PHYC30170: Error Analysis Problems

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Question 1

(a) 50 counts per hour, from this we can calculate the probability of seeing one meteor in the 10% of the sky per second.

$$\frac{(0.1)50}{60(60)} = \frac{5}{3600} = \frac{1}{720}$$

Thus if using a 30s exposure the probability one meteor in 10% of the sky during this exposure is,

$$\frac{1}{720}(30) = \frac{30}{720} = \frac{1}{24}$$

Thus there is a 4.16 % chance of seeing a meteor during a single 30 second exposure.

$$\begin{aligned} P(n) &= \frac{\mu^n e^{-\mu}}{n!} \\ &= \frac{0.04167^1 e^{-0.04167}}{1!} \\ &= 0.0399 \pm 5.893 \times 10^{-3} \end{aligned}$$

Uncertainty on probablity is given by,

$$30\frac{\sqrt{50}}{3600} = 5.893 \times 10^{-3}$$

(b) We can use Poisson's equation to find the probability of seeing at least 2 or more meteors in a single 30 second exposure where $\mu = 0.0416$ and $\sigma = \sqrt{\mu}$.

$$\begin{aligned} P(n) &= \frac{\mu^n e^{-\mu}}{n!} \\ &= \frac{0.04167^0 e^{-0.04167}}{0!} \\ &= 0.95918625 \end{aligned}$$

Probabilities of 0 events occurring, thus all other probabilities are as follows,

$$\boxed{1 - 0.95918625 = 0.0408 \pm 5.893 \times 10^{-3}}$$

Thus the probability of seeing 2 or more meteors in one single 30 second exposure is 0.408%.

(c) Using binomial distribution where $n = 10$ and $p = 0.0399$,

$$1 - (1 - 0.0399)^{10} = 0.335$$

or multiply the answer for part (c) by 10.

$$0.0408(10) = 0.4085.893 \times 10^{-2}$$

Question 2

Note: All relevant code used for this calculation can be found at the end of the document and in the jupyter notebook attached with the submission.

(a) The difference for 100 m race was found to be 0.506 seconds.

(b) The difference for the 400 m race was found to be 2.096 seconds.

Question 3

Given Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Solving for n_2,

$$n_2 = \frac{\sin\theta_1}{\sin\theta_2}$$

Plugging in values,

$$n_2 = \frac{\sin(22.03)}{\sin(14.45)} = 1.503$$

Fractional uncertainty formula is given as follows,

$$\frac{\Delta n_2}{n_2} = \sqrt{\left(\frac{\Delta\sin\theta_1}{\sin\theta_1}\right)^2 + \left(\frac{\Delta\sin\theta_2}{\sin\theta_2}\right)^2}$$

Subbing and evaluating formula,

$$\begin{aligned}
\cot\theta_1 &= 2.4714 \\
\cot\theta_2 &= 3.8807 \\
\frac{\Delta n_2}{1.503} &= \sqrt{(2.4714(3.49 \times 10^{-3}))^2 + (3.8807(3.49 \times 10^{-3}))^2} \\
&= 0.0241
\end{aligned}$$

$$\boxed{n_2 = 1.503 \pm 0.0241}$$

Question 4

Note: All relevant code used for this calculation can be found at the end of the document and in the jupyter notebook attached with the submission.

(a) The first volume was calculated by taking the average of both height and radius and used to compute the volume as follows,

$$V = \pi \bar{h} \bar{r}^2 = 15.815\ cm^2$$

(b) The uncertainty was found as follows,

$$\Delta x = \frac{\sigma_x}{\sqrt{n}},$$

Where $\Delta r = 0.02$ and $\Delta h = 0.038$,

$$\Delta V = V\sqrt{\left(\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta h}{h}\right)^2} = \pm 0.334\ cm^2$$

(c) The volume from calculating each students measurements and averaging each outputted volume was found to be the following,

$$\bar{V} = 15.882\ cm^2$$

The error was then calculated using the following formula,

$$\Delta V = \sqrt{(2\pi r l \Delta r)^2 + (\pi r^2 \Delta l)^2} = \pm 0.318\ cm^2$$

Rough Work

Question 2, first attempt.

(a) **Ireland:** $\mu = 15\ s$, $\sigma_I \ = \ \frac{\sigma}{\sqrt{n_I}}$ and $n_i = 50,000$

$$\begin{aligned}\sigma_I =& \frac{\sigma}{\sqrt{n_i}}\\ =& \frac{1}{\sqrt{50,000}}\\ =& 4.4 \times 10^{-3}\ s\end{aligned}$$

England: $\mu = 15\ s$, $\sigma_e \ = \ \frac{\sigma}{\sqrt{n_I}}$ and $n_e = 500,000$

$$\begin{aligned}\sigma_e =& \frac{\sigma}{\sqrt{n_e}}\\ =& \frac{1}{\sqrt{500,000}}\\ =& 1.4 \times 10^{-3}\ s\end{aligned}$$

The time difference between the runners will be the difference between the two deviations. Thus we can obtain the following,

$$4.4 \times 10^{-3} - 1.4 \times 10^{-3} = 3 \times 10^{-3}\ s$$

(b) If the race was then four times longer we would need to calculate a new mean and standard deviation. This is done as follows,

$$\mu = 15(4) = 60\ s$$

Calculating the standard deviation,

$$\begin{aligned}\sigma \ &= \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2}\\ &= \sqrt{1 + 1 + 1 + 1}\\ &= \sqrt{4}\\ &= 2\end{aligned}$$

Ireland: $\mu = 60\ s$, $\sigma_I \ = \ \frac{\sigma}{\sqrt{n_I}}$ and $n_i = 50,000$

$$\begin{aligned}\sigma_I =& \frac{\sigma}{\sqrt{n_i}}\\ =& \frac{2}{\sqrt{50,000}}\\ =& 8.94 \times 10^{-3}\ s\end{aligned}$$

England: $\mu = 60\ s$, $\sigma_e \ = \ \frac{\sigma}{\sqrt{n_I}}$ and $n_e = 500,000$

$$\begin{aligned}\sigma_e =& \frac{\sigma}{\sqrt{n_e}} \\ =& \frac{2}{\sqrt{500,000}} \\ =& 2.82 \times 10^{-3}\ s\end{aligned}$$

Time difference for 100 second race,

$$8.94 \times 10^{-3} - 2.82 \times 10^{-3} \ = \ 6.12 \times 10^{-3}\ s$$