1) LET X BE A CONTINUOUS RANDOM VARIABLE WITH PDF GIVEN BY

$$f_X(x) = \begin{cases} \frac{c}{2}x^2, & |x| \le 2 \\ 0, & \text{OTHERWISE} \end{cases}$$

a) FIND THE CONSTANT C

$$\int_{-\infty}^{\infty} f_X(u)\,du = 1$$

$$1 = \int_{-\infty}^{\infty} f_X(u)\,du$$

$$= \int_{-2}^{2} \frac{c}{2} u^2\,du$$

$$= \frac{c}{2}\int_{-2}^{2} u^2\,du$$

$$= \frac{c}{2} u^3/3 \Big|_{-2}^{2}$$

$$= \frac{c}{2}\, 2^3/3 - \frac{c}{2}\frac{(-2)^3}{3}$$

$$= \frac{c}{2}\cdot\frac{8}{3} - \frac{c}{2}\frac{-8}{3}$$

$$= \frac{c8}{6} - \frac{-c8}{6}$$

$$= \frac{2c8}{6}$$

$$1 = \frac{c8}{3}$$

$$3 = c8$$

$$3/8 = c$$

B FIND E(X)

$$EX = \int_{-2}^{2} u f_X(u)\,du$$

$$= \frac{3/8}{2}\int_{-2}^{2} u^3\,du$$

$$= \frac{3/8}{2}\frac{u^4}{4}\Big|_{-2}^{2}$$

$$= \frac{3/8}{2}\cdot\frac{2^4}{4} - \frac{3/8}{2}\cdot\frac{(-2)^4}{4}$$

$$= 0$$

C FIND $P(X \ge 1)$

$$P(X \ge 1) = \frac{3/8}{2}\int_{1}^{2} x^2\,dx$$

$$= \frac{3/8}{2}\cdot x^3/3 \Big|_{1}^{2}$$

$$= \frac{3/8}{2}\cdot\frac{2^3}{3} - \frac{3/8}{2}\cdot\frac{1^3}{3}$$

$$= \frac{3/8}{2}\cdot\frac{8}{3} - \frac{3/8}{2}\cdot\frac{1}{3}$$

$$= \frac{3}{6} - \frac{3/8}{6}$$

$$= \frac{24/8}{6} - \frac{3/8}{6}$$

$$= \frac{21/8}{6}$$

$$\simeq 0.4375$$

2 LET X BE A CONTINUOUS RANDOM VARIABLE WITH PDF GIVEN BY

$$f_X(x) = e^{-|x|}, \text{ FOR ALL } x \in \mathbb{R}$$

$$Y = 2X$$

FIND THE CDF OF Y

$$R_Y = [0, \infty)$$

FOR $y \in [0, \infty]$

$$F_Y(y) = P(Y \le y)$$

$$= P(2X \le y)$$

$$= P\left(X \le \frac{y}{2}\right)$$

$$= \int^{y/2} e^{-|x|}\, dx$$

$$= \int_0^{y/2} e^{-x}\, dx + \int_0^{y/2} e^{x}\, dx$$

$$= -e^{-x}\Big|_0^{y/2} + e^{x}\Big|_0^{y/2}$$

$$= (-e^{-y/2}) - (-1) + (e^{y/2}) - (1)$$

$$= 1 - e^{-y/2} + e^{y/2} - 1$$

$$= e^{y/2} - e^{-y/2}$$

$$F_Y(y) = \begin{cases} e^{y/2} - e^{-y/2} & y \ge 0 \\ 0 & \text{OTHERWISE} \end{cases}$$

FIND $P(X \leq 1 \mid X > \frac{1}{2})$

$$P(X \leq 1 \mid X > \tfrac{1}{2}) = \frac{P(\frac{1}{2} < X \leq 1)}{P(X > 1/2)}$$

$$= \frac{\int_{1/2}^{1} 3x^2}{\int_{1/2}^{2} 3x^2}$$

$$= \frac{3\,x^3/3 \Big|_{1/2}^{1}}{3\,x^3/3 \Big|_{1/2}^{2}}$$

$$= \frac{1^3 - 1/2^3}{2^3 - 1/2^3}$$

$$= \frac{1 - 1/8}{8 - 1/8}$$

$$= \frac{7/8}{7\,7/8}$$

$$\approx 0.111$$

1.- LET X BE A CONTINUOUS RANDOM VARIABLE WITH PDF

$$f_X(x) = \begin{cases} x(2x+5), & 0 \le x \le 1 \\ 0, & \text{OTHERWISE} \end{cases}$$

IF $Y = \frac{3}{X} + 2$ FIND VAR(Y)

NOTE - $VAR(X) = E[(X-\mu_X)^2] = EX^2 - (EX)^2$

- FOR $a, b \in \mathbb{R}$

$VAR(aX+b) = a^2 VAR(X)$ / $E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$

$$VAR(Y) = VAR\left(\frac{3}{X}+2\right) = 9\,VAR\left(\frac{1}{X}\right)$$

$$VAR\left(\frac{1}{X}\right) = E\left[\frac{1}{X^2}\right] - \left(E\left[\frac{1}{X}\right]\right)^2$$

$$E\left[\frac{1}{X}\right] = \int_0^1 \frac{1}{x}\cdot x(2x+5)\, dx$$

$$= \int_0^1 (2x+5)\, dx$$

$$= 2\frac{x^2}{2} + 5\frac{x}{1}\Big]_0^1$$

$$= \left(2\frac{1^2}{2} + 5\frac{1}{1}\right) - \left(2\frac{0^2}{2} + 5\frac{0}{1}\right)$$

$$= 6$$

$$E\left[\frac{1}{X^2}\right] = \int_0^1 \frac{1}{x^2}\cdot x(2x+5)$$

$$= \int_0^1 \frac{1}{x}\cdot(2x+5)$$

$$= \int_0^1 2 + 5/x$$

$$= 2x - 5/x \Big|_0^1$$

BECAUSE 5/0 IS UNDEFINED, THIS INTEGRAL DIVERGES

THEREFORE, WE DO NOT HAVE A SOLUTION FOR VAR(Y)

5) LET $X \sim \text{UNIFORM}\left(\frac{\pi}{2}, \pi\right)$. AND $Y = \sin(X)$, FIND $f_Y(y)$

$Y = g(X)$

$R_X = \left[\frac{\pi}{2}, \pi\right]$ $R_Y = [0, 1]$

g IS MONOTONE ON $[0, 1]$.
LOOKING AT $g(x) = \sin(x)$ OVER
THE RANGE $\left[\frac{\pi}{2}, \pi\right]$, FOR $y \in (0,1)$
THERE IS ONE SOLUTION TO
$y = g(x)$. $x_1 = \arcsin(y)$ ~~ARG~~

~~$x_2 = \pi - \arcsin(y)$~~.

NOTE

$$f_Y(y) = \sum_{i=1}^{n} \frac{f_X(x_i)}{|g'(x_i)|} = \sum_{i=1}^{n} f_X(x_i)\left|\frac{dx_i}{dy}\right|$$

$$f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|}$$

$$= \frac{f_X(\arcsin(y))}{|\cos(\arcsin(y))|}$$

$$= \frac{\frac{2}{3\pi}}{\sqrt{1-y^2}}$$

$$f_Y(y) = \begin{cases} \frac{2}{3\pi\sqrt{1-y^2}} & 0 < y < 1 \\ 0 & \text{OTHERWISE} \end{cases}$$