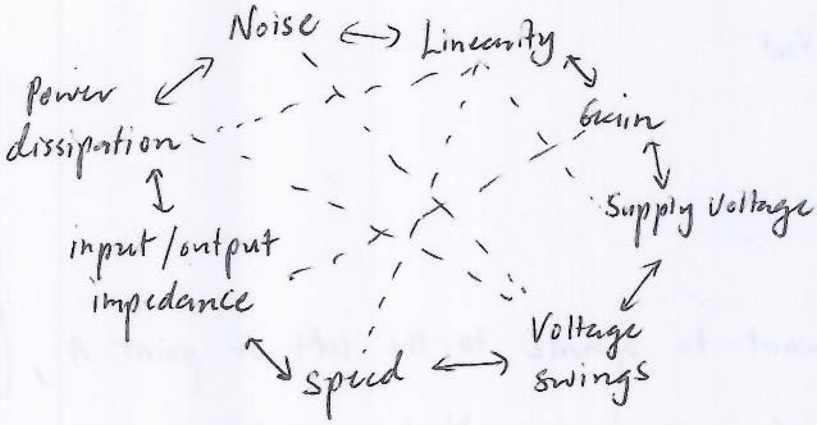
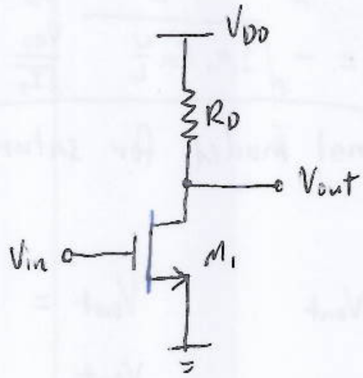
Chapter 2 - Single-Stage Amplifiers

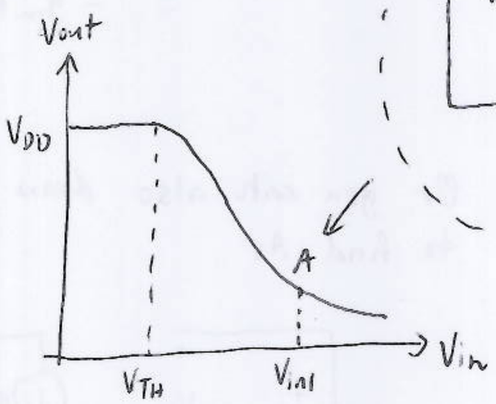
Basic concepts

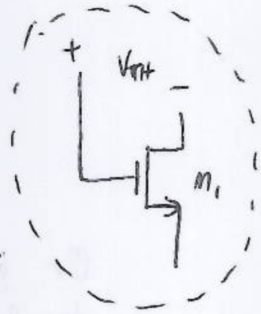


Common-Source Stage

(A.) Common-Source Stage with Resistive Load







If M_1 in saturation,

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

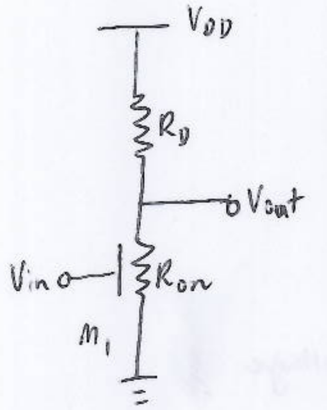
At point A,

$$V_{in1} - V_{TH} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

For $V_{in} > V_{in1}$, M_1 in triode region

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 (V_{in} - V_{TH}) V_{out} - V_{out}^2 \right]$$

If V_{in} is high enough, M_1 is in deep triode region, $V_{out} \ll 2(V_{in} - V$



$$V_{out} = V_{DD} \frac{R_{on}}{R_{on} + R_D} = \frac{V_{DD}}{1 + \mu_n C_{ox} \frac{W}{L} R_D (V_{in} - V_{TH})}$$

Normally we want to operate to the left of point A, $\left[V_{out} > V_{in} - V_{TH} \right]$

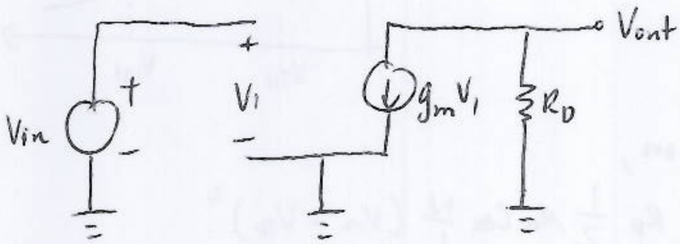
$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

$$A_V = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})$$

$$= -g_m R_D$$

$$\boxed{\begin{aligned} A_V &= -\sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \; \frac{V_{RD}}{I_D} \\ &= -\sqrt{2 \mu_n C_{ox} \frac{W}{L}} \; \frac{V_{RD}}{\sqrt{I_D}} \end{aligned}}$$

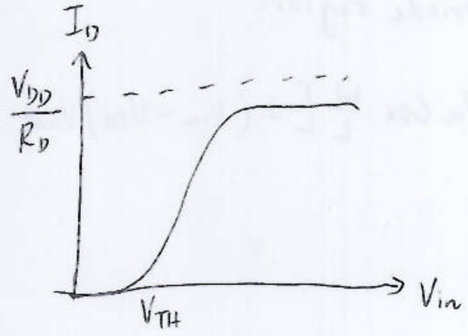
Or you can also draw small-signal model for saturation region. to find A_V

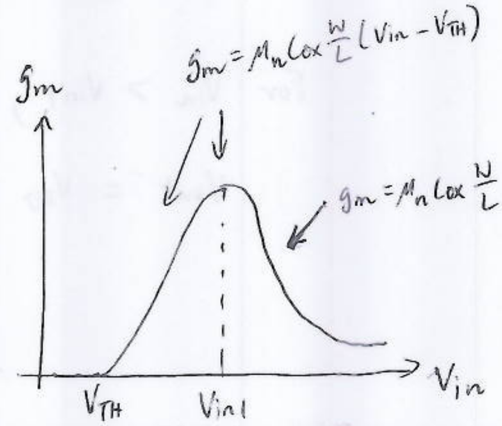


$$V_{out} = -g_m V_{in} \times R_D$$

$$A_V = \frac{V_{out}}{V_{in}} = -g_m R_D$$

E.g. Sketch the drain current and transconductance of M_1 as a function of the input voltage.





Effect of channel length modulation

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 (1 + \lambda V_{out})$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})(1 + \lambda V_{out}) - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 \lambda \frac{\partial V_{out}}{\partial V_{in}}$$

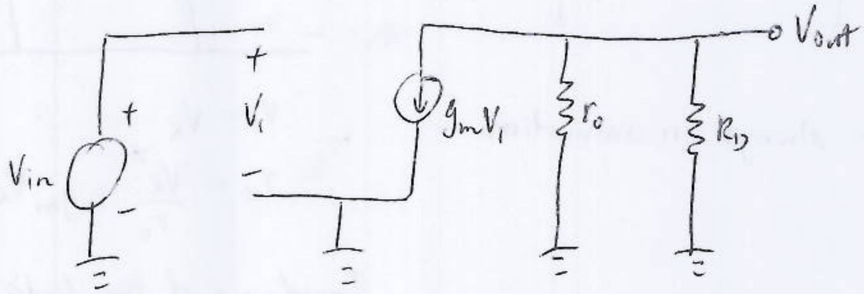
Using $I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$, $g_m = \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})$

$$A_V = -R_D g_m - R_D I_D \lambda A_V$$

$$= - \frac{g_m R_D}{1 + R_D \lambda I_D}$$

Since $\lambda I_D = \frac{1}{r_o}$, $A_V = -g_m \frac{r_o R_D}{r_o + R_D}$

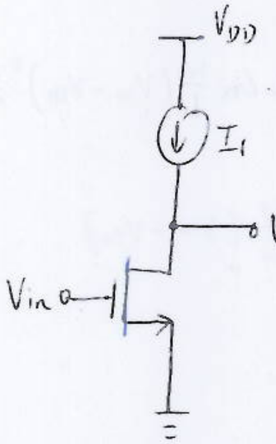
Or use small-signal model to find A_V,



$V_{out} = -g_m V_1 (r_o // R_D)$; $V_1 = V_{in}$

$$\therefore \; A_V = \frac{V_{out}}{V_{in}} = -g_m (r_o // R_D)$$

(E.g.) Assuming M_1 is ~~~~ biased in saturation, calculate the small-signal voltage gain of the circuit.

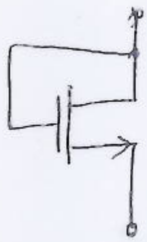


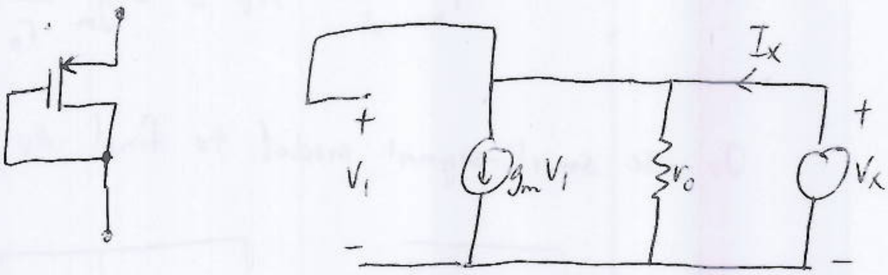
I_1 introduce infinite impedance,

$\therefore$ gain limited by output resistance of M_1.

$$A_V = -g_m r_o$$

(B) CS Stage with Diode-Connected Load.





Transistor always in saturation mode.

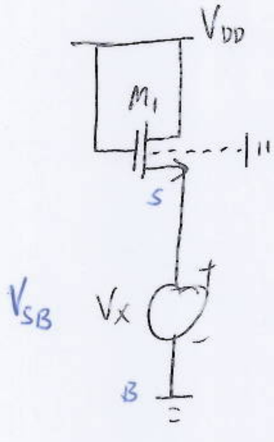
$V_1 = V_X$

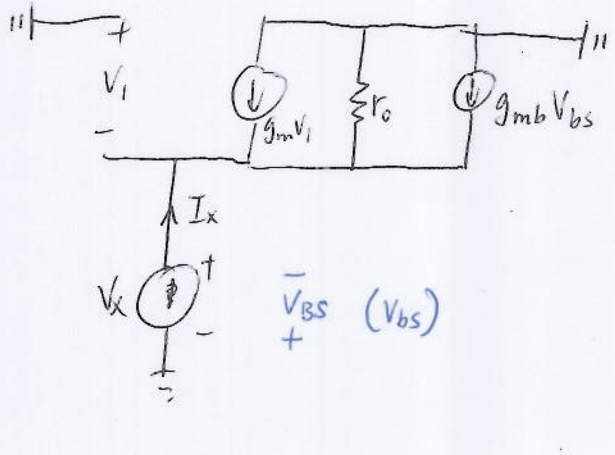
$$I_X = \frac{V_X}{r_o} + g_m V_X$$

Impedance of the diode,

$$\left(\frac{1}{g_m}\right) \parallel r_o \approx \frac{1}{g_m}$$

If body effect exist,



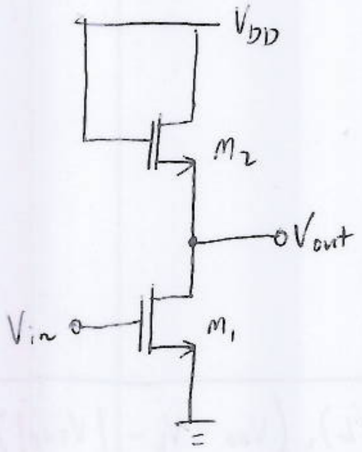


$V_1 = -V_X$, $V_{bs} = -V_X$ $\qquad$ $V_1 = -V_{bs}$, $V_{bs} = -V_{bs}$

$$(g_m + g_{mb})V_X + \frac{V_X}{r_o} = I_X$$

$$\frac{V_X}{I_X} = \frac{1}{g_m + g_{mb} + r_o^{-1}}$$

$$= \left(\frac{1}{g_m + g_{mb}}\right) \parallel r_o \approx \frac{1}{g_m + g_{mb}}$$



$\lambda = 0, \ \gamma \neq 0$

$$A_V = -g_{m1}\left(\frac{1}{g_{m2} + g_{mb2}}\right)$$

$$= -\frac{g_{m1}}{g_{m2}} \frac{1}{1+\eta} \qquad \eta = \frac{g_{mb2}}{g_{m2}}$$

$$= -\frac{\sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}(W/L)_2 I_{D2}}} \frac{1}{1+\eta}$$

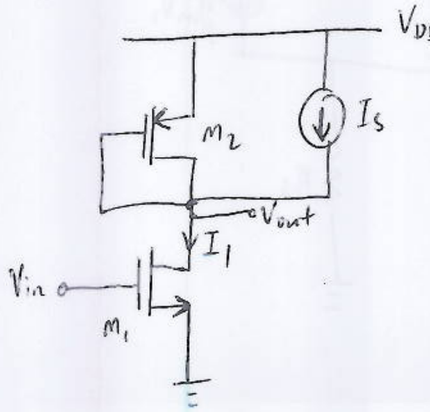
$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1+\eta}$$

If channel-length modulation,

$$A_V = -g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2}\right)$$

[If M_2 is PMOS and no body-effect, $A_V = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}}$]

E.g. M_1 is biased in saturation with a drain current equal to I_1. The current source $I_S = 0.75 I_1$ is added to the circuit. What is the gain?

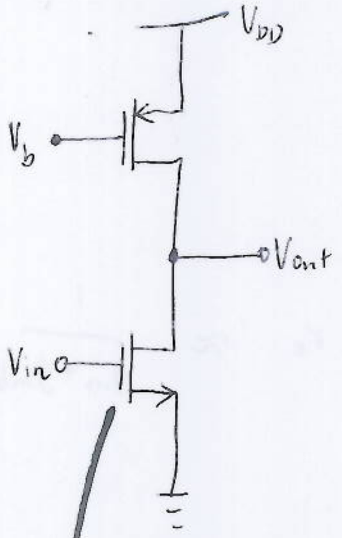


$$|I_{D2}| = I_1/4$$

$$\therefore A_V \approx -\frac{g_{m1}}{g_{m2}}$$

$$= -\sqrt{\frac{4\mu_n (W/L)_1}{\mu_p (W/L)_2}}$$

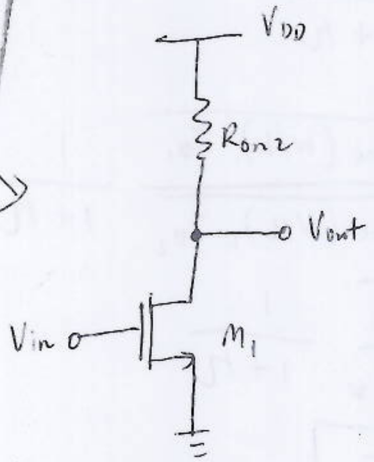
C. CS Stage with Current-Source Load



$\lambda \neq 0$, $\gamma = 0$

$$A_V = -g_{m1}\left(r_{o1} \parallel r_{o2}\right)$$

D. CS Stage with Triode Load.

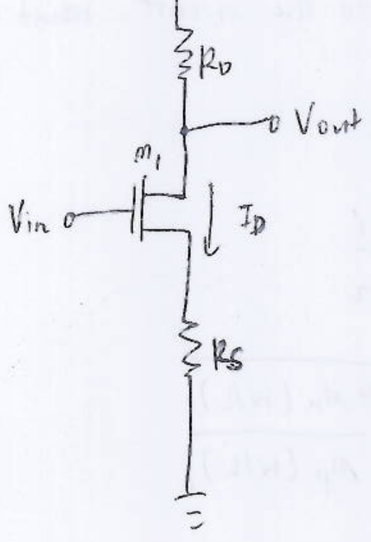


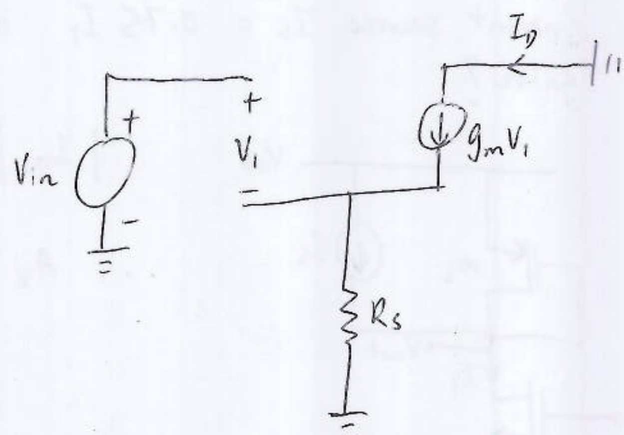
$\lambda \neq 0$, $\gamma = 0$

$$R_{on2} = \frac{1}{\mu_p C_{ox} (W/L)_2 \left(V_{DD} - V_b - |V_{THP}|\right)}$$

$$A_V = -g_{m1}\left(r_{o1} \parallel R_{on2}\right)$$

E. CS Stage with Source Degeneration





$$V_{out} = -I_D R_D$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -\left(\frac{\partial I_D}{\partial V_{in}}\right) R_D$$

let $G_m = \dfrac{\partial I_D}{\partial V_{in}}$,

Assuming $I_D = f(V_{GS})$,

$$G_m = \frac{\partial f}{\partial V_{GS}} \frac{\partial V_{GS}}{\partial V_{in}}$$

Since $V_{GS} = V_{in} - I_D R_S$,

$$\frac{\partial V_{GS}}{\partial V_{in}} = 1 - R_S \frac{\partial I_D}{\partial V_{in}}$$

$$\therefore G_m = \left(1 - R_S \frac{\partial I_D}{\partial V_{in}}\right) \frac{\partial f}{\partial V_{GS}}$$

$$= (1 - R_S G_m)\, g_m$$

$$= \frac{g_m}{1 + g_m R_S}$$

Small-signal voltage gain,

$$A_V = -G_m R_D$$

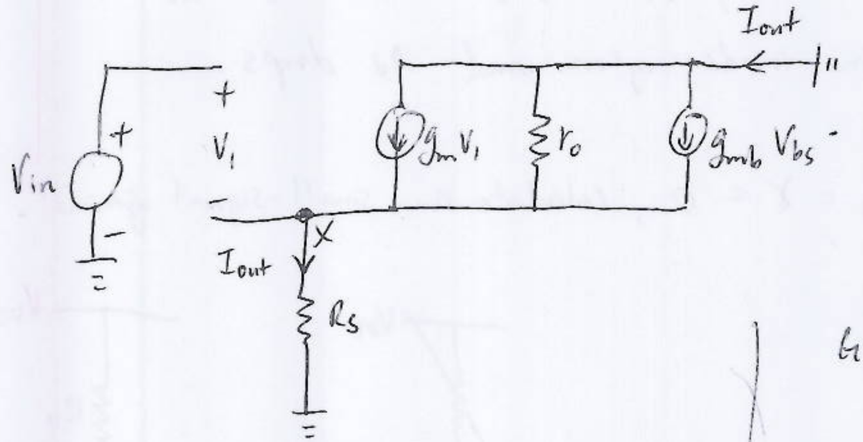
$$= -\frac{g_m R_D}{1 + g_m R_S}$$

For $R_S \gg \dfrac{1}{g_m}$

$G_m \approx \dfrac{1}{R_S}$

i.e. $\Delta I_D \approx \Delta V_{in} R_S$

Determine G_m in the presence of body effect and channel-length modulation.



$$V_{in} = V_1 + I_{out} R_S$$

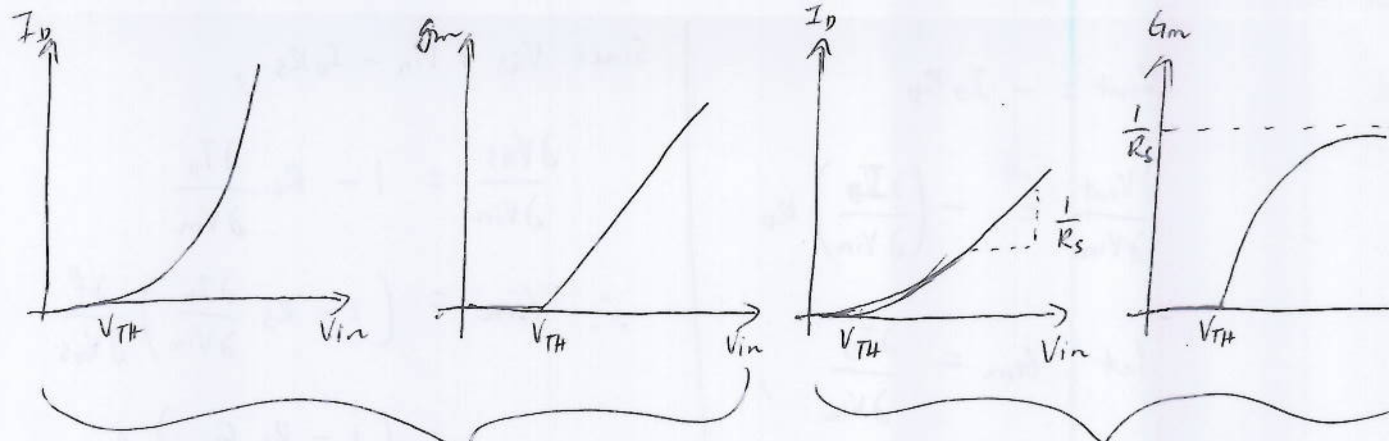
Summing currents at node X,

$$I_{out} = g_m V_1 - g_{mb} V_X - \frac{I_{out} R_S}{r_o}$$

$$= g_m (V_{in} - I_{out} R_S) + g_{mb}(-I_{out} R_S) - \frac{I_{out} R_S}{r_o}$$

$$G_m = \frac{I_{out}}{V_{in}}$$

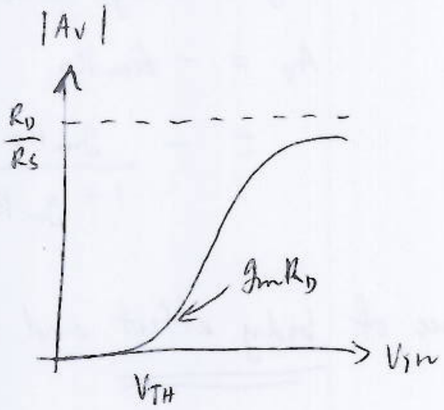
$$= \frac{g_m r_o}{R_S + [1 + (g_m + g_{mb}) R_S]\, r_o}$$



without source-degeneration

with SD

(E.g.) Plot small-signal voltage gain as a function of the input bias voltage.



$$A_V = \frac{-g_m R_D}{1+g_m R_S} = -\frac{R_D}{\frac{1}{g_m}+R_S}$$

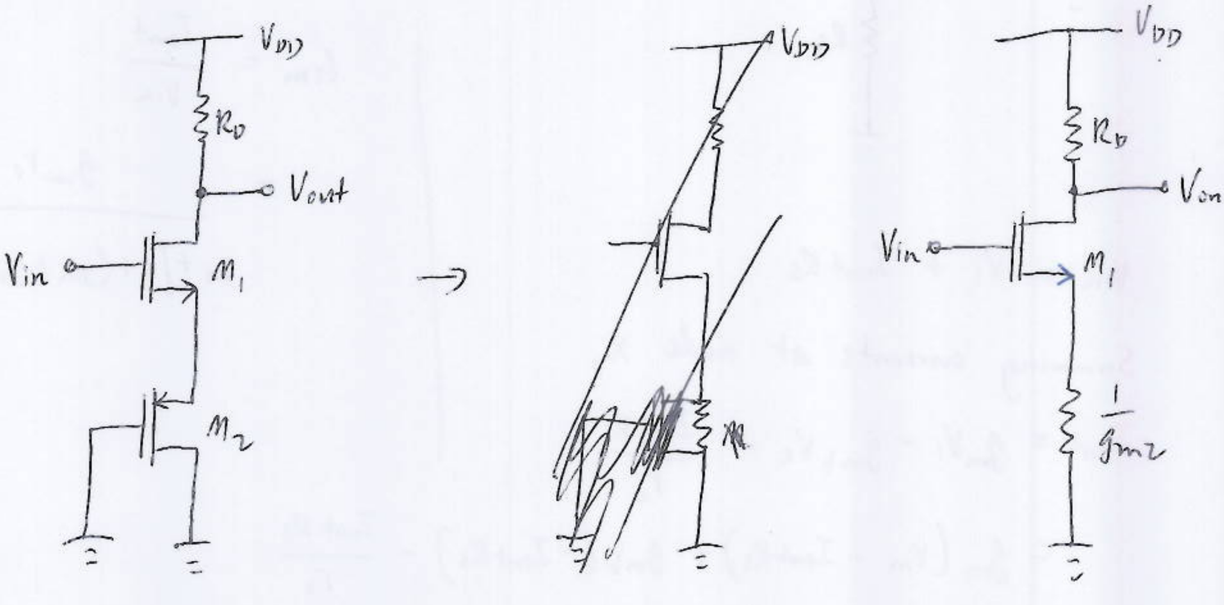
For large values of V_{in},

$$G_m \approx \frac{1}{R_S} \quad \therefore A_V = -\frac{R_D}{R_S}$$

If $V_{in} > V_{out} + V_{TH}$, $R_D I_D > V_{TH} + V_{DD} - V_{in}$

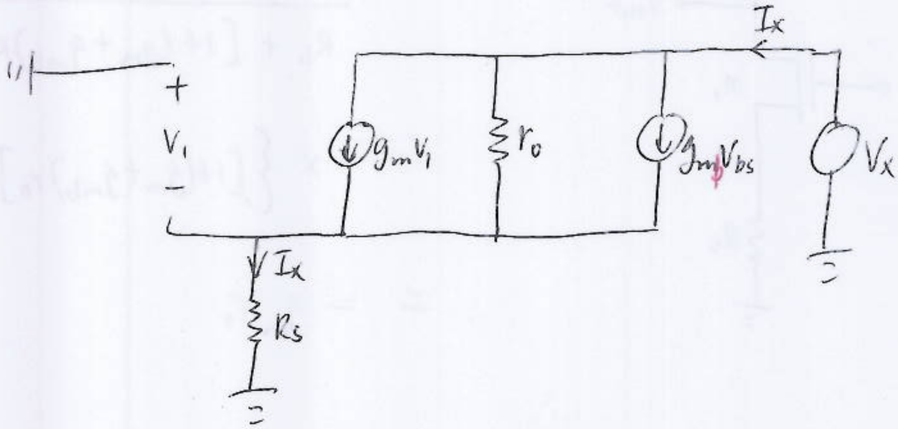
M_1 enters the triode region and A_V drops.

(E.g.) Assuming $\lambda = \gamma = 0$, calculate the small-signal gain.



$$A_V = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

How to calculate output resistance?



$V_1 = -I_X R_S \implies$

Current flowing through r_o, $I_X - (g_m + g_{mb}) V_1 = I_X + (g_m + g_{mb}) R_S I_X$

Adding voltage drop across r_o and R_S,

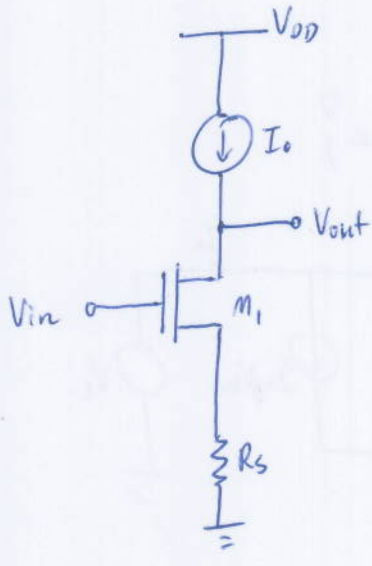
$$r_o \left[I_X + (g_m + g_{mb}) R_S I_X \right] + I_X R_S = V_X$$

$$R_{out} = \left[1 + (g_m + g_{mb}) R_S \right] r_o + R_S$$
$$= \left[1 + (g_m + g_{mb}) r_o \right] R_S + r_o$$

Since typically $(g_m + g_{mb}) r_o \gg 1$,

$$R_{out} \approx (g_m + g_{mb}) r_o R_S + r_o$$
$$= \left[1 + (g_m + g_{mb}) R_S \right] r_o$$

Eg. Calculate the voltage gain of the circuit below. Assume I_o is ideal.



$$A_V = G_m \times R_{out}$$

$$= -\frac{g_m r_o}{R_S + [1+(g_m+g_{mb})R_S]r_o} \times \{[1+(g_m+g_{mb})r_o]R_S + r_o\}$$

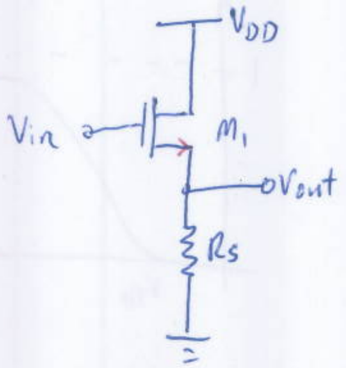
$$= -g_m r_o$$

- If I_o is ideal, the current through R_S cannot change and hence the small-signal voltage drop across R_S is zero.

Source Follower (common-drain stage)

- Common-Source stage → achieve high voltage gain with limited supply voltage, the load must be as large as possible.

- If such a stage is to drive a low-impedance load, then a "buffer" must be placed after the amplifier so as to drive the load with negligible loss of the signal level.



$V_{in} < V_{TH}$, M_1 is off, $V_{out} = 0$

$V_{in} > V_{TH}$, M_1 in saturation, I_{D1} flows through R_S.

$$I_{D1} R_S = V_{out}$$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out})^2 R_S = V_{out}$$

To calculate small-signal gain, differentiate both side with respect to V_{in} (include body effect)

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2 (V_{in} - V_{TH} - V_{out}) \left(1 - \frac{\partial V_{TH}}{\partial V_{in}} - \frac{\partial V_{out}}{\partial V_{in}}\right) R_S = \frac{\partial V_{out}}{\partial V_{in}}$$

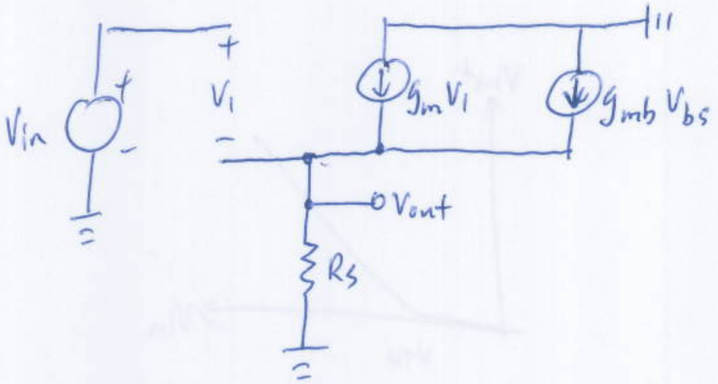
Since $\dfrac{\partial V_{TH}}{\partial V_{in}} = \eta\, \partial V_{out} / \partial V_{in}$

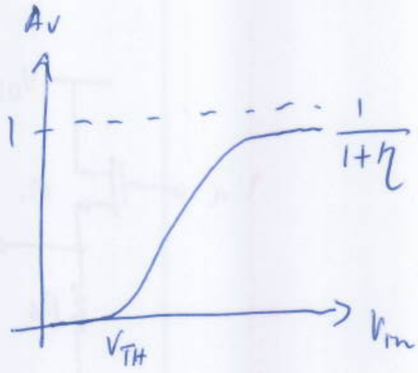
$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out}) R_S}{1 + \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out}) R_S (1 + \eta)}$$

Note that $g_m = \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out})$

$$\therefore A_V = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$

Now prove using small-signal equivalent circuit, $\eta = \frac{g_{mb}}{g_m}$





① ② ③

$V_{in} - V_1 = V_{out}$, $V_{bs} = -V_{out}$, $g_m V_1 - g_{mb} V_{out} = V_{out} / R_S$

$V_1 = V_{out} - V_{in}$

$\cancel{R_S g_m V_1 = R_S g_{mb} V_{out} + V_{out}}$

$\cancel{V_{out} = \frac{R_S g_m V_1}{R_S g_{mb} + 1}}$

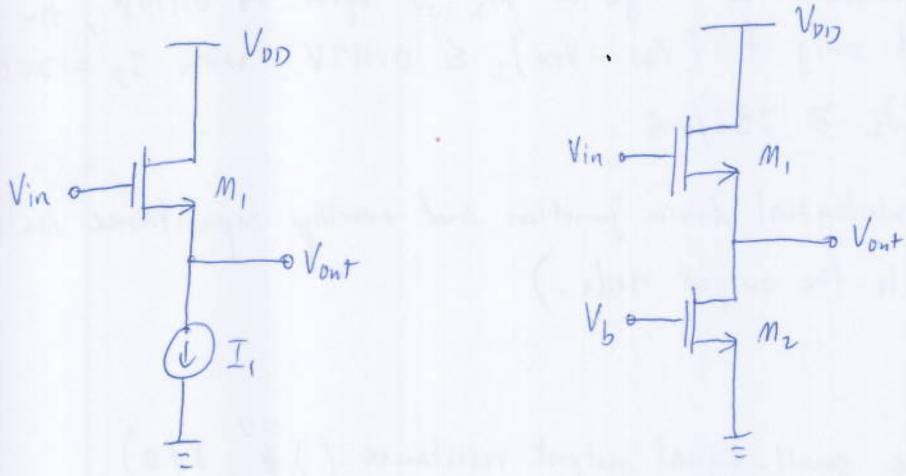
$$A_V = \frac{V_{out}}{V_{in}} = g_m R_S \Big/ [1 + (g_m + g_{mb}) R_S]$$

If g_m increases,

$$A_V = \frac{g_m}{g_m + g_{mb}} = \frac{1}{1+\eta}$$

⊛ Even if $R_S = \infty$, the voltage gain of a source follower is not equal to one.

E.g.



(a) (b)

Suppose in the source follower of Fig(a) Above, $(W/L)_1 = 20/0.5$, $I_1 = 200\,\mu A$, $V_{TH0} = 0.6V$, $2\phi_F = 0.7V$, $\mu_n C_{ox} = 50\,\mu A/V^2$ and $\gamma = 0.4\,V^2$.

a) calculate V_{out} for $V_{in} = 1.2V$.

b) If I_1 is implemented as M_2 in Fig. (b), find minimum value of $(W/L)_2$ for which M_2 remains saturated.

Solution

a) Since the threshold voltage of M_1 depends on V_{out}, we perform a simple iteration. Noting that

$$(V_{in} - V_{TH} - V_{out})^2 = \frac{2 I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}$$

we first assume $V_{TH} \approx 0.6V$, obtaining $V_{out} = 0.153V$. Now we calculate a new V_{TH} as
(V_{SB})

$$V_{TH} = V_{TH0} + \gamma \left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right]$$

$$= 0.635V$$

This indicates V_{out} is approximately 35mV less than that calculated above i.e. $V_{out} \approx 0.119V$.

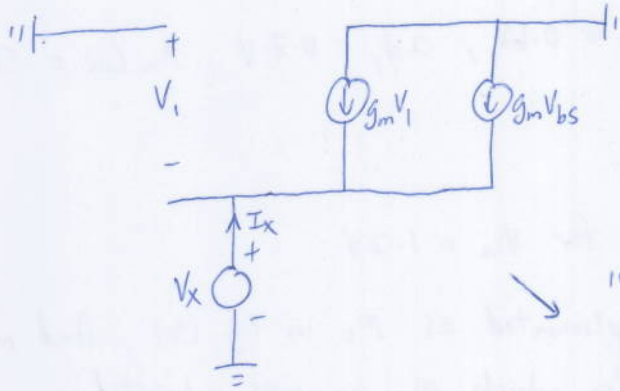
$V_{TH0} = \Phi_{MS} + 2\Phi_F + \frac{Q_{dep}}{C_{ox}}$

E.g. $t_{ox} \approx 50\,\text{Å}$
$C_{ox} \approx 6.9\,fF/\mu m^2$

b) Since the drain-source voltage of M_2 is equal to 0.119V, the device is saturated only if $(V_{GS} - V_{TH})_2 \leq 0.119V$. With $I_D = 200\mu A$, this gives $(W/L)_2 \geq 283/0.5$.

(Note the substantial drain junction and overlap capacitance contributed by M_2 to the output node.)

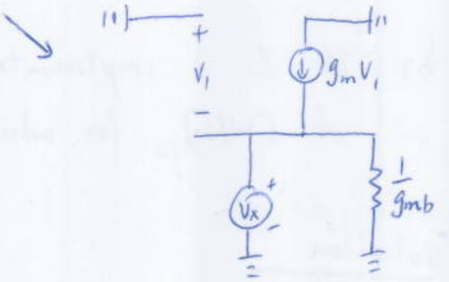
E.g. Calculate small-signal output resistance. ($\lambda \overset{=0}{\cancel{\neq}}$, $\gamma \neq 0$)



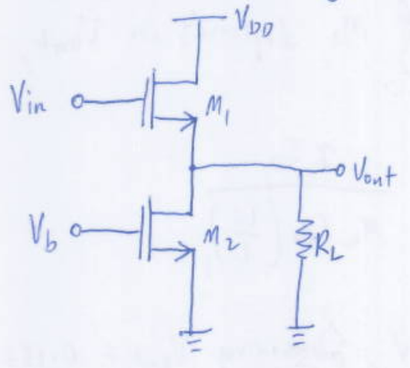
$$V_1 = -V_X$$

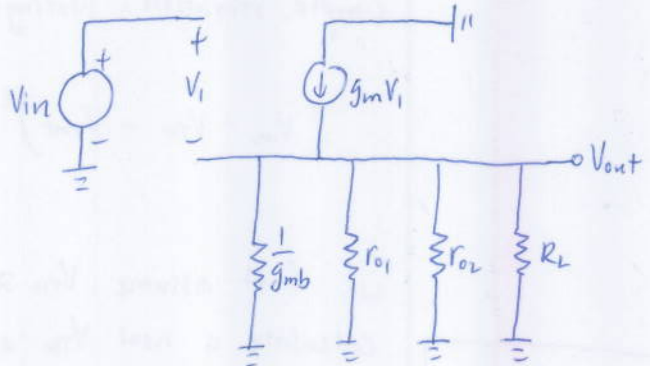
$$I_X - g_m V_X - g_{mb} V_X = 0$$

$$\therefore \; R_{out} = \frac{1}{g_m + g_{mb}}$$



E.g. Find small-signal gain.

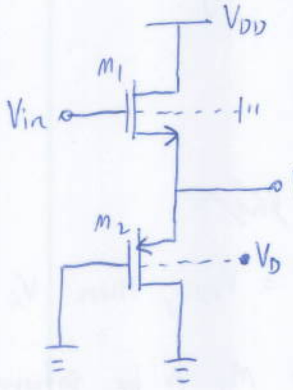




You can use simple voltage divider,

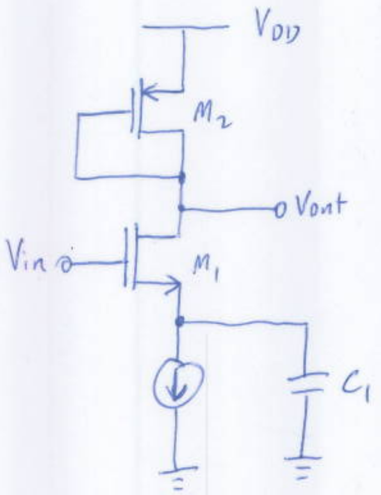
$$A_V = \frac{\frac{1}{g_{mb}} \,//\, r_{o1} \,//\, r_{o2} \,//\, R_L}{\frac{1}{g_{mb}} \,//\, r_{o1} \,//\, r_{o2} \,//\, R_L + \frac{1}{g_m}}$$

E.g. Calculate voltage gain ($\gamma \neq 0$, $\lambda \neq 0$)



$$A_V = \frac{\frac{1}{g_{m2}+g_{mb2}} \parallel r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{mb1}}}{\frac{1}{g_{m2}+g_{mb2}} \parallel r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{mb1}} + \frac{1}{g_{m1}}}$$

E.g. a) Calculate the voltage gain if C_1 acts as an ac short at the frequency of interest. What is the maximum dc level of input signal for which M_1 remains saturated?



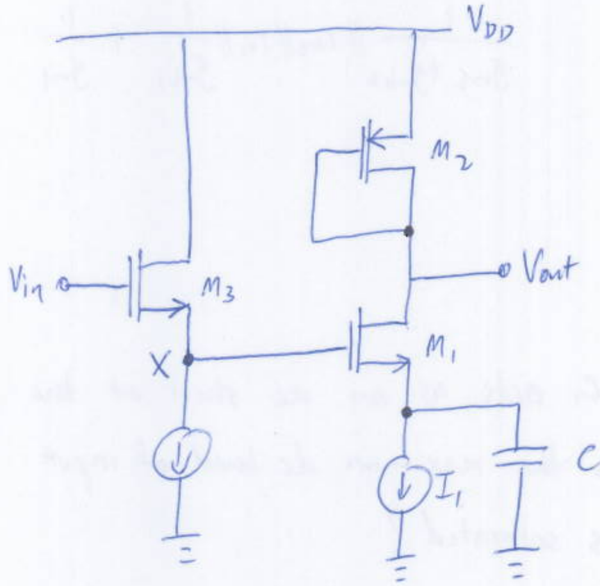
$$A_V = -g_{m1}\left[r_{o1} \parallel r_{o2} \parallel (1/g_{m2})\right]$$

Since $V_{out} = V_{DD} - |V_{GS2}|$, the maximum allowable dc level of V_{in} is equal to $V_{DD} - |V_{GS2}| + V_{TH1}$.

$$V_{out} \geq V_{in} - V_{TH1}$$

$$V_{DD} - |V_{GS2}| + V_{TH} \geq V_{in}$$

b) To accommodate an input dc level close to V_{DD}, the circuit is modified as shown in Fig. below. What relationship among the gate-source voltages of $M_1 - M_3$ guarantees that M_1 is saturated?



If ~~V_{out}~~ =

$V_{in} = V_{DD}$, then $V_X = V_{DD} - V_{GS3}$.

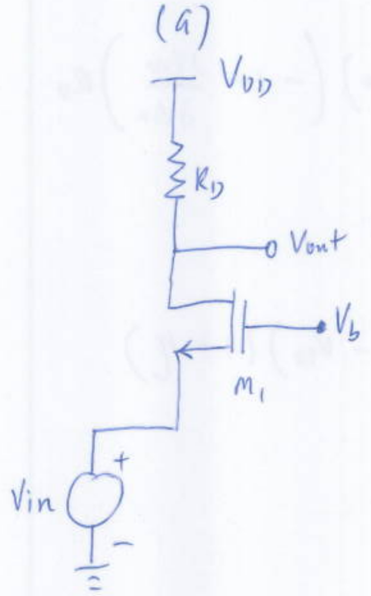
For M_1 to be saturated,

$V_{DD} - V_{GS3} - V_{TH1} \leq V_{DD} - |V_{GS2}|$

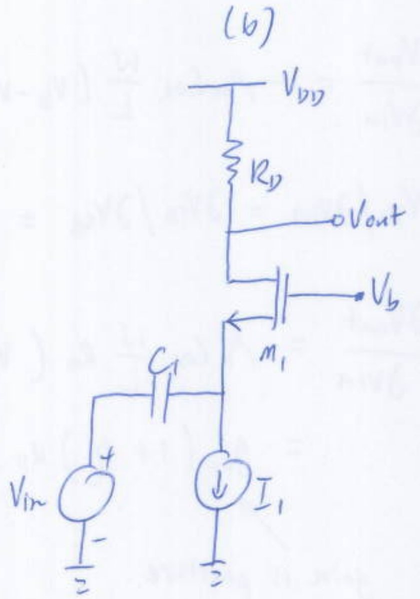
$\therefore \; V_{GS3} + V_{TH1} \geq |V_{GS2}|$.

Common-Gate Stage

gives low input impedance



CG stage with direct coupling at input



CG stage with capacitive coupling at input.

First study large-signal behaviour* for (a),

For simplicity assume that V_{in} decreases from large positive value.

$$V_{in} \geq V_b - V_{TH}, \quad M_1 \text{ is off}$$

$$\therefore \quad V_{out} = V_{DD}$$

For lower values of V_{in},

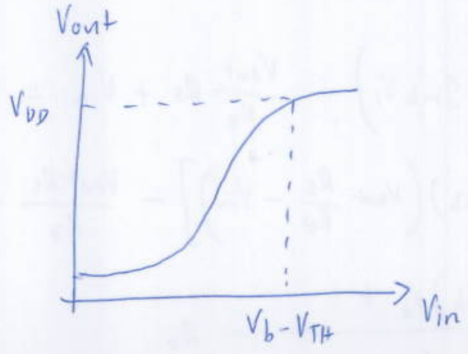
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2, \quad \text{if } M_1 \text{ is in saturation.}$$

As V_{in} decreases, so does V_{out}, eventually drive M_1 into triode region if

$$V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2 R_D = V_b - V_{TH}$$

Input-output characteristic if M_1 in saturation,

$$V_{out} = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2 R_D$$



Small-signal gain ?

$$\frac{\partial V_{out}}{\partial V_{in}} = -\mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH}) \left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right) R_D .$$

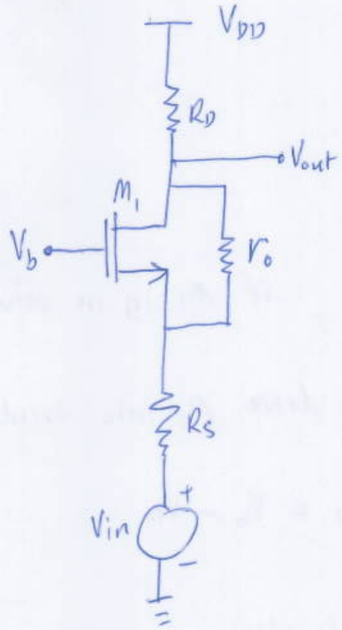
Since $\partial V_{TH} / \partial V_{in} = \partial V_{TH} / \partial V_{SB} = \eta$,

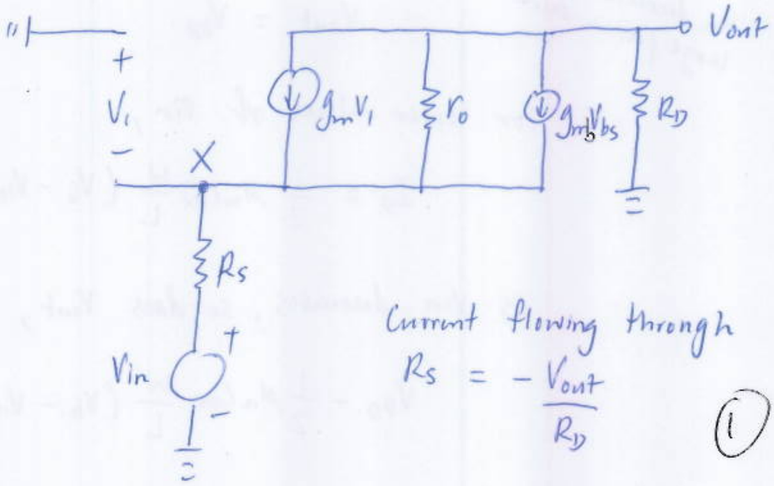
$$\frac{\partial V_{out}}{\partial v_{in}} = \mu_n C_{ox} \frac{W}{L} R_D (V_b - V_{in} - V_{TH})(1 + \eta)$$

$$= g_m (1 + \eta) R_D .$$

gain is positive.

Now study CG stage taking into account both output impedance of the transistor and the impedance of the signal source. $(\lambda \neq 0, \gamma \neq 0)$





Current flowing through $R_S = -\frac{V_{out}}{R_D}$ (1)

$$V_1 - \frac{V_{out}}{R_D} R_S + V_{in} = 0 \quad — \text{(1)} \qquad (2)$$

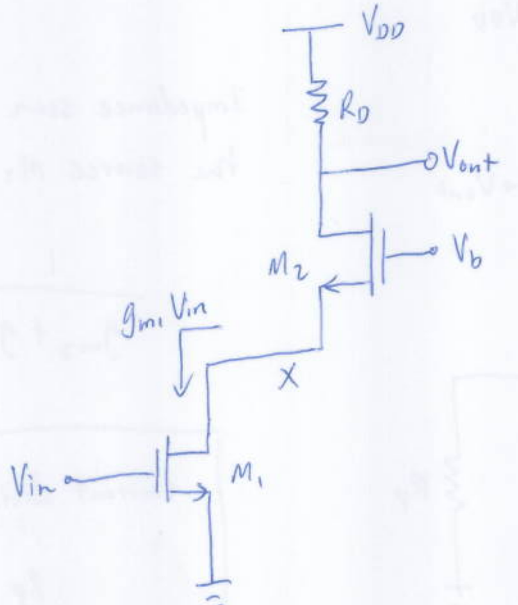
$$I_{r_o} = -\frac{V_{out}}{R_D} - g_m V_1 - g_{mb} V_1$$

$$r_o \left(\frac{-V_{out}}{R_D} - g_m V_1 - g_{mb} V_1 \right) - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out} \quad — \text{(2)} \qquad (3)$$

$$\therefore \quad r_o \left[\frac{-V_{out}}{R_D} - (g_m + g_{mb}) \left(V_{out} \frac{R_S}{R_D} - V_{in} \right) \right] - \frac{V_{out} R_S}{R_D} + V_{in} = V_{out}$$

$$\Rightarrow \therefore \quad \frac{V_{out}}{V_{in}} = \frac{(g_m + g_{mb}) r_o + 1}{r_o + (g_m + g_{mb}) r_o R_S + R_S + R_D} R_D . \quad //$$

Cascode Stage (cascade of a CS stage and a CG stage)



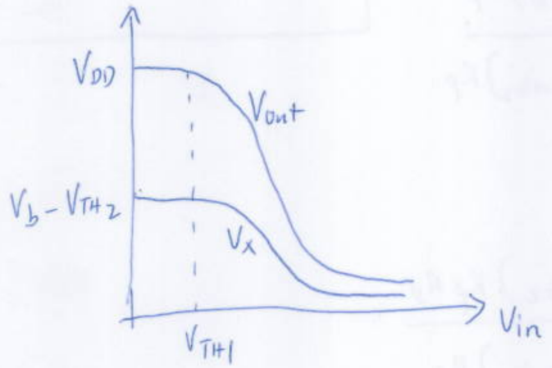
M_1 = input device

M_2 = cascode device

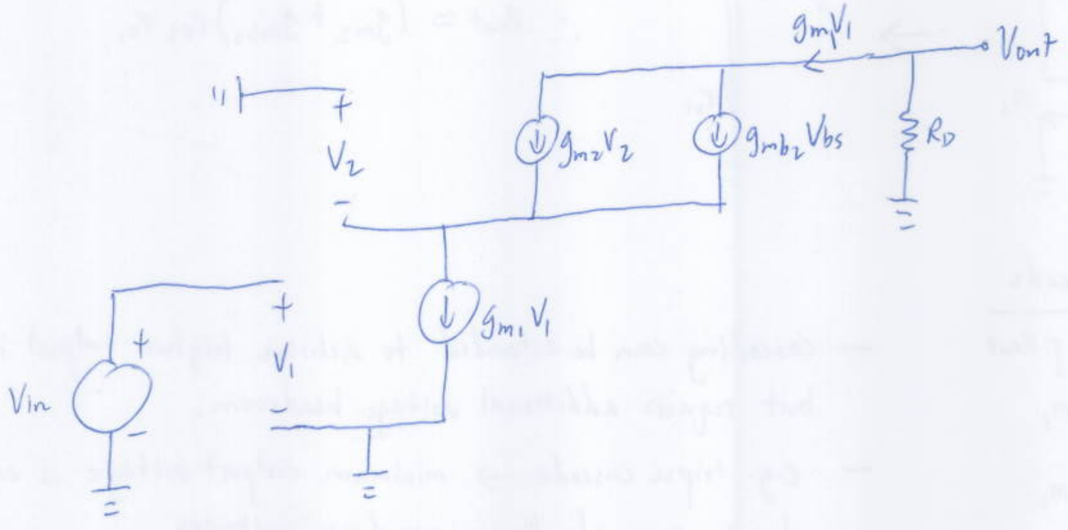
M_1 sat, $V_X \geq V_{in} - V_{TH1}$

M_1, M_2 sat, $V_X = V_b - V_{GS2}$

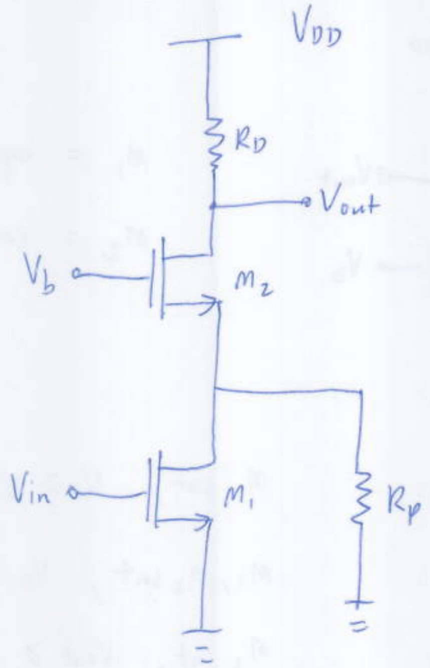
M_2 sat, $V_{out} \geq V_b - V_{TH2}$



Small-signal equivalent circuit $\lambda = 0$, $\gamma \neq 0$



E.g. Calculate voltage gain if $\lambda = 0$, $\gamma \neq 0$



Impedance seen looking into the source M_2,

$$\frac{1}{g_{m2} + g_{mb2}}$$

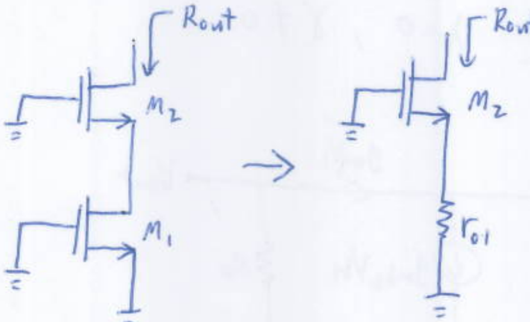
Current divider rule:

$$\frac{R_P}{\frac{1}{g_{m2}+g_{mb2}} + R_P} \cdot g_{m1} V_{in}$$

$$I_{D2} = g_{m1} V_{in} \frac{(g_{m2} + g_{mb2}) R_P}{1 + (g_{m2} + g_{mb2}) R_P}$$

Voltage gain,

$$A_V = -\frac{g_{m1}(g_{m2} + g_{mb2}) R_P R_D}{1 + (g_{m2} + g_{mb2}) R_P}$$

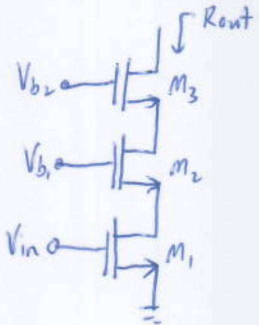


Same as R_{out} for common-source stage with r_{o1} degenera

$$R_{out} = \left[1 + (g_{m2} + g_{mb2}) r_{o2}\right] r_{o1} + r_{o2}$$

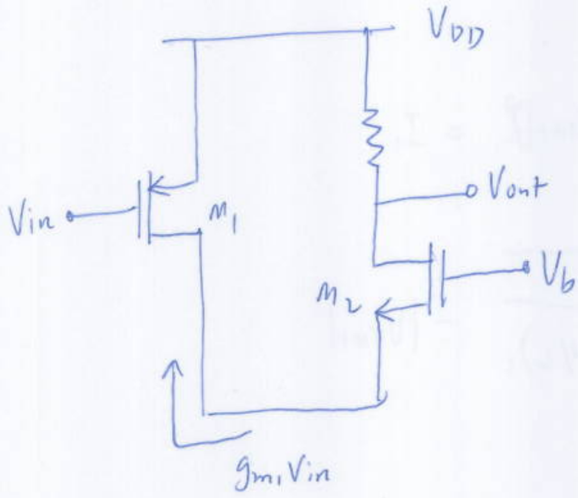
$$\therefore R_{out} \approx (g_{m2} + g_{mb2}) r_{o2} r_{o1}$$

Triple cascode

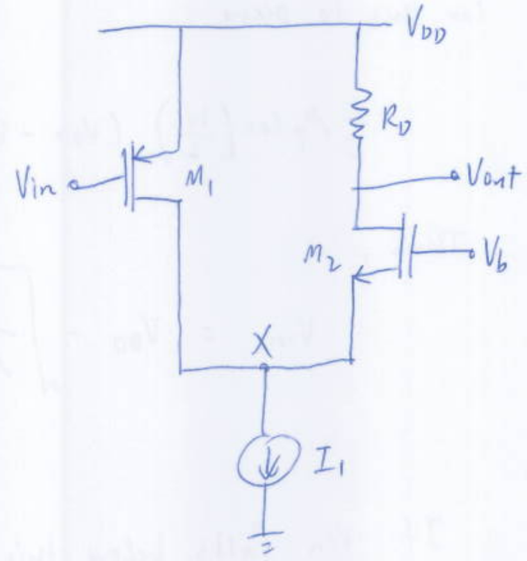


- cascoding can be extended to achieve higher output impedance but require additional voltage headroom.
- e.g. triple cascode → minimum output voltage is equal to the sum of three overdrive voltages.

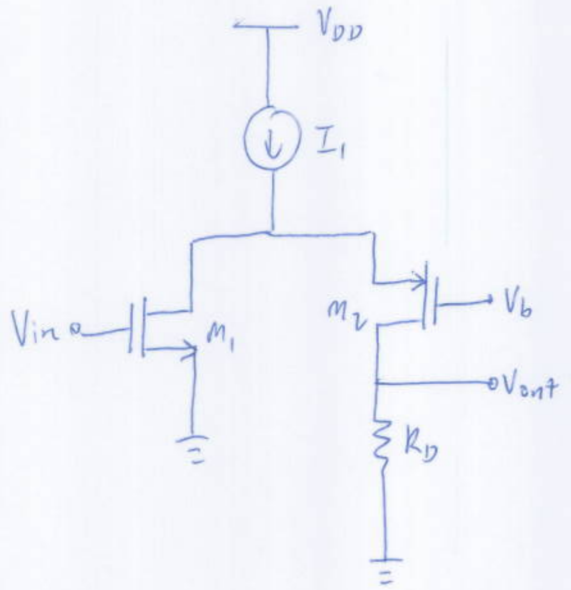
Folded cascode (PMOS - NMOS)



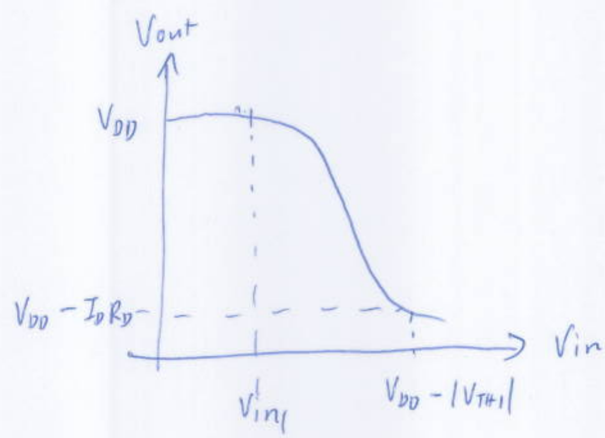
(a) simple

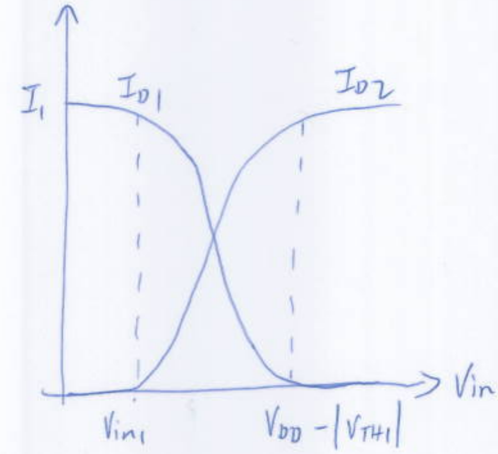


(b) with proper biasing



(c) with NMOS input





(b) $V_{in} > V_{DD} - |V_{TH1}|$, M_1 is off and M_2 carries all of I_1.

$\therefore$ $V_{out} = V_{DD} - I_1 R_D$.

$V_{in} < V_{DD} - |V_{TH1}|$, M_1 is on in saturation.

$$I_{D2} = I_1 - \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1\left(V_{DD} - V_{in} - |V_{TH1}|\right)^2$$

As V_{in} drops, I_{D2} decreases further, falling to zero if $I_{D1} = I_1$.

For this to occur:

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_1 (V_{DD} - V_{in1} - |V_{TH1}|)^2 = I_1$$

Thus,

$$V_{in1} = V_{DD} - \sqrt{\frac{2I_1}{\mu_p C_{ox}(W/L)_1}} - |V_{TH1}|$$

If V_{in} falls below this level, I_{D1} tends to be greater than I_1 and M_1 enters the triode region so as to allow $I_{D1} = I_1$.