Blatt 6

```
clear;clc;
```

Aufgabe 1

$$\frac{d\phi(t)}{dt} = t^2 e^{-5t} - 6\phi(t)$$

$$\phi(t) = e^{-5t}\left(t^2 - 2t + 2\right) - 2e^{-6t}$$

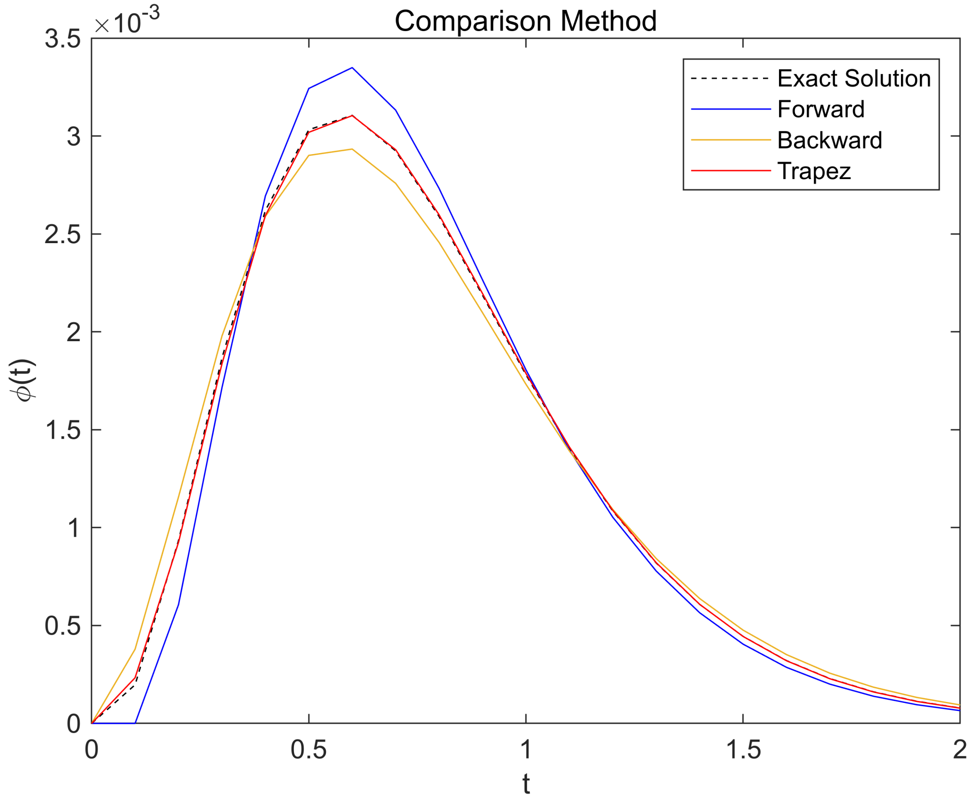
```
% Parameter setting

theta=[0,1,1/2];
% Vorwärts-Euler Verfahren
t0=0:0.1:2;
phi_exact=exactSolution(t0);
phi_forward=forwardEuler(0,2,0.1);
phi_backward=backwardEuler(0,2,0.1);
phi_trapez=trapezEuler(0,2,0.1);
% Exact
plot(t0, phi_exact, '--k', 'DisplayName', 'Exact Solution');
xlabel('t');
ylabel('\phi(t)');
title('Comparison Method');

hold on
% Foreward
plot(t0, phi_forward, '-blue', 'DisplayName', 'Forward');

% Backward
plot(t0, phi_backward, 'LineStyle', '-', 'Color', [0.9290 0.6940 
0.1250], 'DisplayName', 'Backward');

% Trapez
plot(t0, phi_trapez, '-r', 'DisplayName', 'Trapez');
hold off
% 添加图例和坐标轴标签等
legend;
```



Aufgabe 2

2.1 Test mit Einschritt

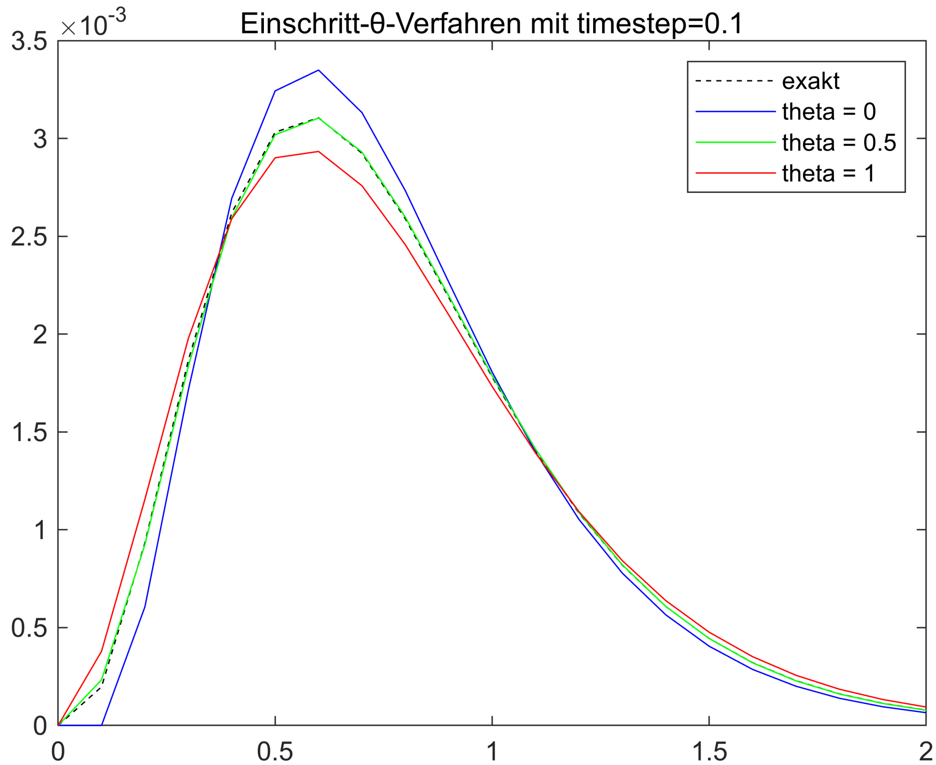
```
% Test Einschritt-        θ-Verfahren für        θ = 0.0, 0.5 und 1.0.
[LHS,RHS] = OST(0.5, 0.2, [1.1], [1.4, 1.5], [1.7, 1.8], [2.0])
```

```
LHS = 0.9600
RHS = 2.8500
```

```
n = size(t0,2);
M = 1; B = [-6,-6];
t0=0:0.1:2;
phi_ost = zeros(n,3);

for    i = 1:n-1
    C = [g(t0(i+1)),g(t0(i))];
    [LHS,RHS] = OST(0.0,0.1,M,B,C,phi_ost(i,1));
    phi_ost(i+1,1) = RHS / LHS;
    [LHS,RHS] = OST(0.5,0.1,M,B,C,phi_ost(i,2));
    phi_ost(i+1,2) = RHS / LHS;
    [LHS,RHS] = OST(1.0,0.1,M,B,C,phi_ost(i,3));
    phi_ost(i+1,3) = RHS / LHS;
end
phiost_zero = phi_ost(:,1);              % theta = 0
faiost_half = phi_ost(:,2);              % theta = 0.5
```

```
faiost_one = phi_ost(:,3);                % theta = 1
% θ = 0.5
figure;
plot(t0,phi_exact,         'k--'     ,t0,phiost_zero,        'b'   ,t0,faiost_half,          'g'   ,t0,faiost_one,          '
r'   );
legend(   'exakt'      , 'theta = 0'        , 'theta = 0.5'          , 'theta = 1'        );
title(      'Einschritt-        θ-Verfahren mit timestep=0.1'              );
```



2.2 Vergleich AB2 AM3 BDF2

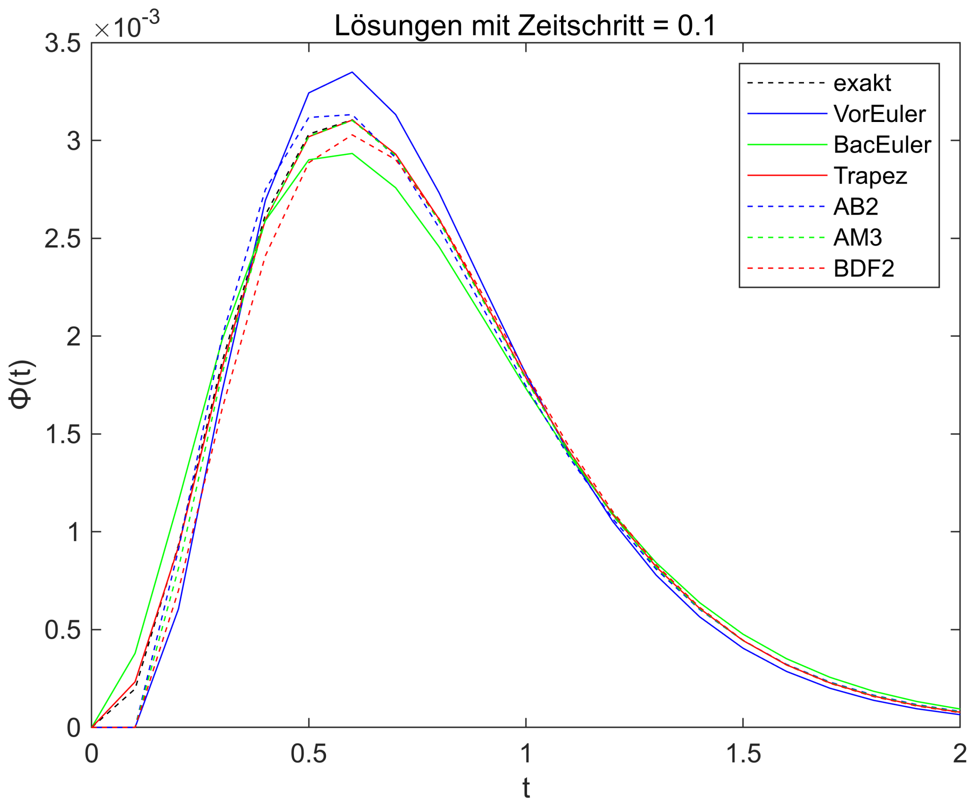
```
%% Adams-Bashforth-Verfahren
phi_ab2 = zeros(1,n);
M = 1;
for    i=2:n-1      % 因为有 i-1    所以要从 2 开始
   M = 1;
   B = [-6,-6];
   C = [g(t0(i)),g(t0(i-1))];
   sol = [phi_ab2(i),phi_ab2(i-1)];
   [LHS,RHS] = AB2(0.1,M,B,C,sol);
   phi_ab2(i+1) = RHS / LHS;
end

%% Adams_Moulton-Verfahren
phi_am3 = zeros(1,n);
for    i=2:n-1
   B = [-6,-6,-6];
```

```
    C = [g(t0(i+1)),g(t0(i)),g(t0(i-1))];
    sol = [phi_am3(i),phi_am3(i-1)];
    [LHS,RHS] = AM3(0.1,M,B,C,sol);
    phi_am3(i+1) = RHS / LHS;
end

%% BDF2-Verfahren
phi_bdf2 = zeros(1,n);
for i=2:n-1
    B = -6;
    C = g(t0(i+1));
    sol = [phi_bdf2(i),phi_bdf2(i-1)];
    [LHS,RHS] = BDF2(0.1,M,B,C,sol);
    phi_bdf2(i+1) = RHS / LHS;
end

figure(2);
plot(t0,phi_exact,'k--',t0,phi_forward,'b',t0,phi_backward,'g',t0,phi_trapez,
'r',t0,phi_ab2,'b--', ...
    t0,phi_am3,'g--',t0,phi_bdf2,'r--');
legend('exakt','VorEuler','BacEuler','Trapez','AB2','AM3','BDF2');
title('Lösungen mit Zeitschritt = 0.1');
xlabel('t');
ylabel('Φ(t)');
```



Funktion

Funktion

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = f(t, \phi(t))$$

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = t^2 e^{-5t} - 6\phi(t)$$

其中，令 $g(t) = t^2 e^{-5t}$ 方便后续的简化计算

```
function    g=g(t)
g=t^2 * exp(-5*t);
end
function    dphi = odeFun(t, phi)
    dphi = g(t) - 6*phi;
end

function    phi = exactSolution(t)
    phi = exp(-5*t) .* (t.^2 - 2*t + 2) - 2*exp(-6*t);
end
```

Vorward Euler

```
function    phi_forward = forwardEuler(t_start, t_end, delta_t)
    t = t_start:delta_t:t_end;
    len = length(t);
    phi = zeros(1, len);
        for   n = 1:len-1
        phi(n+1) = phi(n) + delta_t * (g(t(n)) - 6*phi(n));
        end
    phi_forward = phi;
end
```

Backward Euler

```
function    phi_backward=backwardEuler(t_start, t_end, delta_t)
    t = t_start:delta_t:t_end;
    len = length(t);
    phi = zeros(1, len);
        for   n=1:len-1
            % n  是数组循环的 index， 整数，从 1 到 20， 不是时间
            % t(n)  是时间范围 0 到 2 之间的第 n 个时间
            % 例如在第二次迭代(n=2) 时，t 应该等于 0+0.1*2=0.2s  ， 对应 t(n)
            % 区分到底 g(n+1)  还是 g(t(n+1))
        phi(n+1) = (phi(n)+delta_t*g(t(n+1)))/(1+delta_t*6);                    % 1+delta_t*6    是
简化公式得来的，
            % 这也是为什么没有用 odefunc  这个函数，而是使用 g(t)*  一个东西
        end
    phi_backward=phi;
```

```
end
```

Trapez Euler

```
function    phi_backward=trapezEuler(t_start, t_end, delta_t)
   t = t_start:delta_t:t_end;
   len = length(t);
   phi = zeros(1, len);
      for    n=1:len-1
            % 公式推导见 Notability
         phi(n+1) = ((1-3*delta_t)*phi(n)+0.5*delta_t*(g(t(n))+g(t(n+1))))/
(1+3*delta_t);
      end
   phi_backward=phi;
end
```

Fkt. 9

```
function    [LHS,RHS] = OST(theta,timestep,M,B,C,sol)
%OST
% M: [M],
% B: [B(tn+1),
% B(tn)],
% C: [C(tn+1),C(tn)],
% sol: [phi(tn)]
% phi0=7;
LHS=M-theta*timestep*B(1);
RHS=(M + (1-theta)*timestep*B(2))*sol + timestep*(theta*C(1) + (1 -
theta)*C(2));

end
```

Fkt. 10

```
function    [LHS,RHS] = AB2(timestep,M,B,C,sol)
%AB2 此处显示有关此函数的摘要
% M ... [M ],
% B ... [B(t n ), B(t n                −1 )],
% C ... [C(t n ), C(t n                −1 )],
% sol ... [        ϕ (t n ),        ϕ (t n      −1 )]]
LHS = M;
RHS = (M+3/2*timestep*B(1))*sol(1) + (3*timestep/2)*C(1)                    ...
     - timestep/2*B(2)*sol(2)-timestep/2*C(2);

end
```

Fkt. 11

```
function    [LHS,RHS] = AM3(timestep,M,B,C,sol)
% M ... [M],
% B ... [B(t n+1 ), B(t n ), B(t n                      −1 )],
```

```
% C ... [C(t n+1 ), C(t n ), C(t n                          −1 )],
% sol ... [          ϕ (t n ),          ϕ (t n       −1 )]]
LHS = M - 5/12*timestep*B(1);
RHS =(M + 2/3*timestep*B(2))*sol(1) + 5/12*timestep*C(1) +
2/3*timestep*C(2)        ...
    - timestep/12*(B(3)*sol(2) + C(3));
end
```

Fkt. 12

```
function      [LHS,RHS] = BDF2(timestep,M,B,C,sol)
% M ... [M ],
% B ... [B(t n+1 )],
% C ... [C(t n+1 )],
% sol ... [          ϕ (t n ),          ϕ (t n       −1 )]]
LHS = 3/2*M - timestep*B;
RHS = 2*M*sol(1) - 0.5*M*sol(2) + timestep*C;
end
```