Perspective Games

Now you see me, now you don’t

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Introduction

Perspective Games

Partial Information games : Games where players don't have all the information about the game arena and game state at all times.

Transverse Uncertainty : Sometimes player see all the information, some times they do not, like:

- Concurrent threads given control by a scheduler
- Clients connecting to a server, one at a time.
- Games with concurrent moves

Perspective Strategies or P-strategies are strategies, depending only on the part of the run that is within a player's domain, given by $\rho : V_1^{\oplus} \to V_1$.

Formal Description

Game Graph $G = \langle V_1, V_2, E, v_0, \mathrm{AP}, \tau \rangle$

- AP is a finite set of atomic propositions.
- $\tau : V_1 \sqcup V_2 \to 2^{\mathrm{AP}}$ assigns which atomic propositions are true in which locations.

Given a run $\rho = (\rho_1 \cdot \rho_2 \cdot \rho_3 \cdots) : V^\omega$, the computation of a run $\tau(\rho) : \left(2^{\mathrm{AP}}\right)^\omega$ is defined as

$$\tau(\rho) = \tau(\rho_1) \cdot \tau(\rho_2) \cdot \tau(\rho_3) \cdots$$

Winning condition L decides which runs are winning, given as one of the following:

- An LTL formula
- An ω-automata

Thus $L \subseteq \left(2^{\mathrm{AP}}\right)^\omega$.

Perspective Strategies

Given a game graph G, let the set of all finite runs on it be defined as $V^{\circledast}$.

Given a finite run ρ, we define $\pi_p : V^{\circledast} \to V_p^*$ as the projection map, which drops all vertices in the run not belonging to player p.

> A Perspective strategy σ_p for a player p takes the part of a run restricted to player p vertices and decides a move based on that.
>
> Thus, given a function $f : V_p^* \to V_p$, the following describes a P-strategy:
>
> $$\sigma_p(\rho) :\equiv f(\ \pi_1(\rho)\ )$$

We consider games where each player can have a different kind of strategy,eg. in a PF-game, player 1 has a perspective strategy and player 2 has an ordinary (or full) strategy.

Deterministic Setting

We say that a strategy σ is winning for player 1, if for any strategy τ for player 2, the play induced by the strategies satisfies the winning condition.

Some nice theorems:

- Given a game $\mathcal{G}$
  - Player 1 FF-wins $\mathcal{G}$ iff Player 1 FP-wins $\mathcal{G}$
  - Player 1 PF-wins $\mathcal{G}$ iff Player 1 PP-wins $\mathcal{G}$
- There is a game $\mathcal{G}$ such that player 1 F-wins $\mathcal{G}$ but does not P-win $\mathcal{G}$.
- Perspective Games are not determined.

Probabalistic Setting

A probabalistic strategy for player i is a function $V^{\circledast}V_i \to \mathcal{D}(V)$, where $\mathcal{D}(V)$ is the set of probability distributions on V. Given strategies g_1 and g_2 for both players we define

$$\mathcal{P}_{g_1,g_2}(L) = [\![\rho \text{ is in } L \mid \rho \text{ is generated using } g_1, g_2]\!]$$

A strategy σ is almost winning for player 1 if for every strategy τ we have $\mathcal{P}_{g_1,g_2}(L) = 1$.

Some nice Theorems:

- There is a game $\mathcal{G}$ that is PF-almost winning for player 1, but not P-winning.
- There is a game $\mathcal{G}$ that is PP-almost winning but not PF-almost winning for player 1.
- Perspective games are not almost-determined.

Deterministic Setting Analysis

Results

Deciding whether Player 1 P-wins and finding a P-strategy in a perspective game $\langle G, \mathcal{U} \rangle$ is $\mathrm{EXPTIME}$-complete when $\mathcal{U}$ is a universal automata with a parity or a reachability winning condition. The problem can be solved in time polynomial in $|G|$ and exponential in $|\mathcal{U}|$.

Deciding whether Player 1 P-wins and finding a P-strategy in a perspective game $\langle G, \psi \rangle$ is $2\mathrm{EXPTIME}$-complete when ψ is an LTL specification. The problem can be solved in time polynomial in $|G|$ and doubly exponential in $|\psi|$.

Alternating Tree Automata

Upper Bound

Lower Bound

LTL Winning Condition

Perspective ATL* Model Checking

Perspective ATL*

ATL* is an extension of the logic CTL* which captures the existence of strategies in a game. Perspective-ATL* lets us quantify over perspective startegies.

There are 2 types of formulas

- State formulas φ

$$\varphi ::= \mathrm{AP} \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\langle S \rangle\rangle \psi \mid \langle\langle S \rangle\rangle_p \psi$$

- And there are path formulas ψ

$$\psi ::= \varphi \mid \neg\psi \mid \psi \vee \psi \mid \bigcirc \psi \mid \psi \mathcal{U} \psi$$

There is also the logic ATL (similarly perspective-ATL), which simplifies ATL* by forcing all path operators to be preceeded by path quantifiers. eg $\langle\langle 1 \rangle\rangle \bigcirc \bigcirc p$ is not allowed.

Model Checking

Model Checking is the problem of verifying if a given model M satisfies a given formula φ.

The model checking problem for perspective-ATL^* is $\text{2-EXPTIME-complete}$. The model checking problem for perspective-ATL is PTIME-complete.

Other Fun Stuff

Probabalistic Setting

Structural Winning Condition

Memoryless Strategies

Thank You!

Thank You!

Thank you for attending our presentation. If you are interested in reading the paper, it can be found here:

https://dl.acm.org/doi/10.1145/3627705