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Decoding Default Risk: A Review of Modeling Approaches, Findings, and Estimation Methods

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Abstract

Default risk permeates the behavior of corporate bond returns and spreads, credit default swap spreads, estimation of default probabilities, and loss in default. Pertinent to this review are salient empirical findings and implications of default process estimation from 1974 to 2021. Both structural and reduced-form models are covered. In structural models, default occurs if the value of assets falls below some threshold obligation. The reduced-form models involve assumptions about the default process combined with recovery in default. Default process estimation and measurements of default probability have improved by exploiting data on defaultable bonds, credit default swaps, tally of default realizations, and options on individual equities. Empirical investigations continue to address the relevance of stochastic asset volatility, jumps in asset values, and modeling of default boundary and firm leverage process.

1. MOTIVATING EMPIRICAL FACTS AND RESEARCH QUESTIONS

Much has been learned from the realm of default risk, credit spreads, loss in default, and predicting corporate bankruptcies. This includes the development of sophisticated mathematical models of credit risks as well as elegant estimation approaches that exploit observable economic covariates and latent variables. The scope of this research has expanded to include corporate bonds, traded default risks in the form of credit default swaps (CDS) on individual names and indexes,[1] collateralized debt obligations, empirical default frequencies and recovery in default, and extracting default probability information from options on individual equities and corporate balance sheets. The richness and diversity of these findings make the task of this review both challenging and rewarding.

Indeed, research efforts and big data have paved the way for better risk management of credit-sensitive securities and credit portfolios. This is true whether the modeling approach is structural, namely, based on specification of firm value and default boundary, or reduced-form, namely, based on specification of default intensity process and loss in default. In default risk modeling, the treatment of interest rates appears less pivotal. Most, if not all, structural models assume a flat term structure of default-free interest rates. Moreover, in many reduced-form models, the evolution of interest rates is assumed to be independent of the evolution of default intensity and the evolution of recovery in default. Most empirical models of CDS spreads rely on such a detachment.

1.1. Empirical Facts

The work on explaining movements in the observed credit spreads has highlighted a role for firm-specific fundamentals, macroeconomic fundamentals, liquidity, and tax effects. To get perspective on empirical investigations and key modeling constructs, **Table 1** provides a snapshot of the size of corporate credit spreads in the United States from January 1997 to August 2021.

One observes that the Baa minus the 10-year Treasury yield spread has averaged 252 basis points (bps). Isolating the effect of better credit quality, the Aaa minus the 10-year Treasury yield spread has averaged 152 bps, a full 100 bps lower. **Table 1*b*** further indicates that a compensation of 862 bps is associated with high-yield (below investment grade) corporate debt. Matching some of these spreads has proved tantalizing for many macrofinance models of corporate debt in calibrations and estimation. Such a gap has prompted an appreciation for improved model ingredients.

Table 2 presents the properties of CDS contracts. The index we consider in **Table 2*a*** is the ticker CDX.NA.IG, which is composed of the most liquid 125 CDS referencing North American investment-grade entities. The CDS spreads for the CDX.NA.IG index have averaged 84 bps with a 5th (respectively, 95th) percentile value of 50 (respectively, 144) bps. These entries reflect premiums for credit protection as well as perception of credit conditions and hedging concerns.

Table 2*b* shows the return (and excess return) features of a fully collateralized CDX.NA.HY, which consists of the 100 most liquid high-yield entities. The buyers of this CDS price index earned an annualized average return of 2.8% (standard deviation of 9.39%). Investors adversely affected by worsening credit conditions sell CDS index contracts. Indeed, during the COVID-19–related equity market adjustments over January 2020 to March 2020, sellers of this index experienced a cumulative appreciation of 15.2%.

[1]CDS offer insurance against possible default of a reference issuer. The contract specifies that the CDS seller compensate the CDS buyer in the event of default. In return, the CDS buyer pays a constant payment until the CDS maturity or a default.

Table 1 Size of corporate credit spreads

	Begin date	End date	Mean (%)	SD (%)	Percentiles 5th	25th	50th	75th	95th	ACF_1
Panel A: Corporate credit spreads, Jan. 1997 to Aug. 2021										
Baa minus 10-year Treasury	1/31/1997	8/27/2021	**2.52**	0.74	1.59	1.99	2.45	2.895	3.48	0.95
	10/31/2008	8/27/2021	**2.73**	0.78	1.87	2.22	2.61	3.02	4.29	0.91
Aaa minus 10-year Treasury	1/31/1997	8/27/2021	**1.52**	0.43	0.80	1.205	1.545	1.805	2.17	0.93
	10/31/2008	8/27/2021	**1.65**	0.35	1.05	1.40	1.68	1.87	2.18	0.89
Baa minus federal funds rate	1/31/1997	8/27/2021	**3.91**	1.72	1.25	2.50	4.06	5.27	6.27	0.96
	10/31/2008	8/27/2021	**4.52**	1.46	2.20	3.29	4.62	5.29	7.57	0.94
Commercial paper minus federal funds rate	1/31/1997	8/27/2021	**0.09**	0.38	−0.31	−0.03	0.07	0.19	0.55	0.41
	10/31/2008	8/27/2021	**0.18**	0.33	0.00	0.05	0.10	0.23	0.53	0.26
Panel B: High-yield index and spreads, Jan. 1997 to Aug. 2021										
BofA US high-yield index effective yield	1/31/1997	8/31/2021	**8.62**	2.90	5.43	6.41	7.99	10.15	13.45	0.97
	10/31/2008	8/31/2021	**7.37**	2.99	4.42	5.85	6.51	7.85	14.02	0.92
BofA US high-yield minus 10-year Treasury	1/31/1997	8/31/2021	**5.07**	2.46	2.77	3.40	4.35	6.05	9.11	0.95
	10/31/2008	8/31/2021	**5.06**	2.78	2.85	3.50	4.23	5.47	10.58	0.92
BofA US high-yield minus 5-year Treasury	1/31/1997	8/31/2021	**5.64**	2.60	3.01	3.82	5.07	6.69	9.74	0.95
	10/31/2008	8/31/2021	**5.81**	2.90	3.46	4.00	5.10	6.48	11.71	0.92
BofA US high-yield minus 1-year Treasury	1/31/1997	8/31/2021	**6.43**	2.92	3.23	4.22	5.89	7.80	11.33	0.97
	10/31/2008	8/31/2021	**6.69**	3.12	3.89	4.40	6.02	7.51	13.58	0.93

The credit risk variables are defined as follows: (*a*) Baa minus 10-year Treasury, Moody's seasoned Baa corporate bond yield relative to yield on 10-year Treasury constant maturity; (*b*) Aaa minus 10-year Treasury, Moody's seasoned Aaa corporate bond yield relative to yield on 10-year Treasury constant maturity; (*c*) Baa minus federal funds rate, Moody's seasoned Baa corporate bond yield minus federal funds rate; (*d*) commercial paper minus federal funds rate, 3-month commercial paper rate minus federal funds rate; and (*e*) BofA high-yield index, ICE BofA US high-yield index effective yield. Each data series is from the US Fed. Reserve Bank St. Louis (2021).

Abbreviations: ACF_1, first-order autocorrelation; BofA, Bank of America; ICE, Intercontinental Exchange; SD, standard deviation.

Table 2 Index CDS spreads for investment-grade entities and returns of high-yield index reference entities

	Begin date	End date	Mean	SD	Percentiles 5th	25th	50th	75th	95th	ACF_1
Panel A: CDS spreads (bps) for index of investment-grade reference entities										
CDS spreads for IG Index	10/31/2008	8/31/2021	**84**	34	50	62	75	96	144	0.90
Panel B: Return (%, annualized) features of CDS price index for high-yield reference entities										
Return of HY Index	11/28/2008	8/27/2021	**2.80**	9.39	−51	−12	3	16	53	−0.13
Excess return of HY Index	11/28/2008	8/31/2021	**2.35**	9.41	−51	−12	2	15	53	−0.13

This table presents summary statistics for the following two Markit credit indexes: (*a*) The IG Index [Markit North American Investment Grade CDX Index (Bloomberg ticker symbol CDX.NA.IG)] is composed of the 125 most liquid North American entities with investment-grade credit ratings that trade in the CDS market; and (*b*) the HY Index [Markit North American High-Yield CDX Index (Bloomberg ticker symbol CDX.NA.HY)] is composed of 100 liquid North American entities with high-yield credit ratings that trade in the CDS market. We compute the CDS return as follows: $r_{\{t\to t+1\}} = \frac{1}{I_t}(I_{t+1} - I_t)$ and $r^{\text{excess return}}_{\{t\to t+1\}} = \frac{1}{I_t}(I_{t+1} - I_t) - r^{\text{risk-free}}_{\{t\to t+1\}}$, where I_t is the level of the CDS index at the end of month t. Both indices roll every 6 months, in March and September. Each data series is extracted from Bloomberg (2021).

Abbreviations: ACF_1, first-order autocorrelation; bps, basis points; CDS, credit default swaps; HY, high yield; IG, investment grade; SD, standard deviation.

Table 3 Credit default swap spreads for single-name entities

Moody's / S&P	Caterpillar A2 / A		IBM A2 / A−		General Electric Baa1 / BBB+		Best Buy A3 / BBB+		AIG Baa2 / BBB+	
Maturity	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask
1 year	3.5	8.9	5.1	10.5	16.4	30.9	8.2	18.7	11.6	15.6
3 years	11.8	17.5	15.5	21.3	34.4	51.0	21.7	31.8	29.3	36.8
5 years	26.9	31.6	30.0	34.8	69.1	74.1	39.4	45.0	53.1	58.3
10 years	45.7	71.1	53.9	70.5	102.4	135.5	76.6	96.2	92.8	107.3

Presented are some single-name credit default swap contracts written on the senior unsecured debt of North American obligors with XR (no restructuring) clauses. The bid and ask (in basis points) quoted on September 9, 2021, are obtained from Bloomberg (2021).

Table 3 illustrates the CDS spreads for some single-name entities. The CDS spreads are lower for firms with better credit ratings. Additionally, the bid-ask spreads are tighter for 5-year contracts. Akin to the slope of the Treasury yield curve, the slope of the CDS curve is upward sloping (on average).

Default process estimation draws inferences based on data partitioned by credit ratings and/or firm leverage. Relevant to these exercises is the distribution of firm credit ratings and leverage. To make empirical connections, **Table 4** presents data snapshots from 2006, 2011, and 2016. Compustat discontinued its coverage of the S&P credit ratings in 2017. We note that almost 70% of the firms in the Compustat and Center for Research in Security Prices (CRSP) merged database are unrated. Additionally, 12% of the firm universe is rated investment grade (BBB or higher). For this group, the mean leverage is 17% in 2016. On the other hand, 16% of the firms are rated below investment grade but higher than C. The mean leverage is 39% in 2016. Finally, a paucity of firms are rated at or below C. This group manifests a mean leverage of 62%. Default process

Table 4 US corporations: distribution of firm leverage by credit ratings in Compustat

	Panel A			Panel B			Panel C		
	Leverage^{MVE}			Number of firms			Fraction of firms (%)		
Rating	2006	2011	2016	2006	2011	2016	2006	2011	2016
AAA	0.10	0.04	0.12	9	5	2	0.21	0.14	0.06
AA	0.11	0.18	0.15	28	27	30	0.66	0.75	0.86
A	0.14	0.19	0.18	171	138	115	4.00	3.85	3.30
BBB	0.20	0.23	0.23	271	261	273	6.34	7.29	7.84
BB	0.26	0.32	0.33	351	276	323	8.21	7.71	9.27
B	0.37	0.49	0.46	279	262	245	6.53	7.32	7.03
C	0.33	0.79	0.62	10	6	17	0.23	0.17	0.49
Unrated	0.11	0.15	0.15	3,154	2,606	2,478	73.81	72.77	71.15

We take three yearly snapshots—2006, 2011, and 2016—of Compustat firm credit ratings and capital structure and provide the summary statistics. Among Compustat firms, we exclude utility firms (SIC codes 4900–4949) and financial firms (SIC codes 6000–6999). The credit rating is the Standard & Poor's Long-Term Domestic Issuer Credit Rating (Compustat data item 280 SPLTICRM) in the last quarter of the year. This code was discontinued in 2017. The leverage ratio is defined as $\text{Leverage}^{MVE} = \frac{\text{Book value of debt}}{\text{Book value of debt} + \text{Market value of equity}}$. The book value of debt is the sum of current liabilities (DLCQ) and long-term debt (DLTTQ). Both the book value of debt and the market value of equity are computed in the last quarter of the year. Panel *a* shows the average leverage ratio of firms in each of the seven categories of credit rating together with firms that have no credit rating. Panel *b* is the number of firms in each category, and Panel *c* is the fraction of firms in each category.
Abbreviation: MVE, market value of equity.

estimation and testing frameworks have adapted to such data realities while also tackling the infrequent nature of default outcomes.

1.2. Empirical and Theoretical Questions That Merit Further Consideration

The following empirical and/or theoretical questions continue to draw interest.

1. What is the nature of variables that describe the spread on firms' bonds and the Treasury? Does this spread admit an easy-to-understand decoupling in terms of default risks, recovery risks, liquidity, or differential tax effects? What are the determinants of return correlation between company stock and company credit spreads?
2. How should the default boundary be specified in structural models of default? Should it be constant, deterministic, or vary stochastically with firm leverage? What is an appropriate metric for understanding which notion of default boundary is supported in the data?
3. Can one empirically distinguish which structural model of default risk best describes the data on corporate bond spreads and CDS spreads? Is there a consensus?
4. What is an appropriate criterion to employ if one were to calibrate structural models? The literature has implemented methodologies that minimize a chosen loss function that relies on information from historic expected default and recovery rates by credit ratings and maturity as well as credit spreads.[2]
5. How should reduced-form models be specified to achieve empirically viable representations of credit spreads? What factor structure of the default intensity process is supported by the data? Which notion of recovery in default (i.e., fraction of face value, fraction of equivalent Treasury, fraction of market debt value, or that recovery is a martingale) receives support in the data?
6. How useful are reduced-form models for capturing the variation in the CDS spreads across firms and over time? How useful are latent variables in these exercises? How does one identify variations in default clustering and quantify its impact on the risk premium of default-sensitive securities?
7. What is the relative contribution of asset return volatility and jumps in explaining the data points on the term structure of bond market credit spreads and CDS spreads? Overall, how do reduced-form models fare empirically relative to structural models in terms of pricing and hedging performance (in-sample explanatory power and out-of-sample forecasting ability) of credit-sensitive securities?

Having outlined the basic data facts in the market for credit risks and the research questions we asked, we move on to describe the empirical findings on model implementations. Our focus is on delineating the implications derived from default risk estimation.

2. FINDINGS ON DEFAULT RISK FROM DATA AND MODEL IMPLEMENTATIONS

The task of quantifying default risk of a single name, or multiname, is demanding on theoretical and empirical grounds. The following is what we know from extant research.

[2]Why do structural models underestimate credit spreads for investment-grade firms? This aspect of model shortcoming is associated with the credit spread puzzle. Is it because investment-grade firms embed relatively low leverage and low asset return volatility and, thus, manifest high distance to default?

2.1. General Empirical Findings on Credit Spreads, CDS Spreads, and Default Probabilities

The empirical findings grounded in credit spreads, risk-neutral default probabilities, and CDS spreads provide a background for building credit risk models. Specific applications include managing risk exposures of credit-sensitive bond portfolios.

2.1.1. Finding 1: credit spread determinants. Exploiting Moody's corporate bond yield data, the time series evidence from Longstaff & Schwartz (1995, tables 2 and 3) shows that changes in credit spreads for utilities, industrials, and railroads are negatively related to changes in interest rates. Elton et al. (2001) explore properties of corporate and government bonds and conclude that expected default accounts for a small portion of the credit risk premium. In contrast, their exercises corroborate a role for state taxes and systematic factors.

Collin-Dufresne, Goldstein & Martin (2001, tables 2 and 3) find that variables surrogating default risk inadequately explain variation in individual credit spreads. Variables other than the asset value of the firm appear relevant, which has implications for hedging and managing credit risks.

2.1.2. Finding 2: model-free estimate of risk-neutral default probability. How useful are model-free put-option-based approaches in characterizing default probabilities for individual names? Carr & Wu (2011, equation 4) show the link between deep out-of-the-money American put options on a company stock and a credit insurance contract on the same company bond. Specifically, they develop an option-spread position that replicates a pure-credit contract. In their table 2, they implement their procedure for 121 firms and obtain a mean risk-neutral default probability of 0.111.

2.1.3. Finding 3: cross-sectional variation in CDS spreads. Bai & Wu (2016) examine whether firm fundamentals can explain cross-sectional variations in CDS spreads. They consider CDS valuation by combining the Merton distance-to-default measure with economically motivated firm fundamentals. Their methodology is able to describe 77% of the cross-sectional variation in CDS spreads.

2.1.4. Finding 4: persistent deviations of the CDS bond basis from zero. The CDS bond basis—which reflects the difference between the CDS spreads and corporate yield spreads—is nonzero and varies with economic conditions (Bai & Collin-Dufresne 2019). In theory, the CDS spreads and the yield spreads should not diverge materially. The cross-sectional and time series evidence from Bai & Collin-Dufresne (2019) attributes this divergence to trading liquidity, funding cost, counterparty risk, and collateral quality.

2.1.5. Finding 5: extracting default probabilities from CDS spreads. Conrad, Dittmar & Hameed (2020) develop the link between individual equity options and CDS. They extract risk-neutral moments from options and then compute the default probabilities. These probabilities correlate with estimates of default probabilities extracted from CDS spreads. They show that option-implied default probabilities rise during adverse economic conditions and are higher for firms with more speculative debt.[3]

[3]To implement the CreditGrades model, Cao, Yu & Zhong (2011) build a data set matched by equity options and CDS. Their investigation finds that volatility extracted from equity options is more useful than historic volatility in fitting CDS spreads.

2.2. Findings Related to Structural Models of Default Risk

Structural models connect to the capital structure of a firm and, in particular, the level of the assets in comparison to liabilities. Default is the most significant credit event for a firm. Merton (1974) introduced an approach in which assets follow a geometric Brownian motion and default happens only at debt maturity.

2.2.1. Finding 6: estimation of structural models. Eom, Helwege & Huang (2004) test five structural models of corporate bond pricing, namely, those by Merton (1974), Geske (1977), Longstaff & Schwartz (1995), Leland & Toft (1996), and Collin-Dufresne & Goldstein (2001). They implement some of these models by assuming recovery of coupon. The predominant finding is that the structural model by Merton (1974) generates spreads that are too low, while other structural models (with the exception of that in Geske 1977) predict spreads that are too high on average, compared with the empirical counterparts. Their study highlights a credit spread puzzle, that is, the failure of structural models to explain empirically observed credit spreads.

2.2.2. Finding 7: link of option-implied jump risk premiums and credit spreads. In the model by Cremers, Driessen & Maenhout (2008), the value of firm assets is impacted by both a diffusive component and a jump component. They use prices and returns of equity index and individual equity options to infer the jump parameters. The key result is that incorporating jump risk premium brings predicted credit spread levels close to observed levels.

2.2.3. Finding 8: default probability and distance to default. Bharath & Shumway (2008) show that, adjusting for agency ratings and bond characteristics, default probabilities constructed from CDS and bond yield spreads weakly correlate with the distance-to-default measure. However, they find that distance to default has empirical content for forecasting defaults. Chava & Jarrow (2004) show that bankruptcy prediction is improved using monthly data and emphasize the merits of incorporating industry effects in hazard rate estimation. Duan, Sun & Wang (2012) consider a reduced-form approach based on a forward intensity construction to estimate default probabilities for different forecasting horizons. Their assessment finds that firm-specific leverage, liquidity, profitability, and volatility influence default probabilities.

2.2.4. Finding 9: CDS spreads and structural model with stochastic asset volatility and jumps. Zhang, Zhou & Zhu (2009) calibrate a model with volatility and jumps. They show that their model can help to match credit spreads after controlling for historic default rates. Using high-frequency equity prices, they further show that volatility (respectively, jump) risk predicts 48% (respectively, 19%) of the variation in CDS spreads. Controlling for credit ratings, macroeconomic conditions, and balance sheet information, their approach can describe 73% of the total variation in the CDS spreads.

2.2.5. Finding 10: isolating the size of credit risk. Huang & Huang (2012) ask: How much of the corporate-Treasury spread can be ascribed to credit risk? They empirically examine structural models that incorporate stochastic interest rates, endogenous default, stationary leverage ratios, and strategic default. Additionally, they model time-varying asset-risk premiums and jumps in the firm value process. Their calibrations indicate that credit risk accounts for a small part of the observed yield spreads for investment-grade bonds. On the other hand, credit risk is of higher-order importance for high-yield bonds.

2.2.6. Finding 11: calibrating structural models to historic default rates. Feldhutter & Schaefer (2018) calibrate structural models to historic default rates using the book values of debt. In this regard, they find that the Black & Cox (1976) model can replicate the size of investment-grade spreads. However, generated model spreads for speculative-grade debt are low. Huang & Huang (2012) also calibrate structural models to historic default rates. Bai, Goldstein & Yang (2020) provide an alternative view of Feldhutter & Schaefer (2018) by calibrating the model using the market value of debt and using credit spreads to identify the default boundary. Based on a model with jumps, they provide further evidence for a credit spread puzzle for investment-grade bonds but not for high-yield bonds.

2.2.7. Finding 12: credit risk varies over business conditions. The impact of default and liquidity is examined through the lens of a structural model in the work by Chen et al. (2018). The essential new ingredients are debt rollover and bond price–dependent holding costs. Their investigation shows that the model matches average default rates, credit spreads, and CDS spreads, both in the cross section and over time. They conclude that default-liquidity interactions can account for 10–24% of the level of credit spreads and 16–46% of the changes in spreads over the business cycles.

2.2.8. Finding 13: pseudorisky bond representation and credit spread puzzle. Culp, Nozawa & Veronesi (2018) construct a portfolio of long Treasury and short put options to synthesize the price of a pseudorisky bond. They show that this construction mimics the properties of traded corporate bond prices and manifests a credit spread puzzle that is more severe at short horizons. In their framework, the risk premium for tail and idiosyncratic asset risks helps to reconcile the empirical credit spreads.

2.2.9. Finding 14: stochastic asset risk and volatility risk. Du, Elkamhi & Ericsson (2019) build a credit risk model with priced stochastic asset risk that is able to fit medium- to long-term spreads. They use a parsimonious jump structure to reconcile the behavior of short-term spreads. Estimating the model using firm-level data, they identify significant risk premiums for variance and uncertainty about asset risk.

2.2.10. Finding 15: properties of credit-implied volatility. Kelly, Manzo & Palhares (2019) construct a credit-implied volatility (CIV) surface from the firm-by-maturity panel of CDS spreads. The principal component analysis of leverage-sorted CIV portfolios implies a three-factor structure. They show that the CIV surface exhibits a moneyness smirk in leverage, and the cross section of CDS spreads can be explained by exposures to CIV surface shocks. Their work implements a model of asset growth, which is driven by diffusive volatility and time-varying downside tail risks.

2.2.11. Finding 16: specification testing and estimating structural models. Huang, Shi & Zhou (2020) propose a generalized method of moments (GMM)-based methodology to estimate structural models. Using data on equity volatility and the term structure of single-name CDS spreads, they conclude that (*a*) the Merton (1974) model is rejected, (*b*) other diffusion-based models with flat default boundaries are rejected, and (*c*) models with jumps and stationary leverage ratios improve the fit of CDS spreads.

2.2.12. Finding 17: structural models admitting stochastic debt. Feldhutter & Schaefer (2020) consider a model of stochastic debt with two features. First, leverage is mean-reverting.

Second, debt issuance is stochastic and negatively correlated with changes in the firm's asset value. They estimate models using a loss function that entails minimizing absolute errors between historic defaults and model-implied default probabilities. Using corporate bond yield data, they find that the model produces higher spreads for more credit-worthy firms and lower spreads for riskier firms.

2.3. Findings Related to Reduced-Form Models of Defaultable Debt

Unlike structural models, reduced-form models rely on the notion that the default event is a surprise. Reduced-form models are flexible in specifying the nature of the default intensity rate. Through this approach, one can parsimoniously incorporate the multitude of influences that may determine default probabilities [e.g., macroeconomic conditions (the VIX volatility index), membership to an industry, or firm-specific attributes].

2.3.1. Finding 18: latent variable modeling of default intensity in the recovery of market value model. Duffee (1999) examines the implications of the model from Duffie & Singleton (1999). His empirical design relies on the Kalman filtering estimation. The essential finding is that the considered credit risk models produce term structures of credit spreads that are more steeply sloped for lower-quality firms than for higher-quality firms. His default process parameterizations utilize noncallable bonds.

2.3.2. Finding 19: default risk and recovery are inversely correlated. The model estimation from Jarrow (2001) implies that firms with higher default risks are associated with lower recovery in default. This negative relationship depends on the prevailing state of the macroeconomy. Allowing for nonconstant recovery in empirical implementations can improve characterization of default risks.[4]

2.3.3. Finding 20: role for firm-specific jumps, liquidity, and taxes. The empirical approach from Driessen (2005) provides a decomposition of corporate bond returns into components related to risk premiums associated with firm-specific jumps, liquidity, tax effects, and a risk premium on market-wide credit spread movements. Each firm's default intensity is specified as a function of latent common factors and a latent firm-specific factor. The model parameters are estimated using the Kalman filtering approach.

2.3.4. Finding 21: role of observable covariates in a model of default intensity. The focus of the work by Bakshi, Madan & Zhang (2006) is on studying the pricing and hedging implications of credit risk models with observable covariates. They consider the following firm-specific variables: (*a*) leverage, (*b*) book-to-market value, (*c*) profitability, (*d*) equity volatility, and (*e*) distance to default. They find that interest rate risk is of first-order importance for explaining variations in single-name defaultable coupon bonds and credit spreads, while a credit risk model that takes leverage into consideration reduces absolute yield mispricing. Overall, they empirically assess a family of default risk models driven by a two-factor structure for the short-interest

[4]Altman et al. (2005) show that realized recovery rates are inversely proportional to realized default rates. They further document that recoveries depend on industry and firm value. The study by Acharya, Bharath & Srinivasan (2007) uses data on defaulted firms in the United States from 1982 to 1999 and shows that creditors of defaulted firms recover significantly lower amounts when the industry of defaulted firms is encountering distress.

rate and an additional third factor for firm-specific distress, using the reduced-form modeling framework.

2.3.5. Finding 22: collateralized debt obligations and default clustering.

Longstaff & Rajan (2008) empirically study how the markets perceive corporate default clustering. They propose a model in which three types of Poisson events generate portfolio credit losses.[5] On average, 65% of the CDX spread is due to firm-specific default risk, 27% to clustered sector default risk, and 8% to catastrophic default risk. The credit model explains a predominant portion of time series and cross-sectional variation in CDX index tranche prices. Seo & Wachter (2018) study a model with time-varying probability of disasters. Importantly, their calibration exercise reveals that the model explains the spreads on CDX tranches.

2.3.6. Finding 23: observable covariates are useful for predicting default.

Doshi et al. (2013) assume that default intensity is a quadratic function of several covariates. They show that observable covariates are relevant for explaining credit spreads, CDS spreads, and defaults.

3. STRUCTURAL MODELS OF DEFAULT RISK AND ESTIMATION

The structural modeling framework provides an explanation for default that is tied to the value of assets in comparison to the liabilities. Instrumental to the empirical work on default risk is the foundational model of Merton (1974). The model has a single source of uncertainty and has three building blocks.

Value of assets, A_t. The value of assets reflects the present value of the future free cash flows produced by the firm's assets. Let **c** be the payout ratio of the firm. The dynamics of asset values is (with $A_0 > 0$)

$$\underbrace{\frac{dA_t}{A_t}}_{\text{Percentage change in the value of assets}} = \begin{cases} (\mu_A - \mathbf{c})\,dt + \sigma_A\, d\mathbb{W}_t^{\mathbb{P}} & \text{[under the real-world } (\mathbb{P}) \text{ measure]} \\ (r - \mathbf{c})\,dt + \sigma_A\, d\mathbb{W}_t^{\mathbb{Q}} & \text{[under the risk-neutral } (\mathbb{Q}) \text{ measure],} \end{cases} \quad 1.$$

where $\mathbb{W}_t^{\mathbb{P}}$ (respectively, $\mathbb{W}_t^{\mathbb{Q}}$) is the standard Brownian motion under the $\mathbb{P}$ (respectively, $\mathbb{Q}$) measure. In Equation 1, $\mu_A - \mathbf{c}$ is the payout ratio adjusted drift of A_t under the $\mathbb{P}$ measure, and r is the constant interest rate.

Asset risk, $\sigma_A = \sqrt{\text{var}_t^{\mathbb{P}}(\frac{dA_t}{A_t})/dt}$. Asset risk captures uncertainty about the asset value and is also a measure of the firm's business and industry risk.

Leverage. In this model, the firm is financed by both equity and debt. The empirical measure of leverage is the book value of liabilities relative to the market value of assets.

Let K be the notional value of debt due in T periods. If $A_T < K$, the firm defaults. In this case, the random default time can be depicted. $\mathbb{1}_{\{y\}}$ is an indicator variable that takes a value of 1 if the event y is true and is zero otherwise, as follows:

$$\mathcal{T}_{\text{def}} \equiv \underbrace{T\,\mathbb{1}_{\{A_T<K\}}}_{\text{Default only at } T} + \infty\,\mathbb{1}_{\{A_T \geq K\}}. \quad 2.$$

[5] Multiname credit derivatives have also been analyzed by Giesecke, Goldberg & Ding (2011). In their setting, the value of a multiname derivative is contingent on the distribution of portfolio loss at multiple horizons. Azizpour, Giesecke & Schwenkler (2018) provide an investigation of the sources of default clustering.

In the event of default, the bond holder receives $A_T < K$. Hence, the recovery rate is $\frac{A_T}{K}$ and the loss rate given default is $\frac{1}{K}(K - A_T)$.

Probability of default. The default probability can be computed as follows:

$$\text{Probability of Default}_t = \text{Prob}_t^{\mathbb{P}}[A_T < K] = \mathcal{N}[-\text{DD}_t], \tag{3}$$

where $\mathcal{N}[\bullet]$ is the cumulative distribution function of a standard normal distribution. We define the distance to default as follows:

$$\text{Distance to Default}_t \equiv \text{DD}_t = \frac{\log(\frac{A_t}{K}) + (\mu_A - \mathbf{c} - \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}. \tag{4}$$

Price of risky bond and credit spreads. The price of the zero-coupon bond from Merton (1974) is

$$\underbrace{P_t^T}_{\text{Risky bond price}} = e^{-r(T-t)}\,\mathbb{E}_t^{\mathbb{Q}}(K\mathbb{1}_{\{\mathcal{T}_{\text{def}}>T\}} + A_{\mathcal{T}_{\text{def}}}\mathbb{1}_{\{\mathcal{T}_{\text{def}}<T\}}) \tag{5}$$

$$= e^{-r(T-t)}\,K\mathcal{N}[d_2] + A_t\mathcal{N}[-d_1], \tag{6}$$

where $d_1 \equiv \frac{\log(\frac{A_t}{K}) + (r-\mathbf{c}+\frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}$ and $d_2 \equiv \frac{\log(\frac{A_t}{K}) + (r-\mathbf{c}-\frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}$.

The yield-to-maturity, denoted by Y_t^T, of a corporate bond solves $-P_t^T e^{r(T-t)} + e^{-(Y_t^T-r)(T-t)}K = 0$. Taking logs, one obtains

$$\underbrace{Y_t^T - r}_{\text{Credit spread}} = -\frac{1}{T-t}\log\left(\mathcal{N}[d_2] + \frac{A_t}{K}e^{r(T-t)}\mathcal{N}[-d_1]\right). \tag{7}$$

The value of A_t is presumably unknown. With the estimated values of A_t and σ_A (and input for $\mathbf{c}$), the credit spread implied by the Merton model can be computed and compared to the one in the data for any T.

CDS spreads. In a continuous-time setting, the CDS spread solves

$$0 = \underbrace{-\mathbb{s}_t^T\,K\,\mathbb{E}_t^{\mathbb{Q}}\int_t^T e^{-r(u-t)}\mathbb{1}_{\{\mathcal{T}_{\text{def}}>u\}}du}_{\text{Premium leg}} + \underbrace{K\,\mathbb{E}_t^{\mathbb{Q}}\left(e^{-r(\mathcal{T}_{\text{def}}-t)}\left\{\frac{K - A_{\mathcal{T}_{\text{def}}}}{K}\right\}\mathbb{1}_{\{\mathcal{T}_{\text{def}}<T\}}\right)}_{\text{Protection leg}} \tag{8}$$

and, hence,

$$\underbrace{\mathbb{s}_t^T}_{\substack{\text{CDS spread}\\\text{(Merton)}}} = \frac{e^{-r(T-t)}\,\mathcal{N}[-d_2] - \dfrac{A_t}{K}\mathcal{N}[-d_1]}{(T-t)\{r(T-t)\}^{-1}(1-e^{-r(T-t)})} \approx \underbrace{Y_t^T - r}_{\text{Yield spread}}. \tag{9}$$

3.1. Estimating the Default Probability and Model Credit Spreads

The default probability is parameterized by $(\mu_A, \sigma_A, \frac{A_t}{K})$. The calculation of payout ratio ($\mathbf{c}$) is standard (e.g., Feldhutter & Schaefer 2018, appendix B).

One strand of the literature has suggested the following estimation procedure. Simultaneously solve Equations 10 and 11 to obtain (A_t, σ_A), as follows:

$$\underbrace{S_t}_{\text{Equity price}} = A_t\,\mathcal{N}\Bigg[\underbrace{\frac{\log(\frac{A_t}{K}) + (r - \mathbf{c} + \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}}_{\equiv d_1}\Bigg] - K\,e^{-r(T-t)}\,\mathcal{N}\Bigg[\underbrace{\frac{\log(\frac{A_t}{K}) + (r - \mathbf{c} - \frac{1}{2}\sigma_A^2)(T-t)}{\sigma_A\sqrt{T-t}}}_{\equiv d_2}\Bigg], \qquad 10.$$

$$\underbrace{\sigma_S}_{\text{Equity volatility}} = \frac{A_t}{S_t}\sigma_A\,\underbrace{\mathcal{N}[d_1]}_{\frac{\partial S_t}{\partial A_t}}, \qquad 11.$$

where $S_t = e^{-r(T-t)}\mathbb{E}_t^{\mathbb{Q}}[\max(A_T - K, 0)]$. Specifically, let $\mathcal{D}_1[A_t, \sigma_A] \equiv -S_t + A_t\,\mathcal{N}[d_1] - K\,e^{-r(T-t)}\,\mathcal{N}[d_2]$ and $\mathcal{D}_2[A_t, \sigma_A] \equiv -\sigma_S + \frac{A_t}{S_t}\sigma_A\,\mathcal{N}[d_1]$. The next step is to minimize the criterion function $\{\mathcal{D}_1[A_t, \sigma_A]\}^2 + \{\mathcal{D}_2[A_t, \sigma_A]\}^2$, which recovers A_t and σ_A [e.g., the Kealhofer, McQuown and Vasicek (KMV) approach from Crosbie & Bohn 2003].[6]

Eom, Helwege & Huang (2004) construct the data on asset values from Compustat and obtain μ_A as the average growth rate of assets. Exploiting Equation 11, they infer σ_A from historic equity returns [quadratic variation or GARCH(1,1) specification].

In addition, they work with bond-implied volatility by equating credit spreads to those implied from the model of Merton (1974). Finally, Duffie (2011, chapter 4) suggests estimating σ_A as the standard deviation of $\log(\frac{A_{t+1}}{A_t})$.

With the distance to default computed, one can employ the formula in Equation 3 to compute the default probability. In practice, the distance-to-default value is aligned with data on historic default frequencies. This is accomplished through the likelihood of default and the distance-to-default table [e.g., as made available by commercial providers (Moody's or S&P)].

3.2. Estimation of Alternative Structural Models of Default

The default boundary is an input that determines the level of the assets that causes the firm to default when this value breaches the stipulated default boundary level. These models identify insolvency constraints and recognize the compatibility of default prior to debt maturity.

3.2.1. Collin-Dufresne & Goldstein (2001). They consider the following model:

$$\underbrace{\frac{dA_t}{A_t}}_{\text{Percentage change in the value of assets}} = \begin{cases} (\mu_A - \mathbf{c})\,dt + \sigma_A\,d\mathbb{W}_t^{\mathbb{P}} & \text{[under the real-world } (\mathbb{P}) \text{ measure]} \\ (r - \mathbf{c})\,dt + \sigma_A\,d\mathbb{W}_t^{\mathbb{Q}} & \text{[under the risk-neutral } (\mathbb{Q}) \text{ measure].} \end{cases}$$

[6]Vassalou & Xing (2004) and Bharath & Shumway (2008) follow an iterative procedure. Duan, Gauthier & Simonato (2005) examine the algorithm for estimating the unobserved A_t and the unknown parameters. They show the equivalence between the KMV estimates and the maximum likelihood estimates.

Additionally, they specify the evolution of the liabilities of the firm as follows (under the constant interest rate specification):

$$\textit{Liability dynamics:} \quad d\log(K_t) = \kappa \underbrace{\left(\log\left(\frac{A_t}{K_t}\right) - \nu\right)}_{\text{Firm-specific deviation}} dt, \tag{12.}$$

where $\kappa > 0$ is the speed of adjustment. Let $L_t = \frac{K_t}{A_t}$ be the firm leverage. Hence, $d\log(L_t) = d\log(K_t) - d\log(A_t)$. Ito's lemma implies that

$$\underbrace{d\log(L_t)}_{\text{Leverage dynamics}} = \kappa(\mu_L^{\mathbb{P}} - \log(L_t))\,dt - \sigma_A \underbrace{d\mathbb{W}_t^{\mathbb{P}}}_{\text{under } \mathbb{P}}, \text{ where } \mu_L^{\mathbb{P}} \equiv \frac{1}{\kappa}\left\{-\mu_A + \mathbf{c} + \frac{1}{2}\sigma_A^2\right\} - \nu, \tag{13.}$$

and

$$d\log(L_t) = \kappa(\mu_L^{\mathbb{Q}} - \log(L_t))\,dt - \sigma_A \underbrace{d\mathbb{W}_t^{\mathbb{Q}}}_{\text{under } \mathbb{Q}}, \text{ where } \mu_L^{\mathbb{Q}} \equiv \frac{1}{\kappa}\left\{-r + \mathbf{c} + \frac{1}{2}\sigma_A^2\right\} - \nu. \tag{14.}$$

Define $\mathcal{T}_{\text{def}}$ as the random time at which $\log(L_t)$ hits zero for the first time. Suppose the recovery convention is constant $\mathbb{r}$ of the discounted unit face value.

Then the price of the (unit face) risky bond from Collin-Dufresne & Goldstein (2001) is the following:

$$P_t^T = e^{-r(T-t)}\mathbb{E}_t^{\mathbb{Q}}(\mathbb{1}_{\{\mathcal{T}_{\text{def}} > T\}} + \mathbb{r}\,\mathbb{1}_{\{\mathcal{T}_{\text{def}} < T\}}) = e^{-r(T-t)}(1 - \{1 - \mathbb{r}\}\underbrace{\mathbb{q}[\log(L_t), T]}_{\substack{\text{default probability} \\ \text{(risk-neutral)}}}). \tag{15.}$$

The solution for $\mathbb{q}[\log(L_t), T]$ relies on first-passage density techniques and is displayed by Collin-Dufresne & Goldstein (2001, equations 17–24). The yield spread is $-\frac{1}{T-t}\log(1 - \{1 - \mathbb{r}\}\mathbb{q}[\log(L_t), T])$.

Implementation of this model involves three extra parameters: $(\kappa, \mu_L^{\mathbb{Q}}, \nu)$. These are the parameters of the Ornstein-Uhlenbeck process for $\log(L_t)$. Given the data on leverage L_t, one can estimate κ and $\mu_L^{\mathbb{P}}$ using maximum likelihood, method of moments (e.g., Nowman 1997), or regression.

3.2.2. Black & Cox (1976) and its variants.

Feldhutter & Schaefer (2018) consider a version in which default occurs when the value of assets falls below some constant $d^{\bullet}$ times the value of liabilities (K_t).

From such sources as Chen, Collin-Dufresne & Goldstein (2009), Bjork (2004, chapter 18), and Bao (2009), one may derive the probability of default as follows:

$$\underbrace{\text{Prob}_t^{\mathbb{P}}[A_T < d^{\bullet} K_t]}_{\text{Default probability}} = \mathcal{N}\left[\frac{-\log\left(\frac{A_t}{d^{\bullet} K_t}\right) - \left(\mu_A - \mathbf{c} - \frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{T-t}}\right]$$
$$+ \exp\left(\frac{-2\left\{\log\left(\frac{A_t}{d^{\bullet} K_t}\right)\right\}\left(\mu_A - \mathbf{c} - \frac{1}{2}\sigma_A^2\right)}{\sigma_A^2}\right)\mathcal{N}\left[\frac{-\log\left(\frac{A_t}{d^{\bullet} K_t}\right) + \left(\mu_A - \mathbf{c} - \frac{1}{2}\sigma_A^2\right)(T-t)}{\sigma_A\sqrt{T-t}}\right]. \tag{16.}$$

With data on Moody's default rate for seven rating categories and maturities from 1 to 20 years (7×20 matrix of data), Feldhutter & Schaefer (2018, section 3.2) estimate $d^{\bullet}$ using the following loss function:

$$\min_{d^{\bullet}} \sum_{\substack{7 \text{ credit rating} \\ \text{categories}}} \sum_{T=1}^{20} \frac{1}{T} \left| \text{model default probability} - \text{Moody's default rate} \right|. \qquad 17.$$

This procedure yields an estimate of $d^{\bullet} = 0.8944$. Feldhutter & Schaefer (2018, section 3.2) obtain the grand average of default rate across all firms and all years within a credit rating and maturity category. The model default probability is computed according to Equation 16.[7]

Huang, Shi & Zhou (2020) organize CDS spreads of differing expirations (ranging between 1 and 10 years) of a given reference entity at time t. For each model parameterized by Θ, they consider the deviation between the model CDS spread and the actual CDS spread, specifically, $f_t^j[\Theta] = \widetilde{\mathrm{CDS}}[t, T_j; \Theta] - \mathrm{CDS}[t, T_j]$. Stacking up across different contract maturities, let $\frac{1}{\mathbb{T}} \sum_{t=1}^{\mathbb{T}} \mathbf{f}_t[\Theta] = \mathbf{g}_{\mathbb{T}}[\Theta]$. The GMM criterion function is $\hat{\Theta} = \arg\min(\mathbf{g}_T[\Theta])' \, \Omega_T \, \mathbf{g}_T[\Theta]$, where Ω_T is a weighting matrix. They apply this procedure to five different models. Models with no closed-form solution for CDS spreads (i.e., Longstaff & Schwartz 1995; Collin-Dufresne & Goldstein 2001; and asset dynamics with jumps) are solved numerically.

Kelly, Manzo & Palhares (2019) extend the asset dynamics from Merton (1974) to include stochastic variance, jumps in asset values, and random intensity of jumps. The random intensity of jumps is driven by two latent factors, one of which is variance. The model is estimated using the unscented Kalman filter (Kelly, Manzo & Palhares 2019, appendix E).

3.2.3. Summary.

Structural credit pricing models are based on modeling the stochastic evolution of the balance sheet of the issuer. The default event coincides with the issuer not able to service its debt obligations.

When the firm asset value is a diffusion, default is modeled as the first time the firm's value hit a prespecified boundary (the default boundary could be endogenous as in Geske 1977). Because of the continuity of the process used, the time of default is a predictable stopping time. The models of Merton (1974), Black & Cox (1976), Longstaff & Schwartz (1995), and Briys & de Varenne (1997) are in this class. Such models typically produce small spreads at short maturities, whereas empirically, the credit spreads are positive even for short-maturity investment-grade bonds. One may remedy this shortcoming by introducing jumps into the asset process. Many such approaches with jumps have proceeded along the lines of, among others, Zhou (2001), Kou & Wang (2003), Zhang, Zhou & Zhu (2009), Carr & Wu (2010), and Kelly, Manzo & Palhares (2019).[8]

4. REDUCED-FORM FRAMEWORK FOR MODELING DEFAULTABLE DEBT AND ESTIMATION

In a reduced-form intensity-based modeling framework, the time of default is modeled as the time of the first jump of a Poisson process with random intensity. These models were developed by

[7]Others assume that the value of liabilities is growing deterministically, at constant γ, according to $K_T = e^{\gamma(T-t)} K_t$. Then, Equation 16 must be modified by setting $d^{\bullet} = 1$ and replacing $(\mu_A - \mathbf{c} - \frac{1}{2}\sigma_A^2)$ with $(\mu_A - \mathbf{c} - \frac{1}{2}\sigma_A^2 - \gamma)$. Such models are implemented by Brockman & Turtle (2003), Eom, Helwege & Huang (2004), Hull, Nelken & White (2005), Ericsson & Renault (2006), Bai, Goldstein & Yang (2020), and Feldhutter & Schaefer (2020).

[8]Each of the structural models has a set of parameters to be estimated. Parameters related to firm value and capital structure include the initial levels of debt and assets, the payout parameter, asset return volatility, the speed of a mean-reverting leverage process, and the parameters that characterize the target leverage ratio. In addition, implementation of the models requires estimates of parameters that define the default-free term structure as well as parameters related to bond characteristics.

Jarrow & Turnbull (1995), Madan & Unal (1998), Lando (1998), and Duffie & Singleton (1999). Complementary supportive treatments are from Jarrow, Lando & Turnbull (1997), Duffie & Garleanu (2001), Duffie & Lando (2001), Bakshi, Madan & Zhang (2001), Longstaff, Mithal & Neis (2005), Carr & Linetsky (2006), Bakshi, Madan & Zhang (2006), Linetsky (2006), and Longstaff & Rajan (2008).

The intensity models are allied to a random time $\mathcal{T}_{\text{def}}$ at which default occurs. Let $\mathbb{E}_t^{\mathbb{Q}}(\bullet) = \mathbb{E}^{\mathbb{Q}}(\bullet | \mathcal{G}_t)$ be expectation, under the risk-neutral probability measure $\mathbb{Q}$, conditional on a Brownian subfiltration $(\mathcal{G}_t, 0 \leq t \leq T)$ of the information filtration $(\Im_t, 0 \leq t \leq T)$ of a probability space $(\Omega, \Im, \mathbb{P})$. The filtration $\Im_t$ extends $\mathcal{G}_t$ by the knowledge of the default time process.

In this section, we review a default intensity-based credit risk framework to jointly model default risk and recovery payout. This framework allows for a variety of specifications of recovery in default. Flexibility does occur in models of dynamic interactions between recovery, default intensity, and spot interest rates.

4.1. Defaultable Coupon Bonds in Intensity-Based Models

With a focus on empirical estimation and implementation of credit risk models, the price of a coupon paying defaultable bond can be characterized as follows:

$$\underbrace{P_t^T}_{\substack{\text{Defaultable coupon} \\ \text{bond price}}} = \overbrace{\mathbb{E}_t^{\mathbb{Q}}\left(\int_t^T \exp\left(-\int_t^u (r_s + \lambda_s)\, ds\right) c_u\, du\right)}^{\text{Expected present value of coupon annuity}} + \overbrace{\mathbb{E}_t^{\mathbb{Q}}\left(\exp\left(-\int_t^T (r_s + \lambda_s)\, ds\right)\right) K}^{\text{Expected present value of promised face}}$$
$$+ \underbrace{\mathbb{E}_t^{\mathbb{Q}}\left(\int_t^T \exp\left(-\int_t^u (r_s + \lambda_s)\, ds\right) \mathfrak{R}_u\, \lambda_u\, du\right)}_{\text{Expected present value of probability-adjusted recovery amount}}, \qquad 18.$$

where K is the promised face value of the defaultable bond maturing at date T and $\{c_u : u > t\}$ is the promised continuous coupon payment.

Additionally, λ_t is the default intensity rate (of the intensity function of default time), r_t is the spot interest rate, and $\mathfrak{R}_t$ is the recovery if default were to occur. Intuitively, the default intensity rate reflects the conditional risk-neutral probability of default in a small time interval $[t, t + dt]$, given no prior default.

Typically, the unpaid coupons are not recovered in default. Equation 18 suggests that the value of a defaultable coupon bond is the expected discounted value of the underlying payoff, when the discounting factor is increased to $\{r_t + \lambda_t\}$. When recovery payout is positive, the defaultable coupon bond price, P_t^T, is additive in three components.

The first conditional expectation captures the receipt of bond coupons prior to default, while the second term is due to the promised face value in the absence of default. Finally, the last integral determines the value of the recovery payout if the firm defaults. This payoff represents recovery with probability $\lambda_u\, du$ in the no-prior-default time domain. We assume that $\int_t^T \lambda_s\, ds < \infty$.

The intensity-based modeling is empirically useful, as $(\lambda_u, \mathfrak{R}_u)$ can be functions of the state of the economy while capturing firm-specific information. For example, λ_u can exhibit dependencies on systematic as well as firm-specific variables. Such specifications are pursued in, among others, Duffee (1999), Driessen (2005), Longstaff, Mithal & Neis (2005), Bakshi, Madan & Zhang (2006), Duffie, Saita & Wang (2007), Das & Hanouna (2009), Guo, Jarrow & Zeng (2009), and Duan, Sun & Wang (2012).

4.2. Recovery Conventions in Default

For parsimony of equation presentation, let $B_t^T = \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^T r_s\,ds})$ be the time, t, price of a default-free discount bond with maturity date T.

While modeling defaultable debt claims, the following assumptions are often made about recovery in default, for possibly state-dependent recovery rate $\mathbb{r}_u$:

$$\underbrace{\mathfrak{R}_u}_{\text{Recovery}} = \begin{cases} \mathbb{r}_u\, K & \text{(fraction of face)} \\ \mathbb{r}_u\, B_u^T\, K & \text{(fraction of present value of face), and} \\ \mathbb{r}_u\, P_{u-}^T & \text{(fraction of predefault market value of debt).} \end{cases} \qquad 19.$$

Existing evidence appears to suggest that default probabilities and recoveries are inversely related: The recovery payout tends to be low when actual defaults rise, and the reverse. In light of the empirical evidence, one can postulate the recovery rate $\mathbb{r}_u$ to be a function of the intensity rate or other variables. The concepts of recovery in Equation 19 assume economic content when they are employed to model defaultable claims.[9]

When $\mathbb{r}_u \equiv \overline{\mathbb{r}}$, $0 \leq \overline{\mathbb{r}} \leq 1$, the recovery is a constant proportion. The approach to model recovery as a constant fraction of face is explored by Duffie (1999), whereas the fraction of discounted face is featured in the work by Jarrow & Turnbull (1995), Longstaff & Schwartz (1995), and Collin-Dufresne & Goldstein (2001). Finally, the recovery of market debt value assumption is due to the work by Duffie & Singleton (1999). In this model, the impact of recovery is subsumed within the defaultable discount rate. See, among others, the empirical applications from Duffie & Singleton (1997), Driessen (2005), and Bakshi, Madan & Zhang (2006).

Recovery in default is a crucial determinant of bond values and yields in the default-sensitive high-yield debt market. While relaxed, the recovery conventions postulate that bonds by the same issuer have identical recovery rates regardless of their maturity.

4.3. Empirical Modeling of Default Process

Through the modeling of λ_t, alternative variables can be considered to study the term structure of default correlation (e.g., Yu 2007), differential defaults, and correlation between default and recovery.

4.3.1. Case A.

If the default and recovery process is independent of the interest rate process, then the modeling simplifies to the following:

$$P_t^T = \int_t^T B_t^u\, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^u \lambda_s\,ds})\, c_u\, du + B_t^T\, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^T \lambda_s\,ds})\, K + \int_t^T B_t^u\, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^u \lambda_s\,ds}\, \mathfrak{R}_u\, \lambda_u)\, du. \qquad 20.$$

Any empirically convenient factor structure for default-free discount bonds can be employed.

4.3.2. Case B.

If, in addition, $\mathfrak{R}_u$ is independent of λ_u and $\mathfrak{R}_u$ is a martingale, then

$$P_t^T = \int_t^T B_t^u\, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^u \lambda_s\,ds})\, c_u\, du + B_t^T\, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^T \lambda_s\,ds})\, K + \mathfrak{R}_t \int_t^T B_t^u\, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^u \lambda_s\,ds}\; \lambda_u)\, du. \qquad 21.$$

Hilscher, Jarrow & van Deventer (2020) pursue empirically a model structure in which recovery is a martingale under $\mathbb{Q}$; that is, $\mathbb{E}_t^{\mathbb{Q}}(\mathfrak{R}_u) = \mathfrak{R}_t$.

[9]Elkamhi, Jacobs & Pan (2014) estimate recovery rates from CDS spreads based on 152 firms. While the average recovery rate is 54%, substantial cross-sectional variation occurs. Doshi, Elkamhi & Ornthanalai (2018) exploit the information from the term structure of senior and subordinate CDS and show that the term structure of expected recovery rates slopes downward.

The following specifications for λ_t can be considered, among others, to enable empirical assessments:

$$\lambda_t = \begin{cases} \alpha_0 + \sum_{n=1}^{N} \alpha_n \, \mathbb{f}_t^n & \text{(linear in independent } n \text{ variables)} \\ \left(\frac{S_t}{\frac{1}{\hbar}\int_{t-\hbar}^{t} S_u du}\right)^{-\rho} & \text{(power function of prior gross equity returns)} \\ \{\alpha_0 + \sum_{n=1}^{N} \alpha_n \, \mathbb{f}_t^n\}^2 & \text{(quadratic in independent } n \text{ variables),} \end{cases} \qquad 22.$$

where $\mathbb{f}_t$ is a variable whose evolution can be modeled under $\mathbb{Q}$ (and $\mathbb{P}$ for estimation).

Additionally, one may model the dynamics of λ_t as a single latent, or observable, variable, as follows:

$$d\lambda_t = \begin{cases} \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - \lambda_t)dt + \sigma\sqrt{\lambda_t}\, d\mathbb{W}_t^{\mathbb{Q}} & \text{(Longstaff, Mithal \& Neis 2005)} \\ \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - \lambda_t)dt + \sigma \, d\mathbb{W}_t^{\mathbb{Q}} & \text{(Bakshi, Madan \& Zhang 2006)} \\ \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - \lambda_t)dt + \sigma\sqrt{\lambda_t}\, d\mathbb{W}_t^{\mathbb{Q}} + \underbrace{J^{\mathbb{Q}}}_{\substack{\text{Jump} \\ \text{(additive)}}} \, d\mathbb{N}_t^{\mathbb{Q}} & \text{(Duffie \& Garleanu 2001).} \end{cases} \qquad 23.$$

These models admit tractable expressions for defaultable coupon bond prices and CDS spreads.

4.4. Estimation of Defaultable Process Using Data on CDS Spreads

The value of single-name CDS spread equates to (e.g., Hull & White 2000; Longstaff, Mithal & Neis 2005; Jarrow 2011)

$$\underbrace{\mathbb{s}_t^T}_{\text{CDS spread}} = \frac{\{1 - \bar{\mathbb{r}}\} \int_t^T B_t^u \, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^u \lambda_s ds} \lambda_u) \, du}{\int_t^T B_t^u \, \mathbb{E}_t^{\mathbb{Q}}(e^{-\int_t^u \lambda_s ds}) \, du}. \qquad 24.$$

To outline empirical estimation approaches, we suppose that for each CDS, the model spread deviates from the observed spread. For simplicity, we keep the interest rate constant; that is, $B_t^T = e^{-r(T-t)}$.

4.4.1. Maximum likelihood estimation.

To employ the information in the cross section of CDS spreads and the transition density of the variable $\mathbb{f}_t$ that impacts the evolution of λ_t, suppose $\mathbb{f}_t$ is governed as follows:

$$d\mathbb{f}_t = \kappa^{\mathbb{P}}(\theta^{\mathbb{P}} - \mathbb{f}_t)\, dt + \sigma\, d\mathbb{W}_t^{\mathbb{P}} \quad \text{and} \quad d\mathbb{f}_t = \kappa^{\mathbb{P}}\left(\theta^{\mathbb{P}} - \left\{\frac{\kappa^{\mathbb{P}} + \delta^{\mathbb{P}}}{\kappa^{\mathbb{P}}}\right\} \mathbb{f}_t\right) dt + \sigma\, d\mathbb{W}_t^{\mathbb{Q}}, \qquad 25.$$

where $\mathbb{W}_t^{\mathbb{P}}$ is a standard Brownian motion.

Let $\lambda_t = \alpha_0 + \alpha_1 \, \mathbb{f}_t$, where $\mathbb{f}_t$ could be, for instance, leverage or distance to default. Because the mean of λ_t is higher under $\mathbb{Q}$ than under $\mathbb{P}$, we expect $\delta^{\mathbb{P}} < 0$ and $\alpha_1 > 0$. In this model, one may write $\kappa^{\mathbb{Q}} = \kappa^{\mathbb{P}} + \delta^{\mathbb{P}}$ and $\theta^{\mathbb{Q}} = \frac{\kappa^{\mathbb{P}}\theta^{\mathbb{P}}}{\kappa^{\mathbb{P}} + \delta^{\mathbb{P}}}$. Thus, $d\mathbb{f}_t = \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - \mathbb{f}_t)\, dt + \sigma\, d\mathbb{W}_t^{\mathbb{Q}}$ and $d\lambda_t = \kappa^{\mathbb{Q}}(\theta_\lambda^{\mathbb{Q}} - \lambda_t)dt + \sigma_\lambda d\mathbb{W}_t^{\mathbb{Q}}$ (see Equation A4 of the **Supplemental Appendix**).

Supplemental Material >

For a given model of λ_t, the analytical expression for $\mathbb{s}_t^T$ can be obtained by solving Equation 24. For the solution technique, see the **Supplemental Appendix**.

Since $\mathbb{f}_t$ is normally distributed conditional on $\mathbb{f}_{t-\Delta t}$, the transition density of $\mathbb{f}_t$ (under $\mathbb{P}$) satisfies

$$\underbrace{\Phi[\mathbb{f}_t | \mathbb{f}_{t-\Delta t}]}_{\text{Transition density}} = \frac{1}{\sqrt{2\,\pi\,\Sigma_t}} \exp\left(-\frac{\{\mathbb{f}_t - \mu_t\}^2}{2\,\Sigma_t}\right) \text{ for } \Delta t = \frac{1}{12}, \qquad 26.$$

where $\mu_t = \theta^{\mathbb{P}} + (\mathbb{f}_{t-\Delta t} - \theta^{\mathbb{P}})\, e^{-\kappa^{\mathbb{P}}\Delta t}$ and $\Sigma_t = \frac{\sigma^2}{2\kappa^{\mathbb{P}}}(1 - e^{-2\kappa^{\mathbb{P}}\Delta t})$.

Suppose three CDS spreads are available each month with term-to-maturity, respectively, close to 1, 5, and 10 years. In this case,

$$\Theta \equiv [\alpha_0, \alpha_1, \kappa^{\mathbb{P}}, \theta^{\mathbb{P}}, \delta^{\mathbb{P}}, \sigma] \tag{27.}$$

denotes the parameter vector characterizing the default process. Let the difference between the observed and model CDS spreads of maturity T_n at time t be given by

$$\eta_t^{T_n}[\Theta] \equiv \underbrace{\overline{\mathbb{S}}_t^{T_n}}_{\text{observed CDS spread}} - \underbrace{\mathbb{S}_t^{T_n}[\Theta]}_{\text{from a model}}. \tag{28.}$$

Collect these model errors in a vector $\boldsymbol{\eta}_t[\Theta]$ as follows:

$$\underbrace{\boldsymbol{\eta}_t[\Theta]}_{3\times 1} \equiv \begin{pmatrix} \overbrace{\overline{\mathbb{S}}_t^{T_{1\text{year}}}}^{\text{observed}} - \overbrace{\mathbb{S}_t^{T_{1\text{year}}}[\Theta]}^{\text{from a model}} \\ \overline{\mathbb{S}}_t^{T_{5\text{year}}} - \mathbb{S}_t^{T_{5\text{year}}}[\Theta] \\ \overline{\mathbb{S}}_t^{T_{10\text{year}}} - \mathbb{S}_t^{T_{10\text{year}}}[\Theta] \end{pmatrix}.$$

For estimation, suppose $\boldsymbol{\eta}_t[\Theta]$ is serially uncorrelated but jointly normally distributed with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$. Under these assumptions, the log-likelihood function for a sample of observations on CDS spreads, for $t = 2, \ldots, \mathbb{T}$, is

$$\max_{\Theta, \mathbf{C}} \quad \mathcal{L} = -\frac{1}{2}(\mathbb{T}-1)\log(2\pi) - \frac{1}{2}\sum_{t=2}^{\mathbb{T}} \log(\Sigma_t) - \frac{1}{2}\sum_{t=2}^{\mathbb{T}} \frac{\{\mathbb{f}_t - \mu_t\}^2}{\Sigma_t}$$
$$-\frac{\mathbb{T}-1}{2}\log|\boldsymbol{\Omega}| - \frac{1}{2}\sum_{t=2}^{\mathbb{T}} \boldsymbol{\eta}_t[\Theta]'\,\boldsymbol{\Omega}^{-1}\,\boldsymbol{\eta}_t[\Theta]. \tag{29.}$$

Assume that the variance-covariance matrix $\boldsymbol{\Omega}$ satisfies the Cholesky decomposition $\boldsymbol{\Omega} = \mathbf{CC}'$, where $\mathbf{C}$ is a 3×3 matrix with nonzero elements C_{11}, C_{22}, C_{33}, C_{21}, C_{31}, and C_{32}. Variants of such methods are also employed to obtain the parameters for individual equity option-pricing models (e.g., Bakshi, Cao & Zhong 2021).

The study of Duffie, Saita & Wang (2007) estimates the time t default intensity of a firm that is exponential in (*a*) distance to default, (*b*) prior firm return, (*c*) US 3-month Treasury bill rate, and (*d*) trailing 1-year return of the equity market. Their procedure relies on maximum likelihood estimation and data on corporate defaults and bankruptcies (from Moody's Default Risk Service and CRSP/Compustat). Das et al. (2007) exploit this variable construction of intensities and model estimates to investigate default correlations and test the doubly stochastic assumption underlying credit risk models. A proportional hazard rate model is employed by Hilscher, Jarrow & van Deventer (2020, tables 3 and 4) to estimate default probabilities.

4.4.2. Generalized method of moments estimation approach. If the purpose is to assess the pricing performance of a candidate model of λ_t without identifying default risk premiums, the alternative to maximum likelihood is GMM. In particular,

$$\hat{\Theta}^{\mathbb{Q}} = \arg\min \left(\frac{1}{\mathbb{T}}\sum_{t=1}^{\mathbb{T}} \boldsymbol{\eta}_t[\Theta^{\mathbb{Q}}]\right)' \mathcal{W}_T \left(\frac{1}{\mathbb{T}}\sum_{t=1}^{\mathbb{T}} \boldsymbol{\eta}_t[\Theta^{\mathbb{Q}}]\right), \tag{30.}$$

where $\mathcal{W}_T$ is the GMM weighting matrix and $\Theta^{\mathbb{Q}} \equiv [\alpha_0, \alpha_1, \kappa^{\mathbb{Q}}, \theta_\lambda^{\mathbb{Q}}, \sigma_\lambda]$.

4.4.3. Nonlinear least squares approach.

Under the assumption that $d\lambda_t = \kappa^{\mathbb{Q}}(\theta^{\mathbb{Q}} - \lambda_t)dt + \sigma\sqrt{\lambda_t}d\mathbb{W}_t^{\mathbb{Q}}$, Longstaff, Mithal & Neis (2005) assess the CDS spread implied by the model. Their empirical investigation is based on CDS spreads and bonds for the 5-year contract for 68 firms. Estimating the dynamics of λ_t (for each of the firms) using nonlinear least squares, they compare the model-implied CDS spreads to data-based spreads. They document substantial variation in CDS spreads, both over time and across firms. The work of Doshi et al. (2013) also uses nonlinear least squares when estimating the parameters of the quadratic model of λ_t with observable covariates.

If λ_t were linear or quadratic in latent variables, or admitted stochastic volatility with jumps, the estimation could proceed along the lines of the unscented Kalman filter (e.g., Wan & van der Merwe 2001). These estimation approaches are outlined, for example, by Bakshi, Carr & Wu (2008, section 4.2), Doshi et al. (2013), and Kelly, Manzo & Palhares (2019).

5. COMPARISON BETWEEN STRUCTURAL AND REDUCED-FORM MODELS

Credit risk models provide quantitative tools that bear on risk management practices. Indeed, commercial adaptations such as expected default frequency and calculations of loss given default have influenced the way credit risk is tracked and interpreted in the cross section of individual names and through time. There is ongoing debate as to which modeling approach—structural or reduced-form—is better suited to understanding data realities. Both approaches have their advantages and limitations in fitting the cross section and time series of defaultable coupon bonds and CDS spreads.

Some progress has been made, but more remains to be explored with respect to the formal testing and diagnosis of credit risk models. The idea is to validate models through rigorous empirical testing. For example, Arora, Bohn & Zhu (2012) assess the empirical performance of models based on their ability to discriminate defaulters from nondefaulters using extracted default probabilities. Their exploratory exercises focus on implementing models from Merton (1974), KMV, and Hull & White (2000).

When it comes to single-name entities, both structural and reduced-form models can be onerous to implement. With a deep pool of data on CDS spreads now available across industries and credit ratings, both model types are amenable to in-sample and out-of-sample pricing comparisons. The questions of parameter instability, detecting prospective deterioration in credit quality, and predicting defaulters from nondefaulters remain relevant practical assessment criteria.

Perhaps structural models provide some discipline and transparency by anchoring to data on debt and equity while relying on established methods to determine asset values, asset volatilities, and market risk premiums. In contrast, reduced-form models rely on specifications of default intensity. While this approach is mathematically tractable, there appears no consensus on the functional form or the identity of the variables affecting default intensity. The lack of economic rationale and comprehensive empirical analysis may be hampering real-world applicability. Such empirical gaps need to be closed.

At the same time, there is a recognition that empiricists need to account for estimation noise in model parameters. This will continue to be a challenge given the rarity of defaults. In this regard, the techniques on bootstrapping sparsely occurring events may help to provide complementary supportive evidence.

6. CONCLUSION AND RESEARCH AGENDA IN EMPIRICAL DEFAULT RISK

There are two defining attributes of firm default. First, defaults tend to be sudden and rare, and they generate large and discontinuous price changes. Second, bond holders recoup only a fraction of the promised payments. Mindful of these features of default and recovery, formulaic treatments of default have blended tractability and empirical consistency in pricing and hedging both single-name and multiname credit derivatives. Option-like products on credit derivatives continue to garner interest in light of their information content for understanding the time-varying nature of risk premiums on left-tail events.

In the structural framework for modeling default, the reliance is on the fundamentals of the firm and on diffusion processes. The consequence of this approach is that default events are predictable. In many such models, the short-dated spreads are zero, counter to the understanding that short-dated spreads are observed to be positive even for credit-worthy firms. To remedy, jumps are being added into the asset process in conjunction with richer alternative models of the default boundary.

The reduced-form approaches parameterize the instantaneous risk-neutralized probability of default—the default intensity rate—along with a treatment of recovery in default. The default intensity rate can link the firm-specific distress variables, as well as the stock price and its volatility, to the default process. Some of these approaches hold the promise of modeling equity claims and debt claims together by permitting the individual stock price to jump to zero at the time of default. The task of empirically comparing structural and reduced-form models of default risk remains a first-order question in credit risk valuation.

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