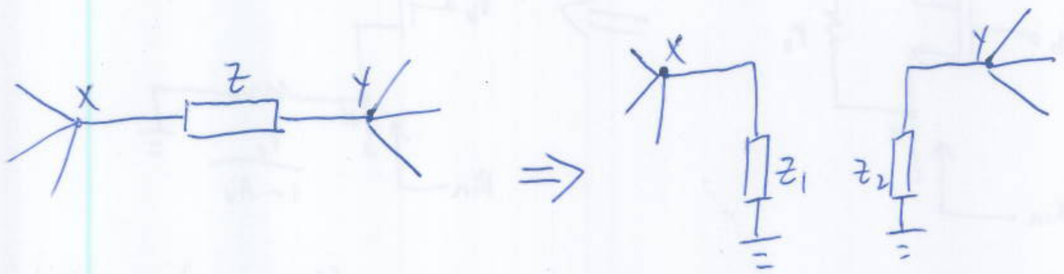
Miller Effect ( High frequency , useful where $Z$ parallel with main signal )

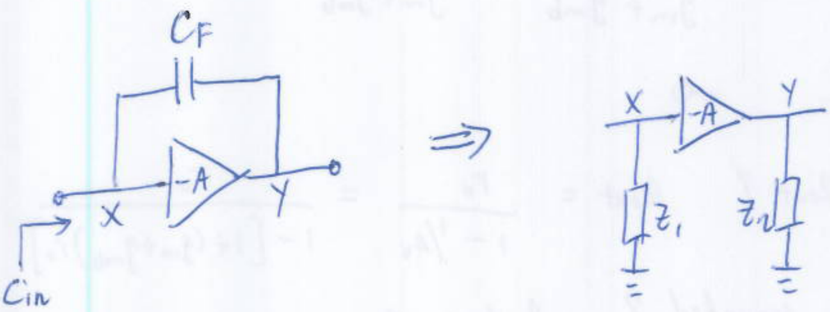
Miller's Theorem $\rightarrow$ $Z_1 = \dfrac{Z}{1-A_v}$ , $Z_2 = \dfrac{Z}{1-A_v^{-1}}$ , $A_v = \dfrac{V_Y}{V_X}$




Proof: Current flowing through $Z$ from X to Y is $(V_X - V_Y)/Z$.
For the two circuits to be equivalent, the same current must flow through $Z_1$.

$$\therefore \quad \frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1} \implies Z_1 = \frac{Z}{1-\frac{V_Y}{V_X}} , \quad Z_2 = \frac{Z}{1-\frac{V_X}{V_Y}}$$

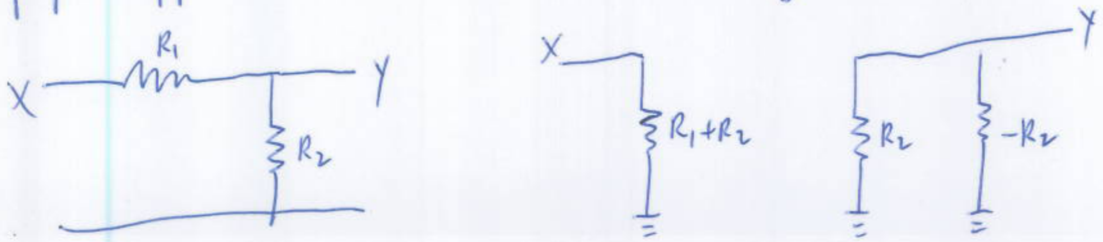
E.g. Consider the circuit below, where the voltage amplifier has a negative gain equal to $-A$ and is otherwise ideal. Calculate the input capacitance of the circuit.



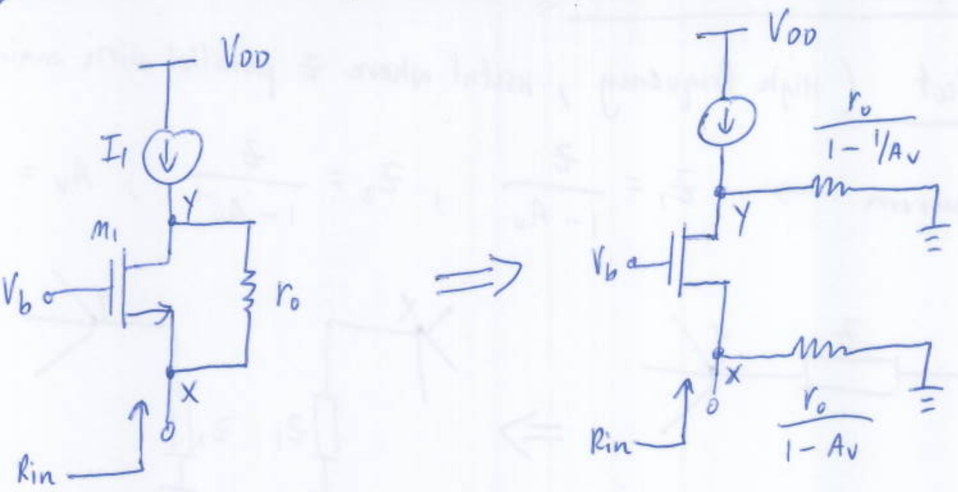

Using Miller's theorem to convert the circuit.

$$Z = \frac{1}{sC_F} \quad \therefore \quad Z_1 = \frac{(1/sC_F)}{1+A} \implies \text{Input capacitance} = C_F(1+A)$$

Improper application of Miller's theorem ( input impedance correct but gain incorrect )



E.g. Calculate the input ~~impedance~~ resistance of the circuit.




C-G stage gain proven before : $A_V = \dfrac{(g_m+g_{mb})r_o+1}{r_o+(g_m+g_{mb})r_oR_S+R_S+R_D}R_D$

For this case, $R_D \to \infty$ $\therefore$ $A_V = 1+(g_m+g_{mb})r_o$

$\therefore$ Input resistance :

$$R_{in} = \frac{r_o}{1-[1+(g_m+g_{mb})r_o]} \parallel \frac{1}{g_m+g_{mb}}$$

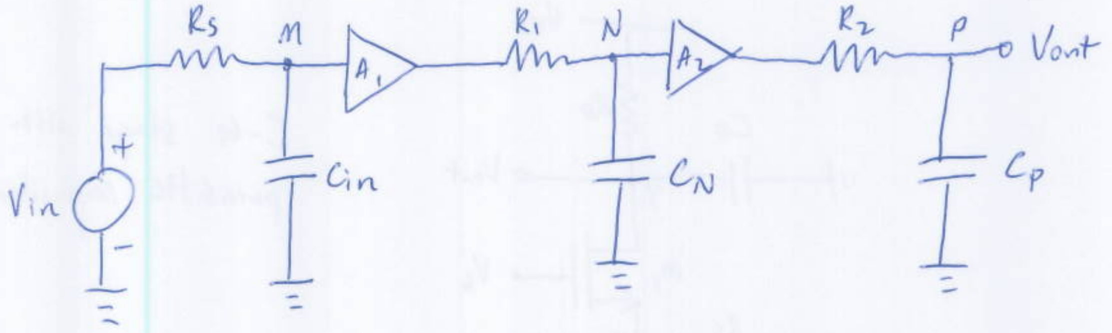
$$= \frac{-1}{g_m+g_{mb}} \parallel \frac{1}{g_m+g_{mb}}$$

$$= \infty$$

What about $R_{out}$? $R_{out} = \dfrac{r_o}{1-1/A_V} = \dfrac{r_o}{1-[1+(g_m+g_{mb})r_o]^{-1}} = \dfrac{1}{g_m+g_{mb}}+r_o$

If X is grounded? $R_{out} = r_o$.

## Association of Poles with Nodes



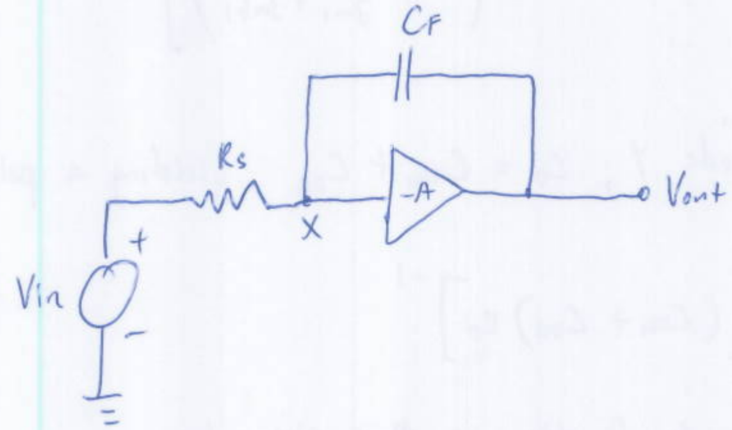

The overall transfer function can be written as

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_p s}$$

Three poles

$$w = \frac{1}{\tau} = \frac{1}{RC}$$

E.g. Calculate the pole associated with node X.



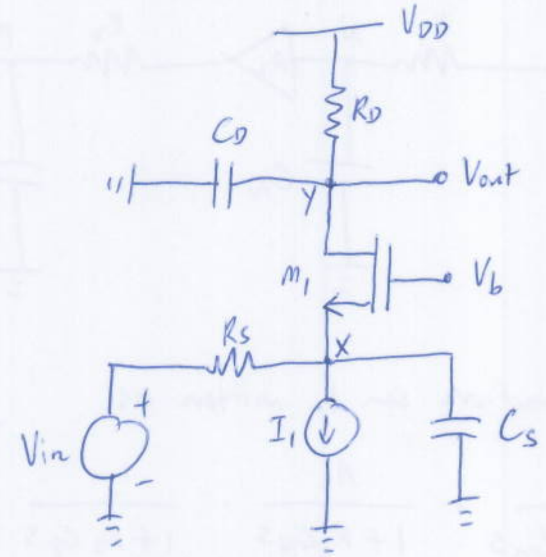

The total equivalent capacitance seen from X to ground is $(1+A)C_F$

Since the capacitance is driven by Rs, the pole frequency is

$$\frac{1}{R_S(1+A)C_F} \quad (\text{in rad/s})$$

input pole.

(E.g.) Neglecting channel-length modulation, compute the transfer function of the common-gate stage shown in Fig. below.




C-G stage with parasitic capacitances.

The capacitances contributed by $M_1$ are connected from input and output nodes to ground. At node X, $C_S = C_{GS1} + C_{SB1}$, giving a pole frequency

$$\omega_{in} = \left[ (C_{GS1} + C_{SB1}) \left( R_S \,\Big\|\, \frac{1}{g_{m1} + g_{mb1}} \right) \right]^{-1}$$

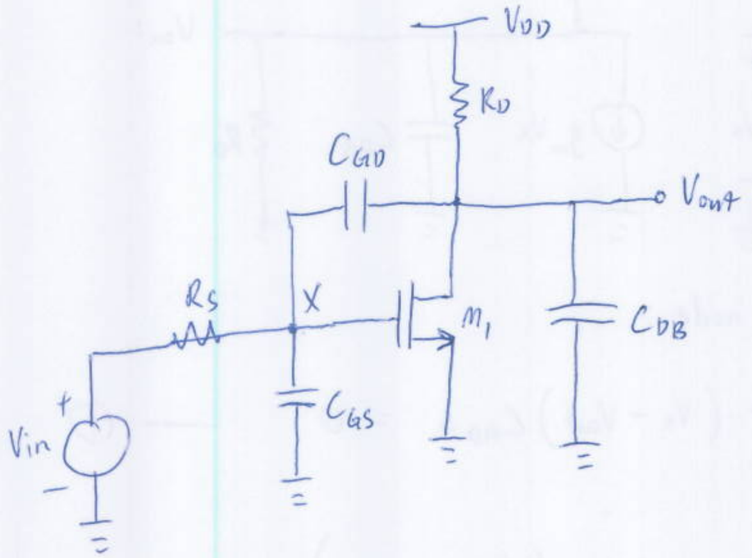
Similarly, at node Y, $C_D = C_{DG} + C_{DB}$, yielding a pole frequency

$$\omega_{out} = \left[ (C_{DG} + C_{DB}) R_D \right]^{-1}$$

The overall transfer function is thus given by

$$\frac{V_{out}}{V_{in}}(s) = \underbrace{\frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_S}}_{\text{low-freq gain}} \cdot \frac{1}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Common-Source Stage




Assume $\lambda = 0$ and $M_1$ operates in saturation. Find/Estimate the transfer function by associating one pole with each node.

High-frequency model.

Total capacitance seen from X to ground is equal to $C_{GS}$ plus Miller multiplication of $C_{GD}$: $C_{GS} + (1 - A_V)C_{GD}$, where $A_V = -g_m R_D$

The magnitude of the input pole is

$$\omega_{in} = \frac{1}{R_S\left[C_{GS} + (1 + g_m R_D)C_{GD}\right]}$$

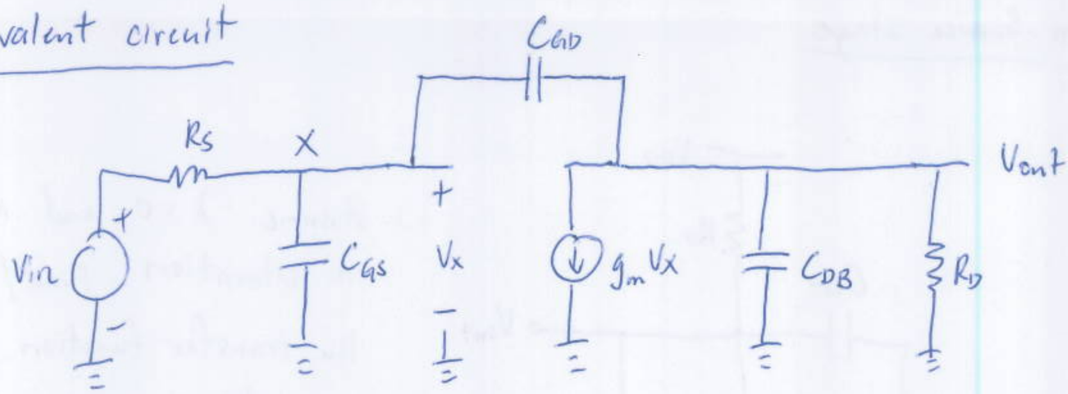
At the output node, the total capacitance seen to ground is equal to $C_{DB}$ plus the Miller effect of $C_{GD}$: $C_{DB} + (1 - A_V^{-1})C_{GD} \approx C_{DB} + C_{GD}$

$$\therefore \quad \omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

Transfer function is

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Equivalent circuit




Sum currents at each node:

$$\frac{V_x - V_{in}}{R_S} + V_x C_{GS} s + (V_x - V_{out}) C_{GD} s = 0 \quad \text{—— ①}$$

$$(V_{out} - V_x) C_{GD} s + g_m V_x + V_{out}\left(\frac{1}{R_D} + C_{DB} s\right) = 0$$

$$V_x = -\frac{V_{out}\left(C_{GD} s + \frac{1}{R_D} + C_{DB} s\right)}{g_m - C_{GD} s} \quad \text{—— ②}$$

Sub ② into ①

$$-V_{out}\frac{\left[R_S^{-1} + (C_{GS} + C_{GD})s\right]\left[R_D^{-1} + (C_{GD} + C_{DB})s\right]}{g_m - C_{GD} s} - V_{out} C_{GD} s = \frac{V_{in}}{R_S}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD} - g_m) R_D}{R_S R_D \xi s^2 + \left[R_S(1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})\right] s + 1}$$

where $\xi = C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}$

Note that the transfer function is of second-order even though the circuit contains three capacitors. This is because the capacitors form a "loop" allowing only two independent initial conditions in the circuit and hence yielding a second-order differential equation for the time response.

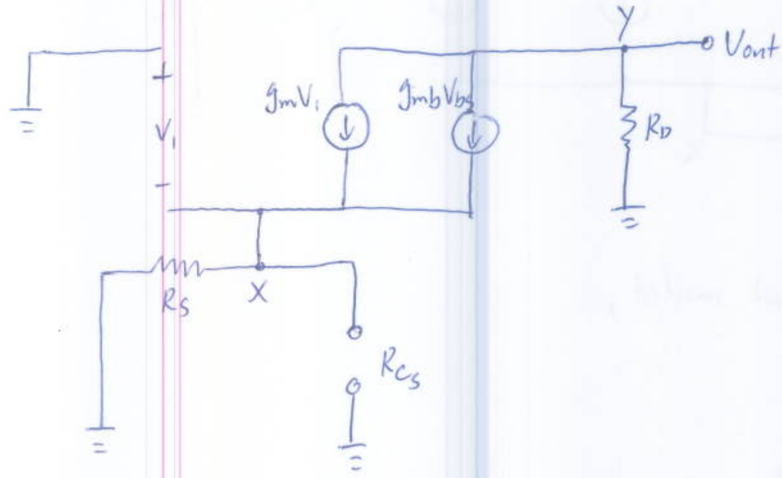
Can express two poles $w_{p1}$ and $w_{p2}$ by the denominator

$$D = \left(\frac{s}{w_{p1}} + 1\right)\left(\frac{s}{w_{p2}} + 1\right)$$

$$= \frac{s^2}{w_{p1} w_{p2}} + \left(\frac{1}{w_{p1}} + \frac{1}{w_{p2}}\right) s + 1$$

## Common-gate stage

Input pole frequency, $w_{in}$, at node X,




From the small-signal model,

$$\frac{V_X}{R_S} - g_m V_1 - g_{mb} V_{bs} - I_X = 0 \quad — ①$$

$$V_1 = -V_X \quad — ②$$

Therefore,

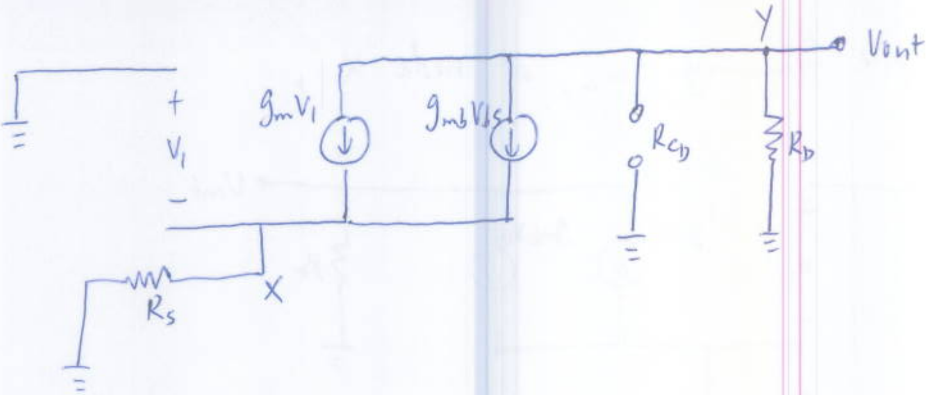
$$\frac{V_X}{R_S} + g_m V_X + g_{mb} V_X - I_X = 0$$

$$R_{CS} = \frac{V_X}{I_X} = \frac{1}{\frac{1}{R_S} + g_m + g_{mb}}$$

$$\therefore \quad w_{in} = \frac{1}{\tau} = \frac{1}{R_{CS} C_S}$$

$$w_{in} = \frac{1}{\frac{1}{\frac{1}{R_S} + g_m + g_{mb}} \cdot C_S} = \left[ (C_S) \cdot \left( R_S // \frac{1}{g_m + g_{mb}} \right) \right]^{-1}$$

Output pole frequency, $w_{out}$, at node Y,



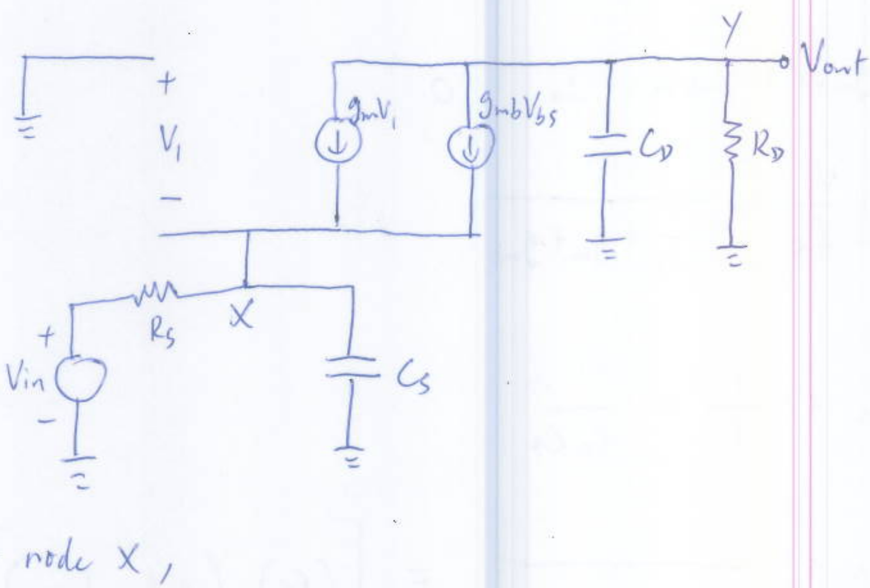

From small-signal model,

$$R_{CD} = R_D$$

$$\therefore \quad W_{out} = \frac{1}{\tau} = \frac{1}{R_{CD} C_D}$$

$$W_{out} = \frac{1}{R_D C_D} = \left[ R_D \, C_D \right]^{-1}$$

The overall transfer function, $\frac{V_{out}}{V_{in}}(s)$




At node X,

$$\frac{V_X - V_{in}}{R_S} + \frac{V_X}{Z_{CS}} - g_m V_1 - g_{mb} V_1 = 0 \quad — ①$$

$$V_1 = -V_X \quad — ②$$

From ① and ②,

$$\frac{V_x - V_{in}}{R_s} + \frac{V_x}{Z_{CS}} + g_m V_x + g_{mb} V_x = 0 \quad — ③$$

At node Y,

$$\frac{V_{out}}{R_D} + \frac{V_{out}}{Z_{CS}} - g_m V_x - g_{mb} V_x = 0$$

$$V_{out}\left(\frac{1}{R_D} + \frac{1}{Z_{CS}}\right) = V_x\left(g_m + g_{mb}\right)$$

$$V_x = \frac{V_{out}\left(\frac{1}{R_D} + \frac{1}{Z_{CD}}\right)}{g_m + g_{mb}} \quad — ④$$

Sub ④ into ③

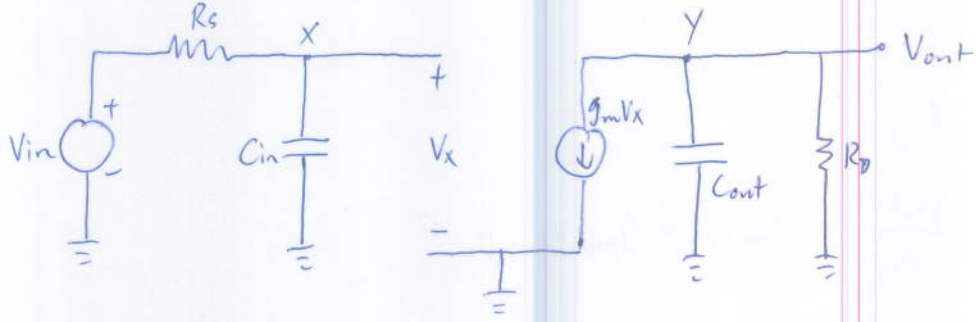
$$\frac{V_{out}\left(\frac{1}{R_D} + \frac{1}{Z_{CD}}\right)}{g_m + g_{mb}}\left[\frac{1}{R_s} + \frac{1}{Z_{CS}} + g_m + g_{mb}\right] = \frac{V_{in}}{R_s}$$

$$V_{out} = V_{in}\left[\frac{g_m + g_{mb}}{1 + \frac{R_s}{Z_{CS}} + g_m R_s + g_{mb} R_s}\right] \cdot \frac{1}{\frac{1}{R_D} + \frac{1}{Z_{CD}}}$$

$$\therefore \frac{V_{out}}{V_{in}}(s) = \left[\frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_s}\right] \cdot \frac{1}{\left(1 + \frac{s}{w_{in}}\right)\left(1 + \frac{s}{w_{out}}\right)}$$

## Common - Source Stage

Using Miller's Theorem,




From equivalent circuit,

$$A_V = -g_m R_D$$

The total input capacitance is,

$$C_{in} = C_{GS} + (1 - A_V) C_{GD}$$

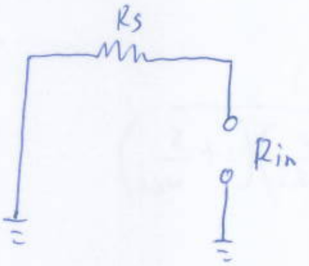
$$= C_{GS} + (1 + g_m R_D) C_{GD}$$

The total output capacitance is,

$$C_{out} = C_{DB} + (1 - A_V^{-1}) C_{GD}$$

$$\simeq C_{DB} + C_{GD}$$

Using open circuit time-constant method, input pole freq $w_{in}$ at node X,

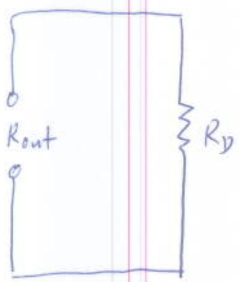



$\therefore \; R_{in} = R_S$

$$w_{in} = \frac{1}{\tau} = \frac{1}{R_{in} C_{in}}$$

$$= \frac{1}{R_S \cdot \left[ C_{GS} + (1 + g_m R_D) C_{GD} \right]}$$

Using open circuit time-constant method, output pole freq. $\omega_{out}$ at node Y,




$$R_{out} = R_D$$

$$\omega_{out} = \frac{1}{\tau} = \frac{1}{R_{out} C_{out}}$$

$$= \frac{1}{R_D \cdot (C_{DB} + C_{GD})}$$