Graded Generalized Algebraic Data Types

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October 12, 2022

Abstract

Write abstract

1 Introduction

Write the intro.

2 The Fundamental Theory

Suppose $\mathcal{C}$ is a category and $(\mathcal{E}, \otimes, I)$ is a strict monoidal category.

Definition 2.1 (Graded F-Algebra)**.** *For a functor $\mathsf{F} : \mathcal{E} \times \mathcal{C} \longrightarrow \mathcal{C}$, a graded F-algebra is a pair (A, h) that consists of a functor $\mathsf{A} : \mathcal{E} \longrightarrow \mathcal{C}$ and a family h of morphisms:*

$$h_{m,n} : \mathsf{F}(m, \mathsf{A}(n)) \longrightarrow \mathsf{A}(m * n)$$

A ***homomorphism*** *between two graded F-algebras (A, h) and (B, h') consists of a morphism*

$$\alpha : (\mathsf{A}, h) \longrightarrow (\mathsf{B}, h')$$

is defined as a natural transformation $\alpha : \mathsf{A}_1 \longrightarrow \mathsf{A}_2$ such that:

$$\mathsf{F}(\mathsf{m}, \alpha_{\mathsf{n}}); h'_{m,n} = h_{m,n}; \alpha_{m \otimes n}$$

Definition 2.2. *If the category of graded* F*-algebras has an initial object, then we call this a* ***graded initial*** F*-algebra denoted by* $(\mu\mathsf{F}, \mathsf{in})$*. That is, for any other* F*-algebra* (A, h) *there must be a unique morphism* $\alpha : (\mu\mathsf{F}, \mathsf{in}) \longrightarrow (\mathsf{A}, h)$*, but this implies that for any object* n*,* $\alpha_n : \mu\mathsf{F}(n) \longrightarrow \mathsf{A}(n)$ *is unique and* $\mu\mathsf{F}(n)$ *is an initial object in* $\mathcal{C}$*.*

Lambek's Lemma guarantees that $\mathsf{in}_{m,n} : \mathsf{F}(m, \mu\mathsf{F}(n)) \longrightarrow \mu\mathsf{F}(m * n)$ is an isomorphism.

References