A REAL-TIME MODEL PREDICTIVE CONTROL APPROACH FOR AUTONOMOUS ACTIVE STEERING

Paolo Falcone, Francesco Borrelli [*,1]
Jahan Asgari, H. Eric Tseng, Davor Hrovat [**]

[*] *Dipartimento di Ingegneria, Università del Sannio, 82100 Benevento, Italy*
{falcone,francesco.borrelli}@unisannio.it
[**] *Ford Research Laboratories, Dearborn, MI 48124, USA*
{jasgari,htseng,dhrovat}@ford.com

Abstract: The problem of controlling the front steering to stabilize a vehicle along a desired path is tackled in this paper. Although a Non-Linear Model Predictive Control (NLMPC) approach can achieve good performance and constraints fulfillment, its computational burden does not allow a real-time implementation. In order to decrease the complexity of the controller, in this paper we propose a suboptimal MPC scheme based on successive on-line linearizations of the non-linear vehicle model. The method stems from an accurate analysis of the vehicle nonlinearities, the constraints and the performance index in the optimal control problem. The simulation results show a significant reduction of the controller complexity, with a small loss of performances compared to a NLMPC controller.

Keywords: Nonlinear Model Predictive Control, Active Steering, Autonomous Systems

1. INTRODUCTION

Recent trends in automotive industry point in the direction of increased content of electronics, computers, and controls with emphasis on the improved functionality and overall system robustness. While this affects all of the vehicle areas, there is a particular interest in active safety. The available commercial active safety systems are mostly based on brake intervention. Anti-lock Braking Systems (ABS) and Electronic Stability Program (ESP) have been introduced to stabilize the longitudinal and yaw motion vehicle dynamics respectively. Future systems will be able to increase the effectiveness of active safety interventions by additional actuator types such as 4WS, active steering, active suspensions, or active differentials, and also by additional sensor information, such as the increased inclusion of onboard cameras, as well as infrared and other sensor alternatives. All these will be further complemented by GPS information including pre-stored mapping. In this context, it is possible to imagine that future vehicles would be able to identify obstacles on the road such as an animal, a rock, or fallen tree/branch, and assist the driver by following the best possible path, in terms of avoiding the obstacle and at the same time keeping the vehicle on the road at a safe distance from incoming traffic (Borrelli *et al.*, 2005), (Fujiwara *et al.*, 2005).

At this stage, we assume this "ultimate" obstacle avoidance system will be possible sometime in the future and we propose a double lane change sce-

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[1] Corresponding author.

nario on a slippery road, with a vehicle equipped with a fully autonomous steering system. The control input is the front steering angle and the goal is to follow the desired trajectory or target as close as possible while fulfilling various constraints reflecting vehicle physical limits. We assume a given desired trajectory and we will design a controller that can best follow the trajectory on slippery road at the highest possible entry speed.

Anticipating sensor and infrastructure trends toward increased integration of information and control actuation agents, it is then appropriate to ask what is the best and optimum way in controlling the vehicle maneuver for given obstacle avoidance situation. This will be done in the spirit of Model Predictive Control, MPC (Garcia *et al.*, 1989; Mayne *et al.*, 2000) along the lines of our ongoing internal research efforts dating from early 2000 (Asgari and Hrovat, 2002).

We use a model of the plant to *predict* the future evolution of the system (Mayne *et al.*, 2000; Borrelli, 2002; Keviczky *et al.*, 2006). Based on this prediction, at each time step t a performance index is optimized under operating constraints with respect to a sequence of future steering moves in order to best follow the given trajectory on a slippery road. The first of such optimal moves is the *control* action applied to the plant at time t. At time $t + 1$, a new optimization is solved over a shifted prediction horizon.

In (Borrelli *et al.*, 2005) a *nonlinear vehicle model* is considered to *predict* the future evolution of the system (Mayne *et al.*, 2000). The resulting MPC controller requires a non-linear optimization problem to be solved at each time step. Although good results have been achieved, even at high vehicle speed, the computational burden is a serious obstacle for experimental validation. In this paper we propose a sub-optimal MPC controller based on successive on-line linearizations of the non-linear vehicle model. The method stems from an accurate analysis of the vehicle nonlinearities, the constraints and the performance index in the optimal control problem. The simulation results show that the computational complexity of the proposed controller allows a real-time implementation. At the same time the expected performance loss, due to the approximation in the prediction model, is acceptable with respect to the NLMPC controller in (Borrelli *et al.*, 2005).

The paper is structured as follows. Section 2 describes the vehicle dynamical model used with a brief discussion on tire models. Section 3 formulates the control problem. The considered scenario and the simulation results are presented in Section 4. This is then followed by concluding remarks in Section 5 which highlight future research directions.

2. MODELING

This section describes the vehicle and tire model we used for simulations and control design. The nomenclature refers to the model depicted in Figure 1. We will also use the following notations:

Subscripts and Superscripts

$(\cdot)_f$	front wheel
$(\cdot)_r$	rear wheel
$(\cdot)_k$	time step
$(\cdot)_{ref}$	reference tracking signals
$\hat{(\cdot)}$	predicted variables
$\overline{(\cdot)}$	upper limit on variables
$\underline{(\cdot)}$	lower limit on variables

2.1 Vehicle Model

We use a "bicycle model" to describe the dynamics of the car and assume constant normal tire load, i.e., $F_{z_f}, F_{z_r} = $ constant. Such model captures the most relevant nonlinearities associated to lateral stabilization of the vehicle. Figure 1 depicts a diagram of the vehicle model, which has the following longitudinal, lateral and turning or yaw degrees of freedom (DOF)

$$m\ddot{x} = m\dot{y}\dot{\psi} + 2F_{x_f} + 2F_{x_r} \qquad (1a)$$

$$m\ddot{y} = -m\dot{x}\dot{\psi} + 2F_{y_f} + 2F_{y_r}, \qquad (1b)$$
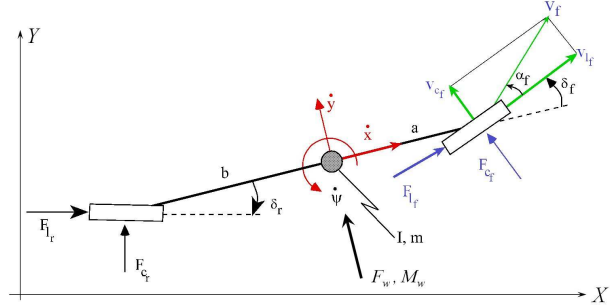
$$I\ddot{\psi} = 2aF_{y_f} - 2bF_{y_r}, \qquad (1c)$$

Fig. 1. The simplified vehicle dynamical model.

The vehicle's equations of motion in an absolute inertial frame are

$$\dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi, \qquad (2a)$$

$$\dot{X} = \dot{x}\cos\psi - \dot{y}\sin\psi. \qquad (2b)$$

Longitudinal and lateral tire forces lead to the following forces acting on the center of gravity:

$$F_y = F_l\sin\delta + F_c\cos\delta, \qquad (3a)$$

$$F_x = F_l\cos\delta - F_c\sin\delta. \qquad (3b)$$

Tire forces for each tire are given by

$$F_l = f_l(\alpha, s, \mu, F_z), \qquad (4a)$$

$$F_c = f_c(\alpha, s, \mu, F_z), \qquad (4b)$$

where α is the slip angle of the tire, s is the slip ratio defined as

$$s = \begin{cases} s = \dfrac{r\omega}{v} - 1 \text{ if } v > r\omega, v \neq 0 \text{ for braking} \\ s = 1 - \dfrac{v}{r\omega} \text{ if } v < r\omega, \omega \neq 0 \text{ for driving} \end{cases}$$
(5)

where v is the vehicle velocity, r and ω are the wheel radius and angular speed respectively. The slip angle represents the angle between the wheel velocity and the direction of the wheel itself:

$$\alpha = \tan^{-1} \frac{v_c}{v_l}.$$
(6)

In equation (6), v_c and v_l are the lateral (or cornering) and longitudinal wheel velocities, respectively, which are expressed as

$$v_l = v_y \sin\delta + v_x \cos\delta,$$
(7a)
$$v_c = v_y \cos\delta - v_x \sin\delta,$$
(7b)

and

$$v_{y_f} = \dot{y} + a\dot{\psi} \quad v_{y_r} = \dot{y} - b\dot{\psi},$$
(8a)
$$v_{x_f} = \dot{x} \quad\quad v_{x_r} = \dot{x}.$$
(8b)

The parameter μ in (4) represents the road friction coefficient and F_z is the total vertical load of the vehicle. This is distributed between the front and rear wheels based on the geometry of the car model, described by the parameters a and b:

$$F_{z_f} = \frac{bmg}{2(a+b)}, \quad F_{z_r} = \frac{amg}{2(a+b)}.$$
(9a)

Using the equations (1)-(9), the nonlinear vehicle dynamics can be described by the following compact differential equation assuming a certain slip ratio s and friction coefficient value μ:

$$\dot{\xi} = f_{s,\mu}(\xi, u),$$
(10a)
$$\eta = h(\xi),$$
(10b)

where the state and input vectors are $\xi = [\dot{y}, \dot{x}, \psi, \dot{\psi}, Y, X]$ and $u = \delta_f$ respectively, and the output map is given as

$$h(\xi) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \xi.$$
(11)

2.2 Tire Model

The model for tire tractive and cornering forces (4) used in this paper are described by a Pacejka model (Bakker *et al.*, 1987). This is a complex, semi-empirical model that takes into consideration the interaction between the tractive force and the cornering force in combined braking and steering. The longitudinal and cornering forces are assumed to depend on the normal force, slip angle, surface friction, and longitudinal slip. Sample plots of predicted longitudinal and lateral force versus longitudinal slip and slip angle are shown

in Figure 2. These plots are shown for the front tire of the "bicycle" model, which represents the two front tires of the actual car.
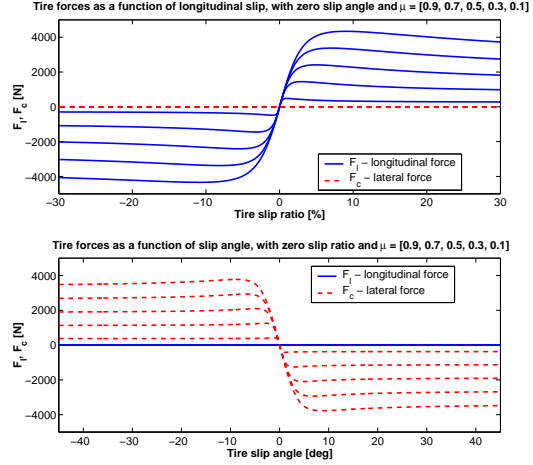

Fig. 2. Longitudinal and lateral tire forces with different μ coefficient values.

3. MODEL PREDICTIVE CONTROL PROBLEM.

In order to obtain a finite dimensional optimal control problem we discretize the system dynamics (10) with the Euler method, to obtain

$$\xi(k+1) = f_{s,\mu}^{dt}(\xi(k), \Delta u(k)),$$
(12a)
$$\eta(k+1) = h(\xi(k)),$$
(12b)

where the Δu formulation is used, i.e., $u(k) = u(k-1) + \Delta u(k)$ and $u(k) = \delta_f(k)$, $\Delta u(k) = \Delta\delta_f(k)$.

We consider the following cost function:

$$J(\xi(t), \Delta\mathcal{U}_t) = \sum_{i=1}^{H_p} \left\| \hat{\eta}_{t+i,t} - \eta_{ref_{t+i,t}} \right\|_Q^2 + \sum_{i=0}^{H_c-1} \left\| \Delta u_{t+i,t} \right\|_R^2$$
(13)

where, as in standard MPC notation (Mayne *et al.*, 2000), $\Delta\mathcal{U}_t = \Delta u_{t,t}, \ldots, \Delta u_{t+H_c-1,t}$ is the optimization vector at time t and $\hat{\eta}_{t+i,t}$ denotes the output vector predicted at time $t + i$ obtained by starting from the state $\xi_{t,t} = \xi(t)$ and applying to system (12) the input sequence $\Delta u_{t,t}, \ldots, \Delta u_{t+i,t}$. H_p and H_c denote the output prediction horizon and the control horizon, respectively. As in standard MPC schemes, we use $H_p > H_c$ and the control signal is assumed constant for all $H_c \leq t \leq H_p$, i.e., $\Delta u_{t+i,t} = 0 \ \forall i \geq H_c$. The reference signal η_{ref} represents the desired outputs, where $\eta = [\psi, Y]'$. Q and R are weighting matrices of appropriate dimensions. In (13) the first summand reflects the desired performance on

target tracking, the second summand is a measure of the steering effort.

The minimization of (13) subject to the model equation (12) is the base of the NLMPC scheme presented in (Borrelli *et al.*, 2005).

In this article we consider a second method based on the solution of following optimization problem (LTV-MPC):

$$\min_{\Delta \mathcal{U}_t} J(\xi_t, \Delta \mathcal{U}_t) \tag{14a}$$

$$\text{subj. to} \tag{14b}$$

$$\Delta \xi_{k+1,t} = \mathcal{A}_t \Delta \xi_{k,t} + \mathcal{B}_t \Delta u_{k,t}, \tag{14c}$$

$$\Delta \eta_{k,t} = \mathcal{C}_t \Delta \xi_{k,t} + \mathcal{D}_t \Delta u_{k,t}, \tag{14d}$$

$$k = t, \ldots, t + H_p$$

$$\delta_{f,min} \leq u_{k,t} \leq \delta_{f,max} \tag{14e}$$

$$\Delta \delta_{f,min} \leq \Delta u_{k,t} \leq \Delta \delta_{f,max} \tag{14f}$$

$$k = t, \ldots, t + H_c - 1$$

$$\alpha_{min} \leq \alpha_{k,t} \leq \alpha_{max} \tag{14g}$$

$$k = t + 1, \ldots, t + H_p$$

$$u_{k,t} = u_{k-1,t} + \Delta u_{k,t} \tag{14h}$$

where the equations (14c)-(14d) are a discrete linear approximation of (12) computed at the current operating point $\xi(t)$, $u(t)$. The optimization problem (14) can be recast as a quadratic program (QP) (details can be found in (Borrelli *et al.*, 2005)) and the resulting MPC controller for a Linear Time Varying (LTV) system will solve the problem (14) at each time step. Once a solution $\mathcal{U}_t$ to problem (14) has been obtained, the input command is computed as $u(k) = u(k-1) + \Delta u_{t,t}$, where $\Delta u_{t,t}$ is the vector of the first m elements of $\mathcal{U}_t$ if system (14c) has m inputs. At the next time step, the linear model (14c)-(14d) is computed based on new state and input measurements, and the new QP problem (14) is solved over a shifted horizon. In constraints (14g) the predictions $\alpha_{k,t}$ of the tire slip angle are computed according to the equations (6)-(8).

Complexity of (14) reduces compared to the NLMPC in ref. (Borrelli *et al.*, 2005), and it is function of the time needed to setup the problem (14), i.e., to compute the linear models $(\mathcal{A}_t, \mathcal{B}_t, \mathcal{C}_t, \mathcal{D}_t)$ in (14c)-(14d) along the trajectory, and of the time to solve it.

In general the stability of the presented control scheme is difficult to prove. Based on an accurate analysis of the vehicle nonlinearities, we obtained a stable and performing controller by a proper choice of the cost (14a) and the constraint (14g).

In particular, without the constraints (14g) the performance of the linear MPC controller (14) is not acceptable and sometime unstable. This is due to the fact that a simple linear model is not able to predict the change of slope in the tires characteristic (see Figure (2)). To overcome this issue we add constraints (14g) to the optimization problem, in order to forbid the system from entering into a strongly nonlinear and possibly unstable region of the tire characteristic. In particular, by looking at the tire characteristics in Figure 2, it is clear that the front slip angle α should be constrained in a range where the lateral forces are linear functions of the front slip angle. The same holds true for the longitudinal tire forces as function of the slip ratio. Note that the (14g) are implicit nonlinear constraints on state and input and they can be handled systematically only in a MPC scheme.

By looking a the model equations (1), it is clear that the higher $\dot{\Psi}$ is, the stronger the nonlinearities in the first two equations are. A yaw rate reference can be used in order to penalize excessive yaw rate. These considerations suggest to add a term in the cost function (13) weighting the deviation of the yaw rate from the computed reference, i.e. $\eta = [\psi, \dot{\psi}, Y]'$ in (13). This modification improves significantly the performance of the MPC (14).

4. SIMULATION RESULTS.

We considered a scenario where the objective is to follow a desired path as close as possible in a double lane change manoeuvre on a snow covered road ($\mu = 0.3$). The control input in our 2WS scenario is the front tire steering angle and the goal is to follow this trajectory as close as possible by minimizing the vehicle deviation from the target path. The experiment is repeated with increasing entry speeds until the vehicle loses control. It should be pointed out that the vehicle is coasting during the maneuver, i.e., no braking or traction torque has been applied to the wheels. The linear MPC controller, presented in Section 3, and a NLMPC controller, designed with the procedure described in (Borrelli *et al.*, 2005) have been tested for different longitudinal speeds, using the front steering angle δ_f as the only control input. In the NLMPC the yaw angle and the lateral position Y are controlled, in addition to these, in the LTV-MPC the yaw rate is controlled as well. In the following these two controllers will be referred to as the NLMPC controller and the LTV-MPC controller respectively. We used the commercial NPSOL software package (Gill *et al.*, 1998) for solving the nonlinear programming problem associated to the NLMPC controller. The simulations results presented in the following have been obtained in the Matlab 6.5 environment running on a 2 GHz Centrino-based laptop. The linear models (14c)-(14d) have been computed with the Matlab function 'linmod'.

The following parameters have been used for the NLMPC and LTV-MPC controllers:

- sample time: $T = 0.05$ sec;
- constraints on maximum and minimum steering angles $-10 \deg \leq \delta_f \leq 10 \deg$
- constraints on maximum and minimum changes in steering angles
  $-30 \deg/s \leq \Delta\delta_f \leq 30 \deg/s$
- prediction horizon: $H_p = 25$
- control horizon: $H_c = 10$
- weighting matrices: $R = 5 \cdot 10^3$, $Q = \begin{pmatrix} 200 & 0 \\ 0 & 10 \end{pmatrix}$ for NLMPC, $Q = \begin{pmatrix} 200 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}$ for the LTV-MPC

The simulations have been performed at different vehicle longitudinal entry speeds. We report simulations at 15 m/s and 20 m/s.
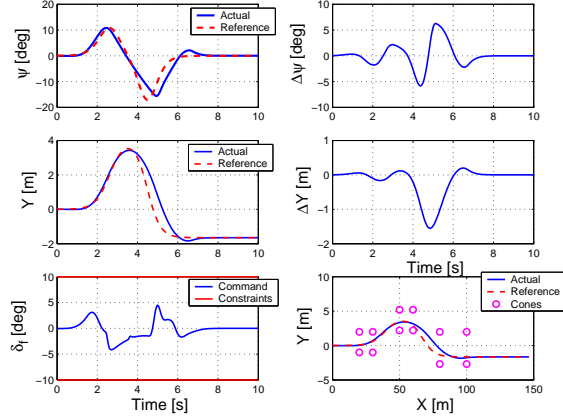

Fig. 3. Double lane change maneuver at 15 m/s with $H_p = 25$ and $H_u = 10$ with the NLMPC controller.

Figures 3 and 4 depict the simulation results when the double lane change manoeuvre is performed by the NLMPC controller. Figures 5 and 6 present the performances of the LTV-MPC controller. By comparing these results we observe that the performance of the LTV-MPC is slightly worse than NLMPC. However the overall performance in the path following is still good. We notice that the NLMPC controller keeps the slip angle α_f in the range $[-3, 3]$ deg, where the tire characteristic is linear and the maximum lateral forces are achievable (see bottom plot in Figure 2) with the selected μ. While this is automatically accomplished by the non-linear optimization solver, the same is obtained in the LTV-MPC controller, as shown in Figure 6, by adding the constraints (14g) and selecting $\alpha_{min} = -\alpha_{max} = -3$ for $\mu = 0.3$.

We remark that the NLMPC controller is too CPU time consuming (see Figure 4) and it cannot be implemented in real time. The suboptimal LTV-MPC scheme decreases significatively the complexity, while performing well. In Figure 6 the
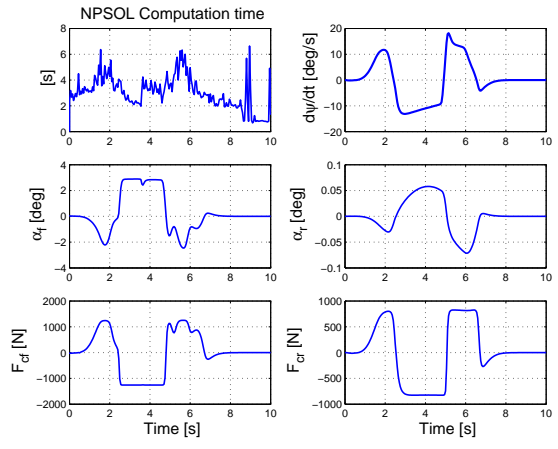

Fig. 4. Computation time, yaw rate, front and rear slip angles, front and rear lateral forces with the NLMPC controller at 15 m/s.
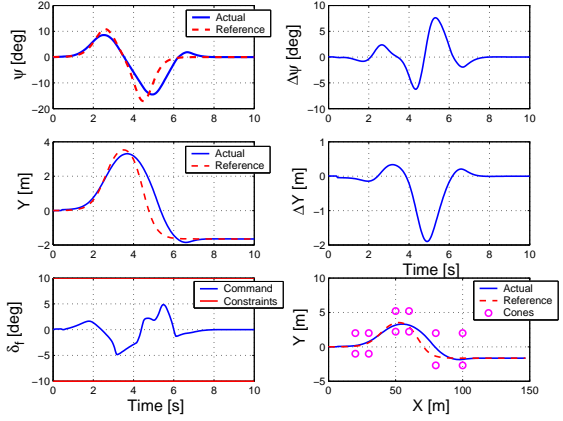

Fig. 5. Double lane change maneuver at 15 m/s with $H_p = 25$ and $H_u = 10$ with the LTV-MPC controller.
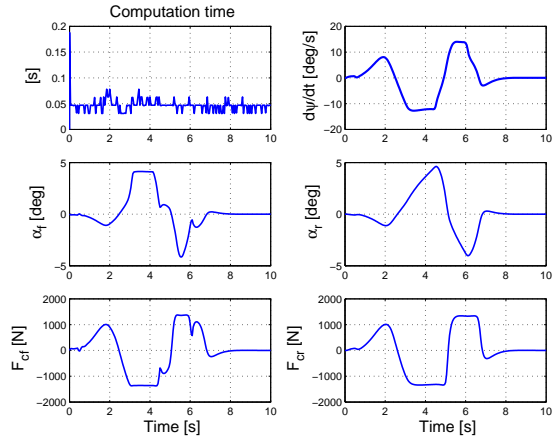

Fig. 6. Computation time, yaw rate, front and rear slip angles, front and rear lateral forces with the LTV-MPC controller at 15 m/s.

time required to compute the input command at each time step is plotted. This time includes the time to set up and solve the QP-problem (14). In our simulations the linear models (14c)-(14d) are computed on-line, but they could be also

computed off-line along the desired trajectory together with the matrices of the QP-problem. This would drastically reduce the set up time, which is higher than the required QP computational time. This is the topic of further investigation.

Table 1 reports the root mean squared value (RMS) of the tracking error $\bar{e}_x$:

$$\bar{e}_x = \sqrt{\frac{1}{T} \int_T x^2(t) dt}$$

and the maximum absolute error $|e_x|_{max}$, associated to the results shown in Figures 3-6.

	$\bar{e}_Y$	$\bar{e}_\Psi$	$|e_Y|_{max}$	$|e_\Psi|_{max}$
NLMPC	.3283	1.5542	1.7256	7.7974
LTVMPC	.3425	1.8447	1.8005	8.1182

Table 1. Summary of results using the different controllers at the entry speed of 15 m/s.

Figures 7 and 8 show the simulation results when the double lane change manoeuvre is performed at a longitudinal entry speed of 20 m/s, by the LTV-MPC.
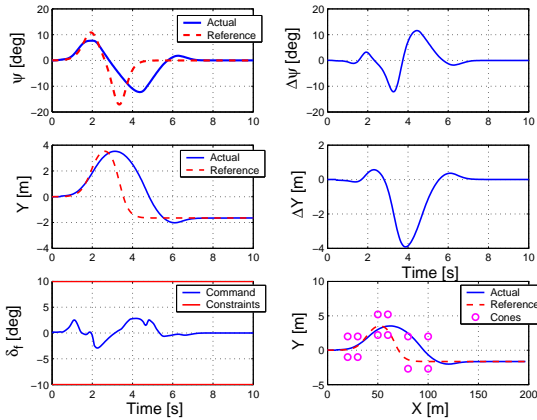

Fig. 7. Double lane change maneuver at 20 m/s with $H_p = 25$ and $H_u = 10$ with the LTV-MPC controller.

At entry speed higher than 21 m/s, since large steering commands are required, the LTV MPC controller violates the constraints on the slip angle α_f losing the control of the vehicle.

5. CONCLUSIONS.

In this paper we presented a Model Predictive Control approach for active steering control. We showed the effectiveness of a suboptimal MPC scheme in reducing the computational burden, while achieving good results compared to a NLMPC controller. The computational complexity of the presented LTV-MPC controller allows real-time implementation for experimental tests.
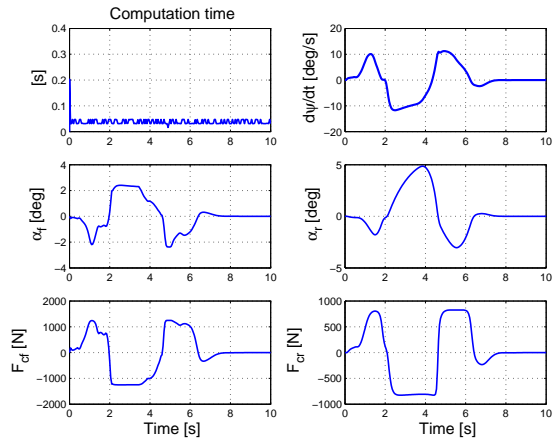

Fig. 8. Computation time, yaw rate, front and rear slip angles, front and rear lateral forces with the LTV-MPC controller at 20 m/s.

Moreover, future steps will aim to study the stability and the performance of the proposed scheme respect to the tuning parameters.

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