

Perspective Games

Now you see me, now you don't

Shubh Sharma, Siddhant Agarwal

2025-11-25

Chennai Mathematical Institute

Introduction

Perspective Games

Partial Information games : Games where players don't have all the information about the game arena and game state at all times.

Transverse Uncertainty : Sometimes player see all the information, some times they do not, like:

- Concurrent threads given control by a scheduler
- Clients connecting to a server, one at a time.
- Games with concurrent moves

Perspective Strategies or **P-strategies** are strategies, depending only on the part of the run that is within a player's domain, given by $\rho : V_1^\oplus \rightarrow V_1$.

Formal Description

Game Graph $G = \langle V_1, V_2, E, v_0, AP, \tau \rangle$

- AP is a finite set of atomic propositions.
- $\tau : V_1 \sqcup V_2 \rightarrow 2^{AP}$ assigns which atomic propositions are true in which locations.

Given a run $\rho = (\rho_1 \cdot \rho_2 \cdot \rho_3 \cdots) : V^\omega$, the computation of a run $\tau(\rho) : (2^{AP})^\omega$ is defined as

$$\tau(\rho) = \tau(\rho_1) \cdot \tau(\rho_2) \cdot \tau(\rho_3) \cdots$$

Winning condition L decides which runs are winning, given as one of the following:

- An LTL formula
- An ω -automata

Thus $L \subseteq (2^{AP})^\omega$.

Perspective Strategies

Given a game graph G , let the set of all finite runs on it be defined as V^* .

Given a finite run ρ , we define $\pi_p : V^* \rightarrow V_p^*$ as the projection map, which drops all vertices in the run not belonging to player p .

A **Perspective strategy** σ_p for a player p takes the part of a run restricted to player p vertices and decides a move based on that.

Thus, given a function $f : V_p^* \rightarrow V_p$, the following describes a **P-strategy**:

$$\sigma_p(\rho) := f(\pi_1(\rho))$$

We consider games where each player can have a different kind of strategy, e.g. in a PF-game, player 1 has a perspective strategy and player 2 has an ordinary (or full) strategy.

Deterministic Setting Analysis

Introduction

We say that a strategy σ is **winning** for player 1, if for any strategy τ for player 2, the play induced by the strategies satisfies the winning condition.

Some nice theorems:

- Given a game \mathcal{G}
 - Player 1 FF-wins \mathcal{G} iff Player 1 FP-wins \mathcal{G}
 - Player 1 PF-wins \mathcal{G} iff Player 1 PP-wins \mathcal{G}
- There is a game \mathcal{G} such that player 1 F -wins \mathcal{G} but does not P -win \mathcal{G} .
- Perspective Games are not determined.

Results

Deciding whether Player 1 P -wins and finding a P -strategy in a perspective game $\langle G, \mathcal{U} \rangle$ is EXPTIME-complete when \mathcal{U} is a universal automata with a parity or a reachability winning condition. The problem can be solved in time polynomial in $|G|$ and exponential in $|\mathcal{U}|$.

Deciding whether Player 1 P -wins and finding a P -strategy in a perspective game $\langle G, \psi \rangle$ is 2EXPTIME-complete when ψ is an LTL specification. The problem can be solved in time polynomial in $|G|$ and doubly exponential in $|\psi|$.

Outline for Upper Bound

- Perform a polynomial conversion from winning condition \mathcal{U} to an alternating tree automaton $\mathcal{A}_{\mathcal{G}}$.
- Show that finding a P -strategy for Player 1 is equivalent to finding an accepting run in $\mathcal{A}_{\mathcal{G}}$.
- Solving the emptiness problem for $\mathcal{A}_{\mathcal{G}}$ gives an upper bound for finding Player 1 P -strategy.

Tree Automata

A Tree Automata is given by the following tuple

$$\mathcal{A} = \langle \Sigma, Q, q_{\text{in}}, \delta, \alpha \rangle$$

where

- $\alpha \subseteq Q^\omega$ is the winning condition.
- $\delta : Q \times \Sigma \rightarrow Q^*$ gives the list of locations corresponding to children of the tree.
- Σ is set of labels for the nodes of the tree.

A run of the automata on a tree T can be thought as mapping the tree along the edges of the automata. It can be given as a function $r : T \rightarrow Q$ such that.

- $r(\text{root}_T) = q_{\text{in}}$
- If v is a node labelled by a with children $v_1, v_2 \dots v_n$ such that $r(v) = q$. Let $\delta(q, a) = q_1, q_2 \dots q_n$ we have $r(v_i) = q_i$.

Add Diagram

Non-deterministic and Universal tree automata transitions return a list of words. Both of these can be combined and represented uniquely by describing an alternating tree automata where the transitions have the type:

$$\delta : Q \times \Sigma \rightarrow \mathcal{B}(Q \times D)$$

Where $\mathcal{B}(Q \times D)$ are positive boolean formulas. Note that runs of ATAs can be over more than 1 trees. One would pick a set nodes, and directions for edge labels.

Upper Bound (Parity Automata)

Consider a universal parity automata cal U

Lower Bound

LTL Winning Condition

Probabilistic Setting

Introduction

A probabilistic strategy for player i is a function $V^*V_i \rightarrow \mathcal{D}(V)$, where $\mathcal{D}(V)$ is the set of probability distributions on V . Given strategies g_1 and g_2 for both players we define

$$\mathcal{P}_{g_1, g_2}(L) = [\![\rho \text{ is in } L \mid \rho \text{ is generated using } g_1, g_2]\!]$$

A strategy σ is **almost winning** for player 1 if for every strategy τ we have $\mathcal{P}_{g_1, g_2}(L) = 1$.

Some nice Theorems:

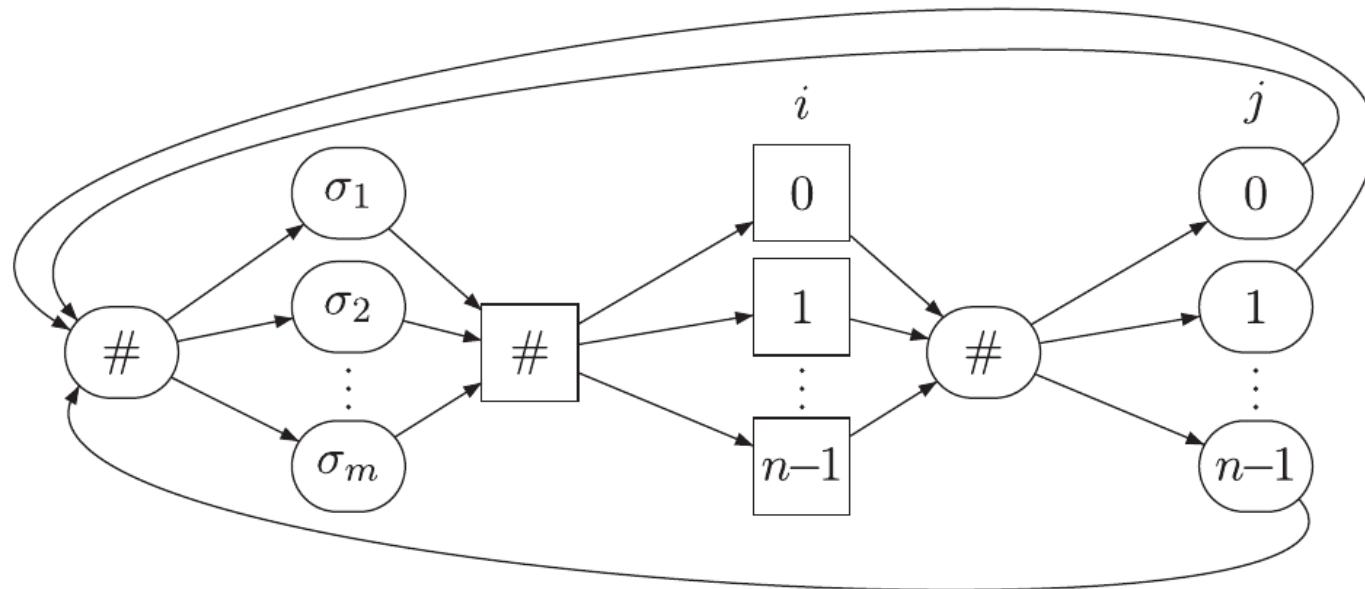
- There is a game \mathcal{G} that is PF-almost winning for player 1, but not P -winning.
- There is a game \mathcal{G} that is PP-almost winning but not PF-almost winning for player 1.
- Perspective games are not almost-determined.

Undecidability

Deciding whether Player 1 (P, F) -almost wins and whether they (P, P) -almost win a perspective game with a deterministic co-Büchi condition is undecidable.

- This can be shown by reduction from emptiness problem of a simplified probabilistic co-Büchi automata (PCW) which has already been proved to be undecidable.
- A simplified PCW is $\mathcal{P} = \langle \Sigma, Q, q_0, \delta, \alpha \rangle$ with the transition function $\delta : Q \times \Sigma \rightarrow \langle q_1, \dots, q_n \rangle$, where $\delta(q, \sigma)$ is the tuple of states that the automata can transition to, each with $\frac{1}{n}$ probability. A word $w \in \Sigma^*$ is accepted by \mathcal{P} if the acceptance probability of w in \mathcal{P} (denoted $\text{Prp}(w)$) is 1.
- We construct a game $\mathcal{G} = \langle G, \mathcal{A} \rangle$, where \mathcal{A} is a deterministic co-Büchi automata such that \mathcal{P} is nonempty iff Player 1 (P, F) -almost wins \mathcal{G} . Intuitively, the probabilistic transitions of \mathcal{P} are simulated by randomized strategies of players in \mathcal{G} .

Undecidability



Undecidability

- We create \mathcal{G} with a game graph that looks like the above. A play in G is an infinite sequence of rounds, such that in each round Player 1 chooses $\sigma \in \Sigma$, Player 2 chooses an index $i \in \{0, \dots, n - 1\}$ and then Player 1 chooses an index $j \in \{0, \dots, n - 1\}$
- Since Player 1 only has perspective visibility, the chose of i and j are independent. So, each player in \mathcal{G} has the possibility to ensure exact simulation of the probabilistic transitions of \mathcal{P} by choosing transitions to the $\{0, \dots, n - 1\}$ vertices in G uniformly at random. If Player 2 chooses i uniformly at random and then Player 1 chooses j without knowing i , the index $(i + j) \bmod n$ is distributed uniformly in $\{0, \dots, n - 1\}$.

Undecidability

- If \mathcal{P} is nonempty and $w \in L(\mathcal{P})$, let g_1 be a randomized P -strategy of Player 1. Since for every random choice of Player 2 the index $(i + j) \bmod n$ is distributed uniformly, we have for every randomized strategy g_2 of Player 2, $\Pr_{g_1, g_2}(L(\mathcal{A})) = \text{Prp}(w) = 1$
- Assume \mathcal{P} is empty. Let g_2 be a randomized P -strategy of Player 2 such that $i \in \{0, \dots, n - 1\}$ is chosen uniformly at random. For every randomized strategy g_1 of Player 1, we have that $\Pr_{g_1, g_2}(L(\mathcal{A}))$ is the probability that P accepts a word w that is drawn according to some distribution that is induced by g_1 . Since $\text{Prp}(w) < 1$ for every w , we also have $\Pr_{g_1, g_2}(L(\mathcal{A})) < 1$
- Every strategy discussed for Player 2 has also been perspective, so we also have \mathcal{P} is nonempty iff Player 1 (P, P) -almost wins.

Perspective ATL* Model Checking

Perspective ATL*

ATL* is an extension of the logic CTL* which captures the existence of strategies in a game. Perspective-ATL* lets us quantify over perspective strategies.

There are 2 types of formulas

- State formulas φ

$$\varphi ::= AP \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\langle S \rangle\rangle \psi \mid \langle\langle S \rangle\rangle_p \psi$$

- And there are path formulas ψ

$$\psi ::= \varphi \mid \neg\psi \mid \psi \vee \psi \mid \bigcirc \psi \mid \psi \mathcal{U} \psi$$

There is also the logic ATL (similarly perspective-ATL), which simplifies ATL* by forcing all path operators to be preceded by path quantifiers. eg $\langle\langle 1 \rangle\rangle \bigcirc \bigcirc p$ is not allowed.

Model Checking

Model Checking is the problem of verifying if a given model M satisfies a given formula φ .

The model checking problem for Perspective-ATL* is 2-EXPTIME-complete. The model checking problem for Perspective-ATL is PTIME-complete.

Conclusion

Structural Winning Condition

Memoryless Strategies

Thank You!

Thank you for attending our presentation. If you are interested in reading the paper, it can be found here:



<https://dl.acm.org/doi/10.1145/3627705>