PHYC30170: Advanced Laboratories

Ramsauer-Townsend Effect

November 19, 2020

Author: Owen Johnson

Student ID: 18305971

Supervisor: Prof. Emma Sokell

UCD School of Physics

Scoil na Fisice UCD

Abstract

A basic review of the theory and demonstration of the Ramsauer-Townsend effect. Using a 2D21 Thyratron apparatus to accelerate electrons through a gas to observe the scattering and collision over an array of electron energies. It was found that the lowest probability of scattering occurred around 1 eV with this point found to be 0.976 ± 0.0574 eV during the experiment. It was also seen that the free mean path of the electrons varied with the change in electron momentum. A method to correct values for electron momentum was also explored and applied to the function in the lab.

Contents

1 Introduction

This experiment investigates and demonstrates the Ramsauer-Townsend effect. This will be done with the use of a 2D21 thyratron apparatus to obtain values for the probability of scattering in a cross-section of Xenon gas, as the momentum of electrons vary in conjunction with how the free mean path of these electrons varies. This experiment will also look at corrections that can be made to measurements obtained in the lab to yield more accurate results suggested by the theory of Ramsauer-Townsend effect.

2 Theory

The Ramsauer-Townsend effect is the process of scattering of low energy electrons as they interact with a particle as the electrons pass through a noble gas. This process can not be described using any classical model; thus, a quantum mechanical wave theory needs to be used to explain the effect in full.
As the electrons pass through the gas, they interact this causes scattering to occur they will be inelastic if the interaction cause ionization or excitation and elastic if this does not occur as the electrons act with wave nature. (8)

It was found that if atoms in a noble gas were under the influence of a potential of typical atomic parameters the solution for the Schrödinger Equation when solved shows that the scattering cross-section will have electron energies near 1 eV.
The 1-D model of electron scattering predicts that whenever half the electron wave-length in a particular wave is a multiple of the width of the well is that the scattering will go to 0. Thus using this model will only show the first distinct minimum. This is because in the case of the one-dimensional square well, some of the waves will be reflected. This reflection could cause destructive interference and a zero probability of scattering if the two waves are not in phase. Thus there will be energy where scattering is at a distinct minimum. (7) (4)

A solution to this problem is the use of three-dimensional square well for the atom of the noble gas, in the case of this demonstration Xenon. Here the scattering cross-section will be much smaller if the width of the well is small. As electron energy increases the dips in the cross-section will not be as prominent. This because the width of the well will contribute non-negligible values[1]. Thus one of the drawbacks of this demonstration is its capacity to produce qualitative results at higher electron energies. (7) (6)

[1]More in-depth analysis of this is available on *pg. 601* of Theory of Atomic Collisions (7).

2.1 Mathematical Interpretation of the Ramsauer-Townsend effect

The equation to describe the wave nature of electrons during this effect can be wrote down as a second order wave function,

$$\left[|\psi^2|\,\Delta V\right] \tag{1}$$

Thus the probability of finding a collision is proportional to finding it in the area, ΔV.

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -(E-V)\psi \tag{2}$$

Where E is the total energy of the particle, and V is the potential energy. Then to apply this to the Ramsauer-Townsend case, the cross-section of scattering must be examined.

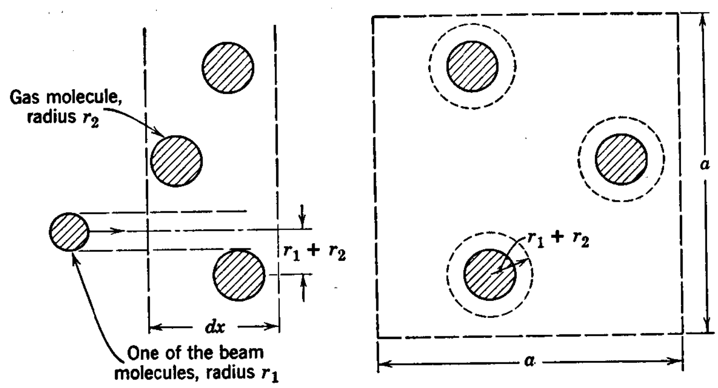

Figure 1 *Graphical representation of the scattering cross-section, along with its width which is labelled a (7)*

Fig. (1) represents the area where the atom is involved in the collision. With r_1 representing the width of the electron beam and r_2 the width of the molecules in the noble gas. Collisions will take place within $r_1 + r_2$. The number of molecules. N is given by a^2. Each point within the enclosed area seen in Fig. (1) has an equal probability of a collision taking place. Each collision that takes place can be thought of as a reduction in the electrons present in the accelerated beam. This reduction from the beam also decreases the current density incident on the plate. The fraction of electrons removed from the beam can be expressed as,

$$\frac{dJ}{J} = -N\pi(r_1+r_2)^2\,dx \tag{3}$$

The above expression is then integrated so that in can be used in the experiment,

$$\int_{J_0}^{J} \frac{dJ}{J} = -N\pi(r_1 + r_2)^2 \int_0^x dx \tag{4}$$

$$J = J_0 \exp\left(-N\pi(r_1 + r_2)^2 x\right) \tag{5}$$

Where J_0 is the magnitude of current density at $x = 0$. This relationship shows that the beam is reduced exponentially as it passes through the gas and the size of the reduction is dependant on the size of the particles in the gas used. Setting the radius of the electron to zero ($r_1 = 0$). Then r_2 is the effective radius. Now Eq. (5) can be written as,

$$dJ = -JN\pi r_2^2\, dx \tag{6}$$

$$J = J_0 \exp\left(-N\pi r_2^2 x\right) \text{ electrons}/m^2 \text{ sec} \tag{7}$$

Where J represents the number of electrons per second per square meter. Recalling that, N is the number of gas atoms per cubic meter, x is the distance in the direction of motion of the beam and electrical current density $j = eJ$ The cross-section can then be experimentally determined using the following equations,

$$J_p = J_0 \exp\left(-l(N\pi r^2)\right) \tag{8}$$

$$J_p = J_0(1 - P_s) \tag{9}$$

Where P_s represents the probability of scattering of a particle in the cross-section and l is the distance between apertures. (7) (8)

2.2 2D21 Thyratron Apparatus

In order to demonstrate this effect, a device is needed that will accelerate electrons through a tube filled with Xenon gas, in the case of this demonstration, a thyratron is being used. Present in the thyratron a shield and plate where the currents can be measured. If the beam intensity at the plate is given by Eq. (7). If the plate is a distance of l from the aperture the intensity incident on the plate can be written as,

$$I_P = I_s f(V)(1 - P_S) \tag{10}$$

Where shield current is represented by I_P and $f(V)$ is a factor which contains ratio for the angle intercepted by the plate to the angle intercepted by the shield and a factor due to thermionically emitted electrons from a negative sheath in the vicinity of the cathode and thereby inhibit further emission effects near the cathode. ()

When Xenon is removed by cooling with use of liquid nitrogen[2], $P_s << 0$ and $f(V) \approx I_P^*/I_S^*$. Probability of scattering can then be represented as,

$$P_s = 1 - \frac{I_P I_S^*}{I_P^* I_S} \tag{11}$$

The mean free path of the electron can be related to the probability of scattering by the following,

$$P_s = 1 - e^{-\frac{l}{\lambda}} \tag{12}$$

Where l is the length between the plate and aperture and λ is the mean free path of the electron[3].

The probability of collision can be similar expressed as seen with Eq. (11),

$$P_c = -\frac{p}{l}\left[\ln\left(\frac{I_p I_s^*}{I_s I_p^*}\right)\right] \tag{13}$$

Where p is the Xenon pressure[4] in the thyatron. (6) (9)

2.3 Correction of Electron Momentum

As outline in the extension paper of the experiments manual(9). The value of $\sqrt{V - V_s}$ does not give a true value of momentum which electrons are propagated through the scattering region at the first aperture.
This miscalculation is due to several factors which can be accounted for. The first contributing factor is due to the difference in work function values for the nickel shield and the cathode in the thyratron. Both of these electrodes are in contact due to the presence of the gas, so there is a net flow of electrons between the shield and cathode when they're at equal

[2]All values where Xenon is removed by cooling is represented by an asterisk superscript. (*)
[3]In this particular case l is the distance from the first aperture to the cathodes (0.7 cm) (6)
[4]Xenon pressure was taken to be pressure describe in manual and 2D21 datasheet as approximately 10^{-3} Torr

temperature and with no potential difference. To mitigate the effects of this, a negative potential needs be added to balance the effect out. Another factor that needs to be taken into consideration is the kinetic energy the electrons have due to thermionic emission before their initial acceleration. The retarding potential (V_P) with respect to the electron source and the current collected is given by,

$$i = i_0 \exp\left(\frac{-3V_p}{2\overline{V}}\right) \tag{14}$$

3 Apparatus

In the lab, a 2D21 Thyratron was used in a circuit as seen in Fig. (2). The thyratron tube contains Xenon gas. The Assembly is vertically mounted so that the filament is exposed at the uppermost part of the assembly[5].

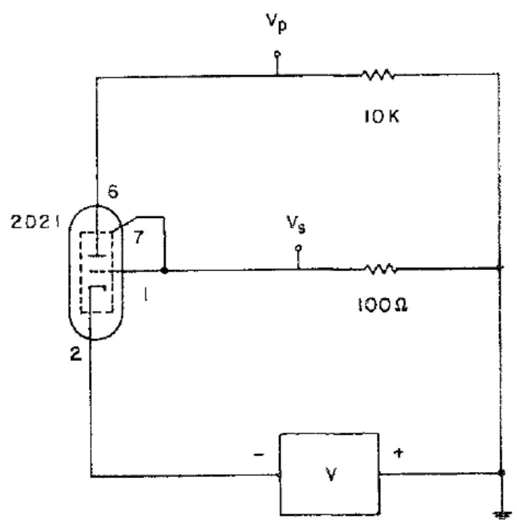

Figure 2 *Layout of the circuit used in conjunction with the 2D21 thyratron throughout the experiment(6).*

The placement of the filament allows for the filament to be directly cooled using liquid nitrogen in the lab so that the Xenon gas can be frozen out. Dewar flask and protective gloves are used in the handling of liquid nitrogen.

A regulated DC power supply is used to adjust the accelerating voltage across the circuit in increments that are easy to control. A transformer set to 4V is used to power the filament in the thyratron.

[5] For more clarification on the Assembly see Fig. (**??**)

Two digital 3-1/2 digit multi-meters are used in the circuit to measure lower voltages with a higher degree of accuracy. This is used for measuring the plate voltage (V_p) and the shield voltage (V_s). (3)

4 Experimental Methodology

The apparatus was connected as displayed and described in Fig. (2) and §3. Three sets of data were taken during the experiment. In each case the filament was heated by 4 V. The first case the accelerating voltage (V), shield voltage (V_s) and plate voltage (V_p). The circuit was used in forward bias. The accelerating voltage was set to 0 V. Initial values of V_s and V_p were taken with multi-meters. The accelerating voltage was then increased in increments of 0.1 until 2V was reached. This was to obtain finer reading around the point of interest of 1 eV as this is where the theorized dip is as discussed in §2. From 2V to 13V in increments of 0.5 V. The reason for going to 13V is this is approximately where ionization occurs(7).

In the second case the same procedure was repeated. However, this time the 2D21 was cooled using liquid nitrogen to freeze out the present Xenon gas. The same values and increments were taken for the same justifications as in the first case.

In the third case, the circuit is placed into reverse and the accelerating voltage is set to zero. The voltage is then increased in small increments until the shield voltage tends to zero. This is done with the Xenon gas frozen out by liquid nitrogen.

All data that was taken in the lab was placed into a series of .dat and .csv files. All manipulation, graphing and other analysis was completed all relevant code used throughout this report can be referenced in §7.2.2. In the case of each set of data the voltage was obtained. Thus the current was found using Ohm's law,

$$I = \frac{V}{R} \tag{15}$$

Where R was taken as the relevant value which can be referenced in Fig. (2).

5 Results

5.1 Results of correction of Electron Momentum

As discussed in §2.3, the electron momentum must be corrected. In the case of this trial of the experiment, the data was first observed without the correction. The corrections plotted with the initial fits before the corrections are shown and discussed in §7.2.2.

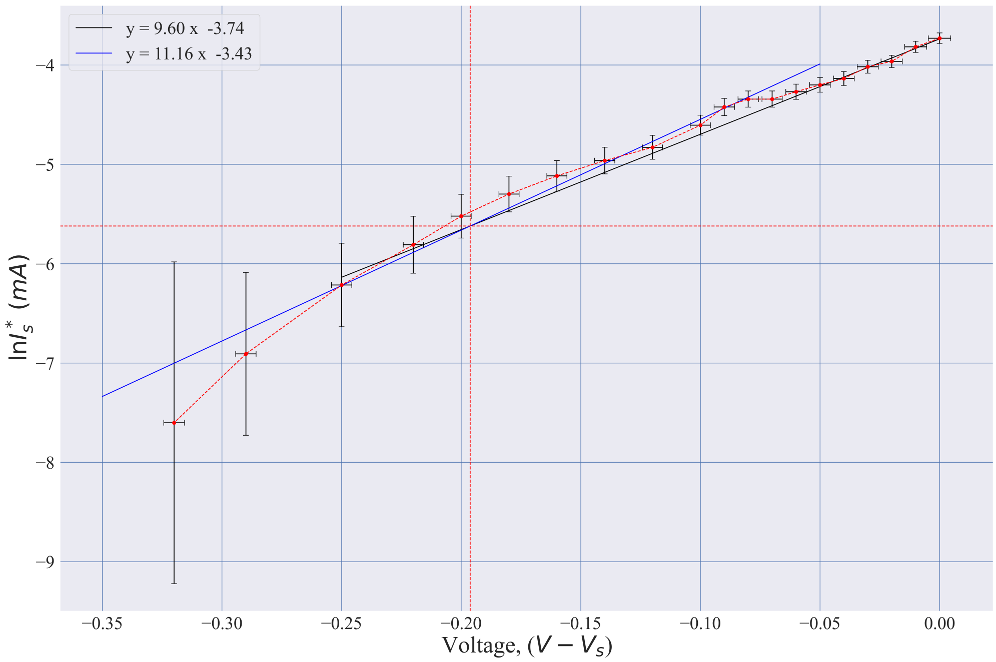

Figure 3 *Electron momentum plotted against the natural log of shield current when Xenon is frozen out.*

In Fig. (3), there are two fits applied to the data. The set of data of each fit is chosen based on the reliability of the fit[6] this process is expanded on in §7.1.1. The contact potential between the electrodes (V_c) is deemed to be the voltage where the log of currents intersect. This was found to be $0.196 \pm U\ V$. This part of the investigation is subject to substantial uncertainty. This is down to the apparatus used in the experiment. The increments used on the multi-meters do not allow for accurate measurements of shield current (I_s^*) this is illustrated in Table. (1). In the case where accelerating voltage yielded the same reading for

[6]This was done by using χ^2 testing methods.

multiple values of shield current, the medium[7] value is taken.
The value for thermionic energy, $\overline{V}$ can be found by using the slope to fit in the following relation from Eq. (14),

$$\frac{3}{2m} = \overline{V}$$

Where m is the slope of the blue fitted function in Fig. (3). The value for $\overline{V} = 0.156 \pm U\ V$. Correction was then made to the electron momentum using the following expression $\sqrt{V - V_s + V_c + \overline{V}}$. This corrected value for electron momentum is applied to all the data in §5.2.

5.2 Corrected Results

As discussed in §2, it was seen that the probability of electron scattering is based on the currents present in the shield and plate as per Eq. (11). In Fig. (4) we can see a visual representation of the current present in the plate when there is a presence of Xenon gas and when there is not. Where the red plot is the measurement of current when the gas is present and the green plot for when the gas in absent.

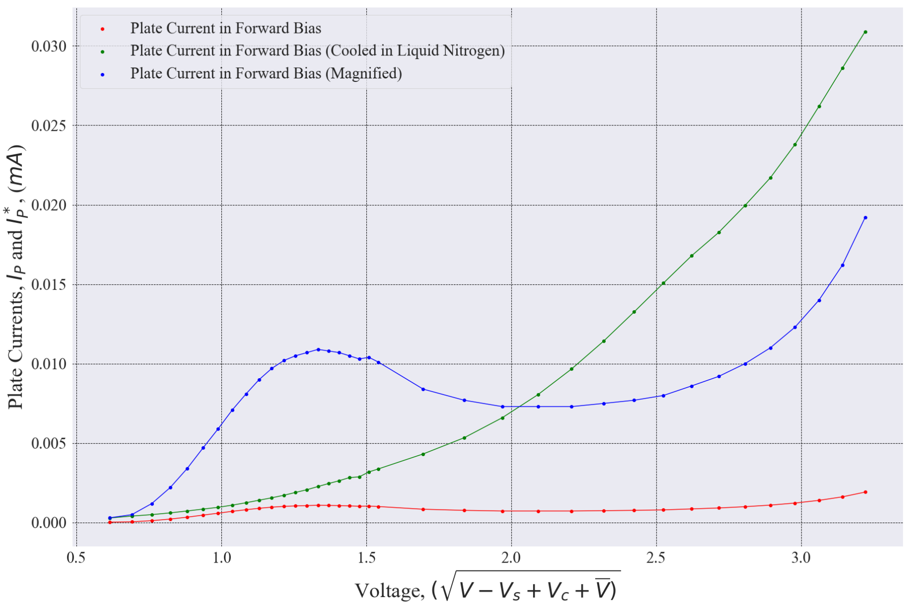

Figure 4 *Plot of values for plate currents with and without the presence of Xenon gas plotted against corrected voltage.*

[7]The true value lies somewhere in the middle of the data set, not in the outliers. This is why mean is not used as outliers would skew the further from the true value than the medium.

From Fig. (4) it can be seen that as the electron momentum increased the plate current rose. This is due to electrons of greater energy reaching the plate from the cathode. When the Xenon gas is frozen out (green line) of the thyratron, the plate currents are substantially higher than that seen when the gas is present (red line).

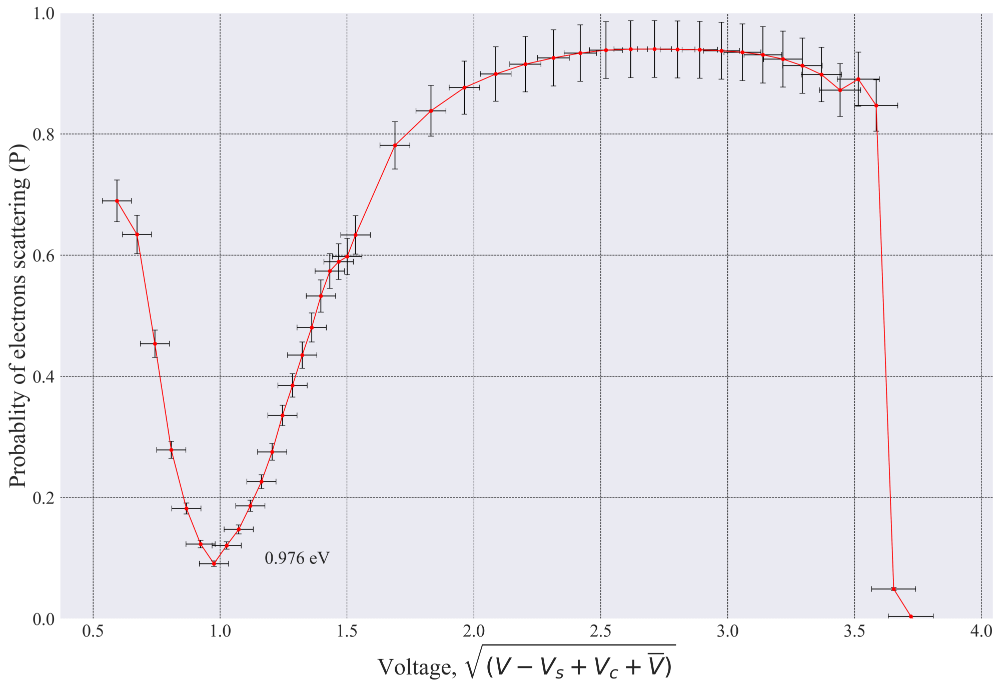

Figure 5 *The probability of electron scattering plotted against the corrected momentum of the electrons* $(\sqrt{volts})$

In Fig. (5) the probability of scattering is in compliance to what is expected from the Ramsauer-Townsend effect with the minimum of the trough in Fig. (5) occurring at 0.976 ± 0.0574 eV. The sudden drop off seen in the last 3 data points in Fig. (5) is due to the ionization[8] of the Xenon gas, which is expected as outlined in theory and manuals.(6)(9) Excluding the region in where the Xenon ionizes (last 3 points) the minimum and maximum value found for the probability of scattering and their associated voltages were found to be 0.091 and 0.94 respectively.

The Ramsauer-Townsend effect was also seen to be demonstrated in Fig. (6). The expected trough occurs at 0.976 ± 0.0574 eV as before. This again illustrates that the scattering

[8]This approximately occurs when the electrons have a momentum of 13 eV

cross-section will contain electron energies that are approximately 1 eV as suggested by the theory in §2

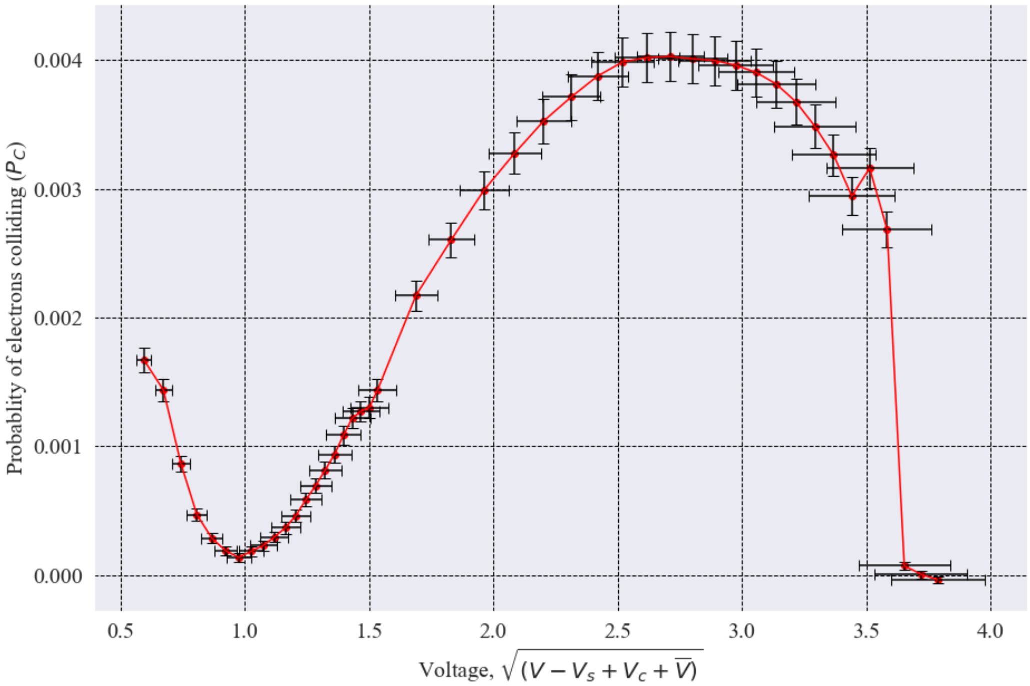

Figure 6 *The probability of electron collisions plotted against the corrected momentum of the electrons* $(\sqrt{volts})$

From the data obtained, it was also possible to determine the mean free path of the electrons (λ) with use of Eq. (12). It was found that for electrons around 1 eV the distance between successive collisions was $(7.358 \pm 0.0338) \times 10^{-4}$ m. It was also seen that this value varied with electron momentum as expected. This can be seen in Fig. (7)
, in this plot it is seen that the shape of the plot corresponds to the plot in Fig. (5). This is expected as the scattering probability is smaller in these regions; the successive distance between regions is larger.

In regards to uncertainty during this demonstration, it is quite varied. While this experiment and method is an appropriate way to demonstrate the Ramsauer-Townsend effect, there is much variance in how the measurements are taken. The first issue arises with the accelerating voltage across the circuit.
Banana plugs are used for wiring the circuit; these traditionally have very high contact resistance which can skew values for shield and plate voltages in the thyratron. This also

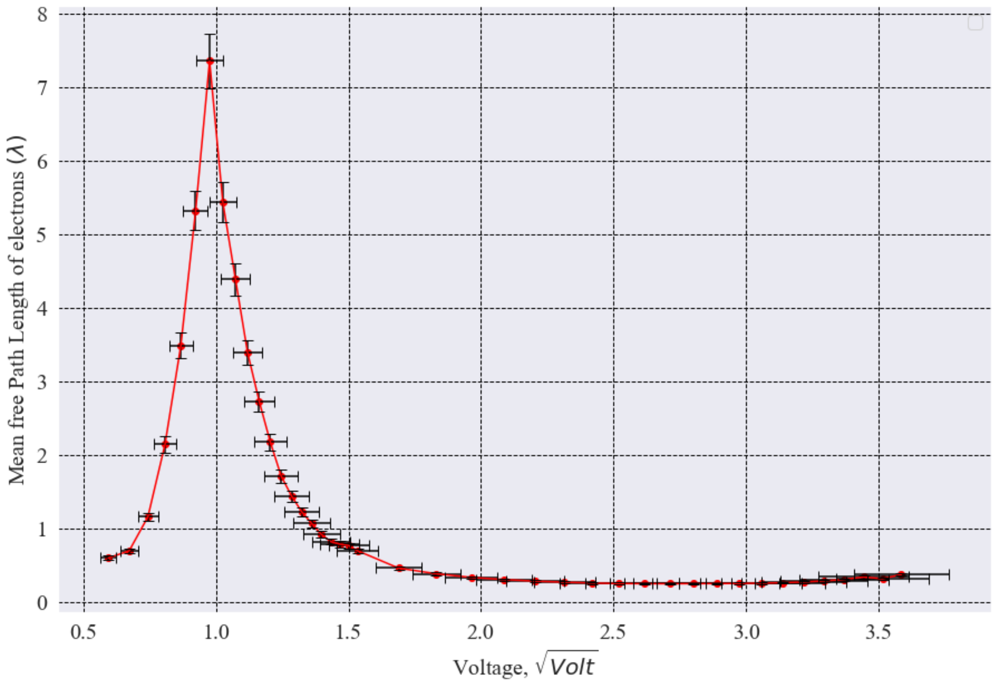

Figure 7 *Free mean path of an electron against the corrected momentum of the electrons* $\left(\sqrt{volts}\right)$

the factor of residual current from the power supplies (**?**). Both of these factors have been neglected when considering errors.

As the multi-meters used during the duration of this lab are only accurate to 3 1/2 digits, accuracy is lost at low readings.

Maintaining a consistent temperature with the liquid nitrogen this difficult to ensure that the Xenon gas is completely frozen out. Finally, the voltage which was passed through the filament was set to 4V, but the power-supply was analogue, so there could be a discrepancy in reading this voltage could also drift and cause variance in the heat of filament. Due to these factors, the errors may be redundant, but always essential to consider and given different means of completing the experiment on how one may approach calculating the errors. More detail on how the errors were calculated and their justification can be found in §7.1.

6 Conclusion

This experiment saw sufficient demonstration of the Ramsauer-Townsend effect. It was sufficiently demonstrated that the electrons exhibit a minimal probability of scattering in a cross-section around 1 eV. The minimum value for scattering in the cross-section was found to be 0.976 ± 0.0574 eV. This result differs from the expected value by 2.4 %. This result was only obtained after a correction of electron momentum where both thermionic energy of the electrons and the contact voltage between the electrodes in the apparatus was accounted for. It was also found that the free mean path of electrons around 1 eV was $(7.358 \pm 0.0338) \times 10^{-4}$ m. With the value for λ varying with electron momentum, peaking at 1 eV which is expected as the probability of collision is at its lowest here thus the distance between successive collisions should be at its highest.

References

[1] *2D21 Data Sheet*, 2018. URL: `http://www.farnell.com/datasheets/233495.pdf`.

[2] *Tenma 72-2535 Data Sheet*, 2018. URL: `http://www.farnell.com/datasheets/2724049.pdf`.

[3] *Tenma 72-2595 Data Sheet*, 2018. URL: `http://www.farnell.com/datasheets/2724052.pdf`.

[4] David J. Griffiths. *Introduction to quantum mechanics*. Pearson Prentice Hall, 2nd ed edition, 2005.

[5] Ifan Huges and Thomas Hase. *Measurements and their Uncertainties: A practical guide to Modern Error Analysis*. Oxford, England University Press, 1st edition, December, 2009.

[6] Stephen. G Kukolich. Demonstration of the ramsauer-townsend effect in a xenon thyratron. *American Journal of Physics*, 36(8), 1968.

[7] Harrie Massey and Nevill Francis Mott. *Theory of Atomic Collisions*. Oxford University Press, 3rd edition, 1965.

[8] J.S. Townsend and V.A. Bailey. Xcvii. the motion of electrons in gases. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 42(252):873–891, Dec 1921.

[9] G.A Woolsey. An extension of the ramsauer-townsend experiment in a xenon thyratron. *American Journal of Physics*, 39(8), 1970.

7 Appendices

7.1 Errors

When numerically finding values to quantify the uncertainty on the data in this experiment. Both the degree of detail that the instruments could measure and the uncertainty on said measurement was taking into consideration. This was done by consultation of data sheets of each of the instruments that contributed to the measurement of necessary data. (5)

They were then propagated appropriately using the following equations dependant on the formula they were derived from. The nature of the data manipulation only required propagation of logarithmic, multiplication, addition and subtraction. (5)

$$\sigma_x = \sqrt{{\sigma_a}^2 + {\sigma_b}^2 + {\sigma_c}^2} \quad \text{(Addition and subtraction)} \tag{16}$$

$$\frac{\sigma_x}{x} = \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2 + \left(\frac{\sigma_c}{c}\right)^2} \quad \text{(Multiplication or division)} \tag{17}$$

$$\sigma_x = \frac{\sigma_a}{a} \quad \text{(Natural Log)} \tag{18}$$

7.1.1 Errors and fit of Fig. (3)

In Fig. (2.3) two linear fits are plotted. The data points present in Fig. (2.3) appear to be a uniform, straight line. While using one linear fit is satiable for determining the value of $\overline{V}$ it is not suitable for determining the contact voltage, V_c. As the contact voltage is determined as the voltage at the junction of the two electrodes (Nickle and barium oxide), (**?**). To determine which data set both linear fits should be applied to the varying combinations of data sets were fitted using loops and put under a χ^2 test with the fits with the least error being taken as the fits used in Fig. (2.3). The error taken on the value for the contact voltage was determined to be the error on the slope. This is illustrated in Fig. (8).
The error on V_c was then calculated using the following method,

$$\Delta x = \frac{m_{\max} - m_{\min}}{2} \tag{19}$$

An error was calculated for both slope fits and propagated to give an error on V_c.

$$\Delta V_c = \sqrt{\left(\frac{\Delta m_1}{m_1}\right)^2 + \left(\frac{\Delta m_2}{m_2}\right)^2} \tag{20}$$

Another method to finding the error would be to take the error on the x and y values in Fig. (2.3) while this would yield a comparable result to the above method. It would be somewhat futile as the apparatus used in the lab does not incremental enough readings that the error would mitigate it as the data would not reflect the true current and voltage accurately enough for the errors to cover it. Also as the contact voltage is calculated based on the intersection on the slopes and how good both fits are it is more appropriate in this case to estimate the error by observing where maximum and minimum intersections occur.

A similar method was applied to the calculation of the uncertainty on $\overline{V}$. Using the slope of the blue line in Fig. (2.3) yielded a marginally differing result to if the slope of the entire data set was taken to calculate $\overline{V}$ (did not affect the first three significant figures). So as before the error of the slope was calculated by Eq. (19). (5) (1) (2)

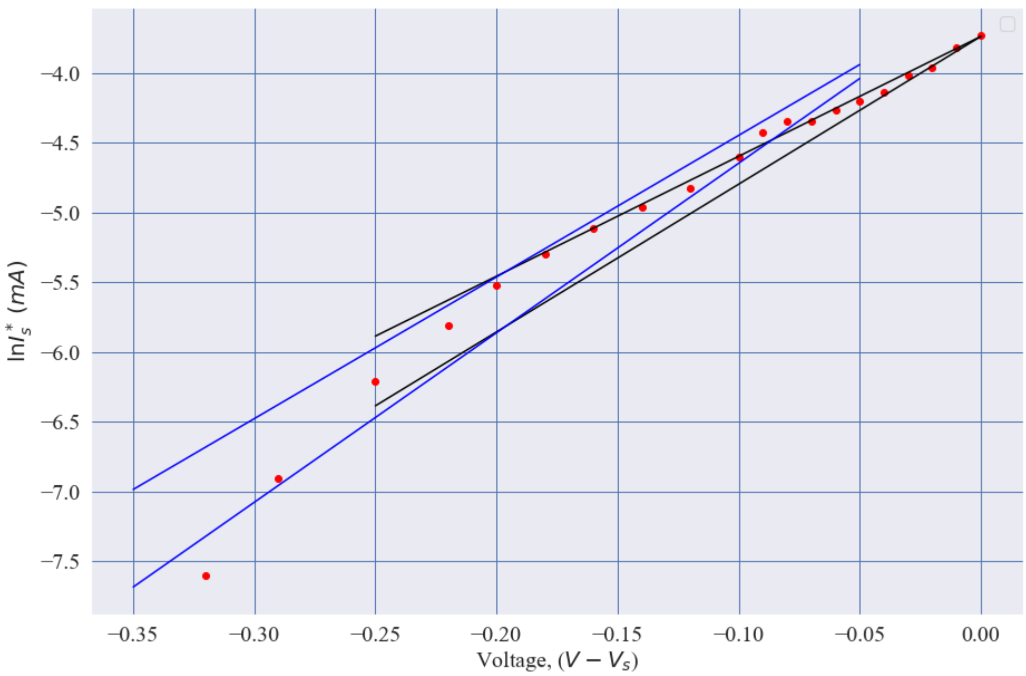

Figure 8 *Illustration of variance in slope fits for data from §5.2*

7.2 General Notes & Data

7.2.1 Raw Data relevant to discussion

V_A (V)	V_S (mV)
0	2.2
0.01	2
0.02	1.9
0.03	1.8
0.04	1.6
0.05	1.5
0.06	1.4
0.07	1.3
0.08	1.3
0.09	1.2
0.1	1
0.11	0.8
0.12	0.8
0.13	0.8
0.14	0.7
0.15	0.7
0.16	0.6
0.17	0.6
0.18	0.5
0.19	0.4
0.2	0.4
0.21	0.3
0.22	0.3
0.23	0.3
0.24	0.2
0.25	0.2
0.26	0.2
0.27	0.1
0.28	0.1
0.29	0.1
0.3	0.1
0.31	0.1
0.32	0

Table 1 *Data points collected the during the experiment with circuit in reverse bias and Xenon frozen out.*

7.2.2 Comparison of initial and corrected data

Below an array of plots can be viewed with the un-corrected electron momentum along with the corrected values.

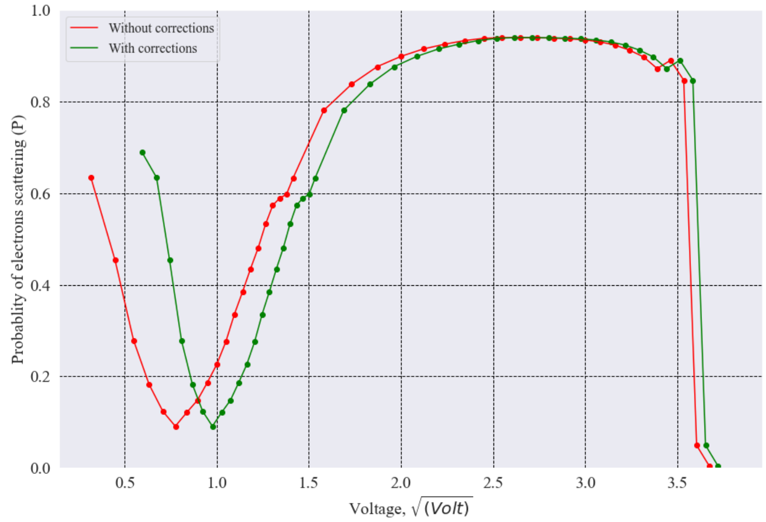

Figure 9 *Plot of the probability of scattering with and without the corrected voltage.*

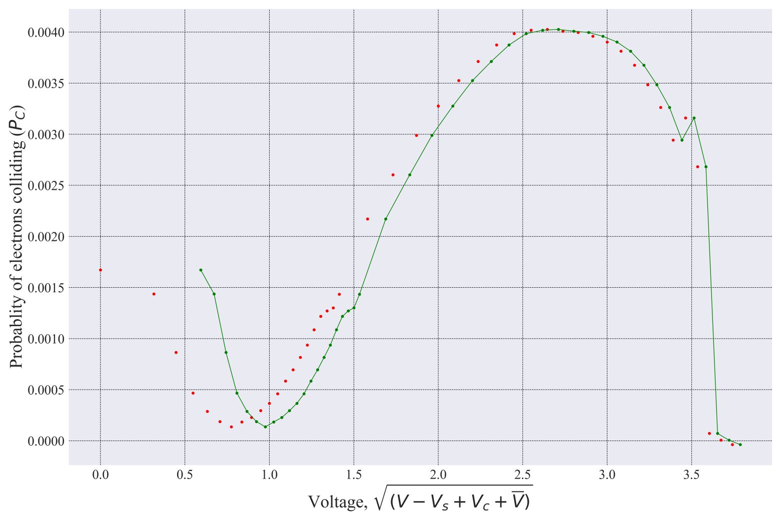

Figure 10 *Plot of the probability of collision with and without the corrected voltage.*

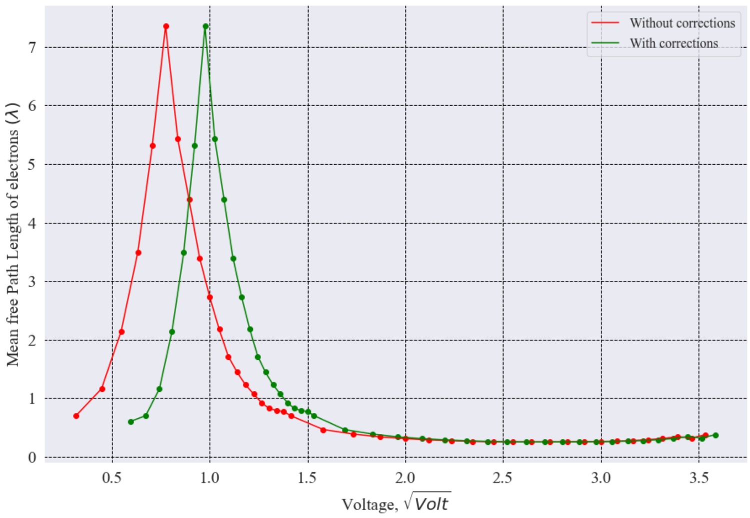

Figure 11 *Plot of mean free path length with and without the corrected voltage.*

As seen in the previous three figures, the correction shifted the results obtained closer to the expected value of 1 eV. The initial dip before the correction was found to be 0.774 eV. This value was 22.6% outside the expected range of 1 eV. Thus the correction outlined in §5.2 improved the final data set.

7.3 Code

```python
import pandas as pd
import scipy
import numpy as np
import os #cwd tools
import matplotlib.mlab as mlab
import seaborn as sns
from astropy.stats import sigma_clip
from matplotlib import pyplot as plt
from scipy.optimize import curve_fit
from scipy import stats
from scipy import optimize
from scipy import interpolate
from scipy.stats import chi2_contingency

%matplotlib inline
sns.set()
```

Listing 1 *Package import.*

```python
# --- function definition ---

def current(V, R):
    return (V/R)

def prob_scat(IP1, IP2, IS1, IS2):
    return 1 - (IP1 IS2)/(IP2 IS1)

def prob_col(IP1, IP2, IS1, IS2):
    p = 1e-3 #Torr
    l = 0.7 #cm
    return (-p/l) (np.log((IP1 IS2)/(IP2 IS1)))

def lin_func(a, b, x):
    return a x + b

def func(a, x):
    return a x

def quad_func(a, b, c, x):
    return a (x 2) + b (x) +c

```

```
def cubic_func(a, b, c, d, x):
    return a (x 3) + b (x 2) + c (x) + d

def quin_func(a, b, c, d, e, f, x):
    return a (x 5) + b (x 4) + c (x 3) + d (x 2) + e (x) + f

def mean_free(P):
    U = 1 - P
    return -0.7/(np.log(U))

def good_fit(data):
    stat, p, dof, expected = chi2_contingency(data)

    # interpret p-value
    alpha = 0.05
    print("p value is " + str(p))
    if p <= alpha:
        print('Dependent (reject H0)')
    else:
        print('Independent (H0 holds true)')
```

Listing 2 *Function Definition*

```
# -- data import --
voltage1, plate_voltage1, shield_voltage1 = np.loadtxt("forward_bias.dat", unpack = True)
voltage2, plate_voltage2, shield_voltage2 = np.loadtxt("forward_bias_cooled.dat", unpack = True)
voltage3, shield_voltage3 = np.loadtxt("reverse_bias_cooled.dat", unpack = True)


# -- intial data manipulation --

IP1 = current(plate_voltage1, 10000)
IS1 = current(shield_voltage1, 100)

IP2 = current(plate_voltage2, 10000)
IS2 = current(shield_voltage2, 100)

vsq = np.sqrt(voltage1 - shield_voltage1 + plate_voltage1 + np.mean(voltage1))
P = prob_scat(IP1, IP2, IS1, IS2)
l = 0.7 #cm
```

```
lam = mean_free(P)

# -- Reverse Polarity ---


IS3 = current(shield_voltage3, 100)
print(shield_voltage3)
print(IS3)
LnIS3 = np.log(IS3)
```

Listing 3 *Data Import*

```
plt.scatter(vsq3, LnIS3, color = 'red')
plt.plot(x_t3, y_t3, color = 'black', label='y = {0:.2f} x  {1:.2f}'.format(
    popt1[1], popt1[0]))
plt.plot(x_t2, y_t2, color = 'blue', label = 'y = {0:.2f} x  {1:.2f}'.format(
    popt2[1], popt2[0]))

#Format label='y = %.2f x + %.2f' %(A, B)

# --- Intercept Code ---

xi = (b1-b2)/(m2-m1)
yi = m1    xi + b1
contact_voltage = xi

print('(xi,yi)',xi,yi)

plt.axvline(x = xi, color = 'red',linestyle='--')
plt.axhline(y = yi, color = 'red',linestyle='--')


plt.errorbar(vsq3, LnIS3, xerr = vsq3err, yerr = LnIS3err, color='red', ls='--
    ', marker='o', capsize = 5, capthick = 1, ecolor = 'black')
plt.xlabel("Voltage, (${V - V_s}$)")
plt.grid(b=True, which = 'major', color='b', linestyle='-')
plt.ylabel("$\ln I_s ^ $ $(m A)$")
# plt.xlim(-.6, 0)
# plt.yscale('log')
plt.legend()
plt.savefig('extension.png', dpi = 300)
plt.show()
```

```
print('Contact␣Voltage␣is:', np.abs(np.round(contact_voltage, 3)))
```

Listing 4 *Example of Plotting Code*