Perspective Games

Now you see me, now you don’t

Shubh Sharma, Siddhant Agarwal

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Chennai Mathematical Institute

Introduction

Perspective Games

Partial Information games : Games where players don't have all the information about the game arena and game state at all times.

Transverse Uncertainty : Sometimes player see all the information, some times they do not, like:

- Concurrent threads given control by a scheduler
- Clients connecting to a server, one at a time.
- Games with concurrent moves

> Perspective Strategies or P-strategies are strategies, depending only on the part of the run that is within a player's domain, given by $\rho : V_1^{\oplus} \to V_1$.

Formal Description

Game Graph $G = \langle V_1, V_2, E, v_0, \mathrm{AP}, \tau \rangle$

- AP is a finite set of atomic propositions.
- $\tau : V_1 \sqcup V_2 \to 2^{\mathrm{AP}}$ assigns which atomic propositions are true in which locations.

Given a run $\rho = (\rho_1 \cdot \rho_2 \cdot \rho_3 \cdots) : V^\omega$, the computation of a run $\tau(\rho) : \left(2^{\mathrm{AP}}\right)^\omega$ is defined as

$$\tau(\rho) = \tau(\rho_1) \cdot \tau(\rho_2) \cdot \tau(\rho_3) \cdots$$

Winning condition L decides which runs are winning, given as one of the following:

- An LTL formula
- An ω-automata

Thus $L \subseteq \left(2^{\mathrm{AP}}\right)^\omega$.

Perspective Strategies

Given a game graph G, let the set of all finite runs on it be defined as $V^{\circledast}$.

Given a finite run ρ, we define $\pi_p : V^{\circledast} \to V_p^*$ as the projection map, which drops all vertices in the run not belonging to player p.

> A Perspective strategy σ_p for a player p takes the part of a run restricted to player p vertices and decides a move based on that.
>
> Thus, given a function $f : V_p^* \to V_p$, the following describes a P-strategy:
>
> $$\sigma_p(\rho) :\equiv f(\ \pi_1(\rho)\ )$$

We consider games where each player can have a different kind of strategy,eg. in a PF-game, player 1 has a perspective strategy and player 2 has an ordinary (or full) strategy.

Deterministic Setting

We say that a strategy σ is winning for player 1, if for any strategy τ for player 2, the play induced by the strategies satisfies the winning condition.

Some nice theorems:

- Given a game $\mathcal{G}$
  - Player 1 FF-wins $\mathcal{G}$ iff Player 1 FP-wins $\mathcal{G}$
  - Player 1 PF-wins $\mathcal{G}$ iff Player 1 PP-wins $\mathcal{G}$
- There is a game $\mathcal{G}$ such that player 1 F-wins $\mathcal{G}$ but does not P-win $\mathcal{G}$.
- Perspective Games are not determined.

Probabalistic Setting

A probabalistic strategy for player i is a function $V^{\circledast}V_i \to \mathcal{D}(V)$, where $\mathcal{D}(V)$ is the set of probability distributions on V. Given strategies g_1 and g_2 for both players we define

$$\mathcal{P}_{g_1,g_2}(L) = [\![\rho \text{ is in } L \mid \rho \text{ is generated using } g_1, g_2]\!]$$

A strategy σ is almost winning for player 1 if for every strategy τ we have $\mathcal{P}_{g_1,g_2}(L) = 1$.

Some nice Theorems:

- There is a game $\mathcal{G}$ that is PF-almost winning for player 1, but not P-winning.
- There is a game $\mathcal{G}$ that is PP-almost winning but not PF-almost winning for player 1.
- Perspective games are not almost-determined.

Deterministic Setting Analysis

Results

Alternating Tree Automata

Upper Bound

Lower Bound

LTL Winning Condition

Perspective ATL^* Model Checking

Perspective ATL^*

Model Checking

Other Fun Stuff

Probabalistic Setting

Structural Winning Condition

Memoryless Strategies

Thank You!