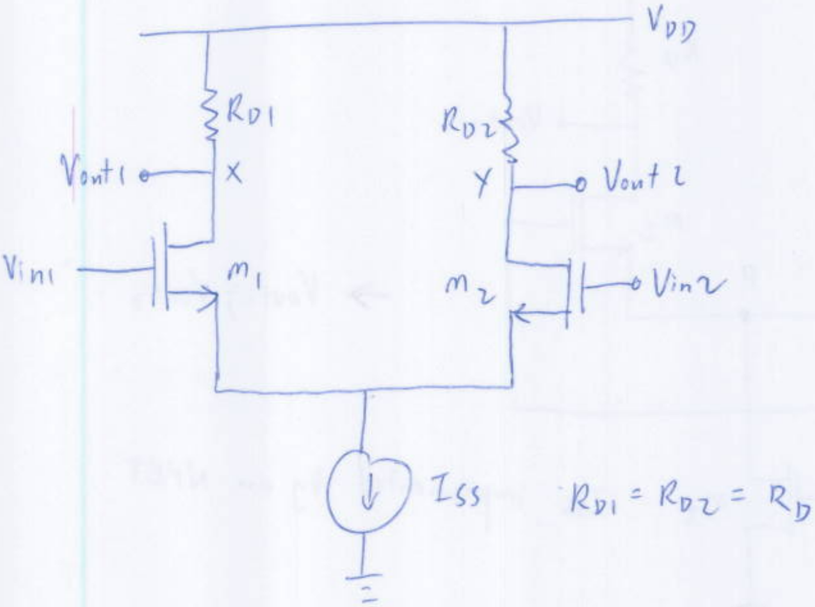
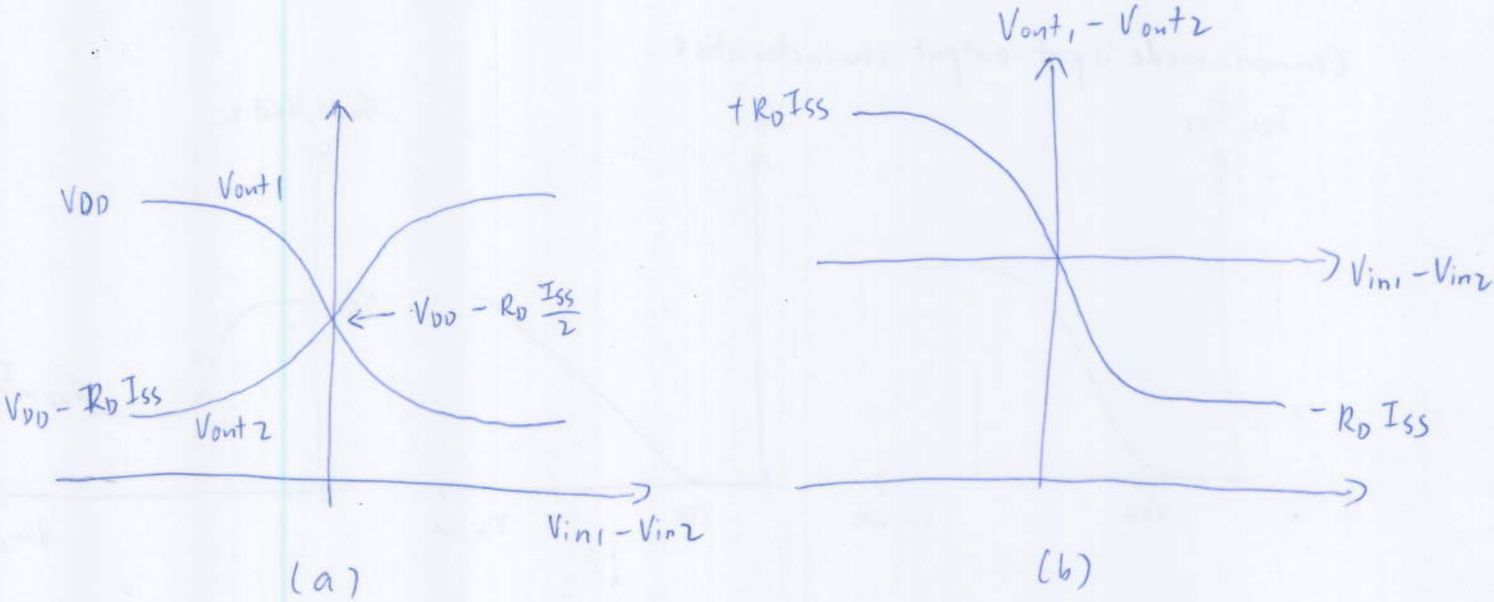
Basic differential pair





Input - Output Characteristics.

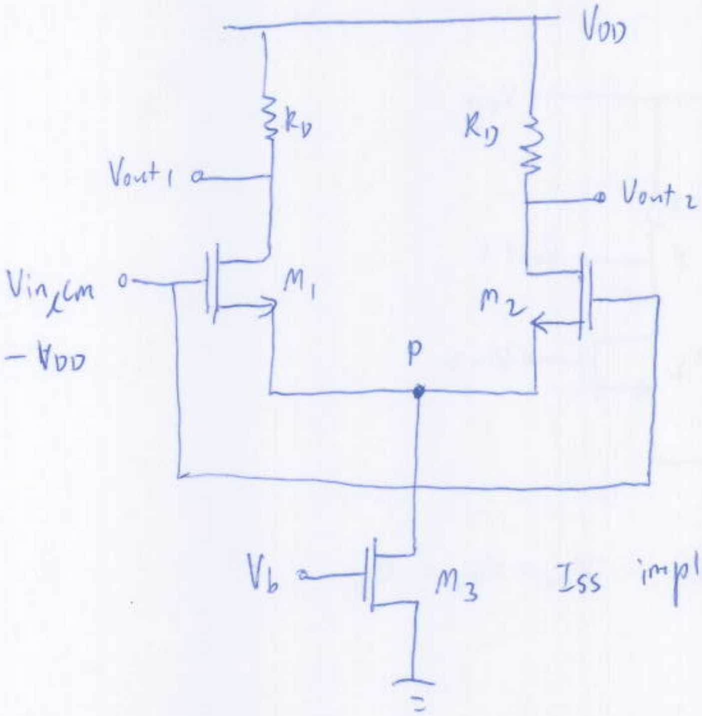
For $\Delta V_{in} = 0$, $G_m = \sqrt{\mu_n C_{ox} (W/L) I_{ss}}$

$V_{out1} - V_{out2} = R_D \Delta I = R_D G_m \Delta V_{in}$,

small-signal differential voltage gain

$$|A_v| = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{ss}} \cdot R_D$$

Differential pair sensing an input common-mode change

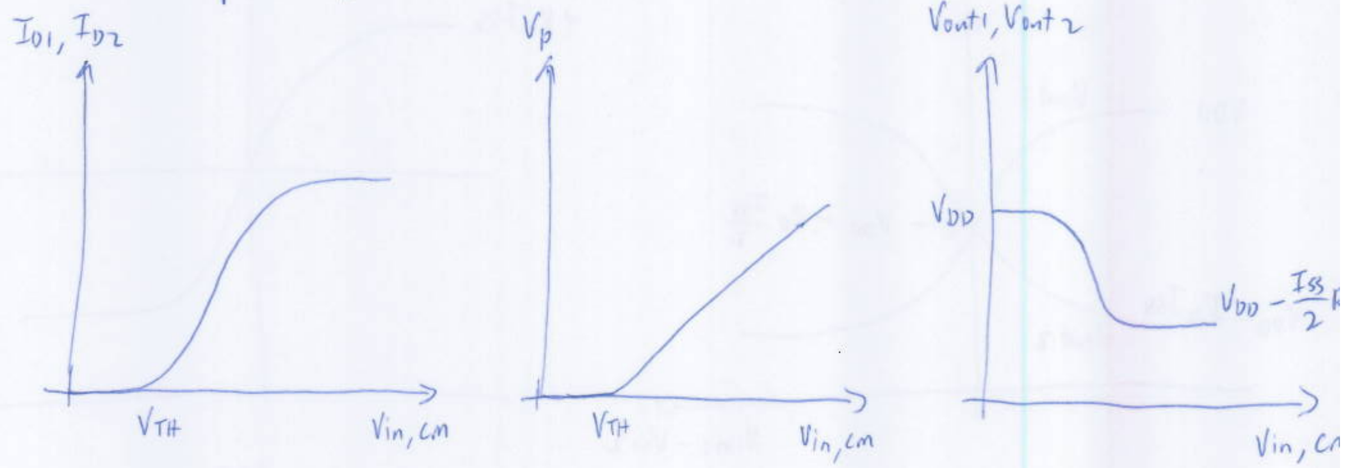


vary $0 - V_{DD}$

$\rightarrow V_{out1} = V_{out2}$

I_{SS} implemented by an NFET

Common-mode input-output characteristics.

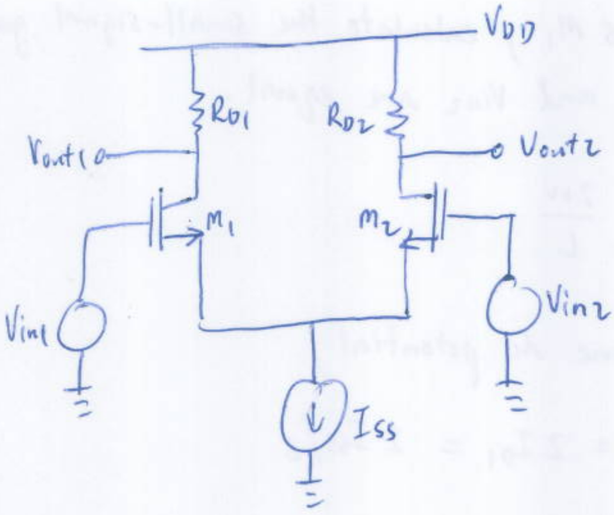


Proper operation $V_{in,cm} \geq V_{GS1} + (V_{GS3} - V_{TH3})$

M_1 and M_2 enters triode region $V_{in,cm} > V_{out1} + V_{TH} = V_{DD} - R_D \frac{I_{SS}}{2} + V_{TH}$

Allowable values of $V_{in,cm}$ is bounded as follows:

$$V_{GS1} + (V_{GS3} - V_{TH3}) \leq V_{in,cm} \leq \min\left[V_{DD} - R_D\frac{I_{SS}}{2} + V_{TH},\ V_{DD}\right]$$



Differential gain ?

$\lambda = \gamma = 0$

$R_{D1} = R_{D2} = R_D$

Superposition: $\dfrac{V_X}{V_{in1}} = \dfrac{-R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$ ← acting like common-source

$\dfrac{V_Y}{V_{in1}} = \dfrac{R_D}{\frac{1}{g_{m2}} + \frac{1}{g_{m1}}}$ ← acting like common-gate

∴ Overall gain due to V_{in1}, $V_X - V_Y = \dfrac{-2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} V_{in1}$

if $g_{m1} = g_{m2} = g_m$,

$(V_X - V_Y)|_{\text{due to } V_{in1}} = -g_m R_D V_{in1}$

$(V_X - V_Y)|_{\text{due to } V_{in2}} = +g_m R_D V_{in2}$

$$\therefore \frac{(V_X - V_Y)_{total}}{V_{in1} - V_{in2}} = -g_m R_D$$

E.g. If M_2 is twice as wide as M_1, calculate the small-signal gain if the bias values of V_{in1} and V_{in2} are equal.

$$M_1 : \frac{W}{L} \qquad M_2 : \frac{2W}{L}$$

M_1 and M_2 gates at same dc potential,

$$V_{GS1} = V_{GS2} \quad \text{and} \quad I_{D2} = 2I_{D1} = 2I_{SS}/3$$

$$\therefore \; g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L} \cdot \frac{I_{SS}}{3}} \;, \quad g_{m2} = \sqrt{2\mu_n C_{ox} \left(\frac{2W}{L}\right) \frac{2I_{SS}}{3}} = 2g_{m1}$$

$$|A_V| = \frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{2g_{m1}}} = \frac{4}{3} g_{m1} R_D$$

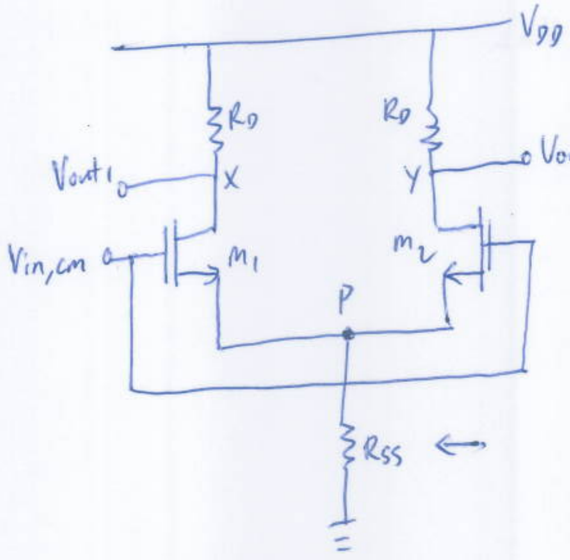
[Note that this value is lower than the gain of a symmetric differential pair (with 2W/L for each device)]

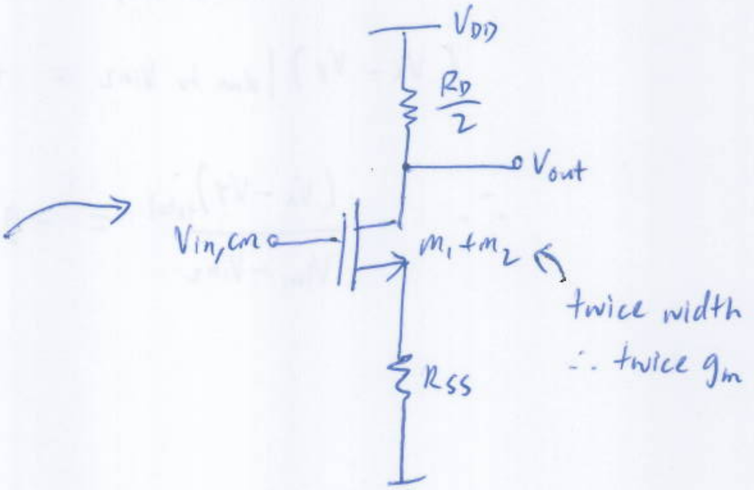
Common-Mode Response

- circuit not fully symmetric in reality
- current source not infinite output impedance

} fraction of CM variation appears at the output.

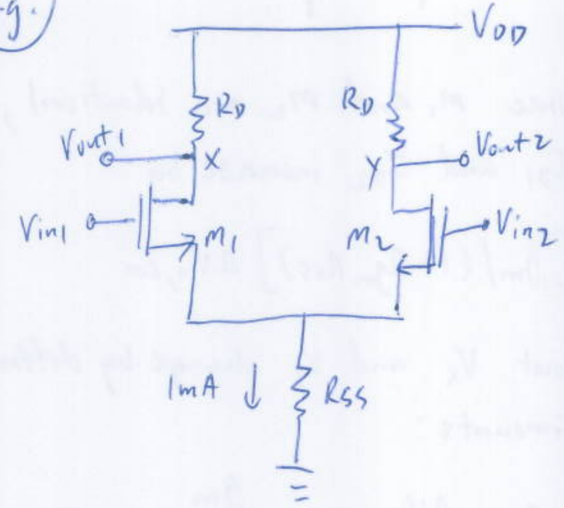
If symmetric but current source has a finite output impedance R_{SS}





$$\therefore A_{V,cm} = \frac{-R_D/2}{1/2g_m + R_{SS}}$$

(E.g.)



Assume $(W/L)_{1,2} = 25/0.5$

$\mu_n C_{ox} = 50\,\mu A/V^2$

$V_{TH} = 0.6V$

$\lambda = \gamma = 0$

$V_{DD} = 3V$

a) What is the required input CM for which R_{SS} sustains 0.5V?

$R_{SS} = 500\,\Omega$

Since $I_{D1} = I_{D2} = 0.5mA$,

$$V_{GS1} = V_{GS2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH}$$

$$= 1.23V$$

$$\therefore \; V_{in,cm} = V_{GS1} + 0.5V = 1.73V$$

b) Calculate R_D for a differential gain of 5.

Note that the output bias level is equal to $V_{DD} - I_{D1}R_D = 1.42V$. Since $V_{in,cm} = 1.73V$ and $V_{TH} = 0.6V$, the transistors are 290mV away from the triode region.

The transconductance of each device is $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_{D1}} = \frac{1}{632\,\Omega}$

$$|A_v| = g_m R_D$$

requiring $R_D = 3.16\,k\Omega$

c) What happens at the output if the input CM level is 50mV higher than the value calculated in (a)?

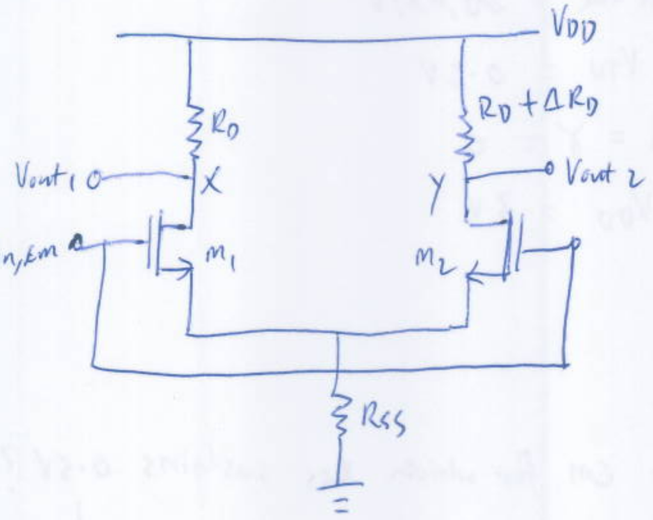
$$|\Delta V_{X,Y}| = \Delta V_{in,cm}\,\frac{R_D/2}{R_{SS} + 1/(2g_m)}$$

$$= 50mV \times 1.94$$

$$= 96.8\,mV.$$

Now M_1 and M_2 are only 143mV away from the triode region because the input CM level has increased by 50mV and the output CM level has decreased by 96.8mV.

Now consider non-symmetric and finite output impedance
asymmetry



Resistor mismatch

Since M_1 and M_2 are identical, I_{D1} and I_{D2} increase by

$$\left[g_m/(1+2g_mR_{SS})\right]\Delta V_{in,cm}$$

but V_X and V_Y change by different amounts:

$$\Delta V_X = -\Delta V_{in,cm}\frac{g_m}{1+2g_mR_{SS}}R_D$$

$$\Delta V_Y = -\Delta V_{in,cm}\frac{g_m}{1+2g_mR_{SS}}(R_D+\Delta R_D)$$

- common-mode change at the input introduces a differential component at the output.

If M_1 and M_2 mismatch? (threshold voltage mismatch)

$$I_{D1} = g_{m1}(V_{in,cm}-V_P)$$

$$I_{D2} = g_{m2}(V_{in,cm}-V_P)$$

$$(g_{m1}+g_{m2})(V_{in,cm}-V_P)R_{SS} = V_P$$

$$V_P = \frac{(g_{m1}+g_{m2})R_{SS}}{(g_{m1}+g_{m2})R_{SS}+1}V_{in,cm}$$

Output voltages,

$$V_X = -g_{m1}(V_{in,cm}-V_P)R_D = \frac{-g_{m1}}{(g_{m1}+g_{m2})R_{SS}+1}R_D V_{in,cm}$$

and

$$V_Y = -g_{m2}(V_{in,cm}-V_P)R_D = \frac{-g_{m2}}{(g_{m1}+g_{m2})R_{SS}+1}R_D V_{in,cm}$$

$$V_X - V_Y = \frac{-(g_{m1}-g_{m2})}{(g_{m1}+g_{m2})R_{SS}+1}R_D V_{in,cm}$$

$$\therefore A_{CM-DM} = \frac{\Delta g_m R_D}{(g_{m1}+g_{m2})R_{SS}+1}$$

$A_{CM-DM} \rightarrow$ common-mode to differential-mode conversion

$\Delta g_m = g_{m1} - g_{m2}$

For meaningful comparison of differential circuits, the undesirable differential component produced by cm variations must be normalised to the wanted differential output resulting from amplification.

Common-mode rejection ratio (CMRR),

$$CMRR = \left| \frac{A_{DM}}{A_{cm-DM}} \right|$$

If only g_m mismatch is considered, we can show

$$|A_{DM}| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{1 + (g_{m1} + g_{m2})R_{SS}}$$

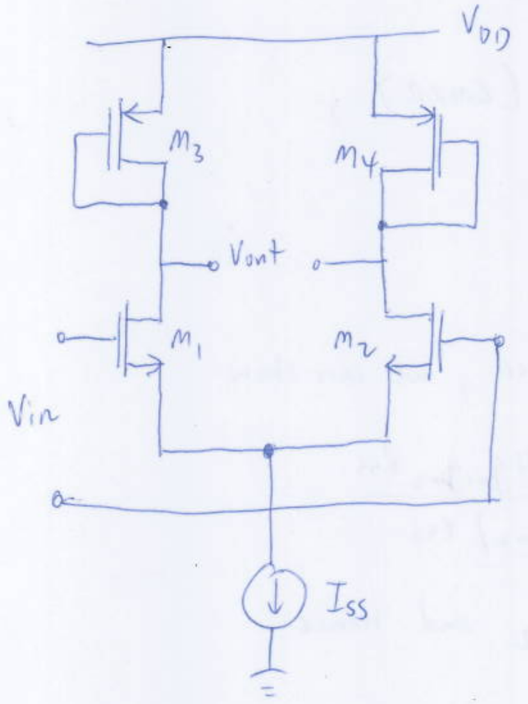
where it is assumed $V_{in1} = -V_{in2}$ and hence

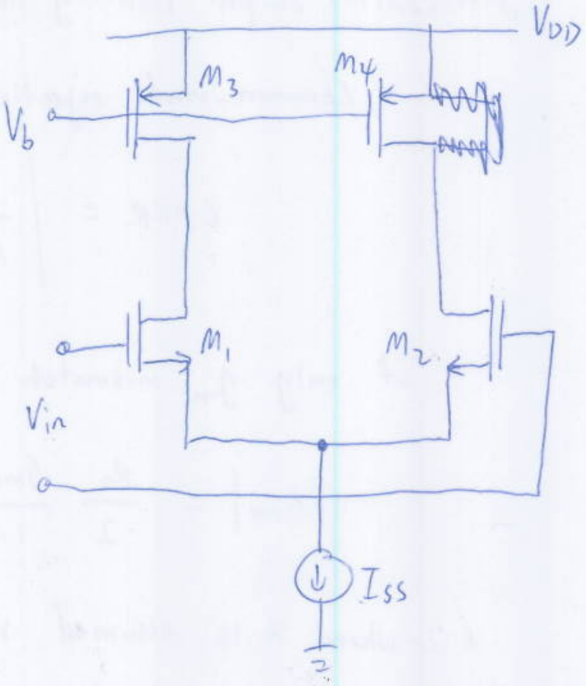
$$CMRR = \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{2\Delta g_m}$$

$$\approx \frac{g_m}{\Delta g_m}(1 + 2g_m R_{SS})$$

where g_m denotes the mean value, i.e. $g_m = (g_{m1} + g_{m2})/2$

Differential Pair with MOS Loads





$$A_V = -g_{mN}\left(g_{mp}^{-1} \,\|\, r_{ON} \,\|\, r_{op}\right)$$

$$\approx -\frac{g_{mN}}{g_{mP}}$$

$$\approx -\sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$$

$$A_V = -g_{mN}\left(r_{ON} \,\|\, r_{op}\right)$$

→ consumes voltage headroom

→ trade-off between the output swings, voltage gain and input CM range.