Homework 3:

Due 4/13 at 9 AM

2.10 Are the following sets of vectors linearly independent?

a.

$$x_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 3 \\ -3 \\ 8 \end{bmatrix}$$

b.

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

2.11 Write

$$y = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

as linear combination of

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

4.3 Compute the eigenspaces of

a.

$$A := \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

b.

$$B := \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

4.4 Compute all eigenspaces of

$$A = \begin{bmatrix} 0 & -1 & 1 & 1 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

4.5 Diagonalizability of a matrix is unrelated to its invertibility. Determine for the following four matrices whether they are diagonalizable and/or invertible

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

4.6 Compute the eigenspaces of the following transformation matrices. Are they diagonalizable?

a. For

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

b. For

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$