

# Perspective Games

Now you see me, now you don't

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# Introduction

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## Perspective Games

**Partial Information games** : Games where players don't have all the information about the game arena and game state at all times.

**Transverse Uncertainty** : Sometimes player see all the information, some times they do not, like:

- Concurrent threads given control by a scheduler
- Clients connecting to a server, one at a time.
- Games with concurrent moves

**Perspective Strategies** or **P-strategies** are strategies, depending only on the part of the run that is within a player's domain, given by  $\rho : V_1^\oplus \rightarrow V_1$ .

## Formal Description

Game Graph  $G = \langle V_1, V_2, E, v_0, AP, \tau \rangle$

- $AP$  is a finite set of atomic propositions.
- $\tau : V_1 \sqcup V_2 \rightarrow 2^{AP}$  assigns which atomic propositions are true in which locations.

Given a run  $\rho = (\rho_1 \cdot \rho_2 \cdot \rho_3 \cdots) : V^\omega$ , the computation of a run  $\tau(\rho) : (2^{AP})^\omega$  is defined as

$$\tau(\rho) = \tau(\rho_1) \cdot \tau(\rho_2) \cdot \tau(\rho_3) \cdots$$

Winning condition  $L$  decides which runs are winning, given as one of the following:

- An LTL formula
- An  $\omega$ -automata

Thus  $L \subseteq (2^{AP})^\omega$ .

## Perspective Strategies

Given a game graph  $G$ , let the set of all finite runs on it be defined as  $V^*$ .

Given a finite run  $\rho$ , we define  $\pi_p : V^* \rightarrow V_p^*$  as the projection map, which drops all vertices in the run not belonging to player  $p$ .

A **Perspective strategy**  $\sigma_p$  for a player  $p$  takes the part of a run restricted to player  $p$  vertices and decides a move based on that.

Thus, given a function  $f : V_p^* \rightarrow V_p$ , the following describes a **P-strategy**:

$$\sigma_p(\rho) := f(\pi_1(\rho))$$

We consider games where each player can have a different kind of strategy, e.g. in a PF-game, player 1 has a perspective strategy and player 2 has an ordinary (or full) strategy.

## Deterministic Setting

We say that a strategy  $\sigma$  is **winning** for player 1, if for any strategy  $\tau$  for player 2, the play induced by the strategies satisfies the winning condition.

Some nice theorems:

- Given a game  $\mathcal{G}$ 
  - Player 1 FF-wins  $\mathcal{G}$  iff Player 1 FP-wins  $\mathcal{G}$
  - Player 1 PF-wins  $\mathcal{G}$  iff Player 1 PP-wins  $\mathcal{G}$
- There is a game  $\mathcal{G}$  such that player 1  $F$ -wins  $\mathcal{G}$  but does not  $P$ -win  $\mathcal{G}$ .
- Perspective Games are not determined.

## Probabalistic Setting

A probabalistic strategy for player  $i$  is a function  $V^*V_i \rightarrow \mathcal{D}(V)$ , where  $\mathcal{D}(V)$  is the set of probability distributions on  $V$ . Given strategies  $g_1$  and  $g_2$  for both players we define

$$\mathcal{P}_{g_1, g_2}(L) = [\![\rho \text{ is in } L \mid \rho \text{ is generated using } g_1, g_2]\!]$$

A strategy  $\sigma$  is **almost winning** for player 1 if for every strategy  $\tau$  we have  $\mathcal{P}_{g_1, g_2}(L) = 1$ .

Some nice Theorems:

- There is a game  $\mathcal{G}$  that is PF-almost winning for player 1, but not  $P$ -winning.
- There is a game  $\mathcal{G}$  that is PP-almost winning but not PF-almost winning for player 1.
- Perspective games are not almost-determined.

# Deterministic Setting Analysis

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# Results

# Alternating Tree Automata

# Upper Bound

# Lower Bound

# LTL Winning Condition

# Perspective ATL\* Model Checking

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## Perspective ATL\*

ATL\* is an extension of the logic CTL\* which captures the existence of strategies in a game. Perspective-ATL\* lets us quantify over perspective startegies.

There are 2 types of formulas

- State formulas  $\varphi$

$$\varphi ::= AP \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\!\langle S \rangle\!\rangle \psi \mid \langle\!\langle S \rangle\!\rangle_p \psi$$

- And there are path formulas  $\psi$

$$\psi ::= \varphi \mid \neg\psi \mid \psi \vee \psi \mid \bigcirc \psi \mid \psi \mathcal{U} \psi$$

There is also the logic ATL (similarly perspective-ATL), which simplifies ATL\* by forcing all path operators to be preceeded by path quantifiers. eg  $\langle\!\langle 1 \rangle\!\rangle \bigcirc \bigcirc p$  is not allowed.

# Model Checking

Model Checking is the problem of verifying if a given model  $M$  satisfies a given formula  $\varphi$ .

The model checking problem for perspective-ATL\* is 2-EXPTIME-complete. The model checking problem for perspective-ATL is PTIME-complete.

## Other Fun Stuff

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# Probabalistic Setting

# Structural Winning Condition

# Memoryless Strategies

**Thank You!**

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