequalities have played a pivotal role in testing the degree of violation of local realism in entangled systems. However, many loopholes have consistently challenged certain conclusions reached by the experiments based on the Bell Theorem (Bell's theorem). In this paper, we will explore a main loophole along with its significance in the discussion of quantum entanglement. Then, we will unravel a newly derived methodology aimed to mitigate the effects of these loopholes on the validity of Bell's Inequalities through changing measurement angles (angle used to measure quantum bits). This is implemented mainly by simulating a quantum experiment then comparing the results with those of a real-time quantum computation. With the closing of certain loopholes, we can reach a higher degree of certainty of the violation of local realism, reinforcing our understanding of quantum entanglement and the validity of the current theories such as local realism regarding it.

## Introduction

In the Einstein-Podolsky-Rosen (EPR) Paradox [3], an argument about the quantum theory's incompleteness was put forth. Mainly, it defers the possibility of nonlocal behavior, the phenomenon achieved by entangled quantum particles. Einstein, Podolsky, and Rosen predicted the existence of certain hidden variables that produced similar effects such that entanglement, or nonlocality would cease to exist. In response to this, John Stewart Bell theorized a set of experiments [2] that could be used to test a created set of inequalities derived in the paper and the predicitons of the local hidden variables theory. This would allow for the determination of the EPR Paradox's validity. Generally, the equation is given by:

$$|S| \leq 2 \tag{1}$$

where S is the quantity related to the cbetween measurements on entangled particles. This is the result deduced by the Clauser, Horne, Shimony, and Holt (CHSH) inequality [8]. A violation of the inequality would imply the failure of local hidden variable theories, indicating a possibility of nonlocality, or nonlocal causality.

The main inequality of Bell's Theorem can be derived using the CHSH inequality, a fundamental result that distinguishes between classical and nonclassical systems. We can assume two observers, Alice and Bob, who each measure one of two entangled particles [8]. Given two measurement settings, $A \; and \; A', B \; and \; B'$ for each observer, the results can amount to -1 or 1. They each represent a measurement outcome of an observable property that can take on the two values above. We can define the correlation function as:

$$E\,(A, B) = P\,(A=+1, B=+1) + P\,(A=-1, B=-1) - P\,(A=-1, B=+1) - P\,(A=+1, B=-1), \tag{2}$$

Where P (A, B) represents the joint probability of Alice obtaining outcome A and Bob obtaining outcome B. Furthermore, the sum of the absolute values of certain correlation functions is defined as:

$$S = |\,E\,(A, B) + E\,(A, B') + E\,(A', B) - E\,(A', B')\,|$$

(3)

E (A, B) represents the expected value of AB between Alice and Bob's measurement outcomes, or the joint probability of Alice obtaining A and Bob obtaining B.

To derive the CHSH inequality, we can consider the following equation:

$$S' = E(A, B) + E(A, B') + E(A', B) - E(A', B') \tag{4}$$

This defines the CHSH inequality, which places an upper limit on the value of S in the local hidden-variable theories. Violations of the CHSH inequality imply the inability of the local hidden-variables theory explaining quantum mechanics, which states that the entanglement phenomenon is only achieved by preexisting variables within the particles that allow for the instantaneous change in information.

In the case of the local hidden variable theory, as suggested by EPR, the expectation value of the product of measurements under different settings is denoted by:

$$|E(A,B) - E(A,B') + E(A',B) + E(A',B')| \leq 2 \tag{5}$$

In quantum mechanics, specifically in Bell Experiments, however, the maximum value possibly achieved is $2\sqrt{2}$ or 2.8284. This discrepancy is a manifestation of entanglement, which can be experimentally confirmed with the CHSH inequality.

## Methods

In the case of Bell's Theorem, a multitude of experiments must be done in order to verify the phenomenon of quantum entanglement. However, in a physical setting, certain error sources often lead to slight bias that led to conclusions supporting nonlocality as opposed to local causality. These error sources are what we call "loopholes", which could lead to the result inequality to be greater than 2, when it could be explained by locally hidden variables phenomenon in reality.

### Detection efficiency loophole

A major loophole in Bell experiments is detection efficiency loophole. It refers to the "quantum effects" produced by the experiments being invalid due to a lack of particle registration of the detectors [5]. Specifically, photons are more likely to pass undetected. The quantum efficiency/detection efficiency is generally given as:

$$\eta = \frac{r_e}{r_p}, \tag{6}$$

Where $r_p$ and $r_e$ are the incident photon rate and corresponding electron rate respectively, or the fraction of incident photons and electrons that can be registered by a photosensitive device.

When certain pairs of particles are not detected, the data would not be representative of the entire sample. Thus, the detector efficiency loophole comes into play in this experiment, as a higher proportion of detected particles would mean a more accurate experiment. According to Garg, a quantum efficiency of 83% or above would be sufficient to refute local realism in relation to 2x2 experiments (referring to two quantum systems), where spin measurements are made on distant pairs of particles [7]. Existing in states such as the "singlet" state, the particle pairs exhibit high levels of correlation. The boundary for nonlocal realism remains to be 2. Instead of the values -1 and 1 however, a third value of 0 is designed to account for non-detected pairs. Thus, we have three possible states after measurement. In this case, $\eta > 0.8284$ (which rounds to 0.83) would be the condition necessary to overcome the loophole at hand.

To test the detection efficiency, we can compare the theoretical CHSH violation value via a computer simulation, then compare the percent error with a real-time experiment on a quantum computer. This could be done by establishing a CHSH circuit for a simulation and a quantum computer, then running the circuit on both ends. In both cases, the parameter would be a value of theta, or the measurement angle (in radians). In the absolute value range larger than 2 (but less than $2\sqrt{2}$), we could find a domain of angle values that provide the highest detection efficiencies with respect to the real-time experiment. Doing so would allow us to find the ideal measurement angles to allow for the most accurate representations of the CHSH value.

To test this, we must first develop a *Qiskit* (IBM's official quantum computing language) program that outputs CHSH witness values (given by the equation) provided the angle of measurement, which is the angle that is used to measure the state of a quantum bit. Using the

CHSH 1 inequality (the general form of the CHSH inequality) given by equation (10), we can find the degree of violation given the measurement angle, assuming an experiment with 10 repetitions. This is likely enough due to the fact that we are using the average values to determine the final outcome, which would not deviate much if we use more terms.

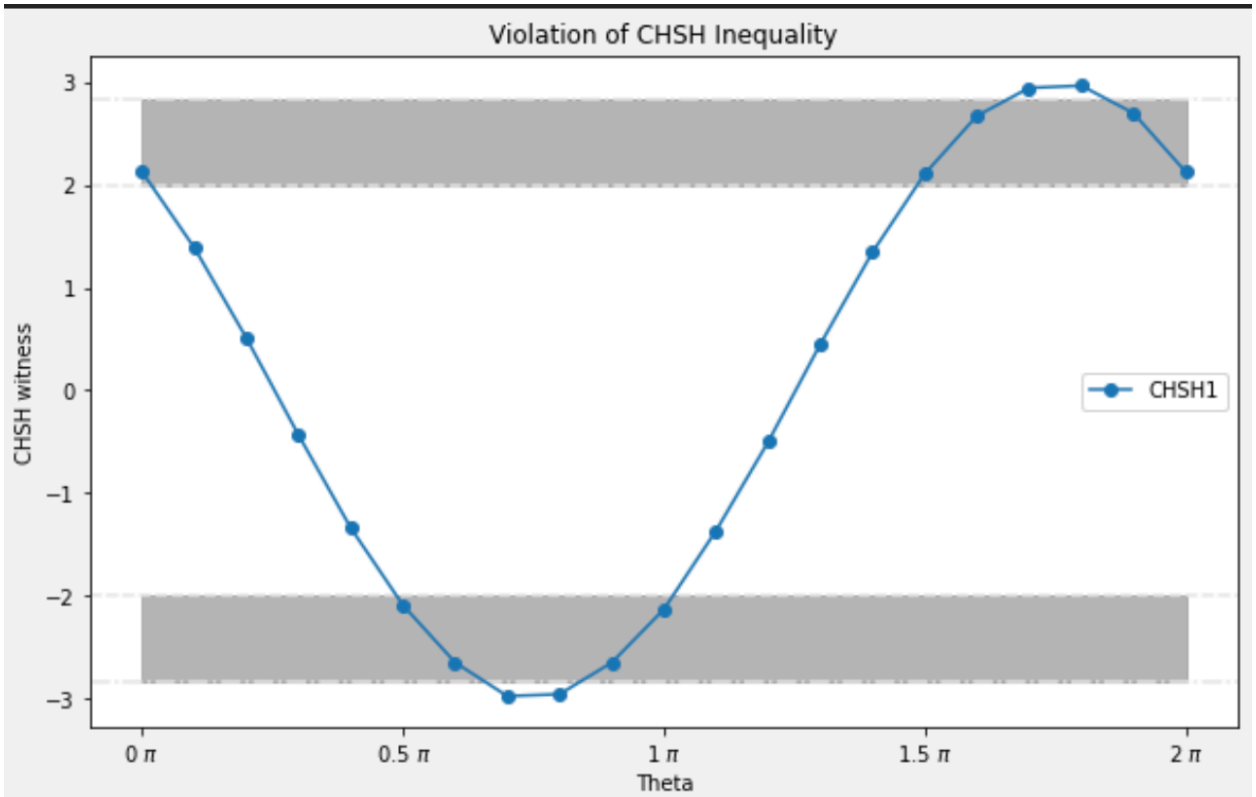


Figure 1. Violation of CHSH1 Inequality given any angle from 0 to 2pi according to quantum simulation. Grey area represents violation of the CHSH inequality.

As shown above, the range of values supporting nonlocal behavior generally lies between $0.5\pi$ and $\pi$ (and symmetrically $1.5\pi$ to $2\pi$). Knowing that the detection inefficiency loophole affects values within this range, we can experiment with values with the range, with a $0.1\pi$ interval in between for precision. In the experiment, we will fire 10000 shots to maximize accuracy. Using a quantum simulation (rather than just deriving the values theoretically), we achieve realistic values that adhere to physical quantum traits rather than just mathematical, while still being theoretically accurate.

The establishment of a CHSH circuit is as shown below:

```
chsh = []
theta = Parameter("$\\theta$")

chsh_circuit_no_meas = QuantumCircuit(2)
chsh_circuit_no_meas.h(0)
chsh_circuit_no_meas.cx(0, 1)
chsh_circuit_no_meas.ry(theta, 0)
chsh_circuit_no_meas.draw("mpl")
```

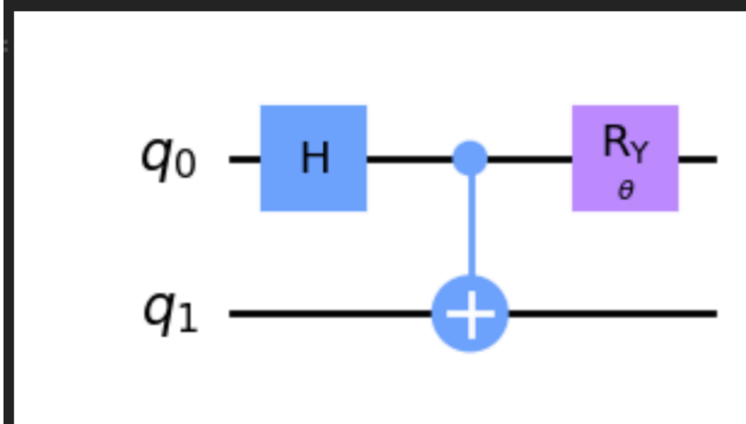


Figure 2. Code and circuit demonstrating the CHSH inequality.

The code constructs a series of logic gates that entangle the quantum bits in context of the CHSH circuit setup. A quantum circuit is printed as a result of the code.

Results

Running the experiment on a quantum computer backend from IBM, we get the values given each angle (in the form of averages and matrices):

0.5 $\pi$: -2.039413770716898 average

[-2.070846650653856, -2.070846650653856, -2.048436673773987, -2.0396230544488714, -2.042022054054054, -2.060425332623736, -2.0267051458885943, -2.031979704016913, -2.0228912579957354, -2.04329446967673, -1.9766418753329782, -2.0217856000000003, -2.0370187667560318, -2.0398738873994637]

0.6 $\pi$: -2.4747315601261114 average

[-2.4778101485842443, -2.468602126468942, -2.464757609921082, -2.4790977107761027, -2.4803040723981904, -2.499136901408451, -2.479334349421394, -2.4525218371372217, -2.495921120448179, -2.468360871995528, -2.4608279200675485]

0.7$\pi$: -2.7839895311780917 average

[-2.765622710230316, -2.786030951739013, -2.7700577017114916, -2.771530255296035, -2.799026926240746, -2.788275982532751, -2.8079766455521895, -2.7958922109866085, -2.7957216630196937, -2.765889064630488, -2.795100623813399]

0.8$\pi$: -2.778808647071839 average

[-2.8362276975361085, -2.8104648648648656, -2.73241222901612, -2.8148119977362764, -2.7687494689770817, -2.7825225245441794, -2.7681733109994413, -2.77855954738331, -2.7709929676511953, -2.757202214839424, -2.7747467714766985]

0.9$\pi$: -2.541674397819171 average

[-2.569759235310655, -2.5369878028404345, -2.587543663561953, -2.501510217755444, -2.53173679165492, -2.5090572864321605, -2.5636900900900903, -2.526863820224719, -2.5150796519786693, -2.5302709677419353, -2.5513485165300933]

1.0$\pi$: -2.040354342527619 average

[-2.010742983915696, -2.0717667704500426, -2.004522905027933, -2.043413213885778, -2.053310484780158, -2.0190199442896937, -2.0844165961049956, -2.037909070548712, -2.048114637559474, -2.0368175626231353, -2.028267112225007]

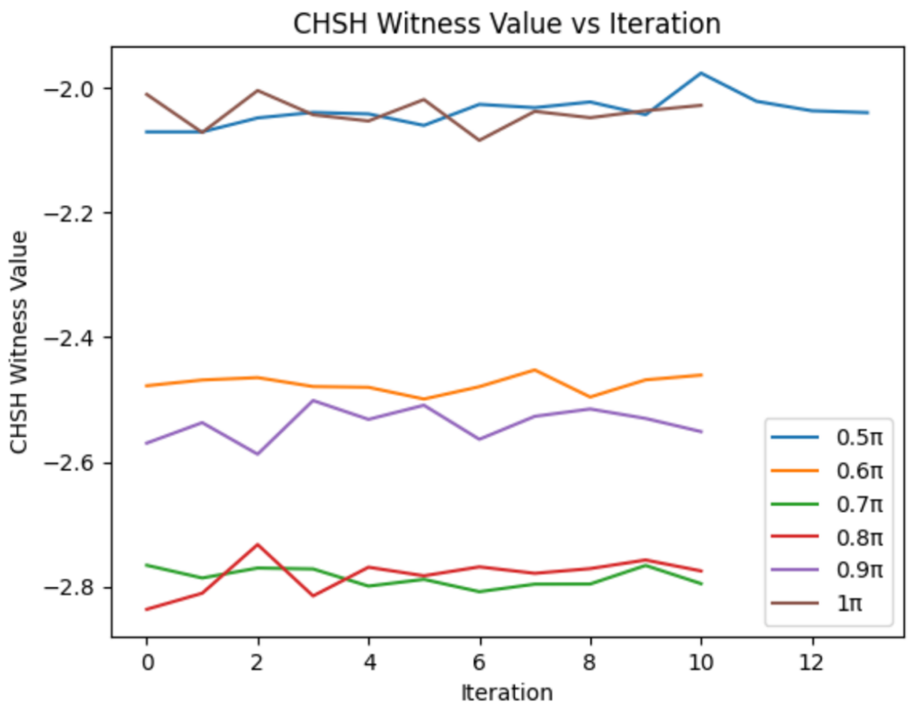


Figure 3. Graph displaying the iteration of values vs. CHSH Witness value given each angle.

Using the IBM *Qiskit* QASM simulator to run the code, we get the values:

$0.5\pi$: -2.0021 average

[-2.009, -2.009, -2.0014, -2.0014, -2.0072, -2.0126, -1.9938, -1.9964, -1.9864, -2.0222]

$0.6\pi$: -2.51954 average

[-2.5092, -2.5208, -2.5152, -2.537, -2.5192, -2.505, -2.5182, -2.5198, -2.5154, -2.5236]

$0.7\pi$: -2.79494 average

[-2.7826, -2.7824, -2.8166, -2.7824, -2.791, -2.7932, -2.7884, -2.7876, -2.809, -2.8]

$0.8\pi$: -2.79786 average

[-2.7876, -2.789, -2.8136, -2.7944, -2.8052, -2.77, -2.7924, -2.7982, -2.8156, -2.7978]

$0.9\pi$: -2.51744 average

[-2.4972, -2.525, -2.5144, -2.5156, -2.5068, -2.5308, -2.5172, -2.52, -2.5206, -2.5412]

1.0$\pi$: -1.99966 average

[-1.9782, -2.0202, -2.0046, -1.9892, -2.0054, -2.0206, -1.9668, -1.9982, -1.9948, -2.0176]

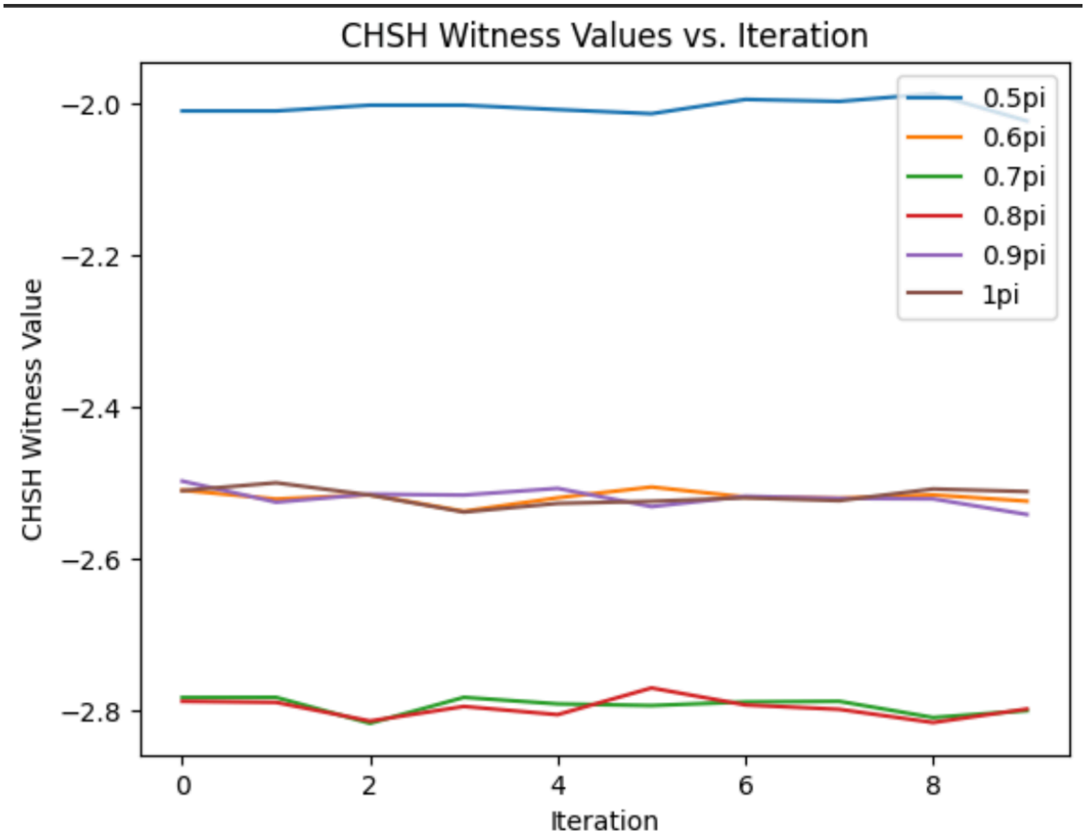


Figure 4. Graph displaying the iteration of values vs. CHSH Witness value given each angle for s imulation.

To approximate the relative detection efficiency at each given angle, we can find percent error given by the averages between the quantum computer experiment and the simulation:

$$0.5\pi: \left|\frac{-2.0021+2.0394}{100}\right| = 0.000373\ \%\ \text{error}$$

$$0.6\pi: \left|\frac{-2.5195+2.4747}{100}\right| = 0.000448\ \%\ \text{error}$$

$$0.7\pi: \left|\frac{-2.7949+2.7840}{100}\right| = 0.000109\ \%\ \text{error}$$

$$0.8\pi: \left|\frac{-2.7979+2.7788}{100}\right| = 0.000191\ \%\ \text{error}$$

$$0.9\pi: |\frac{-2.5174+2.5417}{100}| = 0.000243 \text{ \% error}$$

$$1.0\pi: |\frac{-1.9997+2.0404}{100}| = 0.000407 \text{ \% error}$$

From the data shown above, it is evident that measurement angles taken in the interval $[0.7\pi, 0.8\pi]$ provide the most accurate CHSH witness value in an experiment, as the percent errors with the corresponding angles are the lowest in comparison to others. This indicates the highest probability of minimizing primarily the detection efficiency loophole, a central loophole to modern experiments, within contemporary quantum computing.

## Discussion

In this paper, we established a method of finding the optimal angles to achieve desired CHSH witness values for Bell Experiments. The method proposed provided an optimal angle domain of $[0.7\pi, 0.8\pi]$ that allow for the highest accuracy for verifying quantum entanglement. Although similar codes have been programmed, they have not been utilized for the purpose of optimizing the detection efficiency loophole.

By finding the optimal angles of measurement, quantum experiments regarding Bell's Theorem and entanglement could be vastly expanded. Furthermore, the method proposed in this paper offers a feasible method to discover maximally efficient values without the necessity of physical experiments, as it could be done through simply viable digital premises. Within the scope of the CHSH inequality, we can verify that certain values of theta do indeed result in a

CHSH violation of $[2,2\sqrt{2}]$ but could vary in accuracy given the existence of physical discrepancies.

As the experiment was done through a limited range of 10 values per angle, it is possible that the sample size may introduce a degree of bias in the results. The same procedure done with more iterations would provide a higher accuracy, which, however, would likely result in a similar range achieved in this paper. With the method proposed in this paper, it is probable that future experiments in the quantum theory could be tested with a higher accuracy.

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