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FNCE 926

Empirical Methods in CF

**Lecture 7 – Natural Experiment** *[P2]*

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Professor Todd Gormley

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# Announcements & Informal Survey

- Exercise #3 is due
- Please fill out informal survey
  - Helps me figure out what changes I can make to improve the course for the second half and for future years
    - For example, what topic should I have spent more time on? What topic did you find the most interesting? Is there too much, or too little work? Etc.

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# Background readings

- Roberts and Whited
  - *Sections 2.2, 4*
- Angrist and Pischke
  - *Section 5.2*

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# Outline for Today

- Quick review of last lecture
- Continue to discuss natural experiments
  - How to handle multiple events
  - Triple differences
  - Common robustness tests that can be used to test whether internal validity is likely to hold
- Student presentations of “NE #1” papers

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## Quick Review *[Part 1]*

- Natural experiment provides random variation in  $x$  that allows causal inference
  - Can be used in IV, regression discontinuity, but most often associated with “treatment” effects
- Two types of simple differences
  - Post-treatment comparison of treated & untreated
  - Pre- and post-treatment comparison of treated

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## Quick Review *[Part 2]*

- Difference-in-differences is estimated with...

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 (d_i \times p_t) + u_{i,t}$$

- Compares change in  $y$  pre- versus post-treatment for treated to change in  $y$  for untreated
  - Requires “parallel trends” assumption
- Let’s test your ability to identify a violation of the necessary assumptions for simple diffs and diff-in-diffs...

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## Quick Review *[Part 3]*

- Suppose Spain exits the Euro. And, Ann compares profitability of Spanish firms after the exit to profitability before...
- **What is necessary for the comparison to have any causal interpretation?**
  - **Answer** = We must assume profitability after Spain's exit would have been same as profitability prior to exit absent exit... Highly implausible

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## Quick Review *[Part 4]*

- Now, suppose Bob compares profitability of Spanish firms after the exit to profitability of German firms after exit...
- **What is necessary for the comparison to have any causal interpretation?**
  - **Answer =** We must assume profitability of Spanish firm would have been same as profitability of German firms absent exit...  
Again, this is highly implausible



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## Quick Review *[Part 5]*

- Lastly, suppose Charlie compares change in profitability of Spanish firms after exit to change in profitability of German firms
- **What is necessary for the comparison to have any causal interpretation?**
  - **Answer =** We must assume change in profitability of Spanish firm would have been same as change for German firms absent exit... I.e. parallel trends assumption

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# Natural Experiment [P2] – *Outline*

- Difference-in-difference continued...
  - Using group means to get an estimate
  - When additional controls are appropriate
- How to handle multiple events
- Falsification tests
- Triple differences

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# Standard Regression Format

- Difference-in-differences estimator

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 (d_i \times p_t) + u_{i,t}$$

- $p_t = 1$  if period  $t$  occurs after treatment and equals zero otherwise
- $d_i = 1$  if unit is in treated group and equals zero otherwise

**But, there is another way that just involves comparing four sample means...**

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# Comparing group means approach

- To see how we can get the same estimate,  $\beta_3$ , by just comparing sample means, first calculate expected  $y$  under four possible combinations of  $p$  and  $d$  indicators

# Comparing group means approach [P1]

- Again, the regression is...

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 (d_i \times p_t) + u_{i,t}$$

- And, the four possible combinations are:

$$E(y | d = 1, p = 1) = \beta_0 + \beta_1 + \beta_2 + \beta_3$$

$$E(y | d = 1, p = 0) = \beta_0 + \beta_2$$

$$E(y | d = 0, p = 1) = \beta_0 + \beta_1$$

$$E(y | d = 0, p = 0) = \beta_0$$

What assumption did I  
make in doing this?

Answer:  $E(u | d, p) = 0$ ; i.e.  
the “experiment” is random

# Comparing group means approach [P2]

$$E(y \mid d = 1, p = 1) = \beta_0 + \beta_1 + \beta_2 + \beta_3$$

$$E(y \mid d = 1, p = 0) = \beta_0 + \beta_2$$

$$E(y \mid d = 0, p = 1) = \beta_0 + \beta_1$$

$$E(y \mid d = 0, p = 0) = \beta_0$$

- These can be arranged in two-by-two table

	Post-Treatment, (1)	Pre-Treatment, (2)
Treatment, (a)	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_0 + \beta_2$
Control, (b)	$\beta_0 + \beta_1$	$\beta_0$

# Comparing group means approach [P3]


- Now take the simple differences

	Post-Treatment, (1)	Pre-Treatment, (2)	Difference, (1)-(2)
Treatment, (a)	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_0 + \beta_2$	$\beta_1 + \beta_3$
Control, (b)	$\beta_0 + \beta_1$	$\beta_0$	$\beta_1$
Difference, (a)-(b)	$\beta_2 + \beta_3$	$\beta_2$	

# Comparing group means approach [P4]

- Then, take difference-in-differences!

	Post-Treatment, (1)	Pre-Treatment, (2)	Difference, (1)-(2)
Treatment, (a)	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_0 + \beta_2$	$\beta_1 + \beta_3$
Control, (b)	$\beta_0 + \beta_1$	$\beta_0$	$\beta_1$
Difference, (a)-(b)	$\beta_2 + \beta_3$	$\beta_2$	$\beta_3$



This is why they call it the difference-in-differences estimate; regression gives you same estimate as if you took differences in the group averages



# Simple difference – Revisited *[Part 1]*

- Useful to look at simple differences

	Post-Treatment, (1)	Pre-Treatment, (2)	Difference, (1)-(2)
Treatment, (a)	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_0 + \beta_2$	$\beta_1 + \beta_3$
Control, (b)	$\beta_0 + \beta_1$	$\beta_0$	$\beta_1$
Difference, (a)-(b)	$\beta_2 + \beta_3$	$\beta_2$	$\beta_3$

↑  
This was cross-sectional  
simple difference

When does that simple diff  
give effect of treatment,  $\beta_3$ ?

**Answer** = when  $\beta_2$  equals  
zero; i.e. no difference in level  
of  $y$  absent treatment

# Simple difference – Revisited [Part 2]

- Now, look at time-series simple diff...

	Post-Treatment, (1)	Pre-Treatment, (2)	Difference, (1)-(2)
Treatment, (a)	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_0 + \beta_2$	$\beta_1 + \beta_3$
Control, (b)	$\beta_0 + \beta_1$	$\beta_0$	$\beta_1$
Difference, (a)-(b)	$\beta_2 + \beta_3$	$\beta_2$	$\beta_3$

This was time-series  
simple difference

When does that simple diff  
give effect of treatment,  $\beta_3$ ?

**Answer** = when  $\beta_1$  equals zero; i.e.  
no change in  $y$  absent treatment

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# Why the regression is helpful

- Some papers will just report this simple two-by-two table as their estimate
- But, there are advantages to the regression
  - Can modify it to test timing of treatment  
*[we will talk about this in robustness section]*
  - Can add additional controls,  $X$

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# Natural Experiment [P2] – *Outline*

- Difference-in-difference continued...
  - Using group means to get an estimate
  - When additional controls are appropriate
- How to handle multiple events
- Falsification tests
- Triple differences

# Adding controls to diff-in-diff

- Easy to add controls to regression

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 (d_i \times p_t) + \Gamma X_{i,t} + u_{i,t}$$

- $X$  is some vector of controls

- $\Gamma$  is vector of coefficients

- $E[y | d, p]$  in prior proofs just becomes  $E[y | d, p, X]$

From earlier lecture,  
what type of controls  
should you **NEVER** add?

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# When controls are inappropriate

- Remember! You should never add controls that might themselves be affected by treatment
  - Angrist-Pischke call this a “bad control”
  - You won’t be able to get a consistent estimate of  $\beta_3$  from estimating the equation

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## A Pet Peeve of TG – *Refined*

- If you have a treatment that is truly random, do not put in controls affected by the treatment!
  - I've had many referees force me to add controls that are likely to be affected by the treatment...
  - If this happens to you, put in both regressions (with and without controls), and at a minimum, add a caveat as to why adding controls is inappropriate

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# When controls are appropriate

- Two main reasons to add controls
  - Improve precision (i.e. lower standard errors)
  - Restore ‘random’ assignment of treatment



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# #1 – To improve precision

- Adding controls can soak up some of residual variation (i.e. noise) allowing you to better isolate the treatment effect
  - Should the controls change the estimate?
    - NO! If treatment is truly random, adding controls shouldn't affect actual estimate; they should only help lower the standard errors!
  - If adding controls changes estimates, you might have 'bad controls' *or* worse, non-random treatment ☹

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## *Example* – Improving precision

- Suppose you have firm-level panel data
- Some natural experiment ‘treats’ some firms but not other firms

- Could just estimate the standard diff-in-diff

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 (d_i \times p_t) + u_{i,t}$$

- Or, could add fixed effects (like firm and year FE) to get more precise estimate...

## *Example – Improving precision [Part 2]*

- So, suppose you estimate...

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 (d_i \times p_t) + \alpha_i + \delta_t + u_{i,t}$$

Firm fixed effects

Year fixed effects

- What meaning does  $\beta_1$  have now?
- What meaning does  $\beta_2$  have now?

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## *Example – Improving precision [Part 3]*

- Trick question! They have no meaning!
  - $p_t$  is perfectly collinear with year FE  
*[because it doesn't vary across firms]*
  - $d_i$  is perfectly collinear with firm FE  
*[because it doesn't vary across time for each firm]*
- Stata just randomly drops a couple of the FE
  - The estimates on  $p_t$  and  $d_i$  are just random intercepts with **no** meaning

## *Example – Improving precision [Part 4]*

- Instead, you should estimate...

$$y_{i,t} = \beta_0 + \beta_3 (d_i \times p_t) + \alpha_i + \delta_t + u_{i,t}$$

Firm fixed effects  
control for treatment

Year fixed effects  
control for post-  
treatment

- This is what some call the **generalized difference-in-differences estimator**

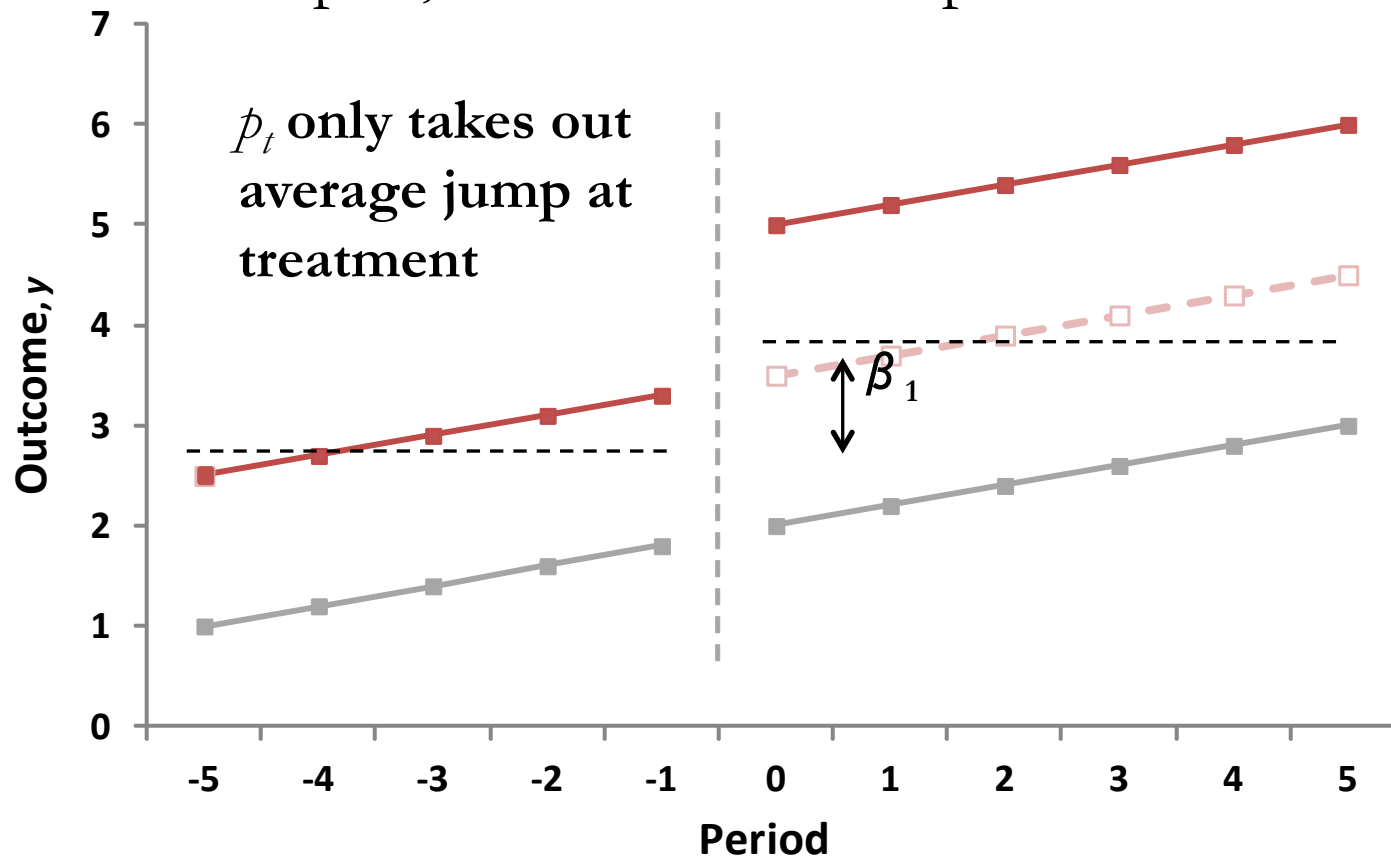
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# Generalized Difference-in-differences

- Advantage of generalized differences-in-differences is that it can improve precision and provide better fit of model
  - It doesn't assume all firms in treatment (or untreated) group have same average  $y$ ; it allows intercept to vary for each firm
  - It doesn't assume that common change in  $y$  around event is a simple change in level; it allows common change in  $y$  to vary by year

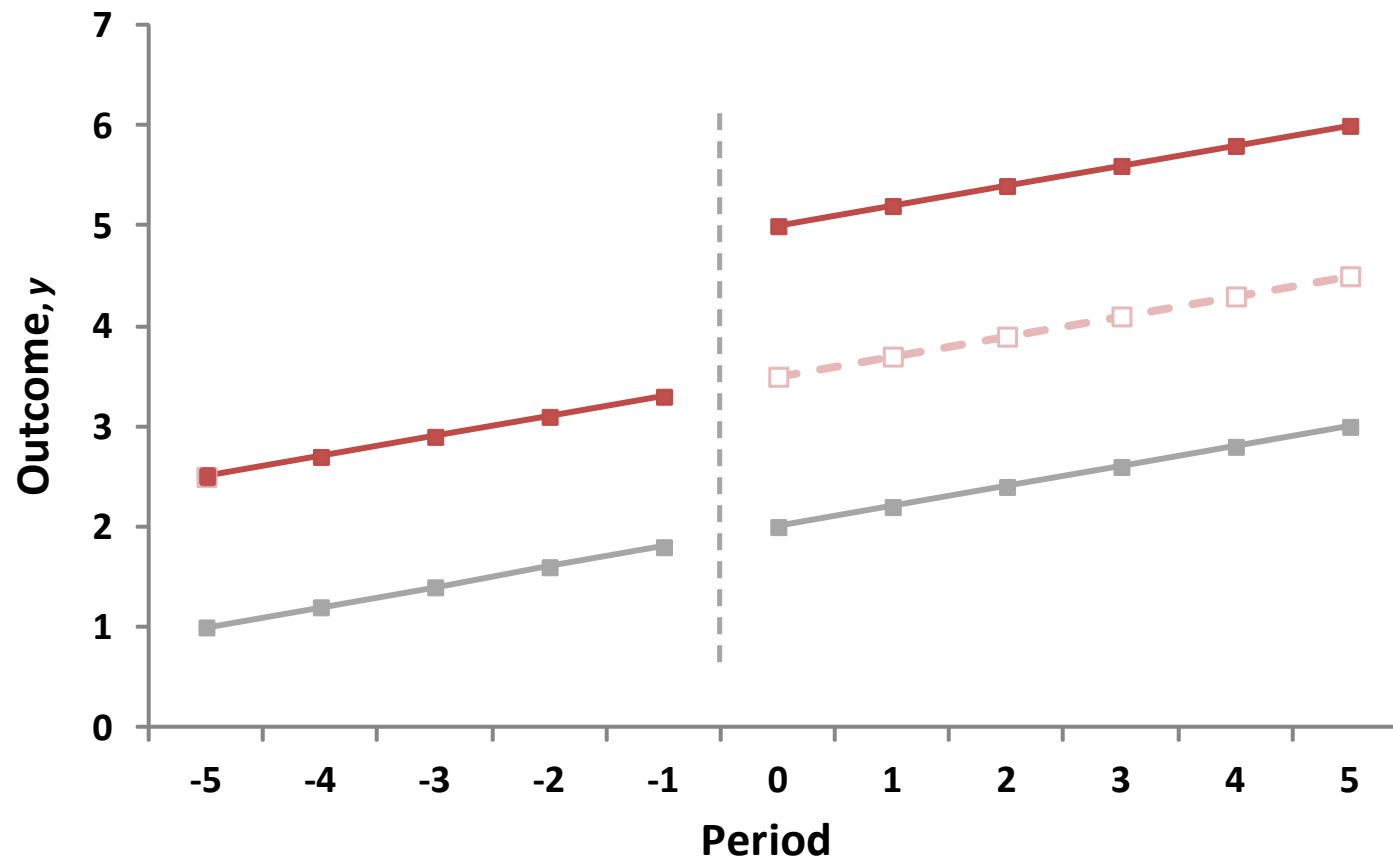
# Generalized D-i-D – *Example [Part 1]*

- To see how Generalized D-i-D can be helpful, consider the example from last week



# Generalized D-i-D – *Example [Part 2]*

□ Year dummies will better fit actual trend





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# When controls are appropriate

- Two main reasons to add controls
  - Improve precision (i.e. lower standard errors)
  - Restore 'random' assignment of treatment

## #2 – Restore randomness of treatment

- Suppose the following is true...
    - Observations of certain characteristic, e.g. high  $x$ , are more likely to be treated
    - **And**, firms with this characteristic are likely to have differential trend in outcome  $y$
  - Adding control for  $x$  could restore ‘randomness’; i.e. being treated is random after controlling for  $x$ !
- I.e. treatment isn’t random
- And, non-randomness is problematic for identification

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## Restoring randomness – *Example*

- Natural experiment is change in regulation
  - Firms affected by regulation is random, except that it is more likely to hit firms that are larger
  - *And*, we think larger firms might have different trend in outcome  $y$  afterwards for other reasons
  - *And*, firm size is not going to be affected by the change in regulation in any way
- If all true, adding size as control would be an appropriate and desirable thing to do

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## Controls continued...

- In prior example, suppose size is potentially effected by the change in regulation...
- What would be another approach that won't run afoul of the 'bad control' problem?
  - **Answer:** Use firm size in year prior to treatment and it's interaction with post-treatment dummy
  - This will control for non-random assignment (based on size) and differential trend (based on size)

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# Restoring randomness – Caution!

- In practice, don't often see use of controls to restore randomness
  - Requires assumption that non-random assignment isn't also correlated with unobservable variables...
  - So, not that plausible unless there are very specific reasons for non-randomness
- **But, regression discontinuity is one example of this; we'll see it next week**

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## One last note... be careful about SEs

- Again, if have multiple pre- and post-treatment periods, need to be careful with standard errors
  - Either cluster SEs at level of each unit
  - Or, collapse data down to one pre- and one post-treatment observation for each cross-section
- We will discuss more about standard errors in lecture on “standard errors”

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# Natural Experiment [P2] – *Outline*

- Difference-in-difference continued...
- How to handle multiple events
  - Why they are useful
  - Two similar estimation approaches
- Falsification tests
- Triple differences

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## Motivating example...

- Gormley and Matsa (2011) looked at firms' responses to increased left-tail risk
  - Used discovery that workers were exposed to harmful chemical as exogenous increase in risk
  - One discovery occurred in 2000; a chemical heavily used by firms producing semiconductors was found to be harmful
- Can you think of any concerns about parallel trends assumption of this setting?



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## Motivating Example – *Answer*

- **Answer:** Yes... This coincides with bursting of technology bubble; technology firms might arguably trend differently after 2000 for this reasons unrelated to chemical
- How might multiple treatment events, occurring at different times (which is what Gormley and Matsa used), help?

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# Multiple treatment events

- Sometimes, the natural experiment is repeated a multiple points in times for multiple groups of observations
  - E.g. U.S. states make a particular regulatory change at different points in time
- **These settings are particularly useful in mitigating concerns about violation of parallel trends assumption...**

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# How multiple events are helpful

- Can show that effect of treatment is similar across different time periods
- Can show effect of treatment isn't driven by a particular set of treated firms
  - I.e. now the “identification police” would need to come up with story as to why parallel trends is violated for each unique event

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# Natural Experiment [P2] – *Outline*

- Difference-in-difference continued...
- How to handle multiple events
  - Why they are useful
  - Two similar estimation approaches
- Falsification tests
- Triple differences

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# Estimation with Multiple Events

- Estimating model with multiple events is still relatively easy to do
  - Use approach of Bertrand and Mullainathan (JPE 2003)
  - Or, used “stacked” approach of Gormley and Matsa (RFS 2011)

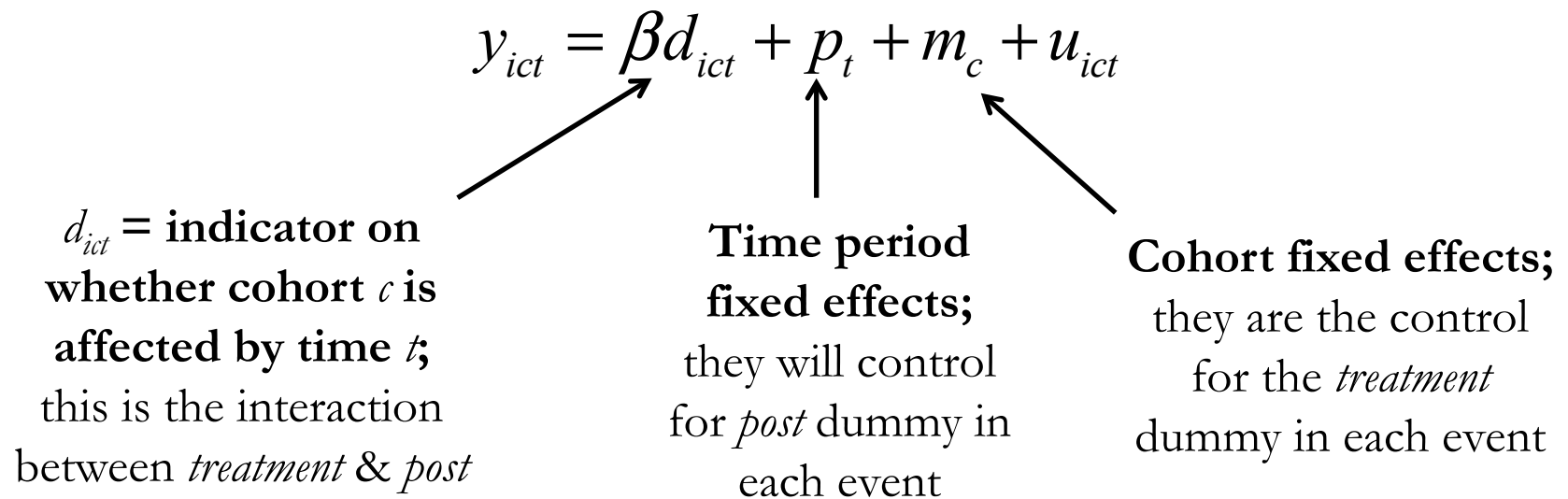
# Multiple Events – Approach #1 [P1]

- Just estimate the following estimation

$$y_{ict} = \beta d_{ict} + p_t + m_c + u_{ict}$$

- $y_{ict}$  is outcome for unit  $i$  (e.g. firm) in period  $t$  (e.g. year) and cohort  $c$ , where “cohort” indexes the different sets of firms treated by each event
  - E.g. different firms might be affected by a change in regulation at different points in time; firms affected at one point in time are a ‘cohort’

# Multiple Events – Approach #1 [P2]



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# Multiple Events – Approach #1 [P3]

- Intuition of this approach...
  - Every untreated observation at a particular point in time acts as control for treated observations in that time period
    - E.g. a firm treated in 1999 by some event will act as a control for a firm treated in 1994 until itself becomes treated in 1999
  - $\beta$  will capture average treatment effect across the multiple events



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## Multiple Events – Approach #2 [P1]

- Now, think of running generalized diff-in-diff for just one of the multiple events...

$$y_{it} = \beta(d_i \times p_t) + \alpha_i + \delta_t + u_{it}$$

- $d_i$  = indicator for unit  $i$  (e.g. firm) being a treated firm in that particular event
- $p_t$  = indicator for treatment having occurred by period  $t$  (e.g. year)
- Unit  $i$  and period  $t$  FE control for the independent effects of  $d_i$  and  $p_t$

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## Multiple Events – Approach #2 [P2]

- But, contrary to standard difference-in-difference, your sample is...
  - Restricted to a small window around event; e.g. 5 years pre- and post- event
  - And, drops any observations that are treated by another event
    - I.e. your sample starts only with previously untreated observations, and if a ‘control’ observation later gets treated by a different event, those post-event observations are dropped

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## Multiple Events – Approach #2 [P3]

- Now, create a similar sample for each “event” being analyzed
- Then, “stack” the samples into one dataset and create a variable that identifies the event (i.e. ‘cohort’) each observation belongs to
  - **Note:** some observation units will appear multiple times in the data [e.g. firm 123 might be a control in event year 1999 but a treated firm in a later event in 2005]

## Multiple Events – Approach #2 [P4]

- Then, estimate the following on the stacked dataset you've created

$$y_{ict} = \beta d_{ict} + \delta_{tc} + \alpha_{ic} + u_{ict}$$

$d_{ict}$  = indicator on whether cohort  $c$  is affected by time  $t$ ; this is the interaction between *treatment* & *post*

**Time-cohort period fixed effects;**  
they control for *post* dummy in each event (i.e. for each 'stack')

**Unit-cohort FE;**  
they control for the *treatment* dummy in each cohort (i.e. in each 'stack')

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## Multiple Events – Approach #2 [P5]

- This approach has same intuition of the first approach, but has a couple advantages
  - Can more easily isolate a particular window of interest around each event
    - Prior approach compared all pre- versus post-treatment observations against each other
  - Can more easily extend this into a triple-difference type specification [*more on that later*]

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# Natural Experiment [P2] – *Outline*

- Difference-in-difference continued...
- How to handle multiple events
- Falsification tests
- Triple differences

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# Falsification Tests for D-i-D

- Can never directly test underlying identification assumption, but can do some falsification tests to support its validity
  - #1 – Compare pre-treatment observables
  - #2 – Check that timing of observed change in  $y$  coincides with timing of event [*i.e. no pre-trend*]
  - #3 – Check for treatment reversal
  - #4 – Check variables that shouldn't be affected
  - #5 – Add a triple-difference

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# #1 – Pre-treatment comparison *[Part 1]*

- Idea is that experiment ‘randomly’ treats some subset of observations
  - If true, then ex-ante characteristics of ‘treated’ observations should be similar to ex-ante characteristics of ‘untreated’ observations
  - Showing treated and untreated observations are comparable in dimensions thought to affect  $y$  can help ensure assignment was random



# #1 – Pre-treatment comparison *[Part 2]*

- If find ex-ante difference in some variable  $z$ , is difference-in-difference is invalid?
- **Answer** = Not necessarily.
  - We need some story as to why units are expected to have differential trend in  $y$  after treatment (for reasons unrelated to treatment) that is correlated with  $z$  for this to actually be a problem for identification
  - **And**, even with this story, we could just control for  $z$  and it's interaction with time
  - **But, what would be the lingering concern?**

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# #1 – Pre-treatment comparison *[Part 3]*

- **Answer = unobservables!**
  - If the treated and control differ ex-ante in observable ways, we worry they might differ in unobservable ways that related to some violation of the parallel trends assumption

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## #2 – Check for pre-trend [*Part 1*]

- Similar to last lecture, can just allow effect of treatment to vary by period to non-parametrically map out the timing
  - “Parallel trends” suggest we shouldn’t observe any differential trend prior to treatment for the observations that are eventually treated

## #2 – Check for pre-trend [*Part 2*]

- Estimate the following:

$$y_{i,t} = \beta_0 + \beta_1 d_i + \beta_2 p_t + \sum_t \gamma_t (d_i \times \lambda_t) + u_{i,t}$$

- $d_i$  and  $p_t$  are defined just as before
- $\lambda_t$  is indicator that equals 1 if event time =  $t$  and zero otherwise, where
  - $t = 0$  is the period treatment occurs
  - $t = -1$  is period before treatment

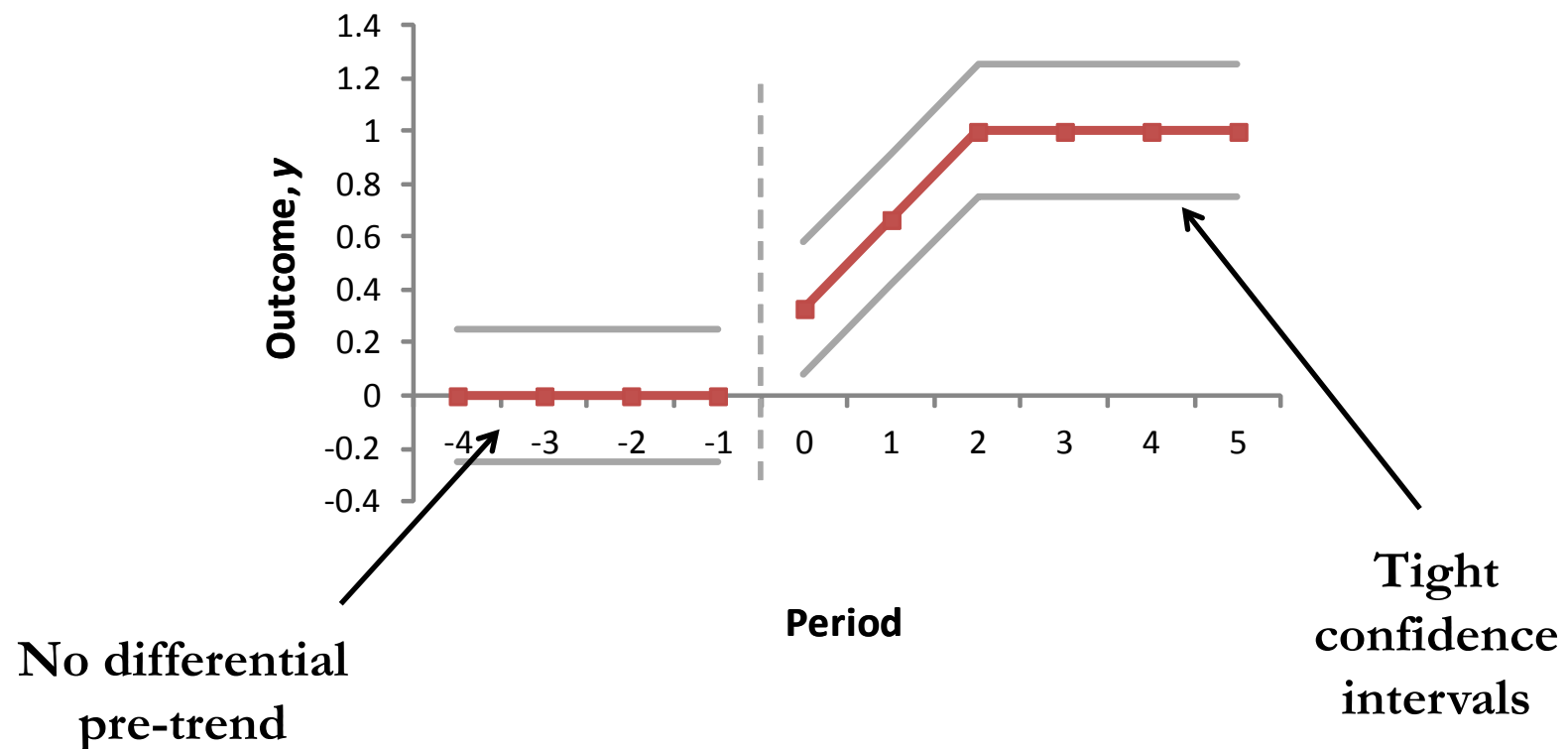
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## #2 – Check for pre-trend [*Part 3*]

- $\gamma_t$  estimates change in  $y$  relative to excluded periods; you then plot these in graph
  - Easiest to **fully saturate** the model (i.e. include  $\lambda_t$  for every period but the very first one); then all estimates  $\gamma_t$  are relative to this period
  - Can also plot confidence interval for each  $\gamma_t$

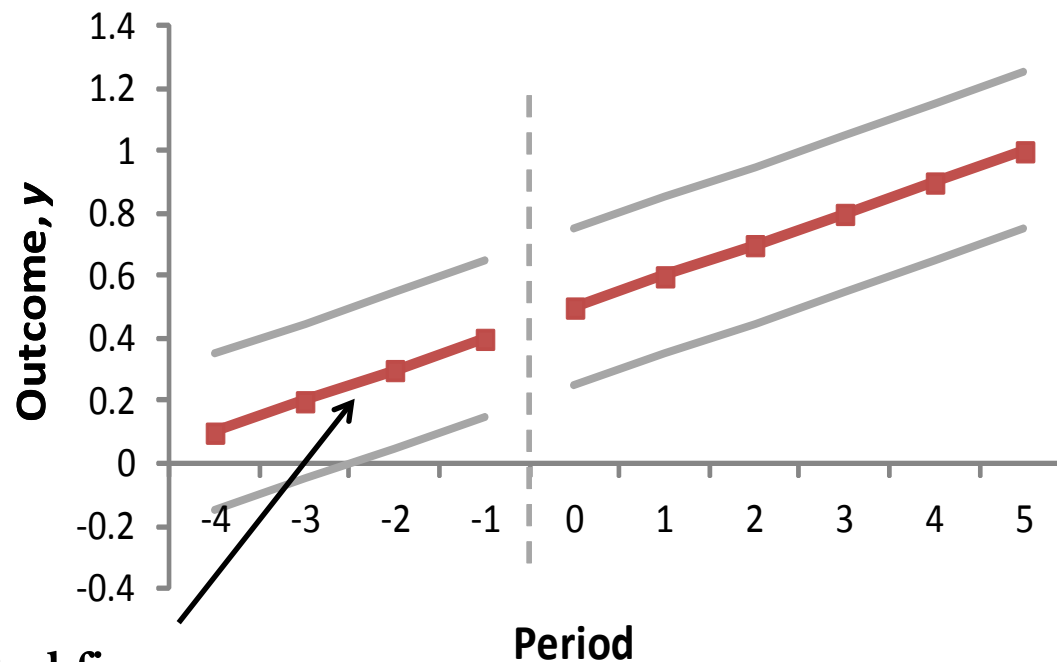
## #2 – Check for pre-trend [*Part 4*]

- Something like this is ideal...



## #2 – Check for pre-trend [*Part 5*]

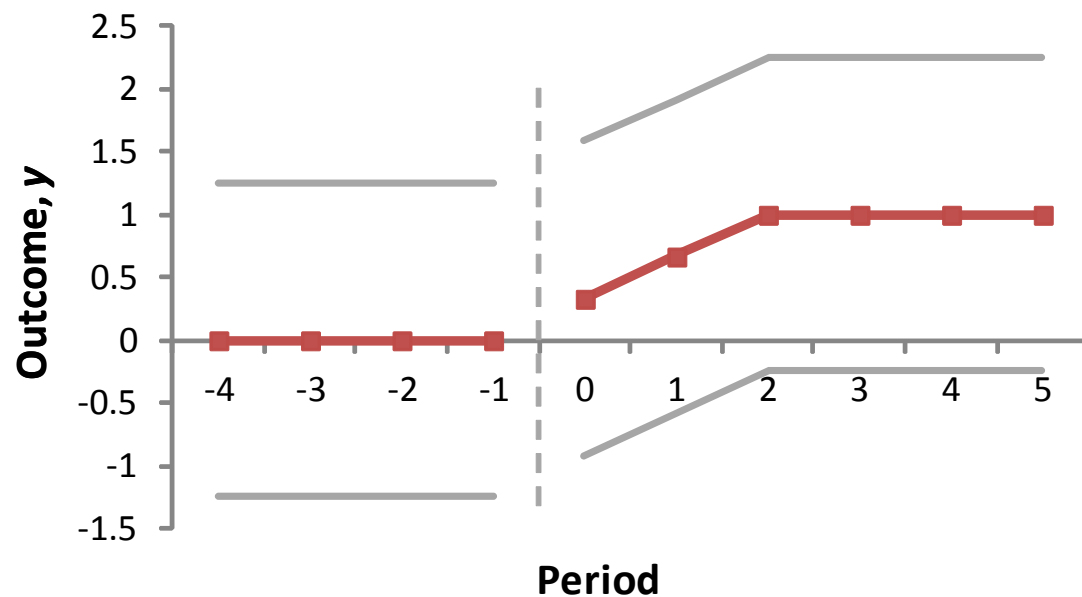
- Something like this is very bad



$y$  for treated firms was  
already going up at faster  
rate prior to event!

## #2 – Check for pre-trend [*Part 6*]

- Should we make much of wide confidence intervals in these graphs? E.g.



**Answer:** Not too much... Each period point estimate might be noisy; diff-in-diff will tell us whether post-average  $y$  is significantly different then pre-average  $y$



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## #2 – Check for pre-trend [*Part 7*]

- Another type of pre-trend check done is to do the diff-in-diff in some “random” pre-treatment to show no effect
  - I’m not a big fan of this... Why?
    - **Answer #1** – It is subject to gaming; researcher might choose a particular pre-period to look at that works
    - **Answer #2** – Prior approach allows us to see what the timing was and determine whether it is plausible

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## #3 – Treatment reversal

- In some cases, the “natural experiment” is subsequently reversed
  - E.g. regulation is subsequently undone
- If we expect the reversal should have the opposite effect, it is good to confirm this

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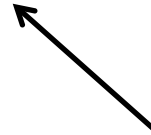
## #4 – Unaffected variables

- In some cases, theory provides guidance on what variables should be unaffected by the “natural experiment”
  - If natural experiment is what we think it is, we should see this in the data... so check

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## #5 – Add Triple difference

- If theory tells us treatment effect should be larger for one subset of observations, we can check this with triple difference
  - Pre- *versus* post-treatment
  - Untreated *versus* treated
  - Less sensitive *versus* More sensitive



This is the third  
difference

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# Natural Experiment Outline – *Part 2*

- Difference-in-difference continued...
- How to handle multiple events
- Falsification tests
- Triple differences
  - How to estimate & interpret it
  - Using the popular subsample approach

# Diff-in-diff-in-diff – Regression

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 h_i + \beta_4 (p_t \times h_i) \\ + \beta_5 (d_i \times h_i) + \beta_6 (p_t \times d_i) + \beta_7 (p_t \times d_i \times h_i) + u_{i,t}$$

- $p_t = 1$  if period  $t$  occurs after treatment and equals zero otherwise
- $d_i = 1$  if unit is in treated group and equals zero otherwise
- $h_i = 1$  if unit is group that is expected to be more sensitive to treatment

# Diff-in-diff-in-diff – Regression *[Part 2]*

- How to choose and set  $h_i$ 
  - E.g. If theory says effect is bigger for larger firms; could set  $h_i = 1$  if assets of firm in year prior to treatment is above the median size
  - **Note:** Remember to use ex-ante measures to construct indicator if you think underlying variable (that determines sensitivity) might be affected by treatment... Why?
    - **Answer =** To avoid bad controls!

## Diff-in-diff-in-diff – Regression *[Part 3]*

$$y_{i,t} = \beta_0 + \beta_1 p_t + \beta_2 d_i + \beta_3 h_i + \beta_4 (p_t \times h_i) \\ + \beta_5 (d_i \times h_i) + \beta_6 (p_t \times d_i) + \beta_7 (p_t \times d_i \times h_i) + u_{i,t}$$

- Easy way to check if done correctly...
  - Should have 8 coefficients (including constant) to capture the  $2 \times 2 \times 2 = 8$  different combinations
  - Likewise, a double difference has 4 coefficients (including constant) for the  $2 \times 2 = 4$  combinations
- What do  $\beta_6$  and  $\beta_7$  capture?



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# Interpreting the estimates [*Part 1*]

- $\beta_6$  diff-in-diff estimate for the less-sensitive obs.
  - Captures average differential change in  $y$  from the pre- to post-treatment period for the less sensitive observations in the treatment group *relative* to the change in  $y$  for the less sensitive observations in the untreated group

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## Interpreting the estimates *[Part 2]*

- $\beta_7$  is the triple diff estimate; it tells us how much larger effect is for the more sensitive obs.
  - $\beta_7$  captures how different the difference-in-difference estimate is for observations considered more sensitive to the treatment
  - What is total treatment effect for these firms?
  - Answer =  $\beta_6 + \beta_7$

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## *Tangent* – Continuous vs. Indicator?

- Can also do the triple difference replacing  $b_i$  with a continuous measure instead of indicator
  - E.g. suppose we expect treatment effect is bigger for larger firms; rather than constructing indicator based on ex-ante size, could just use ex-ante size
  - **What are the advantages, disadvantages of this?**

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# *Tangent* – Continuous vs. Indicator?

- Advantages

- Makes better use of variation available in data
- Provides estimate on magnitude of sensitivity

- Disadvantages

- Makes linear functional form assumption; indicator imposes less structure on the data
- More easily influenced by outliers

# Generalized Triple-Difference

- Similar to diff-in-diff, can add in FE to soak up the various terms and improve precision
- E.g. in firm-level panel regression with firm and year fixed effects, you'd estimate

$$y_{i,t} = \beta_1 (p_t \times h_i) + \beta_2 (p_t \times d_i) \\ + \beta_3 (p_t \times d_i \times h_i) + \delta_t + \alpha_i + u_{i,t}$$

- The other terms (including the constant) all drop out; they are collinear with the FE

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# Natural Experiment [P2] – *Outline*

- Difference-in-difference continued...
- How to handle multiple events
- Falsification tests
- Triple differences
  - How to estimate & interpret it
  - Using the popular subsample approach

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# Subsample Approach

- Instead of doing full-blown triple-difference, you can also just estimate the double-difference in the two separate subsamples
  - Double-difference for low sensitive obs. (i.e.  $h_i = 0$ )
  - Double-difference for more sensitive obs. (i.e.  $h_i = 1$ )
- **Note:** the estimates won't directly match the  $\beta_2$ ,  $\beta_2 + \beta_3$  effects in prior estimation... **Why?**

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# Subsample Approach Differences...

- **Answer** = In subsample approach year FE are allowed to differ by sub-sample
  - Therefore, subsample approach is actually controlling for **more** things
  - However, one can easily recover the subsample estimates in one regression (and test the statistical difference) between subsamples by estimating...



# Matching Subsample to Combined [P1]

$$y_{i,t} = \beta_2 (p_t \times d_i) + \beta_3 (p_t \times d_i \times h_i) + \delta_t + (\delta_t \times h_i) + \alpha_i + u_{i,t}$$



Year FE interacted with  
sensitivity indicator

- Just add interaction between year FE and indicator for being more sensitivity...
- This allows for different year FE for each subsample, which is what happened when we estimated the subsamples in two separate regressions

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# Matching Subsample to Combined [P2]

- **In prior regression...**

- $\beta_2$  will equal coefficient from diff-in-diff using just the subsample of less sensitive observations
- $\beta_2 + \beta_3$  will equal coefficient from diff-in-diff using just the subsample of more sensitive observations
- t-test on  $\beta_3$  tells you whether effect for more sensitive subsample is statistically different from that of the less sensitive subsample

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# Triple Diff – Stacked Regression *[Part 1]*

- Another advantage of stacked regression approach to multiple events is ability to more easily incorporate a triple diff
- Can simply run stacked regression in separate subsamples to create triple-diff or run it in one regression as shown previously

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# Triple Diff – Stacked Regression [*Part 2*]

- Can't easily do either of these in approach of Bertrand and Mullainathan (2003)
  - Some observations act as both 'control' and 'treated' at different points in sample; not clear how create subsamples in such a setting

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# External Validity – Final Note

- While randomization ensures internal validity (i.e. causal inferences), external validity might still be an issue
  - Is the experimental setting representative of other settings of interest to researchers?
    - I.e. can we extrapolate the finding to other settings?
    - A careful argument that the setting isn't unique or that the underlying theory (for why you observe what you observe) is likely to apply elsewhere is necessary

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# Summary of Today *[Part 1]*

- Diff-in-diff & control variables
  - Don't add controls affected by treatment
  - Controls shouldn't affect estimates, but can help improve precision
- Multiple events are helpful in mitigating concerns about parallel trends assumption

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## Summary of Today *[Part 2]*

- Many falsification tests one should do to help assess internal validity
  - Ex. #1 – Compare ex-ante characteristics
  - Ex. #2 – Check timing of observed effect
- Triple difference is yet another way to check internal validity and mitigate concerns about identification

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# In First Half of Next Class

- Regression discontinuity
  - What are they?
  - How are they useful?
  - How do we implement them?
- Related readings... see syllabus



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# Assign papers for next week...

- Gormley and Matsa (RFS 2011)
  - Risk & CEO agency conflicts
- Becker and Stromberg (RFS 2012)
  - Agency conflicts between equity & debt
- Ashwini (JFE 2012)
  - Investor protection laws & corporate policies

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# Break Time

- Let's take our 10 minute break
- We'll do presentations when we get back