Day 1: Introduction to Machine Learning

Summer STEM: Machine Learning

Department of Electrical and Computer Engineering
NYU Tandon School of Engineering
Brooklyn, New York

June 22, 2020

Outline

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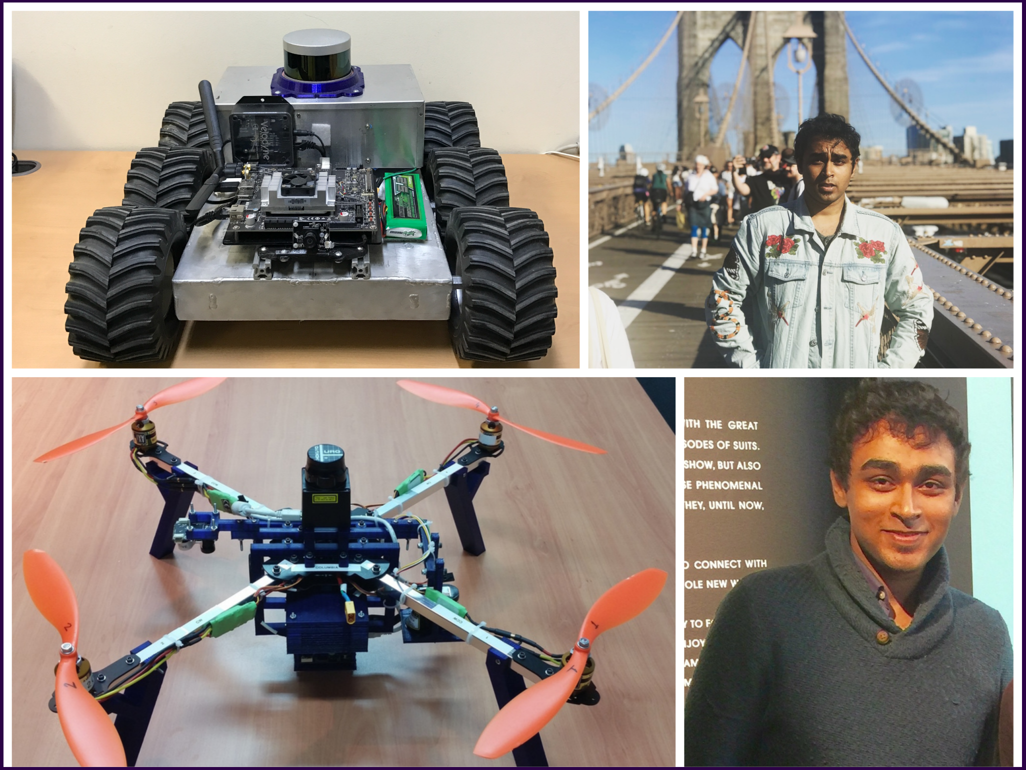
Naman



Huaijiang

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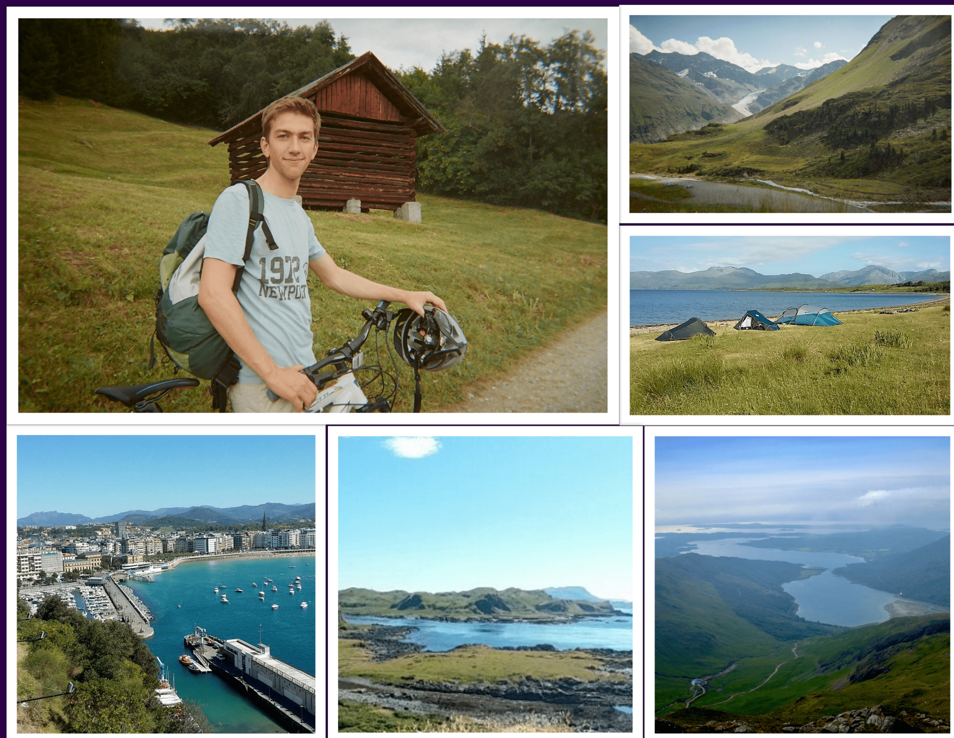
Armand



Virinchi

Tell the class about yourself

- Write down the following information:
  - Name
  - Grade
  - Where are you from?
  - What do you want to get out of this class?
  - What is your favourite movie?
  - What is the IMDB score of this movie!
  - What is the category of this movie? (thriller/drama/action, etc)
  - Rate your coding experience from 1 (no experience) to 5 (plenty of experience)!
- Share your answers with the class!
- We'll visualize this dataset using Python later today
  - Link to excel sheet here

Outline

Machine Learning

- Most recent exciting technology

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- We use these algorithms dozens of times a day

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    - Search Engine

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  - Recommendations

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- Machine Learning is an important component to achieve the big AI dream

Machine Learning

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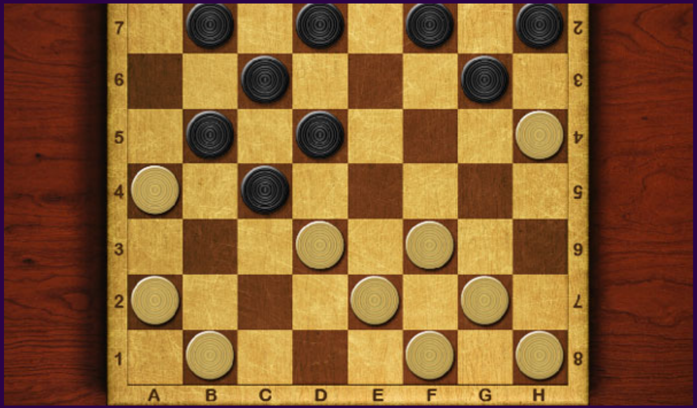
    - Recommendations
- Machine Learning is an important component to achieve the big AI dream
- Practice is the key to learn machine learning

Definition

- Machine Learning is a field of study that gives computers the ability to learn without being explicitly programmed.



Example: Digit Recognition

- Challenges with expert approach
  - Simple expert rule breaks down in practice
  - Difficult to translate our knowledge into code
- Machine Learning approach
  - Learned systems do very well on image recognition problems

```
def classify(image):
    ...
    nv = count_vert_lines(image)
    nh = count_horiz_lines(image)
    ...

    if (nv == 1) and (nh == 1):
        digit = 7
    ...

    return digit
```

Machine Learning Problem Pipeline

1. Gather data

Machine Learning Problem Pipeline

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2. Visualize the data

Machine Learning Problem Pipeline

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2. Visualize the data
3. Formulate ML problem

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   - Regression vs Classification

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   - Choose an appropriate cost function

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5. Evaluate the model on a test set
   - If the performance is not satisfactory, go back to step 4

Outline

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Course Outline

- Day 1: Introduction to ML
- Day 2: Linear Regression
- Day 3: Overfitting and Generalization
- Day 4: Classification and Logistic Regression
- Day 5: Mini Project
- Day 6: Neural Networks
- Day 7: Convolutional Neural Networks
- Day 8: Deep Generative Models
- Day 9: Final Project
- Day 10: Social Impacts of ML and Final Project Presentations

Course Format, Website, Resources

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  - No installation required
- We'll give additional resources at the end of each day based on student interest

Outline

Setting Up Python

- Google Colab
  - Interactive programming online
  - No installation
  - Free GPU for 12 hours
- Your task:

  Register a Google account and set up Google Colab

  Run `print('hello world!')`

  Save the notebook on your computer

Modules/Libraries/Packages

- NumPy: math, vectors and matrices
- MatPlotLib: plotting graphs, visualizing data
- Pandas: convenient for storing and retrieving data
- Sklearn: convenient wrapper for simple ML problems
- PyTorch: deep learning
- Your task:
  Run `import numpy as np`
  Play with `np.add()`
  Open `demo_python_basics.ipynb` from Github

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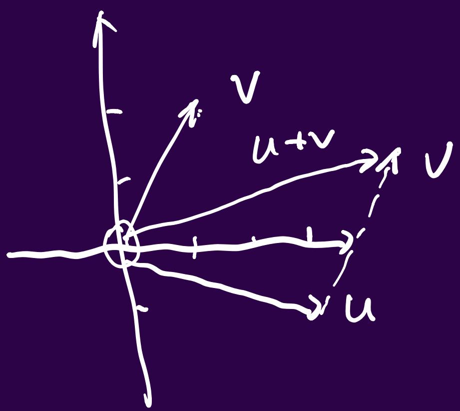
- If-Statements
  - Conditionally execute lines of code
- Functions
  - Reuse lines of code at any time

Outline

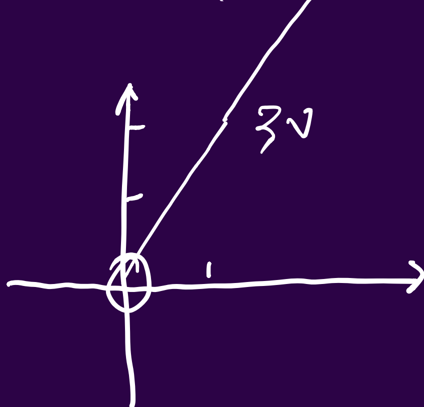
Vectors

- A **vector** is an ordered list of numbers or symbols
  - Represents quantities with magnitude and direction.
  - Ex: Planar force: $\mathbf{v} = (1, 2)$



Vectors

- Vectors of the same size may be added together, element-wise
  - Ex: $\mathbf{u} = (3, -1)$
    $\mathbf{v} + \mathbf{u} = (1 + 3, 2 + (-1)) = (4, 1)$
- Vectors may be scaled by a number, element-wise
  - Ex: $3\mathbf{v} = (3 \times 1, 3 \times 2) = (3, 6)$



Vectors

- Norm of a vector (L2 Norm)
  - Ex: $\|\mathbf{v}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$
- Inner product: sum of element-wise products of two vectors
  - Ex: $\mathbf{u} \cdot \mathbf{v} = 1 \times 3 + 2 \times (-1) = 3 - 2 = 1$
  - Gives the angle between two vectors $\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$

$\cos\theta \|q\|$

$\|p\| = 1$

$q \cdot p = \cos\theta \|q\|$

Exercise: Vectors

Let $\mathbf{p} = (3, 2, 9, 4)$ and $\mathbf{q} = (1, 9, 0, 3)$, calculate

- $3\mathbf{q} + 2\mathbf{p}$
- $\mathbf{q} \cdot \mathbf{q}$ and $\|\mathbf{q}\|^2$
- $\mathbf{p} \cdot \mathbf{q}$ and $\|\mathbf{p}\| \, \|\mathbf{q}\|$
- $\left\| \frac{\mathbf{p}}{\|\mathbf{p}\|} \right\|$

$$3q + 2p = (9, 31, 18, 17)$$

$$p = (3, 2, 9, 4)$$
$$q = (1, 9, 0, 3)$$

$$3q + 2p = 3(1, 9, 0, 3) + 2(3, 2, 9, 4)$$
$$= (3\times 1, 3\times 9, 3\times 0, 3\times 3) + (2\times 3, 2\times 2, 2\times 9, 2\times 4)$$

$$q \cdot q = 91 = \|q\|^2$$
$$q \cdot q = (1, 9, 0, 3) \cdot (1, 9, 0, 3)$$
$$= 1\times 1 + 9\times 9 + 0\times 0 + 3\times 3$$
$$= 1^2 + 9^2 + 0^2 + 3^2$$
$$= \Big(\underbrace{\sqrt{1^2 + 9^2 + 0^2 + 3^2}}_{\|q\|}\Big)^2$$

$$p \cdot q = 33 \qquad \|p\|\,\|q\| = \sqrt{110}\,\sqrt{91}$$

$p \cdot q = 33 \quad \|p\|\,\|q\|$

$$\left\| \frac{p}{\|p\|} \right\| = 1$$

$\dfrac{p}{\|p\|}$: a normalized vector

the norm for such vectors is always 1

$$\left(x, y \quad \left\| \frac{x}{\|y\|} \right\| = \frac{\|x\|}{\|y\|} = 1 \right)$$

Matrices

- A **matrix** is a rectangular array of numbers or symbols arranged in rows and columns. We can conceptualize it as a collection of vectors.

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  - Ex: 2 by 2 matrix, $M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$$\begin{array}{cc} u & v \end{array}$$

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  - Ex: $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 8 \\ 7 & 11 \end{bmatrix}, A + B = \begin{bmatrix} 1 & 9 \\ 9 & 12 \end{bmatrix}$

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- Matrices may be scaled, element-wise
  - Ex: $aB = \begin{bmatrix} 0 & 8a \\ 7a & 11a \end{bmatrix}$, where a is a scalar

Exercise: Matrices

- $\begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 9 & 2 \\ -7 & 6 \\ 3 & 1 \end{bmatrix} = ?$ $\begin{bmatrix} 1+9 & 3+2 \\ 2-7 & -1+6 \\ 4+3 & 7+1 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ -5 & 5 \\ 7 & 8 \end{bmatrix}$
- $2\begin{bmatrix} 1 & 9 \\ 3 & -2 \end{bmatrix} = ?$ $\begin{bmatrix} 2 & 18 \\ 6 & -4 \end{bmatrix}$
- $2\left(\begin{bmatrix} 5 & 1 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 2 & -8 \end{bmatrix}\right) + 3\begin{bmatrix} -2 & 1 \\ 4 & -7 \end{bmatrix} = ?$

$$2\begin{bmatrix} 5 & 1 \\ 1 & -3 \end{bmatrix} + 2\begin{bmatrix} 3 & 4 \\ 2 & -8 \end{bmatrix}$$

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$$2\left(\underbrace{\begin{bmatrix} 5 & 1 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 2 & -8 \end{bmatrix}}\right) + 3\begin{bmatrix} -2 & 1 \\ 4 & -7 \end{bmatrix}$$

$$= 2\begin{bmatrix} 8 & 5 \\ 3 & -11 \end{bmatrix} + 3\begin{bmatrix} -2 & 1 \\ 4 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 13 \\ 18 & -43 \end{bmatrix}$$

distribution law

x, y, z $\quad z(x+y) = zx + zy$

for matrices, α - scalar

A, B - matrices

$$\alpha(A+B) = \alpha A + \alpha B$$

Vectors and Matrices

- We may consider a vector as a matrix

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  - **Row Vector**: shape (1 x N)

$$v = (1, 2, 3)$$

$$v = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

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    Ex: $\mathbf{v} = \begin{bmatrix} 1 & 2 \end{bmatrix}$
  - **Column Vector**: shape (N x 1)
    Ex: $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

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Vectors and Matrices

- We may consider a vector as a matrix
  - **Row Vector**: shape (1 x N)
    Ex: $\mathbf{v} = \begin{bmatrix} 1 & 2 \end{bmatrix}$
  - **Column Vector**: shape (N x 1)
    Ex: $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
  - We'll consider vectors as <u>column vectors by default</u>

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Matrix-vector multiplication

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  - What is 2 times force **u** plus 3 times force **v**?

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  - Let us define matrix-vector multiplication such that

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2 \times \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 3 \times \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

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  - What is 4 times force **u** plus 2 times force **v** as matrix-vector multiplication?

$$\boxed{\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}} = 4 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 8 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \boxed{\begin{bmatrix} 10 \\ 6 \end{bmatrix}}$$

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  - Take a guess: $\underbrace{\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}} = ?$ $\begin{bmatrix} 11 & 10 \\ 1 & 6 \end{bmatrix}$

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  - Take a guess: $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = ?$
  - More in the afternoon.

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  - Words and sentences are treated as lists of characters

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- Classes (advanced)

Python Basics

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- Loops
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- Strings
  - Words and sentences are treated as lists of characters
- Classes (advanced)
  - Making your own data-type. Functions and variables made to be associated with it too.

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Lab: Python Basics

- Write a function to find the second largest number in a list (Hint: use sort())
- Define a class which has at least two methods:
  - getString: to get a string from console input (Hint: use input())
  - printString: to print the string in upper case (Hint: use upper())
- Write a test function to test the class methods.

Outline

More About Matrices

- Matrices may be multiplied together provided their shapes meet the criteria

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- Matrices may be multiplied together provided their shapes meet the criteria
  - $(AB)_{ij} = \sum_{k=1}^{N} A_{ik} B_{kj}$

$$\sum_{\boxed{k=1}}^{N} a_k = a_1 + a_2 + \dots a_N$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A_{11} = 1$

$A_{12} = 2$

$A_{21} = 2$

$A_{22} = 3$

$A_{31} = 1$

$$(AB)_{ij} = A_{i1}B_{1j} + A_{i2}B_{2j} + \dots + A_{iN}B_{Nj}$$

$C = AB$

i : 1 to # rows of C

j : 1 to # cols of C

More About Matrices

- Matrices may be multiplied together provided their shapes meet the criteria
  - $(AB)_{ij} = \sum_{k=1}^{N} A_{ik} B_{kj}$
  - Inner product of the i-th row of A and the j-th column of B

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$u = (u_1, u_2, \ldots, u_N)$$
$$v = (v_1, v_2, \ldots v_N)$$
$$u \cdot v = \sum_{i=1}^{N} u_i v_i$$

$C = AB$

C_{ij} 3 by 2 matrix

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  - Inner product of the i-th row of A and the j-th column of B
  - Criteria: for AB to be valid, # cols of A must equal the # rows of B

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -3 \\ -5 & 2 \end{bmatrix}$

$C = AB$

$C_{11} = (1,2) \cdot (1,-5)$

$C_{12} = (1,2) \cdot (-3,2)$

$C_{21} = (2,3) \cdot (1,-5)$

$C_{22} = (2,3) \cdot (-3,2)$

$C_{31} = (1,3) \cdot (1,-5)$

$C_{32} = (1,3) \cdot (-3,2)$

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  - Result is a matrix with shape (# rows A, # cols B)
  - $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = ?$, $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = ?$

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  - Inner product of the i-th row of A and the j-th column of B
  - Criteria: for AB to be valid, # cols of A must equal the # rows of B
  - Result is a matrix with shape (# rows A, # cols B)
  - $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = ?$, $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = ?$
  - $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = ?$

More About Matrices

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- In general, $AB \neq BA$

$C = \begin{bmatrix} 10 & 2 \\ 7 & 7 \end{bmatrix}$

$C_{11} : (2,4) \cdot (1,2) = 10$
$C_{12} : (2,4) \cdot (3,-1) = 2$
$C_{21} : (3,2) \cdot (1,2) = 7$
$C_{22} : (3,2) \cdot (3,-1) = 7$

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- $\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = ?$ $\begin{bmatrix} 2+9 & 4-3 \end{bmatrix} = \underline{\begin{bmatrix} 11 & 1 \end{bmatrix}}$

T

$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 11 \\ 1 \end{bmatrix}$

T

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- $(AB)^T = B^T A^T$

$v \cdot v = v^T v = \|v\|^2$

Ex. $v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$v^T = \begin{bmatrix} 1 & 2 \end{bmatrix}$

$v^T v = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= 5$

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  - Square matrices only!

Exercise: Playing with NumPy

- NumPy: Python package for linear algebra
- Open `demo_vectors_matrices.ipynb`

Outline

Demo: Plotting Functions

- Generate and plot the following functions in Python:
  - Scatter plot of points: (0,1), (2,3), (5,2), (4,1)
  - Straight Line: $y = mx + b$
  - Sine-wave $y = sin(x)$
  - Polynomial e.g. $y = x^3 + 2$
  - Exponential e.g. $y = e^-2x$
  - Gaussian (Use $\sigma = 0.5$)
  - Choose a function of your own
- Create separate plots for each of the functions, Compute the mean and variance of each function
- Use Wikipedia and Numpy Documentation to search for mathematical formulas and python functions

Outline

Looking at our ice-breaker data in spreadsheets

- Columns have labels in the first row
- Collected data (numbers, words) follow below
- Let's export it to a Comma-Separated Values (CSV) file and open it

These slides have been modified from the original slides provided through the courtesy of Nikola, Akshaj, Aishwarya, and Jack.

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