Chapter 7 - Feedback in Amplifiers

General Considerations

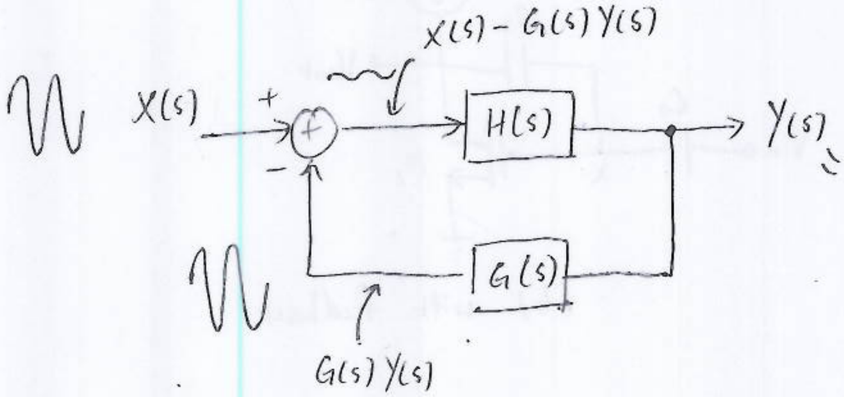
Negative feedback system,

feedback error

=

$H(s)$ = feedforward networks

$G(s)$ = feedback networks



$$Y(s) = H(s)\left[X(s) - G(s)Y(s)\right]$$

$$\therefore \quad \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

$\frac{Y(s)}{X(s)}$ = closed-loop transfer function

$H(s)$ = open-loop transfer function (represents an amplifier)

~~More precise definition~~,

~~$H(s)$ represents an amplifier~~

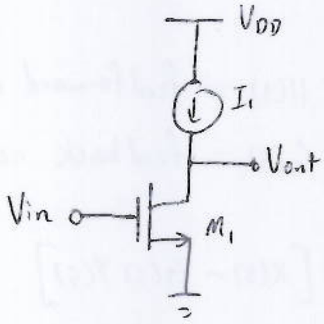
$G(s)$ = frequency-independent quantity. ($B \rightarrow$ feedback factor)

There are four elements in the feedback system:

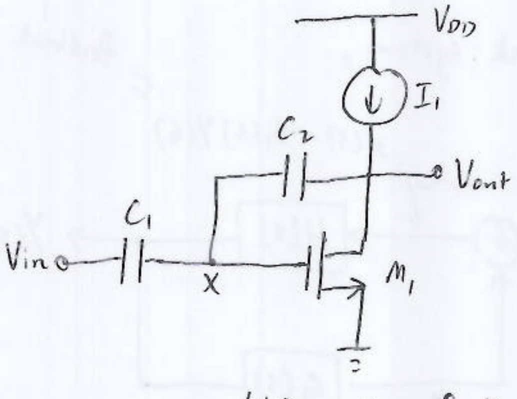
1. feedforward amplifier
2. a means of sensing the output
3. feedback network
4. a means of generating the feedback error.

Properties of Feedback Circuits.

① Gain Desensitization



(a)



(b) with feedback

For (a):

$$\frac{V_{out}}{V_{in}} = -g_{m1} r_{o1}$$

→ poor definition of the gain. g_{m1} and r_{o1} vary with process and temperature.

For (b):

$$\frac{V_{out}}{V_X} = -g_{m1} r_{o1}$$

Node analysis at point X:

$$(V_{out} - V_X) C_2 s + (V_{in} - V_X) C_1 s = 0$$

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\left(1 + \frac{1}{g_{m1} r_{o1}}\right)\frac{C_2}{C_1} + \frac{1}{g_{m1} r_{o1}}}$$

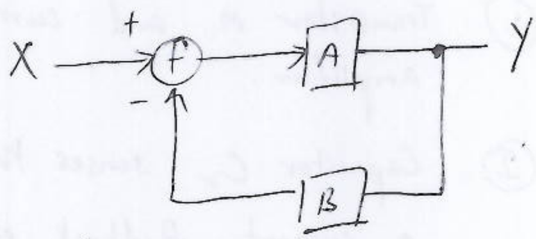
If $g_{m1} r_{o1}$ is sufficiently large,

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}$$

→ Gain can be controlled with much higher accuracy

→ If C_1 and C_2 are made of same material, then process and temperature variations do not change C_1/C_2

For more general case,



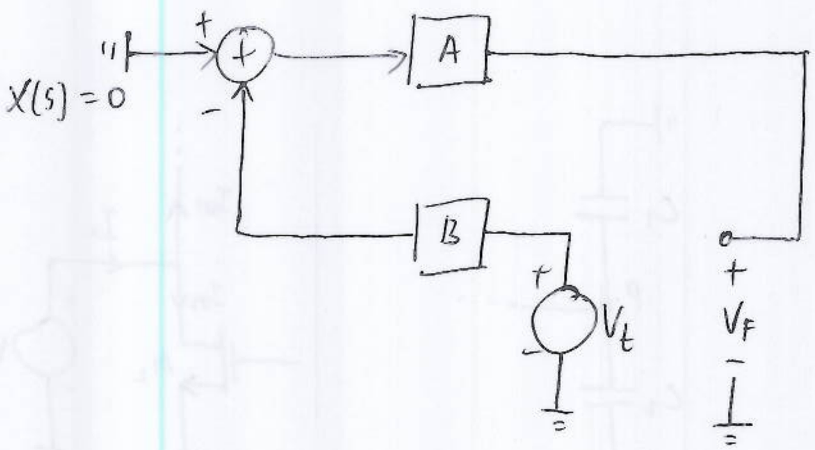
$$\frac{Y}{X} = \frac{A}{1+BA}$$

$$\cancel{\approx \frac{1}{B}\left(1 - \frac{1}{BA}\right)}$$

If $BA \gg 1 \longrightarrow \frac{Y}{X} \approx \frac{1}{B}$

loop gain

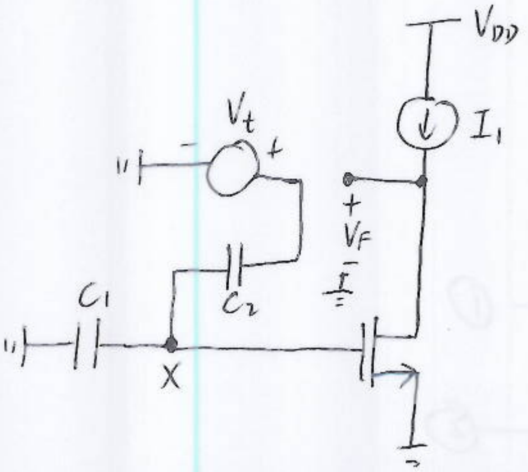
To compute loop gain,



$$V_t\, \beta (-1) A = V_F$$

$$\therefore \frac{V_F}{V_t} = -\beta A$$

For simple feedback circuit,



$$V_t \frac{C_2}{C_1+C_2}(-g_{m1} r_{o1}) = V_F$$

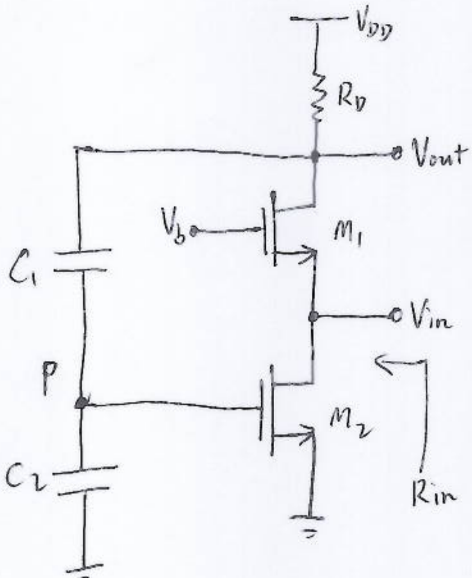
$$\frac{V_F}{V_t} = -\frac{C_2}{C_1+C_2} g_{m1} r_{o1}$$

Four elements of feedback in this circuit ?

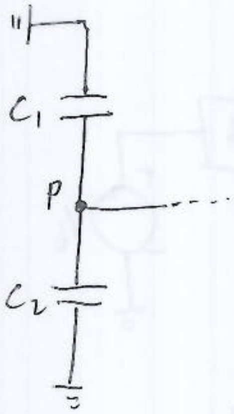
① Transistor M_1 and current source I_1 constitute the feedforward amplifier.

② Capacitor C_2 senses the output voltage and converts it to a current feedback signal, which is then added to the current produced by V_{in} through C_1.

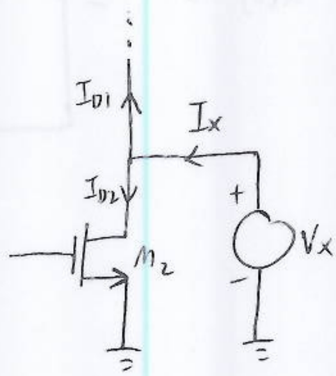
② Terminal Impedance Modification



Common-gate with feedback



open-loop



calculation of input resistance.

a) Consider ~~××~~ open-loop and $\lambda = 0$,

$$R_{in,\,open} = \frac{1}{g_{m1} + g_{mb1}}$$

b) For closed-loop circuit,

$$V_{out} = (g_{m1} + g_{mb1}) V_x R_D \quad \text{— ①}$$

$$V_p = \frac{C_1}{C_1 + C_2} V_{out} \quad \text{— ②}$$

$$\therefore \quad V_P = (g_{m1} + g_{mb1}) V_X R_D \frac{C_1}{C_1 + C_2}$$

Thus, small-signal drain current of M_2

$$I_{D2} = g_{m2}(g_{m1} + g_{mb1}) V_X R_D \frac{C_1}{C_1 + C_2}$$

Adding this current to the drain current of M_1:

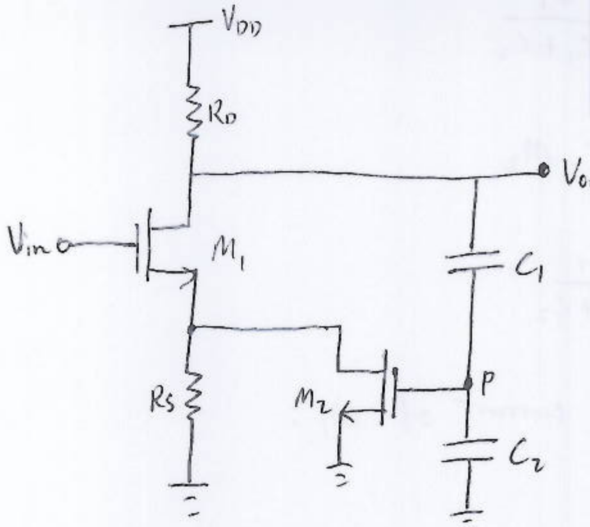
$$\begin{aligned} I_X &= I_{D1} + I_{D2} \\ &= (g_{m1} + g_{mb1}) V_X + g_{m2}(g_{m1} + g_{mb1}) \frac{C_1}{C_1 + C_2} R_D V_X \\ &= (g_{m1} + g_{mb1}) \left(1 + g_{m2} R_D \frac{C_1}{C_1 + C_2} \right) V_X \end{aligned}$$

Therefore,

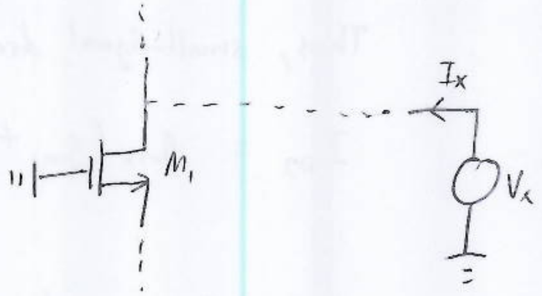
$$\begin{aligned} R_{in,closed} &= \frac{V_X}{I_X} \\ &= \frac{1}{g_{m1} + g_{mb1}} \cdot \frac{1}{1 + \underbrace{g_{m2} R_D \frac{C_1}{C_1 + C_2}}_{\text{loop gain.}}} \end{aligned}$$

Four elements of feedback?

1. M_1 and R_D → feed forward amplifiers
2. C_1 and C_2 → sense output
3. C_1, C_2 and M_2 → feedback network
4. the substraction occurs in the current domain at the input terminal.



CS stage with feedback



Calculation of output resistance

$$I_{D1} = \underbrace{V_X \frac{C_1}{C_1 + C_2} g_{m2}}_{I_{D2}} \underbrace{\frac{R_S}{R_S + \frac{1}{g_{m1} + g_{mb1}}}}_{\text{current divider rule}}$$

Since $I_X = V_X / R_D + I_{D1}$

$$\frac{V_X}{I_X} = \frac{R_D}{1 + \frac{g_{m2} R_S (g_{m1} + g_{mb1}) R_D}{(g_{m1} + g_{mb1}) R_S + 1} \frac{C_1}{C_1 + C_2}}$$

- M_1, R_S and R_D constitute a common-source stage ~~and C~~
- C_1, C_2 and M_2 sense the output voltage
- Returning a current equal to $[C_1/(C_1+C_2)] V_{out} g_{m2}$ to the source of M_1.

③ Bandwidth Modification.

Suppose the feedforward amplifier has a one-pole transfer function:

$$A(s) = \frac{A_0}{1 + \frac{s}{w_0}}$$

where A_0 = low-frequency gain

w_0 = 3-dB bandwidth

What is the transfer function of the closed-loop system?

$$\frac{Y(s)}{X(s)} = \frac{\frac{A_0}{1+\frac{s}{w_0}}}{1 + B\frac{A_0}{1+\frac{s}{w_0}}}$$

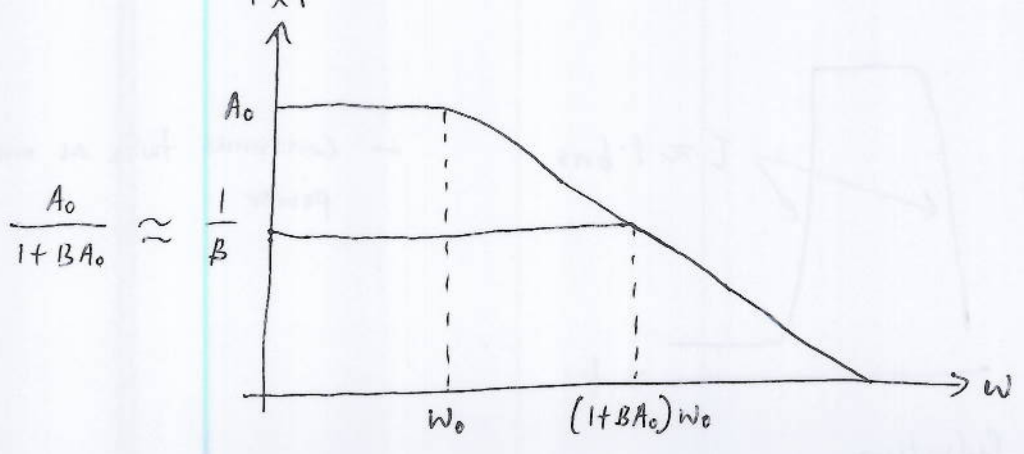
$$= \frac{A_0}{1 + BA_0 + \frac{s}{w_0}}$$

$$= \frac{\frac{A_0}{1+BA_0}}{1 + \frac{s}{(1+BA_0)w_0}}$$

closed-loop gain at low frequency

pole at $(1+BA_0)w_0$

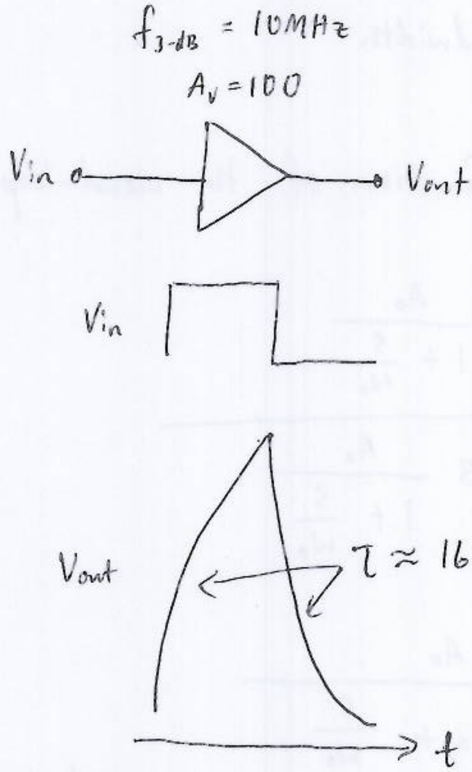
Thus, the 3-dB bandwidth has increased by a factor $1+BA_0$



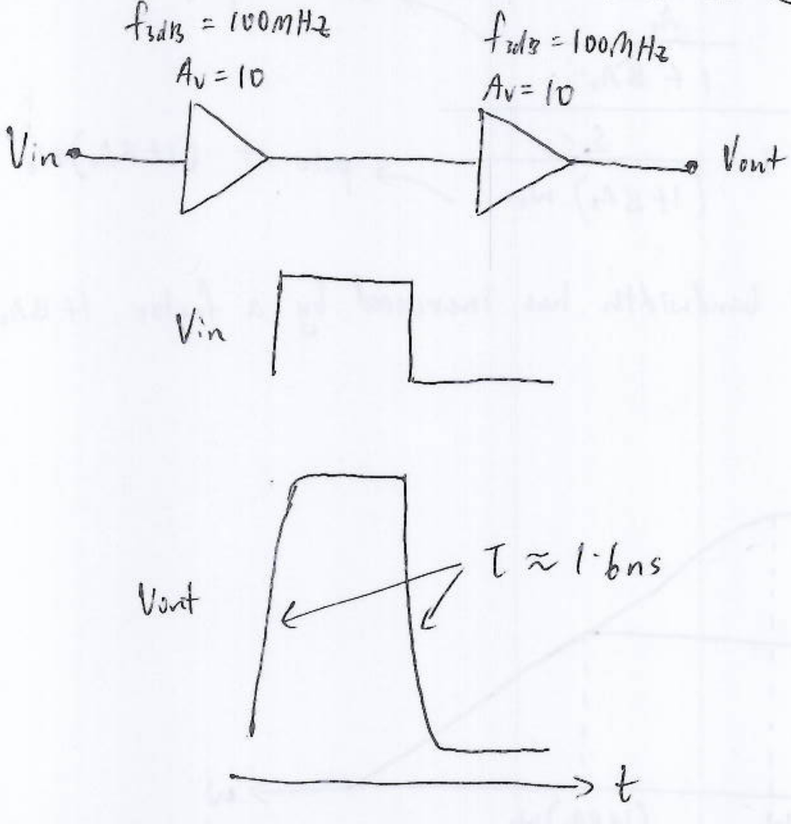
- Gain bandwidth product of one-pole system does not change
- How feedback improves the speed if a high gain is required?

E.g.

Suppose need to amplify a 20MHz square wave by a factor of 100 and maximum bandwidth but we have only a single-pole amplifier with an open-loop gain of 100 and 3-dB bandwidth of 10MHz.



$\tau \approx 16ns$ (long risetime and falltime because time constant is equal to $1/(2\pi f_{3dB}) \approx 16ns$)



Cascade of two 100MHz feedback amplifiers.

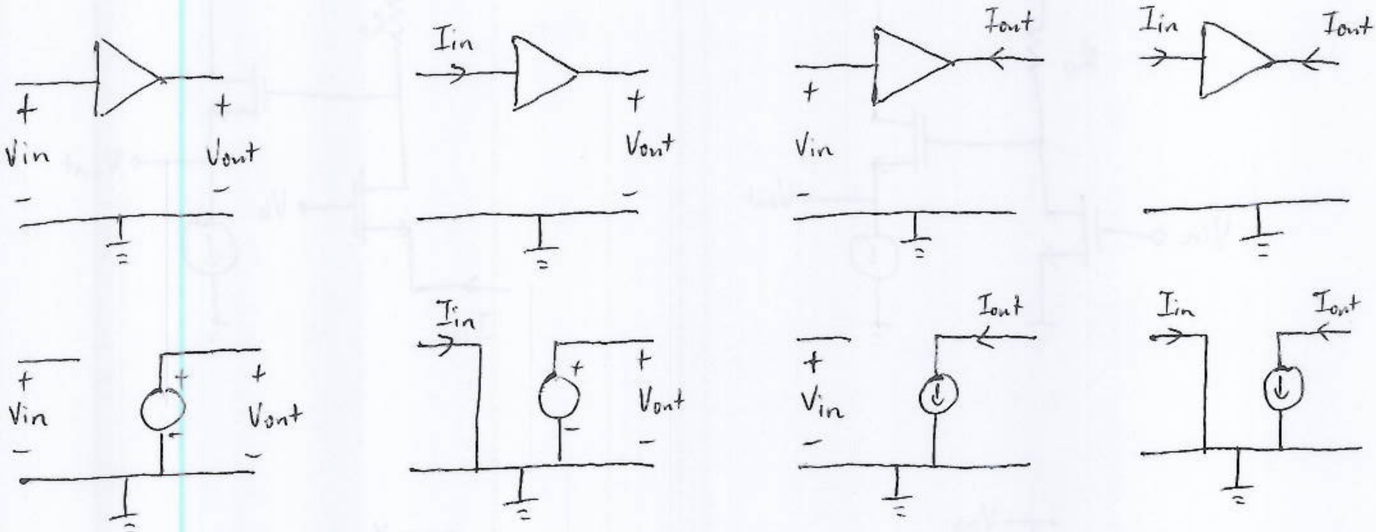
$\tau \approx 1.6ns$

- Consumes twice as much power

④ <u>Nonlinearity Reduction</u>

Types of Amplifiers

Voltage Amp. | Transimpedance Amp. | Transconductance Amp. | Current Amp



Ideal models

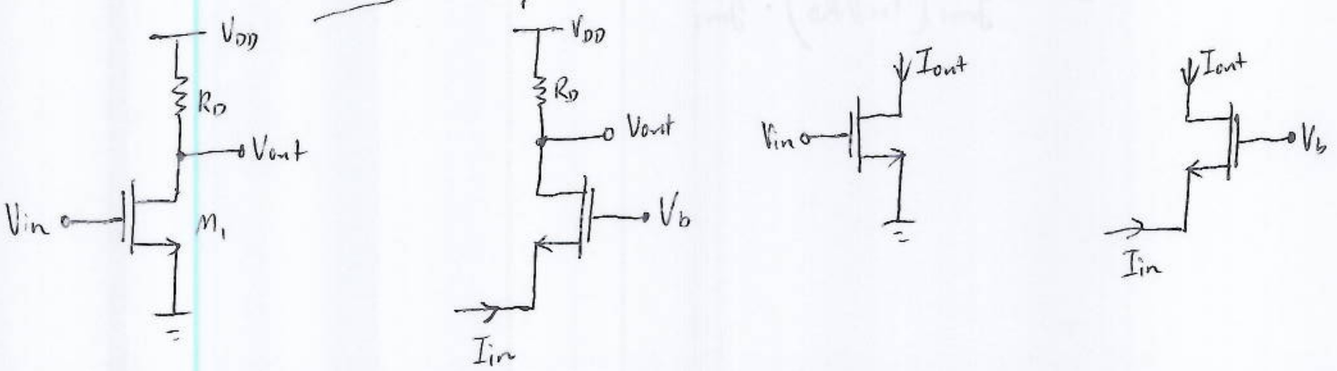
Four configurations have different properties:

1. Circuits sensing a voltage → high input impedance (voltmeter)
2. " sensing a current → low input impedance (current meter)
3. " generating a voltage → low output impedance (voltage source)
4. " generating a current → high output impedance (current source)

Note: Gains of transimpedance (resistance) and transconductance (conductance) amps have different dimensions

Simple implementation

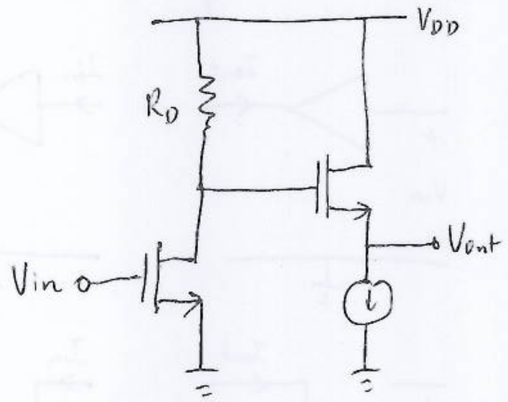
suffer from high output impedance

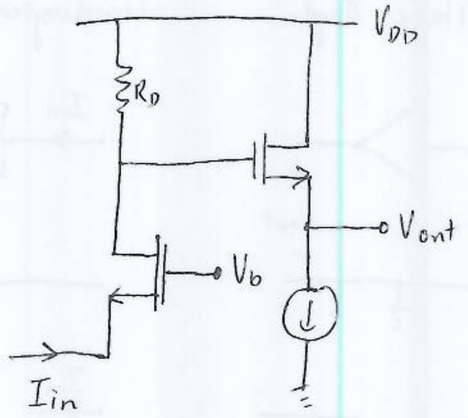


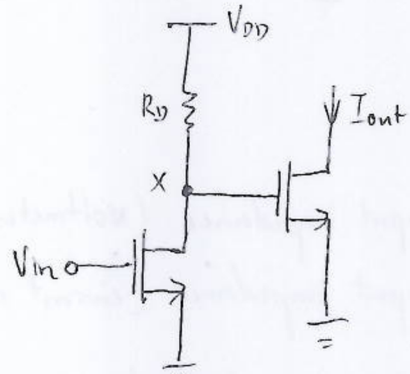
CS: $V \rightarrow V$ | CG: $I \rightarrow V$ | CS: $V \leftrightarrow I$ | CG: $I \leftrightarrow I$

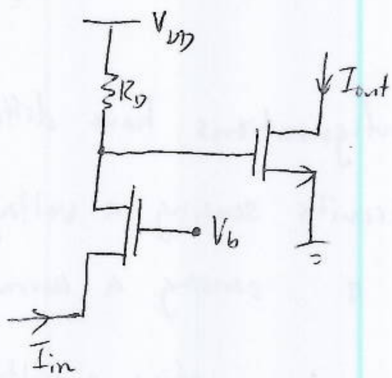
Improved Performance

e.g. input current produced by a photodiode → generates voltage









Gain of the transconductance?

$$G_m = \frac{I_{out}}{V_{in}}$$

$$= \frac{V_x}{V_{in}} \cdot \frac{I_{out}}{V_x}$$

$$= -g_{m1}\left(r_{o1} \parallel R_D\right) \cdot g_{m2}$$

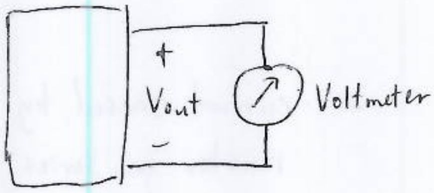
Four types of feedback :

(1) voltage - voltage
(2) voltage - current
(3) current - current
(4) current - voltage

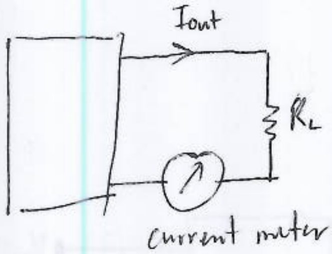
1st term ~~1st~~ quantity sensed at the output

2nd term - type of signal returned to the input

Sense and Return Mechanisms



- sense a voltage
- put in parallel
- when used in feedback system → shunt feedback

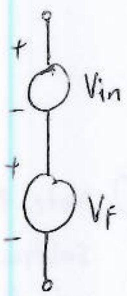


can replace with a resistor

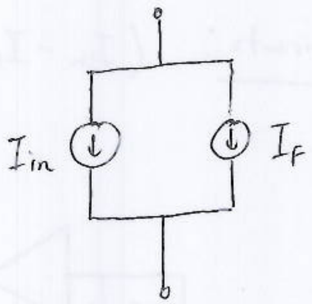
- sense a current
- put in series
- series feedback

Addition of the feedback signal and the input signal :

For voltage domain



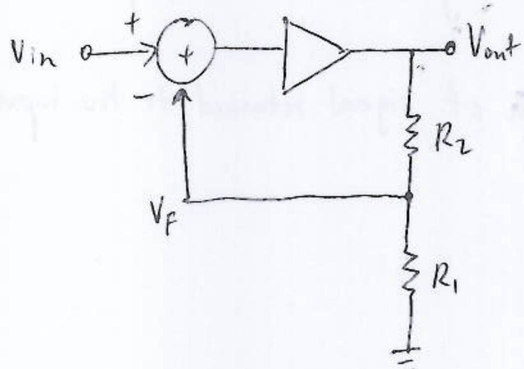
for current domain



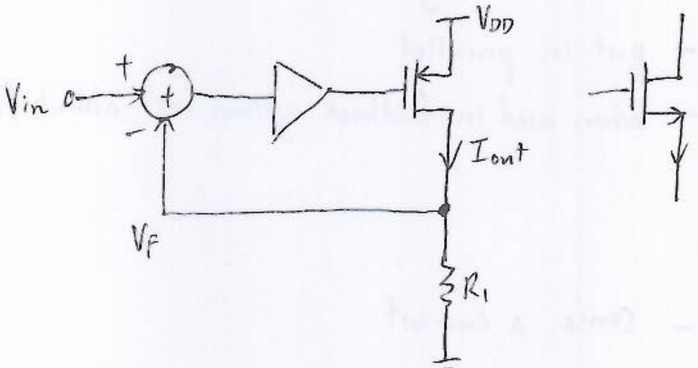
— no influence on the operation of the open-loop amplifier (ideally)

— introduces loading effects that must be taken into account (practically)

Practical implementations

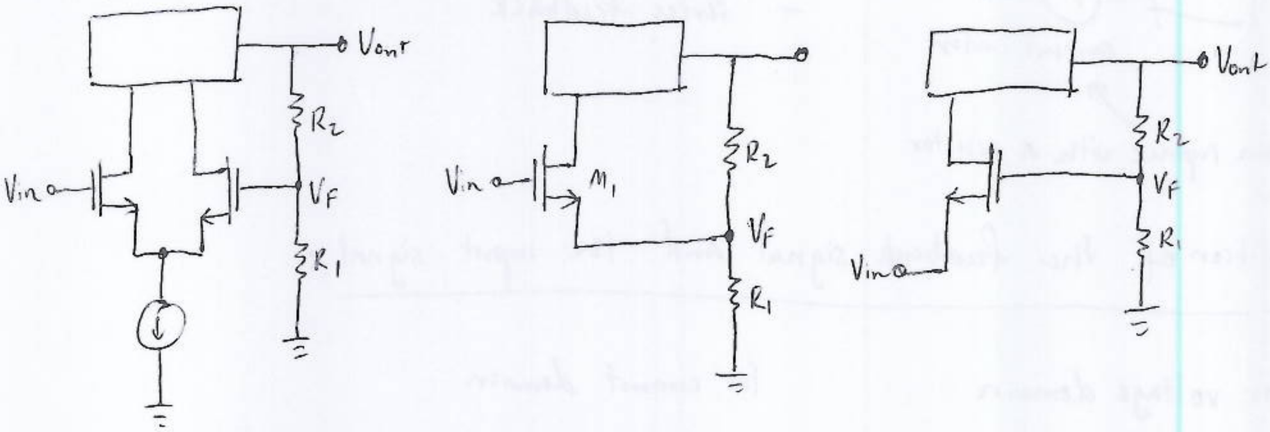


— voltage sensed by a resistive or capacitive divider in parallel with the port.

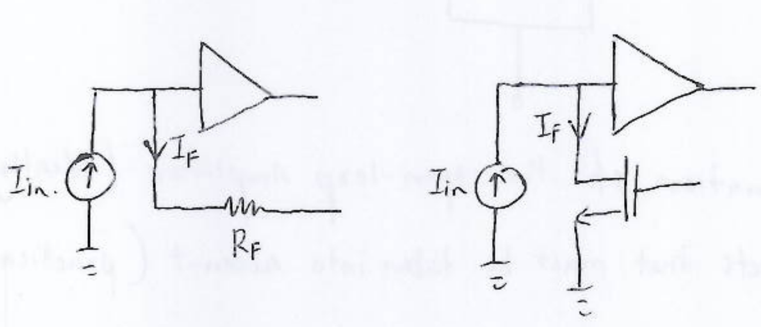


— current sensed by placing a resistor in series with the wire and sensing voltage across it.

To subtract two voltages: $(V_{in} - V_F)$



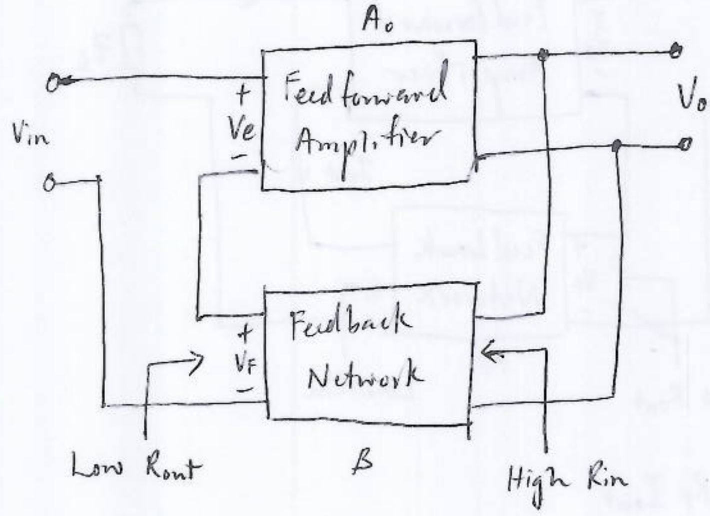
To subtract two currents: $(I_{in} - I_F)$



(✱) Note that voltage subtraction the input and ~~output~~ feedback signals are applied to two distinct nodes whereas current subtraction they are applied to a single node.

Feedback Topologies

① Voltage-voltage Feedback (series-shunt)



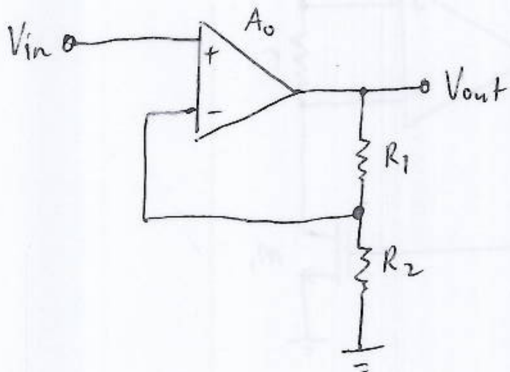
$\therefore \quad V_F = \beta V_{out}$

$V_e = V_{in} - V_F$

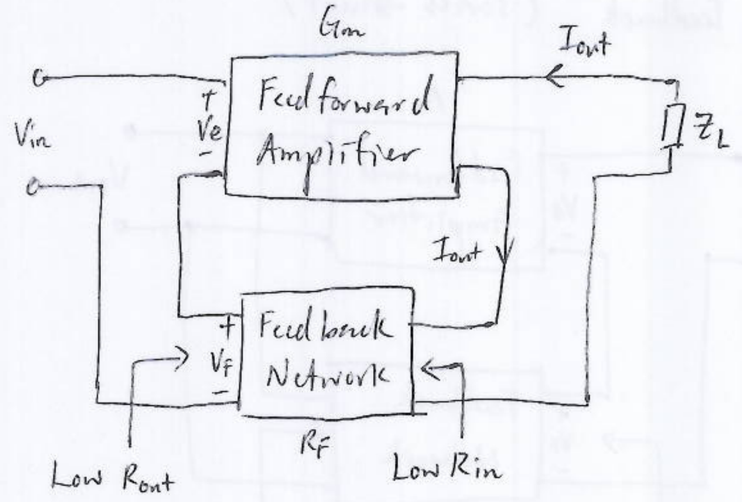
$V_{out} = A_o (V_{in} - \beta V_{out})$

$$\therefore \quad \frac{V_{out}}{V_{in}} = \frac{A_o}{1 + \beta A_o}$$

Simple example:



② Current - Voltage Feedback (series - series)



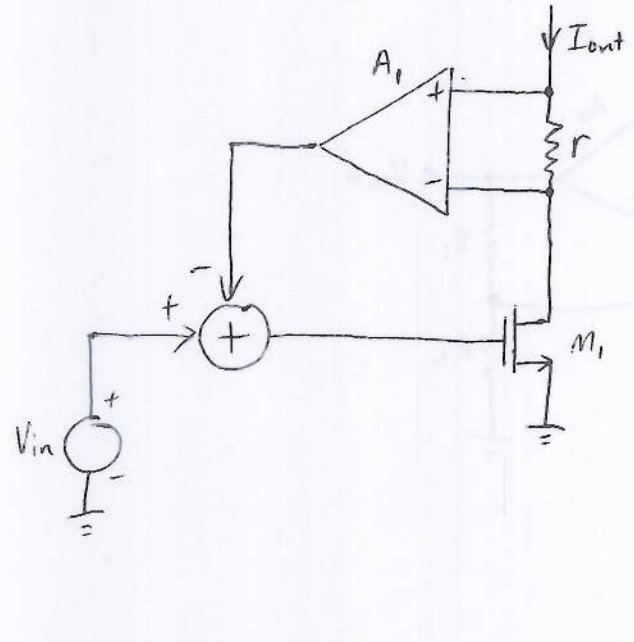
$$\therefore \quad V_F = R_F I_{out}$$

$$V_e = V_{in} - R_F I_{out}$$

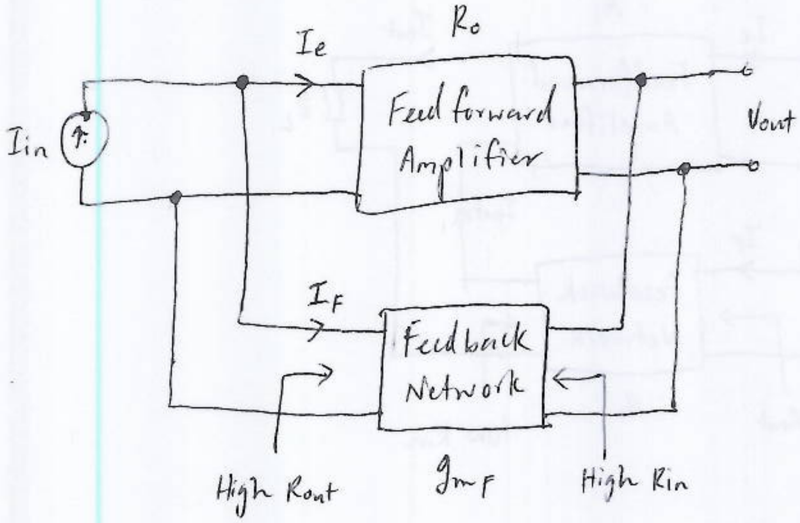
$$I_{out} = G_m \left(V_{in} - R_F I_{out} \right)$$

$$\therefore \quad \frac{I_{out}}{V_{in}} = \frac{G_m}{1 + G_m R_F}$$

Example



③ Voltage - Current Feedback (shunt-shunt)



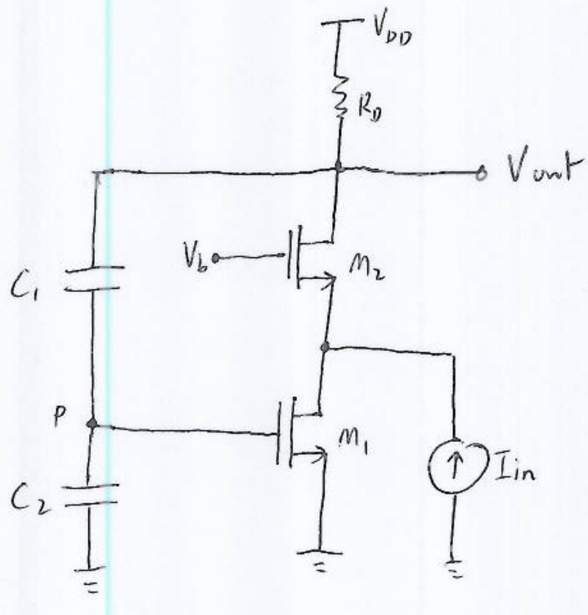
$$\therefore \quad I_F = g_{mF} V_{out}$$

$$I_e = I_{in} - I_F$$

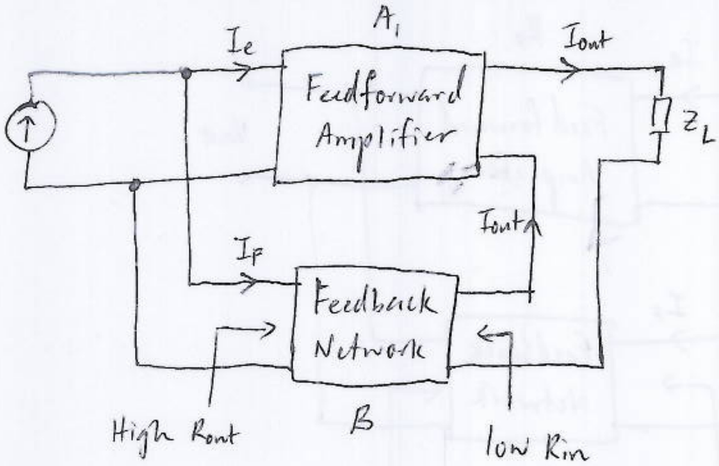
$$V_{out} = R_o I_e = R_o (I_{in} - g_{mF} V_{out})$$

$$\therefore \quad \frac{V_{out}}{I_{in}} = \frac{R_o}{1 + g_{mF} R_o}$$

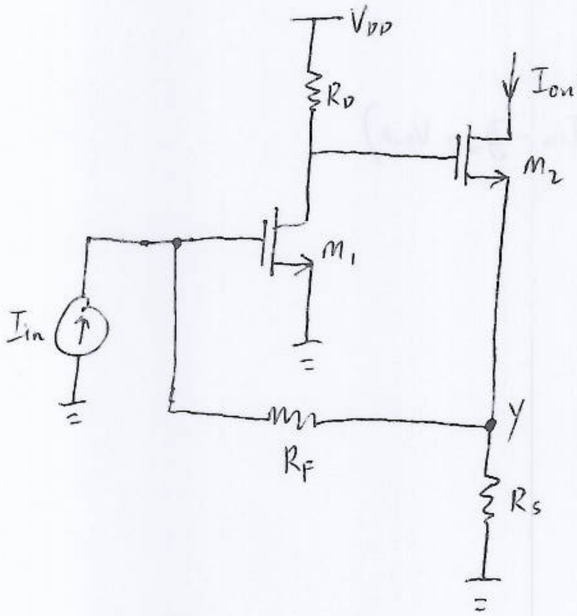
Example



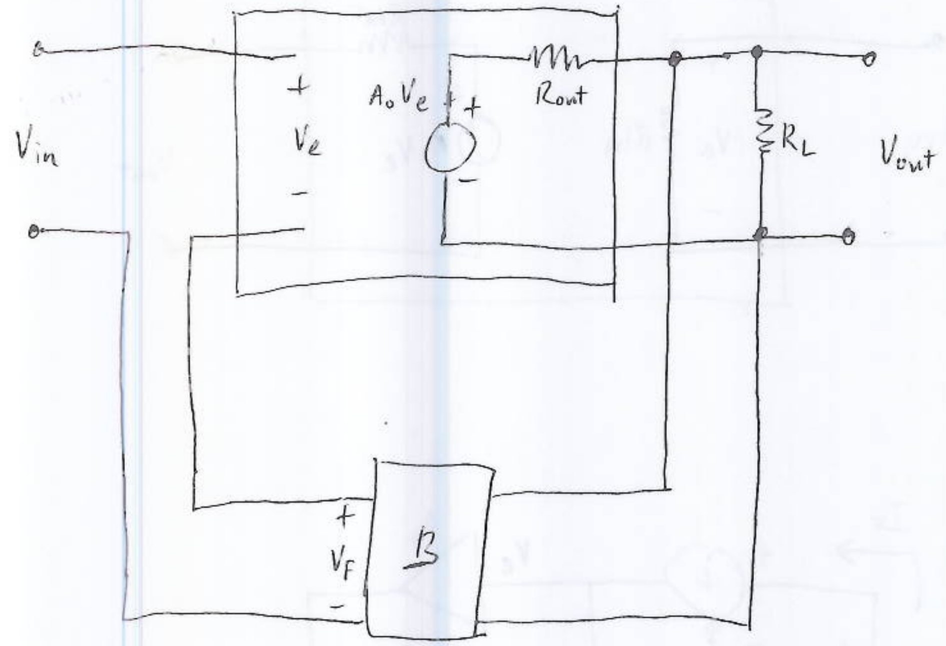
(4) Current_current Feedback (shunt-series)

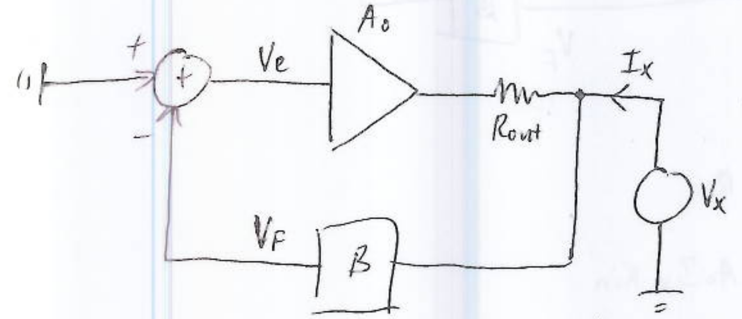


Example



Effect of voltage-voltage feedback on output resistance





$$V_F = \beta V_x$$

$$V_e = -\beta V_x$$

$$V_M = -\beta A_0 V_x$$

$$I_x = \left[V_x - (-\beta A_0 V_x) \right] / R_{out}$$

$$\therefore \quad \frac{V_x}{I_x} = \frac{R_{out}}{1 + \beta A_0}$$

Current - voltage

$$\frac{V_x}{I_x} = R_{out}\left(1 + G_m R_F\right)$$

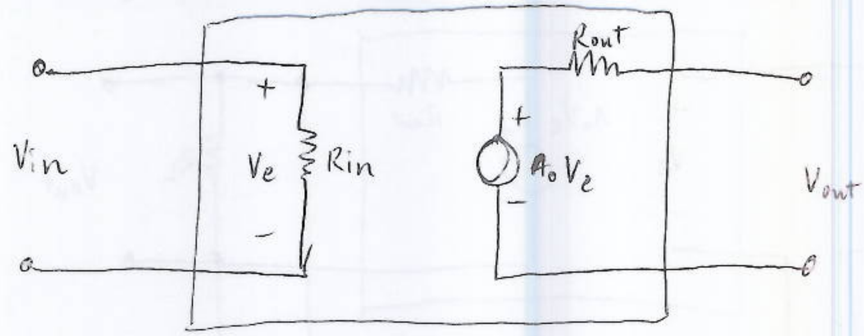
Voltage - current

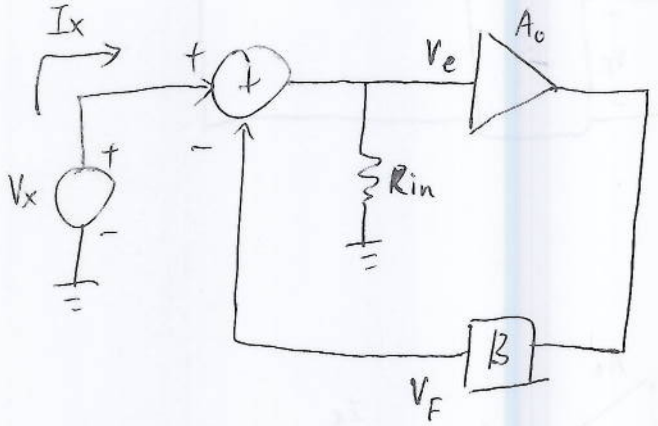
$$\frac{V_x}{I_x} = \frac{R_{out}}{1 + g_{mF} R_o}$$

Current - Current

$$\frac{V_x}{I_x} = R_{out}\left(1 + \beta A_0\right)$$

Effect of voltage-voltage feedback on input impedance





$$V_e = I_x R_{in}$$

$$V_F = B A_o I_x R_{in}$$

$$V_e = V_x - V_F = V_x - B A_o I_x R_{in}$$

$$I_x R_{in} = V_x - B A_o I_x R_{in}$$

$$\frac{V_x}{I_x} = R_{in}\left(1 + B A_o\right)$$

current-voltage

$$\frac{V_x}{I_x} = R_{in}\left(1 + G_m R_F\right)$$

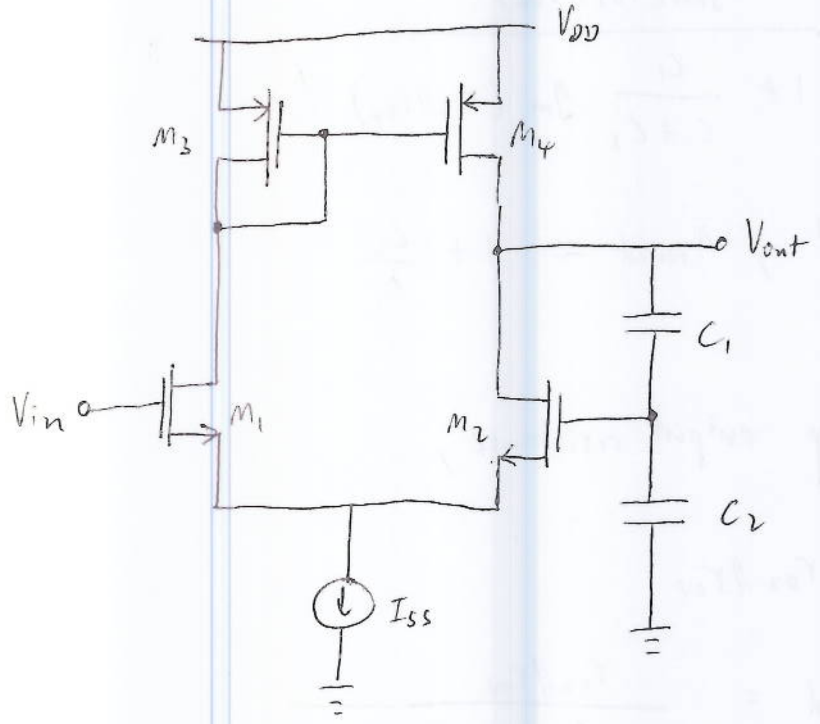
voltage-current

$$\frac{V_x}{I_x} = \frac{R_{in}}{1 + g_{mF} R_o}$$

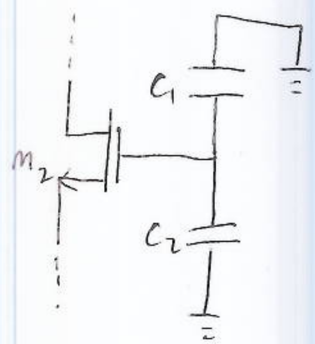
current-current

$$\frac{V_x}{I_x} = \frac{R_{in}}{1 + B A_o}$$

(E.g.) Calculate the closed-loop gain and output resistance of the amplifier.



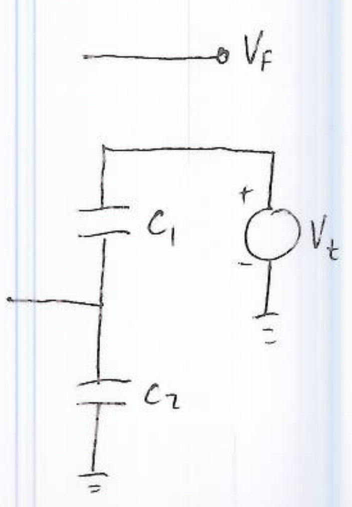
To find open-loop voltage gain, break the feedback loop



The open-loop gain,

$$A_{open} = g_{m1}(r_{o2} \,//\, r_{o4})$$

To find loop gain,



$$V_F = -V_t \frac{C_1}{C_1+C_2} g_{m1}(r_{o2} \,//\, r_{o4})$$

$$BA_o = \frac{C_1}{C_1+C_2} g_{m1}(r_{o2} \,//\, r_{o4})$$

To find closed-loop gain,

$$A_{closed} = \frac{g_{m1}(r_{o2} // r_{o4})}{1 + \frac{C_1}{C_1 + C_2} g_{m1}(r_{o2} // r_{o4})}$$

(*) if $\beta A_o >> 1$, $A_{closed} \approx 1 + \frac{C_2}{C_1}$

To find closed-loop output resistance,

$$R_{open} = r_{o2} // r_{o4}$$

$$\therefore R_{out,closed} = \frac{r_{o2} // r_{o4}}{1 + \frac{C_1}{C_1 + C_2} g_{m1}(r_{o2} // r_{o4})}$$