

User's Manual - Online Pairwise Comparisons Application

by

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An online application developed
to aid decision-makers
to conduct decision-making processes
utilizing MCDM and PC methods

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Contents

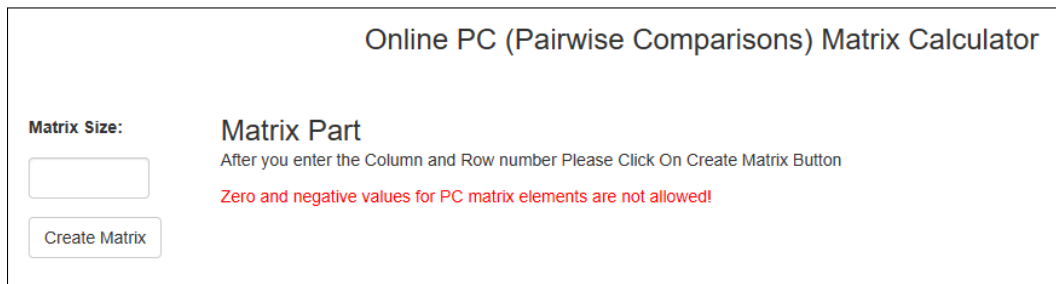
0.1	Application Design	1
0.1.1	Home Page and User Prompt	1
0.1.2	Pairwise Comparisons Calculator	2

0.1 Application Design

In this chapter, we will discuss the design for the pairwise comparisons online application and provide instructions for using such an application.

0.1.1 Home Page and User Prompt

Once users visit our webpage, they will be directed to the homepage of the application, which introduces users to what our online application does and how they can get started. Users are first prompted to input the size of the pairwise comparisons matrix they prefer to initiate with, with size (represented with n) ranging from 3 to 8. We also took care of the usual user input errors, such as submitting anything that is not a number between 3 and 8 with a friendly warning.



The screenshot shows a web interface titled "Online PC (Pairwise Comparisons) Matrix Calculator". It features two main sections: "Matrix Size:" and "Matrix Part". Under "Matrix Size:", there is a text input field and a "Create Matrix" button. The "Matrix Part" section contains instructions: "After you enter the Column and Row number Please Click On Create Matrix Button" and a red warning message: "Zero and negative values for PC matrix elements are not allowed!".

Figure 1: Home screen for the pairwise comparisons online application

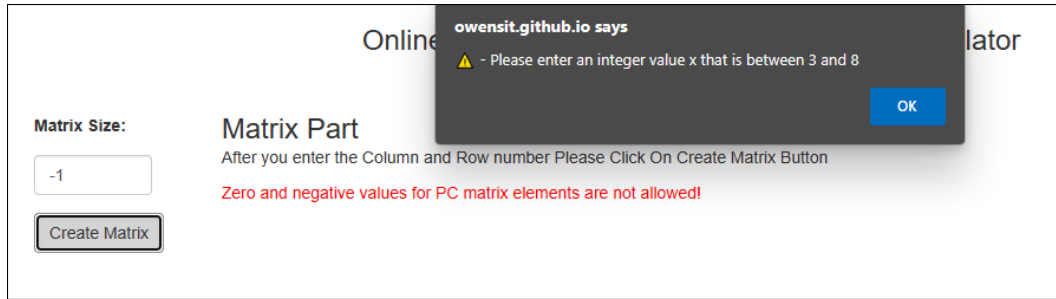


Figure 2: Pop-up warning for exception handling

0.1.2 Pairwise Comparisons Calculator

Once the online application receives a valid input of the matrix size from the user, it will redirect the user to the Pairwise Comparisons Calculator page view, which consists of 4 parts. As indicated in figure 3, these 4 parts are:

1. Pairwise comparisons matrix reset panel
2. Pairwise comparisons matrix modification panel
3. Geometric means indicator and parent node selection panel
4. Treemap weight and hierarchy indicator panel

Online PC (Pairwise Comparisons) Matrix Calculator

Matrix Size:

[Create Matrix](#)

PC (pairwise comparisons) Matrix

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

[Compute KJ](#)

GM	N_GM	Parent
1	0.25	Root▼
1	0.25	Root▼
1	0.25	Root▼
1	0.25	Root▼

1. Pairwise comparisons matrix reset

2. Pairwise comparisons matrix modification

3. Geometric means indicator and parent node selection panel

Root

A 0.25 Root/ 25% of parent 25% of entry 25% of root	C 0.25 Root/ 25% of parent 25% of entry 25% of root
B 0.25 Root/ 25% of parent 25% of entry 25% of root	D 0.25 Root/ 25% of parent 25% of entry 25% of root

4. Treemap weight and hierarchy indicator

Figure 3: The 4 parts in the Pairwise Comparisons Calculator page

We will start with discussing part 1 - Pairwise comparisons matrix reset panel. This module looks the same as the one on the home page. It is implemented to allow users to reset the whole pairwise comparisons matrix shown in part 2, or if they want to set up a matrix with a different size to start with, this is the place to go. In addition, it also checks for valid input from the users and redirects them back to the homepage if invalid inputs are received.

The next module is part 2 - Pairwise comparisons matrix modification panel. This module is the main focus of the online application. It allows the user to modify the pre-initialized pairwise comparisons matrix based on the user-defined matrix size. When the user defines the size of the matrix through module 1, the program initializes a pairwise comparisons matrix of the corresponding size. It fills up the content of the matrix with 1s, so as the geometric means and hierarchy module and the treemap module.

The pairwise comparisons matrix is implemented using a group of several HTML input elements. These input elements are organized with the Bootstrap CSS class so that they form a square matrix (n, n) . As we mentioned in the previous chapter, the pairwise comparisons matrix is a reciprocal matrix with 1s in the main diagonal. In a pairwise comparisons matrix, what we usually care about the user's input are the elements from the triangle above the main diagonal. Once we have the elements from the upper triangle, we can fill up the rest of the elements in the matrix pretty quickly. For this reason, we decided to mark the main diagonal elements and those from the bottom triangle as disabled input boxes and left only elements from the upper triangle activated so that we could eliminate implementation complexity and most of the user errors.

PC (pairwise comparisons) Matrix

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

Figure 4: Pairwise comparisons matrix modification module after initialization with the size $n = 4$

Once the user has finished modifying a specific box on the upper triangle, he/she can hit “enter”, and the following events will take place:

1. The modified box in the matrix will be saved.
2. The element opposite the modified element in the symmetric position will be updated to the reciprocal value of the modified value.
3. The corresponding geometric means and normalized geometric means (in module 3) will get updated based on the new matrix.
4. The treemap weight indicator (in part 4) will get updated based on the new matrix.

PC (pairwise comparisons) Matrix

1	4	5	3
0.25	1	6	2
0.20	0.17	1	1
0.33	0.50	1.00	1

Compute Kii

Figure 5: Pairwise comparisons matrix modification module after sample data input

GM	N GM
2.78	0.54
1.32	0.26
0.43	0.08
0.64	0.12

Figure 6: Geometric means module after sample data input

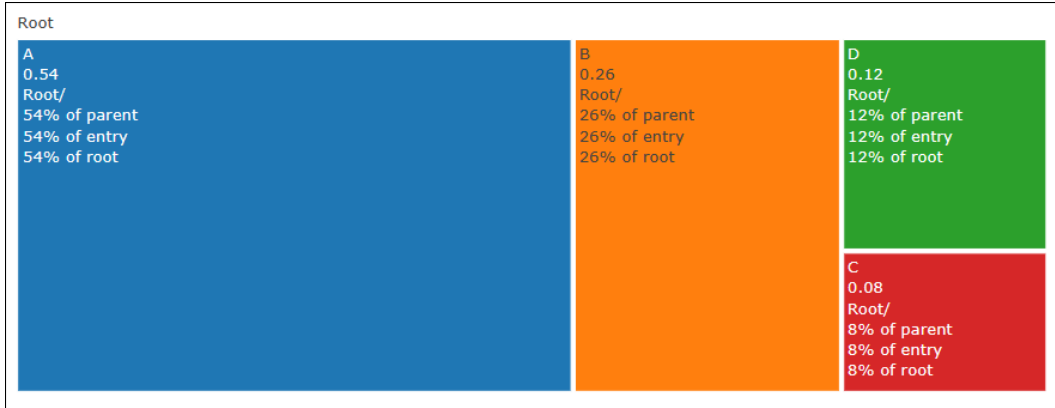


Figure 7: Treemap weight indicator module after sample data input

Another function that has been implemented into module 2 is inconsistency calculation and reduction. Once the user has completed modifying the pairwise comparisons matrix, he/she can move on to the matrix inconsistency calculation and reduction process. Simply click on the “Compute Kii”; the program will then collect all the triads it can find within the given matrix and indicate the triad with the highest Kii inconsistency value either with:

1. Green highlights - which indicate that the triad has a Kii value lower than 0.33, which is the recommended inconsistency threshold proposed by Dr. Koczkodaj.

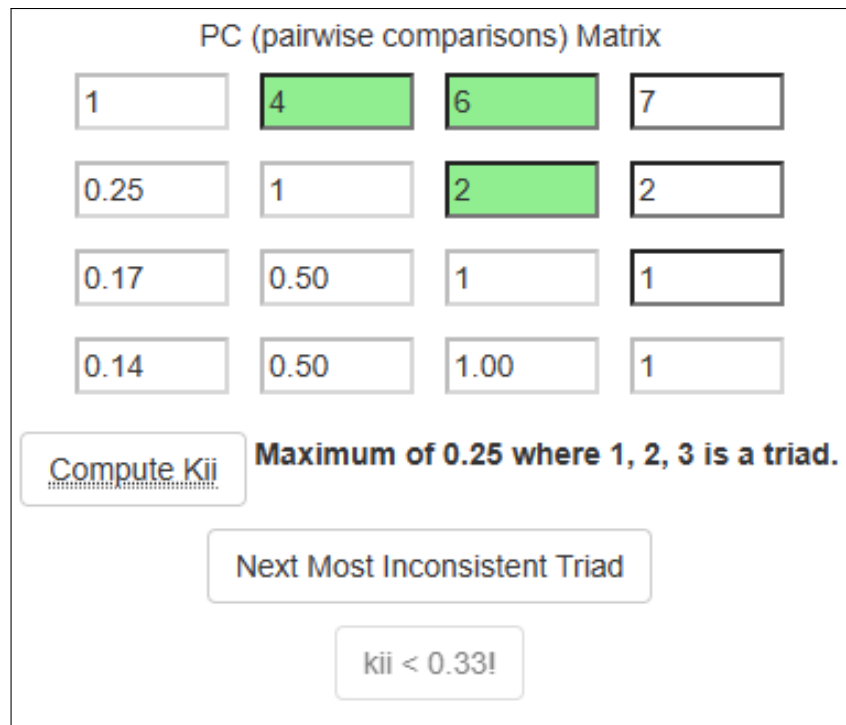


Figure 8: Module 2 Kii evaluator with relatively consistent sample data

2. Red highlights - which indicate that the triad has a Kii value of 0.33 or above.

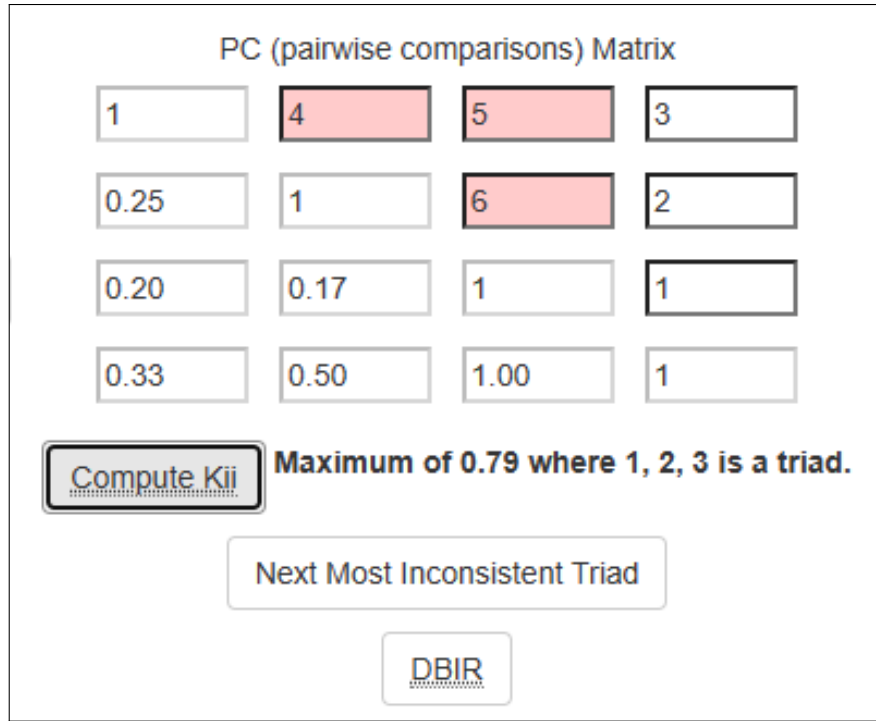


Figure 9: Module 2 Kii evaluator with relatively inconsistent sample data

Immediately below the matrix, you can find information about which matrix has the highest Kii inconsistency value and its Kii value.

The firstly highlighted triad shares the same Kii inconsistency with the matrix, as it is the triad that has the highest Kii inconsistency value. If such a triad is highlighted in green, it indicates that such a matrix has a Kii inconsistency value of lower than 0.33, and it is relatively consistent. Users can then view the triad with the 2nd-highest inconsistent value by following the sign of the button.

However, if the firstly highlighted triad is in red, it means that the most inconsistent triad has a Kii inconsistency value of 0.33 or higher, and the matrix is relatively inconsistent. At this time, users can also choose to view the following most inconsistency triad in the matrix or optimize the most inconsistent triad. To optimize the current triad with an inconsistency of 0.33 or above, simply click on the “DBIR” button, which stands for Distance Based Inconsistency Reduction. The program will then execute the inconsistency reduction algorithm, as mentioned in the previous chapter.

After the inconsistency reduction process, users will notice that the following events have been triggered:

1. The values for the highlighted triad have been updated.
2. The elements opposite the modified elements in the symmetric position are updated to the reciprocal value of the modified value.
3. The corresponding geometric means and normalized geometric means (in module 3) are updated based on the new matrix.
4. The treemap weight indicator (in part 4) is updated based on the new matrix.
5. The “DBIR” button is replaced by a grayed-out “Reevaluation required!” button.

However, the indicated colour and the Kii inconsistency value for the matrix have not yet been updated. In order to do so, users need to click on the “Compute Kii” button to re-evaluate the matrix for the updated Kii inconsistency value.

PC (pairwise comparisons) Matrix

1	2.20	7.25	3
0.45	1	3.30	2
0.14	0.30	1	1
0.33	0.50	1.00	1

Compute Kii **Maximum of 0.79 where 1, 2, 3 is a triad.**

Next Most Inconsistent Triad

Reevaluation required!

Figure 10: Pairwise comparisons matrix from figure 9 after DBIR process

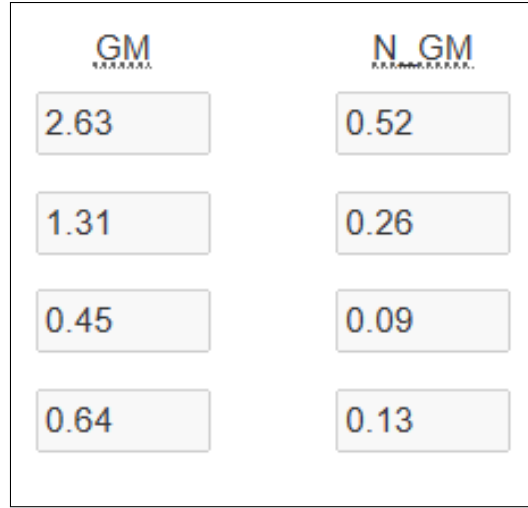


Figure 11: Geometric means module from figure 6 after DBIR process

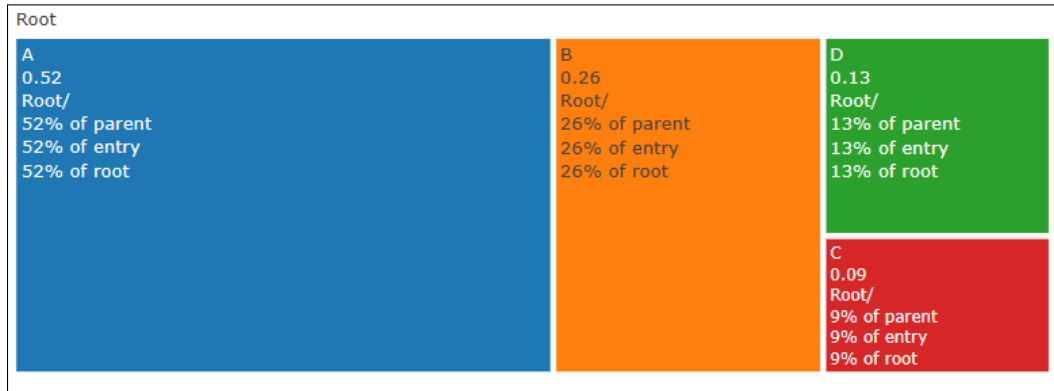


Figure 12: Treemap weight indicator module from figure 7 after DBIR process

Module 3 of the webpage consists of the geometric means indicator and parent node selection panel. As mentioned in the previous chapter, the geometric mean is calculated by taking the n th root of the product of elements from each row, which extract the row-wise weights. The normalized geometric means, on the other hand, compare the geometric means in the vertical faction and eventually convert them into proportion values that add up to 1. The geo-

metric means and normalized geometric means indicator will be automatically updated whenever changes in the pairwise comparisons matrix in module 2 are detected. Because values in this module are not expected to be modified by users directly, we decided to mark out the boxes as disabled to mitigate user errors. Examples of the geometric means and normalized geometric means module are shown previously in figure 6 and figure 11. Module 3 also includes the function of selecting a parent node for each stimulus to add more usability to our online application. This function allows users to place smaller value nodes underneath larger value nodes and supports more than one node in each parent node. When the pairwise comparisons matrix is first initialized, each node or stimulus is, by default, added to the "Root" node. If users want to make any adjustments, they simply need to click on the corresponding row of the "Parent" column and choose the preferred parent node in the drop-down menu. The changes will be applied automatically and can be viewed from the treemap diagram in module 4.

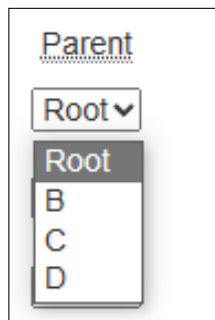


Figure 13: Select a parent node from the drop-down menu



Figure 14: Treemap diagram after selecting node B as the parent node of node A

Module 4 - Treemap weigh indicator is implemented at the bottom of the page with the help of the Plotly JavaScript library to present to the users with the weight distribution between each stimulus in the pairwise comparisons matrix (? , ?). The interactive treemap consists of several blocks, depending on the size of the declared pairwise comparisons matrix. Each block contains a label and different values. The labels are in alphabetical order corresponding to the numeral order of the stimuli in the pairwise comparisons matrix. The values in each block express the weight of the stimuli. Each block is also labelled in different colours for better readability. Like the geometric means and normalized means module, this module will also be automatically updated once changes in the pairwise comparisons matrix are detected. Examples of the treemap weight indicator module can be found in figure 7, figure 12, and

figure 14. The treemap module also provides the feature for downloading the diagram as a PNG file, in case it is required by the users.



Figure 15: Logo of Plotly.js graphing library

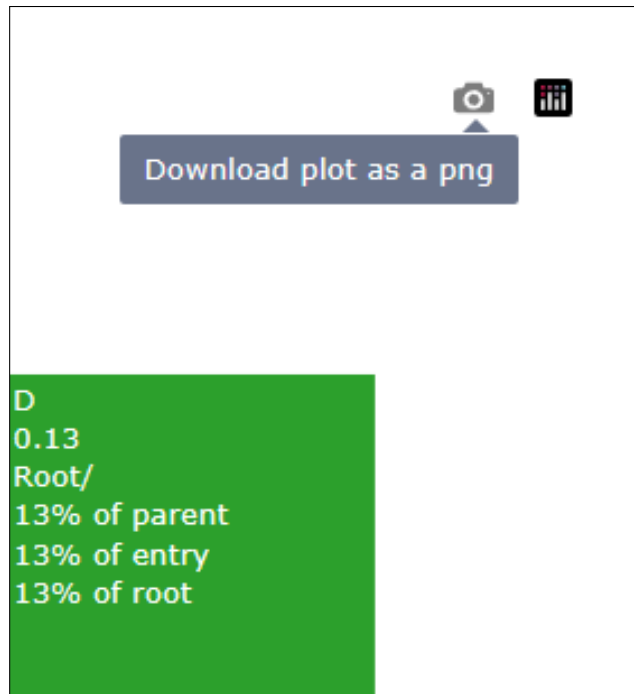


Figure 16: Save as PNG feature of the treemap weight indicator module